PION-NUCLEON PARTIAL WAVE ANALYSIS
AND STUDY OF BARYON STRUCTURE

MASTER

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Abstract

This report details progress toward completion of a long-term pion-nucleon partial wave analysis, summarizing our results and conclusions to date. The report also discusses progress in using our partial wave and resonance parameter results to test dynamical models of the baryon and in better understanding inter-quark forces within baryons.

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I. SCOPE OF WORK

The major thrust of my contract work during the last two years has been devoted to a detailed pion-nucleon partial wave analysis in collaboration with R. E. Cutkosky at Carnegie-Mellon University and R. L. Kelly of Lawrence Berkeley Laboratory. Discussion in this report is limited to our progress and results during the last two years. A secondary phase of my contract work has been to make use of our resonance results to test dynamical models of the baryon, and progress in these theoretical investigations, as well as other phenomenological work carried out under this contract, is also reported.

II. TECHNICAL RESULTS

A. Pion-nucleon Partial Wave Analysis

During the last two years some major changes have taken place in our pion-nucleon partial wave analysis. These revisions have occurred in two stages, and are discussed separately as the 1979 analysis (79PWA) and the 1980 analysis (80PWA).

The 79PWA incorporated new elastic differential cross-section and charge-exchange scattering data¹ and extended the energies of our previous analysis² to a pion laboratory momentum range from 0.42 to 2.0 GeV/c. Partial wave parametrizations were improved over our previous analysis by including physically reasonable isobar-like models to represent inelastic channels, by using inelastic coupling constraints from recent inelastic partial wave analyses³ in our resonance parametrizations, and by including coupled channel effects. A detailed account of the methods and final results of our 79PWA are published in the Physical Review⁴ (copies enclosed), and our partial wave and baryon resonance results appear in the 1980 Review of Particle Properties.⁵ Our qualitative conclusions include finding all expected non-strange members of the $[56, 0^+]$, $[70, 1^-]$ and $[56, 2^+]$ $SU_6 \times O_3$ supermultiplets, most members of the $[70, 3^-]$ supermultiplet

and strong evidence for non-minimal supermultiplets such as the $[56,1^-]$ and $[70,2^+]$.

Our 80PWA refined our 1979 analysis by including new data from KEK⁶ and Argonne⁷ at higher energies and SIN⁸ and Leningrad⁹ at lower energies. The range of our analysis was extended upward, from a laboratory momentum of 0.42 to 2.5 GeV/c. Preliminary results of 80PWA were presented at the conference BARYON-1980 at the University of Toronto in July, 1980, and will appear in the published conference proceedings¹⁰ (see attached preprint). Much of the analysis procedure is similar to our 79PWA, though several different methods for extracting resonance parameters were studied in the latest work.

Extension of our analysis to higher energies has allowed better determination of a number of baryon resonances crucial to the questions of a non-minimal baryon spectrum. Our 80PWA preliminary results found the expected non-strange $[56,0^+]$ *, $[70,1^-]$, $[56,2^+]$, and $[70,3^-]$ minimal spectrum states, plus a number of non-minimal states. A strong D_{35} (1940) state, a second D_{33} resonance at 1940 MeV, a second S_{31} at 1890 MeV, a third S_{11} at 2180, and a third D_{13} at 1880 complete the list of non-strange candidates for a non-minimal $[56,1^-]$ supermultiplet. Additional states found in our analysis such as the F_{17} (1970), P_{11} (1700) and G_{39} (2300) presumably belong to the non-minimal multiplets $[70,2^+]$, $[70,0^+]$, and $[56,3^-]$, respectively.

