A Precision Test of Charge Independence of Hadronic Interactions

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Preface

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Abstract

A precision test of charge independence of hadronic interactions

Broken symmetries are among the richest sources of information about the fundamental interactions: the renewed interest in the study of isospin non-conservation by strong forces is closely related to the effort of understanding some properties of nuclear systems in terms of their basic degrees of freedom.

The hope is to be able to relate the pattern of the dynamical breaking of this symmetry to the mass spectrum of light quarks: to this purpose a more detailed phenomenological knowledge must be provided by a new generation of experiments.

These considerations motivated us to perform a precision test of charge independence of strong nuclear interactions through a measurement of the parameters $\Delta A_{\nu 0}(\theta) = A_{\nu 0}(\theta, {}^3\text{H}) - A_{\nu 0}(\theta, {}^3\text{He})$, (the difference in analyzing power), and $R \equiv d\sigma(\theta, {}^3\text{H})/d\sigma(\theta, {}^3\text{He})$ for the two reactions:

$$\vec{p} + d \longrightarrow {}^3\text{H} + \pi^+$$

$$\vec{p} + d \longrightarrow {}^3\text{He} + \pi^0$$

The observable $\Delta A_{\nu 0}$ is particularly relevant as it probes the spin dependent term of the symmetry breaking interaction, on which so far almost no empirical evidence is available.

The experiment has been performed at the Los Alamos Meson Physics Facility, using the N-type polarized proton beam ($T_{\vec{p}} = 733$ MeV), and detecting the charged heavy particle in the HRS magnetic spectrometer.

The final results are:

$$\Delta A_{\nu 0} = A_{\nu 0}({}^3\text{H}) - A_{\nu 0}({}^3\text{He}) = 0.3930 - 0.3996 = -0.0066 \pm 0.0040 \pm (0.0018)$$

and

$$R = 2.193 \pm 0.007 \pm (0.027).$$

where the first errors are statistical and the second systematic.

Chapter 1

Introduction

1.1 Symmetries in physics

The concept of symmetry can be considered as one of the ‘backbones’ of human culture, as has been pointed out by Weyl in his classic book on the topic¹. Symmetries have always played a fundamental role in our effort to understand the physical world around us. There is a definite trend in the evolution of physics to look for deeper and deeper connections between the dynamical content of theories and the underlying symmetries.

Before the advent of quantum mechanics and quantum field theories of the elementary particles and their interactions, symmetries had a more geometrical nature: the classical model for them¹ was the left-right specularity exhibited by many structures which are part of our daily life. Geometrical symmetries can be classified into two main groups: discrete symmetries, like parity or time-reversal, and continuous ones, like translations or rotations.

The attempt to put these concepts in more quantitative terms lead to the development of group theory. Some fields of physics which study structures exhibiting a high degree of regularity, like crystallography, are based on these mathematical methods.

A deeper dynamical role of symmetry principles was actually built in the Lagrangian and Hamiltonian formulation of classical mechanics and the associated field theory: the connection between invariance of the Lagrangian under

some transformations and the concept of constants of motion. This connection became a fundamental concept in quantum mechanics, allowing the identification of the ‘good quantum numbers’ associated with a physical system. Besides the space-time variables, which are suitable for the description of the macroscopic world, quantum systems have new degrees of freedom. Transformations in some of these ‘quantum’ subspaces which leave a given system invariant are classified as ‘internal’ symmetries.

From the classical theory of fields comes a theorem which has become the starting point for the development of the ‘gauge theories’ of the elementary interactions.² It states that to any continuous one-parameter symmetry of the Lagrangian is associated a conserved current. The 3-space integral of this current can be identified with a conserved charge. The study of the elementary particles and their interactions has led to a further distinction between ‘global’ and ‘local’ symmetries, that is between transformations independent of the space-time point at which they are performed and transformations which are functions of space-time. The latter group represents the actual core of the ‘gauge’ theories of the elementary interactions. Here the symmetry assumed dictates the structure of the coupling between the various fields.

The ‘standard-model’ of the strong and electro-weak interactions marks the great success of this theoretical approach, which was originally proposed by Yang and Mills³ in an attempt to describe the interaction between a proton and a neutron with a Lagrangian invariant under $SU(2)$ ‘gauge’ transformation.

The discovery of these powerful tools awakened the everlasting dream of physicists: a unified description of all the fields and interactions which are known to exist. Theories, so far based more on aesthetic appeal than on empirical evidence, generally known as Grand Unified Theories, have been developed.

This short journey among the more ambitious flights of the modern theoretical physics is bound to end with a reference to supersymmetry, which ‘is the supreme symmetry: it unifies space-time symmetries with internal symmetries,

fermions with bosons and gravity with matter'.⁴

As far as global symmetries are concerned, they are certainly less fundamental aspects of our present understanding of the microworld. They are not required by any theoretical principle: no revolution would occur in our understanding of the basic interactions if some experimental data would prove that they are not exact. However, in the development of phenomenological models which try to describe complex systems, as it is the case, for instance, in nuclear physics, global symmetries act as fundamental constraints. It is therefore of extreme interest to investigate whether they are exact or approximate symmetries, and, if they are approximate, to reach a deeper understanding of the extent and the pattern of the symmetry breaking component of the interaction.

As it will be pointed out in a following section, a great theoretical effort is presently involved in a reformulation of the nuclear theory by including the most relevant features of QCD, which has a good chance of being the correct theory of the strong interaction. The argument developed above therefore suggests that a wider phenomenological knowledge of the internal symmetries of hadronic systems has to be achieved.

1.2 Isospin Invariance

Isospin was the first internal symmetry postulated in nuclear physics: soon after the discovery of the neutron, Heisenberg⁵ proposed the 'nucleon' as the elementary particle sensitive to the strong force. In other words, Heisenberg conjectured that in a hypothetical world in which only the forces responsible for binding nucleons in nuclei are present, there are no dynamical effects which are able to distinguish protons from neutrons. In the isospin formalism they correspond to the fundamental representation of an internal $SU(2)$ group. A rotation in this space is generated by the operator $R_{\vec{\tau}} = \exp(i\theta \hat{n} \cdot \vec{\tau}/2)$, where $\hat{n}$ is the unit vector which gives the direction of rotation and $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ is the set of Pauli matrices, i.e. the generators of the $SU(2)$ group in the fundamental spinor representation.

The two basis states, corresponding respectively to the proton and the neutron, are the eigenstates of the τ_3 operator with eigenvalues $+1$ and -1 respectively. Soon after Heisenberg's conjecture, it was recognized that a nuclear Hamiltonian dependent upon space and spin coordinates of the interacting particles but not upon their projection along the 3-axis in isospin space was consistent with all the known systematics.⁶

After the discovery of the pion⁷, which took place soon after Yukawa⁸ proposed that some 'exchanged particle' was the mediator of nuclear interaction, the isospin formalism was generalized to encompass also the newly discovered mesons. The pions were required to carry an isospin quantum number to generate a nucleon-nucleon interaction consistent with the invariance property discussed above. It is interesting to note that it was exactly the equality in range of the proton-neutron and proton-proton or neutron-neutron interaction which suggested the existence of a neutral pion or, in other words, that the pion is a triplet in isospin space.⁹

There are two fundamental symmetries which can be recognized in isospin space. The first one, known as 'charge independence', corresponds to a rotational invariance in this internal space, in other words, the interaction must be a scalar as far as isospin is concerned. A less restrictive invariance is associated with the 'charge symmetry operator' $P_{cs} = e^{i\pi\tau_2}$, which corresponds to a rotation of 180 degrees around the $\hat{2}$ axis. Its effect is the transformation of a proton into a neutron and vice versa: in a nucleus τ_2 is substituted by $T_2 = \sum_{i=1}^N \tau_2(i)$: it has the effect of reversing the $\hat{z}$ isospin projections of all the nucleons. Charge independence guarantees charge symmetry but charge independence breaking can take place even if charge symmetry is exact. The charge operator, associated with the Coulomb interaction, has an isovector component, as it is defined as $Q = Y/2 + T_3$, where Y stands for the hypercharge and T_3 stands for the total isospin projection of the hadronic system. It breaks charge independence and

charge symmetry as the commutator

$$[e^{i\pi T_2}, T_3] = 2e^{i\pi T_2} \cdot T_3$$

which in the general case is different from 0. However, $T_3 = 0$ for ‘self-conjugate’ systems, with equal number of protons and neutrons, and therefore the charge and isospin parity eigenvalues can be specified simultaneously.

A general pattern which can be identified in the evolution of physics is the fact that the identification of some new symmetry underlying a given interaction is always followed by an experimental effort with the purpose of establishing whether this symmetry is exact or not on the basis of a stronger empirical evidence. If the symmetry turns out to be an approximate one, interest focuses on determining the extent and the patterns of its violation. The first set of ‘dedicated’ experiments performed to test isospin symmetry were measurements of the parameters characterizing the low energy nucleon-nucleon scattering and gave some evidence that this symmetry is not exact: these experiments will be discussed later in this chapter.

Henley¹⁰⁻¹² performed a formal analysis of the possible patterns of isospin violation, identifying the structure of the potential operators associated with them. He recognized four categories of potential operators describing the nucleon-nucleon interaction in isospin space, corresponding to different transformation properties with respect to isospin rotations:

$$V_1(ij) = A [1 + \vec{\tau}(i) \cdot \vec{\tau}(j)] \quad (1.7)$$

This is an isoscalar potential which commutes with all the generators associated with this global symmetry: it therefore represents the bulk of the nuclear interaction, conserving isospin quantum numbers.

$$V_2(ij) = B \left[\tau_3(i)\tau_3(j) - \frac{1}{3} \vec{\tau}(i) \cdot \vec{\tau}(j) \right] \quad (1.2)$$

This is an isotensor operator which commutes with the charge symmetry operator P_{cs} .

$$V_3(ij) = C [\tau_3(i) + \tau_3(j)] \quad (1.3)$$

This is an isovector term with the same structure as the isospin violating term of the Coulomb interaction, as the charge operator is given by $Q = \frac{Y}{2} + T_3$. It breaks isospin independence and isospin symmetry, but is unaffected by the interchange of any two nucleons.

$$V_4 = D [\tau_3(i) - \tau_3(j)] [\vec{\sigma}(i) - \vec{\sigma}(j)] \cdot \vec{L}_{ij} + E [\vec{\tau}(i) \times \vec{\tau}(j)]_3 [\vec{\sigma}(i) \times \vec{\sigma}(j)] \cdot \vec{L}_{ij} \quad (1.4)$$

This potential breaks all the possible symmetries in isospin space. It affects the np system but not the nn or pp systems. It should produce noticeable effects on the polarization observables associated with np pairs in free space or in nuclear environment.

For the sake of completeness we also mention the Coulomb interaction which is the most common agent of isospin breaking. It can be written as.

$$C_{ij} = \frac{e^2}{4r_{ij}} \cdot [1 - \tau_3(i)] [1 - \tau_3(j)] \quad (1.5)$$

It contains two terms which do not conserve isospin: the $\tau_3(i)\tau_3(j)$ term breaks charge independence but not charge symmetry, and the $[\tau_3(i) + \tau_3(j)]$ term which breaks both.

Unfortunately, it is difficult to draw definitive conclusions about the extent and the patterns of isospin breaking. Because of our lack of knowledge of the details of the hadronic interaction, it is generally not possible to extract the effects of the interference between the strong and electromagnetic terms in the scattering

matrix in a model independent way. Moreover the knowledge of the existence of the isospin non-conserving electromagnetic interaction and the aesthetic appeal of isospin symmetry in the treatment of nuclear phenomenology has somewhat biased the theoretical approach to the investigation of its validity . A clear example is the attitude toward the parameter $\Delta = m_p - m_n = -1.2395$ MeV. In a sense this is the first indication that the symmetry under consideration is an approximate one; no theoretical calculation based on the assumption that Δ is purely of electromagnetic origin is able to reproduce even its sign. However, until the advent of QCD the value of Δ was considered an electromagnetic effect¹⁰, even if not understood.

1.3 Isospin in the context of meson theories.

It is interesting to analyze the evolution of the concept of isospin conservation when the meson theory of nuclear force established itself as a successful model of hadronic interactions. In a sense isospin was still considered an intrinsically exact symmetry; all the manifestations of its breaking in nuclear phenomenology were still ascribed to the electromagnetic force.

In this theoretical framework, the nucleon-nucleon potential arises by the exchange of virtual mesons. The example of the pion exchange contribution to the nucleon-nucleon potential is illustrated in Fig. 1.1, which shows the relevant diagrams for OPEP (one pion exchange potential). As the mass of the exchanged particle determines the range of the interaction mediated by it, this term describes the long range part of the interaction.

Figure 1.1: Feynman diagrams for one pion exchange nucleon-nucleon potential.

It is convenient to discuss the issue of isospin violation in meson theory with reference to OPEP, where the analysis is more straightforward, and subsequently generalize to the terms which appear in the description of the interaction at shorter ranges. The so called 'direct electromagnetic effects', which represent the symme-

try breaking terms which vanish if the strong interaction is switched off, will not be considered.

Charge independence would require the equality of the masses $m(\pi^\pm)$ and $m(\pi^0)$ of the charged and neutral pions and the equality of the couplings $g_\pm$ and g_0 associated with the respective nucleon-pion interaction vertex. It is known that, whereas $m(\pi^+)$ is exactly equal to $m(\pi^-)$ as it is required to satisfy the CPT theorem (the two charged pions are related by particle-antiparticle conjugation), there is a relatively large difference between $m(\pi^-)$ and $m(\pi^0)$:

$$\frac{\Delta m(\pi^\pm)}{\langle m(\pi) \rangle} = \frac{4.604 \pm 0.004}{137.27} = 3.4\% \quad (1.6)$$

This mass splitting, in the framework of the meson theories, is attributed to electromagnetic renormalization and is considered to be the major source of isospin violation.

An analogous electromagnetic renormalization of the coupling constants is expected, but it is more difficult to evaluate. However it is expected to be small: the difference between the pseudoscalar coupling constants is estimated to be $\leq 1\%$.¹¹

The other major source of isospin symmetry violation in this approach is the so called 'meson mixing'. Since there are some mesons which have the same space-time symmetry and the same charge, but different isospin quantum numbers, a isospin violating interaction, like the electromagnetic one, can lead to a mixing between these mesons. This is illustrated by the diagram in Fig. 1.2, which describes the ' $\pi - \eta$ ' and the ' $\rho - \omega$ ' mixing.

The description of the isospin breaking in the shorter range of the interaction becomes more complicated, as the number of possible diagrams which contribute becomes large, (the $\rho - \omega$ mixing is illustrated above), corresponding to a more limited understanding of this range of the interaction in this theoretical framework. In the literature various calculations can be found, based on the corrections due to two pion exchange potential¹³, to the $\rho - \omega$ mixing¹⁴⁻¹⁸ and to the difference in

Figure 1.2: Examples of ‘ $\pi - \eta$ ’ and the ‘ $\rho - \omega$ ’ mixing diagrams.

mass between the intermediate isobar excited states¹⁹. Therefore it appears that a quantitative theoretical treatment of the isospin symmetry violation in this range of the interaction not only ‘still escapes us’, as Henley admits¹², but is likely to continue to elude us unless a more fundamental treatment of the interaction is developed.

1.4 Isospin in the context of QCD

It is fair to say that no dramatic change in the role played by isospin symmetry in our understanding of nuclear dynamics took place until new phenomenological models were inspired by quantum chromodynamics (QCD), which is so far the most promising theory of strong interactions. According to QCD, all the hadrons are composite states of quarks. Therefore the quantum numbers characterizing the hadrons arise as the sum of the corresponding ones carried by their elementary constituents.

The isospin quantum number is non-zero for the lightest quarks, up(u) and down (d), which are the ‘valence’ constituents of the nucleons. In this context, isospin symmetry represents a sub-group of a larger symmetry exhibited by a sector of the QCD Lagrangian L , which can be written as:

$$L^{QCD} = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + i\bar{\psi}\gamma^\mu D_\mu\psi + \Delta L^{QCD} \quad (1.7)$$

where:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

is the Yang-Mills field strength,

$$\psi = \begin{pmatrix} q_1 \\ \vdots \\ q_i \\ \vdots \\ q_n \end{pmatrix}$$

is the quark vector (the index i runs over flavors), D_μ is the covariant derivative, and

$$\Delta L^{QCD} = \bar{\psi} M \psi = \Sigma m_i \bar{q}_i q_i$$

The mass term of the Lagrangian, ΔL^{QCD} , has been singled out because it is the only term which breaks a $SU(N)_L \otimes SU(N)_R$ symmetry, where N corresponds to the number of quark flavors. This term is actually an approximation of the mass sector of the full ‘standard model’ of the strong and electroweak interactions, which is valid at energies low with respect to the masses of the W and Z bosons, i.e. ≈ 90 GeV. In fact, in this model the fermion masses can not be introduced as free parameters of the theory, because explicit mass terms would break the $SU(2)_{weak}$ invariance of the Lagrangian. Their dynamical origin is attributed to the spontaneous symmetry breaking produced by the vacuum expectation value of the Higgs fields. This expectation value, together with the couplings of the fermions to the Higgs particles, are the parameters which are equivalent to the bare fermion masses. At low energy it is possible to neglect the weak interaction and describe the dynamics of fundamental particles in terms of L^{QCD} and the analogous Lagrangian density L^{QED} , describing the electromagnetic interaction.

