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AND STABILITY CONSIDERATIONS

By

E.R. Salberta*, R.C. Grimm†, J.L. Johnson,
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ABSTRACT

Investigation of the effect of pressure anisotropy on tokamak equilibrium and stability is made with an MHD model. Realistic perpendicular and parallel pressure distributions, $P_{\perp}(\psi, B)$ and $P_{\parallel}(\psi, B)$, are obtained by solving a one-dimensional Fokker-Planck equation for neutral beam injection to find a distribution function $f(E, v_{\parallel}/v)$ at the position of minimum field on each magnetic surface and then using invariance of the magnetic moment to determine its value at each point on the surface. The shift of the surfaces of constant perpendicular and parallel pressure from the flux surfaces depends strongly on the angle of injection. This shift explains the observed increase or decrease in the stability conditions. Estimates of the stabilizing effect of hot trapped ions indicates that a large fraction must be nonresonant and thus decoupled from the bad curvature before it becomes important.

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I. INTRODUCTION

Ideal MHD equilibrium and stability studies have provided significant guidance for the design and interpretation of toroidal magnetic confinement systems for fusion. Major computational programs have been constructed and are being used for the study of axisymmetric tokamak configuration.^{1,2} The familiar assumption that the plasma pressure in tokamaks is roughly isotropic (i.e., scalar pressure) has usually been involved in these codes. While this is a reasonable approximation for ohmically heated discharges, it becomes quite unrealistic in plasmas with intense auxiliary heating. For example, in discharges heated by neutral beam injection (NBI), the pressure distributions can be strongly anisotropic. Hence, it is obviously of interest to generalize the usual isotropic codes to be able to treat these situations properly. The work reported here represents the beginning of a program to modify the PEST formulation (one of the principal computational tools employed by the fusion community for tokamak equilibrium and stability studies) for this purpose.

Previous studies³⁻⁸ of the equilibrium and stability properties of anisotropic tokamaks have usually employed model fields. In the most comprehensive work up to the present time,^{6,7} the calculations used the solutions of a Fokker-Planck equation with a source term of high energy beam particles to obtain expressions for the variation of the parallel and perpendicular pressures on a magnetic flux surface. However, pitch-angle scattering in velocity space was ignored. This limitation prevented the modeling of equilibria with trapped beam particles, since all beam ions on any given magnetic surface would mirror at exactly the same value of the magnetic field, thereby leading to mirror instability. The method of generating the beam-ion distributions described in the present paper provides a more realistic alternative description of a beam-heated tokamak.

In the next section we describe the equilibrium formalism that has been incorporated into the PEST code. A specific technique for constructing an anisotropic pressure distribution from a calculation with the one-dimensional Fokker-Planck code BALDUR is then presented in Sec. III. Stability studies are carried out in Sec. IV using a ballooning-mode formalism which allows inclusion of finite gyroradius effects as well as kinetic corrections associated with fast ions. A specific application to several typical configurations is presented in Sec. V, and a discussion of the results given in the last section.

II. EQUILIBRIUM

An anisotropic static equilibrium can be described by the system of equations⁹:

$$\underline{J} \times \underline{B} = \nabla \cdot \underline{P} \quad , \quad (1)$$

$$\underline{P} = \frac{1}{2} P_{\perp} - \left(\frac{P_{\perp} - P_{\parallel}}{B^2} \right) \underline{B} \underline{B} \quad , \quad (2)$$

$$\underline{J} = \nabla \times \underline{B} \quad , \quad (3)$$

$$\nabla \cdot \underline{B} = 0 \quad , \quad (4)$$

with $P_{\perp}$ and $P_{\parallel}$ functions of the magnetic flux function, ψ , and the magnitude of the magnetic field. It is convenient to introduce the functions

$$\sigma \equiv 1 - \frac{1}{B} \frac{\partial P_{\parallel}}{\partial B} \bigg|_{\psi} \quad , \quad (5)$$

$$\tau \equiv 1 + \frac{1}{B} \left. \frac{\partial P_{\perp}}{\partial B} \right|_{\psi} , \quad (6)$$

and

$$\underline{K} \equiv \nabla \times \sigma \underline{B} . \quad (7)$$

Hence, Eq. (1) reduces to

$$\underline{K} \times \underline{B} = \left. \frac{\partial P_{\parallel}}{\partial \psi} \right|_B \nabla \psi , \quad (8)$$

and

$$\left. \frac{\partial P_{\parallel}}{\partial B} \right|_{\psi} = 1 - \sigma = \left(\frac{P_{\parallel} - P_{\perp}}{B^2} \right) . \quad (9)$$

The generalized pressure surfaces, which contain the two vector fields $\underline{B}$ and $\underline{K}$, have many properties analogous to constant pressure surfaces in the isotropic pressure case. The magnetic field can be written as

$$\underline{B} = \nabla \psi \times \nabla \psi + g \nabla \phi \quad (10)$$

where $\psi = \Psi/2\pi$ with Ψ being the poloidal magnetic flux between infinity and the magnetic surface passing through the specified point. The $\nabla \phi$ component of Eq. (8) makes $\underline{K} \cdot \nabla \psi = 0$. The use of Eqs. (7) and (10) then shows that σg is a function only of ψ so that we can define

$$G(\psi) \equiv g^2 \sigma^2 / 2 \quad . \quad (11)$$

Taking the $\nabla\psi$ components of Eq. (3) and using the relations between J , B , and K in Eqs. (3) and (7), we obtain the anisotropic Grad-Shafranov equation:

$$\Delta\psi = X^2 \frac{\partial}{\partial X} \frac{1}{X^2} \frac{\partial\psi}{\partial X} + \frac{\partial^2\psi}{\partial Z^2} = - \frac{X^2}{\sigma} \frac{\partial P_{\parallel}}{\partial\psi} \bigg|_B \quad .. \quad \frac{\nabla\psi \cdot \nabla\sigma}{\sigma} - \frac{1}{\sigma^2} \frac{dG(\psi)}{d\psi} \quad . \quad (12)$$

This equation differs little from the well-known isotropic form so that the modification of the PEST equilibrium formulation to tensor pressure is straightforward.