These new resonant states provide strong evidence that the baryon spectrum is not a minimal spectrum and indicate that there are not sizeable ad-hoc exchange forces¹¹ acting within baryons. Furthermore, details of the mass spectrum of our observed states within the same $SU_6 \times O_3$ supermultiplets provide a better empirical understanding of spin-orbit forces within baryons. Baryon states within the $[56,1^-]$ and $[56,2^+]$ supermultiplets show no sign of spin-orbit forces, however the S_{31} (1620) and D_{33} (1710) show some sizeable splitting within the $[70,1^-]$ multiplet, which is currently a subject of some theoretical discussion.¹²

B. Theoretical Problems

A major theoretical question is the nature of the forces between quarks in baryons. Various models for inter-quark forces such as the harmonic-oscillator shell model¹³, the bag model¹⁴, and quantum chromodynamical models¹⁵ all make predictions of the masses and couplings of baryon states. Comparison of various models with our partial wave results reveals that the quantum chromodynamical model of Isgur and Karl¹⁵ is most successful in reproducing our observed baryon spectrum¹⁶. Further calculations by Isgur and Koniuk in a QCD model of the baryon¹⁷ are amazingly accurate in predicting which states couple sufficiently to the elastic channel to be observed in an elastic partial wave analysis such as ours.

A second important theoretical question arising from our partial wave analysis is the reason for such a low-lying $[56,1^-]$ supermultiplet. Calculations in the context of various conventional models of the baryon have been carried out attempting to show that such a state can be explained as a normal $N=3$ baryon state with a lower mass because of mixing with other multiplet states.¹⁸ Unfortunately, a sufficiently low mass has not been found in such models, and the possibility remains that the $[56,1^-]$ may represent the lowest energy signal of gluon excitations within the baryon.¹⁹

A third area of theoretical investigation during the last two years has been a continuation of my work with Ling-Fong Li at Carnegie-Mellon University on neutral current couplings of quarks.²⁰ A great deal of work by many authors has been done to establish model-independent constraints on coupling parameters from neutral current experiments.²¹ My most recent work in this area²² stresses the importance of, and carries out, a simultaneous analysis of neutral current scattering data in determining model-independent bounds on neutral current couplings of quarks, and determines the effects of $s\bar{s}$ contributions to neutrino-nucleon scattering on coupling constant constraints (paper enclosed).

Two new theoretical problems have occupied some of my time during the last year. At the request of experimentalists at Carnegie-Mellon and Fermilab,²³ I have been studying the phenomenology of neutrino trident production²⁴ ($\nu_\mu + Z \rightarrow \nu_\mu + Z + \mu^+ + \mu^-$) to see what constraints on vector and axial-vector muon couplings can be obtained from proposed state-of-the-art neutrino experiments at Fermilab. A second theoretical problem currently under investigation with a graduate student here at St. Bonaventure is the possibility of applying inverse scattering methods,²⁵ which have been successful in revealing inter-quark forces within mesons,²⁶ to baryons.

The progress reported herein indicates full compliance with stipulations of my research contract proposal. Collaboration on the partial wave analysis project and associated theoretical studies continue, and shall continue for the foreseeable future. Our goal is to include additional meaningful pion-nucleon scattering data as it becomes available, extend the energy range and accuracy of our analysis, and address related theoretical questions as they arise.

One hundred percent of my effort has been devoted to this contract during each of the two two-month summer periods. Approximately 50% of my time has been devoted to this contract during the remaining summer month each year, and 15% to 25% of my time has been devoted to this contract during each 9-month academic year. A similar percentage of my time will be devoted to this contract between now and the expiration date of this contract on May 31, 1981.

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Pion-Nucleon Partial Wave Analysis*

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ABSTRACT

We review the methods and results of the 1979 CMU-LBL partial wave analysis, and present preliminary results from a new analysis which extends to 2.5 GeV/c. New data from KEK and ANL in the 2 GeV/c region are used, along with some new low energy data. We find resonances in a number of partial waves in the 2.0-2.3 GeV mass region. The new data also help to clarify several resonances which lie just below this region.