The first feature which emerges from this theoretical approach is the ‘accidental’ nature of isospin symmetry, which is valid, at least approximately, in the hadronic phenomenology. In fact, up and down quarks seem to have no reason to be singled out with respect to heavier quarks, like the charm and the bottom. However QCD is a theory with a scale, which is set by the phase transition which takes place upon hadronization. It is widely believed that in this process the vacuum undergoes a phase transition which breaks the chiral symmetry discussed above according to the pattern: $SU(N)_L \otimes SU(N)_R \longrightarrow SU(N)_V$. The order parameter associated with this phase transition, Λ_{CSB} , $(\approx 1 \text{ GeV})^{20}$, sets the mass scale in hadronic phenomenology: the up, down and, to a lesser extent, the strange quarks, which have bare masses of the order of 4, 10, 140 MeV respectively, are ‘light’ and, to a first degree of approximation, can be treated as massless. This is the reason why the discussion of the flavor symmetry breaking is generally performed with respect to $SU(3)$ and not $SU(2)$, in other words m_s is also considered ≈ 0 . As a consequence of the phase transition discussed above,

Table 1.1: Evaluations of ratios between current algebra masses of the light quarks.

$\frac{m_u}{m_d}$	$\frac{m_d}{m_s}$	Evaluation Method
0.381 ± 0.13	0.045 ± 0.011	Chiral perturbation theory and baryon spectrum ²¹
0.47 ± 0.11	0.042 ± 0.008	Pseudoscalar meson masses and decay constants and $\rho - \omega$ mixing ²²
0.44 ± 0.07	0.05	$\eta' \rightarrow 3\pi$ ²³
0.41 ± 0.15	0.044 ± 0.008	$\eta \rightarrow 3\pi$ ²⁴
0.24–0.73	0.039–0.056	Chiral perturbation theory ²⁵ (2nd order terms $\leq 20\%$)

nine Goldstone bosons arise, corresponding to the generators of the subgroups broken in this process. They are identified with the nonet of lightest pseudoscalar mesons the π 's, η 's, η' 's, and K's. In this approximation they should be massless. They acquire a mass because of the breaking of $SU(3)_V$, which is generated by the unequal values of m_u , m_d , and m_s . The matrix which generates the physical masses of these mesons and their couplings is directly related to these parameters. The empirical values of its elements are the most commonly used method to extract ratios between m_u , m_d , m_s . The mass spectrum of the pseudoscalar mesons and the branching ratios for some specific decays, like $\eta \rightarrow 3\pi$, allow the determination of the quantities $r_+ \equiv (m_u + m_d)/m_s$ and $r_- \equiv (m_d - m_u)/m_s$, from which the ratios $r = m_u/m_d$ and $r' = m_d/m_s$ can be extracted. The information presently available is summarized in Table 1.1.

It can be seen that the general consensus is that at a quark level $SU(2)$ is badly broken. It is interesting to remark that the debate on the current algebra mass ratios for light quarks is still an open subject. Recently ²⁵ a study of second order contribution in the chiral perturbation theory suggests that, under the hypothesis that the second order contribution to the meson masses is equal to

30%, the ratio m_u/m_d can have any value between 0.0 and 0.8. Even if m_u and m_d are actually very different, the large renormalization which takes place upon hadronization washes out this asymmetry almost completely.

It is interesting to note that this is the first time in which a mechanism that is entirely independent of the electromagnetic effect has been identified for isospin nonconservation. This is the reason why it is generally known as 'anomalous isospin breaking'.²⁶⁻²⁷ The task of deriving the consequences of the value of the ratio $r = m_u/m_d$ in hadronic phenomenology shares the problems of all the models which use the theory of the fundamental interaction as 'inspiration' for their guess-work. In other words, as we lack the mathematical ability to solve the problem exactly, it is necessary to resort to models which try to incorporate the relevant dynamical features of the underlying theory in an approximation scheme which is calculable.

The simplest approach which can be taken is to introduce the anomalous isospin breaking in the diagrams involving meson mixing. ($\pi - \eta - \eta'$, $\rho - \omega$).²⁷ These amplitudes were considered also in the meson theories. The latter models derived the couplings phenomenologically from known decays, and therefore obtained essentially correct results. However the new theoretical approach has the advantage of a more consistent theoretical basis, which allows the recognition of new features which were not previously considered (for instance the importance of $\eta' - \pi$ mixing in nucleon-nucleon scattering.)

1.5 A scale for charge symmetry and charge independence violation.

Even a cursory and superficial examination of many of the observables of hadronic physics reveals that CS and CI are obeyed to a surprising degree. A scale for CS and CI violations is easily set by the Coulomb interaction, because, as we have already noted, this interaction violates both CS and CI, and it is essentially omnipresent. The strength of the Coulomb interaction compared to the strong hadronic interaction is often stated as $\alpha = 1/137$, i.e., $\sim 1\%$. Thus CS and CI violations of this order are to be more or less 'expected' because of this one reason alone. The 'true' violations, defined as those in the purely hadronic interaction, must come above and beyond the Coulomb interaction induced violations. Let us look at the general experimental evidence.

1.5.1 Masses of Isospin Multiplets.

Charge independence implies that masses of the different members of an isospin multiplet (same T or t , but different allowed T_3 or t_3) should be the same, except for Coulomb energy in the case of composite systems.

If we ignore the quark-gluon sub-structure of hadrons for the present, CI predicts that masses of different charge members of baryon and meson multiplets must be the same. Experimentally this is not found to be the case. As Table 1.2 shows, the masses differ.

The difference is generally $<1\%$, but in one case, $m(\pi^\pm) - m(\pi^0)$, it is as much as 3.3% .

Nuclei are of course composite. Thus the masses within a nuclear isospin multiplet are meant to be not equal but different by the Coulomb energy, calculated in some simple model of nuclear structure. Two approaches can be taken. One is to determine the functional form of the Coulomb energy based on general principles and the other is to calculate the absolute value of Coulomb energy. Both approaches have been tried.

Table 1.2: Masses of different charge members of baryon and meson multiplets

	$\Delta M/\langle M \rangle$ (%)	ΔM (MeV)	ΔM_{th}^* (MeV)	ΔM_{th}^{**} (MeV)
pn	0.14	-1.239(0)	-1.29	
$\Sigma^+ - \Sigma^0$	0.26	-3.10(14)	-1.83	
$\Sigma^0 - \Sigma^-$	0.41	-4.88(1)	-3.45	
$\Xi^0 - \Xi^-$	0.49	-6.4(6)	-4.02	
$\Xi^{*0} - \Xi^{*-}$	0.42	-2.90(99)	-3.01	
$\pi^+ - \pi^0$	3.35	+4.604(4)	+1.61	+4.0
$\rho^\pm - \rho^0$	≈ 0	-0.3(22)	+0.94	+0.7
$K^\pm - K^0$	0.81	-4.01(13)	-1.62	-4.2
$K^{*+} - K^{*0}$	0.29	-4.1(6)	-1.11	-1.6

* Bag Model calculation (Ref.28)

** Potential model calculation (Ref.29)

First order perturbation theory leads to the simple result³⁰ that

$$m(T_3) = a + bT_3 + cT_3^2 \quad (1.8)$$

where a , b , and c are constants for an isospin multiplet. No higher order terms are expected if CI is assumed. Thus the empirically determined presence of a cubic term dT_3^3 would indicate CI violation. Detailed studies of $T=3/2$ quartets and $T=2$ quintets³¹ have been done but generally no statistically significant d-terms have been found. Thus, for example, for the $A=37$ quartet the value

$$d/b = (0.8 \pm 2.1) \times 10^{-3}, \quad (1.9)$$

which is much smaller than $(\overline{Z}\alpha)^2 = 18 \times 10^{-3}$.

Quantitative calculations of Coulomb energy have been made for nuclei from $A=3$ to $A=41$ within the framework of their known nuclear structure. The simplest of these is of course the ${}^3\text{H}$ - ${}^3\text{He}$ doublet. The experimental beta-decay energy of ${}^3\text{H}$ is $-18.594(8)$ keV, which is equivalent to a difference of $763.745(11)$

Table 1.3: Coulomb energy shift and correction terms (in MeV) (from Ref. 33).

Contribution	$^{15}\text{O}-^{15}\text{N}$	$^{17}\text{F}-^{17}\text{O}$		$^{39}\text{Ca}-^{39}\text{K}$	$^{41}\text{Sc}-^{41}\text{Ca}$	
	$1p_{1/2}^{-1}$	$1d_{5/2}$	$2s_{1/2}$	$1d_{3/2}^{-1}$	$1f_{7/2}$	$2p_{3/2}$
Direct Coulomb	+3.420	+3.390	+3.050	+7.050	+6.850	+6.550
Exchange Coulomb	-0.210	-0.190	-0.150	-0.300	-0.250	-0.210
C.M. motion	-0.070	-0.070	-0.070	-0.040	-0.040	-0.040
Proton size	+0.065	+0.095	+0.085	+0.090	+0.110	+0.100
Neutron size	-0.035	-0.035	-0.035	-0.040	-0.040	-0.040
Magnetic interaction	+0.100	-0.060	+0.025	+0.130	-0.090	-0.020
Vacuum polarization	+0.020	+0.020	+0.020	+0.040	+0.040	+0.040
p-n mass difference	+0.025	+0.035	+0.035	+0.025	+0.035	+0.035
short range correlations	+0.065	+0.045	+0.050	+0.125	+0.075	+0.105
Total(calculated)	+3.38	+3.23	+3.01	+7.08	+6.69	+6.52
Experimental	+3.54	+3.54	+3.17	+7.30	+7.28	+7.05
Discrepancy	+0.16	+0.31	+0.16	+0.22	+0.59	+0.53

keV between the binding energies of ^3H and ^3He . This is about 10% of the total binding energy of either nucleus. The best calculations are only able to account for 683 ± 29 keV of this difference as due to the Coulomb energy of ^3He . The unexplained 81 ± 29 keV, i.e., 1% of the binding energy is often quoted as evidence for CS breaking and or three-body forces.

For heavier nuclei detailed Coulomb energy calculations have been done by many authors. The first of these revealed³² that the calculated Coulomb energies always fall short of the experimental ones. This is the well known Nolen-Schiffer anomaly. A detailed review of the subject has been made by Schlomo³³. Table 1.3, taken from him, summarizes the present situation. Calculations still fall short of the experimental Coulomb energies, by about 7%. We will not go here into the speculations about the possible causes.

1.5.2 Isospin Mixing

Forces which violate isospin conservation mix states with different isospins. At the elementary particle level this should produce mixing between $\pi^0(t=1, m=135 \text{ MeV})$ and $\eta^0(t=0, m=549 \text{ MeV})$, and between $\rho^0(t=1, m=770 \text{ MeV})$ and $\omega^0(t=0, m=783 \text{ MeV})$. The π^0 - η^0 mixing is expected to be small because of the large difference between their masses. On the contrary, the ρ^0 - ω^0 mixing is known to be large. The branching ratio for the decay $\omega^0 \rightarrow \rho^0(\pi^+\pi^-) = 1.4 \pm 0.2\%$.

At the nuclear level isospin mixing is well known. One way of measuring isospin impurity is to measure isospin forbidden Fermi $\beta^\pm$ decays. As Fig. 1.3 (from Ref. 34) shows, the matrix elements for these decays are $\sim 10 \text{ keV}$. Since typical separation between $T(\text{upper})$ and $T(\text{lower})$ states are about 5 MeV , a measure of the isospin violation is $\sim 10 \text{ keV} / 5 \text{ MeV} \approx 0.2\%$.

Figure 1.3: Isospin mixing matrix elements inferred from isospin forbidden Fermi $\beta^\pm$ decays.(from Ref. 34)

In favourable cases large isospin mixing can result. Almost pure neutron and almost pure proton states (which correspond to 50-50 isospin admixtures) have been observed. Some of the well known cases are illustrated in Table 1.4 due to Adelberger.³⁴ The model calculation referred to in this table is one in which the whole mixing is due to single particle Coulomb energy differences. It is somewhat surprising, but gratifying that even such a simple calculation explains the observed

Table 1.4: Isospin mixed doublets (from Ref. 34).

Nucleus		$E_x(\text{MeV})$	$-H_{\text{exp}}^c(\text{KeV})$	$-H_{\text{model}}^c(\text{KeV})$
^8Be	2^+	$\left\{ \begin{array}{c} 16.6 \\ 16.9 \end{array} \right\}$	145	88
		$\left\{ \begin{array}{c} 12.7 \\ 15.1 \end{array} \right\}$		
^{12}C	1^+	$\left\{ \begin{array}{c} 4.9 \\ 8.8 \end{array} \right\}$	130 ± 26	125
		$\left\{ \begin{array}{c} 5.8 \\ 8.9 \end{array} \right\}$		
^{14}N	0^-	$\left\{ \begin{array}{c} 7.1 \\ 13.1 \end{array} \right\}$	$395 \pm 74^{(a)}$	362
^{14}N	3^-	$\left\{ \begin{array}{c} 12.5 \\ 13.0 \end{array} \right\}$	≤ 103	154
		$\left\{ \begin{array}{c} 8.4 \\ 9.5 \end{array} \right\}$		
^{16}O	1^-	$\left\{ \begin{array}{c} 8.4 \\ 9.5 \end{array} \right\}$	591 ± 54	494
^{16}O	2^-	$\left\{ \begin{array}{c} 12.5 \\ 13.0 \end{array} \right\}$	$\geq 155 \pm 30$	163
		$\left\{ \begin{array}{c} 8.4 \\ 9.5 \end{array} \right\}$		
^{24}Mg	4^+	$\left\{ \begin{array}{c} 8.4 \\ 9.5 \end{array} \right\}$	106 ± 40	71

^a Experiment does not fix the sign of H^c but it is expected to be negative.

mixing essentially perfectly.

Now we wish to discuss results of experiments designed for the specific purpose of measuring CI and CS breaking effects. For the purposes of our presentation, we will first consider the weaker principle, CS, and then CI.

There are several excellent reviews of CS and CI breaking.^{10–12,35} They contain detailed descriptions of the experimental data. We will therefore not repeat these descriptions. Instead we will only describe the latest experiments and their impact on our understanding of CS and CI breaking.

1.6 Experimental evidence for charge symmetry.

We will begin by reviewing the latest experiments designed with the specific purpose of testing the weaker principle, namely charge symmetry.

1.6.1 Nucleon – Nucleon scattering.

At low energies nucleon–nucleon scattering takes place almost entirely in the $\ell=0$ partial wave. One can therefore compare the nn, pp, and np scatterings in the $T=1$, 1S_0 state. This comparison is usually done within the framework of the effective range theory.³⁶ Accordingly, the phase shift δ_0 ($\sigma = 4\pi/k^2 \sin^2 \delta_0$) is given by

$$k \cot \delta_0 = \left(-\frac{1}{a}\right) + \frac{1}{2}rk^2 \quad (1.10)$$

where k is the momentum in the center of mass, ‘ a ’ is the scattering length, and ‘ r ’ is the effective range. Because of the near-binding of the 1S_0 state, δ_0 is very close to 90° , and a very small change in the nuclear force results in a very large change in the scattering length (a 1% change in the potential produces ~ 3 fm, or $\sim 18\%$ change in ‘ a ’). Thus the effect of CS and CI breaking is greatly magnified.

In table 1.5 we present the latest results for low-energy nucleon–nucleon scattering. The pp scattering data^{37,38} which was of superb quality already in 1966, re-

Table 1.5: Low energy nucleon-nucleon scattering parameters: effective lengths and effective ranges

	$a(\text{fm})$	$r(\text{fm})$
pp ³⁹	-7.828 ± 0.008	2.80 ± 0.03
pp(corrected for electromagnetic effects)	-17.2 ± 2.0	2.84 ± 0.16
np ³⁶	-23.715 ± 0.015	2.73 ± 0.03
nn ⁴³	-18.6 ± 0.5	2.83 ± 0.11

mains unchanged. There is, however, an improved evaluation of the parameters³⁹. These are: $a_{pp} = -7.828(8)$ fm, $r_{pp} = 2.80(2)$ fm. The main problem concerns Coulomb and other electromagnetic corrections which must be applied to these.

Sauer and Walliser⁴⁰ showed that these corrections depend sensitively on the unknown short-range nuclear force, and reached quite pessimistic conclusions about the uncertainty in these corrections. Even if we temper these conclusions with the criteria of reasonableness⁴¹ and of keeping relativistic effects small⁴², the model dependence of these corrections introduces an error of ~ 2.0 fm in a_{pp}^c . Thus the best results one can quote are: $a_{pp}^c = -17.2 \pm 2.0$ fm, and $r_{pp}^c = 2.84 \pm 0.03$ fm.

The best estimate of nn scattering length to date comes from the absorption of stopped pions on deuterium $\pi^- d \rightarrow nn\gamma$. In this reaction one takes advantage of the fact that in the final state the two neutrons are the only strongly interacting particles. Near the maximum of γ -ray energy, these neutrons have very small relative energy, and one can deduce a_{nn} and r_{nn} from their final state interaction. By doing a high-precision, high statistics experiment Gibioud et al⁴³ have been able to obtain $a_{nn} = -18.6 \pm 0.5$ fm, and $r_{nn} = 2.83 \pm 0.11$ fm. The analysis of these data does not suffer from the problems mentioned with the pp data.

There is report of a direct nn scattering experiment which is under preparation by a Los Alamos–Oak Ridge collaboration⁴⁴ in which an attempt will be made to measure nn scattering cross section by colliding two beams of 14 MeV neutrons from two simultaneously detonated fusion–fission devices (euphemism for bombs). The beams are meant to collide at 3.8° so that a c.m. collision energy of 38 keV is realized for the 14 MeV neutrons. It is claimed that a 10 % measurement of cross sections will determine a_{nn} to better than 3% or ± 0.5 fm. This would certainly be great. It is reported that on April 6, 1986, the first stage of this experiment, i.e. feasibility test for simultaneous detonation, was made. We look forward to the definitive experiment in 1988.

