RF SYSTEM CONSIDERATIONS FOR A LARGE HADRON COLLIDER

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I. Introduction

The radio-frequency (rf) system of a large hadron collider is required to operate in three different modes. It must capture and accumulate in stationary buckets (closed trajectories in longitudinal phase space) successive groups of bunches from its injector; accelerate these bunches in moving buckets up to the design energy; and finally store them, for several hours, while maintaining a minimum ratio of bunch-to-bucket area. In the storage mode it provides the nominal energy gain per turn required to make up for synchrotron radiation losses and for the power loss due to voltages induced in the impedances presented to the beam by the vacuum chamber and accelerating structures.

Each of these modes requires its own set of rf system parameters, i.e. frequency, total gap voltage, power, etc. For conventional machines such as the NAL main ring or the CERN SPS the voltage and power requirements were determined by the acceleration mode since beam storage was not considered in their design. The choice of frequency (53-MHz standing-wave cavities at NAL⁽¹⁾ and 200-MHz traveling-wave structure at CERN)⁽²⁾ was based primarily on rf engineering considerations. For a large hadron collider we shall discover that the storage mode requirements can be the determining factors in the choice of peak voltage and frequency.

In addition to the acceleration rate there are many other machine parameters that can impinge on the choice of voltage and frequency: viz. injection energy, transition energy (but not in a collider since $\gamma_i > \gamma_{tr}$), final energy, initial and final longitudinal phase-space areas of the bunches, number of bunches, the momentum spread at any time, and the bunch length in the storage mode. Another machine parameter that is crucial in determining the design of the rf system is the amount of beam to be accelerated and stored. The total rf power needed and the impedance that the accelerating structures and their drivers are allowed to present to the beam are functions of the beam current. The values of these parameters can also affect the design requirements of other parts of the rf system such as the amount of rf "noise" permitted in the phase and frequency

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control loops. Along with the voltage and frequency, the impedance and power requirements are the primary factors in determining the design of a suitable rf system.

In what follows then, we shall discuss how we arrive at a particular choice of voltage and frequency; the type of acceleration structure that would be suitable for obtaining the required voltage and resonant impedance; static beam loading including a simplified beam stability criterion involving the beam current and total rf system shunt impedance; the basic principle of rf phase and frequency control loops; and the effect of rf noise and its interaction with these loops. Finally, we shall consider the need for and design of rf systems to damp independently coherent oscillations of individual bunches or groups of bunches.

II. Choice of Voltage and Frequency

In this section we shall use the formulas describing synchrotron motion given in reference 3, which are summarized in figure 2.1. The closed trajectories in the $\phi, \Delta W$ phase space are characterized by the parameter N . On the boundary of the stable region $N = 1$ and within the stable region or bucket $N < 1$. The phase-space area (in eV sec) enclosed by such a trajectory is related to the area of a stationary bucket by the factor $\alpha_N(\phi_S)$ which, for ϕ_S (the stable phase angle) equal to $0, \pi$, can be written in terms of the elliptic functions F and E . Its magnitudes for several values of ϕ_S and of N , in steps of 0.1, are tabulated in reference 3. We shall also use the expression $F(\phi_S) = |\cos\phi_S - (\pi/2 - \phi_S) \sin\phi_S|^{1/2}$ and recall that $\eta = (1/\gamma_{tr}^2 - 1/\gamma^2)$ where $\gamma_{tr} E_0$ is the transition energy. In the equations of motion ΔE is the energy deviation from that of the synchronous particle of energy E_S and rotation frequency ω_S on an orbit whose circumference is C_S . The energy gain per turn is given by

$$\delta E = eV \sin\phi_S = eV\Gamma(\phi_S) \quad (2.1a)$$

where V is the total peak rf voltage. The net energy gain per turn is related to the rate of change of magnetic field on the orbit C_S by

$$\delta E_{net} = e\dot{B}_S \rho C_S \quad (2.1b)$$

where ρ is the radius of curvature in the magnets. On the average every particle within a bunch gains the same energy as the phase stable particle. We include

Figure 2.1 Equations for synchrotron motion from reference 3.

the possibility that particles will lose energy on average due to synchrotron radiation and to interaction with longitudinal impedance presented by the vacuum chamber, bellows rf cavities, etc. Hence in general $\delta E \geq \delta E_{\text{net}}$. It should also be noted that for $\dot{B} = 0$ net acceleration (or deacceleration) can, of course, take place with a corresponding change in C_s .

First let us consider the rf voltage requirements at injection, where $\phi_s = \pi$ (we assume $\gamma_{\text{inj}} > \gamma_{\text{tr}}$ and no energy loss due to synchrotron radiation). We can write for the height of a stationary bucket [since $F(\pi) = F(0) = 1$]

$$\Delta E_B = h\beta\omega_0 \frac{A_1(\pi)}{8} \quad (2.2)$$

where $h\omega_s$ is the rf frequency with $\omega_s = \beta\omega_0$ ($\omega_0 = 2\pi c/C_s$). The height of a bunch is given by

$$\Delta E_b = \sqrt{N} E_B \quad (2.3)$$

and the bunch area by

$$A_b = \alpha_N(\pi) A_1(\pi) = \frac{\alpha_N}{\sqrt{N}} \frac{8}{h\omega_0} \left(\frac{\Delta E_b}{\beta} \right). \quad (2.4)$$

By specifying N we determine how much of the bucket is filled by the bunch. The total bunch area is then

$$hA_b = \frac{\alpha_N(\pi)}{\sqrt{N}} \frac{8}{\omega_0} \left(\frac{\Delta E_b}{\beta} \right) \quad (2.5)$$

so that if we also specify $(\Delta E_b/\beta)$ then hA_b is determined. The voltage required for a given value of N and energy spread ΔE_b can be written as

$$V = \left(\frac{\Delta E_b}{\beta} \right)^2 \frac{\pi h |\eta|}{2eNE_s} = \left(\frac{h\omega_s A_b}{8\beta\alpha_N(\pi)} \right)^2 \frac{\pi h |\eta|}{2eE_s}. \quad (2.6)$$

In the case of a large hadron collider at injection, the minimum energy spread will most likely be determined by what value is necessary to stabilize the beam against the transverse single-bunch mode coupling instability.⁽⁴⁾ The maximum permissible energy spread will depend on the linear aperture⁽⁴⁾ of the machine. (However, the maximum tolerable injection energy error, which will depend on the rf bucket height and hence peak voltage, could include the "dynamic" aperture, if this error is damped rapidly enough to prevent longitudinal dilution or transverse smear.⁽⁴⁾ The design of such a system is described in a later section.) A suitable choice for N would be such that the synchrotron frequency spread within a bunch would be sufficient to suppress longitudinal coupled bunch instabilities. The growth of these instabilities will, in general, be driven by the real part of the impedances of the higher-order longitudinal modes in the rf cavities. However, the threshold for these instabilities will most likely be determined by the imaginary part of the broad-band coupling impedance due to the vacuum chamber wall and the various discontinuities in the chamber including the accelerating cavities. This impedance will cause a beam current dependent real frequency shift that can move the coherent frequency outside the incoherent single-particle frequency band thus suppressing the effect of Landau damping that arises from the synchrotron frequency spread within the bunch. We shall describe the requirements for stability at the end of this section.

Now N should not be too large since any nonlinearity in the presence of rf noise can cause dilution of the bunch area.⁽⁵⁾ However, this is primarily a problem in the storage mode, where beam lifetimes of hundreds of hours are desirable. During injection and acceleration a properly designed control system should not produce significant dilution for N values as large as 0.8. In fact, in order to obtain sufficient bunch area at 20 TEV to ensure that the intrabeam scattering growth rate can be compensated by synchrotron radiation damping, it is proposed in the CDR⁽⁴⁾ to intentionally blow up the bunch area during the entire acceleration cycle. Thus, the choice of N at injection is not critical but clearly it cannot be too close to unity since some room within the bucket must be left for injection errors: viz. for $N = 0.8$ Eq. (2.3) indicates a 10% margin in bucket height and the tables in reference 3 give a bunch width 70% of the bucket width.

In order to determine what happens to the energy spread under acceleration we write Eq. (2.2) for $\phi_s \neq 0$, i.e.

$$\Delta E_B = \beta \frac{\hbar \omega_0 A_B}{8} \frac{F(\phi_S)}{\alpha_1(\phi_S)} = \beta \hbar \frac{\omega_0 A_b}{8} k(\phi_S, N) \frac{F(\phi_S)}{\alpha_1(\phi_S)} \quad (2.2a)$$

where the moving bucket area $A_B = \alpha_1 A_0 = k A_b$ with $A_0 = A_1(0)$ and $k = \alpha_1(\phi_S)/\alpha_N(\phi_S)$ ($k \geq 1$). We then have

$$(\Delta E_b)_{acc} = \frac{\beta \hbar \omega_0 A_b}{8} \sqrt{N} \frac{F(\phi_S)}{\alpha_N(\phi_S)} \quad (2.7)$$

which is just Eq. (2.5) transposed if $\phi_S = 0, \pi$. In figure 2.2 we show $k(\phi_S, N)$ for $N = 0.3, 0.5$ and $F(\phi_S)/\alpha_1(\phi_S)$ as a function of ϕ_S . We note that for $\phi_S \neq 0$, $kF/\alpha_1 = F/\alpha_N$

Figure 2.2
Functions of $F(\phi_S)$
and $K(\phi_S)$ vs. ϕ_S .

is always greater than its value at $\phi_s = 0$. Hence if we specify $(\Delta E_b/\beta = c\Delta p)_{inj}$ and N , then, since this determines hA_b , we see from Eq. (2.7) that if N remains constant $(\Delta E_b/\beta)_{acc}$ will always be greater than its initial value. If instead of fixing N at its initial value we keep k constant, then N would decrease slightly as a function of ϕ_s . However, the factor $\sqrt{N} F(\phi_s)/\alpha_N$ will still be larger than for a stationary bucket for the same value of k .

Thus, if $(\Delta E_b/\beta)$ is to remain constant when acceleration begins, k must decrease or N increase from their injection values. This requirement would then limit the initial choice of N (or k).

The voltage needed to accelerate a bunch of energy spread ΔE_b and a given N value can be obtained by rewriting Eq. (2.7) since $A_b = \alpha_N(\phi_s)A_0$, i.e.

$$V = \left(\frac{\Delta E_b}{\beta} \right)^2 \frac{h\pi|\eta|}{2eNF^2(\phi_s)} . \quad (2.8)$$

We can combine Eqs. (2.1a) and (2.8) to obtain

$$\frac{F^2(\phi_s)}{\sin\phi_s} = \left(\frac{\Delta E_b}{\beta} \right)^2 \frac{\pi h|\eta|}{2EN\delta E} \quad (2.9)$$

which relates the stable phase angle ϕ_s to the harmonic number h for a given acceleration rate δE . A plot of $F^2/\sin\phi_s$ vs. ϕ_s is shown in figure 2.2. The value of ϕ_s for any h and initial values of N , E , δE is put into Eq. (2.1a) to find the required rf voltage V . For fixed δE and N the maximum voltage will be needed at the injection energy.

In order to determine typical voltage requirements for the frequency range of 200 to 500 MHz, we use the following SSC parameters as given in reference 4. Injection energy 1 TeV; $\Delta E_b = \sqrt{6} \cdot 1.75 \times 10^{-4} E_{inj}$; $\eta_{inj} = 2.219 \times 10^{-4}$; $\gamma_{tr} = 67$; $f_0 = 3.614$ kc; acceleration time 10^3 sec or $\delta E = 5.26$ Mev/turn. We choose $N = 0.5$ and note that $\Delta E_b/E_{inj}$ is about one half the $\pm 10^{-3}$ momentum aperture specified for the SSC.⁽⁴⁾ In table I we list the required rf voltage and the stable phase angle ϕ_s for several frequencies

Table I RF Voltage Required for Acceleration vs. Frequency

Frequency (MHz)	Voltage (MV)	ϕ_S	$(hA_b)/(hA_b)_{200}$
200	14.2	$(\pi - 21.8^\circ)$	1
300	18.2	$(\pi - 16.8^\circ)$	1.04
400	21.7	$(\pi - 14^\circ)$	1.14
500	25.5	$(\pi - 11.9^\circ)$	1.19

and note that V vs. f_{rf} is linear over this range. In addition we show the ratio of the total bunch area at a given frequency to the bunch area at 200 MHz (assuming all buckets are filled.) This area is proportional to $\alpha_{0.5}(\phi_S) \sqrt{V/h}$ and, although we have specified ΔE_b and N , the total area is not independent of h since we have added the additional constraint of a fixed acceleration rate. Finally, we note that here the bucket-to-bunch area k also increases with frequency; as shown in figure 2.2, $k(\phi_S, N)$ for fixed N is larger at lower values of ϕ_S .

Next we come to the voltage requirements in the storage mode. As mentioned above, the presence of rf noise in conjunction with the spread in synchrotron frequencies caused by nonlinear motion in the bucket will produce bunch area growth. This will, in turn, result in a reduction of the luminosity lifetime of the collider. Experience with the CERN SPS $\bar{p}p$ collider indicates that in order to ensure that the growth time of this effect is in the hundred-hour range one should have $k \geq 3$ or $N < 0.4$. On the other hand, the requirement that the growth rates for longitudinal and horizontal emittances due to intrabeam scattering be considerably less than the damping rate in these coordinates due to synchrotron radiation, dictates a considerable increase in the bunch area in this storage mode over the permissible areas at the injection energy. In order to determine the growth times for intrabeam scattering one must specify the bunch length, transverse emittance, bunch current, lattice properties, and momentum spread. Thus, if we fix k (or N) then the only free parameter for a given growth time is the momentum spread.

