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ABSTRACT

An analytic calculation of the cooling by thermal conduction of a hot, cylindrical symmetric, gas bubble is presented. The results are in good agreement with an exact numerical calculation.

INTRODUCTION

A gas, which has been locally heated by the passage of a charged particle beam, breakdown in a laser beam, or some other means, subsequently cools by a variety of mechanisms. On the most rapid time scale there is some cooling as the expanding bubble does mechanical work against its surroundings. There may also be convective mixing of hot and cool gas by turbulence or other asymmetries. Eventually the system arrives in a state of mechanical equilibrium characterized by uniform

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pressure, and a slow cooling takes place by thermal conduction and radiation. The latter process is dominant for temperatures greater than about 10^4 °K, but may be neglected below 5000 °K.

The phenomenon studied here is the thermal conduction phase of the equilibrium bubble. At some distance from the center of the disturbance, the gas is assumed to be uniform, with ambient temperature T_A . The pressure equals the ambient value P_A everywhere, and the mass density is depressed in the bubble (as required by the equation of state). As the bubble cools there must be mass flow to maintain $P = P_A$. The principal aim of the present work is to derive an analytical formula for the characteristic cooling time, assuming cylindrical symmetry and no variation with z . This goal is achieved with a minimum of numerical approximations. The analytical results are found to agree with a one-dimensional, numerical calculation, within a few percent error.

In section I, the general formulation used for the calculation is reviewed. The equations given here are the basis for both the analytical and numerical study. Section II contains the simplified model equations which apply to a low temperature, nearly ideal gas. The solution of these equations for a small temperature perturbation is discussed in section III. In section IV, the analytic (zero dimensional) model equations are derived and solved. These give the time evolution of the central temperature and scale radius of the bubble. Section V contains a comparison of analytic and numerical results. The properties of the small amplitude disturbance are developed in more detail in Appendix A. In particular, it is shown that an expanding Gaussian temperature profile is the asymptotic state at large time.

It must be emphasized that the results reported here serve primarily as a basis for comparison with experiment. That is, one wants to know what the simplest, anomaly-free, model predicts. There is experimental evidence of significantly faster cooling than the rate predicted here,¹ probably due to convection processes.

I. General Formulation

The general fluid equations for gas flow, including thermal conduction but neglecting viscosity, are²

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0, \quad (\text{mass}) \quad (1)$$

$$\frac{\partial}{\partial t} \rho \mathbf{v} + \nabla \cdot \rho \mathbf{v} \mathbf{v} = - \nabla P, \quad (\text{momentum}) \quad (2)$$

$$\frac{\partial}{\partial t} \left(\rho \epsilon + \rho \frac{v^2}{2} \right) + \nabla \cdot \mathbf{v} \left(\rho \epsilon + P + \rho \frac{v^2}{2} \right) - \nabla \cdot K \nabla T = 0. \quad (\text{energy}) \quad (3)$$

Here we denote

- ρ = mass density,
- $\mathbf{v}$ = velocity,
- P = pressure,
- ϵ = internal energy per gm,
- K = thermal conductivity,
- T = temperature.

It is assumed that mechanical equilibrium prevails; then $\mathbf{v}$ is small, and may be eliminated from Eq. (2). Formally, one defines δ be a small parameter proportional to the thermal conductivity. Then a consistent ordering of terms in Eqs. (1) - (3) is

$$\partial/\partial t = O(\delta),$$

$$\underline{v} = O(\delta),$$

$$\underline{\nabla} \rho = O(1),$$

$$\underline{\nabla} \epsilon = O(1),$$

$$\underline{\nabla} T = O(1),$$

$$\underline{\nabla} P = O(\delta^2).$$

Hence, dropping terms which are small of order δ^2 , one has from Eq. (2)

$$P = \text{constant} = P_A, \quad (4)$$

where P_A is the ambient pressure. Similarly, Eq. (3) becomes

$$\frac{\partial}{\partial t} \rho \epsilon + \underline{\nabla} \cdot \underline{v} (\rho \epsilon + P) - \underline{\nabla} \cdot K \underline{\nabla} T = 0, \quad (5)$$

and Eq. (1) is unchanged.

