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THE NUMERICAL CALCULATION OF THE THREE DIMENSIONAL ELECTRICAL FIELD IN THE CENTRAL REGION OF A CYCLOTRON

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Summary

To provide detailed and accurate electric fields in the ion source-puller region and at the dee dummy-dee gap for a cyclotron, a relaxation method solution of Laplace's equation has been used. A conventional difference equation with variation in mesh size and relaxation factor as well as different schemes for boundary corrections have been developed to achieve roughly 1 percent accuracy for a three-dimensional domain with 10^6 mesh points. Although the computation requires considerable computer time, it is much less expensive than electrolytic tank analogue methods for measuring field distributions around complex electrode configurations.

Introduction

The electric fields in the central region of a cyclotron are complex, and depend on the configuration of the ion source, RF puller electrodes, and dee geometry. The RF electric fields play a significant role in ion extraction from the source and in determining the phase space which is available for injection into the conventional acceleration phase of a cyclotron. In that phase the transverse electric forces acting on the particles are small and the fields primarily act as an impulse for energy gain at dee gap crossings.

The electric field contours in the central region of a cyclotron are usually obtained by mapping in an electrolytic tank, but two problems have always existed. First, near the electrodes the accuracy of the measurement is sensitive to the specific boundary conditions. Second, a two-dimensional map usually takes a number of hours and a three-dimensional map will require a much longer time. There are substantial errors in extended measurements; long periods elapse between designing a new geometry and having the components to make a measurement. These factors provided the motivation for developing a numerical method to compute an electric field map in the central region of the Colorado cyclotron. Even with a numerical method, difficulties are encountered related to the accuracy and computational time needed.

Description of the program

Recently a program called "CR" has been developed which can be used for the calculation of the three dimensional electric field in the cyclotron central region. Briefly, the program calculates from the difference equation for Laplaces equation

$$h^2\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{2y^2}\right) = V_A + V_B + V_C + V_D - 4V_0$$

the value of the residual R at every point V.

$$R = V_A + V_B + V_C + V_D - 4V_0.$$

The value of V_0 is corrected by the value of fR

$$V_0' = V_0 - \frac{fR}{4}.$$

f is a relaxation factor. This process is iterated until suitable values of R are achieved.

This program consists of five successive programs:

1. "Center."

This routine is used to assign the initial potential value to ion source, puller, dee, and dummy dee and anything attached to them, as well as all non-boundary mesh points. Regions where the electric field effectively vanishes are arbitrarily assigned appropriate potentials to limit the total space over which the field is calculated. Usually, the initial potential value at all non-boundary mesh points is set to be one-half the maximum potential value at specific electrodes. In doing so, we found that many mesh points will not be relaxed over many iterations. Therefore, we use "Center" to fix different potential values provided by intuition or results of two-dimensional field maps in the appropriate regions.

2. "CR."

This is the main program and is used to calculate the three-dimensional field map with a boundary smoothing correction and with or without higher order difference correction. There are also three options for boundary correction and an option for choice of mesh size.

3. "Change."

This program compares results of "CR" with results from a previous iteration to determine the approach of the relaxed solution to an asymptotic solution. The best basis for judgement is the magnitude of the residual of the relaxation. In a finite-difference approximation to the Laplace equation, it is obvious that the physical meaning of the residual at any mesh point is equal to a charge placed at that mesh point. The relaxation method may be thought of as a charge elimination process.

4. "Field."

This program is used to provide a two-dimensional electric field map at any vertical and horizontal plane from the three-dimensional potential map.

5. "Vfield."

Since the three-dimensional field map is often calculated by using different mesh sizes in different regions, it is necessary to transfer the non-uniform map into a uniform mesh size map.

Applications

In the application of the relaxation process a number of special problems arose. We first calculated the two-dimensional field of an off-center cylinder, whose analytic function was known, to check the

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accuracy and behavior. We then calculated the two-dimensional field across the dee to dummy-dee gap (Fig. 1) as well as the three dimensional field within the ion source-puller region. Fig. 2 shows a model of the ion source and puller while Fig. 3 shows a section view of the ion-source puller geometry.

