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Stresses and Strains Developed by the Reflection of Seismic Waves at a Free Surface

John R. Banister, D. Maxwell Ellett, Clarence R. Mehl, Frank F. Dean

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STRESSES AND STRAINS DEVELOPED BY THE
REFLECTION OF SEISMIC WAVES AT A FREE SURFACE

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ABSTRACT

Exact and approximate equations have been derived for the stresses and strains beneath a free surface when an incoming longitudinal wave and an incoming shear wave reflect from the surface. Results of the approximate solution for depths much less than the wave length of the incoming wave are given in tabular form and are graphed for Poisson's ratios of 0.25, 0.3, and 0.333. The results should be of use in categorizing the magnitude of near-surface stresses and strains resulting from seismic waves produced by deeply buried explosives or earthquakes.

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STRESSES AND STRAINS DEVELOPED BY THE REFLECTION OF SEISMIC WAVES AT A FREE SURFACE

It is important to understand the behavior and magnitude of the stresses and strains which occur when seismic waves from earthquakes or underground nuclear explosions are reflected at the earth's surface. Such knowledge is especially useful for examining the regions close to the surface where the disturbances might affect buildings or cause such phenomena as soil liquefaction. With this purpose in mind we have made the following study. Although the direction and magnitude of an incident seismic wave is rarely known exactly, we hope that a general understanding of the effects and the trends of the results will help in explaining phenomena which are otherwise difficult to interpret.

Extensive studies of the reflection of a seismic longitudinal or shear wave at a free surface can be found in various forms in many standard texts.^{1,2,3,4} The derivations of the amplitude coefficients of the reflected waves or the expressions for the stress are often used as exercises for the student. However, there are no tables or graphs in one place that give the results collected for the amplitude coefficients, stresses, and strains for a variety of angles of incidence and Poisson's ratios. In this note, we will look particularly into the problem for both incoming longitudinal and shear waves in the region close to the surface (e.g. where the depth is small compared to the length of the seismic wave).

We shall treat the problem in two parts. In the first part we shall consider the problem of a longitudinal wave reflecting at a free boundary. In the second part, we shall treat the problem of a shear wave reflecting at a free boundary. The second part will be further subdivided into two solutions which depend on whether or not a secondary longitudinal wave may be generated at the free surface.

Given an incident longitudinal wave approaching the surface as shown in Figure 1, we may assume the particle displacement of the wave is a sinusoidal function, and write this function as

$$\phi_1 = \text{Re} \left\{ A_1 e^{i(pt + f_1 x + g_1 y)} \right\}$$

where

A_1 = displacement amplitude of the longitudinal wave (cm.)

p = angular frequency, rad/sec

t = time (sec)

$$f_1 = \frac{p \cos \alpha_1}{c_1}$$

c_1 = velocity of body waves in the medium

x = depth

$$g_1 = \frac{p \sin \alpha_1}{c_1}$$

α_1 = angle of incidence

y = distance from origin parallel to the surface

When such a wave strikes the reflecting surface, a longitudinal wave Φ_2 of amplitude A_2 is reflected making an angle of α_2 with the normal and shear wave Φ_3 of amplitude A_3 reflected at an angle of β from the normal. The assumed form for the functions describing the waves is

$$\Phi_2 = \text{Re} \left\{ A_2 e^{[i(pt - f_2 x + g_2 y + \delta_1)]} \right\}$$

$$\Phi_3 = \text{Re} \left\{ A_3 e^{[i(pt - f_3 x + g_3 y + \delta_2)]} \right\}$$

$$f_2 = \frac{p \cos \alpha_2}{c_1}$$

$$g_2 = \frac{p \sin \alpha_2}{c_1}$$

$$f_3 = \frac{p \cos \beta}{c_2}$$

$$g_3 = \frac{p \sin \beta}{c_2}$$

c_2 = velocity of shear waves in the medium

where δ_1 and δ_2 are phase angles which can be shown equal to zero.

Figure 1
Incident Longitudinal Wave, Reflected
Longitudinal and Shear Wave

It can also be shown that $\alpha_1 = \alpha_2$; therefore, we may drop the subscripts. The relation between α and β is a function of Poisson's ratio such that:

$$\frac{\sin \alpha}{\sin \beta} = \frac{c_1}{c_2} = \sqrt{\frac{2-2\nu}{1-2\nu}} = \sqrt{\frac{\lambda+2\mu}{\mu}} = \sqrt{2 + \frac{\lambda}{\mu}}$$

where

$$c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

$$c_2 = \sqrt{\frac{\mu}{\rho}}$$

ν = Poisson's ratio of medium

$\left\{ \begin{matrix} \lambda \\ \mu \end{matrix} \right\}$ = Lamé's Constants

ρ = density of medium

Because the shear stress σ_{yx} at a free surface, must be 0, one equation relates the amplitude of the three waves:

$$2(A_1 - A_2) \cos \alpha \sin \beta - A_3 \cos 2\beta = 0$$

At a free boundary, the stress in the x direction σ_{xx} must disappear. This leads to a second relation between the amplitude of the three waves:

$$(A_1 + A_2) \cos 2\beta \sin \alpha - A_3 \sin \beta \sin 2\beta = 0$$

Dividing both of the last two equations by A_1 , we have two equations to solve for A_2/A_1 and A_3/A_1 , giving

$$\frac{A_2}{A_1} = \frac{\left(\frac{c_2}{c_1}\right)^2 \sin 2\alpha \sin 2\beta - \cos^2 2\beta}{R}$$

$$\frac{A_3}{A_1} = \frac{2}{R} \left(\frac{c_2}{c_1}\right) \sin 2\alpha \cos 2\beta$$

where

$$R = \left(\frac{c_2}{c_1}\right)^2 \sin 2\alpha \sin 2\beta + \cos^2 2\beta$$

The values for A_2/A_1 and A_3/A_1 are given in Table I and Figure 2 for several values of Poisson's ratio which are representative of the usual range of that ratio for geologic materials.

Having solved for the displacement of the reflected wave in terms the displacement of the incident wave and following Kolsky's derivation, we may solve for the value of the stresses on the xx , yy , zz , and xy planes. These are

$$\sigma_{xx} = \frac{2p\mu A_1}{c_1 R} \cos 2\beta \left\{ \sin 2\alpha \sin 2\beta (\cos f_1 x - \cos f_3 x) \cos pt \right. \\ \left. - \left[\frac{c_1^2}{2} \cos^2 2\beta \sin f_1 x + \sin 2\alpha \sin 2\beta \sin f_3 x \right] \sin pt \right\}$$

$$\sigma_{yy} = \frac{2p\mu A_1}{c_1 R} \left\{ \sin 2\alpha \sin 2\beta \left[2 \left(1 - \frac{c_2^2}{c_1^2} \right) \cos f_1 x - \cos 2\beta (\cos f_1 x - \cos f_3 x) \right] \cos pt \right. \\ \left. + \cos 2\beta \left[\cos 2\alpha \cos 2\beta \sin f_1 x + \sin 2\alpha \sin 2\beta \sin f_3 x \right] \right. \\ \left. - \left(\frac{c_1^2}{c_2^2} - 1 \right) \cos 2\beta \sin f_1 x \right\} \sin pt$$

$$\sigma_{zz} = \frac{2p\lambda A_1}{c_1 R} \left[\frac{c_2^2}{c_1^2} \sin 2\alpha \sin 2\beta \cos f_1 x \cos pt - \cos^2 2\beta \sin f_1 x \sin pt \right]$$

$$\sigma_{xy} = \frac{2p\mu A_1}{c_1 R} \sin 2\alpha \left\{ \cos^2 2\beta (\cos f_1 x - \cos f_3 x) \cos pt \right. \\ \left. - \left[\frac{c_2^2}{c_1^2} \sin 2\alpha \sin 2\beta \sin f_1 x + \cos^2 2\beta \sin f_3 x \right] \sin pt \right\}$$

$$\sigma_{xz} = \sigma_{yz} = 0$$

TABLE Ia

Values of Amplitudes, Stresses and Strains for an Incoming
Longitudinal Wave, Poisson's Ratio of 0.25