At the 1976 Oxford Baryon Conference, the CMU-LBL group presented results from the analysis of πN scattering data between 0.8 and 2.0 GeV/c.¹ A revised analysis (79) which incorporated new data from RHEL on elastic DCS² and on charge exchange scattering³ has recently been published.⁴ The 79 analysis extended from 0.42 to 2.0 GeV/c. At present we are carrying out a new revision. The momentum range has been extended up to 2.5 GeV/c. We also include some new data, in particular, from KEK⁵ and ANL⁶ at high energies, from SIN⁷ and the Leningrad cyclotron at lower energies.⁸

Since a complete description of our analysis method has been published,⁴ a very brief summary will be sufficient here. There are three main steps: 1) preparation of an amalgamated data set, 2) energy-independent partial wave analysis incorporating iteratively constructed dispersion relation constraints, 3) parametrization of the partial wave energy dependence in order to identify resonances and determine their properties. One reason for separating these steps is that we want to decouple, as much as possible, such problems as how to cope with discrepant data and how to resolve the ambiguities of a single-energy analysis from the problem of finding resonances. We hope, thereby, to reduce the effect of unconscious biases in the analysis. From the final step of the analysis, we obtain tables of resonances and are led to discussions of resonance models. However, numerical results from the first two steps are available on magnetic tape to anyone who may wish to carry out independent subsequent analyses and draw independent conclusions.

We prepare amalgamated data at a set of selected central momenta by shifting the data in a given momentum-angle bin to a central point where they are statistically combined. Relative normalization and momentum errors

*Presented at the Baryon 1980 Conference, Toronto, July 1980, by
R. E. Cutkosky.

are taken into account, and inconsistent data have their errors stretched. A preliminary normalization of $d\sigma(0^0)/d\Omega$ is made to the dispersion relation prediction. The amalgamated data have quite complicated correlations, arising from the normalization and momentum uncertainties of the original data as well as from the errors of interpolation, but we have found simple ways to represent these correlations approximately. Although it is not customary to include momentum uncertainties in fitting data, we have found that the quality of fits is significantly improved when we do. In almost all cases the momentum adjustments are well within the quoted errors, but in the 79 analysis there were several cases where larger shifts were needed.⁴

After completing the 79 analysis, we added amalgamated data sets at 8 momenta between 2.0 and 2.5 GeV/c and also revised the data at some of the original 35 momenta. Besides including new data which have come along, we have begun a systematic reevaluation of the older data. This is a response to difficulties we had, in the 79 analysis, in fitting some of the data: in particular, data on CEX. We have concluded that it would be desirable to reamalgamate these data after some "pre-stretching" of errors, but so far this has been done only at momenta where there was also new data.

The single energy fits minimize the quantity

$$\chi^2 = \chi^2 + \phi + U + D, \quad (1)$$

where χ^2 is the conventional measure of quality of fit to the amalgamated data, ϕ is a truncation function, U is a unitarity constraint for high partial waves, and D is the contribution of dispersion relation constraints. The real and imaginary amplitudes are expressed separately in the form

$$F(x) = B(x) + R(x) P(z(x)) \quad (2)$$

where $x = \cos\theta$. The "Born term" $B(x)$ includes contributions of ρ , f , and N Regge poles, and of absorptive Pomeron and 2π exchange. The fitted term is P , a polynomial in the elliptic-map variable $z(x)$, modulated by a factor $R(x)$ which gives it the behavior $x^{\alpha(s)}$ at $|x| \rightarrow \infty$. In order to incorporate electromagnetic corrections and impose the unitarity constraints, F is re-expressed in terms of conventional partial wave amplitudes for $J \leq 39/2$. The amplitudes of the low partial waves are the parameters varied in the search for fits: we use 56 parameters for $p_{\text{Lab}} < 1.84$ GeV/c and 64 for $p_{\text{Lab}} > 1.895$ GeV/c, but the use of the truncation function ϕ reduces the effective number of free parameters to about 25 at our lowest momentum, 0.429 GeV/c, and to slightly less than 50 at 2.5 GeV/c.

