MASTER

PROGRESS REPORT

HIGH-RESOLUTION POSITRON Q-VALUE MEASUREMENTS
AND NUCLEAR-STRUCTURE STUDIES FAR FROM
THE STABILITY LINE

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ABSTRACT

This document represents a progress report for contract DE-AS09-79ER10434 between USDOE and USC. The reporting period is 1 March 1980 to 28 February 1981. Extensive data analysis and theoretical analysis has been done to complete the extensive decay scheme investigation of $^{206,208}\text{Fr}$ and the level structures of $^{206,208}\text{Rn}$. A final version of a journal article is presented in preprint form. Extensive Monte Carlo calculations have been made to correct the end point energies of positron spectra taken with intrinsic Ge detectors for annihilation radiation interferences. These calculations were tested using the decay of ^{82}Sr which has previously measured positron branches. This technique was applied to the positron spectra collected at the on-line UNISOR isotope separator. The reactions used were $^{60}\text{Ni}(^{20}\text{Ne};p2n)^{77}\text{Rb}$ and $^{60}\text{Ni}(^{20}\text{Ne};pn)^{78}\text{Rb}$. Values for 5, γ - β^+ coincidence positron end point energies are given for the decay of ^{77}Rb . The implied Q-value is 5.075 ± 0.010 MeV. A complete paper on the calculated corrections is presented. A flow chart of a more complete program which accounts for positrons scattering out of the detector and for bremsstrahlung radiation is also presented. End-point energies of four β^+ branches in ^{77}Rb are given as well as a proposed energy level scheme of ^{75}Kr based on γ - γ coincidence data taken at UNISOR.

I. INTRODUCTION

In this progress report we will present progress during the period from 1 February 1980 to the period 1 February 1981 in research performed under Department of Energy Contract DEAS 0979ER10434. The proposed research is entitled "HIGH RESOLUTION POSITRON Q-VALUE MEASUREMENTS AND NUCLEAR STRUCTURE STUDIES FAR FROM THE STABILITY LINE." The main goal of the original proposal was to develop and thoroughly test methods which utilize thin, planar, intrinsic germanium detectors to make high resolution measurements of positron spectra with end-point energies of approximately 8 MeV. It had been shown earlier¹ that the high energy portion of β^+ spectra result in linear Kurie plots even though a significant fraction of the electrons scatter out of the detector, depositing only part of their energy. This did not indicate that one can utilize these detectors for positrons because of the serious complication associated with the annihilation radiations following the stopping of the positron. This effect was known to lead to serious summing effects which result in significant distortion of the Fermi plots. The original approach described in the 1978 proposal was to surround the Ge detector with a large NaI(Tl) annulus which has two optically independent halves. This approach works very well for strong radioactive sources but leads to a large reduction in efficiency as a direct result of the coincidence requirement. Attempts to use this technique at UNISOR failed repeatedly because the sources from the isotope separator operated on-line to the ORIC cyclotrons are far too weak to accommodate the factor between 10 and 50 loss in efficiency due to the γ - γ - β^+ coincidence requirement. In addition, it is highly desirable, if not absolutely necessary,

to identify a given positron branch with a given γ ray by requiring one more coincidence with a large Ge detector. This now requires γ - γ - γ - β^+ coincidence and further reduces the efficiency by two orders of magnitude making all such experiments virtually impossible. This past year efforts have been mainly devoted to eliminating the necessity for the NaI(Tl) annulus pair detectors, hence eliminating the need for the requirement for γ - γ coincidences between annihilation radiations. This work has been highly successful and is based on our extensive study of the response of Ge detectors to γ rays and positrons. This study involves the development of complex Monte Carlo codes which follow each positron's history in the detector until it either escapes or stops. In addition, it follows the history of both annihilation radiations until they escape or are absorbed, summing the energies of each of their interactions. In this way the details of the distortion of a theoretical, allowed, positron spectrum can be studied with particular emphasis on the shift in the observed end-point energies as a function of end-point energy and any non-linearities of the Kurie plots introduced by summing with annihilation radiations.

It turns out that positron end point energies can be accurately deduced from the experimental data by use of a very simplified version of the code which only accounts for annihilation radiation summing and does not account for positrons scattering out of the detector. This distortion leads to shifts in the measured endpoint energies which are on the order of 10%. It also results in a serious nonlinearity in the very end of the Kurie plot. The Monte Carlo simulations were done for a series of theoretical positron spectra with endpoints ranging from 2.5 to 9.0 MeV. The non-linearity is well predicted and the results of the calculations tell us very accurately what energy range of

experimental data to include in our linear analysis and what the shift in end point energy is. We have carefully tested this procedure using a radioactive source containing $^{82,85}\text{Sr}$ and $^{83,84}\text{Rb}$. The measured end-point energy was 3.19 ± 0.02 which compares well to that measured earlier in a magnetic spectrometer and with a more recent measurement using a scintillation spectrometer. In addition, more recent experiments performed at UNISOR with separated sources following heavy ion reactions clearly show that this technique is efficient enough to use in the singles mode of UNISOR. The greatest advantage of this method lies in the ease with which the detector can be accurately calibrated using γ rays.

In addition to the detector response work discussed above we have investigated the decay scheme of ^{75}Kr following the decay of ^{75}Rb which was formed in the reaction $^{58}\text{Ni}(^{20}\text{Ne}; p2n)^{75}\text{Rb}$. The products were separated in the UNISOR separator, deposited on Mylar tape and viewed by β^+ and γ - γ coincidence spectrometers. We observed 13 new γ rays and the possible existence of an isomeric level in ^{75}Kr . Seven levels in a probable decay scheme are proposed based on γ - γ coincidence data.

We have also completed the analysis of the experiments concerning the decay of $^{206,208}\text{Fr}$ produced in the reactions $\text{Ir}(^{20}\text{Ne}, xn)^{206,208}\text{Fr}$ which led to the detailed construction of the decay schemes of $^{206,208}\text{Rm}$. A final version of the resulting paper appears in an appendix. The major conclusion is that while the Independent Boson Approximation properly predicts most of the nuclear levels, they can also be predicted using a weak coupling shell model scheme. This paper is presently in press in Physical Review C and was finally accepted in January 1981.

Finally we have engaged in a preliminary investigation of the feasibility of performing γ -ray angular distributions from oriented

nuclei which are products of heavy ion reactions separated by the UNISOR isotope separator. We have determined that γ ray angular correlations are not feasible at UNISOR and that we will have to depend on decay systematics and internal conversion coefficients if we do not develop a more reliable technique. Quite aside from fixing the spins and parities of nuclear levels, angular distribution measurements can result in accurate measurements of multipole mixing ratios with an efficiency which is the square root of the efficiency of an angular correlation measurement. This can improve the efficiency by more than two orders of magnitude in a typical case.

The idea is not original with us. Professor Dennis Hamilton at the University of Sussex is using such a technique on-line at Daresbury, and it seems that it can be directly adapted to our application. This study is being done in cooperation with Professor Joseph H. Hamilton of Vanderbilt University.

