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DESIGN ANALYSIS AND PERFORMANCE EVALUATION OF A TWO-DIMENSIONAL CAMERA FOR ACCELERATED POSITRON-EMITTER BEAM INJECTION BY COMPUTER SIMULATION

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ABSTRACT

The characteristics and design of a high-accuracy and high-sensitivity 2-dimensional camera for the measurement of the end-point of the trajectory of accelerated heavy ion beams of positron emitter isotopes are described. Computer simulation methods have been used in order to insure that the design would meet the demanding criteria of ability to obtain the location of the centroid of a point source in the X-Y plane with errors smaller than 1 mm, with an activity of 100 nCi, in a counting time of 5 sec or less. A computer program which can be developed into a 'general purpose' analysis tool for a large number of positron emitter camera configurations is described in its essential parts. The validation of basic simulation results with simple measurements is reported, and the use of the program to generate simulated images which include important second order effects due to detector material, geometry, septa, etc. is demonstrated. Comparison between simulated images and initial results with the completed instrument shows that the desired specifications have been met.

INTRODUCTION

The use of accelerated heavy charged particles for the treatment of malignant tumors is currently being investigated by groups in Europe, Japan and North America. The rationale for using this type of radiation therapy is based on possible therapeutic gains to be achieved as a result of an improved depth-dose distribution and an enhanced biological response when compared to x-rays or electron radiation treatments.^{1,2} The excellent depth-dose distribution of heavy charged particle beams is a result of their well-defined range in matter, and their flat distribution of dose with depth until near the end of their

Fig. 1 Bragg curve (energy deposition curve) for Ne-20 beam showing a very sharp region where cell killing power is high during cancer therapy.

range where there is a market increase in dose (the Bragg peak) (Fig. 1) At the Lawrence Berkeley Laboratory, a large number of clinical trials are underway with helium, neon and silicon beams.

Because of the characteristic sharpness of the Bragg peak, small errors in its placement on a tumor volume could result in very high doses to healthy, sensitive surrounding tissues and in underdose to the target. It becomes imperative, then, to have an independent and safe method to ascertain whether the treatment planning (usually based on X-ray CT scans) is correct before the heavy radiation doses are delivered to a patient.

For this purpose, the fragmentation of accelerated heavy ions, which can convert some stable nuclei into radioactive positron emitters by the loss of one neutron (like Ne-20 to Ne-19) is being exploited at the BEVALAC accelerator³. The energetic positron emitting particle beam can be used as a probing beam before therapy. The annihilation gamma rays emitted after the beam stops can be used for generating an image of the end point of the trajectory for comparison with a known tumor location.

In 1979 we reported on the design of an imaging instrument for the radioactive beam injection method⁴ which was capable of obtaining the end point of a trajectory along the beam line, with the high sensitivity and accuracy needed for the measurement. That instrument has been used with limited success in the intervening time to carry out physical and related medical measurements at the BEVALAC⁵. In preparation for further development of the technique for application to patients, and based on our experiences, it has become necessary to develop a new instrument for imaging of a positron emitter distribution on a plane, with the ability to localize the radioactive end point of a trajectory in two dimensions with accuracies of better than one millimeter (centroid of a distribution), within a measurement time of less than 5 seconds with an injected radioactivity of the order to 100 nanci. The system resolution should also be as high as possible, consistent with a maximum acceptable cost.

This paper will present a discussion of the basic principles used in the design of a camera meeting the indicated demands of accuracy and sensitivity, will describe the computer simulation method used to develop an appropriate camera configuration, its experimental validation, the results of imaging experiments by computer simulation and their comparison with actual camera measurements.

Basic Principles

The problem of imaging a plane of positron emitting activity is very well known, and it has been solved quite successfully by PET detector rings of a variety of configurations. The problem at hand is,

however, significantly different in nature: the injected activity is very small (of the order of 100 nanoCi in volume elements of approximately 1 cm^3), and it can be considered to be limited to a well-defined single plane slice by the beam injection process, without over or underlying activity in other planes. Furthermore, it can be assumed that the activity injected into tissue does not wash out appreciably during the imaging time of a few seconds.

