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TRAPPED PARTICLE DESTABILIZATION OF THE INTERNAL KINK MODE

By

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ABSTRACT

The internal kink mode is destabilized by trapped high energy particles, leading to a new branch of the internal kink dispersion relation with a real frequency near the average trapped particle precession frequency and a growth rate of the same magnitude. This trapped particle branch of the dispersion relation is investigated numerically for a variety of particle distributions. Mode growth rate and frequency is found as a function of plasma β , density, and trapped particle energy and distribution. The high energy trapped particle sources considered are neutral beam injection, ion cyclotron heating, and fusion alpha particles. Relevance for various plasma heating schemes is discussed.

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I. INTRODUCTION

Trapped particle effects on plasma stability have been a subject of much theoretical study, but trapped particle instabilities have thus far failed to play an important role in the description of large scale plasma behavior. Recently, however, instabilities have been observed, associated with neutral beam injection heating in tokamaks, which appear to be partly due to trapped particle effects.¹⁻⁴ In previous work⁵ it was shown that near marginal stability for the internal kink mode, high energy trapped particles introduce an additional destabilizing influence, producing a new branch to the internal kink mode dispersion relation, with a low threshold in the high energy trapped particle β , typically below one percent. Here β is the ratio between particle and magnetic pressures. This trapped particle branch has a frequency near the toroidal precession frequency of the trapped particles and a growth rate of the same magnitude. For a model slowing-down beam particle distribution with a single value of $\alpha = \mu/E$, with μ the magnetic moment and E the energy, the dispersion relation was calculated explicitly.

Further, it was shown⁶ that a mode with low toroidal and poloidal mode numbers, rotating toroidally in resonance with the trapped particles, is capable of rapidly ejecting them from the plasma. These two features provide the basis for a model of the full fishbone instability cycle. Simulations of this cycle using a Hamiltonian guiding center orbit code will be reported in a future publication. In this work we examine the dispersion relation for more general particle distributions. For simulation of the full cycle the particle dynamics under the action of the mode is followed through a Monte Carlo discrete particle representation. In order to study the coupled system of particle distribution and mode, we adopt a discrete particle representation of the dispersion relation. The dispersion relation is first discussed

qualitatively using the Nyquist method and then investigated numerically, and we examine the dependence on hot particle distribution, background plasma beta, and current profile. In Sec. II the dispersion relation for the internal kink mode in a plasma with a hot particle component is derived. In Sec. III the general properties of the solution are discussed, using a Nyquist analysis, and it is shown that they are independent of the analytical form of the distribution function. In Sec. IV the numerical solution of the dispersion relation is discussed as a function of the parameters describing the relatively cold background plasma and the hot particles. Significance for plasma heating is discussed in Sec. V.

II. DISPERSION RELATION

The plasma is considered to consist of two components, a relatively cold MHD part, and a hot trapped particle component, treated with a gyrokinetic description.^{7,8} In the gyrokinetic description each particle is characterized by an energy E , magnetic moment μ , and minor radius r . Finite banana width corrections to the particle dynamics and path integrals are ignored. The magnetic field is approximated by a low β circular equilibrium field, $B = (1 - r \cos \theta)$, where B is normalized to its value on axis, and r to the major radius. The particle energy is

$$E = \frac{1}{2} v_{\parallel}^2 + \mu B + \phi \quad (1)$$

where ϕ is the potential energy associated with an electric field, and we have taken the particle mass to be unity. The volume in velocity space then becomes

$$d^3v = \frac{2\pi v_{\perp} dv_{\perp} dv_{\parallel}}{(2E)^{1/2}} = \sum_{\sigma} \frac{2\pi B dE d\mu}{(2E)^{1/2} [1 - \alpha^2]^{1/2}} \quad (2)$$

where $\sigma = \text{sign}(v_{\parallel})$, and $\alpha = \mu/E$. The volume in the toroidal coordinates ϕ, r, θ is

$$d^3x = X d\phi r d\theta dr \quad (3)$$

with $X = R + r \cos \theta$.

We begin with the expressions for the ideal MHD and the kinetic energy, given in Ref. 5. The ideal MHD energy is well known.⁹ In deriving the expression for the kinetic energy of the trapped particle component, several physical assumptions were made. The calculation is done only to lowest order in the inverse aspect ratio $\epsilon = a/R$. The relative temperatures of the cold and hot components is $T_c/T_h \sim O(\epsilon^2)$. The poloidal beta values of the two components are taken to be of order $\beta_{pc} \sim O(1)$ and $\beta_{ph} \sim O(\epsilon)$. Thus $n_h/n_c \sim O(\epsilon^3)$ and overall charge neutrality may be assumed. For mode frequencies small compared to hot particle transit frequencies and bounce frequencies, the perturbed trapped and untrapped distributions can be found. A variational dispersion functional is then found of the form

$$D(\xi) = \delta W_{\text{MHD}} + \delta W_k + \delta I \quad (4)$$

where δW_{MHD} and δW_k refer to MHD and kinetic contributions and the inertial term is