Now calculations done for the SSC show that at fixed N and bunch intensity the required momentum spread for a given growth rate scales as $f_{rf}^{1/2}$.⁽⁴⁾ Again, in order to determine the required rf voltage as a function of frequency we take the SSC parameter from the CDR.⁽⁴⁾ In the storage mode a growth time of 126 hours

for the horizontal emittance is obtained for a momentum spread of $0.48 \sqrt{6} \times 10^{-4}$ at 20 TeV for $f_{rf} = 374.74$ MHz and $N = 0.5$. Recalling Eq. (2.6), which relates the energy spread to the required voltage for a stationary bucket, we obtain $V_{rf} = 20$ MV and $h = 103,680$. Since the required energy spread scales as $h^{1/2}$ while the necessary voltage scales as $h(\Delta E_b)^2$ we obtain a quadratic variation for V vs. f_{rf} . In table II we show the required voltages and $(\Delta E_b/E)$ for $N = 0.5$ and a 126-hour growth time as a function of frequency.

Table II RF Voltage Required in Storage Mode vs. Frequency

Frequency (MHz)	Voltage (MV)	$(\Delta E_b/E) \times 10^4$
200	5.7	$0.35 \sqrt{6}$
300	12.8	$0.43 \sqrt{6}$
374	20	$0.48 \sqrt{6}$
400	22.8	$0.49 \sqrt{6}$
500	35.6	$0.55 \sqrt{6}$

Thus, we find that the maximum rf voltage can be determined by the storage mode requirements rather than those for the acceleration mode. Our choice of CDR parameters of course makes 375 MHz the frequency that separates the ranges over which each operation mode is the determining factor. If we had specified that $N \leq 0.4$ in the storage mode then the dividing line would be at a lower frequency. That is, the required voltage for the same growth time (assuming the same bunch population) would be greater. Again referring to Eq. (2.6), the smaller N value would increase V but also, since the bunch length would be less at a given frequency, the momentum spread would have to be greater to keep the same growth time. On the other hand, if the required growth time were reduced then $(\Delta E/E)$ could be decreased and the relative voltage increase with frequency would not be as great.

Finally, let us examine how specifying ΔE_b in the storage mode affects the bunch area. Since N is determined by rf noise considerations, fixing ΔE_b uniquely determines A_b . Again we use the CDR case as an example and obtain at 20 TeV for $N = 0.5$ and $(\Delta E_b/E) = 1.17 \times 10^{-4}$ from Eq. (2.5) an $A_b = 4.76$ ev sec for the 95% emittance. Returning to the case at the beginning of

acceleration, where $\Delta E_b = \sqrt{6} \cdot 1.75 \times 10^{-4} \times 10^{12}$, $N = 0.5$, and $\phi_s = (\pi - 15.25^\circ)$, we need to find $\alpha_{0.5}(\phi_s)$ since A_0 for 20 MV and $f_{rf} = 374.74$ MHz is 2.53 eV sec. From reference 3 we obtain $\alpha_{0.5}(15.25^\circ) = 0.26$ so that $A_b(1 \text{ TeV}) = 0.66$ eV sec for the 95% emittance. Thus an increase of more than a factor of seven is required prior to or at the beginning of the storage mode. This can be achieved by intentionally introducing rf noise into the beam control loop as will be described later.

Finally, let us describe how the choice of N at injection can affect the threshold for coupled bunch instabilities. In order to estimate the beam current threshold for stability it is necessary to know the longitudinal impedance below the vacuum chamber cutoff frequency $f_c = c/3.42a$ where a is the tube radius. This is usually called $(Z/n)_\parallel$. It is also necessary to know the bunch area and the phase-space distribution. For a Gaussian distribution the coherent frequency shift for the dipole mode due to $(Z/n)_\parallel$ is given by⁽⁵⁾ ($\phi_s = \pi$)

$$\Delta\omega_1 = \omega_{s0} \sqrt{\frac{\pi}{2}} \frac{I_0 I_m (Z/n)_\parallel}{h V_T (\sigma_z/R)^3} \quad (2.10)$$

where I_0 is the current in one bunch, σ_z is the rms bunch length, ω_{s0} is the zero-current small-amplitude synchrotron frequency due to V_{rf} , $V_T = V_{rf} \omega_s^2 / \omega_{s0}^2$, and ω_s is the incoherent synchrotron frequency. Here V_T includes the time independent part of the induced voltage, which is in general a function of the synchrotron amplitude as is ω_s . In Eq. (2.10) $(Z/n)_\parallel$ is assumed to be independent of n , i.e. a pure space-charge or inductive wall impedance, and $\Delta\omega_1$ is the frequency shift from the incoherent frequency at the center of the bunch.

Although the induced voltage produces a frequency shift that varies along the bunch and hence a frequency spread as well, Landau damping and hence stability can be produced only by the nonlinearity of the external focusing voltage V_{rf} . For a Gaussian bunch the stability limit is given by⁽⁵⁾

$$S_\sigma \geq \frac{\Delta\omega_1}{2} \quad (\text{for } \Delta\omega_1 > 0) \quad (2.11)$$

where S_σ is the synchrotron frequency spread at one standard deviation, which for a sinusoidal rf voltage and $\phi_s = \pi$ can be written as

$$S_\sigma = \frac{1}{16} \left(\frac{\sigma_z}{R} \right)^2 h^2 \omega_{s0} \quad (\text{for } N \lesssim 0.3) . \quad (2.12)$$

From Eqs. (2.10), (2.11), and (2.12) we see that the instability threshold [assuming $I_m(Z/n) > 0$] for a given current is proportional to $(\sigma_z/R)^5$. Thus, in general, the limiting current will depend on the injection parameters since for reasons outlined above the bucket-filling factor N will be smallest at this time. We again consider SSC parameters as given in the CDR⁽⁴⁾ and assume a Gaussian bunch with $\sigma_z = 6$ cm. Taking $V_T = V_{rf} = 20 \times 10^6$, $h = 103,680$, $I_0 = 4.2 \mu\text{A}$, $R = 13.21 \times 10^5$ m, and $I_m(Z/n) = 0.2 \Omega$ we get $\Delta\omega_1 = 5.36 \times 10^{-3} \omega_{s0}$. Since the space-charge impedance is negligible at $\gamma \approx 10^3$, $\Delta\omega_1$ will always be positive for the dipole mode and Eq. (2.11) will apply. Using Eq. (2.12) we find $S_\sigma = 13.8 \times 10^{-3} \omega_{s0}$ so that it is easily satisfied for this distribution and the CDR values.

Next let us consider a phase-space distribution of the form $\psi_0(r) = 3\sqrt{(1-r^2)}/2\pi$, which would result in a line charge of the form $\rho_0(z) = 3(1-z^2)/4$. Here r is the synchrotron amplitude in (r, θ) space, $z = 1$ at $\phi = \phi_{\max}$, the bunch edge, and $r - z = 1$ for $\theta = 0, \pi$. It can be shown that the coherent frequency shift $\Delta\omega_1$ for this case is given by⁽⁵⁾ ($\phi_s = \pi$)

$$\Delta\omega_1 = \frac{\omega_{s0} 3 I_0 I_m(Z/n)_\parallel}{2\pi^2 h V_T (L/2\pi R)^3} \quad (2.13)$$

where $L = (2R\phi_{\max}/h)$ is the full bunch length. Now the stability limit for the distribution which produces a parabolic bunch shape is given by⁽⁵⁾

$$S > \frac{3}{2} \Delta\omega_1 > 0 \quad (2.14)$$

where

$$S = \frac{1}{16} (hL/2R)^2 \quad (2.15)$$

is the incoherent frequency spread. From Eq. (2.14) we see that $\Delta\omega_1 > 0$ for stability or $I_m(Z/n) > 0$, which, as pointed out above, is the case for the SSC.

Now if we choose $\phi_{\max}$ such that the bunching factor $B = I_{\text{avg}}/I_{\text{peak}}$ is the same as for a Gaussian bunch⁽⁶⁾ then we must have

$$\frac{L}{3\pi R} = \frac{\sigma_z}{R\sqrt{2\pi}} \quad (2.16)$$

or $L = 3.71\sigma_z$; thus for the SSC, $L = 22.26$ cm. If we put this value into Eq. (2.13) we obtain $\Delta\omega_1 = 3.19 \times 10^{-3}\omega_{S0}$ whereas Eq. (2.15) yields $S = 47.6 \times 10^{-3}\omega_{S0}$ so again stability is predicted. Here the margin is a factor of ≈ 10 whereas for the Gaussian distribution it was ≈ 5 . This indicates that the stability limit is relatively insensitive to the phase-space distribution when the coherent frequency shift is due primarily to the low frequency wide-band inductive impedance expected in a large hadron collider. For the SSC design parameters then, there is potentially some margin available for a larger Z/n , more beam current, or somewhat smaller bunches at injection.

So far we have ignored the effect of the real part of the impedance of the higher-order longitudinal cavity modes on the stability of the bunches. It has been assumed that the effect of these impedances determines only the growth rate of a given mode if the real frequency shift is large enough. However, in any large collider there will be many similar cavities whose modes will overlap so that the total impedance could be quite large. It will therefore be necessary to incorporate into the cavity design means for reducing the Q 's of these modes and if possible to separate somewhat these frequencies from cavity to cavity. If such precautions are only partially successful then the imaginary frequency shift caused by the real part of $Z_{||}$ can lead to instability. In order to anticipate this possibility the SSC CDR as well as the CERN Large Hadron Collider design report⁽⁷⁾ assume that bunch-to-bunch longitudinal feedback systems will be necessary. For this reason we include a discussion of the design of such a system in a later section.

III. Accelerating Structure Characteristics

In order to provide the large rf voltages required to insure long luminosity lifetimes in the storage ring, highly efficient resonant rf structures must be

employed. A measure of the efficiency of a given structure is its shunt impedance⁽⁸⁾ defined as

$$R_S = V^2/2P_c \quad (3.1)$$

where V is the peak voltage appearing across the accelerating gap and P_c is the power dissipated in the structure boundaries. We also define $V_{\text{gap}} = Ve^{j\omega_{rf}}$ where

$$V = \left| \int_{\text{gap}} E(Z) e^{jKZ} dZ \right|. \quad (3.2)$$

And note that in reference 8, the factor of 2 is omitted in the definition of R_S . Another relevant parameter is the characteristic impedance R_c defined as

$$R_c = R_S/Q = V^2/2W\omega_{\text{res}} \quad (3.3)$$

where W is the stored energy. It is a purely geometrical factor and is one parameter that can be optimized in resonator design. We should remark that the above definitions apply to standing-wave structures resonant at a single frequency ω_{res} and that this is the only type we shall consider. In particular we will discuss the coaxial or shorted transmission line resonator and the reentrant or modified pill-box cavity. As will be seen, the former is better suited for the frequency range below 200 MHz and the latter for frequencies above 350 MHz.

III.1. Coaxial Resonator

This consists of two coaxial cylinders of radii r_1, r_2 of length $\ell < \lambda_{rf}/4$ shorted at one end, as shown in figure 3.1. One can define the capacitance, inductance, and resistance per unit length as

$$L = \mu_0 \ln(r_2/r_1)/2\pi; \quad C = 2\pi\epsilon_0/\ln(r_2/r_1); \quad R = R_{\text{surf}}(r_1 + r_2)/2\pi r_1 r_2$$

Figure 3.1 $\lambda/4$ coaxial resonator.

with $R_{\text{suf}} = \sqrt{\mu_0 \pi f / \sigma}$ where σ is the conductivity of the surfaces of the resonator. For the case of a lossless line we have $R_0 = \sqrt{L/C}$ and can write for the voltage and current along the line

$$i(X,t) = i(t) \cos \beta X, \quad V(X,t) = j i(0,t) R_0 \sin \beta X. \quad (3.4)$$

X is measured from the shorted end of the line where $V(0,t) = 0$ and $i(0,t)$ is the current at that point. We have $\beta = 2\pi/\lambda$ with

$$Z_{\text{in}} = V(X,t)/i(X,t) = j R_0 \tan \beta X \quad (3.5)$$

as the impedance at the point X . Now far away from the gap we have a purely transverse electric and magnetic field while in the gap the electric field is essentially longitudinal. The capacity at the open end of the line is augmented by rounding the surface of the inner conductor to produce a corona roll. Since Z_{in} is inductive for $X \leq \lambda/4$ the gap capacity can be made to resonate the structure at the desired frequency if $C_g = 1/\omega_{\text{res}} R_0 \tan \beta X_{\text{res}}$.

Using the expression for a lossless line, Eq. (3.4), one knows the current distribution and can then calculate the losses given the value of R_{suf} . This approach is valid for a high- Q structure.⁽⁹⁾ One obtains

$$P_\ell = \frac{V^2 R (2\beta \ell + \sin 2\beta \ell)}{8 R_0^2 \beta \sin^2 \beta \ell} \quad (3.6)$$

for the power lost in a line of length $X = \ell$. The power lost in the shorted end is given by

$$P_0 = V^2 R_s / 2 R_0 Z_0 \sin^2 \beta \ell \quad (3.7)$$

where $Z_0 = 377 \Omega$ is the impedance of free space. If we ignore the losses due to the wall at the open end and the added surface necessary to obtain C_g we can readily find R_s from Eq. (3.1) with $P_c = P_\ell + P_0$. In particular this would give us one half the shunt impedance for the case of two such resonators joined so that their gaps oppose one another as shown in figure 3.2. Here we have indicated a possible configuration of resonators that would allow minimum beam separation for a two-ring collider.