From the best present values of a_{nn} and a_{pp}^c we obtain

$$\Delta a = a_{nn} - a_{pp}^c \simeq -1.4 \pm 2.0 \text{ fm} \quad (1.11)$$

The difference is thus not significant.

1.6.2 Pion – Deuteron Total Cross Sections.

Pedroni et al.⁴⁵ have made high precision measurement of π^-d and π^+d total cross sections in the pion energy range 70 – 370 MeV. After correcting their results for the direct effects of the Coulomb potential, they find energy dependent differences, ranging from -6% to $+2\%$, between π^-d and π^+d cross sections (see Fig. 1.4).

Figure 1.4: Difference in $\sigma_t(\pi^-d)$ and $\sigma_t(\pi^+d)$. (from Ref. 45)

They interpret these differences as being due to charge symmetry violation and parameterize them in terms of mass difference, $m(\Delta^0) - m(\Delta^{++}) = 2.7 \pm 0.3$ MeV, or $\sim 0.2\%$, and width difference, $\Gamma(\Delta^0) - \Gamma(\Delta^{++}) = 6.6 \pm 1.0$ MeV, or 6.0% .

1.6.3 Pion - Deuteron Elastic Scattering.

After removal of electromagnetic effects π^+d and π^-d elastic scattering differential cross sections should be equal if charge symmetry holds. Balestri et al.⁴⁶ have measured these cross sections at $T(\pi)=65$ MeV, and Masterson et al. have measured them at $T(\pi)=142$ MeV⁴⁷ and 256 MeV.⁴⁸ Balestri et al. found statistically significant and rather large asymmetries, $A(\theta) = (\sigma(\pi^-) - \sigma(\pi^+)) / (\sigma(\pi^-) + \sigma(\pi^+))$ in their 65 MeV data. On the other hand, Masterson et al. found angle dependent asymmetries in the range $A=(2.5\pm 2.5)\%$ at 142 MeV and $A=(0\pm 2)\%$ at 256 MeV. All the observed effects appear to be quite satisfactorily accounted for by external Coulomb corrections⁴⁹. (see Fig. 1.5)

Figure 1.5: Asymmetry $A(\theta)$ in % for π^+ , π^- elastic scattering cross sections (from Ref. 49). The figure on left is for $T(\pi)=65$ MeV, the one on right is for $T(\pi)=142$ MeV.

1.6.4 $\pi-^3\text{H}$ and $\pi-^3\text{He}$ Scattering.

Charge symmetry requires that the elastic scattering ratios

$$r_1 = \frac{d\sigma(\pi^+ + ^3\text{H})}{d\sigma(\pi^- + ^3\text{He})} \quad (1.12)$$

and

$$r_2 = \frac{d\sigma(\pi^+ + ^3\text{He})}{d\sigma(\pi^- + ^3\text{H})} \quad (1.13)$$

should be separately equal to unity at all angles. Nefkens et al.⁵⁰ have noted that the super-ratio, $R = r_1/r_2$, which should also be unity, has the great advantage that for its determination no absolute cross section normalizations are required and target and beam characteristics mostly cancel out. In their first measurement at $T(\pi)=180$ MeV, Nefkens et al. found that the super ratio varied with angle and reached a maximum of $R=1.31\pm0.09$ at 65° (see Fig. 1.6). This caused a flurry of excitement. Kim⁵¹ rushed in to explain the observations as “manifestation of multiquark compound resonances in interacting hadronic systems”, and Barshay and Sehgal⁵² explained the observations in terms of short-range correlations in the three nucleon system, which are distorted in going from ^3H to ^3He . In the meanwhile, Nefkens et al. have done a more extensive series of measurements⁵³ in which they have measured the super ratio at five angles 40° , 60° , 80° , 90° , and 110° at $T(\pi)=142$, 180 , and 220 MeV. The average of all measurements is found to be $R=1.06(5)$ at 220 MeV, $R=1.09(4)$ at 142 MeV, and $R=1.10(5)$ at 180 MeV. In the $40^\circ - 80^\circ$ interval not a single value larger than $1.11(5)$ is found. At this level it appears entirely likely that external Coulomb effects will explain the observations.

Figure 1.6: Ratios for $\pi^-^3\text{H}(^3\text{He})$ elastic scattering. $r_1 = d\sigma(\pi^+ + ^3\text{H})/d\sigma(\pi^- + ^3\text{He})$, $r_2 = d\sigma(\pi^+ + ^3\text{He})/d\sigma(\pi^- + ^3\text{H})$, $R = r_1/r_2$. (from Ref. 50)

Figure 1.7: Most recent results for the super-ratio R defined in Fig. 1.6 (from Ref. 53)

1.6.5 π - ${}^4\text{He}$ Scattering.

Charge symmetry requires that π^+ and π^- elastic scattering cross sections for ${}^4\text{He}$ should be identical. Recently, Khankhasayev et al.⁵⁴ have done simultaneous phase shift analysis of the available data for $\pi^\pm$ scattering from ${}^4\text{He}$ at $T(\pi)=24$ MeV, 51 MeV, and 75 MeV. They obtain “quite a good description of the angular dependence of the asymmetry parameter $A(\theta)$ by taking account of external Coulomb corrections”. (see Fig. 1.8) They find no significant charge symmetry breaking effects.

Figure 1.8: Asymmetry $A(\theta)$ for $\pi^\pm$ elastic scattering from ${}^4\text{He}$ (from Ref. 54).

At LAMPF experiments are currently in progress to make precision measurements of $\pi^\pm$ ${}^4\text{He}$ elastic scattering in the resonance region.⁵⁵

1.6.6 The Reaction $d + d \rightarrow {}^4\text{He} + \pi^0$.

Obviously this reaction is forbidden by isospin conservation. However, because only self conjugate 'particles' are involved on both sides it only tests charge symmetry, and not charge independence. This is a difficult experiment, but much progress has been made recently. At Saclay Stetz et al.⁵⁶ have been able to establish upper limits of $d\sigma/d\Omega < 5$ pb/sr (at $T_d = 1350$ MeV, $\theta_{cm} = 77^\circ$) and $d\sigma/d\Omega < 0.8$ pb/sr (at $T_d = 800$ MeV, $\theta_{cm} = 100^\circ$). These limits are 142% and 17% respectively of the corresponding cross sections for the isospin allowed reaction $d+d \rightarrow \alpha + \gamma$. Thus although we have come a long way from the 1100 pb/sr limit established by Akimov et al.⁵⁷ in 1962, we are still far from the level of sensitivity required to see charge symmetry breaking effects. As a matter of fact Cheung⁵⁸ has predicted cross sections of the order of 0.03 pb/sr at $T_d = 700$ MeV, $\theta = 0$. Coon and Priedom⁵⁹ predict a forward differential cross section of the order of 0.12 pb/sr at $T_d=1.95$ GeV.

1.6.7 The Reaction $np \rightarrow d\pi^0$.

The differential cross sections for the $T=1 \rightarrow T=1$ reaction $pp \rightarrow d\pi^+$ are symmetric about $\theta_{(c.m.)} = 90^\circ$. If isospin is conserved, the reaction $np \rightarrow d\pi^0$ can only proceed through the $T=1 \rightarrow T=1$ channel and its cross sections must also be symmetric about 90° . Departures from symmetry about 90° therefore provide a measure of isospin symmetry breaking. The latest experiment testing the forward-backward symmetry of $np \rightarrow d\pi^0$ cross sections was done by Hollas et al.⁶⁰ who found the asymmetry parameter $A = -0.15 \pm 0.50$ at $T(n) \approx 795$ MeV. Cheung et al.⁶¹ have predicted $A = -0.11$ at this energy, primarily due to π^0 - η mixing. It would be very useful indeed if the errors on the experimental measurement could be reduced.

1.6.8 Neutron-Proton Elastic Scattering.

As early as 1956, Wolfenstein⁶² noted that in n-p elastic scattering the polarization of the two outgoing particles should be equal:

$$P_n(\theta) = P_p(\pi - \theta) \quad (1.14)$$

This follows from parity conservation and the generalized Pauli principle or the overall asymmetry of the space-spin-isospin wave function. This implies that only like spin states are connected in the initial and final states (i.e., singlet to singlet, triplet to triplet), i.e., the spin dependent part of the transition operator must be even under particle exchange. This gives equal polarization for n and p. The same relation holds for analyzing power if polarized beams or targets are used. For example,

$$\Delta A = A(\vec{n}p, \theta) - A(n\vec{p}, \theta) = 0 \quad (1.15)$$

If charge symmetry is broken, the triplet-singlet transition amplitude f_{ts} is non-zero and leads to non-zero ΔA . As a matter of fact ΔA is directly proportional to f_{ts} . The ideal experiment of this type would consist of scattering polarized neutrons from a polarized proton target. A single experiment could provide ΔA by forming the two analyzing powers, $A(n\vec{p})$ by summing over all the neutron spin directions and $A(\vec{n}p)$ by summing over all the proton spin directions.

Two different experiments have been performed recently. At TRIUMF Abegg et al.⁶³ have made a precision measurement of the difference between the angles at which $A(n\vec{p})$ and $A(\vec{p}n)$ have zero value for $T_n=477$ MeV. The measured difference

$$\Delta\theta = -0.13^\circ \pm 0.07^\circ (\pm 0.03^\circ)$$

is equivalent to

$$\Delta A = 0.0034 \pm 0.0017 \pm (0.0008) \quad (1.16)$$

at that angle. This compares well with the theoretical result of

$$\Delta A = +0.0054 \pm 0.0040 \quad (1.17)$$

due to Miller, Thomas and Williams⁶⁴. In this calculation charge symmetry breaking terms due to one photon exchange, n-p mass difference affecting π , ρ , and 2π exchanges, $\rho - \omega$ mixing, and effects of the up-down quark masses are included. We note that an earlier prediction for ΔA gave a negative result ($\Delta A = -.0035$) due to an error in the sign of the one pion exchange contribution.

The Indiana experiment⁶⁵, which is still under preparation, differs from the TRIUMF experiment in several ways. The experiment will be done at $T_n=200$ MeV with beam and target simultaneously polarized. In this case the left-right asymmetries will be measured at a number of angles ($75^\circ \leq \theta_{cm} \leq 120^\circ$). They should be zero at all angles when the entrance channel neutron and proton spin projections are opposite. It is claimed that the sensitivity to ΔA will be at the level of 0.001.

1.7 Charge Independence

As mentioned earlier, charge independence is the stronger symmetry. However, compared to charge symmetry breaking, many fewer tests of charge independence breaking have been made.

1.7.1 Low Energy np Scattering

Together with the nn and pp scattering lengths, a measurement of the np scattering provides one of the best tests of charge independence. As listed in Table 1.6, the np singlet scattering length is $a_{np} = -23.72 \pm 0.02$ fm. This determination does not suffer from the uncertainties associated with a_{nn} and a_{pp} , and leads to

$$\Delta a = a_{np} - (a_{nn}, a_{pp}) = (-23.72 \pm .02) - (-18.3 \pm 0.5) = -5.4 \pm 0.5 \text{ fm} \quad (1.18)$$

This large difference, amounting to $26 \pm 3\%$, is the basis of the statement that CI is quite evidently broken. We must, of course, remember that this charge independence breaking due type II forces amounts to

$$|\bar{V}_{np}| - |\bar{V}_{nn}| / (|\bar{V}_{np}| + |\bar{V}_{nn}|) / 2 = (2.1 \pm 0.5)\% \quad (1.19)$$

We should also note that one pion exchange alone provides a mechanism for $V_{np} > V_{nn}$ due to $\pi^\pm, \pi^0$ mass difference Δm

$$V_{np} = V_{nn}[-1 + 2(\frac{m^\pm}{m_c})^2 \exp(-\Delta m \cdot r)] \quad (1.20)$$

This contributes about half of the observed difference in scattering lengths. Recently Cheung and Machleidt⁶⁶ have considered Δa within the framework of a consistent meson model of NN interactions. They are able to account for essentially all of the observed difference Δa (see Table 1.7).

It should be noted that the s-wave ($l = 0$) scattering lengths do not permit any insight into Henley's type IV charge independence breaking forces. For these we need to go to a polarization observables.

Table 1.6: Theoretical resolution of the contributions to $\Delta a = a_{np} - \langle a_{nn}, a_{pp} \rangle$ (from ref.66).

Potential	$\delta a(fm)$
OPEP	1.80
TPEP (NN)	0.18
TPEP (N Δ)	3.25
TPEP ($\Delta\Delta$)	-2.58
$\pi\rho$ -exchange (NN)	-0.39
$\pi\sigma, \pi\omega$ -exchanges	1.18
Total	4.44

1.7.2 Pion-nucleon total cross section

If we decompose pion-nucleon total cross sections into cross sections in pure isospin states, as is well known:

$$\sigma(\pi^-p) = \frac{1}{3}\sigma_{3/2} + \frac{2}{3}\sigma_{1/2} \quad (1.21)$$

$$\sigma(\pi^+p) = \sigma_{3/2} \quad (1.22)$$

It follows therefore that according to charge independence:

$$3\sigma(\pi^-p) - \sigma(\pi^+p) = 2\sigma_{1/2} \quad (1.23)$$

$\sigma_{1/2}$ is of course not zero, but at the peak of the (3,3) resonance its contribution is very small. In any case, the quantity $\Delta\sigma/\sigma \equiv [3\sigma(\pi^-p) - \sigma(\pi^+p)]/\sigma(\pi^+p)$ evaluated at the peak of the (3,3) resonance establishes the upper limit of charge independence violation. As Fig. 1.9 shows this upper limit is $\approx 3\%$. Actually $\sigma_{1/2}$ is rather well known from phase shifts analyses, and it can be shown that the CI violation implied by the data is considerably smaller than the upper limit derived above.

Figure 1.9: Percent deviation $[3\sigma(\pi^-p) - \sigma(\pi^+p)]/\sigma(\pi^+p) \cdot 100$.

1.7.3 The Reactions $p+d \rightarrow \pi^+ + {}^3\text{H}$ and $p+d \rightarrow \pi^0 + {}^3\text{He}$

Ruderman⁶⁷ was probably the first to suggest the use of these reactions for testing charge independence. If charge independence is assumed, the ratio

$$R = \sigma(p + d \rightarrow \pi^+ + {}^3\text{H}) / \sigma(p + d \rightarrow \pi^0 + {}^3\text{He}) = 2.0$$

a value obtained easily from the isospin Clebsch-Gordon coefficients. Under the hypothesis that the transition matrix associated with these two reactions does not depend upon isospin quantum numbers, R is given by:

$$R = \left(\frac{\langle 1/2, 1/2 | 1, 1/2, 1, -1/2 \rangle}{\langle 1/2, 1/2 | 1, 1/2, 0, 1/2 \rangle} \right)^2 = \left(\frac{\sqrt{2}/\sqrt{3}}{-1/\sqrt{3}} \right)^2 = 2 \quad (1.24)$$

where the Clebsch-Gordon coefficients have been give according to the notation $\langle j, m | j_1, j_2, m_1, m_2 \rangle$.

Similarly, the analyzing power of the two reactions should be equal, i.e.,

$$\Delta A = A_{\nu_0}(\theta, {}^3\text{H}) - A_{\nu_0}(\theta, {}^3\text{He}) = 0$$

at all angles. It is obvious, but it seems not to have been noticed before that this difference of analyzing powers should be a particularly sensitive measure of the type IV charge independence breaking force of Henley, since it is a spin dependent force.

Several measurements of R have been made before. Their results range from 2.05 to 2.58. No measurements of ΔA have been reported in the literature before the ones which form the subject of the present dissertation. We postpone a detailed discussion of this reaction to a later section.

From the above review of the many experiments done to test isospin invariance we notice that most tests relate to the weaker invariance, namely charge symmetry. Few tests have ever been done to test charge independence. The one precision test which has been done, and which does reveal charge independence breaking unambiguously relates to a very special situation. It measures the difference in ${}^1\text{S}_0$ scattering lengths of the nn (or pp) and the np systems. It sheds no

light on the type IV charge independence breaking forces. There are essentially no precision experiments at intermediate energies which address the question of charge independence. This is the question we have chosen to address in this dissertation.

1.8 Criteria for choice of an experiment.

In the attempt to design an experiment which would give meaningful information about charge independence, there are two main considerations which affect the choice. The first is theoretical in nature and has already been touched upon in a previous section. It is worthwhile, however, to consider it in some more detail, as it plays a crucial role in the correct interpretation of the experimental data. The idea behind each experiment is to find an unambiguous signature of the isospin nonconservation in the strong sector of the interaction between hadrons. Whatever observable is chosen, the expectation is that, even if an effect is present, it is bound to be small. On the one hand it is suggested by all the evidence that in nuclear physics isospin symmetry 'works'. On the other hand, the large rescaling of the quark masses, which takes place upon hadronization, suggests too that, accidental as it may be, isospin has a good chance of surviving as an almost good symmetry even in a reformulation of nuclear theory in terms of the dynamics of its elementary constituents. In this connection, a very simple argument gives some hints about the problems associated with the correct interpretation of the data. The strength associated with the electromagnetic interaction is of the order of its coupling constant, α , i.e. $\approx 1\%$. If we take as a scale of the isospin breaking at a 'elementary' hadronic level the fractional difference between the constituent quark masses $(m_u - m_d)/\langle m \rangle$, we obtain a number of the same order of magnitude, i.e. $\approx 1\%$. However, the two interactions have different dependence upon the energy and momentum variables of the particles involved. In an experimental observable the strong and the electromagnetic effects do not simply add incoherently, but also produce an interference term. In absence of a fundamental theory for the observable, which would enable one to calculate both the Coulomb and the Coulomb-strong interference contributions quantitatively, the safest approach to handle Coulomb contributions is to try to observe phenomena in which Coulomb effects are expected to be small. If this condition is satisfied, a wrong estimate of

the interference term, or even its neglect, will not affect the interpretation of the data significantly.