III. PRESSURE DISTRIBUTIONS

It is reasonable to specify the functional forms of $g(\psi)$ [or of the safety factor $q(\psi)$] and the pressure $p(\psi)$ in the isotropic pressure cases. In the most rigorous sense, these quantities should be obtained from transport-code calculations. However, since the current understanding of transport processes and coefficients is still rather limited, they are usually chosen arbitrarily. A similar approach is also necessary in the anisotropic case. The shape of the current distribution is determined by choosing the function $G(\psi)$, of Eq. (11), and the pressure profile is specified by prescribing a two-dimensional profile of the parallel pressure, $P_{\parallel}(\psi, B)$. The perpendicular pressure, $P_{\perp}(\psi, B)$, is then determined from Eq. (9). In much of the previous work on anisotropic pressure effects, simple analytic expressions for $P_{\parallel}(\psi, B)$ have been assumed. Some studies have made use of results from numerical calculations of the distribution function using a Fokker-Planck operator. In the present paper we describe an approach of this type intended to provide a reasonable model for neutral-beam-heated tokamaks.

We make use of the BALDUR transport code,¹⁰ which solves a radial one-dimensional version of the Fokker-Planck equation. This can be expressed in the form

$$\frac{1}{v} \frac{\partial \phi}{\partial t} + \tau \frac{\partial \phi}{\partial r} = N_e \left[\frac{\partial}{\partial E} U \phi + T \frac{\partial}{\partial \tau} (1 - \tau^2) \frac{\partial \phi}{\partial \tau} \right] + Q, \quad (13)$$

where ϕ is the flux of fast ions through an element of volume in (r, E, τ) space with r a surface radius, E the particle energy, and $\tau \equiv \cos(v_{\parallel}/v)$ a measure of the angle the velocity vector makes with the magnetic field, N_e the electron density, Q the external ion beam source, and the coefficients $U(E)$ and $T(E)$ prescribed energy-dependent functions. The code uses a multigroup algorithm to solve Eq. (13) with a Monte-Carlo source of fast neutral beam particles. Integration of this equation until the time variation becomes small provides a discrete representation of the hot-ion distribution function $f_b(r, E, v_{\parallel}/v)$ on several different surfaces for a finite grid in $E, v_{\parallel}/v$ space. Thus, we have a good representation of the cumulative effect of small-angle collisions. The steady-state distribution functions with different injection angles can be quite different. In particular, there can be a distinct peaking of the higher energy particles in the direction of injection.

Since the BALDUR results depend only on the single spatial variable, r , it is necessary to provide a prescription for constructing reasonable two-dimensional distribution functions. We make this generalization by assuming that $f_b(r, E, v_{\parallel}/v)$ represents the equilibrium distribution function at the outer edge of each flux surface (i.e., at $B_{\min}$, the minimum value of B on each surface). This choice corresponds to neutral beam injection directly along the midplane so that the beam particles are ionized at $Z = 0$. We accordingly set

$$f_b(\psi, E, \mu) \Big|_{B_{\min}} = f_b(r, E, v_{\parallel}/v) \quad , \quad (14)$$

where $\mu = v_{\perp}^2/2B$. This description can be extended to other values of B through

$$f_b(\psi, E, \mu) \Big|_B = \begin{cases} f_b(\psi, E, \mu) \Big|_{B_{\min}} & B < E/\mu \\ 0 & B > E/\mu \end{cases} \quad (15)$$

since μ is a constant of the motion. Thus, the effect of mirroring of the particles as the magnetic field increases is properly represented. We convert back to $f_b(\psi, E, v_{\parallel}/v)$ space at each value of ψ and B and compute the parallel beam component of the pressure with

$$P_{\parallel b} = m_b \int d^3v \, v_{\parallel}^2 f_b(\psi, E, v_{\parallel}/v) \quad , \quad (16)$$

where m_b is the mass of the beam ions. The perpendicular component could be determined in a similar way, but it is more accurate to use Eq. (9) to calculate it.

