MASTER

THREE TOPICAL REPORTS :

- I Thermal studies at Roosevelt Hot Springs, Utah
- II Heat flow above an Arbitrarily Dipping Plane of Heat Sources
- III A Datum Correction for Heat Flow Measurements made on an Arbitrary Surface

by

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Three Topical Reports:

THERMAL STUDIES IN A GEOTHERMAL AREA:

REPORT I. ~~11~~ THERMAL STUDIES AT ROOSEVELT HOT SPRINGS, UTAH;

REPORT II. ~~11~~ HEAT FLOW ABOVE AN ARBITRARILY DIPPING PLANE OF
HEAT SOURCES; *under*

REPORT III. ~~11~~ A DATUM CORRECTION FOR HEAT FLOW MEASUREMENTS
MADE ON AN ARBITRARY SURFACE

By

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ABSTRACT

This volume contains three reports that concentrate on the interpretation of heat flow data in a geothermal area. These three reports include 1) a field study at Roosevelt Hot Springs, Utah which demonstrates how the heat flow map is developed and how it can be enhanced to determine deep reservoir geometry, 2) a technique for interpreting a heat flow profile traversing a fault where fluids are constrained to flow along the fault plane, and 3) a method for correcting heat flow measurements made on an arbitrary surface to a constant elevation datum plane.

Temperature profiles and heat flow values have been determined for 53 drill holes within the Roosevelt Hot Springs KGRA [Known Geothermal Resource Area], Utah. The temperature profiles can be classified in three spatially consistent patterns on the basis of their thermal gradient value. These patterns delineate hydrologic recharge, active convection, and discharge regions respectively. Thermal conductivity values were measured or assigned for all lithologies in the area and a heat flow value computed at each drill site. A heat flow map representing surface conductive heat loss has the following characteristics: heat flow contours are apparently structurally controlled; the 400, 700 and 1000 mWm^{-2} contours enclose areas of 57, 33 and 16 km^2 respectively; the total anomalous surface heat loss of the system (above Great Basin background of 100 mWm^{-2}) is 64 MW.

Effects of rock composition, porosity, water saturation, lateral heat conduction, temperature dependent thermal conductivity, chemical alteration reactions and fluid transport are quantitatively assessed for their importance in causing nonlinear temperature profiles. A downward continuation of the surface heat flow field indicates a geothermal reservoir at Roosevelt Hot Springs approximately 6 km long and 1.5 km wide at a depth of 450 m.

In the Basin and Range physiographic province many geothermal systems are associated with normal faults. The exploitation of such systems as a geothermal resource commonly involves down-dip drilling to tap high temperature and high pressure fluids. We demonstrate that a shallow heat flow profile perpendicular to the strike of such normal fault systems provides critical information on the geometry of the fault and fluid flow in the fault zone. The model is most simply an arbitrarily dipping plane of heat sources. Combining this simple conductive model with inversion theory allows an interpreter to estimate the dip of the fault plane and the vertical depth extent from an observed heat flow profile.

Potential field measurements made on a surface of varying elevation can be biased by topographic effects. In the past, heat flow measurements have not been sufficiently dense to warrant correcting these measurements to a constant elevation datum for modeling purposes, but with the advent of the shallow heat flow survey in geothermal areas, the effect of varying elevation over a survey area can now be determined. By means of an equivalent dipole distribution, surface heat flow measurements can be corrected to a constant elevation datum.

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REPORT I. #4

✓ THERMAL STUDIES AT
= ROOSEVELT HOT SPRINGS, UTAH

Introduction

Roosevelt Hot Springs KGRA [Known Geothermal Resource Area] is one of the more promising new high temperature geothermal discoveries in the western U.S.A. The area has undergone thorough geophysical, geological and geochemical exploration, much of which has been summarized by Ward et al. (1978). An earlier report by Sill and Bodell (1977) provides most of the data base and the initial interpretation for this study. The purpose of this paper is to describe in detail the extensive thermal gradient and thermal conductivity measurements and heat flow determinations that have been made at Roosevelt Hot Springs KGRA and to present the results in the context of heat transfer modes commonly encountered in the exploration for geothermal systems.

The Roosevelt Hot Springs KGRA is located in southwestern Utah at the eastern margin of the Basin and Range physiographic province (Figure 1). The geothermal system is situated on the western flank of the Mineral Mountains approximately 14 km northeast of Milford, Utah. The range consists of schists and gneisses of indeterminate but possibly Precambrian age, Paleozoic and Cretaceous sedimentary rocks, a Tertiary granitic pluton, Tertiary volcanic rocks, and Quaternary rhyolite flows, domes, and ash deposits (Nash, 1976). Recent K-Ar dates by Evans (1979, pers. comm.) indicate an age of 22 m.y. for the pluton which has an areal extent of approximately 250 km². Late Quaternary volcanism produced high silica rhyolite flows and domes which partially cover and intrude the pluton. The youngest flows have been dated at 0.8 m.y. The younger domes, at least ten in number, formed between about 0.6 and 0.5 m.y. The high silica rhyolites

Figure 1. General geology of northern Mineral Mts., Utah (after Evans, 1977) and location of Roosevelt Hot Springs, geothermal area. Numbered areas designate the different thermal regions displayed in Figure 2, and discussed in text.

EXPLANATION

QUATERNARY	Qal	Alluvium
	Qcal	Cemented Alluvium
	Qos	Opal & Opaline Sinter
	Qb	Basaltic Cinder & Flows
	Qrd	Rhyolite Domes
	Qra	Rhyolite Ash
	Qrf	Rhyolite Flows
TERTIARY	Trc	Rhyolite Domes, Corral Canyon
	Tg	Granitic Rocks
PALEOZOIC	IP	Permian Sedimentary Rocks
	ε	Cambrian Sedimentary Rocks
PRECAMBRIAN	pε	Precambrian Rocks

--- Faults
 0.60 M.Y. = K/Ar ages in millions of years
 72-16 = Drill hole Location

suggest that the flows and domes came from a shallow magma differentiate. The total amount of extrusives including flows and domes is estimated to be about 4 km³ (Nash, pers. comm.). Iron titanium oxide and two feldspar geothermometers yield pre-eruption temperatures of 785°C in the flows and 650°C in the domes (Nash and Evans, 1977). The extensive Quaternary Cove Fort basalt-andesite field which has been estimated to be younger than 10,000 yrs (Condie and Barksy, 1972) laps up against the north-eastern flank of the Mineral Mountains.

The Roosevelt Hot Springs are offset approximately 4 km to the west of the axis of the Mineral Mts. igneous activity. The location of the hot springs in relation to mapped faults, suggests strong structural control; they are most likely localized by a set of four major fault systems. These are in order of probable age, 1) large-scale "denudation" faults which dip at shallow angles to the west, 2) northwest-trending fault zones, 3) east-west steeply dipping structures, and 4) north to northeast trending normal faults (Neilson et al., 1978). The two dominant fault zones within the KGRA are the north-northeast trending Opal Mound fault and the east trending Hot Springs fault (Figure 1). The Opal Mound fault has intermittent deposits of siliceous sinter and silica-cemented alluvium and can be traced for approximately 4 km. This eastward dipping fault zone bounds a small horst to the west and a graben to the east (Crebs and Cook, 1976). The Hot Springs fault dips steeply to the south and can be traced for approximately 7 km. Low angle faults which intersect the major fault system may provide permeable conduits for hydrologic

recharge originating in the west central Mineral Mountains. Mylonite zones along the low angle faults are largely impermeable due to silicification, but subsequent faulting and movement along these zones could produce permeable conduits for recharge (Nielson et al., 1978).

Within this geologic setting we have made thermal gradient and heat flow determinations at 53 drill sites spanning an area of 200 km². The impetus for much of the drilling was the exploration and commercial development of a high temperature geothermal reservoir associated with this young igneous system. We have collected temperature-depth and thermal conductivity information and have combined the data to produce a heat flow map from which we can infer the geometry and depth of the reservoir and can place a lower limit on the heat loss of the system. These results will demonstrate the usefulness of such a heat flow survey in delineating and interpreting heat transfer regimes in a geothermal area.

Temperature Measurements

Temperature measurements were made in 53 available drill holes in and around the Roosevelt Hot Springs KGRA. Measurements in forty-nine slim holes (non production wells) were made at discrete intervals with a portable thermistor probe assembly (Sass et al., 1971, Chapman, 1976) connected to a digital ohm meter. This thermistor probe configuration has a time constant of 10 seconds, precision of 0.01°C and an accuracy better than 0.1°C for the temperature range encountered. Temperatures in four production wells were obtained from continuous commercial logs for which the response characteristics and accuracy are less well

known.

Drill hole details including location, elevation, depth and agency responsible for drilling are given in Table 1. Composite temperature-depth plots for about half of the drill holes are shown in Figures 2 and 3. Temperatures range from 12°C which is close to the mean annual temperature for the region to a maximum of 265°C measured in production wells. Near surface thermal gradients vary from 6°C km⁻¹ to 3331°C km⁻¹. For comparison, a typical geothermal gradient to be expected in this lithology in the Utah Great Basin is between 35 and 40°C km⁻¹.

Although temperature profiles for individual holes exhibit great variation and considerable individual complexity, we recognize three spatially consistent patterns which we believe are indicative of heat transfer processes associated with the geothermal system. The three groups of temperature-depth behaviors, labeled I, II, and III are presented in Figure 2. The regions in which boreholes of the three groups are found are similarly labeled in Figure 1.

Group I sites scattered across the east and the west flank of the Mineral Mountains pluton (Figure 1) are characterized by low geothermal gradients, from 6 to 29°C km⁻¹, and temperature profiles exhibiting irregularities commonly attributed to groundwater disturbance. A likely explanation for such low gradients is hydrologic recharge through and flow under the region, whereby a significant fraction of the normal heat flow is intercepted by moving water and transferred laterally towards the geothermal system. The regularity of temperature profiles from GPC7, UUHF3 and GPC9 from this region suggests the flow

Figure 2. Composite plot of temperature-depth curves within the different thermal regimes delineated in Figure 1. Insets show comparison of mean thermal gradients for the different regions. BR represents a typical Basin and Range thermal gradient of $38^{\circ}\text{C Km}^{-1}$.

is taking place at depths exceeding 150 m, thermally decoupling near surface regions of the pluton from deeper regions. Oxygen isotope studies of waters from the geothermal system (Bowman, 1979) indicate that the Mineral Mountains is the probable source region for water in the system, thus supporting our recharge hypothesis.

In contrast to Group I, Group II sites exhibit extremely high near surface geothermal gradients, all above $500^{\circ}\text{C km}^{-1}$, and strong downward curvature in the temperature depth profiles. Deep production drilling in this region has confirmed the existence of a geothermal reservoir of temperature 265°C at depths as shallow as 400 m (Figure 3). The high shallow gradients of Group II therefore result from their proximity to the geothermal reservoir. The nature of heat transfer in this region is discussed later.

Group III sites have internally consistent temperature characteristics (Figure 2) with a mean near surface gradient of $270^{\circ}\text{C km}^{-1}$, intermediate between groups I and II discussed above. However the occurrence of gradients about seven times background, up to 6 kilometers west of the Opal Mound Fault requires either multiple or broad heat sources or alternatively lateral mass and heat transport westward towards the Milford valley.

Thermal Conductivity

Thermal conductivity values were determined on a divided bar apparatus (Sass et al., 1971) calibrated against the conductivity of fused and crystalline quartz (Ratcliffe, 1959). Measurements were made at 20°C on solid rock discs cut from drill core and on drill chips

Figure 3. Deep temperature logs for production scale wells in the Roosevelt Hot Springs, KGRA.

using the cell technique described by Sass et al. (1971). Comparison of solid disc versus chip-cell techniques on the same samples gave an uncertainty of about $\pm 0.1 \text{ Wm}^{-1}\text{K}^{-1}$ over the range 0.8 to $4.2 \text{ Wm}^{-1}\text{K}^{-1}$. Experimental errors in conductivity measurements are typically 3%, however petrologic variability and the problem of choosing representative samples may lead to uncertainties of 10 to 20% in certain cases.