From the pure experimental point of view, the requirements are those generally imposed whenever the evidence for a small effect is sought by means of a comparison between two different reactions. Small differences between the observables measured in the two cases should tell us something about symmetry breaking. Therefore a careful analysis of all the possible sources of bias introduced in the final results by the experimental method employed is needed. To this purpose, the simplest approach to minimize systematic effects is to make as few changes as possible in the experimental conditions in the two measurements to be compared. The best choice would be one which involves a comparison between two reactions which have the same initial state and which have final states whose detection involves a minimal change in the equipment used.

These considerations lead us to the experiment which is the subject of this dissertation. It involves the comparison between the two reactions:

$$p + d \longrightarrow {}^3\text{He} + \pi^+ \quad (1.25)$$

and

$$p + d \longrightarrow {}^3\text{He} + \pi^0 \quad (1.26)$$

The observables to be measured are the cross section ratio:

$$R(\theta) = \frac{d\sigma(\theta, {}^3\text{He})/d\Omega}{d\sigma(\theta, {}^3\text{He})/d\Omega} \quad (1.27)$$

and the analyzing power difference:

$$\Delta A_{p0}(\theta) = A_{p0}(\theta, {}^3\text{He}) - A_{p0}(\theta, {}^3\text{He}), \quad (1.28)$$

where the analyzing power is defined as:

$$A_{p0}(\theta) = \frac{\sigma_{\uparrow} - \sigma_{\downarrow}}{\sigma_{\uparrow} + \sigma_{\downarrow}} \quad (1.29)$$

Our primary emphasis is on a precision measurement of $\Delta A_{\nu 0}$, because of its expected sensitivity to the type IV CI breaking interactions. The advantages of choosing the ratio R as the observable to be measured have already been discussed. This choice was made by all the previous experiments. $\Delta A_{\nu 0}$ was not considered before because polarized proton beams were not available to most of the previous experiments. In many respects $\Delta A_{\nu 0}$ provides a much more suitable observable for a high precision test. In fact, since $A_{\nu 0}$ is a ratio between cross sections, it is relatively insensitive to beam normalization and absolute efficiency of the detector system. Many additional uncertainties, such as variations in beam polarization, cancel out or introduce much smaller errors in the quantity $\Delta A_{\nu 0}$ if the measurements of the two reactions are alternated frequently.

1.8.1 Choice of the incident energy.

In order to facilitate an eventual theoretical explanation of the charge independence breaking effects observed, it is advisable to choose the parameters of the experiment in such a manner as to minimize the effects of both Coulomb and hadronic distortions in both the incoming and outgoing channels. Coulomb distortions in both channels and hadronic distortions in the incoming channel require that the experiment be done at as high an energy as possible commensurate with the other objectives of the experiment. However, because of the resonant πN interaction in the outgoing channel, it is best to avoid outgoing pion energies near the (3,3) resonance, i.e., in the region 180 ± 25 MeV. This could be done by choosing $T_p \leq 500$ MeV or ≥ 700 MeV. Clearly $T_p \geq 700$ MeV is to be preferred because it also minimizes Coulomb effects and incident channel hadronic distortions. Our experiment was done at $T_p = 730$ MeV, which was the highest energy at which a polarized proton beam was made available to us at LAMPF. At this energy the outgoing pion energy was 216 ± 2 MeV. Several of the earlier experiments were done with pion energies much closer to 180 MeV.

1.8.2 Choice of the kinematical point.

In order to do a precision experiment one likes to work at a kinematical point at which derivatives with respect to the many experimental parameters are at or near their minima. This requirement is generally impossible to satisfy and compromises have to be made. For example, one would like to study the reactions in question at angles where $dT_3/d\theta$ is a minimum in order to have the best energy resolution. This would dictate the choice of a forward angle. However forward angles are bad because of many reasons. From a theoretical point of view, they are bad because they maximize Coulomb effects. From the experimental point of view, they are undesirable because it is at forward angles that both the differential cross sections and the analyzing powers change rapidly both with incident energy and angle (see Figs. 1.10 and 1.11). Thus, for example, at $\theta_{cm}(\pi) \approx 7^\circ$, $d\sigma/d\Omega$ changes almost twice as fast with energy ($\approx 0.7\%$ per MeV) as it does at $\theta_{cm}(\pi) = 130^\circ$ ($\approx 0.4\%$ per MeV). The angular dependence of the cross sections is of course much weaker at back angles. For example at 800 MeV (see Fig. 1.10), for angles $\theta(\pi) < 45^\circ$ the cross sections fall at the rate of more than 20% per degree whereas at $\theta(\pi) = 130^\circ$, the rate of change is $< 0.5\%$ per degree. Similar observations can be made about $A_{p0}(\theta)$ (see Fig. 1.11). It is obvious that back angles around $\theta(\pi) \approx 130^\circ \pm 10^\circ$ are to be much preferred over forward angles from the point of view of minimizing effects due to the stability of the beam energy and direction.

Considerations of particle identification also impose strict conditions on the choice of angles. In Figs. 1.12–1.16, we display the detected particle magnetic rigidity, momentum/ z , for several experiments. We notice that while the ^3He rigidities are such that they do not interfere with any other heavy particles (except with pions and highly inelastic scatterings), triton rigidities are in the same general region as the elastically scattered protons and recoil deuterons, both of which have prolific yields. One must therefore avoid the vicinity of the crossing points at which these contaminant particles will flood the magnetic spectrometer and do havoc to

even the best particle identification system. We note parenthetically, that one of the experiments reported in the literature⁸⁰ was run almost precisely at such an undesirable angle at which elastic protons would overwhelm tritons in the focal plane of the spectrometer and another experiment⁷⁸ was run at an angle at which pion yield from the $pp \rightarrow d\pi^+$ reaction (from quasi free protons in deuterium) would compete in the focal plane with ^3He particles. In our choice of angle ($\theta_L = 12^\circ$) we have taken care to be equally far away from protons and deuterons from elastic scattering (see Fig. 1.16). Fortunately, this choice also keeps us far away from pions from both the $pp \rightarrow d\pi^+$ and $pd \rightarrow t\pi^+$ reactions.

Figure 1.10: Differential Cross section for the reaction $p + d \rightarrow {}^3\text{He} + \pi^+$. The data are from Refs. 68-72 and 90.

Figure 1.11: Analyzing power for the reaction $p + d \rightarrow {}^3\text{He} + \pi^+$ at $T_p = 700$ MeV. (from Ref.90)

1.8.3 Choice of the target.

Apart from the choice of incident proton energy and outgoing particle angle, there are several other criteria that should be kept in mind in such a measurement.

The first of these is the choice of a target. In order to minimize the yield from undesirable reactions it is necessary to keep material in the target, other than deuterium, at a minimum. It can be easily shown that this immediately rules out the use either of a solid target (for example CD_2) or a gaseous target at room temperature and pressure (container wall too thick compared to D_2 content in the target). Of the two possibilities super-cooled D_2 gas at high pressure or liquid deuterium, liquid deuterium is preferable because it gives a much better ratio of deuterium to container material. LD_2 is indeed the target of choice in most experiments, including ours.

Ideally the target should be as thin as possible in order to keep to a minimum the difference in energy losses suffered by ^3H and ^3He as they exit from the target. In practice this requirement is compromised by the mechanical design of the LD_2 target as well as the necessity to keep LD_2 /container mass ratio high, and the signal counting rates adequate for the statistical accuracy desired. Target thicknesses used in different experiments range from $\approx 1\text{cm}$ to $\approx 3\text{cm}$. The thickness of our LD_2 target was intended to be $\approx 1\text{cm}$ but ended up being approximately 2.5 cm.

Figure 1.12: Kinematics for the reaction $p d \rightarrow {}^3H({}^3He)\pi^+(\pi^0)$ as measured by Crewe et al.(Ref. 75, 76).

Figure 1.13: Kinematics for the reaction $p d \rightarrow {}^3H({}^3He)\pi^+(\pi^0)$ as measured by Harting et al.(Ref. 77, 78).

Figure 1.14: Kinematics for the reaction $p d \rightarrow {}^3H({}^3He)\pi^+(\pi^0)$ as measured by Booth et al. (Ref. 79)

Figure 1.15: Kinematics for the reaction $p d \rightarrow {}^3H({}^3He)\pi^+(\pi^0)$ as measured by Silverman et al. (Ref. 80)

Figure 1.16: Kinematics for the reaction $p d \rightarrow {}^3H({}^3He)\pi^+(\pi^0)$ for the present experiment.

1.9 Survey of previous measurements of the ratio R .

We now wish to discuss the several measurements of R which have been reported in the literature (see Table 1.7). We shall examine these experiments critically in order to determine how we can improve our own experiment.

1. Bandtel, Frank and Moyer (Berkeley,1957)⁷⁴

This experiment, done with the 340 MeV proton beam of the Berkeley 184 inch synchrotron and a LD_2 target, was the forerunner of all such experiments. It is also the only experiment that did not have the advantage of a magnetic spectrometer. With no momentum selections available dE/dx -E scintillation counters provided minimally successful particle identification. The result was that the authors could only conclude that R was "between 2.0 and 2.6".

2. Crewe et al. (Chicago,1959-60)^{75,76}

This was the first experiment⁷⁷ to use a magnetic spectrometer. Using the 450 MeV proton beam from the Chicago synchrocyclotron and CD_2 and C targets these authors measured differential cross sections for the two reactions at $\theta_L(^3\text{H},^3\text{He})$ between 8° and 13° . A weighted mean of $R=1.91\pm 0.25$ was obtained. The errors are presumably only statistical.

The experiment was refined and repeated⁷⁶ at $\theta_L = 12^\circ$ to obtain $R[\theta_{cm}(\pi)=140^\circ] = 2.17\pm 0.13$. The earlier results at other angles were reanalyzed to obtain $R(135^\circ)=2.08\pm 0.22$ and $R(145^\circ) = 2.34\pm 0.37$. The quoted errors are essentially statistical.

From our experience it would appear that there are two main problems with this experiment. The first is the relative inadequacy of the dE/dx -E system alone to do particle identification at the required level. The second difficulty relates to the method used to determine the momentum acceptance of the detection system. No account of the change in acceptance between ^3H and ^3He detection appears to have been taken. Although no details are given, it would appear that the CD_2 target gave a large yield from the carbon contaminant and the C contaminant

Table 1.7: Experimental results for R.

Authors	Target	T_p MeV	T_3 MeV	T_{π^+} MeV	θ_L^3 deg	$\theta_{cm}^{\pi^-}$ deg	R	Departure from CI %
Crewe et al. ⁽⁶⁹⁾	CD ₂ -C (.8 mm) [†]	450	194	122	12.0	140.0	2.17±.13	8.5± 6.5
Harting et al. ⁽⁷¹⁾	LD ₂ (0.8,2.5 cm)	591	292	164	11.3	128.0	2.26±.11	13.0± 5.5
	LD ₂ (0.8 cm)	591	264	196	14.0	113.0 [§]	2.10±.14	5.0± 7.0
	LD ₂ (0.8 cm)	591	117	346	14.0	37.0*	2.05±.09	2.5± 4.5
Booth et al. ⁽⁷²⁾	D ₂ (gas)	743	388	221	11.7	130.0	2.00±.30	0.0±15.0
Silverman et al. ⁽⁷³⁾	LD ₂ (0.9 cm)	500	221	145	12.5	115.6	2.40±.20	20.0±10.0
			201	165	14.3	102.4 [§]	2.20±.18	10.0± 9.0
			128	238	15.5	56.8*	2.58±.34	29.0±17.0
			117	248	14.5	49.1*	2.52±.26	26.0±13.0
This Experiment	LD ₂ (2.5 cm)	730	381	214	12.0	130.0	2.19±.02	9.6± 1.1

† Large background correction due to carbon in CD₂ target.

* Forward angle measurement. Rapid change in $d\sigma/d\Omega$ with angle. Results likely to be quite unreliable.

§ Large elastic proton contamination expected.

subtraction procedure is far from ideal.

3. Harting et al. (CERN,1959–60)^{77,78}

This was the first experiment to use both a magnetic spectrometer and a liquid deuterium target. Using a 591 ± 3 MeV beam from the CERN synchrocyclotron these authors measured R . Time of flight was used as the means for particle identification. The authors note that while this provided good identification for ^3He particle, triton identification was plagued with large yield of recoil deuterons. In the first experiment at $\theta_L = 11.3^\circ$, corresponding to $\theta_{cm}(\pi) = 128^\circ$ these authors obtained $R = 2.26 \pm 0.11$. The statistical errors were only about 2.5%. Several systematic errors ranging from 1% to 4% were identified, but they were added to the statistical errors in quadrature to yield the final quoted errors of $\approx 5\%$.

In a later set of experiments⁷⁸, these authors measured R at two other angles, $\theta_L = 14^\circ$ [$\theta_{cm}(\pi) = 113^\circ$] and $\theta_L = 14^\circ$ [$\theta_{cm}(\pi) = 37^\circ$]. They report:

$$R[\theta_{cm}(\pi) = 113^\circ] = 2.10 \pm 0.14$$

$$R[\theta_{cm}(\pi) = 37^\circ] = 2.05 \pm 0.09$$

The value of $R[\theta_{cm}(\pi) = 37^\circ]$ has been scaled upwards from the measured value 1.99 ± 0.05 in order to make an estimated correction for the 0.5° difference in the c.m. angles for π^+ and π^0 .

The experiments of Harting et al. are perhaps the most carefully done experiments in the published literature so far. We note however that they introduce rather arbitrary weighting of data to account for target boiling. They suffer (particularly in the case of triton detection) from the fact that they used only time of flight for particle identification. The $\theta_{cm}(\pi) = 37^\circ$ results are the least reliable because of the large variation of $\sigma(\theta)$ with θ at this angle and the use of an estimated correction factor to compensate for the c.m. angle change. In addition it would appear (see Fig. 1.13) that excessive number of pions from the prolific reaction $pp \rightarrow d + \pi^+$ are accepted by the spectrometer at this angle. We also

note that the authors do not provide any discussion of the differential acceptance of ^3H and ^3He by the detection system.

4. Booth et al. (Berkeley,1963)⁷⁹

In this experiment $R=2.00\pm0.10$ is reported for $T_p=743$ MeV, at $\theta_L=11.7^\circ$, $\theta_{cm}(\pi)=130^\circ$. A gas target at 320 psi and liquid nitrogen temperature (77°K) was used. Particle identification was made by time of flight and dE/dx measurements. Since the main purpose of the experiment was to study $\pi-\pi$ interaction, very few details of interest are presented in the paper and it is difficult to evaluate the precision of the experiment. An error of 10% is attributed to the cross sections by the authors on the basis of consistency check between consecutive runs. Even this should lead to an error of $\pm 14\%$ in R as we have listed in Table 1.7. The authors quote a final error of 5% without giving any explanation.

5. Silverman et al. (Saclay,1984)⁸⁰

In this experiment R was measured at $T_p=500$ MeV at $\theta_{cm}(\pi) = 49.1^\circ$, 56.8° , 102.4° and 115.6° . A LD_2 target was used and particle identification was done by measuring both dE/dx and time of flight of the outgoing particles which were momentum analyzed in a magnetic spectrometer. As listed in Table 1.7, these authors report values of R ranging from 2.20 ± 0.18 to 2.58 ± 0.34 at different angles.

These experiment suffer from very poor choice of kinematical conditions. We have already pointed out that because of the rapid variation of cross sections at forward pion angles, these angles are unsuitable for precision measurements. We believe that for this reason the results at $\theta_{cm}(\pi)=49.1^\circ$ and 56.8° are not reliable. The other two angles, $\theta_L(^3\text{H})=12.5^\circ$ and 14.3° do not represent wise choices of angles either. As Fig. 1.15 illustrates, elastically scattered protons should literally flood the spectrometer at $\theta_L=14.3^\circ$, and constitute a serious background even at $\theta_L=12.5^\circ$. Not only should this lead to bad conditions for particle identification but it should also have deleterious effect on chamber efficiencies, accidentals

etc. We also note that in this experiment the momentum acceptance of the spectrometer was considerably smaller than the momentum spread of the ^3He 'peak', and three separate measurements had to be added to completely cover the 'peak'. Because of all the reasons mentioned, we believe that these experiments do not provide reliable measurements of R .

6. Low et al. (LAMPF,1981)⁶⁹

Low et al. have measured differential cross sections for $\text{pd} \rightarrow ^3\text{He}\pi^0$ at 800 MeV in the angular range $\theta_{cm}(\pi)=10^\circ - 130^\circ$. They used a LD_2 target, a C magnet as crude spectrometer and did particle identification by time of flight. They also made measurement of $\text{pd} \rightarrow ^3\text{H}\pi^+$ cross sections at three angles $\theta_{cm}(\pi)=10^\circ, 17^\circ$ and 30° . They obtain $R=2.09(12)$, $2.10(12)$ and $\geq 2.12(22)$ from these measurements. They also compare extrapolations of their $^3\text{He}\pi^0$ data to three datum points due to Aslanides(Saclay)⁶⁸ for the $^3\text{H}\pi^+$ reaction at 809 MeV to obtain $R=2.06(8)$ at $\theta_{cm}(\pi)=20^\circ$, $2.09(8)$ at $\theta_{cm}(\pi)=25^\circ$, and $2.06(8)$ at $\theta_{cm}(\pi)=30^\circ$.

In our opinion these results suffer from the disadvantages common to forward angle measurements. Also comparison of experiments from two different laboratories can be hardly expected to give precision values of R . We believe that the error quoted for the grand average value of $R=2.08(2)$ can not be taken seriously.