It is convenient to represent the pressure profiles as

$$\begin{aligned} P_{\perp} &= P_s(\psi) + P_{\perp b}(\psi, B) \quad , \\ P_{\parallel} &= P_s(\psi) + P_{\parallel b}(\psi, B) \quad , \end{aligned} \quad (17)$$

where $P_s(\psi)$ is the isotropic background plasma pressure, and $P_{\perp b}(\psi, B)$ and $P_{\parallel b}(\psi, B)$ are the beam pressure profiles. We define

$$h(\psi, B) \equiv \frac{P_{\perp b}(\psi, B)}{P_{\parallel b}(\psi, B)} = 1 - \alpha \ln(P_{\parallel b}) / \alpha (\ln B) \quad (18)$$

[from Eq. (9)] and use the profile of Eq. (16) to determine $h(\psi, B)$. This leads to

$$P_{\parallel} = P_s(\psi) + P_b(\psi) \exp \left\{ \int_{B_{\min}}^B [1 - h(\psi, B)] dB/B \right\},$$

$$P_{\perp} = P_s(\psi) + h(\psi, B) P_b(\psi) \exp \left\{ \int_{B_{\min}}^B [1 - h(\psi, B)] dB/B \right\}, \quad (19)$$

where the integrals are evaluated on surfaces of constant ψ . This allows us to use the distribution function approach to determine the pressure variation along the magnetic field lines on a flux surface while retaining freedom to specify the ψ -dependence of both the scalar and beam pressure distributions independently. Since the distribution function obtained with a Monte-Carlo source term for the beam particles is not perfectly smooth, we use a bi-quadratic fit of the form

$$h(\psi, B) = \sum_{i=0}^2 \sum_{j=0}^2 C_{ij} \tilde{\psi}^i \tilde{B}^j, \quad (20)$$

where $\tilde{\psi}$ is normalized to be 0 at the magnetic axis and 1 at the plasma surface, and $\tilde{B}$ is normalized so that $\tilde{B}$ is 0 at the axis and $\tilde{B}_{\max}(\psi_{\lim}) - \tilde{B}_{\min}(\psi_{\lim}) = 1$ at the surface.

This formalism provides a reasonable description of $P_{\perp}(\psi, B)$ and $P_{\parallel}(\psi, B)$ for systems where the beam injection is in a $Z = 0$ plane on the outside of the

tokamak, and B is a monotonic increasing function on a magnetic surface as the poloidal angle is increased from 0 to π . A new prescription needs to be formulated before our one-dimensional results for $f_b(r, E, v_\parallel/v)$ can be used to obtain proper representations for $h(\psi, B)$ for bean-shaped plasmas or for cases where the injection is not along a minimum- B ray. This is due to the fact that a unique way of constructing the distribution function does not presently exist.

The beam pressure function $p_b(\psi)$ is determined by the procedure just described. However, it is convenient to treat both this quantity and $p_s(\psi)$, the background scalar component of the pressure, as arbitrary adjustable variables. In particular, we typically let

$$p_s(\psi) = p_{s0} (1 - \tilde{\psi})^{\eta_s}$$

and

(21)

$$p_b(\psi) = p_{b0} (1 - \tilde{\psi})^{\eta_b} .$$

We define

$$\beta \equiv \frac{2}{3} \frac{\langle 2P_\perp \rangle}{\langle B^2 \rangle} + \frac{1}{3} \frac{\langle 2P_\parallel \rangle}{\langle B^2 \rangle}$$

with the averages taken over the entire plasma.

In the actual application of our analysis, we consider

$$G(\psi) = \frac{R_0^2}{2} \{ 1 + \gamma_1 (1 - \tilde{\psi})^{\eta_g} + \gamma_2 [3(1 - \tilde{\psi})^2 - 2(1 - \tilde{\psi})^3] \} , \quad (22)$$

and use the shaping constants, P_{s0} , P_{b0} , η_s , η_b , η_g , γ_1 , and γ_2 , to fix the total β , the ratio of beam pressure to scalar pressure, the safety factor at the magnetic axis, the total current in the plasma, and the shapes of the pressure and current profiles.

IV. STABILITY

There exist several scalar-pressure stability criteria that have proved useful for the design and interpretation of tokamak experiments. These scalar models have even been used to model tokamaks with significant auxiliary heating. Generalization to anisotropic plasmas of the criteria for localized modes has been made, but much work remains to be done before reasonable modeling of nonlocal low- n modes is possible.

A. Analytical Criteria

The stability criteria that the conditions,

$$\tau = 1 + \frac{1}{B} \frac{\partial P_{\perp}}{\partial B} \Big|_{\psi} > 0, \quad (23)$$

and

$$\sigma = 1 - \frac{1}{B} \frac{\partial P_{\parallel}}{\partial B} \Big|_{\psi} > 0, \quad (24)$$

are satisfied everywhere in the plasma guarantee, respectively, the stability of the system to the well-known "mirror" and "firehose" instabilities.¹¹ Although these are driven by velocity-space effects, they can generate strong nonlocal distortions of the plasma. All of the equilibria that we have generated are indeed stable to both of these modes.

Connor and Hastie have generalized the Mercier criterion to axisymmetric systems with anisotropic pressure.³ An MHD analysis indicates that interchange modes associated with this criterion are stable if

$$\begin{aligned}
 D_I = & \frac{g^2}{q^2} \frac{\langle B^2 \rangle}{\sigma |\nabla\psi|^2} \left[\left\langle \frac{\nabla\psi \cdot \nabla(P_\perp + B^2/2)}{\sigma B^2 |\nabla\psi|^2} \left(\frac{\sigma}{\tau} \frac{\partial P_\perp}{\partial \psi} \Big|_B + \frac{\partial P_\parallel}{\partial \psi} \Big|_B \right) \right\rangle \right. \\
 & + \frac{\sigma \langle \underline{J} \cdot \underline{B} \rangle^2}{B^2 |\nabla\psi|^2} - \frac{\langle \underline{J} \cdot \underline{B} / |\nabla\psi|^2 \rangle^2}{\langle B^2 / \sigma |\nabla\psi|^2 \rangle} + q \left\langle \frac{\sigma \underline{J} \cdot \underline{B}}{B^2} \frac{\partial}{\partial \theta} \frac{\nabla\psi \cdot \nabla \theta}{|\nabla\psi|^2} \right\rangle \\
 & \left. + \frac{q}{\sigma} \left(\frac{\langle \underline{J} \cdot \underline{B} \rangle}{\langle B^2 / \sigma \rangle} - \frac{\langle \underline{J} \cdot \underline{B} / |\nabla\psi|^2 \rangle}{\langle B^2 / \sigma |\nabla\psi|^2 \rangle} \right) \right] - \frac{1}{4} < 0.
 \end{aligned} \tag{25}$$