Conductivity measurements were made on 88 samples from two core drilled and sixteen rotary drilled holes. The core represents one phase of the Mineral Mountains pluton and consists of a biotite hornblende quartz monzonite with interspersed microdiorite and aplite dikes. The chips measured were recovered from holes drilled west of the range front in the prominent alluvial fan. Chips ranged from poorly sorted clayey sands and unconsolidated arkosite alluvium to quartz monzonite, aplite and granodiorite. Thermal conductivity results are given in Table 1 for individual drill holes and are shown collectively in Figure 4. The mean conductivity for the shallow granitic rock of the pluton and for the solid fragments in the alluvium is $2.54 \text{ Wm}^{-1}\text{K}^{-1}$.

Thermal conductivity results from the sixteen shallow rotary drilled holes are also presented in Figure 5 to illustrate the extent of conductivity variation with depth arising from compositional changes in the alluvial fan. Samples from fourteen of the holes exhibit an increase (.6 % per meter) of conductivity with depth while samples from GPC7 and GPC9 exhibit a slight decrease. A deeper sampling of the granitic and quartz monzonite from production well 9-1 showed a slight

Table 1. Heat Flow Data, Roosevelt Hot Springs KGRA

Drill Hole	Location (T/R/S)	H (m)	TD (m)	DI (m)	G (K km ⁻¹)	(Wm ⁻¹ K ⁻¹)		(Wm ⁻¹ K ⁻¹)	(mW m ⁻²)
GPC10	C26/8/6CCD	1841	60	15-45	51±3	5	2.5±.05	1.6±.02	83±5
UUTG2	C26/9/5CCD	1551	67	15-65	66±4	a	2.5±.03	1.6±.01	108±7
UUTG6	C26/9/7CAA	1530	97	15-35	117±51	12	3.0±.16	1.9±.07	216±95
GPC9	C26/9/12CDC	2018	91	30-90	21±1	5	2.6±.12	1.7±.05	36±2
UUTG5	C26/9/14DAA	1865	50	20-50	43±2	a	2.5±.03	1.6±.01	71±4
UUTG1	C26/9/15CBA	1707	60	10-30	254±30	a	2.5±.03	1.6±.01	417±49
UUTG0	C26/9/16BDC	1647	77	15-45	71±4	a	2.5±.03	1.6±.01	116±7
UUTG3	C26/9/19DBC	1572	100	20-50	322±26	a	2.5±.03	1.6±.01	528±42
UU-75-13	C26/9/20ADA	1640	43	10-43	529±91	a	2.5±.03	1.6±.01	868±149
PPC1	C26/9/21CCC	1663	145	10-30	718±92	a	2.5±.03	1.6±.01	1178±151
PPC2	C26/8/30CAC	2377	90	20-90	6±1	a	3.1±.04	3.1±.04	19±4
UUHF3	C26/9/25DCC	2079	150	40-160	29±1	28	2.8±.03	2.8±.03	81±2
GPC8	C26/9/25CCB	2036	2000	10-120	71±2	7	2.9±.11	1.8±.05	129±5
UU75	C26/9/27BBB	1750	124	5-25	649±153	a	2.5±.03	1.6±.01	1064±250
PPC WW4	C26/9/30CAC	1579	34	15-55	369±62	a	2.5±.03	1.6±.01	605±102
PPC WW3	C26/10/25ADA	1558	55	5-55	256±11	a	2.5±.03	1.6±.01	420±19

Table 1 (continued)

Drill Hole	Location (T/R/S)	H (m)	TD (m)	DI (m)	G (K km ⁻¹)	N	K _S (Wm ⁻¹ K ⁻¹)	K _C (Wm ⁻¹ K ⁻¹)	q (mW m ⁻²)
TPC14	C26/9/32ACB	1574	85	10-50	377±62	a	2.5±.03	1.6±.01	618±102
UU-76-1A	C26/9/34CAA	1817	144	5-20	2332±532	a	2.9±.23	1.8±.10	>4244±996
PPC3	C27/8/5AAD	2207	63	30-59	17±4	a	3.1±.04	3.1±.04	53±11
PPC4	C27/9/18DC	1952	59	10-30	205±45	a	2.5±.03	1.6±.01	336±74
TPC6	C27/9/2CAD	1926	32	20-50	186±7	a	2.5±.03	1.6±.01	305±12
14-2	C27/9/28CC	1902	105	18-450	432±1.4	a	2.5±.03	1.6±.01	700±2
UU-75-1B	C27/9/40DA	1811	1900	5-45	643±28	6	2.9±.23	1.8±.10	1170±65
UU-75-1A	C27/9/40AA	1811	65	10-35	1052±143	a	2.9±.23	1.8±.10	1914±281
TPC11	C27/9/4ADC	1804	68	10-50	559±27	a	2.7	1.7	967±46
TPC12	C27/9/58BA	1661	63	10-130	193±8	a	2.5±.03	1.6±.01	317±13
9-1	C27/9/9ABC	1774	150	61-500	207	a	2.3	1.6	323
TPC3	C27/10/10DDD	1547	200	10-50	284±35	a	2.5±.03	1.6±.01	465±58
PPC WW2	C27/10/12DDD	1625	115	20-40	267±20	a	2.5±.03	1.6±.01	438±34
TPC4	C27/9/7CCB	1625	195	10-40	262±2	a	2.5±.03	1.5±.01	430±5
TPC10	C27/9/7DDA	1665	175	10-50	228±25	a	2.5±.03	1.6±.01	374±41
Bearskin	C27/8/8BAA	2341	156	40-100	15±3	20	3.1±.04	3.1±.04	47±9

Table 1. (Continued)

Drill Hole	Location (T/R/S)	H (m)	TD (m)	DI (m)	G (K km ⁻¹)	N	K _S (Wm ⁻¹ K ⁻¹)	K _C (Wm ⁻¹ K ⁻¹)	q (mW m ⁻²)
GPC7	C27/9/13BCC	2109	94	15-90	25±1	7	2.1±.08	1.5±.04	37±2
UU	C27/9/14BCA	1956	100	20-80	79±5	a	2.5±.03	1.6±.01	130±9
TPC15	C27/9/15ACB	1875	64	10-60	437±25	a	2.5±.03	1.6±.01	717±41
TPC7	C27/9/16ADA	1807	90	10-40	1185±58	a	2.0	1.4	1659±81
72-16	C27/9/16AAC	1793	375	10-18	3331±382	a	2.0	1.4	>4663±535
TPC5	C27/9/16BBA	1752	145	20-50	467±15	a	2.5±.03	1.6±.01	766±25
PPC 0B5	C27/9/17ADB	1719	185	30-120	273±8	a	2.5±.03	1.6±.01	448±13
GPC14	C27/9/17CCC	1689	160	20-50	209±15	10	2.4±.08	1.5±.04	336±27
GPC15	C27/9/18DDD	1582	573	24-51	172±14	a	2.5±.03	1.6±.01	282±7
TPC13	C27/10/23CBC	1559	150	10-60	250±17	a	2.5±.03	1.6±.01	410±28
52-21	C27/9/21ABC	1775	2225	10-50	470±227	a	2.5±.03	1.6±.01	771±372
TPC9	C27/9/21DAC	1795	75	10-75	189±14	a	2.5±.03	1.5±.01	310±23
PPC	C27/9/29ADC	1762	110	30-60	218±35	a	2.5±.03	1.6±.01	358±53
GPC6	C27/10/25ACB	1636	91	15-55	250±13	5	2.7±.12	1.7±.05	430±26
GPC4	C27/9/33BAB	1802	91	15-65	59±2	7	2.4±.09	1.6±.04	96±4
GPC5	C27/9/34BAA	1890	55	10-25	45±8	5	2.3±.10	1.5±.05	69±3

Table 1 (Continued)

Drill Hole	Location (T/R/S)	H (m)	TD (m)	DI (m)	G (K km ⁻¹)	N	K _s (Wm ⁻¹ K ⁻¹)	K _c (Wm ⁻¹ K ⁻¹)	q (mW m ⁻²)
PPC5	C27/9/35CBC	1985	36	20-36	15±4	a	2.5±.03	1.6±.01	25±6
PPC6	C27/9/35ACD	2045	70	30-70	10±2	a	2.5±.03	1.6±.01	16±3
GPC3	C28/9/4BBB	1798	91	10-35	82±6	8	2.9±.10	1.8±.04	150±11
GPC2	C28/9/6ABA	1701	91	10-65	56±3	9	2.8±.04	1.8±.04	99±6
GPC1	C28/10/1BAA	1625	109	10-40	132±6	6	2.6±.11	1.7±.05	224±12

Key

- H - drill hole collar elevation
 TD - total depth
 DI - depth interval used for assigning thermal gradient
 G - thermal gradient
 N - number of thermal conductivity measurements
 a - no measurements (thermal conductivity is assumed)
 K_c - composite thermal conductivity
 q - heat flow

Drill hole sponsors

- PPC Phillips Petroleum Company
 TPC Thermal Power Company
 GPC Geothermal Power Company
 UU University of Utah

Figure 4. Histogram of solid rock conductivity measurements. Mean and standard error include an experimental uncertainty of $\pm 0.1 Wm^{-1} K^{-1}$.

Figure 5. Composite plot of thermal conductivity versus depth for several selected drill holes within the Roosevelt Hot Springs KGRA.

decrease of conductivity for the depth range 250 to 1000 m.

In order to characterize completely all lithologies found in the area, additional conductivities were determined for several opaline sinter outcrop samples. Their low conductivity of $2 \text{ Wm}^{-1}\text{K}^{-1}$ reflects the amorphous property of the silica since crystalline quartz at 20°C has a conductivity of about $6.3 \text{ Wm}^{-1}\text{K}^{-1}$ perpendicular to the optic axis.

Thermal conductivity values assigned to the major geologic units in the area are given in Table 2. A major uncertainty in assigning a conductivity value for the alluvial fan material arises because the effective conductivity of the medium depends not only on the solid chip conductivity but also on the porosity and water saturation as is discussed in detail later. A porosity value of 0.22 was obtained from analysis of well logs in a single hole (Glenn and Hulen, 1979). Elsewhere we have attempted to extract porosity values by attributing thermal gradient differences above and below the water table to water saturation differences in a uniform porosity matrix. The calculated porosity values varied from 0.14 to 0.36 in the 10 holes examined. Finally in the absence of any direct or indirect porosity measurements, we could have assigned a value between 0.20 and 0.35 by comparison with values reported elsewhere for a similar geologic environment (DeWiest, 1969). The thermal conductivity assigned for the alluvial fan in Table 2 corresponds to an average solid chip conductivity of $2.54 \text{ Wm}^{-1}\text{K}^{-1}$ and an average porosity of 0.30, and is considered to be a reasonable estimate for the material although variations up to perhaps $0.5 \text{ Wm}^{-1}\text{K}^{-1}$ may be expected in some localities.

Table 2. Thermal conductivities

Unit	Thermal conductivity (Wm ⁻¹ K ⁻¹)
quartz monzonite	2.54
opaline sinter	2.00
biotite gneiss	2.00
alluvial fan	1.64

Table 3. Conductive heat loss

Contour interval (mWm ⁻²)	Area enclosed (Km ²)	Anomalous heat flow (MW)
100-400	>70	>10
400-700	>23	>10
700-1000	17	13
>1000	16	31
		Total
		>64 MW

Heat Flow

Heat flow values have been determined for each drill hole, as the product of the vertical thermal gradient over a specified depth interval times the appropriate thermal conductivity. The depth interval chosen is somewhat arbitrary but we have attempted to choose the uppermost constant thermal gradient region where conductive heat transfer dominates. In doing so we are computing the conductive heat leaking from an obviously convection dominated system. Where actual thermal conductivity measurements were not available we assumed a conductivity based on the lithology (Table 2). Heat flow details for the fifty-three sites are given in Table 1. The surface heat flow pattern is shown in Figure 6. Values vary from 16 mWm^{-2} to more than 4 Wm^{-2} but exhibit a regular spatial pattern which enables contouring of the field. Contour values chosen for Figure 6 have the following significance. Chapman et al. (1979) have shown that characteristic Great Basin heat flow in Utah excluding geothermal systems is $90 \pm 10 (\text{s.d.}) \text{ mWm}^{-2}$. Thus the 100 mWm^{-2} contour is considered to outline the limit of thermal effects associated with this geothermal system. The 400 mWm^{-2} contours enclose regions where heat flow is at least four times greater than background, often considered as a criterion for an economic geothermal prospect (Kappelmeyer and Haenel, 1974, p. 141). The area enclosed by the 400 mWm^{-2} contour in Figure 6 is 57 km^2 . The highest contour shown is 1000 mWm^{-2} which encloses an area of 16 km^2 , in particular a 2 km wide band parallel to the Opal Mound fault where most of the successful production drilling has occurred.