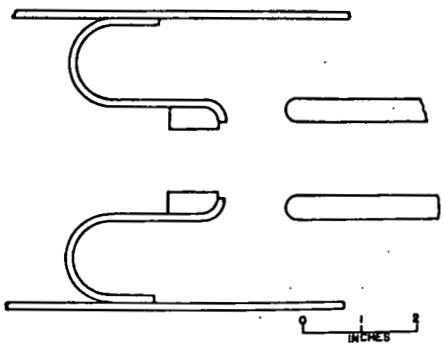

FIG. 1
DEE DUMMY-DEE SECTION

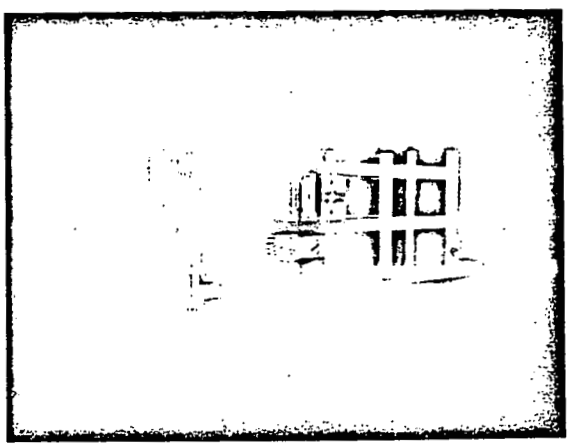

Fig. 2. Ion source puller model, full scale size.

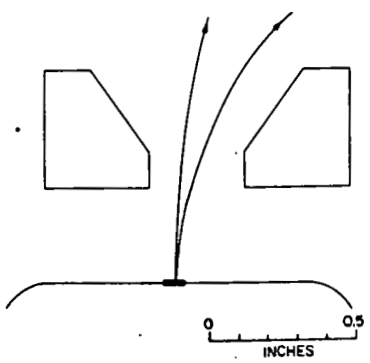

FIG. 3
ION SOURCE MEDIAN PLANE SECTION

The following problems are discussed:

1. Definition of mesh size

A major difficulty encountered in the use of this code is the large number of mesh points necessary to solve a three-dimensional problem. For example, assuming a mesh size of 0.04", a region 4 by 8 by 2.25 inches on x, y, and z coordinates will contain about 10^6 mesh points. It is impossible to store the required data in core, so more time is used in read-write operations from disk files than in actual computation. As a result the mesh size is chosen so that in regions requiring detailed fields appropriate mesh size is used while in regions where the precision is less critical much coarser grids are used.

2. Definition of the boundary

Potentials are specified for all boundaries. However, to restrict the size of the relaxation domain, regions where the field becomes small can be truncated by specifying an artificial boundary with a given potential.

In the ion source-puller region, fields in the dee gap are important for accelerated beam trajectories. Results from a two-dimensional section across the dee-dummy dee gap indicate for our geometry (see Fig. 1) at a distance inside the dee of about two vertical gap heights the potential is within 1 percent of a constant value so the dee and dummy-dee were both truncated at 2.5 vertical gap distances. Errors in these regions are further minimized because of dee voltage reversal when ions are in these regions. Furthermore, after a few revolutions by the ions the electric forces are very small compared to magnetic forces.