For that specific case, it has been shown⁶ that the image reconstruction noise propagation factor is fundamentally better by at least one order of magnitude for a planar camera configuration like that of Fig. 2 than it would be for a transverse detector structure. The main reason for the difference is that gamma-rays travelling from a given pixel to a transverse detector have to traverse other pixels, giving rise to an ambiguity which manifests itself in a bad condition number (CN, largest/smallest eigenvalue) of the blurring matrix which defines the transverse detector. The CN improves considerably, however, by inclusion of time-of-flight (TOF) measurements⁶. If the location of the imaging plane is well defined, that ambiguity does not exist in the planar camera, resulting in blurring matrices of excellent condition number. The problem is equivalent to a PET reconstruction in which exact time of flight information is given a priori and only detector overlap effects have to be considered in the image reconstruction process. For the planar camera case, assuming perfect detectors and pixels with uniform activity it is possible to reconstruct an activity distribution with uncorrelated pixel values, and the excellent noise conditions indicated above, for pixel side dimensions approaching $d/2$ (the sampling limit), where d is the detector side dimension.

Fig. 2 Schematic diagram of two planes of perfect detectors of side dimension d looking at an image plane equidistant from the detector planes with pixels of side dimension s , where s can be smaller than d .

For cameras with blurring matrices of low CN, the pseudo-inverse method of image reconstruction is quite feasible yielding a least-squares solution of minimum norm⁷. For its implementation we require:

- 1) obtaining the system (backprojection) matrix A of the detector and pixel configuration,
- 2) obtaining the symmetric blurring matrix $A' = A^T A$,
- 3) finding the eigenvalues and eigenvectors of A' , and
- 4) obtaining the pseudo-inverse B of A' using the eigen analysis.

The image reconstruction process, after obtaining a vector k of coincidences from a measurement, consists first in carrying out a backprojection

$$k' = A k \quad (1)$$

followed by filtering by the pseudo inverse

$$x = B k \quad (2)$$

where x is the final image. The process is discussed in detail in Ref. 6. This method does not require an invariant point response function, and it is, therefore, possible to accept all possible coincidences without restriction, thereby increasing sensitivity.

The system matrix A for any specific physical arrangement of detectors and imaging region contains a complete description of the imaging capabilities of the camera. The higher the condition number of A' , the worse the camera will behave in the presence of statistical fluctuations in the data. The design of a camera for high sensitivity can then be approached by defining the physical parameters of a prototype and using computer simulation techniques to obtain its system matrix. Once some degree of optimization has been obtained by observation of the eigenvalues, one can further simulate the complete image acquisition and reconstruction and see whether the design objectives are being met.

We have followed that approach in the design of PEBA 11 (Positron Emitter Beam Analyzer). We have focused our attention to planar cameras consisting of two 8×8 detector arrays, with pixels assumed to contain all their activity in their centers, which we call "system points". The total number of detectors (128) was felt would be the maximum that we could plan on using within the budgetary constraints. Bismuth Germanate (BiGeO) was chosen for the scintillator detector material because of its high photoneak efficiency (77.8 for 511 keV gamma-rays incident normally at the center of a 3 cm long crystal of 1.25×1.25 cm cross section vs. 32.3 for a NaI detector of 5 cm length and similar cross section, as calculated by a modified Monte-Carlo simulation program originally written by S. Derezio⁸.

Fig. 3 Program structure for the computer simulation program to calculate system matrices for a general camera. Present implementation of the two sections at right and left are for multiplane cameras of the PEBA II type.