POISSON'S RATIO = .25000

ALPHA	BETA	A3/A1	A2/A1	SXX	SWV	SZZ	SWY	EXX	EYY	EXY
0.00000	0.00000	0.00000	-1.00000	-2.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1.50000	.86598	.06043	-.99895	-1.99917	.00141	.00039	-.06041	.00053	.00158	-.18124
3.00000	1.73152	.12066	-.99578	-1.99670	.00562	.00141	-.12054	.00211	.00632	-.36161
4.50000	2.59629	.18052	-.99053	-1.99257	.01263	.00316	-.18009	.00474	.01421	-.54026
6.00000	3.45988	.23981	-.98320	-1.98681	.02248	.00560	-.23878	.00840	.02520	-.71635
7.50000	4.32187	.29833	-.97382	-1.97941	.03491	.00873	-.29635	.01309	.03928	-.88904
9.00000	5.18167	.35592	-.96242	-1.97040	.05011	.01253	-.35251	.01879	.05637	-1.05753
10.50000	6.03948	.41240	-.94985	-1.95980	.06794	.01698	-.40701	.02548	.07643	-1.22104
12.00000	6.89429	.46758	-.93375	-1.94762	.08833	.02208	-.45960	.03313	.09938	-1.37881
13.50000	7.74588	.52130	-.91658	-1.93388	.11123	.02781	-.51805	.04171	.12513	-1.53015
15.00000	8.59385	.57340	-.89760	-1.91862	.13653	.03413	-.55812	.05128	.15360	-1.67437
16.50000	9.43777	.62372	-.87688	-1.90187	.16416	.04104	-.60362	.06156	.18468	-1.81087
18.00000	10.27722	.67213	-.85449	-1.88364	.19601	.04858	-.64639	.07275	.21826	-1.93906
19.50000	11.11177	.71847	-.83052	-1.86399	.22597	.05649	-.68614	.08474	.25422	-2.05843
21.00000	11.94097	.76263	-.80505	-1.84295	.25993	.06498	-.72284	.09747	.29242	-2.16851
22.50000	12.76439	.80448	-.77817	-1.82056	.29577	.07394	-.75638	.11091	.33274	-2.26890
24.00000	13.58157	.84392	-.74999	-1.79687	.33335	.08334	-.78642	.12500	.37501	-2.35927
25.50000	14.39206	.88085	-.72060	-1.77193	.37253	.09313	-.81311	.13970	.41910	-2.43933
27.00000	15.19539	.91518	-.69012	-1.74578	.41318	.10329	-.83629	.15494	.46483	-2.50886
28.50000	15.99108	.94685	-.65864	-1.71849	.45514	.11379	-.85591	.17068	.51203	-2.56774
30.00000	16.77865	.97578	-.62630	-1.69010	.49826	.12457	-.87196	.18685	.56054	-2.61587
31.50000	17.55762	1.00194	-.59321	-1.66069	.54238	.13560	-.88442	.20339	.61018	-2.65325
33.00000	18.32748	1.02529	-.55950	-1.63030	.58734	.14683	-.89331	.22025	.66075	-2.67993
34.50000	19.08769	1.04580	-.52528	-1.59982	.63296	.15824	-.89868	.23736	.71208	-2.69603
36.00000	19.83778	1.06348	-.49069	-1.56689	.67988	.16977	-.90058	.25465	.76396	-2.70174
37.50000	20.57717	1.07931	-.45586	-1.53401	.72552	.18138	-.89910	.27207	.81620	-2.69729
39.00000	21.30535	1.09332	-.42093	-1.50043	.77209	.19382	-.89433	.28954	.86861	-2.68304
40.50000	22.02178	1.09955	-.38602	-1.46623	.81864	.20666	-.88641	.30699	.92097	-2.65924
42.00000	22.72588	1.10603	-.35128	-1.43148	.86496	.21624	-.87547	.32436	.97307	-2.62641
43.50000	23.41710	1.10982	-.31685	-1.39627	.91187	.22772	-.86166	.34158	1.02473	-2.58498
45.00000	24.09484	1.11099	-.28286	-1.36068	.95619	.23985	-.84516	.35857	1.07571	-2.53547
46.50000	24.75854	1.10962	-.24946	-1.32477	1.00072	.25018	-.82615	.37527	1.12581	-2.47844
48.00000	25.40759	1.10579	-.21679	-1.28863	1.04428	.26107	-.80482	.39161	1.17482	-2.41446
49.50000	26.04141	1.09961	-.18499	-1.25234	1.08668	.27167	-.78139	.40750	1.22251	-2.34418
51.00000	26.65937	1.09118	-.15422	-1.21597	1.12778	.28193	-.75608	.42289	1.26866	-2.26823
52.50000	27.26880	1.08061	-.12463	-1.17960	1.16716	.29179	-.72910	.43768	1.31305	-2.18729
54.00000	27.86532	1.06803	-.09637	-1.14330	1.20484	.30121	-.70869	.45181	1.35544	-2.10206
55.50000	28.44207	1.05356	-.06960	-1.10713	1.24053	.31013	-.68107	.46528	1.39559	-2.01321
57.00000	28.96051	1.03732	-.04458	-1.07115	1.27480	.31850	-.64849	.47775	1.43325	-1.92146
58.50000	29.49003	1.01942	-.02124	-1.03543	1.30502	.32625	-.60917	.48938	1.46815	-1.82751
60.00000	30.00000	1.00000	-.00000	-1.00000	1.33333	.33333	-.57735	.50000	1.50000	-1.73205
61.50000	30.48982	.97915	.01908	-.96490	1.35867	.33967	-.54525	.50950	1.52850	-1.63575
63.00000	30.95889	.95698	.03554	-.93015	1.38072	.34518	-.51309	.51777	1.55331	-1.53927
64.50000	31.40659	.93356	.04936	-.89575	1.39915	.34979	-.48107	.52468	1.57404	-1.44322
66.00000	31.83236	.90896	.06018	-.86168	1.41357	.35339	-.44940	.53089	1.59027	-1.34821
67.50000	32.23561	.88319	.06767	-.82788	1.42396	.35589	-.41826	.53383	1.60150	-1.25477
69.00000	32.61579	.85625	.07143	-.79429	1.42858	.35714	-.38780	.53572	1.60715	-1.16339
70.50000	32.97236	.82807	.07102	-.76076	1.42802	.35781	-.35817	.53551	1.60652	-1.07450
72.00000	33.30479	.79852	.06586	-.72713	1.42114	.35529	-.32948	.53293	1.59879	-.98843
73.50000	33.61261	.76739	.05528	-.69312	1.40706	.35176	-.30181	.52764	1.58292	-.90542
75.00000	33.89534	.73435	.03844	-.65840	1.38459	.34615	-.27520	.51922	1.55766	-.82560
76.50000	34.15254	.69893	.01428	-.62249	1.35237	.33889	-.24964	.50714	1.52142	-.74893
78.00000	34.38381	.66046	-.01855	-.58475	1.30860	.32715	-.22506	.49072	1.47217	-.67518
79.50000	34.58878	.61802	-.06179	-.54433	1.25094	.31274	-.20128	.46918	1.40731	-.60385
81.00000	34.76711	.57832	-.11771	-.50007	1.17639	.29410	-.17803	.44115	1.32344	-.53409
82.50000	34.91852	.51559	-.18932	-.45037	1.08890	.27023	-.15485	.40534	1.21602	-.46456
84.00000	35.04275	.45134	-.24072	-.39383	.95904	.23976	-.13108	.35964	1.07892	-.39323
85.50000	35.13959	.37406	-.39753	-.32495	.80329	.20882	-.10568	.30124	.90371	-.31705
87.00000	35.20889	.27868	-.54770	-.24168	.68386	.15877	-.07714	.22615	.67845	-.23142
88.50000	35.25051	.15776	-.74277	-.13667	.34297	.08974	-.04312	.12861	.38584	-.12936
90.00000	35.26439	.00000	-1.00000	-.00000	.00000	.00000	-.00000	.00000	.00000	-.00000

TABLE Ib

Values of Amplitudes, Stresses and Strains for an Incoming
Longitudinal Wave, Poisson's Ratio of 0.30

POISSON'S RATIO = 0.30000

ALPHA	BETA	A3/A1	A2/A1	SXX	SYY	SZZ	SXY	EYY	EYY	EYY
0.00000	0.00000	0.00000	-1.00000	-2.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1.50000	.80172	.05595	-.99916	-1.99926	.00120	.00036	-.05593	.00063	.00146	-.19577
3.00000	1.60304	.11173	-.99665	-1.99704	.00478	.00143	-.11161	.00251	.00585	-.39065
4.50000	2.40358	.16720	-.99248	-1.99335	.01074	.00322	-.16680	.00564	.01315	-.58378
6.00000	3.20294	.22217	-.98667	-1.98820	.01905	.00571	-.22123	.01000	.02333	-.77630
7.50000	4.00073	.27651	-.97922	-1.98158	.02968	.00890	-.27467	.01558	.03636	-.96135
9.00000	4.79654	.33004	-.97018	-1.97352	.04260	.01278	-.32689	.02236	.05218	-1.14411
10.50000	5.58999	.38263	-.95958	-1.96484	.05775	.01732	-.37765	.03032	.07074	-1.32178
12.00000	6.38066	.43412	-.94745	-1.95313	.07508	.02252	-.42675	.03942	.09197	-1.49361
13.50000	7.16815	.48436	-.93383	-1.94084	.09453	.02836	-.47396	.04963	.11579	-1.65885
15.00000	7.95207	.53323	-.91879	-1.92718	.11602	.03481	-.51909	.06091	.14212	-1.81682
16.50000	8.73198	.58058	-.90237	-1.91217	.13948	.04184	-.56197	.07323	.17086	-1.96449
18.00000	9.50749	.62630	-.88463	-1.89584	.16481	.04944	-.60242	.08653	.20189	-2.10845
19.50000	10.27817	.67025	-.86565	-1.87823	.19193	.05758	-.64028	.10076	.23512	-2.24994
21.00000	11.04339	.71234	-.84549	-1.85936	.22073	.06622	-.67542	.11588	.27040	-2.38397
22.50000	11.80353	.75246	-.82422	-1.83928	.25111	.07533	-.70772	.13183	.30761	-2.50773
24.00000	12.55695	.79051	-.80194	-1.81802	.28294	.08488	-.73708	.14854	.34660	-2.57977
25.50000	13.30401	.82641	-.77872	-1.79562	.31612	.09483	-.76340	.16596	.38724	-2.67190
27.00000	14.04407	.86007	-.75465	-1.77212	.35050	.10515	-.78663	.18401	.42936	-2.75320
28.50000	14.77667	.89144	-.72983	-1.74756	.38596	.11579	-.80671	.20263	.47280	-2.82348
30.00000	15.50136	.92045	-.70434	-1.72201	.42237	.12671	-.82361	.22174	.51740	-2.88265
31.50000	16.21767	.94706	-.67830	-1.69549	.45957	.13787	-.83733	.24127	.56297	-2.93067
33.00000	16.92513	.97122	-.65181	-1.66807	.49742	.14923	-.84788	.26114	.60934	-2.96758
34.50000	17.62326	.99291	-.62496	-1.63979	.53577	.16073	-.85527	.28128	.65632	-2.99346
36.00000	18.31159	1.01212	-.59788	-1.61070	.57446	.17234	-.85957	.30159	.70371	-3.00848
37.50000	18.98961	1.02882	-.57067	-1.58087	.61333	.18408	-.86082	.32200	.75133	-3.01267
39.00000	19.65684	1.04303	-.54345	-1.55035	.65221	.19566	-.85911	.34241	.79896	-3.00689
40.50000	20.31276	1.05475	-.51634	-1.51918	.69095	.20728	-.85454	.36275	.84641	-2.99089
42.00000	20.95689	1.06401	-.48945	-1.48744	.72935	.21881	-.84722	.38291	.89346	-2.96526
43.50000	21.58869	1.07083	-.46292	-1.45517	.76728	.23018	-.83727	.40281	.93989	-2.93044
45.00000	22.20765	1.07525	-.43687	-1.42242	.80468	.24134	-.82483	.42235	.98548	-2.88692
46.50000	22.81326	1.07731	-.41142	-1.38926	.84083	.25225	-.81086	.44144	1.03002	-2.83522
48.00000	23.40498	1.07707	-.38671	-1.35573	.87613	.26284	-.79312	.45997	1.07326	-2.77592
49.50000	23.98228	1.07457	-.36288	-1.32188	.91017	.27305	-.77417	.47784	1.11496	-2.70960
51.00000	24.54463	1.06987	-.34006	-1.28776	.94277	.28283	-.75340	.49495	1.15489	-2.63689
52.50000	25.09151	1.06304	-.31841	-1.25340	.97378	.29211	-.73098	.51119	1.19278	-2.55843
54.00000	25.62237	1.05414	-.29807	-1.21884	1.00276	.30083	-.70711	.52645	1.22838	-2.47488
55.50000	26.13668	1.04323	-.27920	-1.18411	1.02971	.30891	-.68197	.54060	1.26139	-2.38691
57.00000	26.63391	1.03036	-.26197	-1.14922	1.05432	.31630	-.65576	.55352	1.29155	-2.29517
58.50000	27.11354	1.01599	-.24656	-1.11419	1.07634	.32290	-.62866	.56508	1.31852	-2.20033
60.00000	27.57505	.99896	-.23315	-1.07980	1.09550	.32865	-.60087	.57514	1.34199	-2.10388
61.50000	28.01791	.98050	-.22194	-1.04365	1.11151	.33345	-.57254	.58354	1.36160	-2.00390
63.00000	28.44162	.96023	-.21316	-1.00808	1.12406	.33722	-.54387	.59013	1.37697	-1.90353
64.50000	28.84568	.93813	-.20705	-.97225	1.13279	.33984	-.51500	.59471	1.38766	-1.80249
66.00000	29.22960	.91418	-.20387	-.93606	1.13733	.34120	-.48608	.59710	1.39322	-1.70126
67.50000	29.59292	.88831	-.20393	-.89940	1.13725	.34117	-.45723	.59706	1.39313	-1.60031
69.00000	29.93517	.86043	-.20755	-.86212	1.13207	.33962	-.42857	.59434	1.38679	-1.50000
70.50000	30.25591	.83037	-.21511	-.82401	1.12127	.33638	-.40018	.58867	1.37355	-1.40062
72.00000	30.55472	.79795	-.22705	-.78483	1.10421	.33126	-.37210	.57971	1.35266	-1.30236
73.50000	30.83121	.76288	-.24386	-.74426	1.08020	.32406	-.34436	.56710	1.32324	-1.20526
75.00000	31.08508	.72482	-.26610	-.70192	1.04842	.31453	-.31693	.55042	1.28432	-1.10924
76.50000	31.31572	.68330	-.29444	-.65733	1.00794	.30230	-.28972	.52917	1.23473	-1.01403
78.00000	31.52307	.63775	-.32963	-.60989	.95767	.28730	-.26261	.50278	1.17315	-.91914
79.50000	31.70675	.58747	-.37256	-.55849	.89635	.26898	-.23539	.47058	1.09803	-.82386
81.00000	31.86640	.53154	-.42425	-.50344	.82250	.24675	-.20775	.43181	1.00756	-.72712
82.50000	32.00205	.46980	-.48591	-.44230	.73441	.22082	-.17929	.38557	.89966	-.62761
84.00000	32.11324	.39844	-.55892	-.37476	.63012	.18903	-.14948	.33081	.77189	-.52318
85.50000	32.19990	.31832	-.64488	-.29868	.50732	.15219	-.11763	.26634	.62146	-.41171
87.00000	32.26189	.22675	-.74562	-.21239	.36348	.10982	-.08287	.19078	.44516	-.29006
88.50000	32.29912	.12151	-.86323	-.11370	.19539	.05862	-.04411	.10258	.23935	-.15439
90.00000	32.31153	.00000	-1.00000	-.00000	.00000	.00000	-.00000	.00000	.00000	-.00000