Here J is a dimensionless Jacobian $(\nabla\psi \times \nabla\theta \cdot \nabla\phi)^{-1}$ for a system where ϕ is the ignorable toroidal coordinate, θ has been chosen to make the magnetic field lines straight, and the braces $\langle \rangle$ denote an average over a magnetic surface, $\langle A \rangle = \oint A d\theta / \oint d\theta$. These modes are driven by the energy associated with plasma expansion (spatial anisotropy) if the average magnetic field line curvature is sufficiently unfavorable. The basic assumption made in deriving the criterion is that $\partial/\partial\psi \gg \partial/\partial\theta \geq \partial/\partial\ell$ with $\partial/\partial\ell$ measuring the variation along the magnetic field line. With this ordering the instability need not be localized, although the forces driving it are localized near a closed resonant magnetic field line.

B. Ballooning Modes

Ballooning modes, characterized by the ordering $\partial/\partial\psi \sim \partial/\partial\theta \gg \partial/\partial\ell$ are more unstable in tokamaks than the interchange instabilities associated with the Mercier criterion. They too are driven primarily by pressure gradients in the presence of unfavorable magnetic field line curvature. Since most experiments have been observed to operate at or below the critical β associated with them, they remain an important subject of concern.

An anisotropic form of the MHD ballooning equation was derived by Cooper^{8,12} by expanding the Chew-Goldberger-Low form of the potential energy in powers of $1/n$. This equation can also be derived as the MHD limit of the kinetic equations involving appropriate solutions of the gyrokinetic equation.¹³ The basic procedure involves working in a straight magnetic field line coordinate system (ψ, θ, ζ) in which $\underline{B} = \nabla \alpha \times \nabla \psi$ with $\alpha \equiv \zeta - q \theta$ and introducing the eikonal representation,

$$\xi = \bar{\xi} \exp [iS(\psi, \alpha)] \quad , \quad (26)$$

$$S = n \left[\alpha + \int \theta_k(\psi) dq(\psi) \right] \quad , \quad (27)$$

such that the wave vector is

$$\underline{k}_\perp = \nabla S = n \left[\nabla \zeta - q \nabla \theta - (\theta - \theta_k) \nabla q \right]$$

with n being the toroidal mode number and θ_k representing the angle which has the minimum contribution from the shear ∇q . This leads to

$$\begin{aligned} & \underline{B} \cdot \nabla \left(\frac{\sigma k_\perp^2}{B^2} \underline{B} \cdot \nabla \phi \right) \\ & + \frac{a}{B} \left(\frac{1}{\tau} \frac{\partial P_\perp}{\partial \psi} \Big|_B + \frac{1}{\sigma} \frac{\partial P_\parallel}{\partial \psi} \Big|_B \right) (\underline{k}_\perp \times \underline{B} \cdot \underline{\kappa}) (\underline{k}_\perp \times \underline{B} \cdot \nabla \psi) \hat{\phi} + \frac{k_\perp^2 \lambda^2}{B^2 B_0^2} \hat{\phi} = 0 \end{aligned} \quad (28)$$

with $\underline{\kappa}$ the magnetic field line curvature, $\hat{\phi}$ the electric field potential associated with the perturbation ξ and the magnetic fields normalized to B_0 . The first term represents the field line bending and is always stabilizing. The second represents plasma expansion and is stabilizing in regions of

favorable curvature and destabilizing in regions where the curvature is bad. The third term represents inertia, with $\lambda = \rho_M \omega^2 / B_0^2$ the eigenvalue, $\lambda > 0$ being stable. A ballooning mode code, that was developed by Dewar and his collaborators,¹⁴ has been modified to calculate the eigenvalue

$$\lambda = \lambda (\psi, \theta_k) . \quad (29)$$

From this it is easy to determine the ranges of unstable surfaces, the values of θ_k for which each surface is unstable, the growth rate (normalized to the local mass density ρ_M and field strength B_0), and n_c , the value of n above which the modes are unstable. The value of n_c is given by the WKB quantization condition¹⁵

$$n_c = \pi / \oint \bar{\theta}_k (\psi) dq , \quad (30)$$

where $\bar{\theta}_k (\psi)$ is determined from

$$\lambda [\psi, \bar{\theta}_k (\psi)] = 0 . \quad (31)$$

C. Finite Gyroradius Effects

If finite ion gyroradius corrections are retained in the gyrokinetic derivation of the ballooning-mode equation, the basic structure of Eq. (23) is recovered except that now

$$\lambda = \rho_M (\omega^2 - \omega \hat{\omega}_{*p1}) , \quad (32)$$

with $\hat{\omega}_{*p1}$ the ion diamagnetic drift frequency,

$$\hat{\omega}_{*P\perp} = \left(\frac{mc}{|e|B_0^2 \rho_M} \right) \frac{\partial P_\perp}{\partial \psi} \Big|_B (k_\perp \times B \cdot \nabla \psi) = -n \left(\frac{mcB^2}{|e|B_0^2 \rho_M} \right) \frac{\partial P_\perp}{\partial \psi} \Big|_B \quad (33)$$

In practice, we replace the ψ -variation of $P_\perp$ by its average on the surface.