The pattern of the heat flow field is clearly controlled by

geologic structure. The elongated heat flow high is parallel to and encloses the Opal Mound fault. This zone has an abrupt southern termination at a set of west by northwest trending faults, and changes trend at the intersection of the Opal Mound and Hot Springs faults. The easterly spur in the pattern which coincides with the Hot Springs fault, is probably a near surface feature and does not necessarily represent the reservoir geometry. Heat flow contours in many parts of the field parallel mapped or inferred faults. The reemergence of the 400mWm^{-2} contour to the southwest of the area coincides with fault blocks inferred from gravity and electrical resistivity studies (Ward and Sill, 1976) and may result from water leakage over an impermeable upthrown block (Kilty and Chapman, 1980).

Anomalous surface heat loss from the geothermal system has been calculated by integrating the heat flow above background over the areas enclosed by specific contours in Figure 6. Background was taken as 100mWm^{-2} . The results, given in Table 3, show an anomalous surface heat loss of 64MW for the system. This value will be a lower limit as the additional heat loss by subsurface water leakage is not known.

Heat Transfer Characteristics

The density of temperature-depth and heat flow information at Roosevelt Hot Springs KGRA when combined with other geophysical, geological and geochemical studies, affords an opportunity to examine the nature of heat transfer in this geothermal system. The relative importance of conductive versus convective heat transfer can be assessed and a quantitative explanation given for the commonly observed

non-linear temperature-depth profiles.

Non-linear temperature-depth curves can be interpreted in terms of a variety of mechanisms. In order to quantify these mechanisms, it is necessary to separate the heat transfer regimes into predominantly conductive or convective. There is no fine line for this boundary in geothermal systems. However, we can approximate this boundary by considering the region above and below the water table. Above the water table, possible explanations for curvature are as follows: 1) change in rock composition and/or porosity, 2) water vapor movement from evapotranspiration, 3) lateral heat conduction, 4) increasing water saturation as the water table is approached from above, and 5) temperature-dependent thermal conductivity. We will demonstrate the relative importance of these various effects for one borehole, UU-75-1A, plotted in Figure 7, and extend the analysis for the remaining holes with similar thermal characteristics.

As described by Parry et al. (1980) and Nielson et al. (1978) the lithology in UU-75-1A is altered and cemented alluvium to 35 m and altered and brecciated biotite gneiss from 35 m to the bottom of the hole at 66 m. The water table is at a depth of 30 m. The rock composition above the water table is sufficiently constant that mechanism (1) can be discarded as contributing to the curvature.

Water vapor transport in an unsaturated medium has been studied by many hydrologists and soil physicists (Penman, 1940, DeVries, 1950, Van Bavel, 1952, Rollins et al., 1954, Phillip and DeVries, 1957, and Cassel et al., 1969). The approach of DeVries (1950) Van Bavel (1952) and Rollins et al. (1954) was to assume that the water vapor transport

Figure 7. Temperature-depth characteristics for drillhole UU-75-1A. Solid line is least squares fit for measurements above the water table. Dashed line shows fit for different values of V_z in equation (12) below the water table.

was basically a diffusion process governed by Fick's law,

$$V_r = -\alpha\phi D \gamma \frac{P}{P-P_v} \frac{dP_v}{dz} \quad (1)$$

where

V_r = flow rate

α = structure factor

D = diffusion coefficient of water in air

γ = mass of 1 cc of water vapor at 1 mm Hg

P = 1 atm (760 mm)

P_v = partial pressure of water vapor

ϕ = volume fraction of air filled voids

The diffusion concept was later adapted by Phillip and DeVries (1957), to include two phase flow in a region with a thermal gradient. They suggest that the flow rate obeys,

$$V_r = \rho_w (-D_T \nabla T - D_\theta \nabla \theta - K_i) \quad (2)$$

with $D_T = D_{Tliq} + D_{Tvap}$

$D_\theta = D_{\theta liq} + D_{\theta vap}$

where

V_r = flow rate

ρ = density of liquid water

D_{Tvap} = thermal vapor diffusivity

$D_{\theta vap}$ = isothermal water diffusivity

D_{Tliq} = thermal liquid diffusivity

$D_{\theta liq}$ = isothermal liquid diffusivity

θ = volumetric moisture content

K = unsaturated hydraulic conductivity

i = unit vector in positive z direction

This derivation extends Fick's law to include a separation of the 'isothermal' and 'thermal' components of vapor transfer and the effect of relative humidity (or soil water pressure) on the transfer, (Phillip and DeVries, 1957). An experimental investigation by Cassel et al. (1969) suggests that the Phillip and DeVries formulation approximates the measured moisture transfer much more closely than the Fick's law formulation. Since we have no means of estimating the various diffusivities in (2) with the available data, we will approximate the contribution of evapotranspiration to observed heat flow at UU-75-1A by using the Fick's law derivation given in (1). The heat flow resulting from evapotranspiration, q_e , can then be written as,

$$q_e = -L\alpha\phi D\gamma \frac{P}{P-P_v} \frac{dP_v}{dz} \quad (3)$$

where L is the latent heat of vaporization and the remaining parameters are as defined previously.

For

L = 539 cal/g

α = .66

ϕ = .3

D = $4.42 \times 10^{-4} T^{2.3}/P$ (T - degrees K and P in mm Hg)

γ = $.289 \times 10^{-3}/T$ (T - degrees K)

P = 760 mm Hg (1 atm)

P_v = 149 mm Hg (taken from steam tables at $T = 50^\circ\text{C}$)

we find that $q_e = 84 \text{ mWm}^{-2}$. The values of α and D are those suggested by Penman (1940). Since the observed heat flow at UU-75-1A is $1914 \pm 281 \text{ mWm}^{-2}$, the effect of evapotranspiration from this calculation is less than one standard error of the measurement. If the actual transport is less than three times this value, we can ignore evapotranspiration as a major contribution to curvature.

Contributions from conductive lateral heat transfer can be calculated from the heat flow map, Figure 6. For an assumption of constant thermal conductivity and a surface intercept of 12°C , we have computed lateral temperature gradients in the N-S and E-W direction for a 15×15 point grid. Figure 8 shows that the magnitude of the lateral gradient reaches a maximum of $23.5^\circ\text{CKm}^{-1}$ in an area analagous to the previously discussed region II. Since this region has a mean vertical gradient of $1350^\circ\text{CKm}^{-1}$, the lateral heat transfer contribution is less than 2%. In regions where the mean vertical gradient is smaller, the magnitude of the lateral gradients is also smaller by at least an order of magnitude. We are therefore justified in ignoring this contribution in the conductive regime.

Since contributions to curvature have been shown to be minimal for the first three mechanisms, the curvature above the water table can be explained as the combined effect of water saturation and temperature-dependent conductivity. Core was not available for thermal conductivity measurements in UU-75-1A. However, we can approximate the conductivity of the unconsolidated alluvium by using the mixing model proposed by Sass et al. (1971) given as,

$$K_c = K_s^{(1-\phi_0)} K_w^{(\phi_0-\phi_a)} K_a^{\phi_a} \quad (4)$$

Figure 8. Magnitude of conductive lateral heat transfer for the heat flow map in Figure 6. OMF and HSF indicate the traces of the Opal Mound and Hot Springs faults respectively.

where K_C is the composite conductivity, K_S is the solid rock conductivity, K_W and K_A are the conductivities of water and air respectively taken to be $K_W = 0.67 \text{ Wm}^{-1}\text{K}^{-1}$ and $K_A = 0.03 \text{ Wm}^{-1}\text{K}^{-1}$, ϕ_a is the volume percent air, and ϕ_o is the rock porosity. As suggested by DeVries (1958) the water vapor content in the pores will give an additional component to the thermal conductivity, but we will neglect that component for this discussion. Water saturation or moisture content as represented by $\phi_o - \phi_a$ in (4) can be a very complex function of depth (Baver et al., 1972, p. 355). Since no measurements of moisture content versus depth have been made, we will make the simple assumption that there is a linear increase in saturation from the surface to the depth of the water table, z_w , then the volume per cent air is given by,

$$\phi_a = \phi_o (1 - z/z_w) \quad (5)$$

or

$$\phi_o - \phi_a = \phi_o z/z_w$$

The solid rock conductivity, K_S , water conductivity, K_W , and air conductivity, K_A , in (4) are all a function of temperature. For the temperature range observed in UU-75-1A, the greatest temperature dependence is that of the solid rock conductivity. Therefore, we will assume that the temperature dependence of the water and air conductivity is second order and ignore their contribution. Recent measurements on thermal conductivity versus temperature by Sibbitt et al. (1979) can be used to describe the temperature dependence of the

solid rock conductivity in the alluvium which we will assume to be a quartz monzonite corresponding to their sample no. 12-4918. A least squares fit to conductivity as a function of temperature for the quartz monzonite sample yields a solid rock conductivity of,

$$K_s = 3.8 \exp(-(1.7 \times 10^{-3})T) \quad \text{Wm}^{-1}\text{K}^{-1} \quad (6)$$

where T is in $^{\circ}\text{C}$. If we substitute (5) and (6) into (4) assuming $\phi_0 = .3$, the resulting composite conductivity as a function of temperature, moisture content, and depth is given by,

$$K_c = \{3.8 \exp(-(1.7 \times 10^{-3})T)\}^{.7} K_w^{.01} z^{.3} K_a^{.3(1-z/30)} \quad (7)$$

where we've taken $z_w = 30$ m for UU-75-1A. Values of K_c versus depth calculated from (7) for the temperature range in UU-75-1A are listed in Table 4. The conductivity increases from 1.1 to 2.1 $\text{Wm}^{-1}\text{K}^{-1}$ in 22 m. For an assumption of constant conductive heat flow, the increase in conductivity by a factor of 2 can easily account for the observed curvature above the water table. This leads us to conclude that the major mechanisms causing curvature above the water table are temperature-dependent conductivity and water saturation. Since the solid rock conductivity decreases with increasing temperature, as seen in (6), the dominant effect is increasing water content as the water table is approached from above.

We now return to the general problem of assigning a representative gradient and conductivity in the region above the water table. Consider two point thermal gradients with conductivities given by (7) along different portions of the curve above the water table (Figure 7).

Between 10-15m and 25-30m we have the following results,

	(°Cm ⁻¹)	K _C (Wm ⁻¹ K ⁻¹)	q (mWm ⁻²)	% saturation
10-15 m	1.6	1.11	1777	29
25-30 m	0.88	2.0	1758	90

The agreement of heat flow values was improved by using a composite conductivity for the 10-15 m fit corresponding to a depth of 8.5 m. This suggests that increasing saturation with depth is not linear but would best be represented by a polynomial fit of higher order. We can compare these heat flow values with a least squares fit of temperatures versus depth to the top 30m yielding,

$$T = 17.6 + 1.052z \quad (8)$$

If we then take a mean composite conductivity between 0-30m of 1.62 Wm⁻¹K⁻¹ corresponding to 30% porosity and 70% saturation, the calculated heat flow is 1704 mWm⁻² which is in fair agreement with the heat flow calculated for increasing saturation. We conclude from this example that the procedure of computing thermal gradients with a least squares fit and coupling this with a mean composite conductivity with 30% porosity and total saturation is valid in computing heat flow values for holes in spite of observed temperature-depth curvature above a supposed water table.