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SPECTROSCOPIC AND BETA END POINT STUDIES ON LINE AT UNISOR

This past year has been one during which the ORIC cyclotron has been slowly returning to normal operation. This year the new injector Van de Graaff accelerator will become operational which will vastly enlarge the regions of the nuclide chart available for our proposed investigations. Elsewhere in this report we have shown that accurate end point energies of the highest energy positron decay branches can be measured using small intrinsic germanium detectors. The most serious difficulty is the summing of the positron energy with that of the annihilation radiations. This difficulty is also encountered when scintillation detectors are used. The main advantages of using the Ge detector are two fold. First, if there are intense γ rays in the decay, they will show up in the spectra and their effects can be subtracted. The small size of the detectors used, however, make them far more sensitive to positrons than to γ rays. This effect is difficult to detect in plastic scintillators. Second, energy calibration can be facilitated with great accuracy using γ ray sources. The only way a plastic scintillator can be energy calibrated using γ rays is to make a careful Monte Carlo simulation for each geometry and energy. Even then, the accuracy of the calibration is tens of kilovolts as opposed to a fraction of a kilovolt obtainable with the intrinsic Ge detector. This technique has been perfected by our group and we have started to use it in the Rb, Kr, Br, region of the nuclide chart. The reasons for the selection of these nuclides for investigation are several.

First, there have been some direct mass measurements by Epherre

et al¹ of the isotopes $^{74-79}\text{Rb}$ and $^{90-99}\text{Rb}$ using an isotope separator and a thick lens optical system to measure the masses of reaction products using magnetic field techniques. While our measurements will usually be more accurate, we will learn much about our technique by reproducing some of their results. In addition, there is a need to do γ - γ spectroscopy in this region so that decay schemes can be determined. Second, there has been a recent attempt by Campi and Epherre² to calculate the masses of nuclei in this region using self consistent field (Hartree-Fock) techniques. Our results to date have mainly yielded decay scheme details; however, the positron end point energy data were also collected.

The reaction used thus far for these studies was $^{62}\text{Ni} (^{20}\text{Ne}, p3n) ^{78}\text{Rb}$ using 92 MeV, ^{20}Ne ions on both enriched and natural Ni targets. Other competing reactions are $^{62}\text{Ni} (^{20}\text{Ne}, xp, yn) ^{77,78}\text{Kr}$, $^{77,78}\text{Br}$. The most strongly produced products are, in fact, these latter products; however, Kr and Br are not obtainable from the UNISOR ion source. The heavy ion reaction cross section which produces ^{78}Rb was found to be approximately 10 mb. We will extend these studies using the injected sulphur-32 beam from the tandem accelerator at Hollifield. We have determined that we will need 200 MeV³² S ions on targets of $^{46,48}\text{Ti}$. We propose to investigate the positron decays and decay schemes of $^{72,73,74}\text{Kr}$. The main motivation for starting in this region has been the recent direct mass measurements¹ using ISOLDE at CERN. An island of accurate masses are necessary from which we can link new mass measurements. In addition, there is significant disagreement between mass formulae in this region. For example, the liquid drop formula makes better predictions in this region whereas the formulae of Liran and Zeldes³ seems to work better in the region of the tin isotopes. These

issues cannot be settled until better values are obtained for the masses of ^{75}Rb , ^{77}Rb and ^{76}Rb and of their Krypton daughters. In addition, the direct mass measurements of ^{74}Rb and ^{75}Rb could be coupled with decay scheme data to yield accurate masses for ^{74}Kr and ^{75}Kr .

The main results we have to report here concern the γ ray spectroscopy data in the ^{75}Rb experiments. We have found 13 new γ rays in ^{75}Rb whereas previously only one γ ray at 178.85 keV had been observed. We have collected this data in a format whereby each event is tagged by a pulse height from each detector and a time-to-amplitude pulse height. Coincidence relationships were obtained by scanning the data tapes using standard software in our PDP/II system, for example, and the results are shown in the table below. From this table and γ ray intensity data, we have constructed the decay scheme shown below.

The final results of our studies of this region over the next year should result in a series of decay schemes for Kr, isotopes and accurate positron end point energies. The one major failure we have experienced has been in coincidence measurements of positron branches. The sources are still not intense enough to allow adequate statistics to be acquired with such small detectors but the use of small detectors is absolutely necessary if γ ray, positron summing is to be avoided. We propose to try a thin NaI(Tl) detector with a thin window for coincidence measurements. The high resolution singles data will fix the Q value while the branches can be approximately fixed by scintillator. The NaI(Tl) scintillator is easier to energy-calibrate than the plastic scintillator because there are full energy peaks. We shall also try a much larger Ge detector and propose to experimentally determine the

largest Ge detector we can use before γ - β^+ summing becomes a serious problem. This must be determined experimentally because the coincidence requirement tends to eliminate most, and in some cases, all of the summing.

The test of our small 1.6 cm diameter by 0.7 cm thick intrinsic Ge detector for radioactive sources of ^{82}Sr were highly successful; however, we had many early failures in our attempts to measure end-point energies with this detector and its predecessor. The major problem had been the loss of efficiency encountered by requiring annihilation radiation coincidences in an external, double NaI(Tl) annulus. Finally this year, two cyclotron runs have given us singles and coincidence data which show that this technique will work without the annulus requirement.

The ORIC cyclotron was used to produce a ^{20}Ne beam to produce the reactions $^{60}\text{Ni}(^{20}\text{Ne};p2n)^{77}\text{Rb}$ and $^{60}\text{Ni}(^{20}\text{Ne};pn)^{78}\text{Rb}$. The positron decays to excited states of the daughter isotopes were studied using β^+ - γ coincidences using the large Ge γ -ray detector and the small Ge positron spectrometer. While the analysis of the data is still in progress, we can present reasonably final results for the decay of ^{77}Rb to the excited states of ^{77}Kr .

Coincidences of positrons with the 66, 179, 245, 394 and 511 keV γ -rays were investigated. The raw positron end point energies, the coincident γ ray energy and corrected end point energies are given in the table below. The corrections appear in the preprint in the appendix.

$E_0(\text{raw})$	E_0	$E_\gamma(\text{coinc})$
4179 $\pm$ 20	3987 $\pm$ 20	66
4000 $\pm$ 60	3808 $\pm$ 60	179
3995 $\pm$ 30	3803 $\pm$ 30	245
3801 $\pm$ 50	3618 $\pm$ 50	394
4180 $\pm$ 20	3988 $\pm$ 20	511

A variety of combinations of these numbers can be used to obtain the positron Q-value of the decay of ^{77}Rb . These are obtained by adding $E_0(\beta^+)$ to the total γ ray energy required to reach the ground state, we have:

- 1) $(3987 \pm 20) + 66 = 4053 \pm 20$
- 2) $(3808 \pm 60) + 179 + 66 = 4053 \pm 60$
- 3) $(3803 \pm 30) + 245 = 4048 \pm 30$
- 4) $(3618 \pm 50) + 394 + 66 = 4078 \pm 50$
- 5) $(3988 \pm 20) + 66 = 4054 \pm 20$

where we have neglected the small error in the γ ray energies. In this case we can obtain a weighted average, and the corresponding Q-value.

$$Q = (4053 \pm 10) + 1022 = (5.075 \pm 0.010) \text{ MeV}$$

It is possible to make a systematic error in energy calibration here; however, this has been carefully checked. The calculated corrections include interference effects as well as resolution effects.

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⁷⁵Rb COINCIDENCE COUNTS

ENERGY (keV)	GATE														
	179	187	253	314	358	432	450	475	493	511	643	654	671	854	863
171.39		13													
178.86*	1005		148	61		11	78	21	50	693	13	20	8		13
186.86*										55				9	
253.19	44				7					37					
313.70	63				11										
357.96			19	13						45					
432.29	22														
450.10	53														
474.61	32				6										
492.68	77														
511.00	934	96	129	6	27			6	24	1432	29		6	5	
643.15	9									41					
654.24	24														
671.28	25									21					
689.86**	58		17							202					
854.27		10								10					
862.53	13									13					

* Gamma rays identified by lifetime of 18 sec.