TABLE Ic

Values of Amplitudes, Stresses and Strains for an Incoming
Longitudinal Wave, Poisson's Ratio of 0.333

POISSON'S RATIO = .33333

ALPHA	BETA	A3/A1	A2/A1	SXX	SYX	SZZ	SXY	EXX	EYY	EXY
0.00000	0.00000	0.00000	-1.00000	-2.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
1.50000	.74994	.05234	-.99931	-1.99931	.00103	.00034	-.05232	.00069	.00137	-.20929
3.00000	1.49949	.10453	-.99726	-1.99726	.00411	.00137	-.10442	.00274	.00540	-.41768
4.50000	2.24826	.15643	-.99385	-1.99384	.00923	.00300	-.15607	.00615	.01230	-.62429
6.00000	2.99586	.20791	-.98909	-1.98906	.01637	.00546	-.20706	.01091	.02102	-.82824
7.50000	3.74196	.25882	-.98300	-1.98293	.02550	.00850	-.25716	.01700	.03400	-1.02865
9.00000	4.48610	.30903	-.97560	-1.97545	.03659	.01220	-.30617	.02440	.04879	-1.22470
10.50000	5.22791	.35839	-.96693	-1.96665	.04960	.01653	-.35389	.03307	.06614	-1.41556
12.00000	5.96701	.40679	-.95701	-1.95653	.06448	.02149	-.40011	.04299	.08598	-1.60045
13.50000	6.70300	.45408	-.94589	-1.94512	.08117	.02706	-.44466	.05411	.10823	-1.77864
15.00000	7.43547	.50015	-.93368	-1.93243	.09961	.03320	-.48735	.06640	.13281	-1.94941
16.50000	8.16404	.54488	-.92019	-1.91849	.11972	.03991	-.52802	.07981	.15962	-2.11210
18.00000	8.88829	.58816	-.90571	-1.90332	.14143	.04714	-.56653	.09429	.18857	-2.26611
19.50000	9.60783	.62987	-.89023	-1.88694	.16465	.05488	-.60272	.10977	.21954	-2.41087
21.00000	10.32223	.66993	-.87380	-1.86938	.18930	.06310	-.63647	.12620	.25240	-2.54588
22.50000	11.03110	.70822	-.85648	-1.85068	.21528	.07176	-.66768	.14352	.28704	-2.67071
24.00000	11.73400	.74468	-.83835	-1.83066	.24240	.08083	-.69624	.16165	.32330	-2.78496
25.50000	12.43052	.77921	-.81947	-1.80996	.27079	.09026	-.72208	.18053	.36105	-2.88833
27.00000	13.12023	.81174	-.79993	-1.78801	.30010	.10003	-.74514	.20007	.40014	-2.98054
28.50000	13.80271	.84221	-.77980	-1.76505	.33030	.11010	-.76536	.22020	.44040	-3.06142
30.00000	14.47751	.87056	-.75917	-1.74112	.36125	.12042	-.78271	.24083	.48167	-3.13084
31.50000	15.14626	.89674	-.73811	-1.71626	.39283	.13094	-.79718	.26189	.52377	-3.18873
33.00000	15.80823	.92071	-.71673	-1.69050	.42490	.14163	-.80878	.28327	.56653	-3.23511
34.50000	16.45147	.94244	-.69512	-1.66389	.45732	.15244	-.81751	.30488	.60976	-3.27005
36.00000	17.08915	.96190	-.67337	-1.63648	.48995	.16332	-.82342	.32663	.65327	-3.29369
37.50000	17.72091	.97906	-.65157	-1.60829	.52264	.17421	-.82655	.34843	.69686	-3.30621
39.00000	18.34638	.99393	-.62984	-1.57937	.55525	.18508	-.82697	.37016	.74033	-3.30787
40.50000	18.96886	1.00649	-.60827	-1.54977	.58760	.19587	-.82474	.39173	.78347	-3.29898
42.00000	19.58611	1.01676	-.58697	-1.51952	.61955	.20652	-.81998	.41303	.82607	-3.27991
43.50000	20.19150	1.02473	-.56605	-1.48866	.65092	.21697	-.81276	.43395	.86790	-3.25106
45.00000	20.78481	1.03042	-.54563	-1.45723	.68156	.22719	-.80322	.45437	.90874	-3.21289
46.50000	21.36532	1.03385	-.52582	-1.42527	.71128	.23709	-.79148	.47418	.94837	-3.16591
48.00000	21.93262	1.03505	-.50673	-1.39280	.73990	.24663	-.77766	.49327	.98653	-3.11064
49.50000	22.48626	1.03404	-.48850	-1.35985	.76725	.25575	-.76191	.51150	1.02300	-3.04764
51.00000	22.96574	1.03084	-.47125	-1.32644	.79313	.26438	-.74438	.52875	1.05750	-2.97750
52.50000	23.37058	1.02549	-.45511	-1.29260	.81734	.27245	-.72521	.54489	1.08979	-2.90083
54.00000	23.80033	1.01801	-.44021	-1.25832	.83969	.27990	-.70456	.55979	1.11959	-2.81824
55.50000	24.33450	1.00842	-.42670	-1.22362	.85996	.28665	-.68259	.57330	1.14661	-2.73036
57.00000	24.79263	.99674	-.41472	-1.18848	.87792	.29264	-.65945	.58528	1.17056	-2.63779
58.50000	25.23425	.98300	-.40443	-1.15289	.89335	.29778	-.63529	.59557	1.19113	-2.54116
60.00000	25.65891	.96718	-.39600	-1.11680	.90680	.30200	-.61026	.60400	1.20800	-2.44104
61.50000	26.06615	.94928	-.38959	-1.08018	.91561	.30520	-.58450	.61041	1.22082	-2.33802
63.00000	26.45554	.92928	-.38539	-1.04295	.92191	.30730	-.55815	.61461	1.22922	-2.23261
64.50000	26.82665	.90713	-.38360	-1.00504	.92660	.30820	-.53132	.61640	1.23280	-2.12530
66.00000	27.17905	.88278	-.38442	-.96633	.92337	.30779	-.50413	.61558	1.23116	-2.01652
67.50000	27.51235	.85615	-.38809	-.92669	.91787	.30596	-.47666	.61191	1.22383	-1.90664
69.00000	27.82614	.82711	-.39484	-.88595	.90774	.30258	-.44898	.60516	1.21032	-1.79593
70.50000	28.12006	.79551	-.40494	-.84392	.89259	.29753	-.42115	.59506	1.19013	-1.68459
72.00000	28.39375	.76119	-.41867	-.80036	.87280	.29067	-.39318	.58133	1.16266	-1.57271
73.50000	28.64687	.72390	-.43634	-.75499	.84548	.28183	-.36586	.56366	1.12731	-1.46025
75.00000	28.87909	.68337	-.45829	-.70748	.81256	.27085	-.33676	.54171	1.08342	-1.34704
76.50000	29.09014	.63928	-.48487	-.65744	.77269	.25756	-.30819	.51513	1.03026	-1.23276
78.00000	29.27972	.59124	-.51646	-.60445	.72330	.24177	-.27922	.48354	.96707	-1.11689
79.50000	29.44761	.53882	-.55340	-.54800	.66979	.22326	-.24989	.44652	.89305	-.99874
81.00000	29.59358	.48151	-.59633	-.48791	.60550	.20183	-.21935	.40367	.80733	-.87740
82.50000	29.71748	.41875	-.64548	-.42236	.53178	.17726	-.18793	.35452	.70904	-.75172
84.00000	29.81895	.34991	-.70135	-.35183	.44798	.14933	-.15507	.29865	.59731	-.62028
85.50000	29.89888	.27432	-.76437	-.27517	.35344	.11781	-.12036	.23563	.47126	-.48144
87.00000	29.95468	.19128	-.83495	-.19154	.24758	.08253	-.08332	.16505	.33010	-.33327
88.50000	29.98867	.10007	-.91341	-.10011	.12988	.04329	-.04340	.08659	.17317	-.17369
90.00000	30.00000	.00000	-1.00000	-.00000	.00000	.00000	-.00000	.00000	.00000	-.00000

Figure 2. Plot of Ratios of Amplitudes A_2/A_1 and A_3/A_1 vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333.