3. Boundary correction

A problem giving some trouble is the boundary smoothing correction. There are several methods for treating this problem. The linear interpolation of Southwell[1] and the polynomial interpolation of Fox[2] yield the following relations for the finite-difference two-dimensional relation.

$$h^2\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial V}{\partial y^2}\right) = AV_A + BV_B + CV_C + DV_D - (E+F)V_0.$$

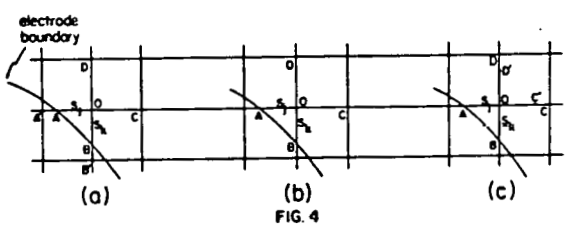

FIG. 4
BOUNDARY MESH POINT INTERPOLATION SCHEMES

Table 1

Comparison of computed and analytics potentials—off center cylinders.*

Method	(a) (with)	(a) (without)	(b) (with)	(b) (without)	(c) (without)	Measured (MSU)
RMS	12.63	7.86	5.75	11.88	1.37	Larger than 26
ΔV	-32	-3	-20	-31	-5	
ΔV/V**	-0.69	-0.75	-0.50	-0.78	-0.125	

* Cylinders are of diameter 0.5 and 2 inches, respectively, and placed 0.2 inches off center.

**Calculated difference between relaxed and analytic values as a fraction of applied potential in percent.

Table 2

Number of iterations to achieve an assigned residual.

Relaxation factor	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
Iteration	84	69	56	46	37	62	68	57	86	200	200

a) See Fig. 4. Southwell's[1] linear interpolation sets

$$A=\frac{1}{S_j}\ ;\ B=\frac{1}{S_k}\ ;\ C=1;\ D=1;\ E=1+\frac{1}{S_j}\ \text{and}\ F=1+\frac{1}{S_k}\ .$$

S_j and S_k are the fraction of the mesh dimension at which the boundary intersects.

b) Likewise, for the polynomial interpolation of Fox[2] one sets

$$A=\frac{2}{S_j(1+S_j)};\ B=\frac{2}{S_k(1+S_k)}\ ;\ C=\frac{2}{1+S_j}\ ;\ D=\frac{2}{1+S_k}\ ;$$

$$E=\frac{2}{S_j}\ \text{and}\ F=\frac{2}{S_k}\ .$$

c) We have also used a modified method[3] for which we set

$$A=\frac{1}{S_j^2}\ ;\ B=\frac{1}{S_k^2};\ C=\frac{1}{S_j};\quad D=\frac{1}{S_k}\ ;\ E=\frac{1+S_j}{S_j^2}$$

$$\text{and}\ F=\frac{1+S_k}{S_k^2}\ .$$

The linear method extrapolates from boundary points A and B to A' and B' (see Fig. 4), whereas the third method interpolates from mesh points C and D to C' and D'.

All three methods used with or without a higher order difference correction have been applied to the off-center cylinder and compared with analytic results. The root mean square (RMS) deviation measures an average fit while ΔV records the largest deviation between the relaxed solution and the analytic solution. (See Table 1.) A comparison with a set of electrolytic tank data[4] is also shown. The method (c) yielded the best result yet did not use the higher order correction. Computational speed is enhanced when higher order equations are not used since less read-write time is required.

4. Relaxation factor

Since the relaxation process involves calculating the residual, R, at every point and correcting the potential V_0 using R, a relaxation factor f such that R'=fR is assigned. Table 2 shows results for convergence of potentials in the two dimensional off-center cylinder problem for different values of f. A value of f=1.4 is optimum and represents a slight overcorrection. In different regions of the map, particularly along the boundaries, the value of f may be varied.

The field map obtained with this program has been used to generate ion orbits between the ion source and through the puller as well as across the dee gap.

The calculation does require substantial computer time, many hours on a PDP 11-60, but offers the convenience of rapid comparison of various geometries and configurations.

Acknowledgements

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References

1. R. V. Southwell and D. G. Christopherson, Proc. Roy. Soc. A 168, 317-350 (1938).
2. L. Fox, Proc. Roy. Soc. A 190, 31-59 (1947).
3. C. U. Cyclotron internal report, M. B. Chen, 1981.
4. These data were measured when M. B. Chen was a visitor at the Michigan State University cyclotron.