Finite Pressure Effects on Sawtooth Oscillations

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Abstract

Nonlinear 3-D simulation of sawtooth oscillations shows that the helically twisted hot spot has a prominent toroidal bulge at the large major radius side, characteristic of a high $\epsilon\beta_p$ instability. This bulge drives other m/n islands just outside the mixing radius resulting in a stochastic annular region. A similar effect is also found in shaped plasmas, even at low β . These effects agree with experimental data, giving detailed support to the Kadomtsev reconnection model. This effect can also be used to explain the experimental sawtooth heat pulse 'anomaly'. At very high $\epsilon\beta_p$, the stochastic region can essentially fill the whole plasma, and can result in a 'high β disruption'.

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Since sawtooth oscillations¹ were first observed in tokamak experiments, Kadomtsev's model² had been successfully used to explain major features of the phenomenon. More recently however, new experimental features of sawteeth have been reported,³⁻⁵ some seemingly contradicting the Kadomtsev model. To explain these results, alternative mechanisms have been proposed, using modified Kadomtsev models,^{6,7} and using entirely different models.^{5,8}

We have previously reported that our 3-D MHD (three-dimensional magnetohydrodynamic) simulation results agreed with the Kadomtsev model, and compared well with experiment if neoclassical resistivity is used.⁹ In the present paper, we will study finite pressure effects on sawtooth oscillations and compare the results with experiment. This will also give a detailed test of the theoretical model. The main effect of pressure is found to be that it generates a stochastic annular region during the sawtooth crash. The simulated sawtooth period was not shortened by pressure even at $\epsilon\beta_p \equiv 8\pi\epsilon \int p ds / \mu_0 \bar{I}_t^2 = 0.5$.

We have used a 3-D MHD code, *MH3D*.⁹ The *MH3D* code solves the full nonlinear compressible resistive MHD equations in a three-dimensional toroidal geometry. The code uses conformal coordinates,^{10,11} transformed from the original toroidal coordinates (r, θ, ϕ) . This allows the code to treat a shifted coordinate origin and shaped plasma boundaries. The θ and ϕ dependencies are represented in a fourier space, while the r dependence is represented in a physical space.

The time step is limited by the shear Alfvén wave instead of the compressional wave. The stabilization of the compressional wave is achieved simply by making implicit the diagonal part of matrices which result from the compressional wave in the difference equations. This 'quasi-implicit' scheme which was also used in our previous codes, *MH2D*¹² and *HIB*,¹³ is found to be at least as effective for our code as the more elaborate 'semi-implicit' scheme.¹⁴ Large parallel thermal conductivity is accurately treated using the technique of artificial parallel sound as explained in Ref. 13.

The computer simulation closely mimics the experiment. In the simulation, just as in an experiment, the toroidal voltage at the wall (a boundary condition) and the initial conditions determine the subsequent evolution of the plasma self-consistently. When auxiliary heated plasmas are simulated, a power deposition profile is required as input.

The simulation produces periodic sawtooth oscillations similar to the experimental sawteeth.⁹ The crash phase corresponds to a Kadomtsev type complete reconnection. In the following, we will use typical TFTR (Tokamak Fusion Test Reactor) machine parameters, with the inverse aspect ratio (ϵ) of 0.33. The β value at the axis is about 1%, and the average $\epsilon\beta_p$ is about 0.2. The Lundquist number (S) used in the simulation is 3×10^5 . Fig. 1 shows the pressure contours on the (toroidal angle) $\phi = 0$ plane toward the end of a crash phase. (The q profile at this time is similar to the one shown in Ref. 9.) The helically twisted hot spot has an additional prominent bulge.

in the large major radius ($+R$) direction. Pressure contours on the $\phi = \pi$ plane in Fig. 2 shows that the hot spot on this plane has a shift also in the $+R$ direction, but smaller. The reason for this bulge is that the hot spot, due to its large pressure gradient, shifts toward the direction of $-\kappa$ ($\kappa \equiv$ magnetic field curvature), which is in the $+R$ direction at both the inside and outside of the torus. (This is due to the fact that the toroidal curvature is a lower order quantity compared to the helical curvature in a tokamak ordering.) This phenomenon is a characteristic finite $\epsilon\beta_p$ effect, somewhat similar to the ballooning mode mechanism.

The toroidal bulge shown in Fig. 1 pushes into the plasma just outside of the mixing radius, producing X-points at various singular surfaces there, and inducing various m/n magnetic islands to grow. These islands eventually overlap to produce an annular stochastic region as shown in Fig. 3, which shows a magnetic field puncture plot on the $\phi = 0$ plane. The X-points induced by the bulge of the hot spot are aligned at $\theta = 0$ line, although no longer directly visible at the stage of Fig. 3. Still, one can deduce from the picture that X-points of both $m = 2$ and $m = 3$ islands lie aligned just outside of the bulge.

This driving mechanism specifies the phase of driven m/n islands relative to the $m = 1$ island shown in the central region of Fig. 3, i.e., the relative phase of the driven islands to the $m = 1$ island is zero at the location of the X-point of the $m = 1$ island in Fig. 3. (There is no apriori known phase

relation between these islands.) If the pressure bulge were helical rather than toroidal, the phase relation would not be clear because a helical bulge would have only given a small or zero drive for island growth, as explained below. The phase of m/n islands is $m\theta - n\phi$, while the position of an $m/n = 1/1$ helical bulge will be determined by $\theta = \phi$. The relative phase between the driven islands and the helical bulge at the toroidal angle ϕ is then $(m - n)\phi$. For $m \neq n$, as ϕ varies $0 - 2\pi$, the helical bulge will push into an equal number of O-points as X-points. When it pushes into an O-point, it will tend to reduce the size of islands.