Computer Simulation

The process of obtaining the system matrix for a given camera configuration and an array of system points is quite simple in principle. A source of coincidence gamma rays of unity strength is placed at system point number 1, and the camera is allowed to count for a fixed period of time. The resulting vector of coincidences forms the first column of the system matrix A . The operation is repeated for all the system points, the columns placed side-by-side and the system matrix is complete. For a simulation, it is therefore necessary to calculate the response of the camera to a point source located arbitrarily in the imaging space, with sufficient sophistication to include all the physical phenomena that can affect the imaging characteristics of the instrument, as are the photoelectric and Compton response of the detectors to gamma-rays incident at different points of the detector surface and at different angles, effects of Compton scattering into neighboring detectors for both of the coincidence gamma-rays, effects due to septa separating the detectors, effects due to energy resolution and energy discriminator thresholds, etc. A computer code of such a magnitude should be usable in a variety of detector geometries with minimum new coding, and require a minimum of Monte Carlo calculations in order to obtain a reasonable computing speed. Hopefully the matrix obtained is sufficiently good to be useful not only for design analysis, but also for obtaining the pseudo-inverses to be used during image

reconstruction with the final instrument. We have generated one such code which will now be described briefly. The program structure is shown in Fig. 3.

On the right of Fig. 3 we show program APXTAL which carries out the only Monte Carlo calculation of the simulation. The detection efficiency of an array of 9 detectors arranged in a configuration which mimics a segment of the instrument is calculated for gamma-rays incident with angles θ and ϕ on a grid of entry points, as shown in Fig. 4 for a gamma-ray incident at $x=y=0$ in a quadrangular arrangement of detectors. Entry points are confined to a quadrant of the center detector and symmetries are exploited later in the main calculation.

For an initial energy of 511 keV the detected energy spectra are convolved with a Gaussian to simulate the effects of finite resolution of a true detector system and a list of efficiencies is generated for a number of detector energy thresholds.

Program FLCVRT, also at the right of Fig. 3, extracts a table of efficiency results for a given energy threshold from the general results of APXTAL and prepares a file that can be loaded into an Array Processor (AP) for a 4-dimensional linear interpolation in x , y , θ and ϕ at the request of the main MATRIX program. The results of the programs on the right of Fig. 3 are then specific to the dimensions, material and packing arrangement of the detectors.

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Fig. 4. Representative 3 x 3 group of detectors for the calculation of detection probabilities as a function of entrance coordinates (x,y) and incidence angles θ and ϕ . Present implementation is for initial energy of 511 keV.

At the left hand of Fig. 3 there are programs CMIDEF and CMIPRT which are specific to a general camera configuration: multiple planes, single ring, multiple rings, etc. At present we have coded only the multiple plane camera. The result of these programs is a listing of the positions and orientations of each detector in space, which detectors are in coincidence with which, and information about the nearest neighbors to each detector, all in a pre-scribed format acceptable to the main program. This information can also be loaded into the AP when required.

The main body of the calculation is shown in the center of Fig. 3 and is independent of camera configuration, detector parameters, etc. It takes the information furnished by the camera dependent programs described above, and it computes the vectors of responses of the complete camera to one unit of activity placed successively at a set of arbitrary points in the imaging space. A detailed description of the MATRIX programs would be too lengthy for presentation here. The resulting vector (columns of the system matrix A) contain the average number of coincidence counts that the camera would register in response to a source of 1 uCi counted during one second. The accuracy with which the vector is calculated depends on the number of gamma-rays that were traced in the initial Monte Carlo calculation and on the adequacy of the physical model selected as input to that same calculation.

The program runs on a 128K PDP-11/34 fitted with a 32K Floating Point Systems AP-120B Array Processor. For PEBA II, with two 64-detector planes, the main MATRIX calculation runs at an average rate of 6.6 minutes for one system point. It should be pointed out that the 11/34 would not be able to handle larger detector systems with any degree of efficiency.