As an example let us consider a 200-MHz resonator with $r_1 = 17$ mm, $r_2 = 124$ mm which would be suitable for beams spaced 144 mm apart inside 3-cm-diameter vacuum chambers. We find that $R_0 = 119.2 \Omega$ and choose $\beta\ell = 72^\circ$ so that the end gap is 7.5 cm. The required gap capacity is $2.16 \mu\text{f}$ and, using Eqs. (3.1), (3.6), and (3.7), we obtain for copper whose $R_{\text{suf}} = 2.6 \times 10^{-7} \sqrt{f_{\text{rf}}} \Omega$ an $R_s = 1.5 \text{ M}\Omega$. If C_g were increased to $4 \mu\text{f}$ then $R_s \equiv 1.3 \text{ M}\Omega$, and $C_g = 6.67 \mu\text{f}$ yields an $R_s \equiv 1 \text{ M}\Omega$. Now, using Eq. (3.4) and the expression for L , one can calculate the energy stored in the resonator. It is found to be⁽⁹⁾

$$E_s = \frac{V^2 T}{2R_0} \frac{(2\beta\ell + \sin 2\beta\ell)}{8\pi \sin^2 \beta\ell} \quad (3.8)$$

where $T = 1/f_{\text{rf}}$. From this one can obtain the Q of the structure since $Q = 2\pi E_s / P_c T$. We find for the resonator shown in figure 3.2

$$Q = \frac{R_0}{R_{\text{suf}}} \frac{(2\beta\ell + \sin 2\beta\ell)}{[(\lambda/4\pi^2)(1/r_1 + 1/r_2)(2\beta\ell + \sin 2\beta\ell) + 4R_0/Z_0]} \quad (3.9)$$

and obtain $Q = 10,990$ for our example.

Next we assume that one could operate the resonator at 150 kV/gap so that for the ~ 14.2 MV required at 200 MHz for the CDR design⁽⁴⁾ parameters one would need 48 of the units shown in figure 3.2 per ring. The total rf power required just to excite the structures would then be $48 \times (300 \text{ kV})^2 / 2 \times 3 \times 10^6 = 720 \text{ kW}$. When losses due to coupling the resonator to its driver and nonperfect surfaces are included, somewhat more than 800 kW would be needed. Again using the CDR design⁽⁴⁾ value of 7.3×10^9 particles in each of 17,280 buckets

and an acceleration rate corresponding to 5.26 MeV/turn, the power delivered to the beam is $P_b = I_b V \sin \phi_s = 0.073 \text{ A } 5.26 \text{ MV} = 384 \text{ kW}$. Hence the maximum required rf power from the driver would be $\approx 1.2 \text{ MW}$ per ring for such a system.

Figure 3.3 Pill-box cavity.

III.2. Reentrant Cavity Resonator

The basic resonator is a pill-box structure as shown in figure 3.3, which can be considered as a circular waveguide operating at cutoff or as a shorted radial transmission line.⁽¹⁰⁾ One characterizes it as a $TM_{0,1,0}$ resonator where the indices refer to the number of waves in the azimuth, radial, and axial direction. The fields are simply

$$E_z = E_0 J_0(Kr) \cos \omega t, \quad H_\phi = -\frac{E_0}{Z_0} J_1(Kr) \sin \omega t \quad (3.8)$$

where J_0, J_1 are Bessel functions and one can show that

$$R_c = \frac{R_s}{Q} = \frac{967 T^2}{2(\lambda/L)} \quad (3.9)$$

Here $T = \sin \pi(L/\lambda) / (\pi L/\lambda)$ is the transit time factor and

$$Q = \frac{Z_0}{R_{\text{suf}}} \frac{2.405}{2[b/L + 1]}$$

with $K = 2\pi/\lambda = 2.405/b$ or $b = 0.3827 \lambda$. Thus, in the ideal case where a charged particle is sent through a small hole along the axis, the electric field is purely longitudinal while H is purely transverse. If several such cavities were coupled together, then, in order to provide optimum acceleration, L would be chosen equal to $\lambda/2$. Adjacent cavities would be excited 180° out of phase and one would have a so-called π -mode cavity. For such an ideal cavity whose $R_c \cong 98 \Omega$ one finds for copper surfaces at 353 MHz a $Q = 5.26 \times 10^4$ and thus an $R_s = 5.15 M\Omega$.

A practical cavity must have a finite opening and by introducing at the same time small reentrant "nose cones" as shown in figure 3.4 it is possible to increase $R_s/Q = V^2/2 W\omega$. The decrease in gap length increases the transit time factor T where now L is the gap spacing. However, as seen from Eq. (3.2), the contribution to V due to $E(Z)dZ$ will decrease for a given stored energy but overall R_c can be increased. Since the electric field now has a radial component, Eqs. (3.8) to (3.10) no longer apply but of course Eqs. (3.1) to (3.3) remain valid. If the nose cones remain small then the diameter of the cavity need be reduced only slightly to obtain the same resonant frequency as for the ideal pill box. In addition to increasing R_c the value of R_s will also be increased in this case and an optimum can be found by using for example the SUPER FISH program.

In particular,⁽¹¹⁾ a single unit of the five-cell cavity shown in figure 3.4 operating at 353 MHz and fabricated from aluminum with $R_{\text{surf}} = 3.3 \times 10^{-7} \sqrt{f_{\text{rf}}}$ has the following parameters (including coupling effects): $R_s = 4.5 M\Omega$, $Q = 29 \times 10^3$, and $R_c = 150 \Omega$. The corresponding values for a pill-box cavity would be $Q = 41.4 \times 10^3$ and $R_s = 4.06 M\Omega$ with $R_c = 98 \Omega$ as before. One sees that although Q is reduced by the introduction of nose cones there is still a net gain in R_s over the ideal resonator. However, b is still >90% of the pill-box value, and it is the large transverse dimensions required to maximize R_s that make the use of these structures in the frequency range around 400 MHz a potential problem in the design of a two-ring collider. For the cavity shown $b \cong 320$ mm so that the spacing between beams would have to be at least 35 cm in those straight sections where the rf cavities are installed. Such a solution is proposed in the SSC conceptual design where so-called utility straight sections are introduced that separate the beams from the regular cell spacing of 140 mm to a maximum of almost 700 mm.

Figure 3.4 PEP five-cell π -mode cavity.

Figure 3.5

Figure 3.5 CERN LHC Septum cavity design.

If maximization of R_s is given up then significant reduction in the radial extent of the cavity can be achieved by increasing appreciably the length of the nose cones, as shown in figure 3.5. This is a "septum cavity" wherein both vacuum chambers are part of the same structure. Calculations made with SUPER FISH for this cavity by a CERN group⁽¹²⁾ give $R_s = 4 \text{ M}\Omega$ per cell and $Q = 18,900$ for copper at 353 MHz. A further reduction in R_s to $\sim 3.4 \text{ M}\Omega$, i.e. 85% of the calculated value, is to be expected when the effects of coupling to other cells and nonideal surface conditions^(4,12) are taken into account. By comparison the cavity for the CERN LEP e^+e^- collider has $R_s = 5.5 \text{ M}\Omega$ and $Q = 40,000$ at the same frequency, and it is expected that a cavity similar to that shown in figure 3.4 made from copper would have $R_s \sim 5 \text{ M}\Omega$ with $Q \sim 40,000$.⁽⁴⁾ In order to compare this structure with the coaxial resonator we shall assume the latter values for $f_{rf} = 374.7 \text{ MHz}$. Then from table II we find that in the storage mode $V_{rf} \cong 20 \text{ MV}$ so that eight 5-cell cavities operating at 500 kV/gap would be required. The total power needed just to provide the gap voltage is then 1 MW. Although this is $\sim 40\%$ more than that required for the 200-MHz case, the efficient generation and transmission of rf power at this frequency has already been established for the SLAC PEP storage ring and the CERN LEP ring.^(11,12) Thus, even if one were to consider the low- R_s structure which would require $\sim 1.5 \text{ MW}$ of excitation power, this, along with the $\sim 400 \text{ kW}$ needed for acceleration, is still a fraction of the 6 MW needed in PEP. Finally, we note that the structure in figure 3.5 is quite similar to that shown in figure 3.2. The primary difference of course is the manner in which the fields are excited for the principal resonant mode. Coupling is always introduced at a point well away from the gap so that an impedance and a voltage step up are realized from the driving source. Thus, the impedance that the beam sees in passing the gap is R_s (assuming $\omega_{res} = \omega_{rf}$) in parallel with the transformed source impedance. In calculating the total rf power at f_{rf} that must be supplied, one must add $V^2/2Z_s$, where Z_s is the transformed driver impedance, to $(P_c + P_b)$.

IV. Static Beam Loading and RF Power Requirements

In the previous section we described a system of coupled " π "-mode cavities whose total shunt impedance at resonance could be as high as $200 \text{ M}\Omega$. For the SSC conceptual design beam current of $\sim 70 \text{ MA}$ the maximum total induced voltage at ω_{rf} for such a system would be $2 \times 0.07 \times 200 \times 10^6 = 28 \text{ MV}$ since,

for bunches that are short compared with the bucket length, $\bar{I}_b \cong 2\bar{I}_b$. Here $\bar{I}_b$ is the ω_{rf} component of the dc beam image current $\bar{I}_b$. As mentioned above, the total shunt impedance at ω_{rf} will be reduced by some factor because of coupling of the resonator to its power source. For the 374-MHz choice of f_{rf} we have a required gap voltage of 20 MV, thus we must determine a suitable value for the total shunt impedance.

In order to investigate the relationship between beam-induced voltage and required gap voltage we shall replace our resonator by a lumped element equivalent circuit (figure 4.1).

Figure 4.1 LRC cavity equivalent circuit.

Here L , C , R_e are the lumped elements with $\omega_{res} = 1/\sqrt{LC}$; $Q_e = R_e/\omega_r L = \omega_r C R_e \gg 1$; $R_c = \sqrt{L/C} = R_e/Q_e$; with $\tau_f = 2Q_e/\omega_r \gg \tau_b$ where τ_b is the time between bunches, R_e the parallel combination of R_s and the transformed source impedance, and Q_e is the loaded Q of the circuit. For the impedance of this circuit we can write

$$Z = R_e \cos\theta e^{-j\theta} ; \quad Y = 1/Z = G \sec\theta e^{j\theta} = G + jB \quad (4.1)$$

with $\tan\theta = (\omega/\omega_r - \omega_r/\omega) Q_e \cong 2Q_e \Delta\omega/\omega_r$ for $\Delta\omega = (\omega - \omega_r) \ll \omega_r$, $Q_e \gg 1$. We take the phase of the cavity voltage V_c as reference and write

$$\bar{V}_c = Z\bar{I}_T, \quad \bar{I}_T = \bar{I}_g + \bar{I}_b, \quad (4.2a,b)$$

$$\bar{I}_b = I_b e^{-j(\pi/2 + \phi_b)} \quad (4.3)$$

We have also

$$P_b = V_c \bar{I}_b \sin\phi_s = I_b V_c \sin\phi_b/2 \quad (4.4)$$

where we see that $\phi_b \equiv \phi_s$ only if the bunches are short compared with $1/f_{rf}$. Now, using Eqs. (4.1), (4.2), and (4.3) we can write

$$\tilde{I}_T = V_c(G + jB)$$

or

$$\tilde{I}_g = GV_c + I_b \sin \phi_b + j(BV_c + I_b \cos \phi_b) . \quad (4.5)$$

Hence if we wish $\tilde{I}_g$ to be in phase with $\tilde{V}_c$ so that the source sees a real impedance we must detune the circuit, i.e. make $\omega_r \neq \omega_{rf}$ the driving frequency. The required detuning is given by

$$BV_c = -I_b \cos \phi_b \quad (4.6a)$$

or

$$\tan \theta = -I_b R_e \cos \phi_b / V_c , \quad (4.6b)$$

then

$$I_g = V_c / R_e + I_b \sin \phi_b = I_{g0} + I_b \sin \phi_b \quad (4.7)$$

where I_{g0} would be the generator current required to develop the voltage V_c with $I_b = 0$. Looking into the load (beam + cavity) from the generator output we see that the beam acts like a conductance and a susceptance given by

$$G_b = I_b \sin \phi_b / V_c , \quad B_b = I_b \cos \phi_b / V_c . \quad (4.8a,b)$$

Using Eq. (4.6b) we can express the fractional detuning as

$$\frac{\Delta \omega}{\omega_r} = \frac{-I_b R_e \cos \phi_b}{2Q_e V_c} = \frac{-I_b \cos \phi_b}{2I_c} = \frac{-I_b R_c \cos \phi_b}{2V_c} \quad (4.8)$$

where $I_c = \omega_r C V_c$ is the circulating current at resonance in the parallel LC circuit.

Problem: Show that for capacitive detuning of the coaxial resonator $\Delta \omega / \omega_r = I_b \sin \beta l \cos \phi_b / \beta I_{r0}$ where I_{r0} is the peak current at the shorted end.

Since $\Delta \omega_r = (\omega_r - \omega_{rf}) = -\Delta \omega$, capacitive detuning of our resonant circuit requires

$$\frac{\Delta C}{C} = \frac{-I_b \cos \phi_b}{I_c} . \quad (4.9)$$

Figure 4.2 shows the relationship of the vectors $\tilde{I}_b$, $\tilde{V}_c$, $\tilde{I}_g$ and the cavity impedance for the case of $\pi \geq \phi_s \geq \pi/2$.

Figure 4.2 Vector diagram for beam loading.