** Sum peak of 179 and 511 gamma rays.

PROPOSED LEVEL SCHEME OF ^{75}Kr

Energy (keV)

CALCULATIONS OF THE INTERFERENCE OF ANNIHILATION
RADIATIONS WITH POSITRON SPECTRA IN A Ge DETECTOR

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The distortion of the Kurie plots of allowed positron spectra due to annihilation radiations was calculated by a simple Monte-Carlo technique for a small intrinsic Ge detector. The experimentally observed non-linearity near the end point is accurately reproduced by the calculations. Corrections were calculated for 15 theoretical allowed spectra with end-point energies ranging from 2.5 to 10 MeV for one small detector 1.6 cm in diameter and 0.7 cm thick and one larger detector 4 cm in diameter and 1 cm thick. The major effect of this source of interference is to shift the measured end-point up in energy 182 keV at 2.5 MeV to 204 keV at 9 MeV in the small detector and from 279 keV at 2.5 MeV to 321 keV at 9 MeV in the larger detector. The method was used to correct the measured end-point energies of the two positron branches in the decay of ^{82}Sr . The corrected data give values of 3.19 ± 0.02 and 2.42 ± 0.02 MeV. The resulting Q-value is 4.21 ± 0.02 MeV.

I. Introduction

One of the most fundamental quantities of interest in nuclear spectroscopic studies is the beta decay Q-value. This quantity can easily be determined from the beta decay end-point energy measurements and a knowledge of the decay scheme of both the parent and daughter nuclides. In the case of ground-state to ground-state beta decays, knowledge of the beta end-point energy is sufficient. In measurements involving long-lived radioactive sources, high resolution magnetic spectrometer measurements result in the most accurate and reliable data. In cases which involve short lived sources produced in nuclear reactions, plastic scintillators and solid state detectors are frequently used. For high energy beta decay measurements, Aleklett¹⁾ introduced a complex system in which the beta flux impinges on a flat cut edge of a Si(Li) detector and in which electrons scattered out of the faces are detected by two other Si(Li) detectors which generate anticoincidence pulses. Moore Hayakawa and Rehfield²⁾ showed that an intrinsic Ge detector can be used with a superconducting magnetic filter to make accurate end-point energy measurements while Girard and Avignone³⁾ showed that careful selection of the detector dimensions will allow accurate measurements even without a magnetic filter. This latter development reduces such measurements to approximately the same simplicity as making x-ray measurements. Recently this technique has been used routinely to measure β^- end-point energies at the TRISTAN isotope separator at the Brookhaven National Laboratory.⁴⁾

The purpose of this note is to report an important extension of this simple technique for high energy positron end-point energy measurements. The added difficulty associated with positrons is the direct sum interference with the annihilation radiations. In our earlier experiments the Ge detector was placed between two wrap-around NaI(Tl) detectors which covered more than 90% of the total solid angle. A γ - γ - β^+ , tripple coincidence requirement eliminates

almost all of the interference from annihilation radiations while a comparison of the annulus on to annulus off data indicated that the Kurie plot suffers a significant non-linearity starting at approximately 250 to 300 keV below the end-point energy, while the NaI(Tl) detectors clearly eliminated this problem, they introduced another. Many of positron branches of interest are in coincidence with γ rays or γ ray cascades, which with significant probability, sum with the annihilation radiations in one or both of the NaI(Tl) detectors and sweep the annihilation radiation from the energy window set on the corresponding differential discriminator. This reduced the efficiency drastically in some cases, and resulted in rendering this technique inadequate for its intended application, namely measuring end-point energies of short-lived sources from the on-line isotope separator at Oak Ridge (UNISOR).

In the present work we report the results of a series of Monte-Carlo calculations which were used to predict the effect of the annihilation radiations on the shape of the observed Kurie plot. These calculations are an application of a more general response code developed by one of us (F.T.A.)⁵⁾ While this work mainly involved calculation of correction factors it also entailed a careful measurement of the end-point energies of the two highest positron branches in the decay of ^{82}Sr . The calculations are shown to accurately predict the non-linearity near the end-point while also resulting in corrections to the data which result in a final result which is in far better agreement with one of the earlier measurements, using a magnetic spectrometer than one which used a plastic scintillation spectrometer. It is possible that either resolution effects or inadequate correction for the effects of the annihilation radiations in the plastic scintillation spectrometer are responsible for this.

II. The Monte-Carlo Code

The flow diagram for the code is shown in Figs. 1 and 2. Very simply this code calculates the Kurie function both with and without the annihilation radiation interference. At the top of Fig. 1 we define NTOP which is the number of channels, the atomic mass and atomic number, the phase of the Fermi function which is (-1) for positrons, the end-point energy EEND, the random number generator seed number IX, the half depth of the detector, the slope of the spectrum in MeV/channel, the intercept and the detector radius. The quantity NPRINT is the number of channels of data to be printed while the array BNETH(NCH) is the actual spectrum. Statement 999 reads a data card which contains a data point of the allowed positron spectrum. The subroutine Kurie computes the Kurie function which can be printed at this point. The quantity NB is the number of positrons of energy EBET. The statement 888 controls how many positrons (NB) will be analyzed at this energy. The next box is an excellent approximation used only for small detector and which saves a great deal of computer time. It assumes all positrons are stopped at the center of the detector. Subroutine EN511 is a simplified version of the code given in ref. (5) and determines how much energy is deposited in the Ge detector. When called, it returns the quantity DPAN which is the γ ray energy deposited which in the next box is added to the positron energy while in the next box the total energy, EDPT of this event is smeared by a Gaussian smearing routine. In the next box, the appropriate element of the array NBETH(NCH) is given one event. If at this point the number of positrons NB at energy EBET has not been exceeded, the code returns to 888 where another annihilation γ ray pair is analyzed. This is repeated until IPOS = NB in which case another card is read. When the last data card is read at statement 999, the code eventually passes the EOF (end of file) and the distorted energy spectrum is printed. In the printing loop, the Kurie plot is computed and the printed array is actually this function.

The approximations made above are excellent for a detector of dimensions, 1 cm thick by 4.0 cm in diameter or smaller. These approximations are easily tested by moving the positron stopping point around inside of the detector volume. It is not very difficult to properly distribute the annihilation events throughout the detector, but the computer time is vastly increased with almost no change in the results.

III. Results and Conclusions

The code was run for positron end-point energies between 2.5 and 10.0 MeV. The points were then plotted so that the break from linearity could be observed visually. A straight line was fit to the linear position of the distorted Kurie plot and the end-point energy corresponding to this best fit straight line was determined and compared to those of the Kurie plots without interference from annihilation radiations. The plots for the larger detector are shown in Fig. (3) for allowed positron spectra with end-point energies of 2.8, 5 and 7 MeV. The calculated energy shifts and correction factors are given in Tables I and II. The shift in the end-point energy as a function of energy was best fit to a series of polynomials and it was found that a second order polynomial reproduces the calculated shifts within a fraction of a percent for end-point energies between 2.5 and 10.0 MeV. The best polynomial were found to be,

$$\Delta E_0 = 0.1572 + 0.0115 E_0 - 0.0007 E_0^2, \text{ (1.6 cm by 0.7 cm)} \quad (1)$$

$$\text{and } \Delta E_0 = 0.2412 + 0.0185 E_0 - 0.00107 E_0^2, \text{ (4.0 cm by 1.0 cm)} \quad (2)$$

where E_0 is the correct end-point energy in MeV.