These expressions are somewhat unwieldy and are not a linear function of x . If we are mainly interested in the stress at depths much less than a wave length down from the surface, we may use the approximations for small x that

$$\sin f_1 x \approx f_1 x$$

$$\sin f_3 x \approx f_3 x$$

$$\cos f_1 x \approx 1$$

$$\cos f_3 x \approx 1$$

Next, rather than using the Lamé constants, we express the equations as functions of Poisson's ratio (or its equivalent c_1/c_2), the density, and the sonic velocity c_1 . The rationale for this use is that both density and longitudinal velocity are relatively easy to measure in the field, and Poisson's ratio is a widely used elastic constant.

These approximations give

$$\sigma_{xx} = p^2 \rho x A_1 \left[-\frac{2}{R} \cos 2\beta \cos \alpha \right] \sin pt = p^2 \rho x A_1 S_{xx} \sin pt$$

$$\sigma_{yy} = p \rho c_1 A_1 \left[\frac{4}{R} \left(\frac{c_2}{c_1} \right)^2 \left(1 - \left(\frac{c_2}{c_1} \right)^2 \right) \sin 2\alpha \sin 2\beta \right] \cos pt = p \rho c_1 A_1 S_{yy} \cos pt$$

$$\sigma_{zz} = p \rho c_1 A_1 \left[\frac{2}{R} \left(\frac{c_2}{c_1} \right)^2 \left(1 - 2 \left(\frac{c_2}{c_1} \right)^2 \right) \sin 2\alpha \sin 2\beta \right] \cos pt = p \rho c_1 A_1 S_{zz} \cos pt$$

$$\sigma_{xy} = p^2 \rho x A_1 \left[-\frac{2}{R} \left(\frac{c_2}{c_1} \right) \sin 2\alpha \cos \beta \left\{ \left(\frac{c_2}{c_1} \right)^4 \sin^2 2\alpha + \cos^2 2\beta \right\} \right] \sin pt$$

$$= p^2 \rho x A_1 S_{xy} \sin pt$$

To study these equations is instructive. Each equation may be divided into two parts. That part within the square brackets is nondimensional and is dependent only on Poisson's ratio and the angle that the incident wave makes with the normal to the surface. It is generally of magnitude between 0 and 2. The other part is a coefficient which has the dimension of stress and involves the magnitude of the incoming wave (A_1), its frequency (p), the density (ρ) of the medium and either the depth (x) beneath the surface or the sonic velocity (c_1) in the medium. In the case of vertical stress σ_{xx} and shear stress σ_{yx} , the coefficient is composed of the particle acceleration of the incoming wave ($p^2 A_1$) times ρx (the mass of the overburden per unit area). In the case of the horizontal stresses, σ_{yy} and σ_{zz} , the coefficient is the product of the particle velocity in the

medium (ρA_1) times the sonic velocity of longitudinal (c_1) waves and density (ρ). (This second coefficient is of the same form as that describing the stress caused by sonic waves moving through a medium.)

As stated earlier, these equations hold only for depths which are small compared to the wave length. The nondimensional parts S_{xx} , S_{xy} , S_{yy} , and S_{zz} are given in Table I for various values of Poisson's ratio for angles between 0 and 90°. These results are also shown in Figure 3.

Figure 3a. Plot of the Coefficients S_{xx} and S_{yy} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Longitudinal Wave.

Figure 3b. Plot of the Coefficients S_{zz} and S_{xy} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Longitudinal Wave.

Another item of interest is the strain (ϵ) caused by the incoming and reflected waves in both the horizontal (ϵ_{yy}) direction and the vertical (ϵ_{xx}) direction, as well as the shear strain ϵ_{xy} . This strain may be of interest from the standpoint of measurements (e.g. those by the firm of Dames and Moore, San Francisco, Calif., on RIO BLANCO) or those strains made along a vertical line-of-sight pipe or emplacement pipe during an underground nuclear explosion. The expressions for these strains for an incident longitudinal wave such as illustrated in Figure 1 are

$$\epsilon_{xx} = \frac{pA_1}{c_1} \left[-\frac{1}{R} \left\{ 1 - 2 \left(\frac{c_2}{c_1} \right)^2 \right\} \sin 2\alpha \sin 2\beta \right] \cos pt = \frac{pA_1}{c_1} E_{xx} \cos pt$$

$$\epsilon_{yy} = \frac{pA_1}{c_1} \left[\frac{1}{R} \sin 2\alpha \sin 2\beta \right] \cos pt = \frac{pA_1}{c_1} E_{yy} \cos pt$$

$$\epsilon_{xy} = \frac{p^2 x A_1}{c_1^2} \left[-\frac{2}{R} \left(\frac{c_1}{c_2} \right) \sin 2\alpha \cos \beta \left(\left(\frac{c_2}{c_1} \right)^4 \sin^2 2\alpha + \cos^2 2\beta \right) \right] \sin pt$$

$$= \frac{p^2 x A_1}{c_1^2} E_{xy} \sin pt$$

These results are also shown in nondimensional form in Table I and in Figure 4.

Figure 4. Plot of the Coefficients E_{xx} , E_{yy} and E_{xy} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Longitudinal Wave.

Next, we treat the subject of an incident shear wave such as is illustrated in Figure 5. First we consider a wave inclined at such an angle that there is possible both a reflected shear and a reflected longitudinal wave. This case may be treated in an analogous manner to the case of the longitudinal wave. Here we describe the incident shear wave as

$$\theta_1 = \text{Re} \left\{ B_1 e^{i(pt + f'_1 x + g'_1 y)} \right\},$$

where

B_1 = displacement amplitude of shear wave

p = angular frequency

t = time (sec)

$$f'_1 = \frac{p \cos \beta}{c_2}$$

$$f'_2 = \frac{p \cos \beta}{c_2}$$

$$f'_3 = \frac{p \cos \alpha}{c_1}$$

x = depth

$$g'_1 = \frac{p \sin \beta}{c_2}$$

$$g'_2 = \frac{p \sin \beta}{c_2}$$

$$g'_3 = \frac{p \sin \alpha}{c_1}$$

y = distance from origin parallel to the surface

β = angle of incidence

Figure 5. Incident Shear Wave, Reflected Shear and Longitudinal Waves

In a treatment analogous to that in the previous section,

$$\theta_2 = \text{Re} \left\{ B_2 e^{i(pt + f'_2 x + g'_2 y + \delta_1)} \right\}$$

$$\theta_3 = \text{Re} \left\{ B_3 e^{i(pt + f'_3 x + g'_3 y + \delta_2)} \right\}.$$

It can be shown, as before, that $\delta_1 = \delta_2 = 0$ and $g'_1 = g'_2 = g'_3 = g$. The same relations as were used in the case of the incident longitudinal wave and reflected longitudinal and shear waves will hold for this case, and we may use the fact that σ_{xy} and σ_{xx} disappear at a free boundary to get expressions that permit us to calculate B_2/B_1 and B_3/B_1 . Here B_2 is the amplitude of the reflected shear wave and B_3 is the amplitude of the reflected longitudinal wave.

$$\frac{B_2}{B_1} = \frac{\cos^2 2\beta - \left(\frac{c_2}{c_1}\right)^2 \sin 2\beta \sin 2\alpha}{\cos^2 2\beta + \left(\frac{c_2}{c_1}\right)^2 \sin 2\beta \sin 2\alpha} = \frac{\cos^2 2\beta - \left(\frac{c_2}{c_1}\right)^2 \sin 2\beta \sin 2\alpha}{R}$$

$$\frac{B_3}{B_1} = \frac{2}{R} \frac{c_2}{c_1} \sin 2\beta \cos 2\beta = \frac{\frac{c_2}{c_1} \sin 4\beta}{R}$$

As was done earlier, we may derive exact expressions for the stresses as

$$\sigma_{xx} = \frac{2\mu B_1}{c_2 R} \left\{ [\cos f'_1 x - \cos f'_3 x] \cos^2 2\beta \sin 2\beta \sin (pt + gy) \right.$$

$$\left. + \left[\frac{c_2^2}{c_1^2} \sin 2\beta \sin 2\alpha \sin f'_1 x + \cos^2 2\beta \sin f'_3 x \right] \sin 2\beta \cos (pt + gy) \right\}$$

$$\sigma_{yy} = \frac{2\mu B_1}{c_2 R} \left(\frac{c_2^2}{c_1^2} \right) \sin 2\beta \left\{ \left[\frac{c_1^2}{c_2^2} \cos^2 2\beta \cos f'_1 x \right. \right.$$

$$\left. + \left(\frac{c_1^2}{c_2^2} - 1 - \cos 2\alpha \right) \cos 2\beta \cos f'_3 x \right] \sin (pt + gy)$$

$$\left. + \left[\sin 2\alpha \sin 2\beta \sin f'_1 x - \left(\frac{c_1^2}{c_2^2} - 1 - \cos 2\alpha \right) \cos 2\beta \sin f'_3 x \right] \cos (pt + gy) \right\}$$

$$\sigma_{zz} = \frac{2B_1 p \lambda}{c_1 R} \left(\frac{c_2}{c_1} \right) \sin 2\beta \cos 2\beta \left[\sin f'_3 x \cos (pt + gy) - \cos f'_3 x \sin (pt + gy) \right]$$

$$\sigma_{xy} = \frac{2B_1 p \mu}{c_2 R} \left(\frac{c_2}{c_1} \right) \left\{ -\cos f'_1 x + \cos f'_3 x \right\} \sin 2\alpha \sin 2\beta \cos 2\beta \sin (pt + gy) \\ - \left[\frac{c_1^2}{c_2^2} \cos^2 2\beta \sin f'_1 x + \sin 2\beta \sin 2\alpha \sin f'_3 x \right] \cos 2\beta \cos (pt + gy) \left\}.$$