The dispersion relation constraints are obtained by fitting the energy dependence of the amplitudes at 0^0 , at 180^0 , and on four hyperbolas which lie inside the physical region above the πN threshold (HDR). The forward C amplitudes are treated separately. For the 22 remaining amplitudes we use a Born term (nucleon and delta poles, πN and πN thresholds) and a fitted expansion term constructed using reproducing kernel techniques. A technical improvement has allowed us to reduce the HDR computer time to 20% of the requirement in the 79 analysis.⁹

In the 79 analysis, our 35 points were supplemented by 9 points from Bugg¹⁰ at low energies and 5 points above 2.0 GeV/c from Ayed.¹¹ We still use the Bugg points, but have now replaced the Ayed points by newer results of our own, which do not go to quite so high a final momentum but are more closely spaced. In the forward hemisphere, some of the Ayed amplitudes had seemed discontinuous with ours at 2.0 GeV/c, and our new amplitudes in this

region are significantly different. In extending our analysis to 2.5 GeV/c, we have found that the HDR predictions could be usefully extrapolated only about 200 MeV/c, and this has required us to extend our analysis region upward in small steps and do quite a bit of iteration. The constraints are also somewhat changed between 1.8 and 2.0 GeV/c. We have made new single energy fits in this region, using the new constraints as well as some new data.

Our amplitudes at the last three momenta, above 2.3 GeV/c, are still somewhat preliminary. In a further iteration step the error bars of the partial wave amplitudes would be reduced, and the central values might be shifted somewhat.

Some examples of fits to the amalgamated data are shown in Figs. 1 and 2 (at end of text). The fit at 820 MeV/c, where we have the highest value of χ^2 per datum (about 2) illustrates some of our problems with fitting in the lower energy region. The fit is actually somewhat worse than it looks, because a large contribution to χ^2 at this momentum is associated with the shifts which accommodate the propagation of systematic errors. The fit at 1437 MeV/c is typical of the intermediate energy region in many respects. Note that the π^-p polarization data has much less precision than the π^+p polarization data. We are not able to fit the charge exchange data as well as one would hope, but the high-precision data are fit reasonably well.

We fit the energy dependence of the partial wave amplitudes using a multichannel coupled-resonance scheme. The scattering matrix is written as:

$$t_{ab}(s) = f_a(s) \gamma_{ia} G_{ij}(s) \gamma_{jb} f_b(s) \quad (3)$$

Here G is a propagator matrix which satisfies a Dyson equation:

$$G_{ij} = G_{ij}^0 + G_{il}^0 \sum_k G_{kj} \quad (4)$$

This approach, summarized by the graphs shown above, is similar to use of a K-matrix, but it is easier to build in the correct analyticity properties. The inverse propagator matrix has the form

$$H_{ij} = e_i(s_i - s) \delta_{ij} - \sum_{ij}(s) \quad (5)$$

where $e_i = \pm 1$ and where the self-energy matrix is given by a sum over channels:

$$\sum_{ij}(s) = \sum_c \gamma_{ic} \phi_c(s) \gamma_{jc} \quad (6)$$

The γ_{ic} are "coupling constants" and ϕ_c is a real-analytic "channel propagator" or "phase space factor" for channel c :

$$\text{Im } \phi_c(s) = f_c^2 \rho_c(s) \quad (7)$$

where ρ is the density of states. There is, roughly speaking, one "bare resonance" (labelled by $i, j, \dots$) for each resonance, but we also include extra terms to represent background effects arising from resonances above our energy region and from long-range potentials. We use a quasi-two-body approximation for such channels as $\pi\Delta$, ρN , $\rho\Delta$. Data on production channels are used, with their errors stretched (usually by a factor of 3) so that the elastic scattering data will dominate the fit to the elastic channel.