These calculated interference effects are significant, hence to gain a more complete understanding of them, the code was run for 1.2×10^6 positrons

at a single positron energy in the larger detector. The energy spectrum deposited by the annihilation radiations was thus calculated. A small peak was observed at $2m_e c^2$ above the unique positron energy which corresponds to a probability that both γ rays will be captured $P(2\gamma) = 0.0035$. The first escape peak was larger and corresponds to a probability $P(1\gamma) = 0.0705$ that one γ ray will be captured while the other escapes. The total probability that some energy will be deposited by the annihilation radiation pair was found to be $P(\gamma) = 0.4932$.

The above results are strictly calculated results and to this point no test of even the most general features of these calculations has been discussed. In this work we have not attempted a complete experimental test of such calculations throughout the energy range of interest; however, several careful measurements of two positron spectra were made and the corrected results were compared with values which appear in the literature. A mixed radioactive source containing 2.4 millicuries ^{82}Sr , 5 millicuries of ^{85}Sr and trace quantities of $^{83,84}\text{Rb}$ and ^{54}Mn was obtained from the Los Alamos Scientific Laboratory. The ^{82}Sr decays by electron capture to ^{82}Rb with a half life of 25 days which in turn decays to ^{82}Kr by positron decay with an electron capture Q-value of approximately 4 MeV. The ground-state decay branching ratio is 0.87. There have been two previous measurements of the two highest energy positron branches in the decay. An early measurement by Castner and Templeton⁶⁾, using a $\pi\sqrt{2}$ magnetic spectrometer, resulted in a value of 3.15 ± 0.03 MeV for the end-point energy of the ground state branch. A later measurement by Beck⁷⁾ employed a two-piece plastic scintillation spectrometer in which the response to monoenergetic electrons was measured using a betatron. An approximate experimental method of correcting for the summing of the positron and annihilation photon energies was used. The approximation was made by having holes drilled in a similar scintillator and placing a source of ^{18}F at various special locations

which simulates a variety of positron energies. Their resulting end-point energies for the two highest energy positrons reported in ref.(7), are 3.35 ± 0.06 and 2.57 ± 0.06 MeV.

The measurements of the present investigation were made to test the calculational techniques both qualitatively and quantitatively as well as to attempt to remove the discrepancy between previous measurements. It can be seen by reference to Fig. 4 that the Monte-Carlo calculations predict the degree of non-linearity near the end-point energy fairly accurately. This allows the calculations to be used as guidance concerning which data points should be included in the Kurie plot. The uncorrected end-point energies, for two the most energetic positron branches, were found to be 3.38 ± 0.02 and 2.60 ± 0.02 MeV respectively. The corrected values are 3.19 ± 0.02 and 2.42 ± 0.02 MeV respectively. The present results agree much better with the measurements which used the $\Pi\sqrt{2}$ spectrometer⁶⁾ and this could well be due to the approximate method used to correct the data from the plastic spectrometer measurements.⁷⁾

In general, for in beam measurements, one can calibrate a positron detector directly using (p,n) reactions resulting in the positron emitters ^{28}P , ^{46}V , ^{50}Mn , ^{54}Co and ^{58}Cu . End-points from 7.05 to 12.56 MeV are available from this set of reaction products which can be obtained using protons from 12 to 16.5 MeV and the appropriate targets.⁸⁾ For use with an isotope separator on-line to a heavy ion cyclotron; however, it is most convenient to calibrate with γ rays. In this case a correction on the order of 1 to 10% must be made and it is somewhat surprising that a very simple approximation leads to a high degree of accuracy. The main advantage of the Ge detector over the magnetic spectrometer is the fact that it records all energies simultaneously while its main advantage over plastic scintillation spectrometers is the superior energy resolution.

Our experience with these detectors strongly suggests that smaller detectors (1.6 cm in diameter and 0.7 cm thick), should be used for single measurements; however, larger detectors (4.0 cm in diameter by 1.0 cm) would be quite acceptable for coincidence measurements. The use of such a large detector for single measurements might well result in coincidence summing of γ rays in coincidence with positrons which can also be a source of serious interference. These interferences are sharply reduced by the coincidence requirement.

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TABLE I. Calculated End-Point Energy Shifts And Correction Factors For A Detector Of Dimensions (1.6 cm in diameter by 0.7 cm thick).

End-Point Energy E_0 (MeV)	Shifted End-Point Energy (MeV)	Energy Shift ΔE_0	Correction [†] Factor F_0
2.500	2.682	0.182	1.0728
2.600	2.782	0.182	1.0700
2.700	2.883	0.183	1.0678
2.800	2.984	0.184	1.0657
2.900	3.085	0.185	1.0638
3.000	3.185	0.185	1.0617
3.300	3.488	0.188	1.0570
3.500	3.689	0.189	1.0540
4.000	4.192	0.192	1.0480
4.500	4.695	0.195	1.0433
5.000	5.197	0.197	1.0394
5.500	5.699	0.199	1.0362
6.000	6.201	0.201	1.0335
7.000	7.203	0.203	1.0290
8.000	8.204	0.204	1.0255
9.000	9.204	0.204	1.0227

[†] E_0 (observed/ F_0 = E^0 (corrected)).

TABLE II. Calculated End-Point Energy Shifts And Correction Factors For
A Detector Of Dimensions (4.0 cm in diameter by 1.0 cm thick).

End-Point Energy E_0 (MeV)	Shifted End-Point Energy (MeV)	Energy Shift ΔE_0 (MeV)	Correction, Factor $F_0^\dagger$
2.500	2.781	0.281	1.1124
2.600	2.882	0.282	1.1085
2.700	2.983	0.283	1.1048
2.800	3.085	0.285	1.1018
2.900	3.186	0.286	1.0986
3.000	3.287	0.287	1.0957
3.300	3.591	0.291	1.0882
3.500	3.793	0.293	1.0837
4.000	4.298	0.298	1.0745
4.500	4.803	0.303	1.0673
5.000	5.307	0.307	1.0614
5.500	5.811	0.311	1.0566
6.000	6.314	0.314	1.0523
7.000	7.318	0.318	1.0454
8.000	8.321	0.321	1.0402
9.000	9.321	0.321	1.0357

$^\dagger E_0 (\text{observed})/F_0 = E_0(\text{corrected})$.

CAPTIONS FOR THE FIGURES

Fig. 1, 2. Flow diagram of the Monte Carlo computer code used to calculate the interference effects of annihilation radiations with positron spectra observed in intrinsic Ge detectors.

Fig. 3. High energy portion of the predicted Kurie Plots of allowed positron spectra observed in a 4 cm. diameter by 1 cm thick intrinsic Ge detector. The straight lines are the Kurie Plots without the annihilation radiation interference.

Fig. 4. Experimental and Monte Carlo prediction of the high energy portion of the positron spectrum in the decay of ^{82}Sr .

SUBROUTINE EN511

SUBROUTINE A511

ELECTRON CAPTURE AND POSITRON DECAY OF
 ^{206}Fr AND ^{208}Fr
 AND THE ENERGY LEVELS OF ^{206}Rn AND ^{208}Rn .[†]

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The isotopes ^{206}Fr and ^{208}Fr were produced by the reactions $\text{Ir}(^{20}\text{Ne}, \text{xn})^{206,208}\text{Fr}$ and mass separated on-line. The electron-capture and positron decays to ^{206}Rn and ^{208}Rn were studied by collecting γ ray and internal conversion electron singles spectra as a function of decay time as well as γ - γ , γ - e^- , and γ -x ray coincidence spectra. The energies and many of the spins were determined for 18 excited, even parity states in ^{208}Rn and for 10 excited, even parity states in ^{206}Rn . These nuclei appear to be excellent candidates for interpretation in terms of a weak coupling shell model. The energy levels were also compared to the predictions of the Interacting Boson Approximation (IBA) model.