7. Dutty et al. (SIN,1981)⁸²

These authors have measured the differential cross sections for the reactions $\text{n d} \rightarrow ^3\text{H}\pi^0$ and $\text{nd} \rightarrow ^3\text{He}\pi^-$ for T_n between 400 MeV and 590 MeV by detecting ^3H and ^3He in a magnetic spectrometer. They find that the ratio

$$R' = \sigma(\text{nd} \rightarrow ^3\text{He}\pi^-) / \sigma(\text{nd} \rightarrow ^3\text{H}\pi^0)$$

has an average value of 1.71 ± 0.10 , instead of the 2.0 expected on the basis of charge independence. It is difficult to relate R' to the ratio R measured in proton experiments without making a model for departures from charge symmetry in

the two cases. To provide an illustration, if the entire difference from the isospin prediction of 2.0 is attributed to differences in wave functions of ^3H and ^3He , then the value $R'=1.7$ for the nd reaction would correspond to $R \approx 2.3$ for the pd reaction.

1.10 The present experiment.

The preceding critical evaluation of earlier experiments enables us to demand the following conditions for our experiment:

- a) The proton beam energy should be ≥ 700 MeV, the higher the better. The beam energy should be stable to well within $\pm 1\%$.
- b) The beam should have a high degree of transverse polarization, $P \geq 0.8$. It should be stable and continuously monitored with a precision of the order $\pm 1\%$.
- c) The target should be LD_2 in a thin-wall flask. Preferably the target should be of uniform thickness of $\approx 1\text{cm}$. The target should be stable, with no boiling or erratic changes in density.
- d) The beam direction and intensity should be stable and continuously monitored.
- e) It is highly desirable to have a stable monitor of beam-target interactions.
- f) Particle identification should be done by both dE/dx and time of flight measurements.
- g) As far as possible particle detection conditions for ^3H and ^3He should be identical. Thus, for example, appropriate changes in chamber voltages and scintillation counter voltages should be made between ^3H and ^3He detection to give comparable pulse heights in the detectors employed.

- h) Measurements of the two reactions should be alternated as many times as possible to cancel out the effects of long term changes in experimental conditions.
- i) A Monte Carlo simulation of the experiment should be done in order to determine parameters which can not be directly measured, e.g., the effect of multiple Coulomb scattering and differential effects on detection efficiency due to different energy spreads in outgoing ^3H and ^3He .

As will be seen later most of these conditions were realized in our experiment. A notable exception was item (e). No independent monitor of beam-target interactions was available.

1.10.1 Difficulties in interpretation.

At the outset we wish to note the simple fact that since no successful theories of pion production are presently available, we will not be able to submit our experimental results to immediate theoretical interpretation. In absence of such a theory, we will not be able to apply reliable corrections for external electromagnetic effects. Knowing this, all we can do, and indeed do, is to choose the parameters of the experiment such that these effects are as small as possible.

In the present dissertation we will be able to present only a phenomenological interpretation of our results. A definitive interpretation will have to wait for the development of a reliable model of pion production in p-d collisions.

Chapter 2

The experimental method.

The present experiment was performed at the high resolution spectrometer (HRS) at the Los Alamos Meson Physics facility (LAMPF). A schematic of the LAMPF accelerator complex is given in Fig. 2.1.

Details of the accelerator are given in Appendix A. In the following each component of the present experimental set-up will be discussed and a critical study of the experimental parameters which have the most significant effect on the observables measured will be performed.

2.1 The beam.

The polarized P^- beam, which is accelerated simultaneously with the main H^+ beam in the LAMPF linear accelerator, is deflected in line X and tailored for use at the HRS in line C. The schematic showing line C is displayed in Fig. 2.2.

During our experiment the beam polarization was of N-type and the nominal kinetic energy of the beam was 733 MeV. The average beam current was in the range of 1-3 na. Details on the polarized proton source are given in Appendix B.

Beam intensity was monitored by means of two ion chambers (*ER02* and *ER04*) located inside the scattering chamber, immediately downstream of the target.

The beam polarization was measured by two independent techniques. The first, known as 'quench ratio'⁸² method, employs the information of the ratio

Figure 2.1: Schematic view of the LAMPF accelerator.

between the unpolarized beam current and the polarized one. At the end of each cycle of 'normal-reverse' polarization, the source is detuned so that only the unpolarized, background part of the beam is transmitted. The ratio of the beam intensities during this 'quenched' period and the 'unquenched' period gives the beam polarization:

$$P_B = 1 - \frac{I_Q}{I_{Pol}} \quad (2.1)$$

where:

$$I_Q = \left(\frac{N(ER04) \text{ or } N(ER02)}{N(\text{BeamGates})} \right)_Q$$

$$I_{Pol} = \left(\frac{N(ER04) \text{ or } N(ER02)}{N(\text{BeamGates})} \right)_{Pol}$$

and

- $N(ER04)$ or $N(ER02)$ represent the readings of the scalers associated with the corresponding beam integrators,
- $N(\text{Beam Gates})$ indicates the readings of the scalers which count the number of beam macropulses,
- the subscripts Q and Pol refer to the polarization status of the beam (Q corresponds to the 'quenched' beam and Pol corresponds to normal or reverse polarized beam).

Another method which is available to measure the beam polarization is the line C polarimeter⁸³. In this polarimeter measurements are made of the left-right and up-down asymmetry in the yields of proton-proton elastic scattering. The left-right asymmetry provides a measurement of N-type polarization (perpendicular to the scattering plane) and the up-down asymmetry provides a measurement of the S-type polarization (perpendicular to the beam direction in the scattering plane).

Each set of monitors is composed of a pair of scintillator counters positioned to detect the coincidence between two protons coming from elastic scattering at

the angles ($\theta = 19.1^\circ$, $\theta' = 63.7^\circ$) corresponding to the maximum value of $A_p(\theta)$ in this energy region. The yields for left(up) and right(down) pairs are integrated and two quantities of interest $\langle P \rangle$ and G_{hor} , are constructed:

$$\langle P \rangle = \frac{1}{A_p} \cdot \frac{(1 - \chi)}{(1 + \chi)} \quad (2.2)$$

where:

$\langle P \rangle$ is the average of the beam polarization for normal and reverse orientation,

$A_p = 0.507 \pm 0.008$ is the analyzing power for pp elastic scattering,

$\chi = \sqrt{\frac{R_\uparrow L_\downarrow}{L_\uparrow R_\downarrow}}$ where L,R refer to the corresponding pair of scintillators,

and

$$G_{hor} = \sqrt{\frac{L_\uparrow L_\downarrow}{R_\uparrow R_\downarrow}} \quad (2.3)$$

G_{hor} corresponds to the difference of effective solid angle between the left and right detector pairs. This parameter is interesting because its relative fluctuations give an indication of the corresponding fluctuations of the beam centroid in the horizontal plane.

The relationships given above also apply to the measurements of the S-type polarization by the change $L(R) \rightarrow U(D)$. The parameter G_{vert} associated with the up and down detectors gives a measurement of the fluctuations of the beam centroid in the vertical plane intercepting the beam direction.

2.2 The Target.

Two main considerations constrained the choice of the deuterium target employed and its geometrical characteristics as used in the present experiment.

Pion production cross sections for the $p+d \rightarrow {}^3H({}^3He) + \pi^+(\pi^0)$ are known to be quite small ($\approx 1\mu\text{b/sr}$ at $\theta(\pi)=130^\circ$). This rules out the use of CD_2 solid target

as well a cooled gas target for which the background from the $(p, {}^3\text{He}({}^3\text{H}))$ reactions on carbon nuclei and nuclei of the container material would be the dominant trigger source. This made the choice of a liquid deuterium target mandatory.

Energy loss effects, which will be described in detail in a subsequent section, give the two criteria for the LD_2 target design. The target should be as thin as possible, consistent with other requirements, e.g., counting rate, desired in the experiment, and the target should present a uniform thickness to the dispersed beam. The target used in the present experiment was designed by Mr. J. K. Novak of MP-7. It consisted of a right circular cylinder of brass with mylar windows for beam entry and exit (see Fig. 2.3). The design volume for LD_2 was 2.560" (6.50 cm) diameter and 0.450" (1.143 cm) thickness. The target was designed to operate with a refrigerator (CTI model 1020) with a nominal cooling capacity of 10 watts at 20°K. The normal operating pressure of LD_2 was 15.0 psia.

The design target thickness was 1.143 cm. However, because the target flask containing liquid deuterium at 15 psia was intended to operate inside a vacuum scattering chamber, its mylar windows were expected to deform as a consequence of the stress induced by the difference in pressure between their two sides. Before installation in the scattering chamber, the target was tested at 38 psia and a maximum deflection of the mylar windows was measured to be 1.080cm. From this measurement, the maximum deflection at the window center (normalized with respect to the window diameter and based on the assumption that the deformed window is segment of a spherical surface) for the operating pressure of 15 psia was calculated to be 0.541cm. It has been suggested ⁸⁴ that the actual value of the maximum deformation was likely to be somewhat more than that calculated in this manner, because of some permanent deformation produced by the initial test performed.

In Table 2.1 the thickness distributions along a transversal direction are given for the two cases under consideration. The calculation was performed up to a deviation of 1 cm with respect to the target center, corresponding to the acceptance

Table 2.1: Thickness distribution as a function of the coordinate ρ

ρ (mm)	Thickness(cm) (P=15 psi)*	Thickness(cm) (P=34psi)
0	2.210	3.200
2	2.205	3.187
4	2.192	3.165
6	2.141	3.073
8	2.103	3.004
10	2.080	2.900

* Pressure during the experiment.

of the HRS spectrometer for the horizontal coordinate at target. This distribution of target thicknesses seen by the beam generates an enhanced sensitivity to fluctuations in the position of the beam centroid with respect to the target center.

Figure 2.3: Schematic of LD₂ target under 15 psi pressure.

2.3 The HRS spectrometer.

The data acquisition was performed at the HRS Facility, employing the standard experimental set-up provided to detect a charged particle in the final state. This detection system performs momentum analysis and particle identification. Momentum analysis is done by a magnetic spectrometer (which also acts as a 'momentum filter') and a set of multiwire drift chambers, which allow complete reconstruction of the trajectory of the detected particle.

The magnetic spectrometer⁸⁵ is of a QDD (quadrupole-dipole-dipole) configuration. The high resolution which is provided by this system for most of the conventional inelastic scattering experiments (≤ 100 KeV FWHM for proton scattering from heavy targets) is achieved through operation in the 'energy loss mode'. Details about this design aspect and on other characteristics of the spectrometer are given in Appendix C.

Fig. 2.4 gives a schematic representation of the detector system associated with the HRS magnetic spectrometer. It consists of 2 sets of multiwire drift chambers and five removable scintillator detectors. Table 2.2 summarizes the relevant geometrical parameters which characterize these detectors.

Table 2.2: Geometrical characteristics of the detectors associated with the HRS magnetic spectrometer.

Detector	Size (cm)
C_i^*	60.0×10.0
S_1	$53.0 \times 9.8 \times 0.953$
S_2	$58.2 \times 10.7 \times 0.953$
S_3	$68.0 \times 12.0 \times 0.953$
S_4	$76.0 \times 13.0 \times 0.953$
S_5	$85.3 \times 14.4 \times 0.953$

* All the drift chambers have this active area.

The front and rear drift chamber modules each consist of a pairs of planes

Figure 2.4: Schematic representation of the HRS detector system. Scintillators S_1 and S_3 were not used in the present experiment and were moved out of position.

(C1,C2 and C3,C4, see Fig. 2.4) sensitive respectively to the vertical and the horizontal position of the detected particle. In each set of adjacent chambers with parallel wires the corresponding anodes are positioned with a relative displacement of .4 cm (length equivalent to one half of a drift cell). This configuration allows one to determine the sign with which the drift distance is combined with the wire position in the reconstruction of the trajectory associated with a given event. The sense wires are soldered on a printed circuit delay line. The two ends of each delay line, which are conventionally referred to as N and P respectively, are connected with constant fraction discriminators located near the detector system. More details on the technical characteristics of these chambers can be found in refs.(86-87).

The scintillator detectors located beyond the drift chambers provide the hardware trigger which defines the occurrence of an 'event' in the detection system. This trigger condition is determined by the coincidence between the scintillators which are on line. During our experiment, the scintillators employed were S_2, S_4, S_5 . These counters also provided the TOF (time-of-flight) and the pulse heights for the particle identification system. The PID test adopted in our experiment involved the pulse height information associated with detector S_2 and the time of flight between S_2 and S_5 . The light signals generated in each scintillator are collected at the two vertical ends, generally referred to as 'up' and 'down' respectively, by two photomultipliers. Their signals are sent to the electronic signal processor which is described in the next section.

2.4 Signal processor for the HRS detector system.

Signals coming from the phototubes at the two ends of the scintillators and from the delay lines associated with the drift chambers are sent to the electronics processor given in Fig. 2.5. This system has the twofold purpose of generating the coincidence $S_2 \cdot S_4 \cdot S_5$ and processing the signals coming from the detectors to obtain the data words used by the HRS event analyzer for the software reconstruction of

the event.

A meantimer circuit generates a logical signal which defines an 'average time' between the times T_{up} and T_{down} defined by the leading edge of the pulses coming from the corresponding ends of each scintillator. The output signals from the meantimers associated with the scintillators which are on line are sent to an AND circuit which produces the coincidence signal which is part of the hardware trigger and generates the event reference time.

The data words which compose the event processed by the HRS analyzer are the relevant times associated with the chambers and the scintillators and the analog signals associated with the two ends of S_2 . The time data words are obtained by digitizing the time differences between stop and start signals sent to a set of LRS 2250T TDC(time-digital-converter) modules. The pulses coming from the two phototubes connected to S_2 are sent to a charge integrating ADC (analog-to-digital-converter) module.

The TDC's associated with the scintillators employ as start signal the time T_0 defined by the 'event' signal, and as stop signal the time defined by the pulse coming from the up or down end of each counter. The average time $T_{ave} = 0.5 * (T_{up} + T_{down})$ contains a term which is proportional to the arrival time of the event at the detector and a constant term which is determined by the delay elements introduced in the timing circuit, the transit time across the photomultiplier and the time taken by the light signals to travel the whole scintillator length. The time difference $T_{diff} = T_{up} - T_{down}$ contains a term which is proportional to the difference in transit time along the scintillator between the photons collected at the up end and the ones collected at the down end. Therefore it gives a rough indication of the vertical position of the event at the detector plane.

Each chamber plane, as mentioned before, has its sense wires connected to a delay line. The signals at the two ends of the delay line (T_p and T_n) provide two independent times from which the wire struck and the distance travelled by the electron pulse generated by the ionizing particle within the drift cell are

Figure 2.5: Block diagram of the HRS signal processor.

reconstructed. At the HRS these two times are processed using two different reference times as T_{start} . The time T_{p0} employs the time T_0 as start and T_p as stop, whereas the time T_{np} employs T_p as start and T_n as stop. The choice of T_{np} against the much larger T_{n0} is determined by the requirement of making the time difference signal independent of the delay between the arrival of the particle at the chamber plane and the event reference time.

A technical feature of the TDCs used at the HRS is relevant to the discussion of the software reconstruction of the event which will be discussed in the next chapter. These TDCs are modified versions of the corresponding Lecroy ADCs 2250L, with an additional input stage composed of a constant current generator connected to an integrating capacitor during the time between the start and stop signal. The gate signal for the ADC stage (trigger input) is provided by the time T_0 . This produces an offset term in the digitized time which is proportional to the time difference $(T_p - T_0)$, corresponding to the time dependent pedestal characteristic of these ADCs. Therefore a correction must be applied for this effect as a first step of the software reconstruction of the chamber information.

Fig. 2.6 shows the 'slow gate' logic which is part of this electronics processor. It generates a set of flags about the beam conditions, computer status (BUSY signal), and data acquisition status (RUN signal). The selection of the kind of event to be processed is coordinated by the trigger module developed at LAMPF, tailored to the structure of the software employed for data acquisition.

An MBD (microprogrammed branch driver) constitutes the interface between the CAMAC crates and the on line computer which, during this experiment, was a PDP 11/45.

Figure 2.6: Block diagram of the HRS slow gate logic.

2.5 The particle identification system.

Three scintillator counters, S_2 , S_4 and S_5 beyond the focal plane (see Fig. 2.4), were part of the detector configuration employed in this experiment: they had the purpose of providing the event trigger and the particle identification information. The variables which were employed for the latter purpose were the energy loss ΔE in the first scintillator and the time of flight between the first and last scintillator.

An investigation of the range of ΔE and time of flight TOF spanned by the detected particles was performed according to a simplified model. At the two angles corresponding to the limits θ_{min} and θ_{max} of the angular acceptance of the spectrometer the range of kinetic energies spanned by all the particles which are transmitted by the magnetic spectrometer was calculated. These particles with kinetic energies equal to $T_{min}(\theta_{max})$ and $T_{max}(\theta_{min})$ were then propagated through the three scintillators. This analysis was performed to identify the major sources of background and to obtain an estimate of the effectiveness of the experimental apparatus in particle detection and identification.

The critical aspect of the particle identification system is the difference in the nature and kinematical parameters of the major sources of background. This is due to the different value of the parameter 'P/z' in the two cases: as all the 'candidate-background' particles have $z=1$, their momentum will scale by a factor two going from the magnetic spectrometer setting suitable for ^3He to the one suitable for ^3H . This is illustrated in Table 2.3, which shows the kinetic energies of the various background particles corresponding to the central momentum in the two cases.

Table 2.4 shows the range of time of flights and energy loss in S_2 for the 'signal' and 'background' particles in the two detection conditions.