As before for small values of x , there are

$$\sigma_{xx} = p^2 \rho x B_1 \left[\frac{2}{R} \frac{c_2}{c_1} \sin 2\beta \cos \alpha \right] \cos pt = p^2 \rho x B_1 S_{xx} \cos pt$$

$$\sigma_{yy} = p \rho c_1 B_1 \left[-\frac{4}{R} \frac{c_2}{c_1} \sin 2\beta \cos 2\beta \left\{ 1 - \left(\frac{c_2}{c_1} \right)^2 \right\} \right] \sin pt = p \rho c_1 B_1 S_{yy} \sin pt$$

$$\sigma_{zz} = p \rho c_1 B_1 \left[-\frac{2}{R} \left\{ 1 - 2 \left(\frac{c_2}{c_1} \right)^2 \right\} \frac{c_2}{c_1} \sin 2\beta \cos 2\beta \right] \sin pt = p \rho c_1 B_1 S_{zz} \sin pt$$

$$\sigma_{xy} = p^2 \rho x B_1 \left[-\frac{2}{R} \left\{ \cos^2 2\beta + \left(\frac{c_2}{c_1} \right)^4 \sin^2 2\alpha \right\} \cos 2\beta \cos \beta \right] \cos pt \\ = p^2 \rho x B_1 S_{xy} \cos pt$$

We have also derived formulae for the strains for small x as

$$\epsilon_{xx} = \frac{p B_1}{c_2} \left[\frac{1}{R} \left\{ 1 - 2 \left(\frac{c_2}{c_1} \right)^2 \right\} \sin 2\beta \cos 2\beta \right] \sin pt = \frac{p B_1}{c_2} E_{xx} \sin pt$$

$$\epsilon_{yy} = \frac{p B_1}{c_2} \left[\frac{1}{R} \sin 2\beta \cos 2\beta \right] \sin pt = \frac{p B_1}{c_2} E_{yy} \sin pt$$

$$\epsilon_{xy} = \frac{pxB_1}{c_2^2} \left[-\frac{2}{R} (\cos 2\beta \cos \beta) \left\{ 4 \left(\frac{c_2^2}{c_1^2} - 1 \right) \sin \beta + 1 \right\} \right] \cos pt = \frac{pxB_1}{c_2^2} E_{xy} \cos pt$$

The final case treated is that of the shear waves which are incident at an angle such that no real reflected longitudinal wave is possible. This occurs wherever β is large enough that for the Poisson's ratio of the material,

$$\sin \beta > \frac{c_2}{c_1} = \sqrt{\frac{1-2\nu}{2-2\nu}}.$$

The treatment of this case is more complicated although the same condition of zero stress σ_{xx} or σ_{yx} on the free surface is the basis for the solution. Following Kolsky's notation we again express the incident shear wave as

$$\psi_1 = \text{Re} \left\{ B_1 e^{[i(pt + f'_1 x + g'_1 y)]} \right\}$$

where

$$f'_1 = p \cos \beta / c_2$$

$$g'_1 = p \sin \beta / c_2.$$

The reflected shear wave is

$$\psi_2 = \text{Re} \left\{ B_2 e^{[i(pt - f'_2 x + g'_2 y + \delta_1)]} \right\}$$

a second resulting wave is

$$\psi_3 = \text{Re} \left\{ B_3 e^{[i(pt - f'_3 x + g'_3 y + \delta_2)]} \right\}$$

where

$$f'_2 = p \cos \beta / c_2 \text{ and } g'_2 = p \sin \beta / c_2$$

$$f'_3 = p(\cos \alpha) / c_1 \text{ and } g'_3 = p(\sin \alpha) / c_1.$$

Here $\sin \alpha$ is not real but complex and everywhere must be replaced by $[(c_1/c_2) \sin \beta]$. Also $\cos \alpha$ is imaginary and

$$(\cos \alpha)^2 = 1 - \left(\frac{c_1}{c_2}\right)^2 (\sin \beta)^2 < 0 .$$

In this case the two phase angles δ_1 and δ_2 are non-zero.

The second wave in this case is a surface wave described by

$$\psi_3 = B_3 e^{-mx} \cos(pt + g'_3 y + \delta_2) ,$$

where

$$m = \left(\frac{p}{c_1}\right) \left[\left(\frac{c_1}{c_2}\right)^2 \sin^2 \beta - 1\right]^{1/2}$$

$$g'_3 = \frac{p(\sin \alpha)}{c_1} = p \frac{(\sin \beta)}{c_2} = g'_1 = g'_2 = g$$

Note that physical considerations require that the exponential exponent be negative.

The values of the variables are

$$\sin \alpha = \frac{c_1}{c_2} \sin \beta$$

$$\cos \alpha = -i \sqrt{\left(\frac{c_1}{c_2}\right)^2 (\sin \beta)^2 - 1}$$

$$B_2 = B_1$$

$$\frac{B_3}{B_1} = \frac{2 \frac{c_2}{c_1} \sin 2\beta \cos 2\beta}{R^*} = \frac{\frac{c_2}{c_1} \sin 4\beta}{R^*}$$

$$\delta_2 = \sin^{-1} \left\{ \frac{2}{R^*} \frac{c_2}{c_1} \sin \beta \sin 2\beta \sqrt{\left(\frac{c_1}{c_2} \sin \beta\right)^2 - 1} \right\}$$

$$\delta_1 = 2\delta_2$$

$$R^* = \left(\cos^4 2\beta + 4 \left(\frac{c_2}{c_1} \right)^2 \sin^2 \beta \sin^2 2\beta \left(\left(\frac{c_1}{c_2} \right)^2 \sin^2 \beta - 1 \right) \right)^{1/2}$$

The values of B_2/B_1 and B_3/B_1 for the incident shear wave are shown in Figure 6 and in Table II for various Poisson's ratios. The values of δ_1 and δ_2 are shown in Table II.

The exact values for the stresses are

$$\begin{aligned} \sigma_{xx} &= \frac{pB_1\mu}{c_2} \left\{ 2 \sin 2\beta \cos(f_1'x - \delta_2) \right. \\ &\quad \left. - \frac{B_3}{B_1} \frac{c_1}{c_2} \cos 2\beta e^{-\frac{px}{c_1} \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1}} \right\} \sin(pt + gy + \delta_2) \\ \sigma_{yy} &= -\frac{pB_1\mu}{c_2} \left\{ 2 \sin 2\beta \cos(f_1'x - \delta_2) + \frac{B_3}{B_1} \frac{c_2}{c_1} \left[2 \left(\frac{c_1^2}{c_2^2} - 1 \right) \right. \right. \\ &\quad \left. \left. - \frac{c_1^2}{c_2^2} \cos 2\beta \right] e^{-\frac{px}{c_1} \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1}} \right\} \sin(pt + gy + \delta_2) \\ \sigma_{zz} &= -\frac{p}{c_1} \left(\frac{c_1^2}{c_2^2} - 2 \right) \mu B_1 \left(\frac{B_3}{B_1} \right) e^{-\frac{px}{c_1} \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1}} \sin(pt + gy + \delta_2) \\ \sigma_{xy} &= -\frac{2pB_1\mu}{c_2} \left\{ \cos 2\beta \sin(f_1'x - \delta_2) \right. \\ &\quad \left. + \frac{B_3}{B_1} \sin \beta \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1} e^{-\frac{px}{c_1} \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1}} \right\} \cos(pt + gy + \delta_2) \end{aligned}$$

As before, we have derived approximate values for the stress for small values of x as

$$\begin{aligned}
 \sigma_{xx} &= p^2 \rho x B_1 \left[\frac{2}{R^*} \left(\frac{c_2}{c_1} \right) \sin 2\beta \sqrt{\left(\frac{c_1}{c_2} \sin \beta \right)^2 - 1} \right] \left[\sin (pt + \delta_2) \right] = p^2 \rho x B_1 S_{xx} \sin (pt + \delta_2) \\
 \sigma_{yy} &= p \rho c_2 B_1 \left[- \frac{4}{R^*} \left(1 - \left(\frac{c_2}{c_1} \right)^2 \right) \sin 2\beta \cos 2\beta \right] \sin (pt + \delta_2) = p \rho c_2 B_1 S_{yy} \sin (pt + \delta_2) \\
 \sigma_{zz} &= p \rho c_2 B_1 \left[- \frac{2}{R^*} \left(1 - 2 \left(\frac{c_2}{c_1} \right)^2 \right) \sin 2\beta \cos 2\beta \right] \sin (pt + \delta_2) = p \rho c_2 B_1 S_{zz} \sin (pt + \delta_2) \\
 \sigma_{xy} &= p^2 \rho x B_1 \left[- \frac{2}{R^*} \cos 2\beta \cos \beta \left\{ 4 \left(\frac{c_2}{c_1} \right)^2 - 1 \right\} \sin^2 \beta + 1 \right] \cos (pt + \delta_2) \\
 &= p^2 \rho x B_1 S_{xy} \cos (pt + \delta_2)
 \end{aligned}$$

The values of the numerical coefficient (S and E) are plotted in Figure 7 and given in Table II. Upon examination, it may be noted that the coefficients for the stresses are essentially of the same form regardless of whether there is a reflected longitudinal wave ($\beta <$ the critical angle) or a surface wave ($\beta >$ the critical angle).

Finally it may be shown that the equations for the strains when β is greater than the critical angle are identical to those previously given when β is less than the critical angle with the exception that R^* must be substituted for R when the surface wave is generated. The values for the coefficients for various Poisson's ratios are shown in Figure 8 and given in Table II.

It is hoped that at some future date the information incorporated in this paper can be used to calculate the maximum stress which results from both incoming transverse and shear waves.