In the region below the water table, non-linear temperature-depth profiles can also be produced by several mechanisms: 1) change in thermal conductivity as a function of composition, temperature, and porosity, 2) convective lateral heat transfer, 3) vertically flowing water, and 4) heat production resulting from exothermic clay

alteration reactions. For UU-75-1A we have assumed that the rock composition below 35 m is approximately constant and equal to that of a biotite gneiss. Therefore, for this model rock composition will have no effect on curvature. A least squares fit to conductivity versus temperature for the biotite gneiss sample no. 9608-1 from Sibbitt et al. (1979) gives

$$K_s = 3.1 \exp(-(1.3 \times 10^{-3})T) \quad (9)$$

This gives a change in thermal conductivity of less than 3% for the temperature range between 30-60 m and eliminates mechanism (1) as a possible factor in producing observed curvature in UU-75-1A below the water table.

In order to evaluate contributions from the last three mechanisms rigorously, it would be necessary to solve a coupled set of differential equations relating conservation of momentum or mass flux, conservation of energy and Darcy's law (Cathles, 1977). However, we can approximate the relative magnitudes of the individual contributions by using a first order approximation for density, heat capacity, and vertical velocity, and by solving the energy equation in one dimension. In effect, this neglects any contribution from free convection, turbulence, and lateral heat transfer. Since no hydrologic information was available to analyze the magnitude of the contribution from convective lateral heat transfer, it will be ignored here. However, this contribution could be substantial in the region above the reservoir.

Steady state conservation of energy for regions of hydrologic

flow can be written as,

$$\nabla^2 T - \frac{\rho_f C_{pf}}{K_c} V \cdot \nabla T = -A/K_c \quad (10)$$

where T is temperature, ρ_f and C_{pf} are the density and specific heat of the fluid, V is the velocity of the fluid, K_c is the composite thermal conductivity and A is a heat generation term representing exothermic clay alteration reactions. For vertical flow, the one dimensional form of (10) is,

$$\frac{\partial^2 T}{\partial z^2} + \frac{\rho_f C_{pf}}{K_c} V \frac{\partial T}{\partial z} = -A/K_c \quad (11)$$

If we make a first order approximation for the coefficient of $\frac{\partial T}{\partial z}$, a solution to (11) via Laplace transforms is,

$$T = T_w + \frac{q_w}{K_c} \left\{ \frac{1 - \exp(-\beta(z - z_w))}{\beta} \right\} - \frac{A}{K_c \beta} \left\{ z - z_w - \frac{1 - \exp(-\beta(z - z_w))}{\beta} \right\} \quad (12)$$

where $\beta = \rho_f C_{pf} V_z / K_c$, T_w is the temperature at the top of the water table, q_w is the heat flow at the top of the water table, and z_w is the depth to the top of the water table. We have used the boundary conditions that $T(z_w) = T_w$ and $T'(z_w) = q_w / K_c$, and we have assumed that A/K_c is approximately constant. In the limit that $V_z \rightarrow 0$, (12) becomes

$$T = T_w + \frac{q_w z}{K_c} - \frac{A z^2}{2 K_c} \quad (13)$$

which is the proper solution to Poisson's equation. Similar solutions to the energy equation have been derived by Stallman (1960), Bredehoeft and Papadopoulos (1965), Sammel (1968), Freeze (1969),

Sorey (1971) and Mansure and Reiter (1979) to estimate vertical groundwater velocities from temperature profiles. In fact, for $A = 0$, $z_w = 0$, $T_w = T_0$, and $T_L = T|_{z=L}$, we find an expression equivalent to that given by Bredehoeft and Papadopoulos (1965) or,

$$\frac{T_z - T_0}{T_L - T_0} = \frac{\exp(-C_p \rho_f V_z / K_c) - 1}{\exp(-C_p \rho_f V_L / K_c) - 1} \quad (14)$$

The solution given by (12) not only takes into account the effect of vertical groundwater motion, but also includes the effect of exothermic clay alteration reactions.

For constant K_c and measured values of A , (12) can be solved for V_z by trial and error or by a generalized inverse. Values of enthalpy per unit volume arising from exothermic clay alteration reactions have been estimated by Parry et al. (1980) for two drill holes in the Roosevelt Hot Springs KGRA. Assuming that none of the sulfate is produced by oxidation of H_2S , they have computed the values tabulated in Table 4, for UU-75-1A. The age of the system is poorly constrained, but if we use a minimum probable age of 10,000 yrs, the heat flow anomaly arising from exothermic reactions will be a maximum. For this assumption, the heat production, A , will be as given in Table 4, and the average value below 30 m is 1.61 mWm^{-3} . From previous arguments the thermal conductivity will remain essentially constant as a function of temperature and we have assigned a solid rock conductivity at 60°C using (9). The composite conductivity is calculated from the mixing model of (4) assuming 10% porosity and complete saturation to give $K_c = 1.85 \text{ Wm}^{-1}\text{K}^{-1}$. We then computed a

Table 4. Thermal conductivity and heat production for UU-75-14

Z(m)	$K_C (Wm^{-1}K^{-1})$ Eq. 7	Total $\Delta H/\text{unit vol.}$ (MJ/M ³)	Heat Production (mW/m ³)
8.5	1.11	481	1.52
15.2	1.36	163	5.14
21.2	1.63	42	1.32
25.9	1.98	234	7.38
30.5	2.12	423	1.33
35.1		795	2.51
39.6		477	1.50
45.7		552	1.74
50.3		364	1.15
54.9		502	1.58
60		347	1.09
64.3		611	1.93

family of curves using (12) for varying V_z . Figure 7 shows that the best fit occurs for $V_z = 5 \times 10^{-9} \text{ ms}^{-1}$, a reasonable value for water flowing through a soil with a large clay content. By comparing the second and third terms in (12), we can evaluate the relative importance of vertical water flow versus exothermic heat production. For the parameters found above, we find

$$T_{60} - T_w = \Delta T_{\text{water}} = \frac{q_w}{K_c} \frac{\exp(-\beta(z_{60} - z_w))}{\beta} = 17^\circ \text{C} \quad (15)$$

$$T_{60} - T_w = \Delta T_{\text{heat gen.}} = \frac{A}{K_c \beta} (z_{60} - z_w - \frac{1 - \exp(-\beta(z_{60} - z_w))}{\beta}) = 8 \times 10^{-3}^\circ \text{C}$$

This demonstrates that the contribution to curvature in UU-75-1A from vertical water flow is four orders of magnitude greater than the contribution from exothermic reactions, and as a result, water flow is the dominant mechanism which causes curvature in the temperature profile below the water table.

The discussion of UU-75-1A illustrates the difficulties in assigning meaningful thermal gradients in regions of curvature. Fortunately, not all of the holes in the Roosevelt Hot Springs KGRA exhibit the same strong curvature and UU-75-1A represents an upper limit for this type of thermal behavior. More commonly, the holes are fairly linear in the upper 30 m section, as in regions I and III (Figure 2). For this reason, we have assigned thermal gradients at a mean depth of 30 m by making a least squares fit to the most linear portion of the hole and constraining the surface intercept throughout the Roosevelt Hot Springs KGRA to a mean annual temperature of $12 \pm 5^\circ \text{C}$.

By choosing a depth of 30 m, we are assured to be above the water table for the majority of the holes and yet we are deep enough to avoid seasonal variations in the temperature wave. A consistent assignment of gradients at 30 m does not define the thermal nature within the deep geothermal system, but it is valuable in outlining near surface geometry and extent of the system. For some holes within the active convective region such as 72-16, the surface intercepts for the least squares fit are much greater than the mean annual temperature (Figure 2). This is expected for areas where warm water flows near to the surface. In order to be consistent with the remainder of the heat flow map, we have used the surface intercept and the temperature near 30 m to determine a maximum gradient.

Downward Continuation

The surface heat flow field at Roosevelt Hot Springs KGRA is sufficiently constrained by the dense drill hole coverage to permit an attempt at processing the surface heat flow map. Digital processing is used extensively for interpretation of potential field, seismic, and electromagnetic data, but less commonly for heat flow data because measurements are seldom made at a spacing small enough to avoid aliasing the anomaly. The objective of this section is to show how a digitizing version of the surface heat flow map can be enhanced to yield estimates of lateral extent and the depth to the top of a hypothesized geothermal reservoir.

Continuation of potential fields is a mathematical method which, in the past, has been primarily used for gravity and magnetic

interpretation (Grant and West, 1965). In a geothermal environment, the same technique can be applied to regions above a hypothesized reservoir where conductive heat transfer dominates. For steady state conductive heat transfer, the energy equation is

$$\nabla \cdot K_C \nabla T = 0 \quad (16)$$

where K_C is the composite solid-fluid thermal conductivity. If we assume that the composite conductivity is sufficiently represented by a constant mean conductivity, (16) reduces to Laplace's equation for the temperature distribution. For an infinite half space with temperature satisfying Laplace's equation, the downward continued heat flow, $Q_S(x,y)$ is given by (Grant and West, 1965),

$$Q_S(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q_H(u,v) e^{2\pi H \sqrt{u^2+v^2}} e^{i2\pi ux} e^{i2\pi vy} du dv \quad (17)$$

where $Q_H(u,v)$ is the Fourier transform of vertical surface heat flow and H is the depth of continuation. A derivation of (17) is given in Appendix I. The form of (17) is particularly amenable to evaluation by the Fast Fourier Transform (FFT).

As is well known (Grant and West, 1965), (17) diverges for H equal to the depth of the anomaly source. The kernel of (17) is plotted in Figure 9 for increasing values of H . As H approaches the depth of the sources, the kernel filter amplifies the high frequency components more and more until (17) finally diverges. Numerically, the effect of the divergence is to increase the noise in the imaginary component of (17) which is theoretically zero. Since it is difficult to quantify the degree of convergence or divergence for this numerical

Figure 9. Plot of continuation filter in equation (17) for unit data spacing "a" and two depths "H" in kilometers. The plot shows increasing amplification at the high frequencies for increasing depth, H.

transform, we have empirically assigned a depth of divergence as when $|I_m Q_s| > 1$. In Appendix I, a comparison of this criterion with the computed results for an analytic model shows good agreement.

The downward continuation formulation was used to continue the surface heat flow map, (Figure 6) to depths of 100 m, 200 m, 300 m, and 400 m. The results are shown in Figure 10. As expected, the contours sharpen as we approach the heat source and the error criterion is exceeded for depths greater than 450 m. The sharpness of the contours at 400 m suggests that the major part of the system is contained in an area approximately 6 km long and 1.5 km wide paralleling the Opal Mound fault. From power spectral arguments given in Appendix I, we show that the continuation technique enhances long wavelength characteristics, and that the short wavelength or high frequency components are spatially aliased. Clearly, we are not guaranteed to intersect the reservoir everywhere within the 6 km by 1.5 km band at 450 m. In fact shallow fractures in 72-16 (350 m) have produced steam with temperatures in excess of 240°C. Since convective heat transfer becomes a critical factor at depths greater than approximately 400 m in 14-2, the absolute magnitudes of the continued heat flow do not represent expected in situ heat flow values. The isothermal nature of the temperature log in 14-2 (Figure 3) below 450 may indicate a major reservoir and this suggests that the continuation technique can be considered successful for this area. The continuation technique may be particularly valuable for assessing the lateral extent of the reservoir and the depth to the top of a long wavelength anomaly. This information can be used to target further

Figure 10. Downward continuation of the surface heat flow map in Figure 6 with contours in mWm^{-2} . OMF and HSF indicate the traces of the Opal Mound and Hot Springs faults.

drilling or to restrict geometric parameters in a more sophisticated model.

Discussion

In this study we have demonstrated the usefulness of a near-surface heat flow survey in outlining the geometry of a geothermal system. Many factors must be considered before assigning conductivities and thermal gradients in a geothermal area. Among these are porosity, degree of saturation, and hydrologic recharge and discharge. For this study, much of this information was unavailable. We have presented arguments to support our method of determining a conductive surface heat flow. Without hydrologic data, extreme care must be used in assigning heat flow values. In order to avoid convective contributions, we have consistently assigned thermal gradients at a depth of 30 m. This depth should be shallow enough to preclude most of the hydrologic disturbances associated with reservoir fluids, but deep enough to avoid seasonal variations. In questionable regions above the active geothermal system, we have attempted to assign upper limits to the conductive heat transfer.