It can be seen that in the case of ^3H detection the almost unique source of back-ground is constituted by the continuum deuterons, whereas in the case of ^3He three additional groups of particles (continuum p,d,t from the lower branch) fall

Table 2.3: Kinetic energy of background particles corresponding to rigidity (p_0/z) range = 1486.3–1523.9 MeV/c/z for ^3H and 739.7–758.4 MeV/c/z for ^3He . (the asterisk denotes those particles which can not be produced by any reaction)

	^3H detection	^3He detection
	T (MeV)	T (MeV)
pion	1492.6–1530.3*	613.2–631.6*
proton	819.4–851.3*	256.5–268.2
deuterons	517.5–541.0	139.5–147.5
tritons	369.0–386.7	95.8–100.6
^3He	1281.0–1336.0*	365.8–383.4

Table 2.4: Time of flight and energy loss ranges for signal and background particles which enter the rigidity acceptance of the spectrometer.

	^3H		^3He	
	TOF (ns)	ΔE (MeV)	TOF (ns)	ΔE (MeV)
protons	–	–	9.9–10.2	3.3–4.8
deuterons	9.9–10.8	3.3–4.9	17.3–17.9	9.4–11.6
tritons	13.1–13.5	5.8–7.6	28.1–30.2	19.3–22.2
^3He 's			13.5–14.2	24.8–28.3

within the acceptance of the magnetic spectrometer. However the former reaction is more critical, as the domains spanned in ΔE and TOF by tritons and deuterons are close to each other.

In the case of ^3He it is easier to achieve background separation as ^3He is well separated either in ΔE or in TOF from the background particles (see Table 2.4).

Table 2.4 clearly shows that, even if the detector system is the same in the two reactions, there are substantial differences for the two cases between the signals which have to be processed and in the spurious events which have to be rejected. It is mandatory to examine these issues carefully in the analysis of systematic errors to be attributed to the final results.

2.6 Monte Carlo simulation of detected particle propagation through the spectrometer

A Monte Carlo code was written (see Appendix D) and a simulation of the experiment was done in order to investigate the correlations which exists between the momentum acceptance of the detector system and the angular spectra of ^3H and ^3He . In principle, the range of scattering angles selected by the spectrometer should not be affected by the momentum of the particle detected. The momentum analysis is associated with the vertical direction of the detector planes and the angular analysis with the horizontal one. The two acceptance boundaries are therefore determined by different physical dimensions and should be independent of each other. However, because of the large kinematic shift ($dT/d\theta \approx 10$ MeV/deg) in the reactions studied by us and the rather substantial momentum spread produced by the energy loss in the target, the edges of the angular spectrum are actually determined by the momentum range selected by the magnetic spectrometer. Fig. 2.7 illustrates this effect: the angular spectra of ^3H and ^3He are compared with the one obtained during a calibration run for the reaction $D(p,p')$ in the deep continuum (at average excitation energy of 264 MeV), which had the main purpose of testing the uniformity in differential efficiency of the drift chambers.

The most problematic aspect which is apparent in this figure is the difference in the distortion induced by this effect in the two reactions to be compared. This is due to the difference in the width of the energy loss distribution for the reaction products caused primarily by the difference in their electric charge. As a consequence, there is a different detection efficiency for the two reactions to be compared.

The program discussed in this section was developed in order to obtain a quantitative evaluation of this effect. It investigates also the effects of multiple Coulomb scattering in the target and in the scintillator detectors and of the finite

Figure 2.7: Experimental angular spectra for the reactions $D(p, {}^3\text{H})\pi^+$ and $D(p, {}^3\text{He})\pi^0$ as compared with the angular acceptance of the HRS spectrometer for an inelastic reaction.

angular resolution associated with the trajectory reconstruction. Another major object of this simulation is the achievement of a deeper understanding of the shape of the distributions of background events for the relevant spectra used to extract the ratio R and $\Delta A_{\nu 0}$.

It is worthwhile to examine the development of an event in the simulation in parallel with the corresponding situation in the data acquisition system in order to understand the model adopted and the approximations introduced in it. Fig. 2.8 illustrates the kinematical configuration and the relevant reference frames for the discussion which follows.

The initial state in a given experiment is characterized by the energy and the direction of the proton beam (particle m_1) and by the target employed (particle m_2). In the data acquisition system this information is given to the event processor as input parameters; no momentum reconstruction is performed for the beam particles. This is motivated by the fact that the spectrometer was designed to operate in 'energy loss mode' described in Appendix 3, and, in principle, the rele-

Figure 2.8: Coordinate system for the two-body reaction.

vant kinematical variables should not be affected by small variations of the beam energy around its nominal value. The effects of assuming a wrong centroid for the beam energy will be discussed later. In the simulation three input parameters are used to describe the beam kinetic energy: the ‘nominal kinetic energy’, T_1^{nom} , corresponding to the value used in the data analysis, the ‘kinetic energy shift’, $\langle \Delta T_1 \rangle$, which represents the difference between the actual average energy and the nominal one, and the percent fractional momentum dispersion of the beam. For each event a ‘true’ beam energy is generated according to the relationship:

$$T_1^{true} = T_1^{nom} + \langle \Delta T_1 \rangle + \Delta T_1^{fluct}$$

where ΔT_1^{fluct} is a random variate uniformly distributed in the interval $|\Delta T_1^{fluct}| \leq \Delta T_1^{max}$. ΔT_1^{max} is determined by the beam dispersion given as input parameter. Therefore it is possible to investigate how the reconstructed variables are affected by the incorrect estimate of the beam energy ($\Delta T_1 \neq 0$) and by its energy spread in situations in which the energy loss compensation does not work. This is actually the case for the two reactions which are the object of our experiment.

The other major approximation performed in processing the data is to assume

that the incident particle direction is always parallel to the $\hat{z}$ axis in Fig. 2.8. There are two main sources of deviation from this direction: the intrinsic angular divergence of the beam and the multiple Coulomb scattering suffered up to the interaction point. The former effect is not simulated, as no reliable information about it is available, whereas the latter effect is introduced in the program.

An interaction point with coordinates (y_t, z_t) is generated for each event: y_t represents the horizontal coordinate referred to the center of the target and z_t the coordinate along the beam direction referred to the entrance window of the target.

The energy of the beam at the interaction point is corrected for the average energy loss and its fluctuations which took place in the thickness traversed Δz_b . The fluctuations are generated according to a gaussian distribution, with a variance proportional to the square root of Δz_t . This functional dependence is suggested by Bohr's treatment of the fluctuations in the energy loss. However the coefficient employed is an empirical parameter consistent with that obtained with the Vavilov calculation of the FWHM in this kinematical domain.

The vertical coordinate x_t is not generated as it is not reconstructed in the event analyzer and therefore no observed correlation with this variable is available for comparison. The direction of the scattered particle trajectory is defined by the two angles θ_t and ϕ_t , which define the orientation of its projections in the $x'z'$ and $y'z'$ planes respectively, in the spectrometer reference frame $\hat{x}' \hat{y}' \hat{z}'$ shown in Fig. 2.8. The $\hat{z}'$ axis lies in the horizontal plane along the central angle θ_{sp} of the spectrometer angular acceptance in this plane. In this connection, it is relevant to observe that there can be an offset between the nominal spectrometer angle θ_{sp}^{nom} and the actual central angle θ_{sp} due to the error in the absolute angle calibration. Therefore the program has as input parameters two different spectrometer angles, in order to simulate the effects of the offset described above. The 'true' θ_{sp} is used in calculating the true scattering angle, whereas the 'nominal' θ_{sp} is used to process the variables to be compared with the experimental ones.

The average energy loss of the scattered particle is calculated and also a simplified model of the fluctuations of energy loss is introduced, through a Gaussian straggling with a variance which is a function of the interaction point. Its functional form was determined in order to reproduce the dependence upon the interaction point of the FWHM associated with the correct Vavilov distribution. In an earlier version of the program the fluctuations in energy loss were actually determined according to a set of Vavilov distributions. However, the complexity of the algorithm employed to generate random variates with this probability distribution required a discretization of the problem which introduced a worse approximation in the simulated spectrum of the kinetic energy of the scattered particles than the solution finally adopted. The uncertainty in the scattering angle associated with multiple Coulomb scattering in the target is also introduced. Moreover a Gaussian straggling in the variables θ_t and ϕ_t is introduced to simulate the finite angular resolution of this detector system, as measured during the tune-up calibrations before the cycle in which this experiment was run.

In the HRS event processor algorithm, the transport properties of the spectrometer are determined empirically through a set of four functions which relate the coordinates at the front detector plane $(x_f, \theta_f, y_f, \phi_f)$ to the coordinates at the target (θ_t, y_t, ϕ_t) and the momentum coordinate $\delta (=100 \cdot (p-p_0)/p_0$, where p is the reconstructed momentum and p_0 is the central momentum of the magnetic spectrometer). This non-linear 'backward transport' system was inverted in order to propagate the scattered particles up to the first detector plane.

The geometrical characteristics of the detectors simulated have been given in Table 2.2. The particles are propagated through two sets of X-Y drift chambers and three scintillator planes. At each plane a test is performed on whether the transverse coordinates of the particle are within the geometrical acceptance of the detector plane. Each detector is considered ideal, i.e. its efficiency is assumed equal to one. Multiple Coulomb scattering and average energy loss effects in the scintillators are simulated.

For the 'good' events, i.e. the ones which pass the acceptance tests at each detector plane, the relevant histograms to be compared to the experimental ones are constructed. The first set of histograms give information about the simulated spectra of the coordinates at the front plane ($x_f, \theta_f, y_f, \phi_f$), allowing the comparison between the simulated and the real spatial distribution in the plane crossing the focal plane of the magnetic spectrometer. Figs. 2.9, 2.10 give the comparison between simulated and experimental spectra for the coordinates at the front plane. Figs. 2.11 gives the comparison between simulated and experimental x coordinate at the front plane for the detection of continuum protons. This spectrum has been singled out because the purpose of studying this reaction was to monitor the efficiency distribution in the focal plane along the dispersion direction x.

The two other relevant spectra, from which the observables of our experiments $\sigma(\theta)$ and $\Lambda_{v0}(\theta)$ are extracted, are the event spectrum as a function of the scattering angle θ_{sc} and the event spectrum as a function of the missing mass MM. The latter is the difference between the mass of the undetected particle calculated by the event analyzer and the theoretical value of m_4 given to the analyzer as an input parameter. The experimental missing mass is a function of the reconstructed momentum of the detected particle p_3 , the reconstructed scattering angle and the beam momentum given as input parameter. For events which are associated with 'signal' reactions, the missing mass spectrum is represented by a peak, for which the centroid position is determined by the precision in the absolute values of the input parameters, and the width is determined by the spread in p_3 and θ_{sc} produced by energy loss effects and the spatial resolution of the drift chambers. Some of these effects are simulated reasonably well. This is illustrated by the comparison of the missing mass spectra from the experiment and the Monte Carlo program. Fig. 2.12 illustrates the comparison for the $p + D \rightarrow {}^3\text{He} + \pi^0$ and $p + D \rightarrow {}^3\text{H} + \pi^+$ reactions and for the reaction $d(p,p')X$ in the deep continuum. Fig. 2.13 illustrates the comparison for elastic scattering runs performed with various targets for efficiency and energy monitoring.

Figure 2.9: Coordinates $(x_f, \theta_f, y_f, \phi_f)$ at the front plane for the reaction $p + d \rightarrow {}^3\text{H} + \pi^+$.

Figure 2.10: Coordinates $(x_f, \theta_f, y_f, \phi_f)$ at the front plane for the reaction $p + d \rightarrow {}^3\text{He} + \pi^0$.

Figure 2.11: Coordinate x_f at the front plane for the reaction $p + d \rightarrow p' + X$.

Figure 2.12: Normalized missing mass spectra from Monte Carlo simulation (continuous line) and from experimental data for the three reactions $d(p, {}^3\text{H})$, $d(p, {}^3\text{He})$ and $d(p, p')$.

Figure 2.13: Normalized missing mass spectra from Monte Carlo simulation (continuous line) and from experimental data for elastic scattering from protons, deuterons and ^{208}Pb . For ^{208}Pb the curve in the figure on the left is without taking into account any intrinsic momentum resolution contribution, and that in the figure on the right is with inclusion of 0.018% intrinsic momentum resolution.

The only missing mass width which is not reproduced correctly is the one associated with elastic scattering from ^{208}Pb (see Fig. 2.13). In this case the experimental width is determined by the intrinsic momentum resolution of the spectrometer, which is determined by its optical properties (non-linearities, aberrations etc.). An empirical Gaussian spread of the variable δ has been introduced in the program in order to simulate this effect. Its variance has been taken from the experimental value determined by the δ spectrum for elastic scattering for ^{208}Pb . However it turns out that this contribution to the missing mass width is already negligible for the spectra corresponding to elastic scattering from deuterium. Overall, the widths of the missing mass distributions are well reproduced. Some discrepancy exists between the centroids of the missing mass spectra obtained from the simulation and the experimental ones. The relative shifts which have been introduced in order to superimpose the experimental and simulated peak are within the error associated with the experimental centroids (≈ 500 KeV for elastic scattering and ≈ 2 MeV for ^3H .)

The distributions of events with respect to the scattering angle are well reproduced, as shown in Fig. 2.14.

Our ability to simulate the missing mass spectra for these 'signal' events gives us confidence in the simulation for the missing mass spectra for the 'background' events generated on the basis of simple, plausible hypotheses about their sources. The steps taken in the generation of 'background' events are the same as discussed before. The hypothesis we use is that the background events are generated as continuum production from impurities in the target (mostly the material of the LD_2 container, aluminized mylar, etc.).

Table 2.5 is a summary of the input parameters employed in the simulation of the two reactions $d(p, ^3\text{H})\pi^+$ and $d(p, ^3\text{He})\pi^0$.

Table 2.6 gives the percentage event rejection by the various detector planes for the specified experimental conditions.

Table 2.7 gives the percent efficiency for two angular regions over which

Figure 2.14: Normalized scattering angle spectra from the Monte Carlo simulation and experimental data.

Table 2.5: Input parameters for the Monte Carlo simulation of the reactions $d(p, {}^3\text{H})\pi^+$ and $d(p, {}^3\text{He})\pi^0$.

	${}^3\text{H}^+$	${}^3\text{He}^{++}$
T_1^{nom}	730. MeV	730. MeV
$\langle\Delta T_1\rangle$	0.13 MeV	0.13 MeV
$\delta p^{in}/p^{in}(\%)$	$\pm.06$	$\pm.06$
target thickness	0.390 g/cm ²	0.390 g/cm ²
HRS central mom.	1501.36 MeV/c	747.153 MeV/c
θ_{sp}^{nom}	12.000 deg	12.000 deg
θ_{sp}^{true}	12.15 deg	12.15 deg
$\sigma(\phi_t)$	1.0 mr	1.0 mr

Table 2.6: Monte Carlo results for detector acceptance in % of the particles which arrive at a detector.

detector plane	^3H	^3He	Ratio
C1	78.01	74.42	1.052
C1-C2	75.27	71.63	1.051
C1-C2-S2	72.75	71.63	1.016
C1-C2-S2-S4	72.75	71.63	1.016
C1-C2-S2-S4-S5	72.65	71.63	1.016

Table 2.7: Integral efficiency for the two angular gates employed in the analysis from the Monte Carlo simulation. (Angles are referred to the spectrometer angle θ_s .)

Gate	^3H		^3He		Ratio $^3\text{H}/^3\text{He}$
	$\Delta\theta(\text{mr})$	Efficiency(%)	$\Delta\theta(\text{mr})$	Efficiency(%)	
$\Delta\theta_1$	-15.8-+8.2	97.8 $\pm$.14	-15.0-+9.0	94.5 $\pm$.14	1.035 $\pm$ 0.20
$\Delta\theta_2$	-13.8-+2.2	98.9 $\pm$.14	-13.0-+3.0	97.7 $\pm$.17	1.012 $\pm$ 0.22

the measured observables were integrated. The intervals used for ^3H are slightly shifted with respect to the ones employed for ^3He in order for their centroids to correspond to the same pion angle in the center of mass system. The reasons for this choice will become clearer when the method of analysis is discussed.

Chapter 3

Data analysis and results.

The presentation of the results obtained in this experiment will be preceded by a discussion of the relevant software employed in processing the information obtained from the data acquisition system. This gives an understanding of the various steps involved and of the sources of uncertainty associated with them. Next a discussion of the method of analysis employed will be given and the results for the observables measured will be presented. Finally the major sources of systematic errors will be analyzed.

3.1 The analysis and calibration software.

There are two main software packages which are employed to handle the data acquisition process and the off-line analysis for experiments performed at the HRS facility.

The first, generally known as 'Q system', is a set of programs and subroutines for data handling and manipulation. They provide the ability of handling the transfer of data words from the Camac modules in the electronics processor to a storage device (typically a magnetic tape), of imposing software cuts on specific data words and of calculating the relevant derived quantities. The latter function requires the input information of a set of parameters which characterize the experimental conditions. They are stored in a large common block shared by the subroutines which manipulate the data. Some of them are obtained in calibration

runs (e.g. chamber parameters), other are entered interactively at the beginning of a run (typically this is the case for kinematics parameters relative to the experiment performed). Detailed information about the various components of this system can be found in ref.⁸⁸

A second set of programs, developed by the MP10 staff⁸⁹, is tailored to the data acquisition system present in the HRS and EPICS facilities. The core of this software package is the 'HRS event analyzer'. The version which is used during data acquisition contains a description of the CAMAC modules which interact with the computer and the set of steps which need to be performed upon the arrival of a specific trigger signal. A version modified in its input-output operations is used for the analysis of the data. Other programs have been developed to perform calibrations, kinematics calculations and data handling. More detailed information about these can be found in reference (89).