We note that as I_b is increased at constant V_c the tuning angle θ must increase if the "load" impedance is to remain real. I_g as given by Eq. (4.7) must also be increased since more power must be delivered to the beam, Eq. (4.4). Let us examine the conditions that occur when $\theta = \phi_b - \pi/2$ so that $\tan \theta = -\cot \phi_b$, which, with Eq. (4.6b), results in the relation

$$I_b \sin \phi_b = V_c / R_e = I_{g0} . \quad (4.10)$$

From figure 4.2 we see that for this value of θ the impedance Z is colinear with $\tilde{I}_b$ as is that part of the total voltage that is due to $\tilde{I}_g$, i.e. $\tilde{V}_g = Z\tilde{I}_g$. Thus, as far as the phase of the external applied gap voltage is concerned, the beam is at an angle $\phi_s^* = \pi/2$. Now the phase stability of the bunch as a whole, in the absence of any closed loops controlling the rf amplitude, tuning, or phase of the beam relative to V_c , depends only on the slope of the externally applied voltage since the beam-induced part of V_c for changes slow compared with Ω_s , the synchrotron frequency, moves with the beam. Hence the operating point for I_b given by Eq. (4.10) with $\theta = (\phi_b - \pi/2)$ is at the limit of stable coherent motion

of the bunch. At a slightly higher current any perturbation of the bunch phase would result in an exponential increase in the phase displacement. In general, incoherent motion about the center of the bunch remains stable because of the beam-induced component of V_c . Equation (4.10) is a special case of Robinson stability limit.⁽¹³⁾ The generalized stability condition (without control loops) is⁽¹⁴⁾

$$0 < I_b < \frac{-2V_c \cos \phi_b}{R_e \sin 2\theta} \quad (\pi \geq \phi_s \geq \pi/2; \theta \geq 0) \quad (4.11)$$

where the generator load impedance is not required to be real.

At the stability limit for a real load impedance Eqs. (4.7) and (4.10) show that the power delivered to the beam is equal to the power P_e dissipated in the accelerating structure and the transformed generator impedance. If one denotes the latter, assumed to be real, by R_g then one requires

$$R_g \leq \frac{R_b R_s}{R_s - R_b} \quad (4.12)$$

where for $R_b > R_s$ one would have stability for any R_g . However, if one requires that the load (cavity + beam) be real and match the transformed source impedance then one needs

$$R_g = \frac{R_b R_s}{R_s + R_b} \quad (4.13)$$

This condition is often referred to as optimum coupling,⁽⁸⁾ and in the case where the generator power is transmitted over coaxial cables or through waveguides it must be satisfied if there is to be no reflected power. One can easily show that for this value of R_g the operating point would always be stable even without control loops.

Equation (4.12) means $P_b \leq P_e$ but, as will be pointed out below, if properly designed control loops are incorporated into the rf system, one can violate the inequality and operate with $P_b > P_e$. As an example we consider the case of the FNAL Main Ring at its peak intensity of 3.25×10^{13} protons/pulse. At an

acceleration rate of 125 GeV/sec or 2.62 MeV/turn and a total gap voltage of $15 \times 213 \text{ kV} = 3.2 \text{ MV}$ we find $\phi_s = 55^\circ$. One has $I_b \approx 0.5 \text{ A}$, $R_e \approx 600 \text{ K}\Omega^{(1)}/\text{gap}$ so that $I_b \sin \phi_s = 0.41 \text{ A} > I_{g0} = V_g/R_e = 0.355 \text{ A}$, and $\theta = \arctan 1.41 \approx 54.6^\circ > (\pi/2 - \phi_s)$. We also find $P_b = 650 \text{ kW}$ and $P_e = 567 \text{ kW}$ for this example. Next let us consider the SSC Conceptual Design for the case of a real load impedance and optimum coupling. At $V_c = 500 \text{ kV/gap}$ or 20 MV total and $\delta E = 5.26 \text{ MeV/turn}$ we have $\sin \phi_s = 0.263$ or $\phi_s = (\pi - 15.25^\circ)$ and $I_b = 0.073 \text{ A}$ so that $R_b = 10.4 \text{ M}\Omega$. Since $R_s = 5 \text{ M}\Omega$ for the 374-MHz copper resonators, the required R_g is $3.37 \text{ M}\Omega$. This results in a total shunt impedance of $2.01 \text{ M}\Omega/\text{gap}$. We already know that $P_b \approx 400 \text{ kW}$ and $P_c = 1 \text{ MW}$ so we have $P_g = 1.4 \text{ MW}$ and hence $P_e \approx 6P_b$.

We see that adjusting the coupling so that the power source is matched to the load during acceleration assures that static beam-loading effects will be under control. However, the hadron collider will be operated the majority of the time in the storage mode, where the power delivered to the beam will be but a few kW, primarily to make up for synchrotron radiation losses. In this case $\phi_s \approx \pi$ and the fractional detuning required to present a real load impedance is a maximum, Eq. (4.8). Then the stability limit given by Eq. (4.12) is always satisfied since $R_b \rightarrow \infty$. Of course, Eq. (4.13) would no longer be satisfied, and mismatch and power reflection in the case of a remote source would take place. Before we consider these effects let us calculate the fractional detuning for SSC CDR cavities. We have $R_c = R_s/Q = 5 \times 10^6/4 \times 10^4 = 125 \Omega = R_e/Q_e$ (assuming R_g real) so that $\Delta\omega/\omega_c = -2 \times 0.073 \times 125/2 \times 0.5 \times 10^6 = -0.18 \times 10^{-4}$ or $\Delta f = -0.18 \times 10^{-4} \times 374.7 \times 10^6 = -6.84 \text{ kc}$. Since $R_e = 2.01 \text{ M}\Omega/\text{gap}$ for optimum coupling, we have $Q_e \approx 16,100$ so that the half bandwidth of the loaded cavity would be $\Delta f = f_r/2Q_e = 11.68 \text{ kc}$. For the Reference Design parameters, f_0 , the rotation frequency, is only 3.614 kc. Hence the resonant impedance of the cavities would occur at a frequency slightly greater than $(f_{rf} - 2f_0)$. With 17,280 bunches in the ring their impedance could drive a longitudinal coupled bunch instability with considerable strength. If we compare this with the FNAL main ring at 3.25×10^{13} we find that during acceleration $|\Delta f/f| \approx 1.33 \times 10^{-4}$ or $|\Delta f| = 7.1 \text{ kc}$ while the half bandwidth of the resonators is $\approx 4.4 \text{ kc}$. However, $f_0 = 47.7 \text{ kc}$ so that the impedance at $f_{rf} \pm f_0$ is significantly reduced. On the other hand, in the CERN SPS the 200-MHz traveling-wave accelerating structures have a considerable bandwidth so that the impedance at $f_{rf} \pm n f_0$ for small n is

appreciable. Coupled bunch instabilities arising from this impedance have been observed.⁽¹⁵⁾

Suppression of coupled bunch instabilities can be achieved by using feedback systems that sense the phase, position, frequency, etc., of the bunches.⁽¹⁶⁾ Usually they are designed to control the growth of coherent oscillations due to the impedance of higher-order resonant modes in the accelerating structures or of other ring components. If such a system were used to suppress the effects of the principal resonant mode of the rf cavity, its gain, in general, would have to be much greater than that required to control the effects of the remaining impedances (see section on bunch-to-bunch damping). A solution to this problem proposed in the SSC Conceptual Design Report is not to detune the cavity. Hence $\theta \equiv 0$ and the load impedance (cavity + beam) is always complex. From Eq. (4.11) we see that as $\theta \rightarrow 0$ the static beam loading current limit approaches ∞ . There will be no possibility of matching a real R_g to the load so that power reflection will occur in all three modes of operation of the rf system. The choice of R_g , i.e. the coupling, can now be based on minimizing the reflected power. It can be shown⁽⁸⁾ that the incident power required is

$$P_i = \frac{V_c^2}{8\beta R_S} \left[\left\{ (\beta + 1) + \frac{I_b R_S}{V_c} \sin\phi_S \right\}^2 + \left\{ \frac{I_b R_S \cos\phi_S}{V_c} + \tan\psi \right\}^2 \right] \quad (4.14)$$

where $\beta = R_S/R_g$ and $\tan\psi = (\beta + 1) \tan\theta$. If $\psi = 0$ then the value of β for minimum P_c is given by

$$\beta_{\min} = \sqrt{1 + 2 \frac{I_b R_S \sin\phi_S}{V_c} + \frac{I_b^2 R_S^2}{V_c^2}} \quad (4.15)$$

Rewriting Eq. (4.13) in terms of β , R_S , and R_b we have

$$\beta_0 = 1 + R_S/R_b = 1 + I_b R_S \sin\phi_S / V_c \quad (4.16)$$

Thus $\beta_{\min} > \beta_0$ and for this value the total shunt impedance will be less than for optimum coupling. For the SSC Conceptual Design rf parameters with $V_c = 500$ kV/gap, $\phi_s = 164.75^\circ$ we find $\beta_0 = 1.382$, $\beta_{\min} = 1.974$ for acceleration and $\beta_{\min} = 1.77$ for storage.

Problem: Derive Eq. (4.14). Hint: remember that the reflected power = $|\Gamma|^2 P_i$ where the reflection coefficient Γ is given by $\Gamma = (Z_L - R_g)/(Z_L + R_g)$.

If we put the $\beta_{\min}$ from Eq. (4.15) into Eq. (4.14) with $\psi = 0$ we obtain

$$P_i = \frac{P_c}{2} [\beta_{\min}(\phi_s) + 1 + I_b R_s \sin \phi_s / V_c] \quad (4.17)$$

where $P_c = V_c^2 / 2R_s$. Using Eqs. (4.6b), (4.16) and (4.14) one can show that for optimum detuning and coupling $P_{i0} = \beta_0 P_c$. Hence, for the values of β given above, the incident power required for accelerations is about 21% greater if the cavity is not detuned to cancel the reactive part of the load impedance presented by the beam. We note also that if ψ is kept zero then the power required for acceleration is always greater than that required for storage (or at injection) when $\beta_{\min}$ is fixed at the acceleration value. In the storage mode $P_i = 1.39P_c$ for $\beta_{\min}(\pi)$ [and $1.487P_c$ for $\beta_{\min}(164.75^\circ)$], and, since $\beta_0(\pi) = 1$ here, $P_{i0} = P_c$ for the optimum case. Thus, considerably more average power is needed if one chooses not to detune the cavities and overcouple them to the power source.

In the above discussion we have assumed that I_b is a fixed value and that it contains frequency components only at $n(\ell f_{rf}/h)$, $n = 0, 1$, etc., where $\ell < h$ is the number of bunches ($\ell = 17,280$ in the SSC Conceptual Design). This is not true at injection where I_b can have a step change and the frequency spectrum can contain components at nf_0 . These can result in transient beam loading effects whose compensation can impinge on the peak rf power requirements. We shall assume that $(1/f_0) \gg \tau_f = 2Q_e/\omega_r \gg \tau_b$ where as before $\tau_b = h/\ell f_{rf} = 1/\ell f_0$ so that at injection when a fraction of ℓ bunches are put into an empty ring the beam-induced voltage will be strongly modulated at f_0 and its harmonics. All the bunches would not see the same gap voltage. If the power source is located at the cavity, it is in principle possible to install fast amplitude and phase correction loops to significantly reduce this modulation.⁽¹⁷⁾ However, a more elegant solution particularly suited to remote power generation is the feed-forward technique.⁽¹⁸⁾ A signal proportional to the rf component of the beam current obtained from a sum pickup electrode is used to generate an additional current $\bar{I}_g$ which is then fed to the cavity to cancel the effect of $\bar{I}_b$.

Let us return to figure 4.2 and consider the conditions at injection where $\phi_s = \pi$ with $\theta = 0$. The additional generator current required to cancel I_b is $|I'_g| = I_b$ in quadrature with I_{g0} so that $|I_g|^2 = |I_b|^2 + |I_{g0}|^2$ is the instantaneous current required to keep V_c constant and at the proper phase relative to I_b . The signal delay from pickup to cavity should equal the beam transit time between these points for ideal compensation. Also the bandwidth of this channel should be greater than the cavity bandwidth in order to cancel the effect of the rotation frequency sidebands. If a tuning loop is present its response will in general be slow compared with $T_0 = 1/f_0$ so that $\theta, (\psi)$ would remain ≈ 0 for a few revolution periods and the peak generator current required would be the same. The incident power required from a remote generator can be found from Eq. (4.14) with $\psi = 0, \phi_s = \pi$. Thus, if one does not detune, the peak power required to compensate transient beam loading is no greater than that required in the storage mode with full beam and, as pointed out above, is less than that required for acceleration. However, if optimum detuning is employed and a $\beta_0 = 1$ is chosen, then the peak incident power required for transient beam loading compensation will be greater than the power required for acceleration.

V. Beam Control Systems

Figure 5.1 is a block diagram of a typical phase-lock beam control system. It shows the control loops for rf amplitude (AGC loop), phase of the rf gap voltage relative to the center of charge of a bunch or bunches (phase-lock loop), phase of the rf gap voltage relative to the phase of the drive signal (tuning loop), and frequency of the rf voltage or radial position of the beam. It omits additional loops or signal paths, such as local fast feedback from the accelerating gap to the input of the power amplifier at the cavity, feedforward from the phase pickup electrode to the power amplifier, or loops to control quadrupole or bunch shape oscillations ($n = 0, m = 2$), etc. We shall discuss the operation of the phase-lock loop, figure 5.2, and of the radial and frequency loops in the limit of low beam current so that coupling to the tuning and AGC loops can be ignored. The operation of these loops under beam loading is treated in references 14, 19, and 20.

Figure 5.1 Simplified block diagram showing control loops.

V.1. Phase Loop Dynamics

The phase of the beam is sensed with a nondestructive pickup device (commonly a cylinder-shaped electrode) and this signal is compared in phase with a signal at ω_{rf} obtained from the accelerating gap(s). The output of the phase detector, figure 5.2a, is then proportional to the phase difference between the center of the bunch and the center of the bucket defined by ω_{rf} .