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I. INTRODUCTION.

The most useful experimental nuclear structure data for the stimulation of our theoretical understanding of nuclei are those which illuminate systematic trends. Of particular interest in this regard are data which yield the energy level structure of a series of isotopes of like Z but differing by an even number of neutrons. This gives rise to more sensitive tests of theoretical models than the knowledge of the level structure of an isolated nuclide on the N - Z plane. Such systematic knowledge is particularly valuable in testing nuclear models which have parameters which must be fixed experimentally. The parameters can be fixed using data on one nucleus in the chain and the accuracy with which properties of other members are predicted can be readily seen.

The experimental study of even-even neutron-deficient Rn nuclei is of particular interest in testing the Interacting Boson Approximation (IBA) of Arima and Iachello,^{1,2} and in testing weak coupling approximations to the full nuclear Hamiltonian, such as that of Hecht et al.³ These models have met with extensive success for several series of nuclei but have not been tested extensively for $Z > 82$ due to the paucity of nuclear structure information. Of the two nuclei selected for this investigation, ^{206}Rn has been studied with in-beam spectroscopy by Inamura et al.,⁴ Backe et al.,⁵ and Horn et al.,⁶ while the only previous study of ^{208}Rn was reported in Ref. 5. The only levels and transitions reported in these previous studies were those of the yrast cascades and in the case of ^{206}Rn , all three level schemes were in conflict.⁴⁻⁶

The major goals of the present investigation were to study the low energy level structure of $^{206,208}\text{Rn}$ populated by the electron-capture and positron decay of $^{206,208}\text{Fr}$ and to attempt an interpretation in terms of shell model states or the level structures predicted by the

IBA model. Earlier theoretical descriptions of these nuclei⁷ were hampered by the lack of experimental nuclear structure information.

II. EXPERIMENTAL PROCEDURE

The isotopes $^{206,208}\text{Fr}$ were produced via the reactions $^{\text{nat}}\text{Ir} ({}^{20}\text{Ne}, \text{xn}) {}^{206,208}\text{Fr}$ with ${}^{20}\text{Ne}^{5+}$ beams from the Oak Ridge Isochronous Cyclotron (ORIC). The data were collected in five runs totaling 150 hours and using Ne beams of 113, 111, 119, 122, and 120 MeV, respectively. The average beam intensities were approximately 200 particle-nanoamperes of heavy ion beam on the target of natural iridium evaporated onto graphite felt. Production rates of from 10^4 to 10^5 nuclei per second were observed. The target was located inside one of two types of thermal ionization sources developed at UNISOR.⁸ The reaction products were separated with the on-line UNISOR, 90° - Danfysik mass separator and deposited on a Mylar tape. The deposited source on the tape was moved periodically under computer control to detector stations after a pre-determined collection time. The beam intensity was periodically monitored by mechanical introduction of a beam stop located a short distance in front of the target. In addition, the activity at the collection point was constantly monitored with a NaI(Tl) detector. The efficiency of the ion source resulted in about 30% of this yield at the collection point of the isotope separator where it was deposited onto aluminized Mylar tape. The collection times were generally chosen to be about three half-lives of the product nuclei of interest.

The detectors used were large Ge(Li) detectors, with energy resolutions (FWHM) of 2.1 keV or less at 1.33 MeV, for γ -ray singles and γ - γ coincidence experiments and a Si(Li) detector for e^- singles and e^- - γ coincidence and x-ray experiments. Standard spectroscopy and coincidence techniques were

used. The coincidence apparatus consisted of two detectors, two fast timing filter amplifiers, two constant fraction discriminators and a time-to-amplitude converter (TAC). The tape was moved between two detectors of the coincidence geometry and three signals were stored on magnetic tape: the spectroscopy amplifier outputs from each detector, and the output of the TAC. The computer based data acquisition system has been described elsewhere.⁹ In addition, the singles data were stored on disk in planes of data representing successive time periods after the activity arrived between the detectors. These data were used to determine the half-lives of the individual spectral lines to aid in identification and association with a given nuclide. The three pieces of information comprising a coincidence event were used to study γ - γ and γ -e⁻ time correlations and coincidences.

Energy calibration of the γ -ray detectors was accomplished with a mixed radioactive source of ^{57}Co , ^{60}Co , ^{85}Sr , ^{88}Y , ^{109}Cd , ^{113}Sn , ^{137}Cs , ^{139}Ce , and ^{203}Hg obtained from the National Bureau of Standards. The Ge(Li) - Si(Li) geometries used for internal conversion coefficient measurements were calibrated with a mixed source of ^{133}Ba and ^{207}Bi . The centroids and areas were determined using a version of the peak fitting routine called "SAMPO"¹⁰ which can accomodate realistic, non-Gaussian shapes. The peak energies were determined to within 0.1 keV while the line intensities were determined to within 5%. The internal conversion coefficients were normalized to the known pure E2 ground state transition in the neighboring even-even polonium nuclei.

The coincidence relationships given in Tables I and II and the γ ray intensities were used to construct the decay schemes of $^{206,208}\text{Rn}$. Feeding of excited levels in Rn directly from the Fr parent was suggested by noting the discrepancies between transition intensities populating and

and depopulating each level, after corrections were made for internal conversion. The ordering of the transitions and levels was assigned, using the observed (γ ray + e^-) intensities, by requiring an intensity balance as well as consistency with the coincidence relationship tables. In this way, internally consistent decay schemes were constructed. Improper intensity balancing eliminates all but a very few possibilities for the correct decay scheme. An additional test was made by determining the transition intensities within the gates and comparing them with the intensities consistent with a given decay scheme.

III. RESULTS

The observed coincidence relationships for the γ rays in ^{208}Rn and ^{206}Rn are given in Tables I and II, respectively. All internal conversion electron measurements, summarized in Tables III and IV, were consistent with M1, E2, or mixtures of these two multipolarities. No evidence for any E0 transition was observed.

The information in Tables I-IV was used to construct the decay schemes discussed in more detail below. The coincidence intensity analyses applied to these decay schemes indicated that the schemes were internally consistent within experimental error. Examples of these analyses are given in Tables V and VI.

A. Decay of ^{208}Fr

Attention is now turned to a discussion of the deduced levels in the level scheme for ^{208}Rn , shown in Figure 1, for which unique spin and parity assignments have been made:

a. the 635.8, 141.3, 1739.5 and 1828.4 keV levels--These levels represent the states previously reported by Backe et al.⁵ The transitions (636, 778, 325, and 88.9 keV) connecting them compose the 8^+ to ground state (0^+) cascade as observed by Backe et al. The E2 multi-

polarities for all four transitions connecting these states unambiguously yield the spins and parity for these four levels.

b. the 1188.9 and 1825.2 keV levels--The 553.1 keV gamma ray is observed to be in coincidence with the 325.2 keV cascade gamma ray only through the 225.5 keV (M1) transition. Since no transition within the radon nucleus are observed to populate the 1825.2 keV level, that level must be fed directly by the electron capture decay of the ground state of ^{208}Fr . Since the spin of the francium parent nucleus has been determined¹¹ to be 7, the minimum physically probable spin for this level would be 6. These observations and the observed E2 multipolarity of the 553.1 and 636.3 keV gamma rays unambiguously determine the spin and parity of their associated levels.

c. other levels--The spins of the levels at 1905.7, 2128.8, 2164.4, and 1459.1 keV, though not uniquely determined, are delimited by the absence of γ feeding within the nucleus--indicating direct ($\text{EC} + \beta^+$) feeding from the parent nucleus--and the angular momentum restrictions dually imposed by the electron capture feeding from the spin 7 ground state of ^{208}Fr and by the multipolarities of their associated transitions. These limits are shown at these levels in Figure 1.