The 53 drill hole data set provided a good base to test digital processing of the anomaly. The ambiguities in interpreting such an analysis are apparent. Actual magnitudes of the continued data are uncertain. However, the results illustrate how a surface heat flow anomaly can be enhanced to infer lateral dimensions of a reservoir and possible depth to the top of a long wavelength anomaly. Since the reservoir is undoubtedly controlled by fractures, shallower short

wavelength anomalies are aliased and therefore are not well represented by the continuation scheme. In this case deep drill hole information confirmed the prediction of depth to a major reservoir.

This study illustrates above all that thermal studies can be used quite successfully to infer geometries and temperature of geothermal systems if the proper precautions are taken. At Roosevelt Hot Springs KGRA, the heat flow mapping has provided probably the least ambiguous data set to guide exploration decisions in the geothermal environment.

Appendix I. Downward Continuation

For an infinite half space with temperature satisfying Laplace's equation, vertical heat flow at a distance H above the surface is given by (Grant and West, 1965, p. 218),

$$Q_H(x,y) = \frac{H}{2\pi} \iint_{-\infty}^{\infty} \frac{Q_S(x',y') dx' dy'}{\{(x-x')^2 + (y-y')^2 + H^2\}^{3/2}} \quad (18)$$

where $Q_S(x',y')$ is the vertical component of surface heat flow and $Q_H(x,y)$ is the continued vertical component. As a convolution, (18) is multiplicative in Fourier transform space, or

$$\hat{Q}_H(u,v) = \hat{Q}_S(u,v) \cdot \hat{K}(u,v) \quad (19)$$

where the $\hat{}$ indicates a 2-D Fourier transform and

$$\hat{K}(u,v) = \exp(-2\pi H \sqrt{u^2 + v^2}) \quad (20)$$

substituting (20) into (19) and back transforming, the upward continued heat flow is

$$Q_H(x,y) = \iint_{-\infty}^{\infty} Q_S(u,v) e^{-2\pi H \sqrt{u^2 + v^2}} e^{i2\pi ux} e^{i2\pi vy} du dv \quad (21)$$

where $Q_S(u,v)$ is the Fourier transform of the surface measurement.

For downward continuation the roles of Q_H and Q_S are reversed. The measured heat flow is taken to be Q_H , and we must solve (21) for Q_S , the continued heat flow at a depth H below the surface. From (19) and (20), the downward continued heat flow, Q_S , is represented by,

$$Q_S(x,y) = \iint_{-\infty}^{\infty} Q_H(u,v) e^{2\pi H \sqrt{u^2 + v^2}} e^{i2\pi ux} e^{i2\pi vy} du dv \quad (22)$$

where $Q_H(u,v)$ is now the Fourier transform of the surface measurement.

The surface heat flow is forward transformed, multiplied by the continuation kernel, $K^{-1}(u,v)$, then back transformed to yield the continued heat flow.

To test the validity of applying (22) in a geothermal environment, a simple conductive model was used to check the accuracy of the continued potential. An arbitrary reservoir was modeled as an infinite line source with a Gaussian surface S shown in Figure 11. Conductive models have limited application for geothermal reservoirs. However, they mimic the surface heat flow, and are used here to illustrate the behavior of the continuation scheme. The analytic test model was made as realistic as possible by choosing model parameters such that they corresponded with deep temperature data. The radius of the cylindrical Gaussian surface in Figure 11 was taken to be 600 m. This corresponds with a depth extent of 1.2 km for the test model system. The depth to the top of the cylinder was 600 m. Heat flow on the cylindrical surface, Q_{GS} , was chosen to correlate with an estimated heat flow at a depth of 600 m from the deep temperature log 14-2 (Figure 3). The heat source strength per unit length, Q_0 , can be solved for quite simply from $Q_{GS} = Q_0 / 2\pi R$. The heat flow above the Gaussian surface for the analytic test model can then be calculated as (Carslaw and Jaeger, 1959),

$$Q = -\frac{Q_0}{2\pi} \left(\frac{(H-H')}{(x-x')^2 + (z-z')^2} - \frac{(H+H')}{(x-x')^2 + (z+z')^2} \right) \quad (23)$$

where primed variables denote source position and unprimed variables are field position.

Surface heat flow for the test model was calculated from (23) and

Figure 11. Test model for downward continuation to two depths above an infinite line source. Dashed lines are calculated from the continuation algorithm, and solid lines are calculated from (23).

then continued downward to depths of 200 m and 400 m via the continuation algorithm. A comparison of continued heat flow and the model analytic solution is shown in Figure 11. The heat flow profile plotted in Figure 11 is for a traverse perpendicular to the axis of the Gaussian cylinder. At 200 m the error is less than 1%. At 400 m the error has increased because of divergence but is still less than 5%. At 600 m our preset criterion for stopping the continuation ($Im > 1$), has been exceeded. This demonstrates that downward continuation gives satisfactory results for conductive models. As predicted by (23), heat flow increases as the source depth is approached.

The power spectrum of the surface heat flow map was calculated for a 15x15 grid with a biased periodogram approximation given by

$$P = \frac{a^2}{N^2} |Q_H(u,v) \cdot Q_H^*(u,v)| \quad (24)$$

where a is the digitizing interval of .8 km, N is the number of input data points in a row, and $*$ indicates complex conjugate. The spectrum is real symmetric about the Nyquist frequencies and therefore only two quadrants of the frequency plane are plotted in Figure 12. At half the Nyquist frequency, $Ny/2$, the power spectrum is four to five orders of magnitude less than the DC frequency value. A low-pass filter with a cutoff frequency of $Ny/2$ could therefore be used to insure that only the long wavelength information was being enhanced. However, since the high frequency contributions were so much less than the low frequency contribution, no low pass filter was implemented.

Aliasing is an inherent problem in the processing of any geophysical data. The drill hole coverage at Roosevelt Hot Springs

Figure 12. Two-dimensional power spectrum of gridded surface heat flow calculated from (24), using scale factor of 10^{-4} . The digitizing interval is 0.8 km and the contours are in MW^2km^2 . Since the power spectrum is symmetric about the Nyquist frequencies, only two quadrants of the frequency plane are plotted.

KGRA averages about .4 holes per square kilometer. This is not sufficient to prevent aliasing in the east-west direction where the cross section length of the target anomaly is only approximately 1.5 km. The surface heat flow measurements have been hand-contoured which in effect smooths the anomaly and reduces high frequency contributions, but this does not guarantee that aliasing will not occur. To avoid this problem in a shallow heat flow survey, care must be taken to choose a proper measurement interval such that aliasing is minimized.

Since the downward continuation formulation given by (22) assumed a semi-infinite half-space, topographic variation introduces error in the processing of the data. In order to reduce this error, we fit the alluvial fan west of the Mineral Mountains to a plane and then used this plane as the surface of the half-space for processing. The calculated vertical and horizontal gradients are then projected onto the normal of the plane by

$$n \cdot \nabla T = \alpha \frac{\partial T}{\partial x} + \beta \frac{\partial T}{\partial y} + \gamma \frac{\partial T}{\partial z} \quad (25)$$

where n is the unit normal vector and α , β , γ , are direction cosines. In order to find lateral gradients a bicubic spline was used to interpolate the data set. The resulting gradients are those plotted in Figure 8. The projected gradient was found to be

$$n \cdot \nabla T = .041 \frac{\partial T}{\partial x} + .014 \frac{\partial T}{\partial y} + .991 \frac{\partial T}{\partial z} \quad (26)$$

for a plane dipping 2° to the southwest. Since the vertical gradients are more than an order of magnitude greater than the lateral gradients, the correction amounts to less than one percent.

Appendix II
Temperature-Depth Curves for
Roosevelt Hot Springs KGRA

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REPORT II.

HEAT FLOW ABOVE AN ARBITRARILY DIPPING
PLANE OF HEAT SOURCES

Introduction

In the Basin and Range physiographic province, many geothermal systems are associated with normal faults (Sammel, 1978). These faults apparently provide permeable zones through which heated fluids rise towards the surface, occasionally discharging as hot springs. As geothermal gradients throughout the Basin and Range province are characteristically $35^{\circ}\text{C km}^{-1}$ and above, ground water need circulate to depths of only three kilometers or so in order to explain the numerous geothermal systems with near surface temperatures of $70\text{--}130^{\circ}\text{C}$.

Such normal fault geothermal systems can be exploited for geothermal energy purposes by extracting hot water from the fault zone. An optimum exploitation procedure which minimizes surface hydrologic effects and maximizes the temperature and pressure of the water extracted, is to drill so that the fault zone is intersected at a depth of several hundred meters. For this strategy it is important to know the dip of the fault, especially in the Basin and Range where normal faults typically dip $75\pm 15^{\circ}$ at the surface. Yet it is in this terrane where geological and structural complexity lead to imprecise dip determinations from more traditional gravity, magnetics and seismic methods.

We demonstrate in this paper that shallow heat flow surveys across normal fault geothermal systems provide valuable information on fault geometry and fluid flow in the fault zone. Although a complete analysis of heat flow-fluid flow would be desirable, the lack of hydrologic measurements at an early exploration stage most often precludes this approach. Alternatively we suggest that it is practical

to model the geometry of the flow via potential theory, in which the geothermal reservoir is modeled as a dipping plane of heat sources. The two dimensional conductive model can be inverted using a generalized linear inverse to yield estimates of dip and depth or lateral extent of the sources.

Forward Model

Assuming that hot water is restricted to a relatively narrow fault plane and that leakage is minimal, the fault zone can be modeled as a plane of heat sources embedded in a conductive medium. Furthermore if a steady state condition and no spatial dependence of thermal conductivity is assumed, the temperature distribution will satisfy Laplace's equation,

$$\nabla^2 T = 0. \quad (1)$$

A solution to (1) in Cartesian coordinates for the temperature distribution $T_L(x,z)$ about an infinite line source striking parallel to the Y-axis with temperature zero at $z = 0$ is (Carslaw and Jaeger, 1959, Page 422),

$$T_L(x,z) = - \frac{Q_0}{4\pi K} \ln \frac{(x-x')^2 + (z-z')^2}{(x-x')^2 + (z+z')^2} \quad (2)$$

where Q_0 is the source strength per unit length of the line source, K is the thermal conductivity of the medium, and primed and unprimed variables indicate source and field positions respectively. The anomalous vertical surface heat flow $Q_L(x,0)$ due to the line source is

$$Q_L(x,0) = K \frac{\partial T}{\partial z} \Big|_{z=0} = \frac{Q_0}{\pi} \frac{z'}{(x-x')^2 + z'^2} \quad (3)$$

For steady state, (3) is independent of thermal conductivity.

Equation (3) can be extended to describe surface heat flow above an arbitrarily dipping plane of heat sources (Figure 1) by requiring x' to obey $x' = x_i + (z'/\tan\alpha)$ and by integrating (3) over z from d to h . The anomalous vertical surface heat flow $Q_p(x,0)$ due to the plane of heat sources is

$$Q_p(x,0) = \frac{Q_0}{\pi(a^2+1)} \left\{ \frac{a^2}{2} \ln \left(\frac{a^2(x_2-x_i)^2 + (x_2-x)^2}{a^2(x_1-x_i)^2 + (x_1-x)^2} \right) \right. \\ \left. + a \tan^{-1} \left(\frac{a^2(x_2-x_i) + (x_2-x)}{a(x-x_i)} \right) \right. \\ \left. - a \tan^{-1} \left(\frac{a^2(x_1-x_i) + (x_1-x)}{a(x-x_i)} \right) \right\} \quad (4)$$

where Q_0 is now the heat source strength per unit area of the plane, x_1 , x_2 , d and h are the surface and depth positions corresponding to the top and bottom of the plane respectively, x_i is the projected surface intercept of the plane, $a = \tan\alpha$, α being the dip of the plane (Figure 1). The spatial coordinates of the top and bottom of the plane are related by

$$d = (x_1 - x_i)a ; \quad h = (x_2 - x_i)a \quad (5)$$

It is convenient to solve (4) directly above the top of the plane at $x = x_1$ and to express the surface heat flow in terms of Q_0 , α , a , H , and d :

$$Q_p(x,0) = \frac{Q_0}{\pi(a^2+1)} \left(\frac{a^2}{2} \ln \left(\frac{h^2}{d^2} \left(1 + \frac{1}{a^2} \left(1 - \frac{d}{h} \right)^2 \right) + a \left(\frac{\pi}{2} - \alpha \right) \right) \right) \quad (6)$$

At $\alpha = \pi/2$ the ratio of the peak anomaly to heat source strength

Figure 1. Cross section XZ plane showing the model geometrical parameters associated with the dipping plane of heat sources (dark line).

approaches $\pi^{-1} \ln (h/d)$ which is the appropriate limit.