$$\delta I = -\frac{1}{2} \omega^2 \int d^3x \rho_m |\xi|^2 \quad (5)$$

There is an inertial singular layer of a width $\Delta = (\omega/\omega_A)a$ at $q(r_s) = 1$, and only in this layer is the contribution of δI significant, due to the singular nature of the plasma flow there. Here $\omega_A = v_A/(\beta^{1/2} R \hat{s})$ and $\hat{s} = r_s q_s'$ is the local shear. Let $D = D_e + D_s$, where D_e and D_s are the contributions from outside and inside the singular layer, respectively. For the case of circular cross sections we have, outside the singular layer, ξ^e given by Bussac *et al.*⁹ We then obtain $D_e = \delta^{(2)} W_{MHD}(\xi^e) + \delta^{(2)} W_K(\xi^e) + O(\epsilon^4)$. These MHD and kinetic contributions to D are given by

$$\begin{aligned} \delta W_{MHD} = \frac{1}{2} \int d^3x \left\{ \frac{|\delta B_\perp|^2}{4\pi} - \frac{j_\parallel}{c} (\xi_\perp^* \times \xi_\parallel) \cdot \delta B_\perp - 2(\xi_\perp^* \cdot \nabla p)(\xi_\perp^* \cdot \kappa) \right. \\ \left. + \frac{B^2}{4\pi} |\nabla \cdot \xi_\perp + 2\xi_\perp \cdot \kappa|^2 + \gamma p_c |\nabla \cdot \xi|^2 \right\} \end{aligned} \quad (6)$$

$$\delta W_K = 2^{9/2} \pi^3 m_n \int R B r dr \int d\mu \int_0^\infty dE E^{3/2} K_b \frac{\bar{J}^* Q \bar{J}}{(\omega_A - \omega)} \quad (7)$$

The function K_b , related to the particle bounce period, is given by

$$K_b = \frac{1}{\pi} \int_0^{\theta_b} \frac{d\theta}{\{1 - \mu E/E\}^{1/2}} = \left(\frac{2}{r\alpha}\right)^{1/2} \frac{K(k^2)}{\pi} \quad (8)$$

where θ_b is the bounce angle, $B(r, \theta_b) = E/\mu$. In addition, we will need the bounce averaged quantity

$$K_2 = \frac{1}{\pi} \int_0^{\theta_b} \frac{d\theta \cos \theta}{\{1 - \mu B/E\}^{1/2}} = \left(\frac{2}{r\alpha}\right)^{1/2} \frac{\{2E(k^2) - K(k^2)\}}{\pi} \quad (9)$$

where $E(k^2)$, $K(k^2)$ are complete elliptic integrals with $k^2 = (E/\mu + r - 1)/(2r)$.

The bounce averaged toroidal precession frequency is¹⁰

$$\omega_d = \frac{[2E(k^2) - K(k^2)]Eq}{mrK(k^2)\omega_o R} = \frac{K_2 Eq}{K_b mr\omega_o R} \quad (10)$$

and ω_o is the cyclotron frequency. The integral Eq. (7) defines $\delta\psi_k$ in the upper half ω plane, with definition in the lower half plane being given by analytic continuation, similar to the definition of the plasma dispersion function.¹¹ The function $J = (\alpha B/2) \vec{\nabla} \cdot \vec{\xi}_1 - [1 - (3\alpha B/2)] \vec{\xi}_1 \cdot \vec{\kappa}$ with $\vec{\kappa}$ the field curvature, $\vec{\kappa} = \partial \vec{e}_\parallel / \partial l$, $Q = \omega \partial F / \partial E - (\partial F / \partial r) / (\omega_o r)$, and F is the hot particle distribution function.

Minimization of $D(\vec{\xi})$ to lowest order in ϵ involves only δW_{MHD} because of the ordering, and gives $\vec{\nabla} \cdot \vec{\xi}_1 = -2\vec{\xi}_1 \cdot \vec{\kappa}$. To evaluate δW_k , the lowest order expression for $\vec{\xi}$ is sufficient, i.e., $\vec{\xi}_1 = (1, i)\xi_o \exp[i(\phi - \theta)]$, for $0 < r < r_s$ and zero for $r > r_s$. Using this expression, we find the bounce averaged J to be

$$\bar{J} = \frac{K_2}{K_b} \frac{\xi_o}{2R} \exp(i\phi) \quad (11)$$

The inertial term δI is found by solving the Euler equation for $\vec{\xi}$ within the singular layer and matching it to the external solution.¹² The internal kink mode is distinct from other ideal MHD modes in that the singular nature of the flow in the singular layer makes the inertial term linear in ω , converting the ideal dispersion relation from an equation for ω^2 to an equation for ω . We thus find

$$D_s = \delta W_{MHD}^{(2)}(\vec{\xi}^s) + \delta I^{(2)}(\vec{\xi}^s) + O(\epsilon^4) = 2\pi R \left(\frac{Br_s}{2R}\right)^2 |\xi_o|^2 \left(-\frac{i\omega}{\omega_A}\right) + O(\epsilon^4) \quad (12)$$

Define the normalized distribution function through $f(r, \alpha, \epsilon) = 2^{5/2} R \epsilon^{1/2}$

F/N_p where N_p is the number of particles. Then $d^3N = N_p f K_b r dr d\alpha dE$. We then find for the average trapped particle β within $q = 1$, $\beta = [f d^3x (8\pi P_\perp)/B^2]/V$ with $P_\perp = \int d^3v EF$, the expression

$$\beta = c_\beta \int r dr d\alpha dE K_b EF \quad (13)$$

with $c_\beta = (4\pi^2 N_p)/(R B r_s^2)$.