3.2 The HRS event reconstruction.

Each event which is stored on tape consists of readings of the TDCs associated with the drift chambers and the scintillators and the ADCs associated with scintillator S_2 . The information from the scintillators is used to generate the 'particle identification test' described in the previous chapter. The time data words from the drift chambers allow the reconstruction of the kinematic variables 'true scattering angle' θ_{sc} and missing mass MM, which have been defined in the section devoted to the Monte Carlo simulation.

The missing mass spectrum allows the identification of the reaction of interest against competing ones which fall within the acceptance of the magnetic spectrometer and satisfy the particle identification test. It can also be employed to obtain the yield from which the differential cross section or the analyzing power are calculated. Alternatively, these observables can be obtained from the integration of the scattering angle spectrum in selected intervals contained within the angular acceptance. The difference between the two approaches will be discussed

in a later section.

The first step in the reconstruction of the kinematic variables for the event processed involves the calculation of the position and orientation of the particle detected at the median plane of the first set of drift chambers. Fig. 3.1 gives a schematic representation of the steps taken to accomplish this task. These operations are performed by processing simultaneously the information from each pair of adjacent chamber planes.

The step which deserves more careful analysis is the interpolation which determines the final value of the 'corrected extrapolated position at the mid-plane'. It involves the definition of variables which are crucial in the characterization of the chamber performance. The geometry pertinent to this discussion is given in Fig. 3.2.

A crude determination of the orientation of the event in the x' - z' plane (or y' - z' plane), characterized by the angle $\theta'(\phi')$, is needed to extrapolate the position at the mid-plane between the two adjacent chambers. Subsequently, four extrapolated positions at plane x_i^* are calculated from the information associated to the two corresponding chamber planes.

$$x_c(I) = x_t(1) + x_{dr}(1) + c_0 \quad (3.3)$$

$$x_c(II) = x_t(1) - x_{dr}(1) + c_0 \quad (3.4)$$

$$x_c(III) = x_t(2) + x_{dr}(2) - c_0 \quad (3.5)$$

$$x_c(IV) = x_t(2) - x_{dr}(2) - c_0 \quad (3.6)$$

where:

$$c_0 = \tan(\theta') \cdot \Delta z'/2$$

$$x_t(i) = \text{truncated position at plane } i$$

$$x_{dr}(i) = \text{drift distance at plane } i$$

Figure 3.1: Flow-diagram of the event reconstruction performed by the IIRS event processor.

Figure 3.2: Geometry for the determination of the corrected extrapolated position of a particle track through the x drift chambers.

Subsequently the four possible differences Δx_{ij} , $[x_c(\text{I}) - x_c(\text{III}), x_c(\text{I}) - x_c(\text{IV}), x_c(\text{II}) - x_c(\text{III}), x_c(\text{II}) - x_c(\text{IV})]$, between the first two and the last two elements of the array x_c are constructed. Ideally, one of these differences should be equal to zero, as it corresponds to the combination of the truncated position with the drift distance performed with the choice of sign corresponding to the correct track orientation. This implies that the choice of the combination of signs which corresponds to the difference with minimum absolute value $\Delta x_{ij}^{\min}$ should be the correct one. The 'corrected extrapolated position' at x_i^* is the average between the extrapolated positions from the two adjacent planes chosen by this criterium. The discussion of this step in the algorithm for event reconstruction has been carried out in detail as the variables $\Delta x_{ij}^{\min}$ play a crucial role in the software test of correct wire identification which was particularly critical for this experiment. These considerations will be developed in the next section.

The coordinates at the front plane $(x_f, \theta_f, y_f, \phi_f)$ are obtained through the relationships:

$$x_f = x_i^* \quad (3.7)$$

$$\theta_f = (x_2^* - x_1^*)/\Delta z \quad (3.8)$$

$$y_f = y_i^* \quad (3.9)$$

$$\phi_f = (y_2^* - y_1^*)/\Delta z \quad (3.10)$$

where:

x_i^* and y_i^* are the 'corrected extrapolated positions' at the mid-planes defined previously.

Δz is the separation between corresponding intermediate planes in the front and rear set of chambers.

The coordinates $(x_f, \theta_f, y_f, \phi_f)$ are used to calculate the variables $(y_t, \phi_t, \theta_t, \delta)$, which characterize the scattering event. The variables (y_t, ϕ_t, θ_t) give the relevant information about the initial trajectory of the detected particle.. The variable y_t corresponds to the horizontal displacement with respect to the target center and

ϕ_t , θ_t correspond to the angular divergences with respect to the spectrometer central ray, in the horizontal and vertical plane respectively. The vertical coordinate at target, x_t , which is proportional to the dispersion of the beam, is unobserved. The variable δ represents the percent deviation $100 \cdot (p_3 - p_0)/p_0$, where p_0 is the spectrometer central momentum. The relationships which are used to calculate the variables $(y_t, \phi_t, \theta_t, \delta)$ from $(x_f, \theta_f, y_f, \phi_f)$ correspond to a nonlinear generalization of the inverse 'transport matrix' of the magnetic spectrometer. They are four second-order polynomials. Their coefficients are determined empirically in calibration runs. In the experiment performed a mapping of the momentum of the detected particle across the whole focal plane was of interest. Therefore the calibration determining the functional dependence $\delta(x_f, \theta_f, y_f, \phi_f)$ was repeated. The method employed and the results obtained will be discussed in the next section.

The knowledge of the momentum and orientation of the scattered particle allows the reconstruction of θ_{sc} and MM. These variables have been defined in the section describing the Monte Carlo simulation performed. The missing mass variable MM is a function of the reconstructed momentum p_3 and scattering angle θ_{sc} and of the input parameters T_1 (nominal beam energy), θ_s (spectrometer angle) and the parameters describing the reaction under investigation.

Each of the reconstructed data words discussed in this section is available for histogram display, if the spectral distribution of events relative to it is judged of interest. Selected regions of the range of values spanned by a data word can be singled out for the generation of software cuts employed to process the events.

3.3 Calibrations.

The calibrations needed to process the time information coming from the drift chambers and to relate the coordinates at the front plane to the ones at the target are obtained during the tune-up runs performed at the beginning of each cycle of experiments. However some of these functions are sensitive to parameters specific of the experiment performed (e.g. magnetic spectrometer field setting, chamber operating voltage etc.). For this experiment new drift time calibrations $s_{drift}(t_{sum}, t_0)$ and $\delta(x_f, \theta_f, y_f, \phi_f)$ calibration were done.

The drift time calibration associates a drift distance to each drift time bin. It is sensitive to the detailed shape of the electric field within the drift cell. In these chambers the boundaries of the drift cells are defined by field-shaping cathode wires located at a distance equal to $w/2$ between two adjacent anode wires. Near the anodes, there is a region of high field (with a space dependence $\approx 1/r$) where the multiplication takes place. When the chambers are illuminated uniformly across their active region, the drift distance spectrum should be flat. However, when the wrong weighting functions are used, these spectra develop spikes corresponding to either of the two boundary regions discussed above. During our experiments the drift chambers were operated at two different voltages, 2000 V for ^3H and 1800 V for ^3He . As a consequence, it was necessary to perform individual calibrations for ^3H and ^3He . A task (named DRT), which is part for the MP10 software package discussed above, allows the determination of $s_{drift}(t_{sum}, t_0)$. Two steps are involved in this procedure. The first one determines the offset term t_0 . For correct values of t_0 , the drift time spectra corresponding to the different chamber planes should exhibit no relative displacement. Fig. 3.3 shows the drift time spectra for the eight chambers X1-X4, Y1-Y4 after the offset calibration performed for ^3H and ^3He respectively. Next the drift distance to drift time dependence is obtained as an array which contains the weighting function which relates each drift time bin t_{drift}^i to the corresponding drift distance s_{drift}^i . These arrays were

obtained for each chamber plane separately. The drift distance spectra obtained after the calibration was performed are given for each chamber plane and for the two reactions in Figs. 3.4-3.7.

The other calibration performed was the one which determines the coefficients of the polynomial function $\delta(x_f, \theta_f, y_f, \phi_f)$. The standard procedure developed at HRS for this purpose was followed. It involves taking data for protons elastically scattered from a heavy target for various settings of the spectrometer magnetic field so that the elastic peak moves across the dispersion plane. In the analysis of these runs some ray files are written. Each ray contains the value of δ as input parameter for each run, the coordinates $(x_f, \theta_f, y_f, \phi_f)$ and the reconstructed scattering angle θ_{sc} for the event processed. The latter variable is used to correct for the kinematic shift associated with scattering events which took place at an angle θ_{sc} different from the spectrometer angle θ_{sp} . The ray file is written by the task FPO running simultaneously with the analyzer. Another task (COF) reads the ray file and a file describing the structure of the polynomial form expected for δ and determines the coefficient of each term by a minimization of the chi-square function (χ^2). Fig. 3.8 shows the missing mass spectrum for elastic scattering from ^{208}Pb analyzed with the $\delta(x_f, \theta_f, y_f, \phi_f)$ obtained from the calibration performed. Fig. 3.9 shows the corresponding δ spectrum.

The intrinsic energy resolution for this experiment was 200 KeV (FWHM) corresponding to a momentum resolution $\Delta\delta = 0.018\%$ (FWHM). A calculation of the quantity ΔMM through the relationship:

$$\Delta MM = \frac{\partial MM}{\partial p_{out}} \cdot 0.01 \cdot \Delta\delta \cdot p_{central} \quad (3.12)$$

with $\partial MM / \partial p_{out} = 0.827 \text{ MeV}/(\text{MeV}/c)$ and $p_{central} = 1382.46$ gives $\Delta MM = 207 \text{ KeV}$, consistent with the result obtained from the fit to the missing mass spectrum.

Figure 3.3: Drift time spectra for the detection of ${}^3\text{H}$ and ${}^3\text{He}$.

Figure 3.4: Drift distance spectra for ${}^3\text{H}$ (X chambers).

Figure 3.5: Drift distance spectra for 3H (Y chambers).

Figure 3.6: Drift distance spectra for ${}^3\text{He}$ (X chambers).

Figure 3.7: Drift distance spectra for ${}^3\text{He}$ (Y chambers).

Figure 3.8: A typical missing mass spectrum for the reaction $^{208}\text{Pb}(p,p)^{208}\text{Pb}$.

Figure 3.9: A typical δ spectrum for the reaction $^{208}\text{Pb}(p,p)^{208}\text{Pb}$

3.4 The good event test.

In the test file there is a macrotest which defines the 'good event', which selects events which are likely to correspond to the detection of a particle coming from the reaction of interest. The combination of microtests adopted to identify the 'good events' in the analysis of this experiment was somewhat different from the usual definition adopted in processing the data taken at the HRS spectrometer facility. This choice was a consequence of the anomalous performance exhibited by the drift chambers during our experiment.

Two set of tests are imposed on the chamber quantities. A test is imposed on the drift time data words to reject overflows and underflows for these variables. The correct wire identification is verified by examining the variables Δx_{ij}^{min} defined in the preceeding section. If the value of this variable is greater than a wire spacing, there is a corresponding error in wire identification at least in one of the chambers considered. Therefore the test of corrected wire identification is a gate on the variables Δx_{ij}^{min} with a width of approximately one wire spacing. This test was the one which was critical in our experiment. As Fig. 3.10 shows, in the plane associated with the dispersion direction, the variable Δx_{ij}^{min} has a distribution characterized by the presence of two additional sizeable peaks (up to 7.5% and $\approx$ 4% of the total event number respectively) centered at $\Delta x_{ij}^{min} = w$ and $\Delta x_{ij}^{min} = 2w$, w being the spacing between signal wires. This effect is present both in the X and in the Y chambers. However, this is critical for our experiment only as far as the X chambers are concerned because of the different momentum spread of the two particles detected.

This effect occurs whenever the tail of the distribution of times associated with one wire exceeds the time shift Δt introduced by the delay line between signals coming from adjacent wires. The fraction of events rejected because of this test was anomalously large (up to 30%) during our experiment. Table 3.1 shows the summary of the fraction of events which passed this test in three different

Figure 3.10: Spectra for the wire identification variables in the dispersion direction.

Table 3.1: Fraction of events passing the wire identification test for the drift chambers used in our experiment.

	^3H detection	^3He detection	Continuum proton
Δx_{12}	85 %	69%	89%
Δx_{34}	89%	73%	92%
Δy_{12}	87%	78%	91%
Δy_{34}	95%	84%	97%

experimental conditions (detection of ^3H , ^3He , continuum calibration run.)

The numbers given represent an average over the active area of the detectors considered. Their difference reflects a difference in overall efficiency of the drift chambers for the detection of the three kinds of particles considered. A difference between ^3H and ^3He is easy to explain, as the chambers were operated at different voltages in the two cases in order to compensate for the difference in ionizing properties of the two particles. It should be corrected by the 'chamber efficiency factor', which will be discussed in a subsequent section.

A careful investigation on the chamber performances showed that not only the differential efficiency distribution was different across the different chamber planes, but it also had a shape dependent upon the kind of particle detected. This effect is illustrated in Figs. 3.11 and 3.12 which compare the relevant spectra for the two detection conditions.

It can be seen that, particularly in the case of ^3H , there is a difference in behaviour exhibited by the front and rear chambers. The front chamber planes are characterized by spectra showing sharp peaks, corresponding to local strong oscillations in differential efficiency. The rear chambers have a smoother behaviour, but exhibit nevertheless a rather sharp discontinuity in the neighborhood of $x=0$. The corresponding ^3He spectra show some weak similarity to the ^3H ones, but have an overall greater uniformity.

The final analysis involves the integration of the data over some angular range, and, consequently, over some x_f interval corresponding to the range of

Figure 3.11: X_f distribution of the events corresponding to 0 (top), 1 (middle), and 2 (bottom) wire difference in extrapolated position for the front chambers C_1, C_2 .

Figure 3.12: X_f distribution of the events corresponding to 0 (top), 1 (middle), 2 (bottom) wire difference in extrapolated position for the rear chambers C_3, C_4 .

Figure 3.13: Scattering angle spectra gated by the conventional good event test.

momenta spanned by the detected particles scattered in this angular region. Unfortunately the x_f regions corresponding to the same angular interval are not the same for ^3H and ^3He because of their different distribution of energy loss in the target. This effect is illustrated by Fig. 3.13, which shows the angular spectra for ^3H and ^3He gated by the conventional good event (corresponding to the selection of the events belonging to the first peak of $\Delta x_{ij}^{\min}$).

The effect on the ^3H spectrum produced by the differential efficiency distribution is evident. As small scattering angles are associated with higher average momenta of the detected particle, there is a discontinuity in the spectrum indicating that in this region there is on an average a lower efficiency.

There is one consideration which makes this effect less critical. As the interest of the experiment is concentrated in the accurate determination of integral quantities which do not have a sharp energy dependence, the momentum resolution of the reconstructed event is not a relevant parameter. The acceptance of events corresponding to $\Delta x_{ij}^{\min}$ equal to two drift cells corresponds to a fractional

Figure 3.14: Normalized x_f distributions for the reactions $D(p, {}^3\text{H})\pi^+$ and $D(p, {}^3\text{He})\pi^0$ gated by the 'extended good event' test.

momentum uncertainty $\Delta\delta$ equal to 0.087% ($\Delta p_{out} \approx 1.3$ MeV/c) which is irrelevant with respect to the other sources of missing mass broadening discussed in the section describing the Monte Carlo simulation performed. This suggests that we can adopt an 'extended good event' test, accepting Δx_{ij} in the range between 0 and two drift cells without any harm to our interests. Fig. 3.14 shows a comparison between the normalized x_f spectra for ${}^3\text{H}$ and ${}^3\text{He}$. It can be seen that the average agreement between the two spatial distributions is very satisfactory.

Further support for this choice is provided by the comparison between the angular spectra corresponding to the two reactions. As seen in Fig. 3.15 the discontinuity present in the case of ${}^3\text{H}$ shown in Fig. 3.13 is removed by adopting this test.

The other conditions applied in the selection of 'good events' are conventional. They involve the particle identification test discussed in the previous chapter and the so-called 'xang cut'. The latter has the purpose of rejecting the particles following peripheral trajectories within the spectrometer, which are likely to have their trajectory altered by scattering from the gap walls. The scatter

Figure 3.15: Scattering angle spectra gated by the 'extended good event' test. Compare with Fig. 3.13.

plots corresponding to these two tests are given in Figs. 3.16 and 3.17 respectively. ,

Figure 3.16: Dot plots for ^3H and ^3He identification, abscissa are pulse heights and ordinates are time of flight. (box 1 contains the particles of interest in both cases.)

Figure 3.17: Dot plot for XANG cut. The abscissa is x_f and the ordinate is related to the angle θ_t . Box 4 defines the fiducial space.

3.5 The method of analysis.

A choice is available in order to obtain the yields from which the observables of interest are calculated. It is possible to integrate the angular spectra in selected intervals or to analyze the missing mass peaks gated by a specific angular interval.

The latter solution was adopted in the analysis of this experiment. The motivation for this choice is illustrated in Fig. 3.18 which shows the tails of the missing mass peaks for normal and reverse polarization superposed and normalized with respect to the beam monitor counts accumulated during the respective polarization states and the peaks normalized with the same method. It can be seen that in the tail regions displayed, the ratio normal/reverse ≈ 1 , which implies an analyzing power very close to zero ($|A_y| \leq 0.1$). In the region of the main peaks $A_{y0} \approx 0.4$ which gives normal/reverse ≈ 2 . Near-zero analyzing power indicates the dominance of background events in the tail regions. These events are most likely produced in the material (mostly mylar and aluminized mylar) surrounding the active volume of the target. One needs to subtract these background events in order to obtain the true results. This is possible only with the missing mass spectra, as the background produces no structure in the scattering angle spectrum.