The temperature distribution about an arbitrarily dipping plane can be calculated from (2) by making the substitution $x_1 = x_i + (z'/\tan\alpha)$ and by integrating over z' . This yields

$$T_p(x,z) = - \frac{Q_0}{4\pi K} \int_d^h \ln \frac{(x-z'/a-x_i)^2 + (z-z')^2}{(x-z'/a-x_i)^2 + (z+z')^2} dz' \quad (7)$$

For a medium of constant conductivity (1) will be linear, and a regional background temperature contribution can be incorporated into (7) by adding a temperature gradient and intercept. The modified temperature distribution is

$$T_p(x,z) = - \frac{Q_0}{4\pi K} \int_d^h \ln \frac{(x-z'/a-x_i)^2 + (z-z')^2}{(x-z'/a-x_i)^2 + (z+z')^2} dz' + \Gamma z + T_0 \quad (8)$$

where T is the regional thermal gradient, and T_0 is the surface intercept. Given the dip and vertical depth extent, (8) can be evaluated anywhere in the half-space by fitting the integrand to a cubic spline and then by using the spline coefficients to integrate analytically the area under the curve.

The shape and amplitude of the heat flow anomalies given by (4) and (6) and the temperature distributions given by (7) and (8) are functions of heat source strength and of geometric parameters such as dip, depth of burial, d , and vertical depth extent, h , of the plane. As our prime concern is in modeling the geometry of the plane it is convenient to normalize the peak anomaly by adjusting the value of Q_0 appropriately. A set of normalized heat flow anomalies and the corresponding isotherm cross sections for a set of dipping planes of heat sources is shown in Figures 2 and 3. The peak heat flow anomaly

Figure 2. Surface heat flow above a series of dipping planes of heat sources, calculated using (4), (5) and (6). Dip angles are indicated in the figure. Other parameters include burial depth, $d = 5$ m; depth extent, $h = 500$ m; heat source strength normalized to 3500 mWm^{-2} peak.

Figure 3. Temperature cross sections calculated from (8) for the four dipping planes of heat sources shown in Figure 2. Burial depth, $d = 5$ m; depth extent $h = 500$ m, surface temperature $T_0 = 10^\circ\text{C}$ and background gradient $\Gamma = 30^\circ\text{C Km}^{-1}$. Heat source strengths Q_0 necessary to produce maximum surface heat flow anomaly of 3500 mWm^{-2} are shown on the diagrams.

of 3500 mWm⁻², shallow burial depth $d = 5$ m and steep dips are characteristic of several Basin and Range systems we have examined but will be particularly useful in comparing model predictions with field observations later in this paper.

The Generalized Inverse

Having demonstrated the forward calculation of surface heat flow and isothermal cross sections for dipping planes we now treat the estimation of geometric parameters using inversion theory (see Wiggins, 1972, or Jackson, 1972, for general discussion of geophysical applications of inversion theory). We will use the ridge regression technique described by Marquardt (1963); the formalism parallels that described by Inman (1975) and Petrick et al. (1977) for electrical resistivity studies, although our observations are in the form of a heat flow profile.

Small changes in the dip of the plane α or equivalently in a ($a = \tan \alpha$) and in the lateral extent of the plane ($x_2 - x_1$) given in (4) can be related to small changes in the observed surface heat flow through a derivative matrix (Bevington, 1969). Since (4) is non-linear with respect to a and $x_2 - x_1$, the relationship between parameter change and observed data change must be linearized first. The linearized equation is,

$$\Delta G = A \Delta P + \epsilon \quad (9)$$

where ΔG is the data change matrix, A is an $n \times m$ derivative matrix with n equal to the number of observations and m equal to the number of parameters, ΔP is the parameter change matrix, and ϵ is a matrix

containing non-linear contributions. Since the derivative matrix is calculated by a forward difference algorithm, analytical expressions for the derivatives are not necessary. The ridge regression method is a curve matching algorithm which minimizes the least squares error between the observed and calculated data. This requires finding the minimum of,

$$\chi^2 = \Delta G^T \Delta G \quad (10)$$

where χ^2 is the least squares error, and ΔG^T is the transpose of the data change matrix. For small ϵ , an ordinary least squares estimate of ΔP is,

$$\Delta P = (A^T A)^{-1} A^T \Delta G \quad (11)$$

where A^T is the transpose of the derivative matrix. Heat flow measurements in a geothermal environment can be uncertain to within as much as $\pm 20\%$. In this case, statistical significance can be gained only by weighting the data by the uncertainty of the individual measurement. The parameter change matrix, ΔP , can then be calculated as (Petrick et al., 1977),

$$\Delta P = (A^T W^T W A)^{-1} (A^T W^T W) \Delta G \quad (12)$$

where the weighting matrix W is given by

$$W_{ii} = \frac{1}{\sigma_i} \quad i = 1, n \quad (13)$$

with σ_i equal to the standard deviation of the i th measurement.

Parameter statistics can be evaluated by calculating covariance and correlation matrices. For weighted data, these are estimated by (Beck and Arnold, 1977),

$$\text{cov}(P) = \chi_v^2 (A^T W^T W A)^{-1}$$

$$\text{cor}(P_{ij}) = \frac{\text{cov } P_{ij}}{(\text{cov } P_{ii} \cdot \text{cov } P_{jj})^{1/2}} \quad (14)$$

where $\text{cov}(P)$ is the covariance matrix of the inverted parameters for a given χ^2 fit, $\text{cor}(P_{ij})$ is the correlation coefficient between the i and j parameters for the same χ^2 fit, and χ_v^2 is the reduced χ^2 estimate given by,

$$\chi_v^2 = \frac{\chi^2}{n-m} \quad (15)$$

which is an estimate of the true variance of the data. The diagonal of the covariance matrix gives estimates of parameter uncertainties for a particular fit. The correlation matrix indicates the linear dependence of the parameters. A correlation coefficient close to +1 indicates that only the ratio of the two parameters is being resolved, whereas a correlation coefficient close to -1 means that only the product has been resolved.

The ridge regression method involves weighting the diagonal components of $A^T A$ to stabilize the inverse. The expression for parameter change, (11), becomes,

$$\Delta P = (A^T W^T W A + \lambda I)^{-1} (A^T W^T W) \Delta G \quad (16)$$

where I is the identity matrix, and λ is a weighting factor initially set equal to .001 but decreases as the minimum of the least squares fit is approached. This method combines the best features of the gradient search with the method of linearizing the fitting function (Bevington, 1969). For large λ , (16) approximates the expression for the gradient search method which converges slowly to the minimum. For small λ , (16) approximates the expression for a linearized fitting function which

converges rapidly to the minimum. The parameter statistics are only valid for $\lambda = 0$.

The Monroe Geothermal System

As a field test, we will consider the Monroe geothermal system located approximately 1 km east of Monroe, Utah. It is situated on the eastern edge of an alluvial filled intermontane valley bounded on the east by the Sevier Plateau and on the west by the Pavant Range. The steeply dipping Sevier fault seems to serve as a conduit for waters heated by a normal Basin and Range thermal gradient. A more detailed summary of the geology can be found in Mase et al. (1978). Kilty et al. (1979) and Mase et al. (1978) have proposed that hydrologic recharge in the Sevier Plateau circulates to depths of approximately 2.5-4 km. The hydraulic head forces the water to the surface along the Sevier fault and it is discharged at the Monroe hot springs. The geometry of the hot springs, suggest that a two-dimensional model may be applicable for interpreting the geometry of the flow. The 2-D inversion was therefore tested in this geological setting.

The Monroe geothermal system was being studied for feasibility as a space heating project for homes and public buildings in the City of Monroe. As a result detailed thermal information is available for thermal gradient-heat flow drillholes crossing the Sevier Fault at the Monroe hot springs. Temperature-depth results from the five drill holes are shown in Figure 4. All drill holes show the influence of the geothermal system; the least geothermal gradient is $300^{\circ}\text{C km}^{-1}$ in drillhole M6, about seven times background. Further, the thermal

Figure 4. Temperature-depth curves for drill holes comprising the Monroe profile which crosses the Sevier Fault. Drill hole locations are given on insert. [Figure modified from Mase, 1978]

gradients systematically increase towards the inferred trace of the Sevier Fault, as is to be expected if the fault zone acts as a permeable conduit for hot water discharge.

Thermal conductivity values have been determined for samples from all the drillholes and surface heat flow values were computed as the product of the harmonic mean thermal conductivity and the near surface thermal gradient (Mase, 1978). In order to investigate these heat flow data for geometric properties of the geothermal system, the anomalous heat flow caused by heat sources in the fault zone must be separated from a background heat flux. Although all the local heat flow sites are affected by the system, Chapman et al. (1979) have shown that 90 mWm^{-2} can be taken as representative of Great Basin heat flow in Utah, and so we subtract a background 90 mWm^{-2} from the profile. The resulting heat flow profile and the accompanying error bars are shown in Figure 5. All distances are measured normal to strike from the inferred trace of the fault.

The parameters chosen for inversion are the tangent of the dip, a , and the lateral position x_2 as given in equation (4). The point x_1 is assumed to be known for shallow burial depths and is selected to correspond with the inferred trace of the fault. For simplicity, the heat source strength, Q_0 , is calculated from (6) for an estimated h/d ratio and an observed peak heat flow anomaly. For fixed parameters $d = 5 \text{ m}$, $x_1 = 467 \text{ m}$, and $Q_0 = 1200 \text{ mWm}^{-2}$ and an initial guess of $a = 3.25$ and $x_2 = 600 \text{ m}$, the solution for the six observation points in Figure 5 is $a = 5.76 \pm 4.4$ and $x_2 = 575 \pm 60 \text{ m}$ corresponding to a dip of $80 \pm 7^\circ$ and depth extent of 600 m respectively. The convergence rate is shown in

Figure 5. Two-dimensional heat flow inversion of the Monroe Hot Springs profile. Solid dots are observed residual heat flow with accompanying error bars. Dashed curve is computed from the solution parameters. Convergence is shown in inset.

the inset of Figure 5.

The actual average dip of the Sevier Fault at the Monroe hot spring has been determined from drilling tests and therefore can be used as a field check on the inversion results. The appropriate lithology logs for drillholes MC1, MC2 and MC3 are shown in Figure 6. In each case the drillhole was started in hot spring sinter and progressed through volcanic alluvium into a latite volcanic breccia bedrock. In addition MC3 intersected 100 m of an arenaceous limestone most likely of the Sevier River Formation. Determining the exact fault intersection in each hole was difficult because differences in drill chips from the volcanic breccia footwall were quite subtle. However, by combining drill chip examination, with alteration intensity, drilling rates and caliper logs we have assigned fault intersection depths for the three holes to be 88 m, 181 m and 308 m respectively. The average dip of the fault, assuming a strike of $N10^{\circ}W$ is 72° .