Substituting $\xi_\perp^+$ and F into δW_k , normalizing δW through $\hat{\delta W} = 2R\delta W/(\pi^2 B^2 r_s^2 \xi_0^2)$, dropping terms of higher order in ϵ , and integrating by parts, we find

$$\hat{\delta W}_k = \frac{c_\beta}{2} \int r dr d\alpha dE f \left[E \omega \alpha \frac{\partial}{\partial \alpha} \left(\frac{K_2^2}{(\omega_d - \omega) K_b} \right) + \frac{E^2}{\omega_0 r} \frac{\partial}{\partial r} \left(\frac{K_2^2}{(\omega_d - \omega) K_b} \right) \right] \quad (14)$$

Part of the contribution of δW_{MHD} is due to the hot component pressure $P_\perp$, coming from the term $-\int d^3x (\xi_\perp \cdot \nabla P_\perp) (\xi_\perp^* \cdot \kappa)$. Substituting $P_\perp$, we find this contribution to be

$$\hat{\delta W}_{MHD, h} = -\frac{c_\beta}{2} \int r dr d\alpha dE f \frac{K_2^2 E}{r} \quad (15)$$

Combining the MHD and kinetic parts, we then find for the dispersion relation

$$0 = D(\omega) \equiv -\frac{i\omega}{A} + \frac{\hat{\delta W}_c}{\omega} + \frac{c_\beta \omega}{2} \int r dr d\alpha dE f E \left(\frac{(A-B)}{\omega_d - \omega} - \frac{D\omega}{(\omega_d - \omega)^2} \right) \quad (16)$$

where

$$A = \left[\frac{RK_2^2}{r} + R \frac{dK_2^2}{dr} + \alpha \frac{d}{d\alpha} \left(\frac{K_2^2}{K_b} \right) \right] \quad (17)$$

$$B = \left(K_2 R \frac{d\omega_d}{dr} + \alpha \frac{K_2^2}{K_b} \frac{d\omega_d}{d\alpha} \right) \left(\frac{1}{\omega_d} \right) \quad (18)$$

Energy dependence in Eq. (16) appears through $f(r, \alpha, E)$ and $\omega_d = g(r, \alpha)E$. Evaluating this expression at threshold, $\omega = \omega_r + i\gamma$, $\gamma \rightarrow 0$, $\gamma > 0$, we find from the imaginary part $1/\omega_A = \pi C_{\beta 0} R(\omega_r)/2$, giving for the threshold β ,

$$\beta_c = \frac{2}{\pi \omega_A R(\omega_r)} \int r dr d\alpha dE K_b E f \quad (19)$$

with the residue given by

$$R(\omega_r) = \int r dr d\alpha \left[\frac{(A-B)fE}{q} - \frac{\omega_r}{2} \frac{d}{dE}(fE) \right] \frac{\omega_d}{|\omega_d|} \quad (20)$$

evaluated at $E = \omega_r/g$. The threshold frequency ω_r is determined by the real part of Eq. (16), which contains $\delta \hat{\omega}_c$, i.e., it is given by solving

$$0 = \delta \hat{\omega}_c + \frac{C_{\beta 0}}{2} \int r dr d\alpha dE f E \left[\frac{(A-B)}{1-\Omega} - \frac{B\Omega}{(1-\Omega)^2} \right] \quad (21)$$

with $\Omega = \omega_r/\omega_d + i\delta$, $\delta \rightarrow 0$, $\delta > 0$. Thus ω_r is given by a weighted integral of ω_d , and in general the threshold β value depends on $\delta \hat{\omega}_c$ through ω_r . In the lower half plane $2\pi iR$ must be added to the integral in Eq. (16), with R evaluated at the complex frequency ω .

The solution to Eq. (21) is clearly dependent on the particle distribution. However, we have found that the general features of the solution, and to a large degree its quantitative properties, are independent of the detailed form of the distribution f . The cases we have examined numerically include model distributions for neutral beam injection, ion cyclotron heating, and fusion product α -particles. In addition, we have used

particle distributions¹³ generated by codes which simulate realistic neutral beam injection, including slow down and scattering, as well as the modification of these distributions produced by the fishbone mode. The essential properties of the solutions are identical for all these cases. We illustrate the behavior of the solution using two examples, model distributions for neutral beam injection and ion cyclotron heating. In the case of neutral beam injection, the initial pitch of a particle is determined by the field at the point of ionization and the injection angle. In the large aspect ratio approximation this value is approximately the same for all beam particles, $v_{\parallel}/v = \lambda_0$, which corresponds to a fixed value of $\mu/E = (1-\lambda_0^2)/B$. Slowing down of the beam through electron drag, described by $dE/dt = -\nu E$ then gives a distribution of the form $f(E) \sim 1/E$, $E < E_{\max}$.

In the case of ion cyclotron heating the banana tips of a resonant particle must remain on the resonant surface, $R = R_L$, which implies a unique value of $\alpha = R_L/R_0$ for heating. In this case, we assume that the energy distribution assumes the form of a Maxwellian, $f(E) \sim e^{-E/E_m}$.