Figure 6. Plot of Ratios of Amplitude B_2/B_1 and B_3/B_1 vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333.

Figure 7a. Plot of the Coefficients S_{xx} and S_{yy} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Shear Wave.

Figure 7b. Plot of the Coefficients S_{xy} and S_{zz} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Shear Wave.

Figure 8. Plot of the Coefficients E_{xx} , E_{yy} and E_{xy} vs. Angle of Incidence for Poisson's Ratios of 0.25, 0.30 and 0.333, Incident Shear Wave.

TABLE IIa

Values of Amplitudes, Stresses and Strains for an Incoming Shear Wave, Poisson's Ratio of 0.25

POISSON'S RATIO = .25000 R = 1.73205 CRITICAL ANGLE = 35.264309603 R31CRT = 3.265916324 SYVCRT = 7.542472333														
BETA	ALPHA	R3/R1	R2/R1	SVT	SXX	SZZ	SKV	EXX	EYY	EXY	DEL1	DEL2		
0.00000	0.00000	0.00000	1.00000	-0.00000	0.00000	-0.00000	-2.00000	0.00000	0.00000	-2.00000	0.00000	0.00000		
1.50000	2.59867	.06042	.99684	-.13353	.06044	-.03488	-1.99524	.01744	.05232	-1.81285	0.00000	0.00000		
3.00000	5.70052	.12060	.98736	-.27351	.12076	-.06963	-1.98099	.03481	.10444	-1.62423	0.00000	0.00000		
4.50000	7.81070	.18029	.97162	-.41637	.18085	-.10409	-1.95738	.05205	.15614	-1.43489	0.00000	0.00000		
6.00000	10.43064	.23927	.94971	-.55256	.24057	-.13814	-1.92459	.06907	.20121	-1.24559	0.00000	0.00000		
7.50000	13.06624	.29729	.92173	-.68657	.29981	-.17164	-1.88289	.08502	.25746	-1.05784	0.00000	0.00000		
9.00000	15.72095	.35415	.88785	-.81787	.35844	-.20447	-1.83263	.10223	.30670	-.87088	0.00000	0.00000		
10.50000	18.39953	.40962	.84826	-.94597	.41633	-.23649	-1.77421	.11825	.35474	-.68538	0.00000	0.00000		
12.00000	21.10717	.46351	.80318	-1.07043	.47334	-.26761	-1.70813	.13388	.40141	-.50344	0.00000	0.00000		
13.50000	23.84973	.51565	.75287	-1.19084	.52931	-.29771	-1.63494	.14866	.44557	-.32515	0.00000	0.00000		
15.00000	26.63338	.56588	.69765	-1.30685	.58409	-.32671	-1.55525	.16336	.49107	-.15181	0.00000	0.00000		
16.50000	29.46745	.61410	.63788	-1.41828	.63752	-.35455	-1.46973	.17728	.53183	.01041	0.00000	0.00000		
18.00000	32.35972	.66023	.57396	-1.52473	.68935	-.38118	-1.37913	.19059	.57177	.16267	0.00000	0.00000		
19.50000	35.32196	.70427	.50636	-1.62644	.73948	-.40661	-1.28423	.20338	.60391	.34144	0.00000	0.00000		
21.00000	38.36811	.74632	.43565	-1.72355	.78739	-.43089	-1.18588	.21544	.64533	.49453	0.00000	0.00000		
22.50000	41.51525	.78662	.36247	-1.81663	.83297	-.45416	-1.08496	.22708	.68123	.64282	0.00000	0.00000		
24.00000	44.78824	.82564	.28763	-1.90674	.87572	-.47658	-.98242	.23834	.71503	.78436	0.00000	0.00000		
25.50000	48.17164	.86422	.21214	-1.99583	.91583	-.49896	-.87925	.24948	.74844	.92262	0.00000	0.00000		
27.00000	51.84428	.90385	.13741	-2.08173	.95801	-.52184	-.77653	.26092	.78276	1.05897	0.00000	0.00000		
28.50000	55.73712	.94729	.06552	-2.16768	.99721	-.54692	-.67543	.27346	.82038	1.19768	0.00000	0.00000		
30.00000	60.00000	1.00000	0.00000	-2.25394	1.00000	-.57735	-.57735	.28668	.86603	1.34715	0.00000	0.00000		
31.50000	64.82248	1.07425	-.05195	-2.43087	1.06665	-.62822	-.48428	.31811	.91033	1.52729	0.00000	0.00000		
33.00000	70.62168	1.20496	-.07078	-2.75274	.98382	-.69568	-.48841	.34784	1.04353	1.79432	0.00000	0.00000		
34.50000	78.82435	1.56565	.04896	-3.61571	.84665	-.98393	-.34589	.45196	1.35589	2.42718	0.00000	0.00000		
36.00000	0.00000	2.17625	1.00000	-5.02584	1.34580	-1.25646	-.25231	.62823	1.88469	3.49469	-104.47751	-52.23876		
37.50000	0.00000	1.21970	1.00000	-2.81677	1.57551	-.78419	-.82840	.35210	1.05629	2.82855	-147.11751	-73.55876		
39.00000	0.00000	.75431	1.00000	-1.74208	1.57362	-.43558	-.85825	.21775	.65325	1.28464	-164.03713	-82.81856		
40.50000	0.00000	.46665	1.00000	-1.07767	1.53661	-.26942	-.87763	.13471	.44413	.81464	-172.66825	-86.33813		
42.00000	0.00000	.26657	1.00000	-.61583	1.49484	-.15391	-.86692	.07695	.23886	.47582	-177.21924	-88.68962		
43.50000	0.00000	.11711	1.00000	-.27045	1.45272	-.06761	-.83883	.03381	.13142	.21326	-179.39893	-89.69547		
45.00000	0.00000	0.00000	1.00000	-0.00000	1.41421	-0.00000	-0.00000	0.00000	0.00000	-0.00000	-180.00000	-90.00000		
46.50000	0.00000	-.09486	1.00000	.21987	1.37859	.05477	-.84565	-.02738	-.05215	-.17884	-179.58664	-89.75332		
48.00000	0.00000	-.17354	1.00000	.88877	1.34546	.18819	-.89568	-.05818	-.15029	-.33212	-178.18985	-89.89493		
49.50000	0.00000	-.23989	1.00000	.55400	1.31147	.13858	-.94886	-.06925	-.20775	-.46546	-176.22875	-88.11438		
51.00000	0.00000	-.29647	1.00000	.88468	1.28084	.17117	-.98558	-.25675	-.58252	-.73.74309	-86.87155			
52.50000	0.00000	-.34504	1.00000	.79684	1.25643	.19921	-.99961	-.09961	-.68588	-170.81516	-85.48758			
54.00000	0.00000	-.38582	1.00000	.89331	1.22872	.22333	-.98863	-.11166	-.77701	-167.58243	-83.75121			
55.50000	0.00000	-.42267	1.00000	.97611	1.20137	.24403	-.96828	-.12201	-.85733	-163.84523	-81.92262			
57.00000	0.00000	-.45722	1.00000	1.04666	1.17482	.26166	-.94988	-.13883	-.92755	-159.87169	-79.93545			
58.50000	0.00000	-.47889	1.00000	1.10596	1.14634	.27649	-.94567	-.13824	-.98820	-155.60887	-77.80843			
60.00000	0.00000	-.50800	1.00000	1.15470	1.11883	.28868	-.94888	-.14434	-.98301	-151.84498	-75.52249			
61.50000	0.00000	-.51673	1.00000	1.19334	1.08878	.29833	-.95392	-.14917	-.944750	-146.21892	-73.18546			
63.00000	0.00000	-.52920	1.00000	1.22213	1.05878	.30553	-.95756	-.15277	-.85838	-141.10140	-70.55870			
64.50000	0.00000	-.53744	1.00000	1.24117	1.02622	.31029	-.96459	-.15515	-.74654	-135.71580	-67.85790			
66.00000	0.00000	-.54146	1.00000	1.25045	.99228	.31261	-.96295	-.15631	-.64892	-130.05089	-65.02545			
67.50000	0.00000	-.54120	1.00000	1.24934	.95615	.31246	-.96473	-.15623	-.46869	-124.10147	-62.05873			
69.00000	0.00000	-.53656	1.00000	1.23914	.91748	.30979	-.96510	-.15489	-.46468	-117.86895	-58.93848			
70.50000	0.00000	-.52745	1.00000	1.21818	.87596	.30453	-.96365	-.15226	-.45679	-111.32286	-55.66103			
72.00000	0.00000	-.51374	1.00000	1.18644	.83125	.29661	-.96856	-.14830	-.44491	-104.47751	-52.23876			
73.50000	0.00000	-.49531	1.00000	1.14386	.78385	.28596	-.96493	-.14288	-.42895	-97.32882	-48.66841			
75.00000	0.00000	-.47203	1.00000	1.09011	.73187	.27253	-.96295	-.13626	-.40879	-89.84724	-44.92362			
76.50000	0.00000	-.44385	1.00000	1.02582	.67587	.25626	-.96810	-.12813	-.38438	-82.05840	-41.82740			
78.00000	0.00000	-.41073	1.00000	.94854	.61487	.23714	-.96416	-.11857	-.35570	-73.94549	-36.97274			
79.50000	0.00000	-.37273	1.00000	.86873	.55838	.21528	-.95188	-.10768	-.32280	-.65.52638	-32.76319			
81.00000	0.00000	-.33001	1.00000	.76213	.48164	.19053	-.94639	-.09527	-.28580	-.56.81885	-24.48543			
82.50000	0.00000	-.28285	1.00000	.65221	.40879	.16383	-.94856	-.08165	-.24496	-.62388	-47.81956	-23.98978		
84.00000	0.00000	-.23166	1.00000	.53499	.33218	.13375	-.93834	-.06687	-.20862	-.51160	-38.58111	-19.29856		
85.50000	0.00000	-.17701	1.00000	.40870	.25227	.10228	-.92576	-.05118	-.15329	-.39128	-29.13227	-14.56613		
87.00000	0.00000	-.11961	1.00000	.27622	.16973	.06986	-.91212	-.03453	-.18358	-.26457	-19.51754	-9.75877		
88.50000	0.00000	-.06029	1.00000	.13924	.08534	.03481	-.89696	-.01741	-.05222	-.13342	-9.78809	-4.89405		
90.00000	0.00000	-.00000	1.00000	.00000	.00000	.00000	-.88000	-.00000	-.00000	-.00000	-.00000	-.00000		