In general, the change in guide field with time is taken as the independent variable, and the frequency program is derived from measurement of B at a radius of ρ_0 where the orbit length $L_0 = 2\pi\rho_0 + \text{total length of straight sections}$ so that⁽²¹⁾

$$f_{rf} = h \frac{c}{L_0} \frac{cB\rho_0}{\sqrt{(cB\rho_0)^2 + E_0^2}} \quad (5.1)$$

where $eB_0\rho_0 = \beta E/c$.

The ideal frequency program for the orbit L_0 would be such that the energy gain per turn would be given by

$$\delta E = e\rho_0 \dot{B}_0 L_0 \quad \text{or} \quad \sin \phi_{S0} = \rho_0 L_0 \dot{B}_0 / V_{rf} \quad (5.2a,b)$$

Figure 5.2 Phase-lock loop.

a) Block diagram.

b) Equivalent circuit for loops.

i.e. one that keeps the stable phase angle constant or, at most, allows it to vary adiabatically. Since $\dot{B}_0/V_{rf}$ does not appear in Eq. (5.1), any nonadiabatic change in these parameters (variations that occur within a phase oscillation period or less) can result in coherent oscillations about the center of the bucket.⁽²¹⁾ In addition, modulation of Φ_e , see Eq. (5.4b), arising from rf noise or ripple in the cavity tuning system can result in coherent dipole oscillations.

We shall assume the linearized form of the equation for synchrotron motion and denote by Φ_b , Φ_c the deviations of the phase of the center of the beam bunch(es) and of the gap voltage(s) from the ideal synchronous value defined by Eq. (5.2b). Thus, we can write

$$\ddot{\Phi}_b = \omega_S^2 (\Phi_c - \Phi_b) \quad (5.3a)$$

or, using the Laplace transform,

$$s^2 \bar{\Phi}_b - s\Phi_b(0) - \dot{\Phi}_b(0) = \omega_S^2 (\bar{\Phi}_c - \bar{\Phi}_b) \quad (5.3b)$$

From figure 5.2b we have

$$\bar{\Phi}_c = \bar{\Phi}_e K/s + \bar{\theta}_{rf}, \quad \bar{\Phi}_e = \bar{\Phi}_b - \bar{\Phi}_c \quad (5.4a,b)$$

where Φ_e is the output of the phase detector and K , which is a function of frequency, includes the transfer functions of the phase detector, the loop filter, and the cavity as well as the time delay of the entire loop. The transfer function of the idealized voltage-controlled oscillator is k_0/s where k_0 is absorbed into K and θ_{rf} represents any sources of phase noise or variation in the system but referred to the output of the cavity.

Solving for $\bar{\Phi}_e$ and $\bar{\Phi}_b$ we obtain

$$\bar{\Phi}_e = \frac{-s^2 \bar{\theta}_{rf}}{(s^2 + Ks + \omega_S^2)} + \frac{s\Phi_b(0)}{(s^2 + Ks + \omega_S^2)} + \frac{\dot{\Phi}_b(0)}{(s^2 + Ks + \omega_S^2)}, \quad (5.5a)$$

$$\bar{\Phi}_b = \frac{\omega_S^2 \bar{\theta}_{rf}}{(s^2 + Ks + \omega_S^2)} + \frac{(s + K)\Phi_b(0)}{(s^2 + Ks + \omega_S^2)} + \frac{(s + K)\dot{\Phi}_b(0)}{s(s^2 + Ks + \omega_S^2)} \quad (5.5b)$$

where $\Phi_b(0)$, $\dot{\Phi}_b(0)$ are initial values, for example at injection, where a bunched beam is first introduced into a ring or if the phase-lock loop had been closed after being disabled. They represent phase and frequency (momentum) errors from the ideal values for the orbit L_0 .

Let us consider the operation of this loop for injection errors $\Phi_b(0)$, $\dot{\Phi}_b(0)$ and the case where $K(\omega) \gg 2\omega_S$ for $(\omega \leq 2\omega_S)$ i.e. an overdamped loop. Then for a phase error

$$\Phi_e(t) \equiv e^{-Kt} \Phi_b(0), \quad \Phi_b(t) \equiv \Phi_b(0) \quad (5.6a,b)$$

and a momentum error

$$\Phi_c(t) = \frac{1}{K} (1 - e^{-Kt}) \dot{\Phi}_b(0) , \quad \Phi_b(t) \equiv \dot{\Phi}_b(0)t \quad (5.7a,b)$$

for $t < 1/\omega_s$. We see that the loop acts rapidly compared with a phase oscillation period to move the bucket defined by f_{rf} so that the bunches are centered. This prevents any dilution that would result from nonlinear oscillations when a bunch is not injected into the center of a bucket. For $t > K/\omega_s^2$ it is easily shown that $\dot{\Phi}_b(t) \rightarrow 0$ so that the bunch and bucket slowly approach the ideal orbit. Thus, as long as there is sufficient aperture to accommodate the momentum error, the phase loop will, in principle, correct completely any injection error in longitudinal phase space. We assume that the momentum error corresponding to $\dot{\Phi}_b(0)$ or $\omega_s \Phi_b(0)$ represents only a fraction of the rf bucket height.

Next, let us consider how the phase loop corrects for static phase or frequency errors. These can be represented by $\bar{\theta}_{rf} = K\delta\theta/s^2$ and $\bar{\delta}_{rf} = s\bar{\theta}_{rf} = \delta\dot{\theta}/s$ where we have assumed that the phase error $\delta\theta/s$ occurs before the VCO input, and that the frequency error originates in the VCO. We are interested in $\dot{\Phi}_b(t)$ for large t and, using Eq. (5.5b), we find that $\dot{\Phi}_b(t) \rightarrow K\delta\theta$ for a step phase error and becomes equal to $\delta\dot{\theta}$ for a static frequency error. The corresponding change in beam position is given by the relation

$$\frac{\Delta R}{R_0} = \frac{\gamma^2}{(\gamma_{rf}^2 - \gamma^2)} \frac{\Delta f}{f_0} \quad (f_{rf} = hf_0) \quad (5.8)$$

where $2\pi R_0 = L_0$. We see that the larger we make the static phase loop gain the smaller the tolerable static phase error must be for a given permissible radial displacement. We also see that in a machine where one accelerates through the transition energy the tolerance on frequency or phase errors is very tight, Eq. (5.8).

V.2. Radial Position and Frequency Loops

In order to reduce the amplitude of any radial excursions arising from phase or frequency program errors one can introduce either a radial or frequency control loop. Let us first consider the action of an ideal radial position loop. In Eq. (5.4a) we add a term $-sG_r(K_r/s)\tilde{\Phi}_b$ to $\tilde{\Phi}_c$ where with $2\pi h\Delta f = \dot{\Phi}_b$ and Eq.

(5.8) we can write $G_r = GR_0/\gamma_{tr}^2 \eta h 2\pi f_0$. This is equivalent to closing the loop shown by the dashed line in figure 5.1 with G being the transfer function of the loop amplifier and filter. The result is to multiply the ω_S^2 term of the quadratic factor ($s^2 + Ks + \omega_S^2$) appearing in the denominators of Eq. (5.5b) by $(1 + K_r G_r)$, where K_r includes k_0 plus radial and phase loop cable delays and the cavity transfer function. One can then show that for constant G the effects of static phase or frequency errors are reduced by $1/(1 + K_r G_r)$. In principle G_r can vary over a wide range, but in a large hadron collider $\eta \approx 1/\gamma_{tr}^2$ and f_0 are essentially constant. One cannot make G_r too large since the radial loop reduces the fast damping of the phase loop for fixed G_r ; i.e. for a given value of K_r the size of the large root of $s^2 + Ks + (1 + K_r G_r)\omega_S^2$ is decreased. However, by proper choice of the filters in both loops plus programming their static gains, it is possible to obtain fast phase response and minimal radial excursions.

Since the relation between Δf and ΔR , Eq. (5.8), is essentially constant for $\gamma \gg \gamma_{tr}$, precise control of the beam radial position can be achieved by using a frequency loop, as shown in figure 5.1. The reference is obtained from a stable low noise frequency synthesizer which can be programmed, but only at a very slow rate. One retains the VCO in order to maintain the fast response of the phase loop. For this case we must add $-sG_f \tilde{\Phi}_c$ to the right-hand side of Eq. (5.4b) where G_f (rad/rad/sec) is the transfer function of the frequency discriminator filter combination. As before, the quadratic factor in the denominators of Eq. (5.5b) is changed. It becomes $(K'G_f + 1) \times [s^2 + sK/(K'G_f + 1) + \omega_S^2]$ so that, for constant G_f , static phase and frequency errors are reduced by $(1 + K'G_f)$ where $K' = (K/k_d)$ with k_d being the phase detector transfer function. Again, the speed of the phase loop would be affected unless G_f is made to decrease with frequency from its static or dc value. Discussions of overall loop stability and the effects of time delay can be found in references 20, 22, and 23.

V.3. Interaction Between Control Loops and RF Noise Sources

In the storage mode, the beam control system will keep the bunches at a fixed frequency (radius) and phase relative to this frequency to a high degree of accuracy. Since a large hadron collider will contain only superconducting magnets, the effect of magnetic fluctuations can be neglected. Therefore, the principal sources of "rf noise" are expected to arise from elements of the control

system itself. Figure 5.3 shows the system in figure 5.1 redrawn with only the frequency and phase loops active. Here we have included three sources of signal fluctuation: $\delta\theta_{rf}(t)$ is frequency "noise" appearing at an accelerating gap due to the VCO, the loop amplifiers, or power amplifiers; $\delta\theta_p(t)$ is phase noise arising from the phase detection process; and $\delta\theta_f(t)$ is frequency noise in the reference oscillator itself.

Figure 5.3 Simplified phase and frequency control loops with noise sources.

If we solve the loop equations for the frequency noise seen by the beam $\Phi_c(t)$ or its transform $\tilde{\Phi}_c(s)$ we obtain

$$\tilde{\Phi}_c = \frac{\tilde{\delta\theta}_{rf} + K(\tilde{\delta\theta}_p + G_f \tilde{\delta\theta}_f)}{1 + K(B + G_f)} \quad (5.9)$$

where $-B = -s/(s^2 + \omega_s^2)$ is the transfer function between the rf frequency Φ_c and the phase error Φ_e . For $s = j\omega$ and $\omega \ll \omega_s$, $G_f(\omega) > B(\omega)$ with $KG_f \gg 1$, we see that if we can make $\delta\theta_p < G_f \delta\theta_f$ then the reference oscillator will determine the beam frequency stability since here we can always have $KG_f \delta\theta_f > \delta\theta_{rf}$.

Next let us consider the region around $\omega \approx \omega_s$, where B has a pole if all the particles in a bunch have the same synchrotron frequency ω_{s0} (ω_{s0} being the frequency for zero-amplitude phase oscillations if the applied voltage is sinusoidal). In this case, the frequency noise would be completely cancelled by the action of the phase loop since the denominator in Eq. (5.9) becomes infinite. However, because of the nonlinearity present in a stationary bucket generated by a sinusoidal rf voltage, there will always be a finite spread in synchrotron

frequency for a finite bunch width. $B(\omega) = j\omega/(\omega_s^2 - \omega^2)$ can be related to the dispersion integral for the bunch,⁽²⁴⁾ figure 5.4, with $|1/B(j\omega)| \approx S$ the spread in ω_s (see Appendix A). One still has $KB \gg 1$ and by increasing the gain $K(\omega_s)$ the contribution of the VCO noise δ_{ff} can be suppressed relative to the other noise sources. Once this is achieved then additional gain is not necessary. In the case of the CERN SPP control system, it was also possible to have $G_F(\omega_s)\delta_F(\omega_s) < \delta\theta_p$ so that the phase detector noise dominated the contribution to $\tilde{\Phi}_c$. Then the beam sees

$$\tilde{\Phi}_c = \frac{\delta\tilde{\theta}_p}{B} \quad \text{or} \quad \tilde{\Phi}_c \equiv \delta\theta_p (\omega_s) S. \quad (5.10a,b)$$

As long as the bunch half-width remains below one radian the noise around ω_s will be the principal cause of diffusion in longitudinal phase space. Once the bunch area approaches $\approx 1/3$ the bucket area, the noise at $3\omega_s$ can also contribute to bunch diffusion. Here $K(3\omega_s) B(3\omega_s)$ will not be nearly as large as before so that even for a flat noise spectrum $\delta\tilde{\theta}_p(j\omega)$ the contribution to $\tilde{\Phi}_c$ can be significant. As the bunch becomes even larger the noise at $5\omega_s$ or higher-order odd harmonics can enhance the growth. Hence, in addition to the phase detector noise, one must consider again the VCO noise (see below). If one wishes to take full advantage of the manner in which the phase loop reduces the effect of the VCO noise on bunch area growth then this area must be kept well below $1/3$ the bucket area at the beginning of the storage mode of operation. In addition, the phase detector noise must be made as low as possible.

Figure 5.4
Plot of $1/|B|S$ for $\psi_0 = 3(1 - r^2)^2/\pi$.

Figure 5.5 SSB phase noise of SPS $\bar{p}p$ VCO.