III. GENERAL PROPERTIES OF THE SOLUTION

While the exact solution of Eq. (16) requires the fully numerical evaluation of δW_k , examined in the next section, we first discuss the general properties of the solution. Our aim is to derive some general criteria for the occurrence of the instability as well as to check the dependence on the particular form of the distribution function used. We use a Nyquist analysis, which is probably the most suitable among the analytical approaches. Consider a path in the complex ω plane which runs between $-\infty$ and $+\infty$ just above the real axis and then closes along a half circle in the upper half plane, shown in Fig. 1. The number of times the equivalent path in the complex D plane

encircles the origin equals the number of unstable roots of the equation $D(\omega) = 0$. In order to obtain a simple expression for $D(\omega)$ we evaluate the contribution of the trapped particles retaining only the average effect of the curvature and so we assume that the arguments of the elliptic integrals are not functions of r and α , but can be replaced by some mean value. Under this approximation the radial dependence of the toroidal drift frequency is retained through the poloidal wave number only [the explicit $1/r$ dependence in Eq. (10)]. In addition, we take a distribution function which is zero except for $\alpha = \alpha_0$. Writing $f(r, \alpha, E) = n_0(r) f(E) \delta(\alpha - \alpha_0)$ and using Eq. (13), Eq. (16) can be written

$$D(\omega) = -i \frac{\omega}{\omega_A} + \delta \hat{W}_C + \frac{3}{4a} \frac{1}{K} \frac{\beta}{r_s} \frac{\omega}{\omega_{ds}} \left[\hat{A} - \hat{B} - \hat{B} \omega \frac{\partial}{\partial \omega} \right] \int_{\eta}^1 dt t^{1/2} H\left(t \frac{\omega}{\omega_{ds}}\right) \quad (22)$$

where $t = r/r_s$,

$$\omega_{ds} = \frac{(2E-K) E_m q}{r_s K \omega_0},$$

$$H(\xi) \equiv \int_0^\infty dE \frac{E f(E)}{E - \xi},$$

$$\hat{A} = \frac{1}{2} (2E-K) \left[1 - 2 \frac{d}{dk} \frac{E}{K} \right] - \left[\frac{E}{K} + k^2 - 1 \right] \frac{d}{dk} \frac{(2E-K)}{2}, \quad (23)$$

$$\hat{B} = - (2E-K) - 2 \left[\frac{E}{K} + k^2 - 1 \right] K \frac{d}{dk} \frac{E}{K},$$

$$a = \int_0^\infty dE E f(E),$$

and all elliptic integrals are independent of r . In the following we shall assume $n = |1/\alpha_0 - 1| R/r_s \ll 1$. In the expression for ω_{ds} , E_m is a

characteristic energy, equal to the temperature in the case of a Maxwellian distribution function $f(E) \sim \exp(-E/E_m)$, and to the maximum energy in the case of a slowing down distribution function, $f(E) \sim 1/E$.

A. Slowing-down beam population

For the beam case the function $H(\xi)$ reduces to

$$H(\xi) = \ln \left(1 - \frac{1}{\xi} \right). \quad (24)$$

Carrying out the radial integration, the final expression for $D(\omega)$ becomes

$$D(\omega) = -i \frac{\omega}{\omega_A} + \delta \hat{w}_c + \frac{1}{2} \frac{1}{K} \frac{\beta}{r_s} \{ (\hat{A} - \hat{B}) \ln \left(1 - \frac{1}{\Omega} \right) + \right. \\ \left. + \left(\hat{A} + \frac{1}{2} \hat{B} \right) \left[\Omega^{-1/2} \ln \frac{1 + \Omega^{1/2}}{1 - \Omega^{1/2}} - 2 \right] \right\} \quad (25)$$

where $\Omega = \omega/\omega_{ds}$. If the $(\hat{A} + \hat{B}/2)$ term is neglected, $D(\omega)$ reduces to the expression studied in Ref. 5. The effect of the new term is negligible for $\omega < \omega_{ds}$. It is due to particles at small r with $\omega_d > \omega_{ds}$. Due to the nonanalyticity of the function $H(\xi)$, we have to deform the path in the complex ω plane around the points $\omega = 0$ and $\omega = \omega_{ds}$, as shown in Fig. 1. Looking first at the Nyquist diagram and assuming $\omega_{ds} > 0$ (beam particles predominantly in the bad curvature region), it is seen that the imaginary part of D is given by

$$-\omega/\omega_A$$

$$\omega < 0$$

$$\text{Im} D = -(\omega/\omega_A)(1-\beta/\beta_C)$$

$$0 < \omega < \omega_{ds}$$

$$(26)$$

$$-\omega/\omega_A + \frac{\pi}{2} \frac{\beta}{r_s} \frac{1}{K} \left(\hat{A} + \frac{1}{2} \hat{B} \right) \left(\frac{\omega}{\omega_{ds}} \right)^{-1/2}$$

$$\omega > \omega_{ds}$$

if in Eq. (25) the $\hat{A} + \hat{B}/2$ term is neglected, then $\text{Im } D=0$ for $\omega \in C_1$ if $\beta < \beta_C$ and for $\omega \in C_2$ if $\beta > \beta_C$, (see Fig. 1) where β_C is defined by

$$\beta_C = \frac{2}{\pi} \frac{r_s}{R} \frac{\omega_{ds}}{\omega_A} \frac{K}{\hat{A}-\hat{B}} \quad (27)$$