Then,

$$\omega = \frac{\hat{\omega}_{*P\perp}}{2} \pm [B_0^2 (\lambda - \lambda_c) / \rho_M]^{1/2} \quad (34)$$

with

$$\lambda_c \equiv -\rho_M^2 \hat{\omega}_{*P\perp}^2 / B_0^2, \quad (35)$$

so that ω is real and the equilibrium is stable if $\lambda > \lambda_c$. We can therefore use the same basic program to determine stability. However, it is now necessary to solve iteratively

$$\lambda_c [\psi, \bar{\theta}_k(\psi, n_c)] = -n_c^2 \left[\frac{c}{2\omega_{pi}} \left\langle \frac{\partial P_\perp}{\partial \psi} \right\rangle \Big|_B \right]^2, \quad (36)$$

with

$$n_c = \pi / \oint \bar{\theta}_k(\psi, \lambda_c) dq. \quad (37)$$

Initially, $\lambda_c = 0$ is assumed, $\bar{\theta}_k$ is calculated from $\lambda = \lambda_c$, and n_c is determined from Eq. (37). A new estimate of λ_c is then obtained from Eq. (36), and the process is repeated. For any given equilibrium there are two possible outcomes of this iteration process. The first occurs when the equilibrium is only partially stabilized by finite gyroradius effects. Since

the stabilization is proportional to n^2 , there will be two values of n_c that satisfy Eqs. (36) and (37), n_1 and n_2 , with $n_1 < n_2$. Then the system is unstable with respect to ballooning modes for $n_1 < n < n_2$, and stable otherwise. Since our numerical iteration scheme is unstable around n_2 , we use a binary search technique to determine this value. The other possible case is that finite gyroradius effects stabilize all values of n . In this eventuality, there is no solution for n_c , and the iteration procedure diverges to $n_c \rightarrow \infty$.

D. Energetic Ion Corrections

For the case of a bi-Maxwellian distribution with fast ions being very energetic, many of the fast ions do not contribute to the ballooning-instability-driving mechanism because they drift across the destabilizing^{8,16-18} curvature region at a rate that is faster than the oscillation of the mode. Cooper has derived a ballooning mode equation for this case;⁸ i.e.,

$$\begin{aligned}
 & \mathbf{B} \cdot \nabla \left(\frac{\sigma k_{\perp}^2}{B^2} \mathbf{B} \cdot \nabla \hat{\phi} \right) \\
 & + \frac{\sigma}{B^4} \left[\frac{1}{\tau} \frac{\partial (P_{\perp} - P_{\perp h})}{\partial \psi} \right] \Big|_B + \frac{1}{\sigma} \frac{\partial (P_{\parallel} - P_{\parallel h})}{\partial \psi} \Big|_B \left[(\mathbf{k}_{\perp} \times \mathbf{B} \cdot \nabla) (\mathbf{k}_{\perp} \times \mathbf{B} \cdot \nabla \psi) \right] \hat{\phi} \\
 & - \frac{1}{B^4} \frac{\partial P_{\perp h}}{\partial \psi} \Big|_B \frac{\partial (P_{\perp} - P_{\perp h})}{\partial \psi} \Big|_B (\mathbf{k}_{\perp} \times \mathbf{B} \cdot \nabla \psi)^2 \hat{\phi} \\
 & + \frac{k_{\perp}^2}{B^2 B_0^2} \hat{\phi} = 0
 \end{aligned} \tag{38}$$

with

$$\lambda = \rho_M \omega \left[\omega - \omega_{*PC1} - \frac{\rho_h \omega_{*h}}{\rho_M} \right] \quad (39)$$

Here $P_{\perp h}$, $P_{\parallel h}$, ρ_h , and ω_{*h} refer to the hot ions and ω_{*PC1} refers to the cold ions. The effects of these high-energy particles appear in two places. First, those that do not stay in the regions of bad field line curvature sufficiently long do not contribute to the destabilization. Thus, they are removed from the second term in Eq. (38). Second, they introduce an additional stabilization associated with the third term of this equation.

A reasonable estimate of the stabilizing effect of the fast ions can be made by assuming that a fixed fraction f of the ions that contribute to the anisotropy are hot. Then $P_{\perp h}$ and $P_{\parallel h}$ can be set equal to $fP_{\perp b}$ and $fP_{\parallel b}$ in Eqs. (38) and (39).

V. APPLICATION

Application has been made to several series of equilibria, all with roughly PLT-type parameters. The purpose here is primarily to investigate the effect of changing the injection angle of the neutral beam source. In each set the equilibria are characterized by physically similar parameters; i.e., the same cross-sectional shape, the same aspect ratio, nearly the same q profile, etc. In this way we can have some confidence that the effects are really representative of differences in the pressure anisotropy and not due to some other equilibrium quantity.