TABLE IIB

Values of Amplitudes, Stresses and Strains for an Incoming Shear Wave, Poisson's Ratio of 0.30

POISSONS RATIO = .30000 R = 1.87083 CRITICAL ANGLE = 32.311533237 R31CRT = 2.253744679 SVVCRT = 6.023386019													
BETA	ALPHA	R3/R1	R2/R1	SVY	SXX	SZZ	SVY	EXX	EYY	EXY	DEL1	DEL2	
0.00000	0.00000	0.00000	1.00000	-0.00000	0.00000	-0.00000	-2.00000	0.00000	0.00000	-2.00000	0.00000	0.00000	
1.50000	2.80705	.05594	.99707	-.14952	.09595	-.04486	-1.99521	.02243	.05233	-1.88689	0.00000	0.00000	
3.00000	5.61892	.11170	.98830	-.29854	.11178	-.08956	-1.98889	.04478	.10449	-1.61267	0.00000	0.00000	
4.50000	8.44057	.16710	.97374	-.44659	.16735	-.13398	-1.95714	.06699	.15630	-1.41880	0.00000	0.00000	
6.00000	11.27714	.22195	.95348	-.59318	.22252	-.17795	-1.92418	.08898	.20761	-1.22352	0.00000	0.00000	
7.50000	14.13410	.27609	.92764	-.73787	.27717	-.22136	-1.88226	.11068	.25626	-1.02988	0.00000	0.00000	
9.00000	17.01740	.32936	.89639	-.88025	.33115	-.26488	-1.83174	.13204	.30889	-.83769	0.00000	0.00000	
10.50000	19.93363	.38163	.85994	-1.01995	.38429	-.30598	-1.77303	.15299	.35698	-.64754	0.00000	0.00000	
12.00000	22.89025	.43278	.81852	-1.15665	.43643	-.34780	-1.70864	.17350	.40483	-.45999	0.00000	0.00000	
13.50000	25.89583	.48273	.77245	-1.29015	.48738	-.38785	-1.63312	.19352	.45155	-.27553	0.00000	0.00000	
15.00000	28.96047	.53144	.72207	-1.42034	.53692	-.42610	-1.55311	.21305	.49712	-.09457	0.00000	0.00000	
16.50000	32.09631	.57894	.66782	-1.54728	.58480	-.46418	-1.46738	.23209	.54155	.08260	0.00000	0.00000	
18.00000	35.31827	.62533	.61021	-1.67127	.63069	-.50138	-1.37647	.25069	.58494	.29586	0.00000	0.00000	
19.50000	38.64516	.67086	.54988	-1.79296	.67422	-.53789	-1.28143	.26894	.62754	.42530	0.00000	0.00000	
21.00000	42.10142	.71598	.48765	-1.91355	.71884	-.57486	-1.18311	.28703	.66974	.59141	0.00000	0.00000	
22.50000	45.71989	.76146	.42457	-2.03510	.75184	-.61053	-1.08249	.30526	.71228	.75527	0.00000	0.00000	
24.00000	49.54666	.80864	.36215	-2.16118	.78418	-.64835	-.98068	.32418	.75641	.91895	0.00000	0.00000	
25.50000	53.65011	.85988	.30269	-2.29813	.80986	-.68944	-.87898	.34472	.80434	1.08639	0.00000	0.00000	
27.00000	58.13979	.91970	.25009	-2.45799	.82591	-.73748	-.77986	.36870	.86030	1.26531	0.00000	0.00000	
28.50000	63.21226	.99769	.21213	-2.66645	.82559	-.79993	-.68354	.39997	.93326	1.47243	0.00000	0.00000	
30.00000	69.29519	1.11905	.20971	-2.99079	.79129	-.89774	-.59816	.44862	1.04678	1.75175	0.00000	0.00000	
31.50000	77.82427	1.39013	.32512	-3.71529	.64582	-.111459	-.59478	.55729	1.30035	2.28888	0.00000	0.00000	
33.00000	0.00000	2.03294	1.00000	-5.43326	.97702	-1.62998	-.53240	.04199	1.90164	3.49406	-64.29831	-32.14915	
34.50000	0.00000	1.51475	1.00000	-4.04834	1.48153	-1.21458	-.28859	.68725	1.41692	2.70341	-114.10837	-57.05819	
36.00000	0.00000	1.08507	1.00000	-2.89998	1.60612	-.06999	-.02224	.43588	1.81499	2.88175	-141.48783	-78.74391	
37.50000	0.00000	.76555	1.00000	-2.04603	1.61216	-.61381	.06920	.30698	.71611	1.45427	-157.87481	-78.93748	
39.00000	0.00000	.52871	1.00000	-1.41302	1.58021	-.42391	.18338	.21195	.49456	1.03088	-167.93165	-83.96583	
40.50000	0.00000	.34857	1.00000	-.93160	1.53771	-.27948	.10297	.13974	.32686	.69572	-174.07951	-87.03975	
42.00000	0.00000	.20740	1.00000	-.55431	1.49417	-.16629	.08096	.08315	.19401	.42274	-177.66321	-88.83168	
43.50000	0.00000	.09370	1.00000	-.25042	1.45272	-.07513	.04505	.03756	.08765	.19464	-179.47363	-89.73682	
45.00000	0.00000	0.00000	1.00000	-0.00000	1.41421	-0.00000	-0.00000	0.00000	0.00000	-0.00000	-180.00000	-90.00000	
46.50000	0.00000	-.07865	1.00000	.21019	1.37859	.06306	-.05185	-.03153	-.07357	-.16883	-179.55819	-89.77989	
48.00000	0.00000	-.14561	1.00000	.38916	1.34549	.11675	-.18592	-.08337	-.13621	-.31707	-178.35947	-89.17973	
49.50000	0.00000	-.20323	1.00000	.54316	1.31449	.16295	-.16302	-.08147	-.19011	-.44834	-176.54915	-88.27458	
51.00000	0.00000	-.25317	1.00000	.67663	1.28513	.20299	-.22111	-.10149	-.23682	-.56522	-174.22931	-87.11466	
52.50000	0.00000	-.29662	1.00000	.79276	1.25698	.23783	-.27928	-.11891	-.27747	-.66952	-171.47263	-85.73631	
54.00000	0.00000	-.33446	1.00000	.89388	1.22966	.26816	-.33645	-.13408	-.31286	-.76256	-168.33119	-84.16559	
55.50000	0.00000	-.36731	1.00000	.98164	1.20281	.29450	-.39212	-.14725	-.34359	-.84525	-164.84219	-82.42189	
57.00000	0.00000	-.39564	1.00000	1.05740	1.17687	.31722	-.44553	-.15861	-.37009	-.91823	-161.03175	-80.51588	
58.50000	0.00000	-.41978	1.00000	1.12190	1.14911	.33657	-.49685	-.16829	-.39267	-.98191	-156.91755	-78.45878	
60.00000	0.00000	-.43994	1.00000	1.17579	1.12163	.35274	-.54388	-.17637	-.41153	-1.03654	-152.51067	-76.25533	
61.50000	0.00000	-.45628	1.00000	1.21946	1.09331	.36584	-.58681	-.18292	-.42681	-1.08221	-147.81687	-73.98843	
63.00000	0.00000	-.46886	1.00000	1.25309	1.06382	.37593	-.62428	-.18796	-.43858	-1.11888	-142.83765	-71.41883	
64.50000	0.00000	-.47771	1.00000	1.27674	1.03285	.38302	-.65728	-.19151	-.44686	-1.14644	-137.57099	-68.78550	
66.00000	0.00000	-.48280	1.00000	1.29034	1.00085	.38710	-.68443	-.19355	-.45162	-1.16465	-132.01198	-66.00599	
67.50000	0.00000	-.48406	1.00000	1.29371	.96588	.38811	-.70512	-.19406	-.45280	-1.17323	-126.15342	-63.07671	
69.00000	0.00000	-.48138	1.00000	1.28656	.92758	.38597	-.71877	-.19298	-.45029	-1.17181	-119.98644	-59.99322	
70.50000	0.00000	-.47465	1.00000	1.26855	.88718	.38057	-.72478	-.19028	-.44399	-1.15998	-113.50108	-56.75854	
72.00000	0.00000	-.46371	1.00000	1.23931	.84351	.37179	-.72257	-.18590	-.43376	-1.13731	-106.68707	-53.34354	
73.50000	0.00000	-.44841	1.00000	1.19842	.79622	.35953	-.71161	-.17976	-.41945	-1.10337	-99.53470	-49.76735	
75.00000	0.00000	-.42861	1.00000	1.14552	.74494	.34366	-.69141	-.17183	-.40093	-1.05773	-92.03586	-46.01793	
76.50000	0.00000	-.40421	1.00000	1.08028	.68938	.32409	-.66159	-.16204	-.37410	-1.00009	-84.18524	-42.09262	
78.00000	0.00000	-.37512	1.00000	1.00254	.62929	.30076	-.62190	-.15038	-.35089	-.93025	-75.98179	-37.99889	
79.50000	0.00000	-.34135	1.00000	.91229	.56452	.27369	-.57227	-.13684	-.31930	-.84820	-67.43017	-33.71509	
81.00000	0.00000	-.30300	1.00000	.80981	.49584	.24294	-.51287	-.12147	-.28343	-.75421	-58.54239	-29.27119	
82.50000	0.00000	-.26031	1.00000	.69570	.42099	.20871	-.44415	-.10435	-.24349	-.64887	-49.33910	-24.66955	
84.00000	0.00000	-.21363	1.00000	.57095	.34267	.17178	-.36689	-.08564	-.19983	-.53314	-39.85075	-19.92538	
85.50000	0.00000	-.16351	1.00000	.43699	.26862	.13110	-.28222	-.06555	-.15295	-.40842	-30.11800	-15.05900	
87.00000	0.00000	-.11062	1.00000	.29565	.17553	.08869	-.19162	-.04435	-.10348	-.27650	-20.19143	-10.09572	
88.50000	0.00000	-.05581	1.00000	.14915	.08832	.04475	-.09688	-.02237	-.05220	-.13955	-10.13023	-5.06511	
90.00000	0.00000	-.00000	1.00000	.00000	.00000	.00000	-.00000	-.00000	-.00000	-.00000	-.00000	-.00000	