As mentioned, mass flow appears in the theory to maintain constant pressure. Hence the situation is somewhat more involved than ordinary thermal diffusion. The derived flow velocity must be small compared with the speed of sound for the ordering in δ to hold.

It is convenient to eliminate the internal energy (ϵ) by introducing the enthalpy per gm

$$w \equiv \epsilon + \frac{P}{\rho}. \quad (6)$$

Then, using the fact that P is constant, one obtains from Eq. (5)

$$\frac{\partial \rho w}{\partial t} + \underline{\nabla} \cdot \rho \underline{v} w - \underline{\nabla} \cdot K \underline{\nabla} T = 0. \quad (7)$$

Equations (1), (4) and (7) determine the evolution of the system if they are supplemented by a table of gas properties. For numerical work it is easiest to use P and w as the independent thermodynamic variables. Table 1 gives the properties of air at $P_A = 1$ atm.

For a cylindrically symmetric, z independent problem, we have

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r \rho v_r = 0, \quad (8)$$

$$P = P_A = \text{constant} \quad (9)$$

$$\frac{\partial \rho w}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} r \rho w v_r - \frac{1}{r} \frac{\partial}{\partial r} \frac{K \partial T}{\partial r} = 0. \quad (10)$$

These equations are solved numerically by the code ENTHEL using the tabulated air properties.³

II. Low Temperature Formulation

Considerable analytical progress can be made in solving Eqs. (8) - (10) if the temperature is low enough that there is no excitation or dissociation of the gas molecules. For air this requires $T \leq 2000^\circ\text{K}$.

In this range we have

$$K = (.690 \times 10^{11}) T \equiv \alpha_1 T, \quad (11a)$$

$$w = (.992 \times 10^7) T \equiv \alpha_2 T, \quad (11b)$$

$$\rho = (.353) \left(\frac{P_A}{1 \text{ atm}} \right) T^{-1} \equiv \alpha_3 P_A T^{-1}. \quad (11c)$$

Here the units of T are $^{\circ}\text{K}$ and P_A are atm. The units of K , w and ρ are standard cgs. The only significant error of Eq. (11) is in K , which is accurate only to $\pm 15\%$.

Note first that

$$\rho w = (\alpha_3 P_A T^{-1})(\alpha_2 T) = \alpha_2 \alpha_3 P_A . \quad (12)$$

It is now elementary to integrate Eq. (10): we have

$$r \left(\rho w v_r - K \frac{\partial T}{\partial r} \right) = 0 , \quad (13)$$

with the constant of integration vanishing because the flow cannot be singular. Hence, the velocity is

$$v_r = \frac{K}{\rho w} \frac{\partial T}{\partial r} = \frac{\alpha_1 T}{\alpha_2 \alpha_3 P_A} \frac{\partial T}{\partial r} = \lambda T \frac{\partial T}{\partial r} , \quad (14)$$

where

$$\lambda = \frac{\alpha_1}{\alpha_2 \alpha_3} \frac{1}{P_A} = (1.97 \times 10^{-6}) \left(\frac{1 \text{ atm}}{P_A} \right) \frac{(\text{cm}^2)}{(^{\circ}\text{K})^2 (\text{s})} \quad (15)$$

Using Eqs. (11c) and (14) we eliminate ρ and v_r from Eq. (8) in favor of T to obtain

$$\frac{\partial}{\partial t} \left(\frac{\alpha_3 P_A}{T} \right) + \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{\alpha_3 P_A}{T} \right) \left(\lambda T \frac{\partial T}{\partial r} \right) = 0 , \quad (16)$$

which may be written in the convenient form

$$\frac{\partial T}{\partial t} = \lambda T^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial T}{\partial r} . \quad (17)$$

This equation is a significant result because it indicates that the inward flow of gas results in a more rapid decay of temperature. If there were no flow and K were linear in T , then only two powers of T would appear on the right hand side of Eq. (17), instead of three powers.