We have used the results¹⁹ from a study of beam injection into a cylindrical model with plasmas parameters roughly analogous to those in the PLT tokamak to obtain distributions for $f_b(r, E, v_{\parallel}/v)$ from which we can determine $h(\psi, B)$. Since the beam was composed of several different energy components and the density and current distributions obtained from a

transport model, the resulting distribution functions provide a reasonable description of what should be expected in an actual device. We consider three sets of cases with different injection angles. The cases where the injection angle is 56.9° from the magnetic field are called "parallel" cases since at this angle $P_{\perp b} = 0.5 P_{\parallel b}$ at the magnetic axis and the majority of the beam ions are completely circulating. We chose an injection angle of 68.0° for our "intermediate" cases where $P_{\perp b} = P_{\parallel b}$ at the axis. Here the bulk of the hot ions are still circulating. Our "perpendicular" cases have an injection angle of 81.0° . This gives $P_{\perp b} = 2 P_{\parallel b}$ on the axis with the majority of the beam ions strongly trapped in the region of unfavorable curvature. In all the cases, $h(\psi, B)$ varies with both ψ and B , so that different behavior can be observed in different regions in the plasma. This is particularly evident for the intermediate injection angles.

We have considered three sets of equilibria for each of these injection angles. In particular, cases one and two deal with circular cross sections with major radius $R = 1.35$ m, minor radius $a = 0.42$ m and $B_0 = 30$ kG. Case three has a D-shaped cross section with $R = 1.35$ m, $a = 0.32$ m, ellipticity $b/a = 1.7$, and $B_0 = 30$ kG. In all three cases we adjust γ_1 and γ_2 of Eq. (22) so that $q(0) = 1$ and $q(a) = 2.3$. We choose $\eta_g = 1.5$ for all three cases, obtaining the $q(\psi)$ profile of Fig. 1 for our case 1 and quite similar profiles for the other two cases. These profiles change only slightly with different values of β . We assume that $\eta_g = 1.5$ in all three cases which provides a reasonably shaped pressure profile for the background plasma. We set $\eta_b = 1.8$ in case 1 to get a peaked beam pressure profile and $\eta_b = 1.35$ in cases 2 and 3 to obtain a broad one. The $P_b(\psi)$ profiles for these cases are also given in Fig. 1. The constants P_{S0} and P_{b0} are adjusted as the total β is varied from 2.0% to 3.2% in the first two cases and from 2.8% to 4.6% to keep the ratio of

beam β to scalar β equal to 0.8 while varying the total β . These limits were selected so that instabilities could be found in ballooning mode stability studies and extrapolation could be made to determine critical β 's. Cases spanning the same range of β , but with the scalar β fixed at 1.6% and the beam β varied were also run for case 1.

Flux surfaces and pressure distributions along the midplane for a case 1 equilibrium with the scalar β equal to 1.45% and a total β of 2.6% are shown in Fig. 2. Results for a case 2 equilibrium for the same β values are given in Fig. 3. The results for case 3 with isotropic β equal to 2.08% and total β of 3.75% are shown in Fig. 4.

As noted earlier, all of these configurations are stable everywhere with respect to the "mirror and "firehose" stability criteria, Eqs. (23) and (24). They satisfy Eq. (25), the generalized Mercier criterion, except in a few high- β cases near the magnetic axis. Even then, raising the value of $q(0)$ slightly above unity provides stabilization.

Values of the critical β , β_c , are determined by evaluating the absolute value of $|\lambda|$ from Eq. (28) for a set of identical equilibria except with P_{S0} and P_{b0} adjusted to obtain different values of the total β and then extrapolating to where $\lambda = 0$, as in Fig. 5. This has been done for all three cases for all three injection angles and for an isotropic case with $h(\psi, B) = 1$, keeping $\beta_b/\beta_s = 0.8$. It has also been done for the three injection angles in case 1 for a series with $\beta_s = 1.6\%$. The results are summarized in Table 1.

We have used Eqs. (28) and (36) to investigate finite ion gyroradius effects on these modes. Contours of constant $\lambda(q, \theta_k)$ for case 2, the circular cross section tokamak with a broad pressure profile, with $\beta_s = 1.45\%$, $\beta_b = 1.15\%$, $\beta_{\text{total}} = 2.6\%$ are shown in Fig. 6 for the three injection angles and for the case with $h(\psi, B) = 0$. Here the magnetic surfaces are labeled by their

value of q , and the choice of θ_k affects the orientation of the mode. The evaluation of Eq. (36) depends on the value of ω_{pi} that is assumed, which in turn is dependent on the electron density. Results for different plasma densities are given in Table 2.

We use Eq. (38) with the assumption that a fixed fraction f of the particles that contribute to the anisotropy is hot in order to investigate the stabilizing effect of the nonresonant ions. The values of $|\lambda|$ as a function of f are given in Fig. 7 for injection into case 2, the broad pressure profile case, with $\beta_b = 0.8 \beta_s$ and $\beta = 4.5\%$. In this case the hot ions would completely stabilize the instability if $f > 0.6$, even at this large value of β . Values of the critical β as a function of f are given in Table 3 for case 2 for all three injection angles and for the isotropic beam case where $h(\psi, \beta) = 1$. As one should expect, the effect is large for the $\beta_b = 0.8 \beta_s$ case where a large part of the plasma can be decoupled. The effectiveness of the hot ions is somewhat larger in the isotropic beam and intermediate injection cases than in the more strongly anisotropic perpendicular and parallel injection cases.

VI. DISCUSSION

The main purpose of this work was to modify appropriately the usual isotropic pressure formalism and accordingly revise the PEST code¹ so that it could be used to study the effects of pressure anisotropy on tokamak equilibrium and stability. In the course of carrying out this task, we have improved our understanding of the equilibrium and stability properties of neutral-beam-driven tokamaks.