TABLE IIc

Values of Amplitudes, Stresses and Strains for an Incoming Shear Wave, Poisson's Ratio of 0.333

POISSONS RATIO = .33543 R = 2.00000 CRITICAL ANGLE = 30.00000025 R3ICRT = 1.732051012 SYCYRT = 5.196152430													
BETA	ALPHA	B3/B1	B2/Y1	SVY	SXX	SZZ	SKY	EXX	EYY	EXY	DEL1	DEL2	
0.00000	0.00000	0.00000	1.00000	-0.00000	0.00000	-0.00000	-2.00000	0.00000	0.00000	-2.00000	0.00000	0.00000	
1.50000	3.00163	.05234	.99726	-.15781	.05234	-.05234	-1.99521	.02617	.05234	-1.00386	0.00000	0.00000	
3.00000	6.00828	.10453	.98906	-.31359	.10453	-.10453	-1.98086	.05226	.10453	-1.60529	0.00000	0.00000	
4.50000	9.02806	.15644	.97545	-.46932	.15643	-.15644	-1.95789	.07822	.15644	-1.40726	0.00000	0.00000	
6.00000	12.06709	.20794	.95654	-.62381	.20789	-.20794	-1.92489	.10397	.20794	-1.20953	0.00000	0.00000	
7.50000	15.13252	.25890	.93246	-.77678	.25874	-.25890	-1.88213	.12945	.25890	-1.01263	0.00000	0.00000	
9.00000	18.23221	.30922	.90338	-.92765	.30881	-.30922	-1.83157	.15461	.30922	-.81711	0.00000	0.00000	
10.50000	21.37503	.35883	.86955	-1.07648	.35792	-.35883	-1.77285	.17941	.35883	-.62343	0.00000	0.00000	
12.00000	24.57110	.40766	.83125	-1.22298	.40583	-.40766	-1.70647	.20383	.40766	-.43281	0.00000	0.00000	
13.50000	27.83266	.45573	.78882	-1.36718	.45230	-.45573	-1.63301	.22786	.45573	-.24320	0.00000	0.00000	
15.00000	31.17395	.50388	.74272	-1.50924	.49782	-.50388	-1.55313	.25154	.50388	-.05720	0.00000	0.00000	
16.50000	34.61381	.54988	.69348	-1.64964	.53961	-.54988	-1.46757	.27494	.54988	.12596	0.00000	0.00000	
18.00000	38.17271	.59432	.64181	-1.78927	.57957	-.59432	-1.37715	.29821	.59432	.30652	0.00000	0.00000	
19.50000	41.88316	.63422	.58861	-1.92965	.61628	-.63422	-1.28278	.32161	.63422	.48515	0.00000	0.00000	
21.00000	45.78564	.66914	.53517	-2.07341	.64854	-.66914	-1.18552	.34557	.66914	.66339	0.00000	0.00000	
22.50000	49.93964	.70147	.48334	-2.22581	.67505	-.70147	-1.08668	.37083	.70147	.84318	0.00000	0.00000	
24.00000	54.43660	.73193	.43612	-2.39247	.69317	-.73193	-.98768	.39874	.73193	1.02988	0.00000	0.00000	
25.50000	59.43155	.76379	.39097	-2.59137	.69885	-.76379	-.89882	.43189	.76379	1.22378	0.00000	0.00000	
27.00000	65.22781	.79522	.38366	-2.85666	.67888	-.79522	-.80855	.47611	.79522	1.47338	0.00000	0.00000	
28.50000	72.61488	.82748	.32943	-3.29243	.62029	-1.09748	-.72901	.54874	.82748	1.81551	0.00000	0.00000	
30.00000	89.99770	.87193	.17393	-5.19581	.08013	-1.73193	-.86597	.73193	.87193	3.03889	0.00000	0.00000	
31.50000	0.00000	1.61902	1.00000	-4.85785	1.00179	-1.61902	-.56888	.00951	1.61902	2.92721	-68.83713	-34.41856	
33.00000	0.00000	1.37015	1.00000	-4.11844	1.45887	-1.37015	-.27699	.68507	1.37015	2.62235	-104.01684	-52.48842	
34.50000	0.00000	1.08151	1.00000	-3.24452	1.80618	-1.08151	-.07170	.54875	1.08151	2.14623	-138.94235	-65.47118	
36.00000	0.00000	.81991	1.00000	-2.45973	1.63982	-.81991	.05088	.48995	.81991	1.67975	-149.09913	-74.54956	
37.50000	0.00000	.68408	1.00000	-1.81225	1.62182	-.68408	.11891	.38284	.68408	1.27388	-161.36988	-88.68498	
39.00000	0.00000	.43844	1.00000	-1.29131	1.50235	-.43844	.12866	.21522	.43844	.93817	-169.58188	-84.75854	
40.50000	0.00000	.29027	1.00000	-.87882	1.53814	-.29027	.11860	.14514	.29027	.64157	-174.72988	-87.36498	
42.00000	0.00000	.17562	1.00000	-.52646	1.49422	-.17562	.09888	.08781	.17562	.39612	-177.88469	-88.94234	
43.50000	0.00000	.08835	1.00000	-.24185	1.45273	-.08835	.04928	.04018	.08835	.18459	-179.51745	-89.57872	
45.00000	0.00000	0.00000	1.00000	-0.00000	1.41421	-0.00000	0.00000	0.00000	0.00000	-0.00000	-188.80888	-90.88888	
46.50000	0.00000	-.06865	1.00000	.28594	1.37859	-.06865	-.05475	-.03432	-.06865	-.16279	-179.58774	-89.79367	
48.00000	0.00000	-.12791	1.00000	.38372	1.34551	-.12791	-.11385	-.08395	-.12791	-.38754	-178.45942	-89.22971	
49.50000	0.00000	-.17947	1.00000	.53842	1.31456	-.17947	-.17340	-.08974	-.17947	-.43788	-176.74228	-88.37118	
51.00000	0.00000	-.22458	1.00000	.67375	1.28529	-.22458	-.23462	-.11229	-.22458	-.55328	-174.52772	-87.26386	
52.50000	0.00000	-.26415	1.00000	.79246	1.25731	-.26415	-.29574	-.13288	-.26415	-.65762	-171.88248	-85.94124	
54.00000	0.00000	-.29886	1.00000	.89659	1.23023	-.29886	-.35594	-.14943	-.29886	-.75132	-168.85482	-84.42741	
55.50000	0.00000	-.32922	1.00000	.98766	1.20368	-.32922	-.41448	-.16461	-.32922	-.83518	-165.47953	-82.73977	
57.00000	0.00000	-.35559	1.00000	1.06678	1.17732	-.35559	-.47868	-.17788	-.35559	-.90988	-161.78115	-80.89857	
58.50000	0.00000	-.37825	1.00000	1.13474	1.15084	-.37825	-.52391	-.18912	-.37825	-.97481	-157.77616	-78.88888	
60.00000	0.00000	-.39736	1.00000	1.19208	1.12398	-.39736	-.56735	-.19868	-.39736	-1.03127	-153.47465	-76.73732	
61.50000	0.00000	-.41305	1.00000	1.23914	1.09619	-.41385	-.61097	-.20652	-.41385	-1.07892	-148.88149	-74.44874	
63.00000	0.00000	-.42536	1.00000	1.27687	1.06739	-.42536	-.65968	-.21268	-.42536	-1.11770	-143.99719	-71.99868	
64.50000	0.00000	-.43438	1.00000	1.30293	1.03715	-.43438	-.69481	-.21715	-.43438	-1.14745	-138.81864	-69.40932	
66.00000	0.00000	-.43983	1.00000	1.31953	1.00514	-.43983	-.72397	-.21992	-.43983	-1.16792	-133.33968	-66.66984	
67.50000	0.00000	-.44188	1.00000	1.32565	.97898	-.44188	-.74645	-.22094	-.44188	-1.17878	-127.55168	-63.77580	
69.00000	0.00000	-.44034	1.00000	1.32185	.93431	-.44034	-.76161	-.22017	-.44034	-1.17961	-121.44379	-60.72198	
70.50000	0.00000	-.43507	1.00000	1.30521	.89473	-.43587	-.76888	-.21754	-.43587	-1.16997	-115.88428	-57.58214	
72.00000	0.00000	-.42592	1.00000	1.27776	.85184	-.42592	-.76738	-.21296	-.42592	-1.14936	-110.22949	-54.11825	
73.50000	0.00000	-.41272	1.00000	1.23817	.80523	-.41272	-.75673	-.20636	-.41272	-1.11726	-101.88818	-50.54889	
75.00000	0.00000	-.39532	1.00000	1.18597	.75851	-.39532	-.73629	-.19766	-.39532	-1.07328	-93.57253	-46.78627	
76.50000	0.00000	-.37358	1.00000	1.12875	.69934	-.37358	-.70559	-.18679	-.37358	-.981674	-85.68958	-42.84475	
78.00000	0.00000	-.34748	1.00000	1.04221	.63948	-.34748	-.66427	-.17378	-.34748	-.94768	-77.42748	-38.71378	
79.50000	0.00000	-.31675	1.00000	.95026	.57451	-.31675	-.61220	-.15838	-.31675	-.86568	-68.78874	-34.39437	
81.00000	0.00000	-.28169	1.00000	.84588	.50458	-.28169	-.54947	-.14885	-.28169	-.77115	-59.78868	-29.89288	
82.50000	0.00000	-.24241	1.00000	.72724	.42972	-.24241	-.47652	-.12121	-.24241	-.66455	-50.43348	-25.21674	
84.00000	0.00000	-.19925	1.00000	.59774	.35023	-.19925	-.39412	-.09962	-.19925	-.54683	-40.78964	-20.38432	
85.50000	0.00000	-.15269	1.00000	.45886	.26665	-.15269	-.30349	-.07634	-.15269	-.41941	-30.83287	-15.41644	
87.00000	0.00000	-.10340	1.00000	.31819	.17975	-.10340	-.20623	-.05170	-.10340	-.28428	-20.68129	-10.34864	
88.50000	0.00000	-.05219	1.00000	.15658	.09848	-.05219	-.10431	-.02618	-.05219	-.14351	-10.37928	-5.18964	
90.00000	0.00000	0.00000	1.00000	.00000	.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	

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