All other levels for which possible spins are shown are based solely on the angular momentum restrictions imposed by the multipolarities of the associated transitions. The TAC spectrum of the 88.9 - 325.2 keV cascade was analyzed by the slope method¹² and the half-life of the 8^+ isomeric level at 1828.4 keV was determined to be $0.35 \pm 0.22 \mu\text{sec}$, consistent with a pure E2 transition. The magnitude of this half-life precluded the use of other techniques because of the limited TAC range used. The error quoted is the statistical error in the least-squares fit to the decay curve.

B. Decay of ^{206}Fr

Based on the information contained in Tables II and IV, the decay scheme shown in Figure 2 was deduced. The scheme was determined to be energy and intensity balanced. Additional confirmation was provided by a coincidence intensity analysis. The results obtained with the coincidence intensity analysis for several transitions are shown in Table 6.

The previous observations⁴⁻⁶ of the four γ -ray cascade from the 1924.3 keV level, verified by the present data, allowed the following unique assignments: $J^\pi(575.3) = 2^+$, $J^\pi(1134.3) = 4^+$, $J^\pi(1762.9) = 6^+$, and $J^\pi(1924.3) = 8^+$. It was also determined that γ -ray feeding did not account for all of the intensities of the 559.0 and the 628.6 keV γ rays; hence, it is concluded that direct feeding of the 1134.3 and 1762.9 keV levels by electron capture is very probable. This implies that the spin of the ^{206}Fr ground state is most probably 5.

The spin and parity of the 2206.9 keV level were delimited by the internal conversion data. The half-lives of the 8^+ level at 1924.3 keV and the 6^+ level at 1762.7 keV were measured by analysing the TAC spectra from the 282.6 - 161.4 - 628.6 keV cascades. The spectra were analyzed using the centroid shift and deconvolution moment methods¹² with the results: $T_{1/2}(1924.3) = 6.3 \pm 2.4$ and $T_{1/2}(1762.9) = 1.8 \pm 1.3$ ns.

IV. THEORETICAL CONSIDERATIONS

A general shell model treatment of nuclei in the region of the doubly magic ^{208}Pb nucleus is in principle very attractive because of the established stability of the ^{208}Pb core. The fact that there are many low-lying single particle levels with relatively high spins results in very large Hamiltonian matrices beyond the capability of current

diagonalization techniques. McGrory and Kuo¹³ have pointed out that the large neutron excess in this region leads one to the reasonable simplifying assumption that the neutron-proton interaction plays a relatively minor role. The plausibility of this assumption rests on the fact that the valence neutrons and protons mainly occupy very different orbits. The protons are filling the $g_{9/2}$ level while the neutron holes exist in the top of the shell where $J = 1/2$ and $3/2$, hence the principle and orbital quantum numbers are not similar, which are the criteria for strong n-p interactions. The general approach is to treat the neutrons (or neutron holes) and protons separately in a standard shell model calculation and then to couple them via a so-called "weak coupling" of the protons, and in this case, neutron holes.

No such detailed calculations exist in the literature for $^{206,208}\text{Rn}$; however, very simple arguments can be made from which it can be concluded that these nuclei are excellent candidates for such an interpretation. The ^{208}Rn nucleus can be pictured as the ^{208}Pb core with four neutron holes and four protons. In the weak-coupling scheme ^{208}Rn may be considered in a first approximation as being a mixture of the four neutron holes of ^{204}Pb and four protons of ^{212}Rn . The weak n-p coupling then must lower the first excited $2+$ state, for example, and with the correct interaction give the experimental spectrum of ^{208}Rn . The energy levels of $^{206,208}\text{Rn}$ along with their particle or hole neighbors which would participate in the weak coupling scheme are shown in Figure 3.

Another approach to the interpretation of these nuclei is the Interacting Boson Approximation (IBA). We will not give a lengthy theoretical discussion here; however, for completeness, it is necessary

to give a condensed introduction to the salient features of the model as well as a brief description of the symmetry limits which can be related to well known macroscopic models, for example, rotational or vibrational models. The IBA is based on the assumption that like valence nucleons couple through a dominant pairing interaction as they do, for example, in the BCS model. In the simplest version of the model nucleon pairs are assumed to couple to angular momentum $L = 0$ or $L = 2$. The coupled fermion pairs are called s and d bosons, respectively.

Using the s- and d- bosons, their six components span a six dimensional space and provide a linear basis for an $SU(6)$ group representation. The group $SU(6)$ has only three decompositions: $SU(6) \rightarrow SU(5) \times U(1)$, $SU(6) \rightarrow SU(3) \times SU(2)$ and $SU(6) \rightarrow O(6) \rightarrow O(5) \rightarrow O(3)$. These decompositions represent three limits of the Hamiltonian. If the s-boson degrees of freedom are neglected, only the second and third terms of the Hamiltonian are maintained involving only four parameters. This is called the $SU(5)$ or d-boson limit.^{14,15} The energy spectra of this limit are typically vibrational. The other symmetry limits lead to rotational spectra¹⁶ and a spectrum resembling a γ -soft vibrator.¹⁷

The use of the IBA in analyzing the deduced decay schemes must be undertaken with some caution. The nuclei studied lie near ^{208}Pb and the application of any collective model in fitting the nuclear level schemes may not represent a reasonable test of the overall value of the model or its underlying assumptions. An alternative approach such as weak coupling, may provide a more complete explanation of the observed features of the nuclei, but such calculations have not been undertaken due to the lack of data in the region. The application of the IBA to

the nuclei discussed here, then, represents only an initial theoretical explanation of the observed phenomena.

As a first approximation to a description of the more detailed level scheme of ^{208}Rn , analytical expressions representing each of the symmetry limits of the full Hamiltonian were applied to the spectrum of ^{208}Rn . This nucleus, with four protons and four neutron holes outside the ^{208}Pb core, would have four bosons. When the limit formulae were applied, only the SU(5) limit was found to properly order the levels observed experimentally by energy and spin. The 8^+ level at 1828.4 keV was not included in these or subsequent calculations since its probable two particle nature⁵ would preclude an explanation of that state as the four d-boson, 8^+ level in an IBA description.

Using a set of initial SU(5) parameters, the computer program "PHINT"¹⁸ was used to calculate the theoretical spectrum using the full IBA Hamiltonian. The program allows the computer to vary any combination of the nine parameters to minimize the energy-weighted chi-square difference between the experimental and theoretical spectra. In the analysis used in this study, initially only the 0^+ , 2^+ , 4^+ , and 6^+ levels were fit.

It was found that only five of the nine parameters are needed to achieve good agreement between theory and experiment for ^{208}Rn . These five parameters are the four SU(5) parameters and the coefficient for the term

$$(d^+d^+)(0)_{(ss)}(0) + (dd)(0)_{(s^+s^+)}(0) \quad (0)$$

The theoretical spectrum obtained is compared with the experimental spectrum in Figure 4. It is seen that the theoretical spectrum shows very good agreement with the observed spectrum, correctly predicting the known spins for the low lying states or predicting spins within

the range of possible spins. Many more low spin states are predicted than are observed and are not shown in Figure 4. The presence of these states cannot be ruled out based on this study since they would not be populated from the Fr ground state and would be "shielded" by the higher spin states. Additional efforts to find them must await further in-beam and decay studies.