III. Linearized Solution

For small temperature perturbations a useful, self-similar solution of Eq. (17) is found. Let

$$T = T_A + \delta T. \quad (18)$$

where

$$\frac{\delta T}{T_A} \ll 1.$$

Then the linearized equation for δT is

$$\frac{\partial \delta T}{\partial t} = \lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial \delta T}{\partial r}. \quad (19)$$

It is easily verified that

$$\delta T_s = \frac{A_0 T_A}{(1 + t/\tau)} \exp \left[- \frac{r^2}{R_0^2 (1 + t/\tau)} \right], \quad (20)$$

is a solution for properly selected decay time constant τ . Here A_0 and R_0 are the initial amplitude and scale radius. The time constant is

$$\tau = \frac{R_0^2}{4\lambda T_A^2} = (1.41) \left(\frac{R_0}{1 \text{ cm}} \right)^2 \left(\frac{P_A}{1 \text{ atm}} \right) \left(\frac{300^\circ \text{K}}{T_A} \right)^2 \text{ sec}. \quad (21)$$

It is proved in Appendix A that an arbitrary initial profile $\delta T(r, t = 0)$ evolves into a self similar solution of the described type, with scale radius and amplitude known functions of t .

IV. Approximate Non-Linear Solution

For $\delta T/T_A$ of arbitrary magnitude we seek an approximate solution of the equation

$$\frac{\partial \delta T}{\partial t} = \lambda (T_A + \delta T)^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \delta T}{\partial r} . \quad (22)$$

The quantities of greatest interest are the central temperature and scale radius; hence we expand around $r = 0$:

$$\frac{\delta T}{T_A} = C_0(t) + C_2(t) r^2 + C_4(t) r^4 + \dots , \quad (23)$$

and the constants C_0 and C_2 are to be determined. We introduce the notation for time dependent amplitude and scale radius:

$$A = C_0 = \frac{\delta T(r=0)}{T_A} \quad (24a)$$

$$\frac{1}{R^2} \equiv -\frac{C_2}{C_0} = -\frac{1}{4} \frac{\nabla^2 \delta T(r=0)}{\delta T(r=0)} . \quad (24b)$$

Equations (22) and (23) generate an infinite coupled set of equations for the C_n . In order to truncate the expansion, a relation between C_4 and A and R is imposed. It is assumed that a Gaussian profile is a good approximation for δT around $r^2 = 0$:

$$\begin{aligned} e^{-r^2/R^2} &= 1 - \frac{r^2}{R^2} + \frac{r^4}{2R^4} \dots \\ &\approx 1 + \frac{C_2}{C_0} r^2 + \frac{C_4}{C_0} r^4 + \dots = \frac{\delta T}{C_0 T_A} . \end{aligned} \quad (25)$$

The expansions are equal (by the definition of R and A) in the first two terms.

Equating the third terms gives

$$C_4 = \frac{C_0}{2R^4} = \frac{A}{2R^4} \quad (26)$$

Equation (26) is the principal approximation of the theory. It is justified by convenience, the desire to gain contact with the self-similar solution δT_s , and comparison with exact numerical results.

Equation (22) is now expanded in powers of r^2 :

$$\begin{aligned} \frac{\partial}{\partial t} (C_0 + C_2 r^2 + \dots) &= \lambda T_A^2 (1 + C_0 + C_2 r^2 + \dots)^2 \\ &\times (2^2 C_2 + 4^2 C_4 r^2 + \dots) \end{aligned} \quad (27)$$