Taking into account the expression for $\text{Re } D$, we find that for instability either $\delta \hat{W}_C < 0$ and $\beta < \beta_C$, or simply $\beta > \beta_C$. These two conditions refer to two different branches. Note that in both cases the path in the D plane encircles the origin only once and so for a given value of β and $\delta \hat{W}_C$ at most one branch of the dispersion relation can be unstable. The unstable branch is a MHD mode or a trapped particle (T) mode depending on the value of γ at $\beta=0$. The Nyquist diagrams for $\beta < \beta_C$ and $\beta > \beta_C$ are shown in Fig. 2. We can gain further insight into the behavior of the solution of Eq. (25) by examining the threshold frequency for $\beta = \beta_C$, which lies in the range $0 < \omega < \omega_{ds}$. To this aim we can use the simplified form of the real part of Eq. (25),

$$\gamma = \delta \hat{W}_C - \frac{1}{\pi} \frac{\omega}{\omega_A} \ln \left| 1 - \frac{\omega_{ds}}{\omega} \right| \quad (28)$$

The second term on the r.h.s. of Eq. (28) has a maximum for $\omega = (2/9) \omega_{ds}$, a zero for $\omega = (1/2) \omega_{ds}$ and goes to $-\infty$ for $\omega \rightarrow \omega_{ds}$. Thus Eq. (28) defines two branches for $0 < \delta \hat{W}_C < \delta \hat{W}_{C_1} \approx -(\omega_d/\pi \omega_A) (2/9) \ln (7/2)$ while for

$\delta\hat{W}_C < \delta W_{C1}$ no marginal modes are present. Linearizing Eq. (25) around the threshold solution we find for the growth rate

$$\gamma = -\omega_r \frac{\beta - \beta_C}{\beta_C} \frac{\pi \omega_r}{\omega_{ds}} \frac{1}{\ln(\omega_{ds}/\omega_r - 1) - \omega_r/(\omega_{ds} - \omega_r)} \quad (29)$$

where ω_r is the solution of Eq. (28). From Eq. (29) it is possible to see that the MHD mode is unstable for $\beta < \beta_C$ while the T mode is unstable for $\beta > \beta_C$. For $\delta\hat{W}_C > 0$ Eq. (28) admits only one solution in the range $0 < \omega < \omega_{ds}$ which corresponds to a T mode and for $\delta\hat{W}_C \rightarrow \infty$ ($\omega_0 \rightarrow \omega_{ds}$) the growth rate becomes exponentially small, $\gamma \sim \exp(-\pi [\omega_A/2\omega_{ds}] \delta\hat{W}_C)$. If the second term in Eq. (25) is taken into account, the behavior of the solution of the dispersion relation changes only for $\omega > \omega_{ds}$. In particular, the only relevant effect is that the T mode is stable for

$$\beta < \frac{\pi \beta_C \delta\hat{W}_C \omega_A}{\omega_d [1 + (2\hat{A} + \hat{B})/(\hat{A} - \hat{B})]} \quad (30)$$

and $\beta \gg \beta_C$. In fact for smaller β , in the domain given by Eq. (30) and $\beta \gtrsim \beta_C$, the mode, while unstable, has an exponentially small growth rate, and thus in practice Eq. (30) can be regarded as defining a stable regime for all $\beta < \beta_C$.

B. Maxwellian distribution

In the case of a Maxwellian distribution function, $H(\xi)$ is related to the plasma dispersion function

$$H(\xi) = \sqrt{\pi} \left[\frac{1}{2} + \xi + \xi^{3/2} Z(\xi^{1/2}) \right] \quad (31)$$

and the usual analytic continuation for $\text{Im } \xi < 0$ holds. Using the relation $Z'(t) = -2[1+t Z(t)]$, it is easy to carry out the radial integration in Eq. (22). The expression for $D(\omega)$ is

$$D(\omega) = -i \frac{\omega}{\omega_A} + \frac{\hat{W}_C}{K} + \frac{1}{K} \frac{\beta R}{r_s} \left\{ \left(\hat{A} + \frac{1}{2} \hat{B} \right) i \sqrt{\pi} \Omega^{-1/2} - \left(\hat{A} + \frac{1}{2} \hat{B} \right) \Omega \left[1 + \frac{4}{\Omega} \Omega^{-3/2} (\Omega^2 + 2\Omega + 2) Z(\Omega^{1/2}) \right] - \hat{B} \Omega \left(\frac{1}{2} + \Omega + \Omega^{3/2} Z(\Omega^{1/2}) \right) \right\} \quad (32)$$

Besides the $\omega=0$ solution, the real zeros of the imaginary part of $D(\omega)$ are given, after some algebra, by the roots of the following equation

$$\frac{\omega_{ds}}{\omega_A \sqrt{\pi}} \frac{r_s}{\beta R} = \Omega^{-3/2} \left\{ 2(\hat{A} + \hat{B}/2) - \left\{ (\hat{A} + \hat{B}/2)(\Omega^2 + 2\Omega + 2) + \hat{B}\Omega^3 \right\} e^{-\Omega} \right\} \equiv h(\Omega) \quad (33)$$

The r.h.s. of Eq. (33) is positive and has a maximum for $\Omega = \Omega_0 \sim 1$. So Eq. (33) admits solutions only for β larger than some minimum value β_m given by

$$\beta_m = K \frac{\omega_{ds}}{\omega_A \sqrt{\pi}} \frac{r_s}{R} h^{-1}(\Omega_0) \quad (34)$$

Using the asymptotic expansion of $h(\Omega)$, the two solutions for $\beta \gg \beta_m$ are

$$\Omega = \left[\frac{3K \omega_{ds} r_s}{(\hat{A} - 5\hat{B}/2) (\sqrt{\pi} \omega_A \beta R)} \right]^{2/3} \quad (35)$$

and

$$\omega = \left[\frac{2(\hat{A} + \hat{B}/2)(\sqrt{\pi} \omega_A \beta R)}{K \omega_{ds} z_s} \right]^{2/3} \quad (36)$$