Validation of the camera simulation

The most important characteristics which we have been interested in validating are the absolute efficiency of the instrument and spatial response of the detectors. We have checked these two parameters by simulating a camera with one single detector per bank and calculating the absolute intrinsic response of the instrument as a point source is made to sweep between the two detectors. For the corresponding measurements we have chosen an arrangement of 3 detectors at each side, in the geometry described in Fig. 5, each of dimensions 1.25 x 1.25 x 3 cm with the center detector of each group coupled to a Hamamatsu R647-04 phototube. The two detector groups were separated by 30 cm. This arrangement was used also for evaluating experimentally the effect of septa of different materials on the detector response to a point source. A substantial number of relative measurements were also made with finely collimated annihilation gamma-rays in order to confirm the results of the APXIAL Monte Carlo calculations of efficiency versus entrance point and angle.

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Fig. 5. Schematic representation of the set-up to measure intrinsic resolution and validate the calculations of absolute detection efficiency (not to scale).

Fig. 6 shows the results of the absolute calculation of intrinsic detector efficiency in coincidence counts per sec-uCi as a function of a hypothetical point source position. Detector energy, resolution was assumed to be 20 FWHM, and the threshold was placed at 375 keV, just below the photopeak. Shown is also the measured response from the central elements of the 3 x 3 test set. No septa were used between crystals. The broadening of the skirts of the measured response can easily be accounted for by the range of the positrons emitted by the Na-22 source used, which in plastic makes a point source appear more like a triangular distribution with a base of 0.3 cm approximately, in one dimension. The approximately 17% discrepancy in the absolute coincidence efficiency, (8% in a single detector) has been found to be due to the difference between a theoretical sharp threshold at 375 keV and a "soft" threshold in

the experiment due to high frequency noise of the wide band input amplifier and photoelectron statistics. Centering the threshold at approximately 375 keV results in some photopeak count losses.

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Fig. 6. Absolute detection efficiency of BiGeO detectors of $1.25 \times 1.25 \times 3$ cm for 511 keV coincidence gamma-rays for the set-up of Fig. 5 with detector group separation of 30 cm. The results of the computer simulation are compared to measurement. An intrinsic resolution of approximately 5.3 mm FWHM is obtained.

Very substantial agreement was obtained between the calculated single gamma-ray response to a perfectly collimated beam of 511 keV gamma-rays and the measured response to a reasonably well collimated (0.5 x 0.5 cm) source. Fig. 7 shows relative count rate for coincidence gamma-rays entering a $1.25 \times 1.25 \times 3$ cm BiGeO detector at the center of its entrance face at an angle θ ranging between 0 and 45 degrees, $\phi = 0$. Shown are also the photopeak count rate, and the count rate of a directly adjacent detector. Detector center-to-center separation was 1.58 cm, no septa were included.

Sampling and the process of image formation

The quadrangular array of detectors shown in Fig. 2 results in a sampling of the image plane at an interval $\Delta x = d/2$. System matrices generated by placing system points in the image plane with that separation result in good condition numbers, although an attempt at using $\Delta x = d/2$ is disastrous, as the sampling theorem is violated. If only one set of system points is considered in the image plane, as, for example, those labelled "A" in Fig. 8, a continuous line source cannot be identified as being different from a set of point sources placed at the system points. In order to generate a continuous image, it is necessary to use an interpolation method. As an extension of the method that was used in the one-dimension instrument of Ref. 2, we have defined 15 sets of system points whose centers are shown by letters "A" through "P" in Fig. 8. Each set of system points is used to generate an independent system matrix, and the vector of results of a measurement k of Eq. (1) from a fixed

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Fig. 7. Comparisons between measured and calculated detection efficiencies for collimated 511 keV gamma-rays for entrance coordinates $x=y=0$ (Fig. 4), angle θ ranging between 0 and 45 degrees, $\phi=0$. Energy thresholds of approximately 10 keV and 375 keV have been investigated, and both the results for the center crystal and for the adjacent crystal to the right (in Fig. 4) are shown for gamma-rays entering the center crystal.

