CLASSICAL BEHAVIOR IN HIGH TEMPERATURE CHROMODYNAMICS

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In searching for tools to describe physical systems consisting of hadronic matter at high temperature, it is worthwhile to consider the application of classical chromodynamics. Classical non-Abelian gauge theories have been extensively studied¹ and continue to attract theoretical interest. However, the thrust of most work has been to consider classical dynamics as merely a guide to the quantum mechanical path integral. Attention has therefore focussed on particle-like field configurations or on topological structures which may be important in the presence of color confinement.

Confinement in low-temperature QCD provides a substantial barrier to the use of any classical approximations. With color fields confined to isolated spatial regions, it is very implausible that any classical approximation can be made for bulk hadronic matter. However, at temperatures above the postulated deconfining phase transition² there are reasons to believe that classical physics would be a valid approximation. Statistical fluctuations at high temperature can dominate quantum fluctuations and it is possible that the behavior of a large system can be described by averaged fields which obey classical equations.

In order to familiarize ourselves with the properties of bulk hadronic matter we will assume the existence of a neutral system characterized by a gauge-covariant polarization tensor, $M_{\mu\nu}^a$, which is suitably smooth. The Lagrangian for the color fields interacting with the matter and with themselves is

$$\mathcal{L}^{cl} = -\frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{2} G_{\mu\nu}^a M_{\mu\nu}^a, \quad (1)$$

where $G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc}A_\mu^b A_\nu^c$. Except for the color indices, the Lagrangian is the same as that for the electrodynamics of a polarizable medium. However, because of the nonlinearities associated with the self-coupling of the gauge fields, the classical limit of chromodynamics is quite distinct from that of electrodynamics. We can exploit some further simplifying assumptions to see examples of this difference. Choose an SU_2 subgroup of SU_3 and introduce the tensors

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$$\begin{aligned}
\rho_{ia} &= \hat{r}_i \hat{r}_a \\
\delta_{ia}^T &= \delta_{ia} - \hat{r}_i \hat{r}_a \\
\epsilon_{ia}^T &= \epsilon_{iak} \hat{r}_k
\end{aligned} \tag{2}$$

which connect space ($i, k=1\dots 3$) and color ($a, b=1, \dots, 3$). If we have explicit spherical symmetry, we can write the covariant polarization current

$$\begin{aligned}
j_0^a &= q(r, t) \hat{r}_a \\
j_i^a &= m(r, t) \rho_{ia} + p_1(r, t) \delta_{ia}^T + p_2(r, t) \epsilon_{ia}^T
\end{aligned} \tag{3}$$

and the vector potential

$$\begin{aligned}
gA_0^a &= A_0(r, t) \hat{r}_a \\
gA_i^a &= A_1(r, t) \rho_{ia} + \frac{a(r, t)}{r} \sin \omega(r, t) \delta_{ia}^T \\
&\quad + \frac{a(r, t) \cos \omega(r, t) - 1}{r} \epsilon_{ia}^T.
\end{aligned} \tag{4}$$

To formulate a comparison with electrodynamics it is convenient to make projections of the covariant field strength tensor,³

$$\begin{aligned}
gE_L &= \partial A_0 / \partial r - \partial A_1 / \partial t & gB_L &= (a^2 - 1) r^2 \\
grE_{(+)} &= -\partial a / \partial t & grB_{(+)} &= -a(\partial \omega / \partial r - A_1) \\
grE_{(-)} &= -a(\partial \omega / \partial t - A_0) & grB_{(-)} &= \partial a / \partial r,
\end{aligned} \tag{5}$$

where the subscripts $(\pm)$ refer to chiralities of the transverse fields as given by projections with the gauge-dependent tensors

$$\begin{aligned}
e_{ia}^{(+)}(\omega) &= \delta_{ia}^T \sin \omega + \epsilon_{ia}^T \cos \omega \\
e_{ia}^{(-)}(\omega) &= \delta_{ia}^T \cos \omega - \epsilon_{ia}^T \sin \omega .
\end{aligned}
\tag{6}$$

The decomposition of the fields in (5) is such that under a radially-directed gauge transformation the field components remain invariant and the tensors (6) rotate by changing the arguments of the sine and cosine functions. By making a similar decomposition of the transverse currents p_1 and p_2 in Eq.(3) we can formulate the dynamics without recourse to transverse gauge fixing.

Because of the assumption of spherical symmetry it is convenient to introduce a two-dimensional notation for the time and radial degrees of freedom. The dynamics of the classical gauge theory can be specified by two coupled two-dimensional wave equations³

$$\square_{(2)} \epsilon - \frac{2\epsilon}{r} = 2g^2 r^2 E_i^a B_i^a - \epsilon^{2m} \partial_\ell J_m \tag{7}$$

$$\square_{(2)} \beta - \frac{2\beta}{r} - \frac{1}{2} (\beta^2 + \epsilon^2) = g^2 r^2 (B_i^a B_i^a - E_i^a E_i^a) + (1 + \beta)^{1/2} r p^- \tag{8}$$

where $\epsilon = r^2 E_L$ and $\beta = r^2 B_L$ are the longitudinal degrees of freedom. The two dimensional current $J_\ell = (r^2 q, r^2 m)$ is constrained by covariant current conservation

$$\partial_\ell J_\ell = 2ar p^{(+)} . \tag{9}$$

Lorentz invariants such as $G_{\mu\nu}^a G_{\mu\nu}^a = 2E_i^a B_i^a$ and $G_{\mu\nu}^a G_{\mu\nu}^a = (B_i^a B_i^a - E_i^a E_i^a)$ are written in noninvariant form to emphasize that the assumption of spherical symmetry specifies a particular Lorentz frame. The transverse fields are given by

$$\begin{aligned}
2ar E_{(+)} &= \partial\beta/\partial t & 2ar B_{(+)} &= J_1 - \partial\epsilon/\partial t \\
2ar E_{(-)} &= J_0 + \partial\epsilon/\partial r & 2ar B_{(-)} &= \partial\beta/\partial r
\end{aligned}
\tag{10}$$

with $a(r,t) = [1 + \beta(r,t)]^{1/2}$.

Some study of simple solutions to (7)-(10) can be found in Ref.(3). In comparing the dynamics of a non-Abelian gauge theory to electrodynamics it is useful to keep in mind that the fields themselves carry a color charge. This is the underlying source of

the nonlinearities in the field equations. When we characterize a possible physical state it is necessary to use order parameters such as $G_{\mu\nu}^a G_{\mu\nu}^a$ which include the color fields. It will take a lot of effort to uncover the chromodynamic properties of high temperature nuclear matter even if there is experimental evidence for a transition to a deconfined phase. The use of the classical approximation for the non-Abelian dynamics may provide an important tool.

REFERENCES

1. See, for example, A. Actor, Rev. Mod. Phys. 51, 461 (1979).
2. See, for example, the many papers on the phase transition at this conference.
3. J. P. Ralston and D. Sivers, Phys. Rev. D (to be published) and J. P. Ralston and D. Sivers, Phys. Rev. D28, 953 (1983).

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