The corresponding values of the real part of D are

$$\text{Re } D = \hat{\omega}_C + 1/(3^{1/3} \sqrt{\pi} h^{1/3} (\Omega_0)) \frac{\omega_{ds}}{\omega_A} (\hat{A} - \hat{B}) / (\hat{A} - 5\hat{B}/2)^{2/3} (\frac{\beta}{\beta_m})^{1/3} \quad \Omega \ll 1 \quad (37)$$

$$\text{Re } D = \hat{\omega}_C - \frac{3}{2\sqrt{\pi}} \frac{\hat{A}}{h(\Omega_0)} \frac{\omega_{ds}}{\omega_A} \frac{\beta}{\beta_m} \quad \Omega \gg 1 \quad (38)$$

and the behavior of the Nyquist diagram can be understood in terms of these expressions.

For $\delta \hat{\omega}_C$ positive the MHD mode is stable, while the T mode reaches marginal stability for $\beta \sim \beta_m (\omega_A/\omega_d) \delta \hat{\omega}_C$ at $\omega \gg \omega_d$. For $\delta \hat{\omega}_C$ negative the MHD mode is unstable and the T mode becomes unstable for $\beta \sim \beta_m (\omega_A |\delta \hat{\omega}_C|/\omega_d)^3$. Above this value both the modes are unstable. The Nyquist diagrams for $\beta < \beta_m$ and $\beta > \beta_m$ are shown in Fig. (3).

We want to stress here that the behavior of the Nyquist diagram as well as the asymptotic behavior of the threshold value of β , show a more than qualitative agreement for the two cases studied, in spite of the different distribution functions. This is probably an indication that the occurrence of the instability is determined only by the value of β and $\delta \hat{\omega}_C$ and not by the details of the distributions. In general, a minimum threshold value of $\beta \sim (\omega_d/\omega_A) r_s/R$ exists for $|\delta \hat{\omega}_C| < \omega_d/\omega_A$; at larger and positive value of $\delta \hat{\omega}_C$ the threshold value increases almost linearly $\beta \sim \beta_m (\omega_A |\delta \hat{\omega}_C|/\omega_d)$, while for larger and negative values of $\delta \hat{\omega}_C$, i.e., under conditions of instability for the MHD mode, the dependence of the threshold value of β on $\delta \hat{\omega}_C$ is cubic $\beta \sim \beta_m (\omega_A |\delta \hat{\omega}_C|/\omega_d)^3$.

IV. NUMERICAL SOLUTIONS

Solutions of the dispersion relation, Eq. (16), were obtained numerically using an interactive scheme making use of the Stokes plot¹⁴ in the complex ω plane associated with $D(\omega)$. The particle distribution was given by the discrete representation

$$f = \frac{1}{N} \sum_{n=1}^N \delta(u - \alpha_n) \delta(E - E_n) \delta\left(\frac{r^2}{2} - \frac{r_n^2}{2}\right) \quad (39)$$

with N the number of particles used in the simulation. A continuous distribution $F(E, \xi, r)$ would normally give rise to a cut in $D(\omega)$ along the real axis from zero to ω_{dm} , the maximum drift frequency in the distribution. Because of the discrete particle representation of F this cut is represented as a sum of N singularities. In addition, there are a large number of extraneous roots to $D(\omega) = 0$, confined to a neighborhood of the real axis (within a distance given by the spacing between singularities). These Dawson,¹⁵ or streaming solutions, make it impractical to solve the dispersion relation for $|\gamma| \lesssim \omega_{dm}/N$ where N is the number of particles in the Monte Carlo simulation. For small values of γ it is necessary to use an analytic expression obtained by a first order solution of the dispersion relation around the threshold.

The particle distributions were generated using Monte Carlo techniques and random number sequences available on the CRAY 1. The details of the method are illustrated for the first case considered.

A. Slowing-down population

We first consider the slowing-down population used in Ref. 5,

$$F(E, \mu, r) = \frac{c(r)}{E^{3/2}} \delta\left(\frac{\mu}{E} - \alpha\right) \quad (40)$$

This approximates the distribution found in a neutral beam heated plasma during beam injection. The number of particles in an element of phase space is $F d^3v d^3x$, giving the number in energy range dE proportional to $1/E$. Writing $dn/dE = (dn/dx)(dx/dE) = a/E$ and using a randomly generated uniform distribution in x in the interval $(0, x_{\max})$, i.e., $(dn/dx) = \text{const}$, the correct energy distribution is generated through

$$E = E_{\max} \exp(x - x_{\max}) \quad (41)$$

with range $E_{\max} \exp(-x_{\max}) < E < E_{\max}$. In addition, the particles are chosen to be distributed uniformly in volume, i.e., $r_n \sim (n/N)^{1/2} r_s$.