The effects of anisotropy on plasma equilibrium can be seen in Figs. 2 through 4. Note that the surfaces of constant pressure are quite

significantly shifted in relation to the surfaces of constant ψ , which vary little with changes in injection angle.

In all the cases studied, the surfaces of constant $P_{\perp}$ were found to be shifted to a greater degree than those of constant $P_{\parallel}$. The direction of shift is strongly dependent on the injection angle. This can be understood by noting that in the parallel injection case the hot beam ions are nearly all circulating particles. The adiabatic invariance of the magnetic moment, $\mu = v_{\perp}^2/2B$, dictates that they will have a larger ratio of $P_{\perp}$ to $P_{\parallel}$ in high field regions than in low field regions. Since the smaller $v_{\parallel}$ in regions of small B allows the ions to remain there longer, the pressure is accordingly enhanced. The increased perpendicular velocity in the mirror regions will enhance the perpendicular pressure even more than the parallel unless the injection is at such a large angle that there are very few particles with large perpendicular velocity injected. In such a case all these effects tend to provide an inward shift of the surfaces. On the other hand, with perpendicular injection, most of the particles are trapped in the outside region leading to an outward shift.

Our intermediate case is one where many of the particles are barely trapped since the inward shifts of the $P_{\perp}$ surfaces on the innermost surfaces suggest circulating particles, and the outward shift (half way out) suggests that the bulk of the beam ions are trapped. Since fewer particles have been trapped near the plasma surface, the effect in the outer region is weaker than it would be well inside the plasma.

There is very little difference in surface shifts between cases with peaked and broad pressure profiles. Plasma with circular and D-shaped cross sections also exhibit no appreciable variation in this respect. However, changes in the injection angle can produce significant differences.

The ballooning mode stability studies of Table 1, which ignored the stabilizing effects of both finite gyroradius and the tenuousness of the energetic ions, indicate that both parallel and intermediate injection provide considerable stabilization. However, perpendicular injection appears to be destabilizing. This is consistent with the picture that the main effect of the anisotropy is either the increase or the reduction of the pressure gradients in regions of bad magnetic field line curvature due to the surface shifts. Varying only the beam- β , while maintaining a constant isotropic plasma produces results that are qualitatively the same as those when the total β is changed with a fixed ratio of the beam to background components. In general, the anisotropic effects are found to be more pronounced for the broad profile systems. This is due to the fact that the steeper pressure gradients are located in the outer regions of the plasma where the variations of the magnetic field magnitude are larger.

Our results indicate that parallel injection is stabilizing while perpendicular injection lowers the critical β from an equivalent isotropic configuration. This is in disagreement with Cooper's conclusion that perpendicular injection provides more stabilization than parallel. A possible explanation is that his parallel injection case is one where essentially all the beam ions are circulating particles with a very small ratio of $v_{\perp}$ to $v_{\parallel}$. Hence, their effect on the equilibrium properties will also be weak. In fact, Cooper's perpendicular injection case is similar to our parallel injection case where the majority of the beam ions are almost trapped. As a result, most of them spend a large amount of time in regions of favorable curvature and have a large $v_{\perp}$ -component. They thus provide an inward shift of the $p_{\perp}$ surfaces with respect to the ψ surfaces, thereby enhancing stability. As the injection angle becomes more perpendicular, the critical β will be lowered because increasingly more ions are trapped in the unfavorable region.

It can be seen from Table 2 that finite-gyroradius stabilization becomes significant only in relatively high density plasmas. The effect is strongest for the perpendicular injection case which indeed corresponded to conditions where the effects of anisotropy were found to be more deleterious.

The modification of stability associated with the hot ion component^{8,16-18} is even more interesting. As can be seen in Table 3, the effect becomes very important when a large part of the beam ions is assumed to drift rapidly enough to be decoupled. The critical β 's are not increased as much for the cases where the scalar pressure component is closer to what might be expected in an operating fusion device. In such cases, more closely modeled by $\beta_b < \beta$ ($\beta_b = 0.15 \beta_s$ or $\beta_b = 0.56\%$ in these studies), the fraction f of the particles that have high energy and are nonresonant must be large to get a strong improvement. The most important feature of these studies is that the effects of these ions are much more pronounced in the isotropic beam and intermediate cases than in the more anisotropic perpendicular and parallel injection cases. Our results indicate that the stabilization associated with these high energy particles, especially in cases where the plasma is highly anisotropic, may not be as strong as has been thought. This could seriously impact design considerations for future tokamak devices. More careful work with the kinetic modeling to determine $P_{\perp h}$ and $P_{\parallel h}$ is needed since the exact identification of the behavior of specific hot ions is complicated by the background plasma, the geometry, and the exact nature of the particle trapping.

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CASE	FEATURES	INJECTION ANGLE			
		ISOTROPIC	PERPENDICULAR	INTERMEDIATE	PARALLEL
1	Peaked, fixed ratio	2.17	2.10	2.40	2.46
1A	Peaked, fixed β_{iso}		2.22	2.36	2.44
2	Broader, fixed ratio	2.25	2.00	2.49	2.51
3	D-shaped, fixed ratio	3.13	2.81	3.60	3.60

Table 1: Critical β values for varying beam injection angles and plasma parameters extrapolated down to the point at which $|\lambda| = 0$. The corresponding value of β is taken to be β_c .