Above the energy of the 8^+ isomeric level, the experimental level density in Figure 1 can be seen to increase significantly. Such an increase generally occurs as a critical energy is reached above which non-collective effects dominate. In particular, at this energy it may be possible to break the closed core and excite nucleons from it. Since the IBA requires that boson number be conserved, spectra above this critical energy are not completely explicable in terms of the model. The IBA predicts only one state above this point, the 8^+ level, which appears to have been found in the experimental spectra.

There is insufficient experimental information at this time to assess predictions of the IBA for electromagnetic transitions. It can be noted, however, that the 778 keV, $4_2^+ \rightarrow 2^+$ transition is ruled out by the model: The 4_2^+ state is a three d-boson state, while the 2^+ state is a single d-boson state. As can be seen from the selection rules for E2 transitions,¹⁴ the number of d-bosons in a nucleus cannot change by 2 with an E2 transition. This state presents an important problem in interpreting the observed ^{208}Rn level scheme in terms of the IBA. There are two possible explanations: (a) the level may not be an IBA state, or (b) it may represent a new mode of excitation within the model, namely a (L=4) or "g"-boson. A definite conclusion concerning the nature of this state must await further experimental data. Of particular value would be the measurement of the half-life of this state.

The level scheme deduced for ^{206}Rn in this study and that deduced for ^{204}Rn by Backe et al⁵ do not yet possess sufficient detail to permit as complete an analysis of their structures with the IBA as was possible for ^{208}Rn . Figure 5 shows the results of the IBA calculations performed for these nuclei using PHINT. For those levels deduced in the two nuclei, good agreement exists between theory and experiment. However, much greater detail in these two level schemes, particularly in ^{204}Rn , will be required if conclusions concerning the applicability of the IBA to these nuclei are to be complete. Also, as in the case of ^{208}Rn , electromagnetic transitions predicted by the IBA cannot be extensively tested at this time because there is still insufficient experimental data.

V. SUMMARY AND CONCLUSIONS

Nuclear spectroscopic techniques with mass separated sources of $^{206,208}\text{Fr}$ have permitted the unambiguous determination of the level schemes of $^{206,208}\text{Rn}$ with significantly more known levels, spins and parities than previously determined. The theoretical interpretation of the levels of $^{206,208}\text{Rn}$, in terms of a weak coupling shell model looks very promising. These level schemes also compared well to those predicted by the Interacting Boson Approximation; however, there is still insufficient data on transition probabilities to test this aspect of the theory. Future experimental investigations of these and other nuclei in this region should probably concentrate on electromagnetic transition probabilities to subject the IBA to more stringent tests.

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Table I. Coincidence relationships established by gamma-gamma gates for ^{208}Rn transitions. "x" denotes appearance of given line in gated spectra. "?" denotes marginal significance. Only gates with adequate statistics to deduce coincidence relationships are shown.

Gated energy (keV)	Gamma energy (keV)																		
	89	225	299	325	335	389	470	492	511	553	636	671	691	717	720	778	887	943	990
225				x						x	x								
299								x	x										
325	x	x			x		x		?		x			?	x	x			
335				x					x		x								
389				x					x	x	x					x			
470										x	x	x							
492			x						x										
553		x	x	x		x	x				x	x		x					x
636	x	x		x	x	x	x	x		x	x		x	x	x	x	x	x	x
671							x			x	x								
690				x					?	x	x								
717				?						x	x					?			
720				x						x	x					x			
778	x			x	x	x		x			x		x						
943											x					x			
990										x	x								

Table II. Gamma ray transitions observed in ^{208}Rn . Theoretical internal conversion coefficients (ICC's) are from the Hager-Seltzer algorithm ³³ for the multipolarity shown in the last column.

Gamma ray Energy (keV)	Relative Photon Intensity	Internal shell	Conversion Coefficient ($\times 10^{-2}$)		Multi-polarity
			experimental	theoretical	
88.9 $\pm$ 0.1	1.5 $\pm$ 0.3	L	1110 $\pm$ 170	1050	E2
225.5 $\pm$ 0.2	0.77 $\pm$ 0.10	K	122 $\pm$ 12	112	M1
298.7 $\pm$ 0.1	0.95 $\pm$ 0.10	K	29.1 $\pm$ 4.3	29.0	M1 + E2 ^a
325.2 $\pm$ 0.2	53.2 $\pm$ 3.8	K	4.4 $\pm$ 0.2	5.8	E2
335.0 $\pm$ 0.3	12.3 $\pm$ 5.0	K	39.5 $\pm$ 16.0	38	M1
389.3 $\pm$ 0.3 ^b	4.0 $\pm$ 2.2	K	27.9 $\pm$ 10.0	25	M1
389.3 $\pm$ 0.3	2.8 $\pm$ 2.2	K	4.0 $\pm$ 2.0	4	E2
469.8 $\pm$ 0.1	5.7 $\pm$ 0.4	K	14.6 $\pm$ 1.5	15.1	M1
491.9 $\pm$ 0.1	2.8 $\pm$ 0.2	K	2.5 $\pm$ 0.5	2.5	E2
553.1 $\pm$ 0.1	31.0 $\pm$ 2.2	K	1.7 $\pm$ 0.1	2.0	E2
635.8 $\pm$ 0.2	100.0	K	1.4 $\pm$ 0.1	1.5	E2
636.3 $\pm$ 0.2	6.4 $\pm$ 2.8	K	1.4 $\pm$ 0.1	1.5	E2
671.6 $\pm$ 0.1 ^c	1.3 $\pm$ 0.9				
690.5 $\pm$ 0.1 ^{c,d}	2.2 $\pm$ 0.2				
716.8 $\pm$ 0.1	4.0 $\pm$ 2.8	K	2.5 $\pm$ 1.0	1.2	E2
719.6 $\pm$ 0.1	6.6 $\pm$ 0.5	K	1.1 $\pm$ 0.2	1.2	E2
778.5 $\pm$ 0.1 ^{c,d}	68.9 $\pm$ 4.9	K	1.0 $\pm$ 0.1	1.0	E2
887.3 $\pm$ 0.1 ^{c,d}	1.7 $\pm$ 0.4				
942.5 $\pm$ 0.1 ^{c,d}	4.6 $\pm$ 0.3				
990.1 $\pm$ 0.1 ^{c,d}	4.1 $\pm$ 0.8				

- Notes: a Mixing ratio (δ^2) is 1.0 $\pm$ 0.2.
 b Doublet line not resolvable. ICC's are based on M1 and E2 pure components.
 c Insufficient data to deduce ICC.
 d Line could not be assigned in decay scheme.

Table III. Analysis of coincidence gates for ^{206}Rn transitions.
"x" denotes appearance of given line in gated spectra.

Gate Energy (keV)	Gamma Energy (keV)												
	161	198	274	283	346	356	559	575	629	684	720	890	926
161				X			X	X	X				
198							X						
283	X						X	X	X				
346	X						X	X	X				
356								X		X			
559								X	X	X		X	
575	X		X	X			X		X	X	X		X
629	X			X	X		X	X					
684					X		X	X					
890				X			X	X					
926								X					

Table IV. Gamma ray transitions observed in ^{206}Rn . Theoretical internal conversion coefficients (ICC's) are from the Hager-Seltzer algorithm.³³ Energies are in keV, and have errors of ± 0.1 keV. ICC's are for the multipolarities shown in the last column.