Using $f(r, \alpha, E) = a/E$, we find from Eqs. (19) and (20) for the critical beta,

$$\langle \beta \rangle_{\text{crit}} = \frac{2 \langle K_b \rangle (E_{\max} - E_{\min})}{\pi \omega_A \langle (A-B) E^2 / \omega_d \rangle} \quad (42)$$

where the brackets indicate averages over the distribution, i.e., $\langle H \rangle = \int r dr d\alpha dE f H$. If the dependence of ω_d on r and α , and the dependence of K_b on r is neglected, we find $B = 0$ and

$$\langle \beta \rangle_{\text{crit}} = \frac{2 \omega_{dm}}{\pi \omega_A} \frac{K_b}{\langle A \rangle} \quad (43)$$

in agreement with Ref. 5. These approximations typically give $\langle \beta \rangle_{\text{crit}}$ too large by a factor of two. However, as shown in Sec. III, the fact that this threshold value is independent of $\hat{\delta W}_C$ is misleading, because for values of $\hat{\delta W}_C > \omega_d / \omega_A$ and $\beta \gtrsim \beta_C(\omega_A |\hat{\delta W}_C| / \omega_d)$, the growth is exponentially small. Thus the threshold behavior of the neutral beam and ion cyclotron cases can be regarded as essentially the same.

Perturbing β and ω about the threshold values, linear expressions for the growth rate and frequency can be obtained. The term $\partial\omega/(\omega_d - \omega)^2$ in the dispersion relation is easily shown to be negligible. We then find

$$\frac{\gamma}{\omega_r} = \frac{(\beta - \beta_c)}{\beta_c} \frac{S_1}{(S_3 - S_2)} \quad (44)$$

$$\frac{\delta\omega_r}{\omega_r} = \frac{(\beta - \beta_c)}{\beta_c} \frac{S_2}{(S_3 - S_2)} \quad (45)$$

where the numerical factors S_1, S_2, S_3 are given by

$$S_1 = \langle B^2 \frac{(A-B)}{\omega_d} \rangle, \quad S_2 = \langle \frac{(A-B)E^2 \ln}{\omega_d} \left| \frac{E_{\max} - \frac{\omega_r E}{\omega_d}}{E_{\min} - \frac{\omega_r E}{\omega_d}} \right| \rangle, \quad (46)$$

$$\text{and } S_3 = \langle \frac{\omega_r (A-B)E (E_{\max} - E_{\min})}{\left(\omega_r - \omega_d \frac{E_{\min}}{E} \right) \left(\omega_d \frac{E_{\max}}{E} - \omega_r \right)} \rangle.$$

From Eq. (21) we then have that for $\beta \approx 2\beta_c$ the growth rate, aside from a numerical factor, is given by $\langle \omega_d \rangle$.

Shown in Figs. 4 through 6 are the results of a numerical investigation of the dispersion relation for the slowing-down distribution, with $\alpha = 0.99$, $\omega_A = 3.8 \times 10^6 \text{ sec}^{-1}$, and various $\hat{\omega}_c$. We have used the equilibrium of Ref. 5, with the $q = 1$ surface located at $r_s \approx a/2$, and $B = 10 \text{ kG}$. We considered a 50 keV deuterium ion injection. For these parameters $\omega_{dm} = 1.7 \times 10^5 \text{ sec}^{-1}$ and the average value $\langle \omega_d \rangle = 5 \times 10^4 \text{ sec}^{-1}$. In Fig. 4 are shown the MHD (M)

and trapped particle (T) branches of the dispersion relation, as a function of β , for various values of $\hat{\delta W}_C$. The subscripts indicate the values of $\hat{\delta W}_C$ in multiples of 2.5×10^{-3} , i.e., the curve $T_{\pm n}$ has $\hat{\delta W}_C = \pm 2.5n \times 10^{-3}$. The trapped particle branch is located at $\gamma = -\infty$ for $\beta = 0$, and appears in the lower half plane as β approaches the threshold value β_C , which is independent of $\hat{\delta W}_C$ for this distribution. Figures 5 and 6 give the dependence of γ and ω_r on β for the same curves shown in Fig. 4.

The MHD branch, located at $\gamma = -\omega_A \hat{\delta W}_C$ and $\omega_r = 0$ for $\beta = 0$, is stabilized by the trapped particles for $\hat{\delta W}_C > 0$. For $\hat{\delta W}_C < 0$ the mode takes on a positive frequency as β increases, but is stabilized as long as $\gamma < \langle \omega_d \rangle$. As $\hat{\delta W}_C$ becomes more negative, and the MHD and trapped particle branches coalesce. As observed in Sec. III, at most one branch is unstable for given values of $\hat{\delta W}_C$ and β . For sufficiently negative $\hat{\delta W}_C$ this is the MHD branch, as displayed in Fig. 5 by the curves M_{-2} and T_{-2} .

In Fig. 7 is shown the dependence of γ on $\hat{\delta W}_C$ for various values of β , given by $\beta = n \times 10^{-3}$ for curve T_n . The trapped particle branch shows essentially exponential behavior, $\gamma \sim \exp(-\alpha \hat{\delta W}_C)$, in agreement with the analysis of Sec. III. In Fig. 8 is shown the frequency for the curves of Fig. 7.