One can expect from Fig. 3 that the electron heat released by the sawtooth crash process will be conducted outward quickly through the stochastic annular region. This effect is clearly seen in the experiment as shown in Fig. 4. This figure shows the temperature contours as functions of time, as measured by the ECE (electron cyclotron emission) signals. The large oscillation in the middle is directly due to the hot spot which rotates toroidally with the bulk plasma rotation. The small upward spikes at the $+R$ side denoted by (B) are due to the magnetic field distortion just outside of the bulging hot spot. If the spikes were due to a helical distortion, one would expect similar spikes at the $-R$ side, this time downward rather than upward. However, it is clear from the figure that such spikes are absent at the $-R$ side. The simulation result in Fig. 2 indicates small upward spikes at the $-R$ side, but these are probably too small to be detected in the signal.

The steeper gradients of temperature contours right after the crash phase as denoted by (Q) represent a fast heat pulse propagating through a annular region just outside of the mixing radius, probably due to the annular stochastic region as shown in Fig. 3. This interpretation can resolve the sawtooth heat pulse 'anomaly', as described in Ref. 15.

The agreement between experiment and theory seen here gives additional detailed support to the Kadomtsev reconnection model of a sawtooth crash. The experimental signal shown in Fig. 4 translates to a stochastic region, about 30% smaller than the one shown in Fig. 3. Accurate quantitative comparison, however, is very difficult because the amount of the stochasticity depends on many parameters, e.g., the durations of the sawtooth crash and precursor oscillation, the stability margins of various m/n modes before the start of the crash, and the Lundquist number (S) used.

To clearly show that the stochasticity in Fig. 3 is due to a finite $\epsilon\beta_p$ effect, we have run a similar case with zero β . This case, as shown in Fig. 5, produced little stochasticity. We have also run a shaped boundary case as shown in Fig. 6. This case, although the $\epsilon\beta_p$ was negligible, produced a larger stochastic region than in the case of Fig. 5, but not nearly as much as in Fig. 3. As explained before, an accurate quantitative estimate of the stochasticity for a given set of experimental parameters is very difficult. However, it is clear from these studies that a larger stochastic region will be generated as the pressure and/or the shaping factor at the singular surface increase. It

is rather obvious then, that at a higher $\epsilon\beta_p$ and/or with strong shaping, the stochastic region generated by the sawtooth crash can be so large that it virtually fills the whole outside region of the plasma. A similar drastic loss of confinement will also occur with the same parameters as the case of Fig. 3, if the wall is located near the $q = 2$ surface. In these cases, the electron thermal energy will be lost so quickly that the discharge will be unable to sustain itself without an application of extremely high loop voltage. This scenario can explain a form of 'high β disruption' where the discharge is terminated right after a large sawtooth crash.

In Fig. 3, the inside region retains good flux surfaces. This is a general character which depends only on the fact that the magnetic shear (s) is small in the region during a sawtooth crash. Since the distance between singular surfaces goes as s^{-1} while the island size goes as $s^{-1/2}$, magnetic islands do not overlap easily for small s . This good flux surface region, however, cannot prevent the major disruption described above, because of the complete reconnection process.

In our simulation, the crash phase corresponds to a complete reconnection process with the Sweet-Parker scaling ($\sim \tau_A^{1/2}\tau_R^{1/2}$, or equivalently, $\sim \eta_x^{1/2}$), as in the 2-D case of Ref. 16. Here η_x is the resistivity at the X-point. In TFTR discharges, the sawtooth crash time varies anywhere between a few msec to about one hundred μ sec. The slower time matches the Sweet-Parker time with non-enhanced η_x , whereas the fast time could result from an η_x

enhancement of two orders of magnitude. This enhancement may result from a microinstability in the current sheet at the X-point, due to strong current density and gradient. Another possible source of the enhanced effective η_x is electron parallel viscosity at the stochastic X-point region. However, this second possibility seems unlikely, because our present numerical simulations have shown that the severity of the stochasticity increases with $\epsilon\beta_p$, whereas, we could not find any experimental correlation between the crash time and $\epsilon\beta_p$.

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Figures

FIG. 1. Pressure contours at $\phi = 0$. The helically twisted hot spot has an additional bulge in the $+R$ direction

FIG. 2. Pressure contours at $\phi = \pi$, showing a small shift also in the $+R$ direction.

FIG. 3. Puncture plot of magnetic fields, showing a stochastic annular region.

FIG. 4. Experimental temperature contours as functions of time, from ECE measurement. R denotes the major radius direction, and MR the mixing radius.

FIG. 5. Puncture plot of magnetic fields in a $\beta = 0$ case.

FIG. 6. Puncture plot of magnetic fields in a shaped plasma with a negligibly small β .

PPPL#90X6332

Fig. 1

Fig. 2

PPPL#90X0329

Fig. 1

Fig. 4

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Fig. 5

Fig. 6