Gamma ray Energy	Relative Intensity	shell	ICC ($\times 10^{-2}$)		Multi- polarity
			experimental	theoretical	
161.4 ^a	10.0 $\pm$ 2.2	L	0.65 $\pm$ 0.07	0.681	E2
197.8 ^a	4.9 $\pm$ 1.0	K	0.34 $\pm$ 0.17	0.546	E2
274.9 ^a	2.4 $\pm$ 0.4				
282.6	8.3 $\pm$ 0.9	K	0.51 $\pm$ 0.07	0.60	M1
345.6	3.3 $\pm$ 1.4				
356.0	3.1 $\pm$ 2.0				
559.0	70.0 $\pm$ 2.1	K	0.015 $\pm$ 0.005	0.019	E2
575.3	100.0	K	0.013 $\pm$ 0.005	0.018	E2
628.6	31.0 $\pm$ 2.7	K	0.016 $\pm$ 0.005	0.015	E2
684.0	8.3 $\pm$ 2.1				
890.6	6.1 $\pm$ 2.0				
926.5	8.0 $\pm$ 2.0				

Notes: a. Not placed in decay scheme.

Table V. Examples of coincidence intensity analysis for selected ^{208}Rn gates. Energies are in keV; errors are approximately 25% for all intensities.

Gated Energy	Coincident Transition Energy	Relative Intensity In Gated Spectrum	Relative Intensity In Gated Spectrum Calculated From Decay Scheme
325	89	3.0	2.8
	225	1.9	1.4
	389	6.6	7.5
	469	0.7	0.0
	553	1.9	3.0
	635	100	100
	720	2.3 ^a	12.8
	778	100	100
389	325	51	59
	511	28	-
	553	47	41
	636	100	100
	778	76	59
553	225	4.6	2.4
	325	4.4	5.2
	389	7.5	10
	469	18	18
	511	4.1	
	636	100	100
	671	3.1	4.0
	718	35 ¹	13
	990	9.1	13

^a Coincides with a sum peak; difficult to ascertain intensity.

Table VI Examples of coincidence intensity analysis for selected ^{206}Rn gates. Energies are in keV; errors are approximately 25% for all intensities.

Gated Energy	Coincident Transition Energy	Relative Intensity in Gated Spectrum	Relative Intensity In Gated Spectrum Calculated from Decay Scheme
282	161	84	100
	511	35	-
	559	90	100
	575	100	100
	629	77	100
629	161	26	31
	282	23	23
	511	15	-
	559	100	100
	575	97	100
684	356	36	37
	559	100	100
	575	88	100

FIGURE CAPTIONS

- Figure 1. Deduced level scheme for ^{208}Rn . Transition and level energies are in keV. Deduced transition multipolarities are indicated. Small numbers following transition energy denote relative transition intensities. The 942.5 keV line and the 235.7 keV level are suspect due to poor statistics and contamination in that gate.
- Figure 2. Deduced level scheme for ^{206}Rn . Deduced transition multipolarities are indicated. Small numbers following transition energies denote relative transition intensities.
- Figure 3. Experimentally determined level schemes for ^{202}Pb , ^{206}Rn , ^{212}Rn , ^{208}Rn and ^{204}Pb from Reference 19.
- Figure 4. Comparison of deduced level scheme for ^{208}Rn with the results of the IBA calculations. Energies are in keV. IBA levels which correspond to experimental levels are shown.
- Figure 5. Comparison of deduced level schemes for ^{204}Rn and ^{206}Rn with the results of IBA calculations. The ^{204}Rn level scheme is from Backe et al.⁵

^{208}Rn

 ^{206}Rn

²⁰⁸Rn

EXPERIMENTAL RESULTS

IBA CALCULATIONS

(7,8)+	2164	2159	(8+)
<hr/>			

6+	1825	1833
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6+	1739	1731
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(3-5)+	1659	1693	(4+)
<hr/>			

+	1578	1560	(5+)
<hr/>			

4+	1414	1418
<hr/>		

4+	1189	1186
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2+	636	648
<hr/>		

0+	0	0
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^{204}Rn

EXPT. THEO.

$8+$ 2101 / 2132

$6+$ 1802 / 1900

$4+$ 1128 - 1128

$2+$ 542 - 544

$0+$ - 0

^{206}Rn

56

EXPT. THEO.

$(6+)$ 1818 / 1891
 $6+$ 1763 - 1753

$(4+)$ 1501 / 1550

$4+$ 1135 / 1158

$2+$ 575 - 565

$0+$ - 0

PROGRAM TO CALCULATE THE RESPONSE OF Ge to BETA PARTICLES

This program is designed to calculate the beta response function in a pure Ge detector. The multiple elastic scattering formula given by Molière is used to calculate the angular distribution and the step thickness. A lower limit of the scattering beta energy is 50 keV. The energy loss at each step is calculated using a Landau distribution as well as the most probable energy loss with the polarization effect. The total energy deposited in the detector is smeared by the Gaussian noise distribution.

While all functions of the program have been documented, we are only now beginning to obtain results on the predictions of the response of Ge detectors to positrons. The program described in Appendix A accounts only for the interference due to annihilation radiations. The present code accounts for electrons or positrons scattering out of the detector as well as for bremsstrahlung radiation. This level of sophistication of this program is not necessary to correct Q-value measurements but will be necessary to interpret branching ratio data when more than one β^+ branch is present. The flow chart is given because this program has wide application. This code will be used to predict a variety of responses which will also have to be experimentally tested.

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1-2 Initial condition

1-3 General Definition

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3-3 Subroutine RMTX (THET, FI, SX, SY, SZ)

3-4 Subroutine LANDAU (E, THICK, ED)

3-5 Subroutine ESTEP (E, THICK, ED)

3-6 Subroutine SMEAR (ETOT, SLP, EB, IDEP)

3-7 Subroutine RANDM (RN)

1. DEFINITION

1-1 Initial procedure

COMMON IX: Random number
 RCL: Detector radius
 H: Detector half height
 R (3,3): Rotation matrix of each step in the multiple scattering.

DIMENSION

NBETA (NTOP): Beta counts in a channel.
 NTOP: Maximum channel number
 N: Trial number
 NLIMIT: Maximum trial number
 B= NBACK: Beta escape number by back scattering
 S= NSID: Beta escape number by side scattering
 F= NFOR: Beta escape number by forward scattering
 SLP: Slope of calibration curve
 EB: Intercept of calibration curve

1-2 Initial condition

EI: Initial beta energy in MEV
 PI=(0,0,-H): Initial point
 SI=(0,0,1): Initial direction unit vector
 R(3,3)=1 : Reset of rotation matrix in Sub.
 RMATX by SI=R(3,3)*(0,0,1)
 N=NBACK=NSID=NFOR=0: Reset of trial and escape numbers.
 NBETA(L)=0: Reset of beta spectrum

1-3 General Definition

EN: E Beta energy after n-th step
 ENP1: E_{n+1}
 SN=(SX,SY,SZ): S_n Direction unit vector at n-th step
 SNP1: S_{n+1}
 PN=(XN,YN,ZN): P_n Point after n-th step
 PNP1: P_{n+1}
 THET = θ , FI = ψ : Multiple scattering angle
 THICK=X: Scattering thickness at each step
 EDN: ED Deposit energy at n-th step
 EDNP1: $E_{D,n+1}$
 ETOT, ENGY: Total deposit energy
 IDEP, NCH: Channel number at ETOT
 EDA: Deposit energy by annihilation
 PA=(PAX, PAY, PAZ): Annihilation point