B. Maxwellian distribution

The second particle distribution we consider is a Maxwellian distribution function with a single value of μ/E

$$F(\mathbf{E}, \mu, r) = n(r) e^{-E/T_h} \delta\left(\frac{\mu}{E} - \alpha\right) \quad (47)$$

This distribution is a good approximation to the hydrogen distribution function found during the minority species ion cyclotron heating experiments. The value of n is typically of the order 10^{-2} times the average density and the value of the temperature ranges between 30 and 100 keV depending on the injected power. Both these quantities can be assumed constant inside the $q=1$ surface. The value of α is obtained by requiring that the tips of the trapped particle banana orbits lie in the resonant layer. If R_L is the distance of the layer from the axis of the torus, this condition can be written

$$\alpha = R_L/R. \quad (48)$$

We have used the Monte Carlo techniques, as in the slow down case, to investigate the stability of this distribution function. The random variable x is related to the energy by the equation

$$x(E) = \frac{\text{erf } E^{1/2} - \text{erf } E_1^{1/2} + (2/\sqrt{\pi})[E_1^{1/2} \frac{E_1}{E} - E^{1/2} \frac{E_1}{E}]}{\text{erf } E_2^{1/2} - \text{erf } E_1^{1/2} + (2/\sqrt{\pi})[E_1^{1/2} \frac{E_1}{E} + E_2^{1/2} \frac{E_2}{E}]} \quad (49)$$

where E is normalized to the temperature, $0 < x < 1$, and $E_1 < E < E_2$. A qualitative agreement of the behavior of the dispersion relation with the slow down case has been found.

A plot of the stable and unstable regions in the $\beta, \delta\hat{W}_C$ plane is given in Fig. 9 for the same equilibrium parameters and various positions of the resonant layer. Note that the range of $\delta\hat{W}_C$ covered is considerably larger than that given by the curves in Figs. 4 and 5, which, although describing the beam injection case, show the same behavior. The dependence of the threshold

beta on α_0 is easily interpreted physically and has important consequences for plasma heating. Large α_0 corresponds to $v_{\perp} \gg v_{\parallel}$, i.e., near perpendicular beam injection and ion cyclotron heating in the deeply trapped, bad curvature region, where ω_d is largest. Not only is the threshold lower and the range of unstable $\delta\hat{W}_C$ larger when the particles are deeply trapped, but the particle ejection induced by the mode is more efficient.⁶ Thus experiments producing fewer deeply trapped particles should observe less frequent and less violent fishbone oscillations.

C. Fusion alpha particles

Particle distributions simulating fusion alpha particles were also generated using Monte Carlo techniques. The distribution was assumed uniform in pitch λ , and in initial location, r and θ . The distribution in $\alpha = \mu/E$ is then given by $\alpha = (1-\lambda^2)/B(r,\theta)$. A slowing-down distribution was assumed for the energy. Plasma parameters corresponding to a fusion reactor were used. This model produces threshold values of β of a fraction of a percent for $\delta\hat{W}_C = 0$. This is approximately the value of β due to fusion alpha particles expected in a steady-state reactor.¹⁶ Whether or not fusion α particles will induce fishbone oscillations will thus depend on details such as the plasma β obtained and the threshold for the internal kink mode, which depends also on plasma shaping and device parameters.

V. CONCLUSION

The destabilization of the internal kink mode by trapped high energy particles has been investigated for a number of distribution functions. The mode threshold and growth rate was found to be fairly independent of the form of the distribution function. The threshold trapped particle beta, β_C has a

minimum for $\delta\hat{W}_C = 0$ and increases linearly in $\delta\hat{W}_C$ for $\delta\hat{W}_C > 0$. For typical fusion experiment parameters the minimum β_C is a fraction of a percent. The range of $\delta\hat{W}_C$ in which the mode is unstable for a given β decreases as the particle population becomes less deeply trapped. These results indicate two effective means of avoiding the instability. Plasma deformation can raise the threshold of the internal kink, effectively increasing $\delta\hat{W}_C$ for a fixed particle distribution, and the heating mechanism can be modified to produce a population which is less deeply trapped. Fishbone oscillations induced by fusion alpha particles are a possibility, but their occurrence will depend on the details of the reacting plasma.

Other MHD modes, besides the internal kink, must be regarded as candidates for fishbone-like instabilities. In particular, there is frequently observed a high frequency precursor to the fishbone oscillation, which has been tentatively identified as a localized interchange mode¹⁷ or ballooning modes¹⁸ excited by energetic trapped particles. The attainment of higher beta plasmas will undoubtedly reveal other, related, instabilities.

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FIGURE CAPTIONS

- FIG. 1 The complex w plane, showing the closed curve used in the Nyquist analysis.
- FIG. 2 The complex $D(w)$ plane for the slowing-down distribution, showing the Nyquist plots for $\delta\hat{w}_c = 5 \times 10^{-3}$, $\alpha_0 = 1.05$, and $\beta < \beta_c$, $\beta > \beta_c$.
- Fig. 3. The complex $D(w)$ plane, for the Maxwellian distribution, $\delta\hat{w}_c = 0$, $\alpha_0 = 1.05$, and $\beta < \beta_c$, $\beta > \beta_c$.
- FIG. 4. Trapped particle (T) and MHD (M) branches of the dispersion relation for the slowing-down distribution, for various values of $\delta\hat{w}_c$, given by $\delta\hat{w}_c = \pm 2.5n \times 10^{-3}$ for curve $T_{\pm n}$.
- FIG. 5. The growth rates γ of the curves of Fig. 1 as functions of β .
- FIG. 6. Frequencies ω_r of the curves of Fig. 1 as functions of β .
- FIG. 7. Growth rate as a function of $\delta\hat{w}_c$ for various values of β .
- FIG. 8. The frequency as a function of $\delta\hat{w}_c$ for the curves of Fig. 7.
- FIG. 9. Critical beta, β_c vs $\delta\hat{w}_c$ for a Maxwellian distribution, for two values of α_0 .

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Fig. 1

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Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

fig. 7

Fig. 8

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Fig. 9

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