DENSITY $\times 10^{19}$	ISOTROPIC		PERPENDICULAR		INTERMEDIATE		PARALLEL	
	n_1	n_2	n_1	n_2	n_1	n_2	n_1	n_2
1.0	None	None	None	None	None	None	None	None
3.3	None	None	None	None	None	None	None	None
10.0	None	None	2.9	17.0	None	None	None	None
33.0	4.2	23.0	2.6	37.0	5.5	22.0	7.8	21.0
100.	3.9	48.0	2.5	69.0	4.9	46.0	6.6	44.0

Table 2: Critical n values for varying beam anisotropies and assumed plasma densities.

f	Isotropic	Perpendicular	Intermediate	Parallel
$\beta_b = 0.8 \beta_s$				
0.0	2.22	1.91	2.41	2.65
0.1	2.45	2.31	2.61	2.84
0.2	2.73	2.58	3.05	3.16
0.3	3.09	2.95	3.42	3.43
0.4	3.64	3.29	3.83	3.80
0.5	----	3.66	----	----
0.6	----	4.30	----	----
0.7	----	----	----	----
$\beta_b = 0.15 \beta_s$				
0.0	2.24	2.27	2.27	2.28
0.1	2.27	2.29	2.44	2.43
0.2	2.43	2.43	2.49	2.48
0.3	2.49	2.49	2.53	2.52
0.4	2.55	2.54	2.58	2.57
0.5	2.61	2.58	2.63	2.61
0.6	2.67	2.61	2.68	2.65
0.7	2.74	2.67	2.73	2.70
0.8	2.80	2.72	2.79	2.75
0.9	2.87	2.77	2.85	2.81
1.0	3.12	2.81	3.05	2.86
$\beta_b = 0.56\%$				
0.0	2.27	2.34	2.40	2.39
0.1	2.36	2.42	2.48	2.47
0.2	2.46	2.48	2.56	2.54
0.3	2.54	2.54	2.63	2.60
0.4	2.62	2.62	2.69	2.66
0.5	2.71	2.68	2.76	2.72
0.6	2.79	2.73	2.82	2.78
0.7	2.97	2.79	2.96	2.91
0.8	3.05	2.85	3.02	2.97
0.9	3.13	2.97	3.09	3.04
1.0	3.27	3.04	3.23	3.11

Table 3. Critical β values as functions of the fraction of the beam that has high energy and is nonresonant in case 2. Critical β 's are not available for the $\beta_b = 0.8 \beta_s$ cases at higher values of f because the necessary higher- β equilibria have not been constructed.

FIGURE CAPTIONS

FIG. 1. The $q(\psi)$ profile and the beam pressure parameter $p_{b0}(\psi)$ for case 1 (peaked) and case 2 (broad).

FIG. 2. Flux surfaces and midplane pressure profiles for "perpendicular" injection (upper right), "intermediate" injection (lower left), and "parallel" injection (lower right) for equilibria with case 1 parameters with $\beta_s = 1.45\%$ and $\beta_b = 1.15\%$. The solid curves are the scalar pressure component, the dashed curves are the perpendicular pressure, and the broken curves are the parallel pressure.

FIG. 3. Flux surfaces and midplane pressure profiles for "perpendicular" injection (upper right), "intermediate" injection (lower left), and "parallel" injection (lower right) for equilibria with case 2 parameters with $\beta_s = 1.45\%$ and $\beta_b = 1.15\%$. The solid curves are the scalar pressure component, the dashed curves are the perpendicular pressure, and the broken curves are the parallel pressure.

FIG. 4. Flux surfaces and midplane pressure profiles for "perpendicular" injection (upper right), "intermediate" injection (lower left), and "parallel" injection (lower right) for equilibria with case 3 parameters with $\beta_s = 2.08\%$ and $\beta_b = 1.67\%$. The solid curves are the scalar pressure component, the dashed curves are the perpendicular pressure, and the broken curves are the parallel pressure.

FIG. 5. Most unstable eigenvalue $|\lambda|$ as a function of β for case 2, with a broad pressure profile. Here $\beta_b/\beta_s = 0.8$. The uppermost curve is for "perpendicular" injection, the next is an "isotropic" case with $h(\psi, B) = 1$, the next is for "intermediate" injection, and the lowest is for "parallel" injection.

FIG. 6. $\lambda(q, \theta_k)$ contours for ballooning mode instabilities in case 3 with $\beta_s = 1.45\%$ and $\beta_b = 1.15\%$. The modes are unstable for eigenfunctions inside the outside solid curves. The upper left figure is for an isotropic case with $h(\psi, B) = 1$ and has $|\lambda|_{\max} = -0.00837$. The upper right figure, for "parallel" injection, has $|\lambda|_{\max} = 0.00179$. The lower left figure, for "perpendicular" injection, has $|\lambda|_{\max} = 0.0242$, and the "intermediate" injection case (lower right figure) has $|\lambda|_{\max} = 0.00263$.

FIG. 7. Most unstable eigenvalue $|\lambda|$ as a function of f , the fraction of hot ions that are treated as highly energetic and nonresonant for injection into a case 2 equilibrium with $\beta = 4.5\%$ and $\beta_b = 0.8\beta_s$. The uppermost curve is for "perpendicular" injection, the next lower one is the isotropic case, then "intermediate" injection, and at the bottom "parallel" injection.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

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Fig. 5

Fig. 6

Fig. 7

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