SINGLE PASS COLLIDER MEMO

CN- 243

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DATE: 8/17/83

REPLACES CN²⁴³

TITLE: FURTHER EFFECTS OF IONS IN THE ARCS

SLAC-CN--243

DE84 011761

One of the effects of ions in the SLC arcs was considered by A.W. Chao in CN-121. He estimated that the extra betatron phase shift of particles in the tail of a bunch caused by ions produced by the head was probably tolerable, provided the vacuum was 10^{-4} torr or better. In this note I shall suggest that a more thorough study of ion effects in the arcs is warranted.

First let us consider the dispersion function n . It is zero by design in the linac, but not so in the arcs, and the small gradients produced by the ion cloud perturb it. We are depending on exceedingly precise adjustment of the dispersion to make it zero at the final focus--an adjustment we intend to make by tuning the focussing. But when the adjustment is right for the head, it is wrong for the tail because of the ions. The question is: how much?

We can try to approach the question through perturbation theory. For illustrative purposes, let us restrict ourselves to the horizontal degree of freedom for which the equation of free betatron motion for an on-momentum particle is

$$x'' - k(t)x = 0 \quad (1)$$

Following Karl Brown, SLAC-75, Rev. 4, we observe that the two independent solutions (sine-like and cosine-like) may be combined in a Green's function

$$G_0(t, \tau) = s_0(t)c_0(\tau) - c_0(t)s_0(\tau) \quad (2)$$

for the inhomogeneous form of Eq. (1). I am using the variables t and τ as longitudinal positional coordinates, not time; although they do represent the independent variable.

Now, the dispersion function obeys the inhomogeneous equation,

$$n_0'' - k(t)n_0 = h(t) \quad (3)$$

(The subscripts, zero, appear in anticipation of perturbations to come.) and we could write it explicitly as a Green's-function integral, but there is no need just now. Rather let us move on to the equation obeyed by particles in the tail of the bunch, symbolizing by Δk the small focussing due to ions and labeling perturbation orders by powers of ϵ .

$$n'' - kn - h = \epsilon \Delta k n \quad (4)$$

Introducing the perturbation expansion of the dispersion function

$$n = n_0 + \epsilon n_1 + \dots \quad (5)$$

we obtain the first order perturbation equation

$$n_1'' - kn_1 = \Delta kn_0 \quad (6)$$

which admits a Green's-function solution.

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$$\eta_1(t) = \int_0^t \Delta k(\tau) \eta_0(\tau) G_0(t, \tau) d\tau \quad (7)$$

We have reduced the solution to quadrature--cause for rejoicing. Or is it? We need to carry the integral all the way through the arc to the interaction point and we need to get it accurate to 100 microns. The energy spread will increase the spot size for the tail intolerably if η_1 is that large at the interaction point. Although Δk never changes sign and η_0 seldom changes sign, G_0 oscillates tirelessly. I do not see how to make a useful estimate of the integral easily. More fertile minds are needed.

Next let us turn to the free betatron motion. The perturbed version of (1) for tail particles is

$$x'' - kx = \epsilon \Delta k x \quad (8)$$

and the first order perturbation equation obeyed by the s- and c-functions has the form

$$s_1'' - ks_1 = \Delta k s_0 \quad (9)$$

The solutions are

$$s_1(t) = \int_0^t \Delta k(\tau) s_0(\tau) G_0(t, \tau) d\tau \quad (10)$$

$$c_1(t) = \int_0^t \Delta k(\tau) c_0(\tau) G_0(t, \tau) d\tau \quad (11)$$

Once we have thus obtained the perturbed s- and c-functions, we can form the perturbed R-matrix and with it the perturbed σ -matrix. From the latter we can get finally the perturbed envelope, $\beta^{1/2}$. However, the means of carrying out these integrals are not evident to me. Once again cleverness is called for.

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