The predicted dip of $80 \pm 7^{\circ}$, although it overlaps the measured dip, is somewhat high. The discrepancy may be the combined effect of several factors. These include, 1) the large error and limited number of heat flow measurements, 2) heat refraction across the fault zone resulting from the thermal conductivity contrast between alluvium and volcanics, 3) deviation from the idealized 2-D planar geometry, and 4) hydrologic leakage along the fault zone. The magnitude of each contribution cannot be evaluated without going to a more sophisticated model. However, the quality and number of measurements for the Monroe hot springs profile does not warrant a more complicated model. Since the predicted and observed dip overlap, the application of the dipping

Figure 6. Structure section across the Sevier Fault at Monroe Hot Springs based on lithology logs from drill holes MC1, MC2, and MC3. Dip of the fault on this section is 72° .

plane model can be considered to be at least partially successful in this geologic environment.

Discussion

We have utilized a two-dimensional inversion scheme for estimating the dip and depth extent of an arbitrarily dipping plane of heat sources. The field test at Monroe hot springs suggests that for this geological setting, the forward model is adequate for matching the observed heat flow profile. The dipping plane model ignores heat refraction and hydrologic leakage, and as such it only estimates the dip for a given heat flow profile. However, its simplicity and ease of use makes it practical for preliminary exploration modeling in a Basin and Range environment where warm water can be constrained to flow along a fault plane.

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REPORT III.

A DATUM CORRECTION FOR HEAT FLOW MEASUREMENTS
MADE ON AN ARBITRARY SURFACE

Introduction

Until recently, drill hole coverage in a geothermal area has been too sparse to warrant digital processing or quantitative modeling of a heat flow anomaly. With the advent of the shallow heat flow survey for exploration in a geothermal area, the number of measurements per square kilometer has increased to the point where quantitative modeling of the heat flow anomaly to delineate reservoir geometry is now practical. In the Western United States, the topography surrounding a geothermal site can vary as much as 500 m in elevation. For these regions, terrain corrections such as those proposed by Bullard (1940) or Birch (1950) can be used to correct the temperature boundary conditions for varying surface temperature changes related to erosion and the atmospheric lapse rate. The terrain correction allows one to compute temperature or thermal gradient versus depth as if the surface was a plane normal to the drill hole and intersecting the drill hole collar elevation. In a geothermal area, heat flow can exceed 400 mWm^{-2} . The terrain correction for such high heat flow is typically less than 2%. Once this correction is made, the interpreter is still faced with the problem of modeling a set of heat flow measurements which correspond with varying surface elevations.

As summarized by Bhattacharyya and Chan (1977), the importance of adjusting potential field measurements to a constant datum level has been studied extensively for gravity and magnetic surveys. The study by Bhattacharyya and Chan (1977) is particularly adaptable to a heat flow survey. Their approach is to model surface measurements made on an arbitrary surface as an equivalent dipole distribution and then to

continue the measurements upward to either a plane of constant elevation or a parallel surface. This is an appealing method because, in a geothermal area, the datum cannot be a relatively low elevation, since a low elevation may be below the convecting part of the geothermal system which is the modeling target. The purpose of this study is to extend the formulation of Bhattacharyya and Chan (1977) to include heat flow measurements and to apply the method to a heat flow survey at Roosevelt Hot Springs KGRA (Wilson and Chapman, 1980).

Potential Theory

For a distribution of dipoles on a surface, the potential field has a discontinuity on that surface of 4π times the surface potential. If the potential field is known everywhere on the surface, this feature of the dipole distribution allows one to find an equivalent dipole distribution in terms of the surface measurements. Once the dipole distribution is known, the surface potential can be continued to any position in the whole space and in particular to a plane above the surface. I will demonstrate the analysis first for a temperature dipole and then extend the formulation to include heat flow measurements. The formalism and notation follows closely that presented by Bhattacharyya and Chan (1977) for gravity and magnetic measurements.

For steady state conduction and a homogeneous medium the temperature distribution satisfies Laplace's equation,

$$\nabla^2 T = 0. \quad (1)$$

Since the function $1/r$ is harmonic in Cartesian coordinates, the

solution for a dipole satisfies Laplace's equation with the temperature given by,

$$T(x,y,z) = \frac{Q_0}{4\pi K} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \quad (2)$$

where Q_0 is a heat source strength with units of cal/s, K is the thermal conductivity and r_1 and r_2 are the position vectors to the plus and minus charge. A heat source dipole is analagous to an electrical dipole where a source and sink give rise to a potential field which in this case is temperature. For a small source-sink separation (2) becomes

$$T(x,y,z) = - \frac{2Q_0\epsilon}{4\pi K} \frac{\cos\theta}{r^2} = \frac{2Q_0\epsilon}{4\pi K} \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \quad (3)$$

where ϵ is the source-sink separation, θ is the angle between the position vector r and the normal n which is directed along the axis of the dipole, and $\frac{\partial}{\partial n}$ is the normal derivative. Extending the argument to a number of dipoles distributed over a surface S with dipole density u , the temperature at a field point is

$$T(x,y,z) = \int_S u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS \quad (4)$$

If a point on the surface S is denoted by coordinates (α, β, γ) and the field point by coordinates (x, y, z) then

$$r^2 = (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2$$

and

$$\frac{\partial}{\partial n} = n_\alpha \frac{\partial}{\partial \alpha} + n_\beta \frac{\partial}{\partial \beta} + n_\gamma \frac{\partial}{\partial \gamma}$$

where n_α , n_β , and n_γ are the direction cosines between the surface normal and the source vector. An expression for the potential T at a

point on the surface S in the limit as the surface is approached from the external side can be found from Green's theorem which is

$$\int_V (w \nabla^2 u - u \nabla^2 w) dV = \int_S (w \frac{\partial u}{\partial n} - u \frac{\partial w}{\partial n}) dS \quad (6)$$

where u and w are continuous functions which have continuous and integrable first and second derivatives in the volume V . If the source point $(\alpha', \beta', \gamma')$ is isolated with a hemisphere of radius ρ , for $w = 1/r$ and u equal to the dipole density in (4), (6) can be written as,

$$\begin{aligned} \int_{V'} \frac{1}{r^2} u dV - \int_{S'} \frac{1}{r} \frac{\partial u}{\partial n} dS + \int_{S'} u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS \\ - \int_S \frac{1}{r} \frac{\partial u}{\partial n} dS + \int_S u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS = 0 \end{aligned} \quad (7)$$

where V' and S' denote the region outside the hemisphere and S is the region within the hemisphere. Taking the limit as $\rho \rightarrow 0$, gives

$$\lim_{\rho \rightarrow 0} \int_S u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS = \lim_{\rho \rightarrow 0} \frac{u(\alpha', \beta', \gamma')}{\rho^2} \int_S dS = 2\pi u(\alpha', \beta', \gamma') \quad (8)$$

By assumption, u has a continuous first derivative. Therefore

$$\lim_{\rho \rightarrow 0} \int_S \frac{1}{r} \frac{\partial u}{\partial n} dS = \lim_{\rho \rightarrow 0} \frac{1}{\rho} \frac{\Delta u(\alpha', \beta', \gamma')}{\Delta n} \int_S dS = 0 \quad (9)$$

Substituting (8) and (9) into (7) results in

$$\int_V \frac{1}{r^2} u dV - \int_{S^*} \frac{1}{r} \frac{\partial u}{\partial n} dS + \int_{S^*} u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS = -2\pi u(\alpha', \beta', \gamma') \quad (10)$$

where S^* is the entire arbitrary surface except for the source point $(\alpha', \beta', \gamma')$. For the entire region except the source point $(\alpha', \beta', \gamma')$, Green's theorem also yields,

$$\int_V \frac{1}{r^2} u \, dV - \int_{S^*} \frac{1}{r} \frac{\partial u}{\partial n} \, dS = - \int_{S^*} u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \, dS = - T_S(\alpha', \beta', \gamma') \quad (11)$$

where T_S is the total potential at $(\alpha', \beta', \gamma')$ due to all other dipoles except the one at the source point. Combining (10) and (11), gives the final expression for the surface potential, T_S , as

$$T_S(\alpha', \beta', \gamma') = 2\pi u(\alpha', \beta', \gamma') + \int_{S^*} u \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \, dS \quad (12)$$

where

$$r^2 = (\alpha' - \alpha)^2 + (\beta' - \beta)^2 + (\gamma' - \gamma)^2$$

and $u(\alpha', \beta', \gamma')$ is the equivalent dipole distribution. If equation (12), which is a Fredholm integral equation of the second kind, is solved for the equivalent dipole distribution, u , in terms of T_S , the measured variable, the temperature anywhere above the surface can be calculated from (4).

In a heat flow survey, the equivalent dipole distribution for the vertical heat flow must be found instead of the distribution for the potential. Equations (4) and (12) can be adapted for heat flow in the following manner. The heat flow in the z direction, z , is (from (4)),

$$Q_z = -K(z \cdot \nabla)T = - \int_S u \{K(z \cdot \nabla) \frac{\partial}{\partial n} \left(\frac{1}{r} \right)\} \, dS$$

where

$$z \cdot \nabla \equiv \frac{\partial}{\partial z} \quad (13)$$

Since T is a function of (x, y, z) it stands that

$$-K(z \cdot \nabla_o)T = 0 \quad (14)$$

where

$$z \cdot \nabla_o \equiv \frac{\partial}{\partial \alpha}$$

Substituting (4) into (14) results in

$$\int_S K \{ (z \cdot \nabla_o) u \} \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \, dS = \int_S K u \{ (z \cdot \nabla) \frac{\partial}{\partial n} \left(\frac{1}{r} \right) \} \, dS \quad (15)$$

Equation (13) then gives,

$$Q_z(x,y,z) = - \int_S v \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS \quad (16)$$

where

$$v \equiv K(z \cdot \nabla_0)u$$

and

$$r^2 = (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2$$

An equivalent expression for the surface heat flow results from a similar derivation demonstrated for surface temperature, T_S . This is

$$Q_{z_S}(\alpha', \beta', \gamma',) = 2\pi v(\alpha', \beta', \gamma') + \int_S v \frac{\partial}{\partial n} \left(\frac{1}{r} \right) dS \quad (17)$$

with

$$r^2 = (\alpha' - \alpha)^2 + (\beta' - \beta)^2 + (\gamma' - \gamma)^2$$

which is also a Fredholm integral equation of the second kind. As with the expressions for temperature (4) and (12), this integral equation can be solved for the equivalent dipole distribution in terms of surface vertical heat flow measurements, Q_{z_S} . Once the distribution is known, it can be substituted into (16) to determine the heat flow at any point above the surface, S .

Before (16) and (17) can be evaluated, the normal derivative must be calculated in terms of the surface elevation and the surface coordinates (α, β, γ) . If the vertical elevation at the point (α, β, γ) is defined as $\gamma = f(\alpha, \beta)$, then the direction cosines will be,

$$n_\alpha = \frac{-f_\alpha}{(1+f_\alpha^2+f_\beta^2)^{1/2}}, \quad n_\beta = \frac{-f_\beta}{(1+f_\alpha^2+f_\beta^2)^{1/2}}, \quad n_\gamma = \frac{1}{(1+f_\alpha^2+f_\beta^2)^{1/2}} \quad (18)$$

where f_α and f_β are the first derivatives of elevation in the α and β direction. The normal derivatives for a given surface S can now be

evaluated from (5) and (18). The final step is to project the surface element dS onto the α, β plane. This projection is given by

$$dS = \frac{d\alpha d\beta}{n_\gamma} = (1 + f_\alpha^2 + f_\beta^2)^{1/2} d\alpha d\beta \quad (19)$$

Substituting (18), (19) and (5) into (16) and (17), the final form of the heat flow equation is,

$$Q_z(x, y, z) = - \int_S v(\alpha, \beta, \gamma) \frac{-f_\alpha(x-\alpha) - f_\beta(y-\beta) + (z-\gamma)}{\{(\alpha'-\alpha)^2 + (\beta'-\beta)^2 + (\gamma'-\gamma)^2\}^{3/2}} d\alpha d\beta \quad (20)$$

and

$$Q_{z_s}(\alpha', \beta', \gamma') = 2\pi v(\alpha', \beta', \gamma') + \int_S v(\alpha, \beta, \gamma) \frac{-f_\alpha(\alpha'-\alpha) - f_\beta(\beta'-\beta) + (\gamma'-\gamma)}{\{(\alpha'-\alpha)^2 + (\beta'-\beta)^2 + (\gamma'-\gamma)^2\}^{3/2}} d\alpha d\beta \quad (21)$$

Numerical Techniques

Before the surface heat flow can be continued upward from an arbitrary surface to a datum plane using (20), the equivalent dipole distribution must be found in terms of the surface heat flow measurements. Since (21) has a $1/R^2$ singularity at $\alpha'=\alpha$, $\beta'=\beta$, and $\gamma'=\gamma$, it is difficult to evaluate numerically. The calculation can be simplified if a first order approximation for the dipole distribution given by

$$v(\alpha', \beta', \gamma') = \frac{Q_{z_s}(\alpha', \beta', \gamma')}{2\pi} \quad (22)$$

is used.