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Fig. 8. Schematic of interpolation scheme for obtaining continuous images with PEBA II. Centers of 16 sets of independent system points are shown separated by 0.1875 cm.

camera is successively backprojected and filtered by all sixteen matrices. The images obtained are superimposed with the displacements corresponding to the position of the system point array centers. If the image plane is sufficiently sampled by the detector array, this method is equivalent to having a moving detector array which scans the image plane in a raster, but the desired result is accomplished without mechanical motion.

The requirement for frequent sampling cannot be bypassed, however, and analysis of the simulation results shows that the imaging accuracy which is desired for the PEBA II camera can only be obtained if the sampling distance is decreased to $s=d/4$ from $s=d/2$. This requirement can also be met without mechanical motion if the detector array is configured as shown in Fig. 9. A simple geometrical construct shows that the detector centers are now sampling the imaging plane at the desired higher rate, except at the extreme periphery of the plane. Simulation results show that the efficiency and accuracy of this fixed arrangement is identical to that of a camera in which quadrangular detector arrays would be moving so as to result in identical sampling.

Effect of septa

The presence of high atomic number septa between detectors can be simulated in the APXTAL Monte Carlo program. It is found that tungsten septa reduce considerably the number of false events caused by Compton scattering between crystals. The importance of these events has to be put in the proper perspective by considering (a) that the high photofraction of BiGeO allows setting a high threshold (approximately 375 keV) for the event discriminators and meet the sensitivity requirements set for PEBA II, and (b) with

BiGeO DETECTOR ARRAYS FOR PEBA II

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Fig. 9 Schematic of the method utilized for sampling the image plane at an interval of $s=d/4$ (where d is detector center-to-center spacing) in two dimensions for a stationary PEBA II camera.

that high threshold, relatively few events can scatter from one detector and deposit enough energy in a second one to trigger the discriminator. Also, the camera will be used in practice with a human absorber between the radiation source and the detectors, and the Compton scattering effects in the human can be expected to introduce more errors than the detector-to-detector effects. In order to ascertain the relative importance of the above considerations, a number of measurements were carried out. The set-up of Fig. 5 was used with two blocks of lucite plastic each of 10 cm thickness between the source and the detectors. It was found that the presence of the absorber reduced the photopeak coincidence count rate by a factor of 3. Curves like that of Fig. 6 were plotted for a number combinations of energy threshold and septum materials and the resulting conclusion was that with the high threshold and the absorber, septum material made no significant effect on the detector response curve. The measured intrinsic full-width resolution at 1/10th maximum increased only from 12.5 to 13.5 mm when the absorber was used. For that reason, simple aluminum separators were designed for the actual camera.

Simulated imaging results

The sixteen matrices and pseudo inverses for the camera described in Fig. 9 were obtained from the simulation programs. Separation between detector planes was taken to be 25 cm. Matrix condition numbers were found to be between 10.35 and 11.5, which were expected to be very favorable. The simulation program MATRIX was also used to generate single vectors k (Eq. 1) for two kinds of images: a 3×3 array of points separated by 1.25 cm and a line of length 5.66 cm formed by 25 points. Statistical fluctuations were introduced to each of the coincidence entries of k to correspond to some desired activity of the sources and counting time.

Fig. 10 (a) Simulation image of a 9-point array of sources with a strength of 90 microCi each, counted during 1 second. Separation between sources is 1.25 cm in both directions. The grid of points shown corresponds to set "A" of system points (Fig. 8), with a separation of 0.75 cm in both directions.

Fig. 10 (a) shows the 9-point array for source strengths of 90 microCi each counted during one second, while Fig. 10 (b) shows one typical image of the same array for a source strength of 150 nanoCi, one second counting.