In the current revision, we have been experimenting with three alternative ways to construct the ϕ_c . In scheme A, the ϕ_c are the same as used in the 79 analysis. In scheme B, $\text{Im } \phi_c$ is given by the decay matrix elements for meson emission as calculated using harmonic oscillator quark model wave functions. In scheme C, as suggested by boundary-condition models,¹² we use

$$\begin{aligned} \phi_c(s) &= \psi'(R)/\psi(R) \quad , \\ \psi(R) &= p R h_\ell^{(1)}(pR) \quad , \end{aligned} \quad (8)$$

where R is the baryon radius, ℓ is the angular momentum, and p is the momentum in channel c . To define p for quasi-two-body channels, we use an approximation in which the second-sheet two-body thresholds are treated correctly but in which the physical three or four body thresholds are somewhat weakened. In scheme C, $\text{Im } \phi_c(s) \sim p$ at high energies, in scheme B, $\text{Im } \phi_c(s)$ falls exponentially and for low partial waves has a maximum in our energy region, while in scheme A it approaches a constant.

In schemes B and C we also include multiplicative background absorptive phase-shift factors in the S matrix. As a variant of scheme C, we have experimented with the background factor suggested by the R -matrix approach,¹² but have found that this does not provide an economical parametrization, as the background repulsion is too large and needs to be cancelled by additional resonance terms.

A summary of the resonances found in this analysis is given in Table I, and their parameters are listed more completely in Tables II and III. The errors on the parameters depend as much on the method of parametrization as on statistical errors in the partial wave amplitudes. We have used the variations obtained with use of different functions ϕ_c for an estimate of this problem. It is hard to estimate the degree of significance of some of the weaker resonances. Our "quality factor" list in Table I gives our best estimate, and we also refer the reader to the plots shown at the end of the text.

In Fig. 3 three fits to S_{31} are shown; the first has three resonances, and the second has resonance #2 omitted. This shows that the second resonance (at 1890 MeV) is a highly significant effect. The fit to P_{33} (Fig. 3) also has three resonances.

The D and F waves are shown in Figs. 3-4. With addition of the new data, the resonant signals seen before in the D_{35} and F_{17} partial waves are even more pronounced. These resonances are noteworthy because they are most naturally associated with the "non-minimal" multiplets $(56, 1^-)$ and $(70, 2^+)$. The D_{35} wave seems to have a second broad resonance at about 2400 MeV. The D_{15} wave has, besides the well-known lower resonance, a second at about 2200 but nothing in between. Thus, the other odd-parity resonances found in the

1800-2000 MeV region probably belong in the same multiplet as the $D_{35}(1940)$. The third D_{13} resonance (at about 1900 MeV) is the weakest of these - it is of marginal significance, and we have not yet excluded the possibility that it is caused by some artefact in the data.

The fits shown to the F_{35} and F_{37} waves include second resonances at the upper end of the energy range. However, we do not have evidence for a second F_{15} resonance.

Several aspects of the results listed in the tables are of significance to baryon models. We have already mentioned states we identify with the $(56, 1^-)$ and $(70, 2^+)$ multiplets; the $P_{11}(1700)$ and $G_{39}(2300)$ resonances presumably belong in the $(70, 0^+)$ and $(56, \frac{3}{2}^-)$ multiplets. The $(70, 1^-)$ states show no sign of spin-orbit forces, and this is also true of the $(56, 1^-)$ and $(56, 2^+)$ states (although one expects these states to mix strongly with other multiplets). Note, however, that the $S_{31}(1620)$ and $D_{33}(1710)$ are significantly different in mass. (For discussions of the significance of this, see the talks by Kelly¹⁵ and by Reinders¹⁴ in these proceedings). The masses above 2000 MeV are not yet precise enough to allow simple conclusions about the forces in these states to be drawn at the present time. Many of these states are also likely to be strongly mixed.