As will be shown later, the error of this approximation is

sufficiently small for heat flow calculations. Using (22), equation (20) can be approximated with a midpoint rule for a gridded set of data having unit spacing. This is,

$$Q_z(x,y,z) = -\frac{1}{2\pi} \sum \sum Q_{zs}(\alpha_i, \beta_j, \gamma_{ij}) \frac{-f_{\alpha_i}(x-\alpha_i) - f_{\beta_j}(y-\beta_j) + (z-\gamma_{ij})}{\{(x-\alpha_i)^2 + (y-\beta_j)^2 + (z-\gamma_{ij})^2\}^{3/2}} \quad (23)$$

where α_i and β_j are the spatial coordinates of the grid midpoints, γ_{ij} is the elevation at (α_i, β_j) , f_{α_i} and f_{β_j} are the first derivatives of elevation at α_i and β_j , and I, J are the number of points in the columns and rows respectively. The first derivatives are calculated with a cubic spline approximation given by

$$P_{i+1,j} + 4P_{i,j} + P_{i-1,j} = \frac{3}{h}(z_{i+1,j} - z_{i-1,j}) \quad \begin{matrix} i=2, I-1 \\ j=1, J \end{matrix} \quad (24)$$

where P_{ij} is the horizontal derivative in either the α or β direction and h is the grid spacing. This ensures a continuous first and second derivative. The mid point values of f_α , f_β and Q_{zs} , are calculated with a four-point interpolation approximation given by (Abramowitz and Stegun, 1970),

$$\begin{aligned} f(x_0+rh, y_0+sk) &= (1-r)(1-s)f_{0,0} + r(1-s)f_{1,0} \\ &+ s(1-r)f_{0,1} + rsf_{1,1} + O(h^2) \end{aligned} \quad (25)$$

where the equation is as shown in Figure 1.

The first step in the evaluation of (23) is to digitize the heat flow anomaly and topography. Edge effects which arise from calculating (23) for a finite surface are minimized by adding to the digitized grid ten points on a side which have a value equal to that on the grid edge. This 'padded' grid is then used to calculate f_α and f_β with (24). Once, f_α , f_β , and $Q_{zs}(\alpha, \beta, \gamma)$ have been determined,

Figure 1. Notation for linear interpolator used for determining heat flow and lateral derivatives at the midpoint.

the heat flow can be continued to a constant elevation using equation (23).

Test models

Three tests have been conducted to determine the accuracy of the numerical approximation, (23), for a heat flow data set. The tests were constructed to evaluate the accuracy and edge effects on a two-dimensional and three-dimensional surface. As a model, the observed heat flow was calculated for a line source parallel to the grid axis and positioned in the center of the data set. A solution to Laplace's equation for heat flow above an infinite line source is,

$$Q_L = - \frac{Q_0}{2\pi} \frac{(z-z')}{(x-x')^2 + (z-z')^2} \quad (26)$$

where primed quantities indicate source position, unprimed quantities indicate field position, and Q_0 is the source strength per unit length. No image was included in the test model because it is a continuation into a whole space which has a thermal conductivity equal to the mean conductivity of a survey area.

On a planar surface, $f_\alpha = f_\beta = 0$; for which equations (20) and (21) result in

$$Q_z(x,y,z) = - \frac{1}{2\pi} \int_S \frac{Q_{zs}(\alpha,\beta,\gamma)(z-\gamma) d\alpha d\beta}{\{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2\}^{3/2}} \quad (27)$$

which is the upward continuation integral discussed for gravity by Grant and West (1965).

The observed heat flow on a plane, Q_{zs} for a line source with $z' = 5$ km, $z = -1$ km, and $Q_0 = 9000 \text{ mWm}^{-1}$ is plotted in Figure 2 as the

Figure 2. The solid line is observed heat flow on a planar surface at -1 km. The dotted line is observed heat flow continued to a surface at -2.5 km with (23). The long dashed line is the theoretical heat flow on a surface at -2.5 km calculated from (26).

soild line. This surface heat flow continued upward to a plane -2.5 km above the surface by means of (23) is compared with the analytic solution calculated from (26). The results are plotted in Figure 2 as the dotted line and long dashed line respectively. The numerical and analytical curves differ only in a small region at the peak of the anomaly. At the center of the anomaly the maximum error is 2%. This means that the edge effects have been minimized for the planar surface. This good agreement is expected for a planar surface since the first order approximation, (22), is exact.

For another 2-D surface test, a line source at a depth of .5 km was set beneath a hemi cylindrical surface which has the form

$$z = \{.0093(x+5)^2H - .1934(x+5)\}H \quad (28)$$

where x is the lateral grid position in Figure 3 and H is the grid spacing. With a grid spacing of .8 km, (28) results in a maximum relief of .33 km. The observed heat flow on this surface, calculated from (26) with $Q_0=9000\text{mWm}^{-1}$, is plotted in Figure 3 as the solid curve. Continuing the observed heat flow to a plane 1 km above the highest elevation by means of (23) and comparing the numerical calculation with the analytic solution given by (26), results respectively in the dotted and dashed curves of Figure 3. As in the last test, the numerical and analytical curves differ appreciably only near the peak of the anomaly. The maximum error is only 4%.

For a three-dimensional hemispherical surface, the edge effects were tested by comparing calculated versus analytical continued heat flow for a traverse through the center of the hemisphere and a

Figure 3. The solid line is observed heat flow on the hemicylindrical surface shown in inset. The dotted line is observed heat flow continued to a surface at -2 km with (23). The long dashed line is the theoretical heat flow on a surface at -2 km calculated from (26).

traverse on the edge of the hemisphere. The vertical elevation is calculated from

$$z = H\{6507 - (x-7)^2 - (y-7)^2\}^{1/2} - 79.4 \quad (29)$$

where x is the lateral grid position in Figure 4, y is the vertical grid position, and H is the grid spacing. Again the grid spacing is .8 km. This gives a maximum relief of 500 m. A plot of (29) and the two traverses AA' and BB' are shown in Figure 4. Heat flow on this surface calculated from (26) with $Q_0=9000\text{mWm}^{-1}$ and $z'=.5$ km is also shown in the adjacent figure. The slight bend in the heat flow contours of Figure 4 shows that the heat flow from a line source with a large heat source strength is only slightly affected by topography. The modeled observed heat flow on the surface of the hemisphere for both traverses is shown in Figure 5 as the solid line. The observed heat flow in Figure 5 was continued to a plane 1 km above the highest elevation and compared with the analytical continuation. The numerically continued heat flow for the traverses is shown in Figure 5 as the dotted line.

For the traverse along the edge of the hemisphere, AA', the maximum error is 4% at the peak anomaly and drops to less than 2% away from the peak. For the traverse through the center of the hemisphere, BB', the maximum error is, again, 4% at the peak anomaly, but drops to a negligible value away from the peak.

These results suggest that the edge effect of the numerical approximation is small, and that for the idealized topography models just considered, the error will be minimized by placing the peak anomaly in the center of the grid. Since the maximum error occurs at

Figure 4. The elevation and observed heat flow for a hemispherical surface calculated from (29) and (26) respectively.

Figure 5. Heat flow results for the two traverses AA' and BB' shown in Figure 4. The solid line is observed heat flow on the hemispherical surface shown on insets. The dotted line is observed heat flow continued to a surface at -2 km by (23). The long dashed line is the theoretical heat flow on a surface at -2 km calculated from (26).

the peak heat flow anomaly, the maximum anomaly in a geothermal area will govern the accuracy of the numerical approximation. For the test models, the maximum error was 4%. This error could be reduced by going to a second order approximation in (23). However, this would double computation time, and the gain in accuracy is probably not worth the loss in efficiency.

Roosevelt Hot Springs

The Roosevelt Hot Springs geothermal system is located on the western flank of the Mineral Mountains in the Basin and Range province of southwestern Utah. The thermal anomaly has been discussed in detail in a previous paper, Wilson and Chapman (1980). Of particular interest here is the heat flow anomaly shown in Figure 6 and how this anomaly can be corrected using the methods of this paper.

The continuation approximation given by (23) has a singularity at $x=\alpha$, $y=\beta$, $z=\gamma_{ij}$. In order to avoid this singularity the datum plane is selected to be at a height just above the highest elevation. Surface heat flow and continued surface heat flow on a plane 2.6 km above sea level are shown in Figure 7 as the solid and dotted lines respectively. Since upward continuation is a smoothing operation, the dotted contours in Figure 7 are a smoothed version of the surface heat flow anomaly. This can be seen as the disappearance of the 400 mWm^{-2} contour in the southwest part of the system and the 1000 mWm^{-2} contour in the center of the anomaly. The area enclosed by the 700 mWm^{-2} contour has also been reduced. By analogy with the results of the test cases, the expected error above the maximum anomaly should be on

Figure 6. Surface heat flow at Roosevelt Hot Springs, Utah (after Wilson and Chapman, 1980).

Figure 7. The solid lines are observed surface heat flow at Roosevelt Hot Springs. The dotted lines are the observed heat flow continued to a planar surface at 2.6 km with (23). Contours are in mWm⁻².

the order of 4-5%.

In order to compare the continued heat flow with the observed, the datum plane can be lowered by means of downward continuation assuming that no heat sources are present between the original datum plane and the downward continued plane. Such a comparison will allow one to determine the effect of adjusting surface heat flow measurements to a constant elevation datum plane. The method of downward continuation for heat flow measurements is described by Wilson and Chapman (1980). Heat flow on the datum plane downward continued to a plane at an elevation of 1.9 km and observed surface heat flow are shown in Figure 8 as the dashed and solid lines respectively. The downward continued plane corresponds with the elevation of Thermal Power Co. Utah State Production well 14-2 also shown in Figure 8. The continued 400 mWm^{-2} contour has characteristics similar to the observed contour with the exception that the isolated high southwest of the system is now included in the main anomaly. The area enclosed by the 700 mWm^{-2} contour has been decreased in the northwest. This results from an elevation difference of .33 km between the surface and the datum plane in this part of the system. The area enclosed by the 1000 mWm^{-2} contour has been decreased substantially in size. This may be the result of elevation differences between 100-200 m over the area of the anomaly, but downward continuation also introduces an error of 10-15% above the maximum anomaly. Observed and continued contours are in fair agreement in the vicinity at Utah State well number 14-2. Quite clearly, the shape of the anomaly will depend on whether or not the

Figure 8. The solid lines are observed heat flow. The dashed lines are the datum corrected heat flow downward continued to a plane surface at 1.9 km. Contours are in mWm^{-2} .

anomaly is corrected to a datum plane. The results of this section suggest that heat flow measurements made on a surface with a topographic relief of greater than 300 m should be corrected to a datum plane before modeling.

Summary

The formulation of Bhattacharyya and Chan (1977) for a gravity and magnetic datum correction has been extended to include heat flow. An equivalent dipole distribution for an arbitrary surface allows one to continue upward surface heat flow measurements to a datum plane of constant elevation. A first order approximation for the equivalent dipole distribution results in a maximum error of 4% above the peak heat flow anomaly in test cases. At Roosevelt Hot Springs a comparison of the observed versus downward continued heat flow shows a decrease in the area enclosed by the 1000mWm^{-2} contour and an extension of the 400mWm^{-2} to include the isolated high southwest of the main system. These results suggest that topographic reliefs of greater than 300 m can affect the apparent geometry of the system.