Now for $\omega \gg \omega_s$ we see, again from Eq. (5.9), that eventually the VCO noise $\hat{\theta}_{rf}$ will be the dominant term in $\hat{\theta}_c$. Figure 5.5 shows the single sideband phase noise for the VCO and reference oscillator used in the CERN SPS low level rf system for $\bar{p}p$ operation. $L(f_m)$ is defined as the power density in one phase modulation sideband divided by the carrier power. If one assumes that the peak phase fluctuation ϕ_{peak} for a given frequency is $\ll 1$ radian then it can be shown that $L(f_m) = [\phi_{peak}/2]^2 = S_\phi(f_m)/2$ where S_ϕ is the power spectral density for phase modulation.⁽²⁹⁾ The corresponding power spectral density for frequency modulation is given by $S_f = f_m^2 S_\phi(f_m)$. Note that the slope of the VCO plot in figure 5.5 is greater than 20 db/decade below 10 kc so that in this region the frequency noise $\hat{\theta}(j\omega) = \omega \sqrt{S_\phi(\omega)}$ will actually decrease with ω . In order to determine the relative importance of the VCO and phase detector noise in this region, let us calculate $\hat{\theta}_{rf}$ and $K\delta\theta_p$ at ω_s and $5\omega_s$ for the CERN $\bar{p}p$ system. We assume $\omega_s = 2\pi 200$, $\delta\theta_p = 1.6 \times 10^{-6}$ rad/ $\sqrt{\text{Hz}}$ from 20 Hz to beyond 500 kHz, $\hat{\theta}_{rf}(\omega) = 2\pi f \sqrt{2L(f)}$, and that the phase loop transfer function in this region is given by $K = K_0/(1 + j\omega\tau)$ where $\omega\tau = 1$ at $f = 10$ Hz.

From the VCO plot in Figure 5.5 we find $L(200) = -85 \text{ db} = 10^{-8.5}$, $L(1 \text{ kc}) = -105 \text{ db} = 10^{-10.5}$ so that $\hat{\theta}_{rf} = 0.1 \text{ rad/sec } \sqrt{\text{Hz}}$ at ω_s and $0.05 \text{ rad/sec } \sqrt{\text{Hz}}$ at $5\omega_s$. If we assume $K_0 = 100 \text{ kc/degree}$ or $36 \times 10^6 \text{ rad/sec rad}$, then, at ω_s , $|K| \approx K_0/20$ and at $5\omega_s$ ($K_0/100$). This then gives $(K\delta\theta_p)_{\omega_s} = 2.88 \text{ rad/sec } \sqrt{\text{Hz}}$ and $\approx 0.58 \text{ rad/sec } \sqrt{\text{Hz}}$ at $5\omega_s$. With this choice of K_0 one easily satisfies the criterion $K(\omega)\delta\theta_p(\omega) \gg \hat{\theta}_{rf}(\omega)$ for $\omega = \omega_s$ and apparently for as large as $5\omega_s$.

However, we should note here that the phase loop is locked on a single bunch (in the SPS case) or possibly a group of bunches (SSC case) that occupies a small fraction of the ring circumference. Since $\omega_{s0}/\omega_0 \ll 1$, the control "bunch" will sample the rf noise of the cavity at f_0 , the rotation frequency. Hence, noise at $f_{rf} \pm (nf_0 + f_s)$, $n = 1, 2, 3, \dots$ cannot be distinguished from that around f_{rf} . The contributions from each line in the spectrum, within the cavity bandwidth, should be added quadratically since no correlation effects are expected in the VCO output.

For the SPS, $n \leq 20$ even though $f_0 \approx 43$ kc, and for the SSC design ($Q_\ell = 1.6 \times 10^4$), $n \leq 4$. Thus our calculation of the $\tilde{\theta}_{rf}$ contribution is an underestimate. For the region around ω_s , the relation $K\delta\theta_p > \tilde{\theta}_{rf}$ will still hold, but the effective VCO noise will again become important below 1 kc for the SPS system. In any event, the main phase loop will not be able to cancel the effects of VCO noise for $n > 0$ on other than those bunches that are part of the control group. It will be necessary to have an additional loop (or loops) that can act on individual bunches or groups of bunches independently. The gain of this loop need not be as great as that of the main loop since a well designed VCO should have a decreasing frequency noise spectrum up to ≈ 20 kc from the operating frequency.

Let us return to figure 5.2a, where we indicated with the dashed lines an additional loop with a filter F' and a phase shifter PS located in the VCO output line. We can again use the loop diagram shown in figure 5.2b with K/s replaced by $K'F'$ where $K' = k_d k'_0$. We assume that there is a device in the output of the phase detector that allows us to measure the phase error $\tilde{\Phi}'_e$ of a bunch other than the control bunch (or bunches). Then Eq. (5.5a) will be valid for this bunch with the denominator replaced by $(s^2 + \omega_s^2 + s^2 K'F')$. Next, we assume that the bandwidths of the phase detector and the phase shifter are much greater than that of the accelerating cavities so that k_d and k'_0 , the phase shifter gain, are frequency independent. Then, in order to obtain damping around the synchrotron frequency, the filter must have a transfer function of the form $[\omega'/(s + \omega')]$ where $\omega' \ll \omega_{s0}$. Since we are concerned primarily with the loop response at $\omega_s, 3\omega_s$, etc., we make the approximation $\tilde{F}' \equiv \omega'/s$ and obtain

$$\tilde{\Phi}'_c = \frac{\tilde{\theta}'_{rf} + K'_0 \delta \tilde{\theta}'_p}{1 + K'_0 B} \quad (K'_0 = K'\omega') \quad (5.11)$$

using the loop equations and assuming $\tilde{\Phi}_c = s\tilde{\Phi}_c$. Here $\tilde{\delta}'_{rf}$ and $\delta\tilde{\delta}'_p$ are the effective frequency and phase noise at the frequencies $\pm(nf_0 + f_s)$ within the cavity bandwidth. In principle K'_0 can be increased so that around ω_s the phase discriminator noise would dominate (in order to keep the roots of the denominator real ω' must be decreased at some point as K'_0 is increased). However, this may not be necessary if the VCO noise can be made low enough. In the case of the SPS $\bar{p}p$ system the auxiliary loop gain at ω_s was 10^{-3} of the main loop gain so that although the VCO noise was dominant the resultant noise $\tilde{\delta}'_{rf}(j\omega_s)/K'_0 B(j\omega_s)$ seen by the other proton bunches did not contribute to additional longitudinal emittance growth.

We have considered only one additional loop acting on one bunch or group of bunches independently of the main control loop. In the SPS $\bar{p}p$ system there are only a total of six bunches (3p and 3 $\bar{p}$) so that individual loop filters for each bunch with multiplexed output from the phase detector and a multiplexed input stage for the phase shifter are feasible.⁽²⁴⁾ However, in a large hadron collider there will be many thousands of bunches per ring (>17,000 in the SSC conceptual design) and one must consider some form of compromise if the above scheme is to be maintained. For the SSC design there will be fifteen separate groups of bunches each occupying somewhat less than 1/15 the circumference. Thus, if the cavity bandwidth is such that $n \leq 5$, then a measurement of the average phase error of each group of bunches would be equivalent to a sampling rate of at least $3n$ (the nominal minimum required sampling rate being $2n$). This reduces the number of channels to a manageable value and, depending on the VCO quality, it could represent a viable solution.

V.4. Bunch-to-Bunch Feedback

So far we have considered control systems that apply the same correction to all bunches (main phase and frequency loops) or to a group of bunches (auxiliary phase loops). Also needed are feedback systems that act on the individual bunches which for the CDR parameters will be spaced only 4.8 meters apart or occur at a 62.5-MHz rate. At injection, when, after capture of the first group of bunches by the action of the main phase loop, additional groups of bunches will not always arrive in the center of the rf buckets defined by this system, it will be necessary to damp the resulting phase oscillations independently of the bunches

already present on the injection orbit. In addition, a wideband feedback system will be required to suppress the growth of coupled bunch dipole instabilities driven by the higher-order resonant modes present in the rf cavities and the net effect of other longitudinal impedances present in the vacuum chamber. We shall discuss some of the characteristics of such systems and how they may interact with the main phase/frequency control loop.

V.4.i. Injection Error Damping

First let us consider measuring the radial position variation of each bunch and using this to control the phase oscillations arising from injection errors. In order to obtain damping, this information will be used to vary the phase of the net rf voltage per turn seen by the bunch. Equation (5.4a) becomes

$$\tilde{\Phi}''_c = -sG_r k'_0 \tilde{\Phi}''_b + \tilde{\theta}''_{rf} \quad (5.12)$$

where $G_r = GX_p/\omega_{rf}\eta$ with X_p being the value of the dispersion at the location where $\Delta\tilde{R} = -s(dR/d\omega_{rf})\tilde{\Phi}''_b$ is measured and we have substituted X_p for (R/γ_{tr}^2) . Then using Eq. (5.3b) we obtain

$$\tilde{\Phi}''_b = \frac{\omega_S^2 \tilde{\theta}''_{rf}}{(s^2 + G\alpha k'_0 s + \omega_S^2)} + \frac{s\Phi''_b(0)}{(s^2 + G\alpha k'_0 s + \omega_S^2)} + \frac{\Phi''_b(0)}{s^2 + G\alpha k'_0 s + \omega_S^2} \quad (5.13)$$

where $\alpha = \omega_S^2 X_p/\omega_{rf}\eta$. Again G is the transfer function of the loop amplifier filter and k'_0 is the gain of the phase modulator (yet to be specified). For the CDR parameters at injection $\alpha = 28.8$ cm/sec with $X_p = 4$ m.

Next let us calculate the necessary gain and phase modulation (or voltage requirement) to adequately damp the expected peak injection momentum error of $(\delta p/p)_{inj} = 1.5 \times 10^{-4}$ in the SSC.⁽⁴⁾

We assume that it is not necessary to have a damping rate $(G\alpha k'_0/2) > 2\omega_S$ as is the case for the main phase loop or the auxiliary loop discussed above. In order to determine a suitable value we must consider the dilution rate that will occur due to the nonlinearity of the phase oscillations arising from the injection error. We have a bunch with a given momentum spread $\pm(\Delta p/p)$ and momentum error $(\delta p/p)$. Thus in the bucket the initial momentum extremes are

$(\Delta p + \delta p)$ and $(-\Delta p + \delta p)$ and we consider maximum dilution to occur when these particles overlap (i.e. have the same phase in r, θ space). If $\Delta\omega_s$ is the difference in synchrotron frequency of the two particles then the time required will be $(2\pi/2\Delta\omega_s)$. Now in a stationary bucket for small amplitudes we can write

$$\omega_s^2 = \omega_{s0}^2 \left(1 - \frac{\hat{\phi}^2}{8}\right) \quad (5.14)$$

so that if $\hat{\phi}$ is the bunch width and $\delta\hat{\phi}$ is the amplitude of the phase oscillation due to the momentum error $(\delta p/p)$ we have for $\hat{\phi} \geq \delta\hat{\phi}$

$$\Delta\omega_s \equiv \frac{\omega_{s0}}{16} [(\hat{\phi} + \delta\hat{\phi})^2 - (\hat{\phi} - \delta\hat{\phi})^2] = \frac{\omega_{s0}}{4} \hat{\phi} \delta\hat{\phi}$$

For small errors we can write $\delta\hat{\phi} = (\omega_{rf}/\omega_{s0})(\delta p/p)$ where $\eta \equiv 1/\gamma_{tr}^2$ since $\gamma_{inj} \gg \gamma_{tr}$, and for the CDR parameters ($\gamma_{tr} = 67$, $\omega_{s0} = 2\pi 30.9$, $\omega_{rf} = 2\pi 374.7 \times 10^6$) we obtain $\delta\hat{\phi} = 0.4$ rad for $(\delta p/p) = 1.5 \times 10^{-4}$.

We calculate $\hat{\phi}$ from the 95% bunch area at injection which is $6\pi \times 0.035$ eV sec = 0.66 eV sec. For $V = 20 \times 10^6$ volts at injection $A_0 = 2.53$ eV sec so that $\alpha_N(\pi) = 0.26$ and, using the tables of Cole and Morton, we find $N \equiv 0.32$, $\hat{\phi} = 1.21$ rad, and $(\Delta p/p) = 4.2 \times 10^{-4}$. This gives $\Delta\omega_s = 0.121 \omega_{s0}$ and $\Delta t = 0.133$ sec to produce a 180° phase slip between the extreme particles. If we take the required damping rate to be $1/\Delta t = (\alpha Gk'_0/2)$ or $Gk'_0 = 2$ rad/28.8 $\times 0.133$ cm, and note that for $X_p = 4$ m, $\Delta R_{in} = 4 \times 10^2 \times 1.5 \times 10^{-4} = 0.06$ cm, then $(Gk'_0)_{in} = 0.06 \times 2/3.83 = 0.0313$ rad. Since $\phi_s = \pi$ in the stationary bucket, the maximum voltage seen by the bunch due to the feedback would be $V \sin \Delta\Phi_c \equiv V \Delta\Phi_c = 20 \times 10^6 \times 0.0313 = 626$ kV if the phase of the accelerating cavities were modulated with the error signal. However, as pointed out above, this modulation must be removed during the gap between bunches, which is 0.15 μ sec for the CDR. Because of the high Q_c of these cavities ($\approx 10^4$) this is not possible, and the necessary correction voltage must be supplied by an auxiliary rf system with a greater bandwidth but lower power requirements.

This cavity need not be driven at the rf frequency but must be driven at some integral multiple of the bunch frequency for the following reason. In order to effectively "phase modulate" the total voltage seen by the bunch, the correction voltage must always be in quadrature with the accelerating cavity

voltage. Since the frequency of the latter is at some multiple of f_b (the bunch frequency; $f_{rf} = 6f_b$ for the CDR machine) the frequency of the former must also be Mf_b (M an integer) and synchronized with f_{rf} . The amplitude of the feedback cavity voltage will be modulated by the variations in radial position. Individual bunch signals can be digitized and stored in a first-in first-out memory bank that is clocked at the bunch frequency. The number of memory locations is chosen so that when the other delays from signal processing and amplification plus the pickup and cavity spacing are included the correction voltage generated by each bunch appears at the appropriate time.