Finally, we should like to comment on some weaknesses in the data used in the present analysis, and how these weaknesses affect our present knowledge of the resonances. In the 0.5-1.0 GeV/c region, the data in all channels need improvement, (especially CEX). The $I=1/2$ resonances have distorted shapes as a result. An interesting question is whether isospin violations are significant. In principle, measurable effects should exist,¹⁵ but they are masked by systematic errors in the present data. In the 1.0-2.0 GeV/c region, the π^-p polarization data have large errors, and improved CEX data would also be helpful. The $I=1/2$ partial wave amplitudes in this region have large errors, making it hard to see weak resonances. In the 2.0-3.0 GeV/c region, data (especially on polarization) is needed at more closely spaced momenta. There are likely to be, in this region, many closely spaced resonances which cannot be resolved at the present time.

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CAPTIONS FOR FIGURES

- Fig. 1. Fit to amalgamated data at 0.82 GeV/c. The data have been adjusted to accommodate systematic shifts. The differential cross sections are in mb/sr.
- Fig. 2. Fit to amalgamated data at 1.437 GeV/c. See also the caption to Fig. 1.
- Fig. 3. Examples of partial wave amplitudes. Three different fits to the S31 amplitude are shown, and also the P33, D13, and D15 amplitudes. The fits are shown as solid lines, and are also projected into the Argand plot. The arrows on the Argand plot are 5 MeV long and are at intervals of 50 MeV.
- Fig. 4. The D33, D35, and F-wave amplitudes. See also the caption to Fig. 3.

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Fig. 1

XBL 793-9027

Fig. 2

XBL 793-9023

Fig. 3

Fig. 4

Table I

Summary Resonance Comparison, PDG (April, 1980) with CMU/LBL (July, 1980)

	<u>PDG</u>		<u>CMU/LBL</u>			<u>PDG</u>		<u>CMU/LBL</u>	
	<u>Mass</u>	<u>Quality</u>	<u>Mass</u>	<u>Quality</u>		<u>Mass</u>	<u>Quality</u>	<u>Mass</u>	<u>Quality</u>
P31	1910	4	1910	4	P11	938	4		
P33	1232	4	1232	4	P11	1470	4	1440	4E
P33	1690	3	1600	4	P11	1710	4	1700	4E
P33	1960	2	1920	3	P11			2125	1E
					P13	1810	4	1700	4E
F35	1890	4	1910	4	F15	1688	4	1680	4
F35			2200	1Q					
F37	1950	4	1950	4	F17	1990	3	1970	4E
F37			2350	1H	H19	2220	4	2230	4Q
H39	2300	1	2400	2H					
H311	2420	4	2400	4H					
S31	1650	4	1620	4	S11	1535	4	1550	4
S31	1900	2	1890	3	S11	1650	4	1650	4
S31			2150	2	S11	2100	1	2180	1E
D33	1670	4	1710	4	D13	1520	4	1525	4
D33			1940	2	D13	1700	4	1675	4
D35	1960	3	1940	4	D13			1880	1
D35			2400	2H	D13	2040	2	2060	3
G37			2200	2Q	D15	1670	4	1675	4
G39	2500	1	2300	2Q	D15	2100	2	2180	3Q
					G17	2190	4	2200	4EQ
					G19	2200	4	2250	4Q

The PDG mass values quoted are the nominal values they use for identification (not necessarily their best average).

The numerical rating refers to the number of "stars".

H: Half of the resonance lies above our range; Q: About 1/4 lies above our range; the accuracy of our determination of parameters is reduced in these cases, even if the existence of the resonance is clear.

E: The partial wave amplitudes have large errors or are erratic in the neighborhood of the resonance. This affects the accuracy of determination of parameters. There may also be additional real structure.

Table II.