Equating coefficients of r^n on each side of Eq. (27) gives

$$\frac{dC_0}{dt} = \lambda T_A^2 4(1 + C_0)^2 C_2, \quad (28a)$$

$$\frac{dC_2}{dt} = \lambda T_A^2 \left[16(1 + C_0)^2 C_4 + 8(1 + C_0) C_2^2 \right], \quad (28b)$$

and similarly for higher C_n . With the aid of expression (26) we eliminate C_4 and obtain a closed set of equations. Using the definitions (24a) and (24b), these are

$$\frac{dA}{dt} = - \frac{4 \lambda T_A^2}{R^2} (1 + A)^2 A, \quad (29)$$

$$\frac{d}{dt} \left(\frac{A}{R^2} \right) = - \frac{8 \lambda T_A^2}{R^4} \left[(1 + A)^2 A + (1 + A) A^2 \right]. \quad (30)$$

The derivative of A may be eliminated from Eq. (30) with the aid of Eq. (29) to obtain

$$\frac{dR^2}{dt} = 4\lambda T_A^2 (1 + 4A + 3A^2) \quad (31)$$

The coupled Eqs. (29) and (31) are most easily solved by first eliminating t as an independent variable; we have

$$\frac{dA}{dR^2} = - \frac{(1 + A)^2}{R^2} \frac{A}{(1 + 4A + 3A^2)} \quad (32)$$

This equation is of separable form and may be integrated immediately:

$$\frac{d}{dR^2} \log [R^2 A(1 + A)^2] = 0 \quad (33)$$

Taking A_0 and R_0 to be the values of A and R at $t = 0$, we obtain the relation

$$\frac{R^2}{R_0^2} \frac{A(1 + A)^2}{A_0(1 + A_0)^2} = 1 \quad (34)$$

Returning to Eq. (29), R^2 is eliminated with Eq. (34) to get

$$\begin{aligned} \frac{dA}{dt} &= - \left[4\lambda T_A^2 (1 + A)^2 A \right] \left[\frac{A(1 + A)^2}{R_0^2 A_0(1 + A_0)^2} \right] \\ &= - \frac{1}{\tau} \frac{A^2(1 + A)^4}{A_0(1 + A_0)^2} \quad (35) \end{aligned}$$

This equation is also integrable, i.e.,

$$\frac{t}{\tau} = - A_0(1 + A_0)^2 \int_{A_0}^A \frac{dx}{x^2(1 + x)^4} \quad (36)$$

It may be verified that the value of the integral is

$$\frac{t}{\tau} = -A_0(1+A_0)^2 \left[-\left(\frac{1+x}{x}\right) + 4 \log\left(\frac{1+x}{x}\right) + 6\left(\frac{x}{1+x}\right) - 2\left(\frac{x}{1+x}\right)^2 + \frac{1}{3}\left(\frac{x}{1+x}\right)^3 \right]_{A_0}^A \quad (37)$$

$$\begin{aligned} &= (1+A_0)^2 A_0 \left\{ \left(\frac{1+A}{A} - \frac{1+A_0}{A_0} \right) - 4 \log \left[\left(\frac{A_0}{A} \right) \left(\frac{1+A}{1+A_0} \right) \right] - 6 \left(\frac{A}{1+A} - \frac{A_0}{1+A_0} \right) \right. \\ &\quad \left. + 2 \left[\left(\frac{A}{1+A} \right)^2 - \left(\frac{A_0}{1+A_0} \right)^2 \right] - \frac{1}{3} \left[\left(\frac{A}{1+A} \right)^3 - \left(\frac{A_0}{1+A_0} \right)^3 \right] \right\} \quad (38) \end{aligned}$$

For small A , Eq. (38) becomes

$$\frac{t}{\tau} \approx A_0 \left(\frac{1}{A} - \frac{1}{A_0} \right) ,$$

or

$$A = \frac{A_0}{1+t/\tau} , \quad (39)$$

and Eq. (34) gives

$$R^2 = R_0^2 \frac{A_0}{A} = R_0^2 \left(1 + \frac{t}{\tau} \right) . \quad (40)$$

Thus the parameters of the self-similar solution are recovered, as expected.