Fig. 10 (b) Simulation for a strength of 150 nanoCi counted during 1 second.

Figures 11 (a) and (b) show the line source with a strength of 440 microCi per cm and 440 nanoCi per cm, one second count time. The points appearing in the image plane, separated by 0.75 cm, correspond to the system points of the "A" matrix of Fig. 8. The results of this simulation show that there is some structure in all the images which is particularly visible with good statistics. This is due to a limited sampling of the image plane, carried out at $s=d/4$ by the stationary design of Fig. 9. A simulation with sampling at $s=d/3$ results in better images, but careful observation of Fig. 10(b) and a number of similar ones indicates that the design objectives of positional accuracy and sensitivity of PEBA II can be met at $s=d/4$ and a more complicated design has not been considered at this time.

Initial experimental images

PEBA II has been constructed following the design criteria indicated above. Details of the instrument design and a careful evaluation of its capabilities will be published in the near future. At this time, it is appropriate to show some preliminary results obtained by imaging a point source in plastic and a 0.8 cm. diameter line source enclosed in Pb foil. Detector groups were separated by 34 cm for these experiments, energy thresholds set somewhat below 375 keV, no substantial absorbers were used.

Fig. 12 (a) shows the response of the camera to a 1.57 microCi point source positioned near the center of the field, counted during 100 seconds. Fig. 12 (b) shows the same source counted during 0.1 second (approximately 100 counts in the image). A correlation analysis between the 100 second image and sixteen 0.1 second images shows a standard deviation in

Fig. 11 (a) Simulation of a line source with a strength of 440 microCi per cm counted during one second. Some structure due to limited sampling is observable.

Fig. 11 (b) Ditto for 440 nanoCi per cm, counted during 1 second. Statistical fluctuations obscure sampling structure.

position of a 0.4 mm radially for the 0.1 second images. A similar analysis carried out for a point source placed at a radius of 2.1 cm from the center of the image plane shows a standard deviation of 0.77 mm. Fig. 13 shows a continuous image of the line source obtained with 120,000 counts, in which any structure due to undersampling is obscured by statistical fluctuations and threshold non-uniformities.

Fig. 12 (a) Measured response of the PEBA II camera to a source of 1.67 microCi in plastic, counted during approximately 100 seconds.

Fig. 12 (b) Sample image of the same source counted during 0.1 seconds. See text for results of statistical analysis.

DISCUSSION AND CONCLUSION

The design of a new complex instrument necessarily requires the acceptance of a number of trade-offs and compromises. It is often possible to examine some of the important conditions governing the necessary decisions by carrying out simple bench experiments. However, it is felt that there are a number of questions in positron emission tomography in general, and there were in regard to the design of our instrument, which could only be answered by either constructing a substantial part of the instrument, or by a reliable computer simulation which would include some important second order effects. We have shown that it is possible to write a computer code of sufficient sophistication that can be useful in the design of a new instrument of rather demanding characteristics.

Fig. 13 Response of PEBA II to a line source of 0.8 cm, encased in Pb foil. Image contains 120,000 counts. Some structure is observable with irregularities mainly at the camera edges, where detection efficiency is low. Dark peripheral areas correspond to negative values in the reconstruction results.

The basic MATRIX program which we have developed can be adapted to run on bigger computers than the PDP-11/34, with or without an array processor, and we hope that the concept can be useful to the PET community in developing a "computer aided" method of camera design. In the case of PEBA II, we have shown that the design criteria developed by simulation analysis have resulted in an instrument which will allow us to locate the centroid of a weak point source in air, with an activity of less than 200 nanoCi in 0.1 seconds, with expected errors smaller than 1 mm. For a more realistic situation, when radioactive heavy ions will be injected into a patient, we have found PEBA II capable of locating a 0.15 μ Ci Na-22 source in a 19 cm lucite cylinder to a radial standard deviation of 0.8 mm with a counting time of 5 seconds.

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