In principle the error signal will be the same for all bunches within a given injected batch. Hence, the maximum modulation rate will be determined by the interbatch spacing. In general, the cavity bandwidth will be the limiting factor. We can calculate this by specifying that the excitation voltage should fall or rise to a given value during the time gap between groups of bunches. If we choose 10% for the decay or 90% for the rise of the required correction voltage, then we must have $\exp(-\omega_{res}\tau/2Q_\ell) = 0.1$ where $\tau = 0.15 \mu\text{sec}$ for the CDR case. A suitable choice for ω_{res} is $2\pi 562 \text{ MHz}$, i.e., $M = 9$, since the maximum variation of the voltage during the maximum excursion of $\delta\hat{\phi}$ is given by $(1 - \cos\delta\hat{\phi}_c)$. Here $\delta\hat{\phi}_c = 1.5 \delta\hat{\phi} = 0.6 \text{ rad}$, giving a 10% greater change than for $M = 6$, $\delta\hat{\phi} = 0.4 \text{ rad}$. We obtain then a $Q_\ell = 115$ and a bandwidth $\Delta\omega_{res} = 2\pi \times 4.9 \text{ MHz}$.

Now if we assume this cavity is to be driven from a broadband klystron, the required incident peak power can be calculated by using Eq. (4.14) with $I_b = \phi_s = \psi = 0$, so that

$$\hat{P}_i = \frac{V_c^2}{8 \beta R_s} (1 + \beta)^2 \quad (5.15)$$

where the necessary bandwidth is obtained by strongly overcoupling the power input. Hence, β and $Q_0/Q_\ell \gg 1$ and we can write ($V_c = 626 \text{ kV}$)

$$\hat{P}_i \equiv V_c^2 / 8 Q_\ell R_c = 426 \times 10^6 \text{ W} / R_c \text{ (ohms)}$$

where R_c is the characteristic impedance of the cavity, Eq. (3.3). Although we have put $I_b = 0$ in this calculation, we cannot ignore the voltage induced in this cavity due to the component of beam current at $\omega_{res} = 9 \omega_{beam}$. It will be given by $Q_\ell R_c I_b(\omega_{res})$ so that for $R_c = 125 \Omega$ (the value assumed for the 374-MHz

accelerating cavities) and $I_b(\omega_{\text{res}}) \cong 1.5 I_b$, which is valid for the relatively large bunches used in our calculation ($N = 0.32$), we would have $V_{\text{ind}} \geq 1.5 \text{ kV}$ for the $I_b = 0.073 \text{ A}$ of the CDR. For six-bit digitization of the error signal corresponding to minimum detectable position error of $\approx 10^{-3} \text{ cm}$, the induced voltage would be one quarter of the LSB value of $9.8 \text{ kV} = 626 \text{ kV}/64$. In order to suppress any effect due to this voltage, a feedforward compensation signal derived from a sum pickup electrode, as mentioned at the end of Section IV, could be added to the error signal. Finally, we note that since this cavity is centered on a harmonic of f_b it cannot excite coupled bunch oscillations because the upper and lower sidebands of a given mode within its bandwidth will see the same impedance.

Before discussing the requirements for a system to suppress instabilities, we should point out that Eqs. (5.12) and (5.13) must be interpreted differently in the case of damping with an auxiliary cavity. Here Φ''_c represents the effective phase deviation of the accelerating voltage seen by the bunch and is proportional to $\Delta V_c/V_{\text{acc}}$; $\Phi''_e = (\Phi''_b - \Phi''_c)$ where Φ''_c is the phase deviation of V_{acc} , which is here assumed to be zero so that $\Phi''_e = \Phi''_b$; and Eq. (5.3b) becomes

$$s^2 \Phi''_b = \omega_S^2 \Phi''_c - \omega_S^2 \Phi''_b + s \Phi''_b(0) + \dot{\Phi}''_b(0) \quad (5.16)$$

which is essentially Eq. (5.13) by virtue of Eq. (5.12). However, if we were to use $-K\Phi''_e$ as the error signal then, since $\Phi''_c = 0$, damping would be obtained if K contained a differentiation. The difference between this case and the auxiliary phase loop described previously, where an integration is needed in the loop to provide damping, can be seen as follows. Here the change in effective phase is seen only by the beam and not by the Φ''_c input to the phase detector as is the case for a loop that controls the phase of V_{acc} directly.

V.4.ii. Coupled Bunch Dipole Instability Damping

The two principal parameters of a feedback loop to suppress coupled bunch instabilities are the damping rate (or gain) and bandwidth. The former depends on the expected growth rate of the instability and the latter on the number of modes to be damped.

If there are N bunches equally spaced around the ring then N coupled bunch modes are possible where $(2\pi l/N)$, $l = 0, 1, 2, \dots (N-1)$, is the phase shift

per bunch for the mode l . When we look at the frequency spectrum of a sum pickup electrode we observe, for a given l , lines at $f_p = l(l + pN)f_0 + mf_s$ with $-\infty < p < \infty$.⁽²¹⁾ A similar spectrum would be seen if a position-sensitive signal were analyzed. Here $m = 1$ for dipole, 2 for quadrupole, etc., and we will be concerned only with the dipole mode. Unless the bunches are all of equal intensity in the sum signal case and in addition the closed orbit error is zero at the detector in the difference signal case, spectrum lines at all multiples of f_0 would in general also be present. In order to see how these lines are generated let us consider one of the coupled bunch modes l as seen in a coordinate system moving with the beam. A standing wave along the beam (for the moment assumed continuous) in the radial direction can be written as

$$X(\theta_m) = A \cos \frac{2\pi z}{\lambda} \sin Q_S \Omega_0 t \quad (5.17)$$

where $\theta_m = z/R_0$ is the azimuth in the moving system, $\lambda = 2\pi R_0/l$, $Q_S = f_s/f_0$, $\Omega_0 = 2\pi f_0$. In the laboratory system we observe at some location θ_L where $\theta_L = (\Omega_0 t + \theta_m)$ so that we can write

$$X_L(\theta_L) = \frac{A}{2} [\sin(l\theta_L - l\Omega_0 t + Q_S \Omega_0 t) - \sin(l\theta_L - l\Omega_0 t - Q_S \Omega_0 t)] \quad (5.18a)$$

which for $\theta_L = 0$ becomes

$$X_L = \frac{A}{2} [\sin(l + Q_S)\Omega_0 t - \sin(l - Q_S)\Omega_0 t] \quad (5.18b)$$

Now our standing wave in Eq. (5.17) can be decomposed into two traveling waves moving with phase velocity $(1 \pm Q_S/l)f_0$ around the ring. Above the transition energy it is the wave with the $+$ sign that is potentially unstable⁽²⁵⁾ and hence we need only consider the first term in Eq. (5.18b).

Next let us consider a bunched beam whose frequency spectrum for N equal intensity equally spaced bunches can be written as

$$B(t) = \sum_{k=0}^{\infty} B_k \cos(kN\Omega_0 t) \quad (5.19)$$

where we have ignored the sidebands due to phase modulation, Eq. (5.20a).

Problem: Show that if one includes these sidebands, then for small $\delta\hat{\phi}_k$ there exist another pair of sidebands at twice the modulating frequency $(l + Q_s)\omega_0$ whose amplitude is $\delta\hat{\phi}_k/2$ smaller than those given by Eq. (5.20).

Then for no closed orbit error at $\theta_L = 0$ and for a given l the observed spectrum will be given by the product

$$\sum_{k=0}^{\infty} \sin(l + Q_s)\Omega_0 t \cos kN\Omega_0 t \quad (5.19a)$$

which we plot in figure (5.6) for the N possible modes. We note that the envelope of the amplitude of the lines will be proportional to the bunch spectrum itself (the B_k) and that for $k > 0$ there will be a pair of sidebands around each multiple of Nf_0 for any value of l given by

$$[\sin(kN + l + Q_s)\Omega_0 t - \sin(kN - l - Q_s)\Omega_0 t] . \quad (5.20)$$

Figure 5.6 Coupled-bunch mode spectra ($m = 1$).

In particular if $kN\Omega_0 = \omega_{rf}$ then we obtain the same first-order sidebands that would be present if the accelerating voltage were phase modulated with the radial error signal. As we have shown previously this will produce damping of coherent dipole oscillations. Hence ideal bunch-to-bunch damping could be achieved by amplifying the signal from a wideband radial position detector and applying it to a wideband cavity so that the signal from each bunch arrives at the same time as the bunch. Because $Q_s \ll 1$ (0.0085, 0.0014 at injection and at 20 TeV for the CDR) the time delay could be as much as one revolution without significantly affecting the damping rate. However, it is usually feasible to transmit a wideband signal only when the time delay is of the order of a few microseconds. [We remark here that with the use of fiber optics and laser diodes

this could be extended somewhat.⁽²⁶⁾] Thus, this type of system would be limited to relatively small machines.

In order to damp any of the N coupled bunch modes we will show that it is necessary in principle only to transmit at most the signals within a bandwidth of $Nf_0/2$. The necessary delay can be obtained by digitization of the individual bunch position error signals and storing this information in a FIFO memory as described earlier. One obtains effective phase modulation by the standard procedure of amplitude modulating the "carrier" frequency shifted by 90° and adding it to the "carrier." (We assume balanced modulation here so that the "carrier" is suppressed and only the required sidebands are present.) In our example above the addition is accomplished with the auxiliary cavity, and the "carrier" frequency is not ω_{rf} but $1.5\omega_{rf}$.

Before we discuss the implementation of such a system let us consider the nature of the sidebands that would be obtained from a sum pickup electrode for mode l . In this case the bunched beam spectrum, Eq. (5.19), would be phase modulated by $\cos(l + Q_s)\Omega_0 t$ since the bunch phase motion is in quadrature with its energy or radial position variation in the rf bucket. Thus one would have terms of the form

$$\cos [kN\Omega_0 t + \hat{\delta\phi}_k \cos(l + Q_s)\Omega_0 t]$$

which will generate first-order sidebands given by

$$-\sin [(kN + l + Q_s)\Omega_0 t] - \sin [(kN - l - Q_s)\Omega_0 t] . \quad (5.20a)$$

If the sum signal is to be used to damp the mode l , then we see that the sign of one sideband relative to the other would have to be reversed. This is essentially the scheme employed on the CERN Booster Synchrotron.⁽²⁷⁾ However, this method requires $N/2$ feedback channels to damp all N modes.

At this point we should examine the digitization, storage, and modulation processes in some detail. We note that the frequency spectrum shown in figure 5.6 is what we would obtain if we "sampled" the function $f(t)$ given by the first term in Eq. (5.18b) at the frequency $f_b = Nf_0$ where the sampling function is given by Eq. (5.19), i.e. the bunched beam provides the samples of $f(t)$. The existence of the lower sidebands ($N - l$) is due to the

overlapping of the spectra generated by the sampling process. This effect is sometimes called aliasing and is due to the fact that the signal $f(t)$ can contain frequencies up to essentially the sampling frequency itself. Ideally of course the sampling rate should be at least twice the maximum frequency present at the input. The process of digitization of the individual sampled (bunch) signals, and storage, retrieval, and reconstruction of the function $f(t)$ is what characterizes a sampled-data system. Thus we must introduce some of the concepts of such a system.

The simple process described above is shown in figure 5.7, where $f_h(t)$ is the reconstruction of $f(t)$ by means of a zero-order data hold.⁽²⁸⁾ The output

Figure 5.7 Reconstruction of $f(t)$ by a zero-order data hold.

consists of a series of steps occurring at the sampling frequency $(1/T)$ (f_b in our case) whose amplitude is proportional to each individual bunch displacement. In order to include the effect of a "zero-order data hold" in the loop one must be able to assign it a transfer function. It can be shown that this is given by

$$G_h(s) = \left(\frac{1}{s} - \frac{e^{-Ts}}{s} \right) \quad (5.21)$$

or, in terms of frequency, with $s = j\omega$

$$G_h(j\omega) = \frac{(1 - e^{-j\omega T})}{j\omega} = \frac{T \sin(\omega T/2)}{(\omega T/2)} \bigg/ \frac{(\omega T/2)}{(\omega T/2)} \quad (5.21a)$$

which we plot in figure 5.8. Thus the signal that will be used to amplitude modulate the quadrature component of the carrier will be the product of G_h and

the spectrum shown in figure 5.6. In other words, as far as the loop is concerned, the zero-order data hold acts as a filter with the transfer function G_h .

Figure 5.8 Amplitude and phase response of zero-order data hold.

As a consequence this system will not provide much damping for the modes l close to $N/2$ unless some form of compensation is introduced in the feedback path. We can understand this by first considering the case of the lowest frequency mode $l = 1$ which will generate sidebands at $\omega_c \pm (1 + Q_s)\Omega_0$ and a much smaller pair at $\omega_c \pm (N - 1 + Q_s)\Omega_0$. Here ω_c is the "carrier" frequency $\cos 9\omega_b t$ (assumed suppressed) and we ignore any transmission by G_h above f_b . If the phase is adjusted so that the low frequency sidebands produce damping, then the high frequency pair will antidamp since they will be shifted by about 180° by G_h . Now for $l \approx N/2$ (N even) the phase shift will be close to $\pi/2$ for both pairs of sidebands and the amplitudes will be approximately the same. Hence for these modes one will have essentially no damping. The system will produce a real frequency shift for these modes which could affect their stability. This can be seen as follows. The modulating signal $(A/2) \sin(l + Q_s)\Omega_0 t$ for $l = N/2$ is shifted by $-\pi/2$ in phase so that it becomes $(-A/2) \cos(N/2 + Q_s)\Omega_0 t$, which, when multiplied by $\cos \omega_c t$, generates the sideband pair $[\cos(\omega_m + \omega_c)t + \cos(\omega_c - \omega_m)t]$ where $\omega_m = (N/2 + Q_s)\Omega_0 t$. However this is just the pair of sidebands that would be generated if we phase modulated $\sin \omega_c t$ with $\cos \omega_m t$.