V. Comparison with Numerical Results

The code ENTHEL has been run to study the following case:

$$\begin{aligned} T_A &= 300^\circ\text{K} , \\ P_A &= 1 \text{ atm (air)} , \\ \delta T(t=0) &= T_A \left[1 + \left(\frac{r}{.5 \text{ cm}} \right)^2 \right]^{-2} . \end{aligned}$$

For the analytic comparison we thus use

$$\begin{aligned} A_0 &= \frac{\delta T(0,0)}{T_A} = 1.0 , \\ R_0 &= .5/\sqrt{2} = .35355 \text{ cm} , \\ \tau &= (1.41) \left(\frac{.5}{\sqrt{2}} \right)^2 = .176 \text{ sec.} \end{aligned}$$

Table 2 gives the analytic values of t/τ and R/R_0 for a broad range of A .

Table 3 gives the on-axis perturbed temperature computed with ENTHEL for a range of t . An inferred radius of curvature is tabulated, but this quantity is in error by about +3% caused by the finite radial mesh size. The quantity $R\sqrt{A} (1+A)$, which the analytic theory predicts to be constant, is also calculated from the perturbed temperature and radius; it is found to decrease about 8% from start to finish of the run.

Figure 1 is a graph of the analytic and computational perturbed temperature plotted vs real time. The agreement is excellent.

Finally, Table 4 gives some analytic decay times for other values of A_0 and A .

Appendix A

Evolution of a Small Amplitude Bubble

We wish to show that a general solution of the linearized equation

$$\frac{\partial \delta T}{\partial t} = \lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \delta T}{\partial r} \quad (A-1)$$

evolves into the self similar solution

$$\frac{\delta T_s}{T_A} \propto \frac{A_0}{1 + t/\tau} \exp \left[- \frac{r^2}{R_0^2 (1 + t/\tau)} \right] \quad (A-2)$$

for large t .

First, note that there is a conservation law; let

$$U = \int dr 2\pi r \delta T . \quad (A-3)$$

Then from Eq. (A-1) we have

$$\begin{aligned} \frac{dU}{dt} &= \int dr 2\pi r \frac{\partial \delta T}{\partial t} = \int dr 2\pi r \lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \delta T}{\partial r} \\ &= \left[2\pi \lambda T_A^2 r \frac{\partial \delta T}{\partial r} \right]_0^\infty = 0 . \end{aligned} \quad (A-4)$$

Next we define an rms bubble radius:

$$\overline{r^2} = U^{-1} \int dr 2\pi r \delta T r^2 . \quad (A-5)$$

Substituting from Eq. (A-1) and integrating by parts twice, we have

$$\begin{aligned}
 \frac{d}{dt} \overline{r^2} &= U^{-1} \int dr 2\pi r^3 \frac{\partial \delta T}{\partial t} = U^{-1} \int dr 2\pi r^3 \lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \delta T}{\partial r} \\
 &= - U^{-1} \lambda T_A^2 \int dr 4\pi r^2 \frac{\partial \delta T}{\partial r} = + U^{-1} \lambda T_A^2 \int dr 8\pi r \delta T \\
 &= U^{-1} \lambda T_A^2 4U = 4 \lambda T_A^2 .
 \end{aligned} \tag{A-5}$$

$\overline{r^2}$ is thus known at time t if its value $\overline{r^2}_0$ is known at time $t = 0$:

$$\overline{r^2} = \overline{r^2}_0 + 4 \lambda T_A^2 t = \overline{r^2}_0 (1 + t/\bar{\tau}) , \tag{A-6}$$

where $\bar{\tau} = (\overline{r^2}_0)/(4\lambda T_A^2)$ is a generalization of the previously defined decay time τ .