Even parity delta resonances								Even parity nucleon resonances							
Wave	Mass	Γ	x	Re W	-Im W	$ r $	θ	Wave	Mass	Γ	x	Re W	-Im W	$ r $	θ
P31	1910	225	0.19	1880	100	20	-90	P11	1440	340	0.68	1375	90	52	-100
	40	50	03	30	20	4	30		30	70	04	30	20	5	35
P33	1232	120	1.00	1210	50	53	-47	P11	1700	90	0.20	1690	40	8	175
	3	5	00	1	1	2	1		50	30	04	20	10	2	35
P33	1600	300	0.18	1550	100	17	-150	P11	2125	260	0.12	2120	120	14	35
	50	100	04	40	30	4	30		75	100	03	40	40	7	25
P33	1920	300	0.20	1900	150	24	-150	P13	1700	125	0.10	1680	60	8	-160
	80	100	05	80	50	4	30		50	70	04	30	20	2	30
F35	1910	400	0.14	1830	140	25	-50	F15	1680	120	0.62	1667	55	34	25
	30	100	03	40	30	8	20		10	10	05	5	5	2	5
F35	2200	400	0.07	2150	175	16	150								
	125	125	04	100	50	5	90								
F37	1950	340	0.39	1890	130	50	-33	F17	1970	350	0.06	1900	130	9	-60
	15	50	04	15	20	7	8		50	120	02	30	30	3	30
F37	2350	300	0.08	2350	130	12	-90								
	100	100	04	100	50	6	60								
H39	2400	425	0.06	2370	210	10	-20	H19	2230	500	0.15	2160	240	45	-45
	125	150	02	80	80	4	30		80	150	03	80	50	20	25
H311	2400	450	0.08	2360	210	18	-30								
	125	150	03	100	50	6	40								

The estimated errors (~90% confidence level) are listed below the values of the parameters. All quantities are in MeV, except for the elasticity, x , and the phase angle of the residue of the pole, θ (which is in degrees). The pole position is W and the residue is r .

Table III

Odd parity delta resonances								Odd parity nucleon resonances							
Wave	Mass	Γ	x	Re W	-Im W	$ r $	θ	Wave	Mass	Γ	x	Re W	-Im W	$ r $	θ
S31	1620 20	140 20	0.25 03	1600 15	60 10	15 2	-110 20	S11	1550 40	240 80	0.50 10	1510 50	130 40	120 40	15 45
S31	1890 50	170 50	0.10 03	1870 40	90 25	10 3	20 40	S11	1650 30	150 40	0.65 10	1640 20	75 15	60 10	-75 25
S31	2150 100	200 100	0.08 02	2140 80	100 40	7 2	-60 90	S11	2180 80	350 100	0.18 08	2150 70	175 50	40 20	0 90
								D13	1525 10	120 15	0.58 03	1510 5	57 5	35 2	-12 5
D33	1710 30	280 80	0.12 03	1675 25	110 20	13 3	-20 25	D13	1675 25	90 40	0.11 05	1660 30	45 20	6 3	0 50
D33	1940 100	200 100	0.05 02	1900 100	-100 30	8 3	135 45	D13	1880 100	180 60	0.10 04	1880 100	80 40	10 5	100 80
								D13	2060 80	300 100	0.14 07	2050 70	100 40	30 20	0 100
D35	1940 30	320 60	0.14 04	1890 50	130 30	18 6	-20 40	D15	1675 10	160 20	0.38 05	1660 10	70 5	31 5	-30 10
D35	2400 125	400 150	0.20 10	2400 125	200 75	15 8	-70 70	D15	2180 80	400 100	0.10 03	2100 60	180 40	20 10	-90 50
G37	2200 80	450 100	0.06 02	2100 50	170 40	8 3	-70 40	G17	2200 70	500 150	0.12 06	2100 50	200 80	25 10	-30 50
G39	2300 100	330 100	0.05 02	2260 60	160 80	8 4	-25 15	G19	2250 80	480 120	0.10 02	2150 50	180 50	20 6	-50 20

See the notes following Table II.