Since the phase error of the bunches is proportional to $\cos\omega_m t$, this is equivalent to phase modulation of the accelerating voltage as a function of the phase error in the rf bucket. Hence it will alter the coherent dipole frequency of the bunches.

If we assume that the time delay is to be one revolution period $T_0 = NT$ then we must include in the loop transfer function the factor $\exp(-sT_0)$. Then for $s = j\omega$ with $\omega = (l + Q_s)\Omega_0$ there would be an additional but negligible phase shift of $\omega_s T_0$ since factors of 2π are not relevant here. If the overall time delay were reduced by $(T/2)$ then the factor would be $\exp[-s(T_0 - T/2)]$ and this combined with Eq. (5.21a) would give

$$e^{-j\omega(T_0 - T/2)} \frac{(1 - e^{-j\omega T})}{j\omega} = \frac{T \sin(\omega T/2)}{(\omega T/2)} / (\omega_s T_0) . \quad (5.21b)$$

The result would then resemble the ideal system mentioned above but with a filter of the form $(\sin x/x)$ centered on the carrier frequency. Note that for frequencies between f_b and $2f_b$, $3f_b$, and $4f_b$, etc., antidamping would occur but with considerably reduced gain relative to the nominal passband of f_b .

Next we must consider the effect of the rf cavity on the loop response. It can be shown that the transfer function of a resonant circuit, of medium or high Q tuned at ω_c , to phase or amplitude modulation of ω_c can be written as⁽¹⁴⁾ $[1/(1 + s\tau_c)]$ where $\tau_c = (2Q/\omega_c)$. It is possible to compensate for this factor by introducing a circuit with the following transfer function after the D/A converter: $(1 + s\tau_c)/(1 + s\tau)$ where $\tau < \tau_c$. The attenuation provided by the denominator is necessary to reduce the amplitude of any modulating signal outside the required bandwidth which here is nominally $(\pm f_b/2)$. In principle, however, one does not need to generate both the upper and lower pairs of sidebands to produce damping of all the modes. The production of single sidebands can be achieved by amplitude modulating the carrier frequency $[\sin kN\Omega_0 t]$ ($k = 9$ for our example) with a signal proportional to $[\cos(l + Q_s)\Omega_0 t]$ thus generating the sidebands:

$$[\sin(kN + l + Q_s)\Omega_0 t + \sin(kN - l - Q_s)\Omega_0 t] . \quad (5.22)$$

One then adds or subtracts this pair of sidebands to those given by Eq. (5.20) (including of course the lines generated by aliasing) to obtain a single set of sidebands. Since we already have $\cos kN\Omega_0 t$ the sine is easily obtained. One obtains a signal proportional to $\cos(l + Q_s)\Omega_0 t$ by measuring the phase error of

each bunch relative to f_{rf} and processing it in a manner similar to the position information. A block diagram of such a system is shown in figure (5.9).

Figure 5.9 Block diagram of single-side-band bunch-to-bunch feedback.

As far as the bunches are concerned the effect of the single sideband signal around ω_c is equivalent to simultaneous phase and amplitude modulation of ω_{rf} . The phase modulation will produce damping while the amplitude modulation will result in modulation at ω_s of the focussing force for motion about the center of the bunch. This modulation is not harmful (modulation at $2\omega_s$ can of course produce $m = 2$ or quadrupole oscillations of the bunches). We can see this by considering the case when both sidebands are present and there is no phase shift at the modulating frequency. The envelope of the signal that appears at the gap will vary at $2\omega_m$ (ω_m = modulating frequency). As long as the bunches are not too wide, the voltage will not change significantly as a bunch passes the gap. The variation will be a minimum when the bunch phase error in the ω_{rf} bucket is at a maximum. When one sideband is eliminated the voltage at the center of the maximum phase error bunch will still be zero but the change along the bunch will be a maximum hence the slope of V_{total} at the center will be altered. Any phase shift in the modulating signal will not affect this result. We note here that the peak amplitude of a single sideband would have to be twice that of an upper and lower pair to produce the same damping rate.

In order to calculate the required gain of the damping system we take for the maximum growth rate the value of 1.6 sec^{-1} as calculated in the CDR⁽⁴⁾ at 1 TeV assuming no Landau damping. This result is based on averaging the effect of computed impedances for the higher-order modes in PEP type cavities. This is

about 1/5 the damping rate estimated for the injection error system described earlier. The necessary gain is given by $(\alpha Gk'_0)/2 = 1.6 \text{ sec}^{-1}$ or $Gk'_0 = 3.2 \text{ rad}/28.8 \text{ cm} = 0.11 \text{ rad/cm}$ at 1 TeV while the required voltage would be $20 \times 10^6 \times 0.11 \text{ volts/cm}$ for either double or single sideband operation. For single sideband operation one must generate in the phase error channel the same voltage for the comparable radial position variation. For small amplitude oscillations we can write

$$\delta\hat{\phi} = (\eta\omega_{rf}/X_p\omega_{s0}) \delta\hat{R} \quad (5.23)$$

which for the CDR parameters at 1 TeV gives $\delta\hat{R}/\delta\hat{\phi} = 0.148 \text{ cm/rad}$ at $X_p = 4 \text{ m}$. Hence the required gain would be $20 \times 10^6 \times 0.11 \times 0.148 = 326 \text{ kV/rad}$. For equal bunch lengths the growth rate is $\sim\omega_{s0}$ or $1/\gamma^{1/2}$ so the gain in the position channel would have to be increased by $\sqrt{20}$ at the end of the acceleration cycle, since $\alpha \sim \omega_{s0}^2$, if the relative damping rate is to remain the same, again assuming no Landau damping. The gain in the phase channel would not change since $\delta\hat{R}/\delta\hat{\phi} \sim \omega_{s0}$.

Finally let us consider the power requirements for this system. It is necessary to calculate the incident power for only one sideband in order to determine the total power needs. We again use Eq. (4.14) with $\phi_s = \pi$ and ignore the rf component of beam current at the sideband frequency. Then, since $\beta \gg 1$, we can write

$$P_i \cong \frac{V_c^2}{8 R_s} \left[\beta + \frac{\tan^2 \psi}{\beta} \right] \quad (5.24)$$

where $\tan \psi \cong 2 Q_0(\Delta\omega/\omega_r)$. Here $\omega_r = 9\omega_b$ and $\Delta\omega = (l + Q_s) \Omega_n$, the sideband frequency. If we compute $(dP_i/d\beta)$ we obtain the relation $(\beta = \tan \psi)$ for minimum incident power. Thus, if we wish to minimize the power at the nominal bandwidth of $(f_b/2)$, we must have $(\beta = Q_0/9)$. The power required at other sidebands $< f_b/2$ would of course be less for the same V_c . One would have $Q_\ell \cong Q_0/\beta = 9$ for this choice so that $\tau_c = 2Q_\ell/\omega_r$ would be just (T/π) . The compensation network mentioned above $(1 + s\tau_c)$ would then provide the necessary preemphasis. There would be a factor of two increase in required power from the lowest frequency mode $l = 1$ to the mode $l = N/2$ for the same V_c . We note that because of the $(\sin x/x)$ factor the damping rate for the $N/2$ mode would be 1.27 times as large as that for the $l = 1$ mode. The SSC growth

rate mentioned above was for the mode number $l = 7381$ or $f \approx 0.427f_0$ and $0.573f_0$ ($N = 17,280$) so that with no other loop compensation the gain again would be ≈ 1.27 times as great as for the $l = 1$ mode since for the latter the lines at $\pm(N - 1)f_0$ are negligible.

In conclusion we remark that if one had chosen to minimize P_i at $(f_p/4)$, say, then β would have been one half the value found above. The power for $l = 1$ would also have been one half that calculated above but it would have been 25% greater at $(f_p/2)$ and the min-to-max ratio would have been five instead of two. Hence the choice of $\beta = (Q_0/9)$ is to be preferred. We see also that there is no strong argument for choosing the single sideband option as far as power considerations are concerned. Thus the additional low level phase measurement channel shown in figure (5.9) can be dispensed with.

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Appendix A

Calculation of the open loop transfer function $B(\omega)$ between Φ_e and $\hat{\theta}_{rf}$ for a bunched beam, Eq. (5.5a). We assume the distribution function in longitudinal phase space (r, θ) can be written as

$$\psi(r, \theta, t) = \psi_0(r, \theta) + \psi_1(r, \theta)e^{j\omega t}$$

where ψ_1 is a small perturbation which oscillates at the frequency ω . Next we employ the linearized Vlasov equation in the form⁽³⁰⁾

$$j\omega\psi_1 - \omega_s \frac{\partial \psi_1}{\partial \theta} + \frac{F_1}{\omega_s} \sin \theta \frac{d\psi_0}{dr} = 0 \quad (A-1)$$

where

$$F_1 = \ddot{\phi} + \omega_s^2 \phi \quad (A-2)$$

is the linearized equation for synchrotron motion. Here F_1 is the source of the perturbation, which we will assume is an additional rf voltage given by

$$V = V_1 e^{j(\omega + \omega_r)t} \quad (A-3)$$

where ω can take on + or - values. Then we can write Eq. (A-2) as

$$\ddot{\phi} + \omega_{S0}^2 \phi = \frac{+\omega_{S0}^2 V_1 e^{j\omega t}}{V_0 \cos \phi_S} \quad (A-4)$$

where ω_{S0} is the zero-amplitude synchrotron frequency and we have assumed low current so that any induced voltage can be neglected. Finally, then, Eq. (A-1) becomes (with the $e^{j\omega t}$ factor suppressed)

$$j\omega\psi_1 - \omega_S \frac{\partial \psi_1}{\partial \theta} - \frac{\omega_{S0}^2 V_1}{V_0 \omega_S} \frac{d\psi_0}{dr} \sin \theta = 0 \quad (A-5)$$

for $\phi_S = \pi$. Now ψ_1 must be periodic in θ so we can expand in a Fourier series,

$$\psi_1 = \sum_{-\infty}^{\infty} f_m e^{jm\theta}, \quad (A-6)$$

put it into Eq. (A-5), multiply by $e^{-jm\theta}/2\pi$, and integrate from $-\pi$ to π to obtain

$$f_m = \frac{-j\omega_{S0}^2 V_1 (d\psi_0/dr)}{2\pi V_0 \omega_S (\omega - m\omega_S)} \int_{-\pi}^{\pi} \sin \theta e^{-jm\theta} d\theta. \quad (A-7)$$

Thus the general solution to Eq. (A-5) can be written as

$$\psi_1(r, \theta) = \sum_{-\infty}^{\infty} -j \frac{\omega_{S0}^2 V_1 (d\psi_0/dr) e^{jm\theta}}{2\pi V_0 \omega_S (\omega - m\omega_S)} \int_{-\pi}^{\pi} \sin \theta e^{jm\theta} d\theta. \quad (A-8)$$

Since we are interested in dipole oscillations only for $\omega > 0$, we take $m = 1$ in Eq. (A-6) and calculate the dipole moment of the bunch in (r, θ) space:

$$\Phi_b = \int_{-\infty}^{\infty} r dr \int_{-\pi}^{\pi} r \cos \theta \psi_{1,1} d\theta \quad (\text{A-9})$$

where $\psi_{1,1}$ is just $\psi_1(r, \theta)$ for $m = 1$, Eq. (A-8). We obtain

$$\Phi_b = \frac{-\omega_{S0}^2 V_1}{V_0} \pi \int_0^{\infty} \frac{r^2 (d\psi_0/dr) dr}{2\omega_S [\omega - \omega_{S0}(r)]} \quad (\text{A-10})$$

(if we wanted to consider $\omega < 0$ then we would choose $m = -1$). Finally, using Eqs. (5.3b) and (5.4b) we can write $\bar{\Phi}e = -\bar{\Phi}_b s^2 / \omega_{S0}^2$, thus giving

$$\Phi_e(j\omega) = \frac{j\omega V_1}{V_0} j\omega \int_0^{\infty} \frac{\pi r^2 (d\psi_0/dr) dr}{2\omega [\omega - \omega_S(r)]} \quad (\text{A-11})$$

The voltage V_1 can be considered as effectively both phase and amplitude modulating $V_0 e^{j\omega_{rf} t}$ at the frequency ω . Amplitude modulation will not affect the $m = 1$ mode but phase modulations will. The phase modulation amplitude θ_{rf} is given by (V_1/V_0) so that $j\omega(V_1/V_0) = \dot{\theta}_{rf}$ is the effective frequency modulation of f_{rf} due to the perturbing voltage V_1 . Next we consider the second part of the right-hand side of Eq. (A-11) and write it as

$$\pi \int_0^{\infty} r^2 (d\psi_0/dr) dr \left\{ \frac{-j\omega}{2\omega_S [\omega_S(r) - \omega]} \right\} \quad (\text{A-12})$$

where the bracket expression is essentially $-B(j\omega)$, i.e.

$$-B(j\omega) = \frac{-j\omega}{\omega_S^2 - \omega^2} = \frac{-j\omega}{(\omega + \omega_S)(\omega_S - \omega)} \equiv \frac{-j\omega}{2\omega_S(\omega_S - \omega)} \quad (\text{A-13})$$

for $\omega \approx \omega_S$. The negative of the integral on the right-hand side of Eq. (A-11) is the so-called bunched beam dispersion integral.

Problem: Assume that $\omega_S = (\omega_{S0} - Sr^2)$ and that $\omega_0/\omega_S = 1$. Then for $\psi_0 = (3/\pi)(1 - r^2)^2$ obtain figure 5.4 using Eq. (A-11).