Third, we define the functional

$$H = \int dr 2\pi r \delta T \log \delta T . \tag{A-7}$$

Then,

$$\begin{aligned}
 \frac{dH}{dt} &= \int dr 2\pi r \left(\frac{\partial}{\partial \delta T} \delta T \log \delta T \right) \frac{\partial \delta T}{\partial t} \\
 &= \int dr 2\pi r (1 + \log \delta T) \lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \delta T}{\partial r} \\
 &= - \lambda T_A^2 \int dr 2\pi r \frac{1}{\delta T} \left(\frac{\partial \delta T}{\partial r} \right)^2 .
 \end{aligned} \tag{A-8}$$

It is easily shown that δT , if initially positive, remains positive, so for problem at hand

$$\frac{dH}{dt} < 0 . \tag{A-9}$$

If $\delta T(r)$ is specified at $t = 0$, we know U and $\overline{r^2}$ at any later time. When the evolution of differing profiles is compared, they should have the same values of U and $\overline{r^2}$ so we do not "compare apples with oranges". The minimum value of H for given U and $\overline{r^2}$ is found by the usual variational method, using Lagrangian multipliers:

$$0 = \Delta h + \lambda_1 \Delta U + \lambda_2 \Delta(\overline{U r^2}) = \int dr 2\pi r \Delta(\delta T) \left[(1 + \log \delta T) + \lambda_1 + \lambda_2 r^2 \right]. \quad (A-10)$$

Hence, minimal δT satisfies

$$0 = 1 + \log \delta T + \lambda_1 + \lambda_2 r^2. \quad (A-11)$$

This is just the self-similar solution δT_s , i.e.,

$$\delta T = e^{-(1+\lambda_1+\lambda_2 r^2)} = \frac{A_0}{(1+t/\tau)} e^{-\frac{r^2}{R_0^2(1+t/\tau)}}, \quad (A-12)$$

with

$$R_0^2(1+t/\tau) = \frac{1}{\lambda_2} = \overline{r^2}, \quad (A-13)$$

$$\frac{A_0}{(1+t/\tau)} = e^{-(1+\lambda_1)} = \frac{U}{\pi \overline{r^2}}, \quad (A-14)$$

and $\tau = \overline{r^2}$.

It remains to show that δT_s also gives a minimum of the functional $(-dH/dt)$. As before we take the variation with U and $\overline{r^2}$ fixed:

$$\begin{aligned} 0 &= \Delta \left(\frac{-dH}{dt} \right) + \beta_1 \Delta U + \beta_2 \Delta(\overline{Ur^2}) \\ &= \int dr 2\pi r \left[\Delta \frac{\lambda T_A^2}{\delta T} \left(\frac{\partial \delta T}{\partial r} \right)^2 + \beta_1 \Delta \delta T + \beta_2 r^2 \Delta \delta T \right] \\ &= \int dr 2\pi r (\Delta \delta T) \left[-\lambda T_A^2 \left(\frac{1}{\delta T} \frac{\partial \delta T}{\partial r} \right)^2 \right. \\ &\quad \left. - 2\lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{1}{\delta T} \frac{\partial \delta T}{\partial r} \right) + \beta_1 + \beta_2 r^2 \right]. \end{aligned} \quad (A-15)$$

Minimal δT satisfies

$$0 = -\lambda T_A^2 \left(\frac{1}{\delta T} \frac{\partial \delta T}{\partial r} \right)^2 - 2\lambda T_A^2 \frac{1}{r} \frac{\partial}{\partial r} r \left(\frac{1}{\delta T} \frac{\partial \delta T}{\partial r} \right) + \beta_1 + \beta_2 r^2. \quad (A-16)$$

It is easily verified that this equation is satisfied by δT_s , with

$$\beta_1 = -\frac{8\lambda T_A^2}{R_0^2(1+t/\tau)} \quad (A-17a)$$

$$\beta_2 = -\frac{4\lambda T_A^2}{R_0^4(1+t/\tau)^2} \quad (A-17b)$$

We have found that δT_s minimizes both H and $(-dH/dt)$. The H computed for any other profile having the same $\overline{r^2}$ and U is larger than that of δT_s and is decreasing more rapidly. Hence for large time the H functions converge. The minimum of H is unique, so δT_s is the asymptotic state.

Captions

- Table 1. Properties of air at one atm. T is temperature, K is thermal conductivity, ρ is mass density, and σ is electrical conductivity. Source of data is J. Jos et al.³
- Table 2. Decay time (t) of a thermal pulse, calculated analytically. $A(t)$ is the ratio of the on-axis temperature perturbation divided by the ambient temperature, τ is the scale decay time (defined in the text), and R_0 is the initial scale radius of the perturbation.
- Table 3. Decay of a thermal pulse computed with ENTHEL. The ambient temperature is 3000K. $\delta T(0)$ is the on axis perturbation and R is the scale radius. The quantity $R\sqrt{A(1+A)}$ is predicted (analytically) to be constant.
- Table 4. Decay times and radii for selected initial and final amplitudes of a thermal pulse, computed analytically. A is the ratio of the on-axis temperature perturbation and ambient temperature.
- Figure 1. Comparison of analytical and numerical calculations of the decay of a thermal pulse. The on axis temperature perturbation is plotted. $R_0 = .35355$ cm and $T_A = 3000$ K.

Table 1

$T(^{\circ}K)$	$K(\frac{\text{erg}}{\text{cm}^2\text{s}^{\circ}K})$ $\times 10^{-4}$	$w(\frac{\text{erg}}{\text{gm}})$ $\times 10^{-11}$	$\rho(\frac{\text{gm}}{\text{cm}^3})$ $\times 10^{+6}$	$\sigma(\frac{\text{mho}}{\text{cm}})$
0.0	0.0	.0000	∞	0.0
500.	.345	.0496	705.6	0.0
1000.	.690	.0992	352.8	0.0
1500.	.980	.1488	235.2	0.0
2000.	1.27	.2289	176.4	0.0
2500.	1.93	.3021	140.7	0.0
3000.	4.01	.4075	115.0	.0002
3500.	6.47	.5774	93.3	.0037
4000.	5.85	.7699	76.6	.0254
4500.	4.11	.9037	66.0	.1000
5000.	6.10	1.0251	58.2	.281
5500.	11.3	1.2008	51.5	.638
6000.	20.3	1.4937	44.7	1.24
6500.	30.6	1.9665	37.9	2.17
7000.	35.1	2.6150	31.6	3.66
7500.	29.1	3.2803	26.6	6.17
8000.	19.7	3.7823	23.4	9.83
8500.	13.9	4.1086	21.3	14.1
9000.	11.8	4.3429	19.8	18.5
9500.	11.7	4.5480	18.5	22.7
10,000.	12.6	4.7698	17.4	26.8
10,500.	14.0	5.0292	16.3	30.7
11,000.	15.7	5.3472	15.3	34.6
11,500.	17.5	5.7404	14.2	38.3

Table 2

A	t/τ	R/R_0
1.0	0.0	1.0
.9	.0309167	1.10957
.8	.0786630	1.24225
.7	.155418	1.40615
.6	.285107	1.61374
.5	.518731	1.8856
.4	.977634	2.25876
.3	2.00014	2.80883
.2	4.80491	3.72677
.1	16.7739	5.7495
.05	47.4198	8.51835
0.0	∞	∞

Table 3

$t(\text{ms})$	$\delta T(o) (^{\circ}\text{K})$	$R \text{ (cm)}$	$\frac{R}{R_0} \sqrt{\frac{A}{A_0}} \left(\frac{1+A}{1+A_0} \right)$
0.0	300.0	.36409	1.0
10.0	254.49	.43417	1.01501
20.0	227.49	.47767	1.00440
30.0	208.43	.51011	.98959
40.0	193.77	.53698	.97545
50.0	181.93	.56006	.96216
60.0	172.03	.58135	.95073
70.1	163.52	.59948	.93909
80.1	156.20	.61573	.92783
90.1	149.74	.63190	.91909

Table 4

A_0	1.0	2.0	2.0	3.0	3.0	4.0	4.0	5.0	5.0
A	.5	1.0	.5	1.5	.5	2.0	.5	2.5	.5
t/τ	.51873	.31466	2.6489	.21043	7.1254	.15038	14.866	.11274	26.770
R/R_0	1.8856	2.1213	4.0000	2.2627	6.5319	2.3570	9.4280	2.4243	12.649

Figure 1

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