In order to evaluate the correctness of our background subtraction procedure the distribution of the analyzing power as a function of the missing mass examined by calculating $A_{y0}(MM)$ for the two reactions of interest, with intervals of integration 6 MeV wide. The results for ^3H and ^3He respectively are shown in Fig. 3.19 where the shapes of the MM spectra have been also superposed.. It can be seen that these distributions allow an identification of the region where 'signal' events are dominant and the region where 'background' events are dominant. Because of the wider range of energy loss at the target, the ^3He 'signal' peak has a greater spread in missing mass. This feature makes it more difficult to obtain a reliable separation of the background and it also induces a greater sensitivity of the final results to the subtraction performed. One consideration which makes

Figure 3.18: Comparison of the yields in the tail region the missing mass spectra for normal and reverse polarization

Figure 3.19: $A_{\nu 0}$ as a function of missing mass for the two reactions (no background subtraction has been done). The shape of the missing mass peak is shown in each case to illustrate the weight of different regions of missing mass.

this problem less severe is the similarity between the shape of the tails of the missing mass spectra for ^3H and ^3He . This property is illustrated in Fig. 3.20, which shows the missing mass spectrum for ^3H scaled by a factor M with respect to the corresponding one for ^3He , where M is given by:

$$M = \left(\frac{ER04}{CF} \right)_{^3\text{He}} / \left(\frac{ER04}{CF} \right)_{^3\text{H}}$$

where:

- ER04 refers to the counts of the beam integrator and
- CF is the correction factor for chamber efficiency and computer livetime.

This comparison led us to the conclusion that the most reliable way to proceed was to extract the background distribution from ^3H , where the 'signal' peak is less wide and to subtract a distribution with the same shape scaled by a factor M from the corresponding ^3He spectrum. This choice is supported also by the Monte Carlo simulation, which was performed for ^3H and ^3He background events separately and showed no difference between the distribution in missing mass for the two cases. The analysis of the tails of the ^3H spectrum allowed an estimate of the analyzing power of the background to be -0.03 ± 0.06 . The shape of missing mass spectrum associated with this kind of event was obtained from the Monte Carlo program. This spectrum was normalized with respect to the experimental one in the tail region on the low missing mass (between $-70.$ and $-40.$ MeV). The normalized background spectrum was empirically corrected at the missing mass boundary regions in order to match the experimental distribution. The limited ability of the program in reproducing the MM distribution in these regions can be understood in terms of the fact that the Monte Carlo simulations for x_f spectra show significant differences from the experimental distributions at the extreme of the acceptance (see Figs. 2.11 and 2.12). Fig. 3.21 shows the missing mass spectra for ^3H and ^3He . The background distribution subtracted is indicated in dotted lines.

Perhaps a more satisfactory way to determine background would have been

Figure 3.20: Comparison between the tails of the normalized missing mass spectra for ${}^3\text{H}$ and ${}^3\text{He}$ detection.

Figure 3.21: Missing mass spectra for the two reactions. The dotted curves indicate the background level.

to measure it. This would have required us to take data with the liquid target container filled with hydrogen and the magnetic spectrometer at the two field settings corresponding to ^3H and ^3He detection. The hydrogen filling is required to reproduce the spatial distribution of the target flask (bulging) and to generate the same energy loss distribution as during the runs corresponding to the reactions of interest. During this experiment it was not possible to perform these background runs because of failure of the mechanism for emptying the target and refilling it with hydrogen.

Another choice to be made in analyzing the data was the angular gate to apply to the missing mass spectra. The statistical accuracy is proportional to the width of the interval of angular integration $\Delta\theta_{ac}$. In addition, the region of the drift chambers which is illuminated by the events increases proportionally to $\Delta\theta_{ac}$. The efficiency correction, which will be discussed later is an integral quantity, determined by the average behaviour of the individual chamber planes. It is therefore more appropriate for wider x_f intervals, over which the effects of the details of the differential efficiency are averaged out. These considerations suggest the choice of a wider angular gate. On the other hand, the effect of the different distortions of the angular acceptance curve for ^3H and ^3He due to the difference in their energy loss at the target becomes more serious for wider angular acceptance. The choice of a single angular interval is not easy. We have therefore analyzed the data with two different angular cuts. The efficiency for each angular gate $\epsilon(\Delta\theta)$ was obtained from the Monte Carlo simulation. The difference between the results obtained in the two angular gates should give us a quantitative indication of the uncertainty of the correction applied. As we shall see later the difference was found to be well within the specified errors.

Another remark must be made in connection with the angular integration performed. Equal laboratory angles of the heavy reaction products correspond to different π angles in the center of mass reference frame, because of the difference of mass between π^+ and π^0 of the two reactions. In order to remove this kinematical

Table 3.2: Average angles for the observables measured.

${}^3\text{H}$			${}^3\text{He}$		
$\Delta\theta_{lab}(\text{deg})$	$\langle\theta_{lab}\rangle(\text{deg})$	$\langle\theta_{cm}^\pi\rangle(\text{deg})$	$\Delta\theta_{lab}(\text{mrad})$	$\langle\theta_{lab}\rangle(\text{deg})$	$\langle\theta_{cm}^\pi\rangle(\text{deg})$
11.21 - 12.13	11.674	130.520	11.26 - 12.17	11.715	130.528
11.09 - 12.47	11.778	130.033	11.14 - 12.52	11.810	130.085

effect, the angular regions of integration were chosen with a relative displacement such that the corresponding average π angle in the center of mass was the same for the two reactions to be compared. Table 3.2 gives the ranges employed for ${}^3\text{H}$ and ${}^3\text{He}$ respectively and the corresponding average laboratory angle of the detected particle and the π angle in the center of mass.

3.6 Calculation of $R(\theta)$ and $A_{\nu 0}(\theta)$.

The normalized yields used in the calculation of the ratio $R(\theta)$ and $A_{\nu 0}(\theta)$ were obtained through the relationships:

$$Y'_n = \frac{CF_n \cdot Y_n}{ER04_n \cdot \epsilon_n(\Delta\theta)} \quad (3.13)$$

$$Y'_r = \frac{CF_r \cdot Y_r}{ER04_r \cdot \epsilon_r(\Delta\theta)} \quad (3.14)$$

where:

- the subscripts n and r refer to the beam polarization state (normal or reverse),
- Y is the yield obtained by integrating the missing mass spectrum in a selected interval after having performed background subtraction,
- CF is the correction factor for chamber efficiency and computer livetime,
- $ER04$ is the reading of the scalers associated with one of the beam integrators located in the scattering chamber,
- $\epsilon(\Delta\theta)$ is the detection efficiency in the angular gate considered as obtained from the Montecarlo simulation.

The computer livetime τ_c is obtained by taking the ratio between a scaler which counts the number of hardware triggers which take place when the computer is not busy and a scaler which counts the total number of hardware triggers. The chamber efficiency CE is determined as the fraction of events which fall within the PID box and simultaneously satisfy the 'good chamber test' discussed in a previous section. The correction factor CF is given by:

$$CF = \frac{1}{\tau_c \cdot CE} \quad (3.15)$$

Table 3.3 gives the values of the parameters discussed above as employed to obtain the normalized yields for the reactions of interest.

The efficiency parameters in a given angular gate obtained from the Monte Carlo simulation are quoted with the statistical errors associated with them.

Table 3.3: Parameters employed to normalize experimental yields.

	^3H		^3He	
	normal	reverse	normal	reverse
τ_c	0.865	0.886	0.886	0.889
CE	0.733	0.742	0.601	0.611
$ER04$	3414901	3084917	9831786	8850427
$\epsilon(\Delta\theta_1)$	0.9780 ± 0.0014		0.9450 ± 0.0014	
$\epsilon(\Delta\theta_2)$	0.9890 ± 0.0017		0.9770 ± 0.0017	

The ratio between the cross sections, R is calculated as:

$$R = \frac{[J \cdot (Y'_n + Y'_r)]_{^3\text{H}+\pi^+}}{[J \cdot (Y'_n + Y'_r)]_{^3\text{He}+\pi^0}} \quad (3.16)$$

We note that the yields $Y'_{n,r}$ above are integrated over the full peaks (effectively $-40 \geq MM \leq 40$ MeV).

The analyzing power is obtained as:

$$A_{y0} = \frac{1}{P_B} \cdot \frac{(Y'_n - Y'_r)}{(Y'_n + Y'_r)} \quad (3.17)$$

The beam polarization was obtained from the average between the polarization values calculated with the quench method from the beam monitors $ER02$ and $ER04$. The average value of the polarization of the beam during this experiment was 0.840.

The final distributions of the analyzing power as a function of the missing mass for the two reactions are given in Fig. 3.22. It should be noted that the deviant values of A_{y0} come from regions of MM which have very little statistical weight.

In order to obtain the observable ΔA_{y0} the data were integrated over the missing mass intervals corresponding to stable values of the analyzing power. These are respectively -4 . MeV - 20 . MeV for ^3H and -10 . MeV - 20 . MeV for ^3He .

Figure 3.22: A_{y0} as a function of missing mass for the reactions $\bar{p} + d \rightarrow {}^3H + \pi^+$ and $\bar{p} + d \rightarrow {}^3He + \pi^0$ (after background subtraction. The dotted curve indicates the missing mass shape from Fig. 3.20).

3.7 Error analysis.

We will now examine the sources of systematic errors and evaluate their effects on the precision of the observables measured.

1. Beam Energy Variation.

The relative stability of the beam energy was determined by comparing the positions of the centroid of the missing mass peak corresponding to elastic scattering from deuterium in two runs taken respectively near the beginning and the end of the experiment. This difference was ± 0.250 MeV, which corresponds to a variation $\Delta T_{beam} = \pm 0.300$ MeV. As mentioned before, at ≈ 700 MeV and $\theta_{cm}(\pi) = 130^\circ$ a change in the beam energy by 1 MeV causes less than 0.5% change in the cross section. Therefore $\Delta T_{beam} = \pm 0.300$ MeV corresponds to less than $\pm 0.15\%$ change in the cross section. This variation, already negligible, is further reduced, as far as the ratio R is concerned, as data were taken in a ‘cycling’ mode, alternating between ^3H and ^3He detection approximately each 5 hours. Therefore fluctuations on the beam energy have on the average the same effect on the ^3H and ^3He cross sections and cancel out in R .

ΔA_{y0} is not affected by this source of error as the analyzing power is defined as a ratio between cross sections and therefore is not sensitive to variations of their absolute value.

2. Beam Position.

No monitor of fluctuations of beam position at the target was available. However a quite sensitive monitor of the beam position (≈ 2 m upstream of the target) is provided by the polarimeter which is equipped with Up–Down and Left–Right detectors. Changes on the geometrical factors G_{hor} and G_{vert} (defined in chapter 2) reflect the displacement of the beam spot in the horizontal and vertical plane respectively. A calibration in the changes of these geometrical factors with

Figure 3.23: Measured geometrical factors G_{hor} for Line-C polarimeter. The open circles correspond to ^3He detection and the filled circles correspond to ^3H detection.

respect to the beam position has been done by the MP10 staff and the results are:

$$\frac{dG_{vert}}{dy} = 0.26/\text{cm}, \text{ or reciprocally } 0.38 \text{ mm}/\% \text{ change in } G_{vert} \quad (3.18)$$

$$\frac{dG_{hor}}{dx} = 0.43/\text{cm}, \text{ or reciprocally } 0.22 \text{ mm}/\% \text{ change in } G_{hor} \quad (3.19)$$

Changes in beam position have two effects. The first is to change the effective thickness of the target, according to Table 2.2 and the second is to change the geometrical acceptance of the spectrometer. During the course of our experiment it was found that the geometrical factors G_{hor} , G_{vert} changed by up to $\pm 5\%$ about their mean values, as is shown in Fig. 3.23. This corresponds to up to ± 1.2 mm horizontal displacement and ± 1.9 mm vertical displacement. As Table 2.2 shows one millimeter horizontal displacement changes the target thickness by well under 1%. However, this variation is of no consequence as far as the error in our determination of R is concerned. What matters is the ratio of the average values

of G between ^3H and ^3He runs:

$$\rho = \frac{\langle G \rangle_{^3\text{H}}}{\langle G \rangle_{^3\text{He}}} = \frac{\left(\sum_{i=1}^N \text{ER04}_i \cdot G_i / \sum_{i=1}^N \text{ER04}_i \right)_{^3\text{H}}}{\left(\sum_{j=1}^{N'} \text{ER04}_j \cdot G_j / \sum_{j=1}^{N'} \text{ER04}_j \right)_{^3\text{He}}} \quad (3.20)$$

where the numerator represents a weighted average of the geometrical factor over the ^3H runs and the denominator represents a weighted average over the ^3He runs.

Because we alternated ^3H and ^3He runs most fluctuations effectively cancel out. For example for the horizontal displacement we obtain the average value:

$$\rho_{hor} = \frac{\langle G_{hor} \rangle_{^3\text{H}}}{\langle G_{hor} \rangle_{^3\text{He}}} = \frac{1.0024}{1.0074} = 0.995 \quad (3.21)$$

The ratio ρ_{hor} corresponds to a relative horizontal beam displacement of $\approx 0.1\text{mm}$ or to relative target thickness changes well under $\pm 0.1\%$. We have therefore made no correction for this effect. Because the beam spot at the target is a vertical line $\approx 3\text{ cm}$ in height, the effect of vertical displacement of the same order is even less. It also has been neglected.

ΔA_{y0} is not affected by target thickness variations due to beam spot movements. Monte Carlo simulation shows that there is no change in spectrometer acceptance for such small displacements in beam position either.

3. Beam Polarization.

The uncertainty in the determination of the polarization of the beam produces an error in the difference of analyzing power. A $\pm 1\%$ error in beam polarization⁸⁰ gives an error in ΔA_{y0} equal to $0.01 \cdot \Delta A_{y0} = 0.06 \times 10^{-3}$. This is completely negligible.

4. Beam Integration

In order to evaluate the effects of errors in beam integration on R , we can write:

$$R = \frac{(CF_N)_{^3\text{H}}}{(CF_N)_{^3\text{He}}} \cdot \frac{(\text{ER04}_N)_{^3\text{He}}}{(\text{ER04}_N)_{^3\text{H}}} \cdot \frac{\left[\frac{J}{\epsilon(\Delta\theta)} \cdot \left(Y_N + \frac{CF_R}{CF_N} \frac{\text{ER04}_N}{\text{ER04}_R} Y_R \right) \right]_{^3\text{H}}}{\left[\frac{J}{\epsilon(\Delta\theta)} \cdot \left(Y_N + \frac{CF_R}{CF_N} \frac{\text{ER04}_N}{\text{ER04}_R} Y_R \right) \right]_{^3\text{He}}} \quad (3.22)$$

We can safely assume that $ER04_n = ER04_r$ and $CF_n = CF_r$, as the data were taken by alternating the sign of the beam polarization each three minutes and therefore relative fluctuations were averaged out. As a consequence the two quantities within square brackets can be considered constant so that we can write:

$$R \propto \frac{(CF_N)_{^3H}}{(CF_N)_{^3He}} \cdot \frac{(ER04_N)_{^3He}}{(ER04_N)_{^3H}} \quad (3.23)$$

From this relationship we can deduce that:

$$\left(\frac{\Delta R}{R} \right)_{ER04} = \left[\left(\frac{\Delta ER04}{ER04} \right)_{^3He} - \left(\frac{\Delta ER04}{ER04} \right)_{^3H} \right] \quad (3.24)$$

This shows that the absolute precision of the beam monitor $\Delta ER04/ER04$ does not affect the error in R ; only the difference during 3H and 3He measurements matters. In order to determine the reliability of the stability of the beam monitor $ER04$ with time we consider the ratio $(ER04/ER02)$. $ER02$, the other beam integrator located in the scattering chamber, was known to have a small gas leak and therefore a monotonic drift in its gain. Therefore a linear least square fit has been made to the ratio $ER04/ER02$ as a function of time. The standard error of this fit is found to be 0.15%, which shows that there were no significant short-term fluctuations in $ER04$ (or $ER02$).

The ratio R is sensitive to long term drifts of the beam monitors employed. As Eq. (3.23) shows, if data for the two reactions had been taken in two separate long runs this source of systematic error would not be negligible. However, since we have alternated many 3H and 3He runs, the summed data are averaged over the same experimental conditions and no error due to beam integration survives. This argument also applies to the determination of ΔA_{y0} . Therefore no error associated with beam integration uncertainty has been attributed to these observables.

5. Target density.

A possible source of error in the measurement of cross sections using a liquid target is a fluctuation of the target density produced by partial boiling of the liquid

deuterium. An indication of the stability of the density is given by the target pressure. During our experiment the target pressure was monitored continuously and had a stable value of 14.5 psi ($\pm \leq 0.05$ psi)

6. Correction Factor CF .

Uncertainties in the correction factor CF are an important contributor to the final uncertainties in R . The correction factor is the product of the computer livetime correction $1/\tau_c$ and the chamber efficiency correction $1/CE$.

The dependence of the relative error upon this parameter is again:

$$\frac{\Delta R}{R} = \left[\left(\frac{\Delta CF}{CF} \right)_{^3H} - \left(\frac{\Delta CF}{CF} \right)_{^3He} \right] \quad (3.25)$$

The livetime τ_c is determined as the ratio between two scalers which count respectively the number of taped triggers and the total number of triggers. Therefore we can apply to it the same argument by which we demonstrated that no error is associated with the beam integrator. A more careful analysis must be performed for the chamber efficiency CE , as it is determined from ratios of tests applied to data words associated with detectors operating at different voltage settings and detecting particles with different ionizing properties. Therefore the assumption of equal accuracy of the chamber efficiency correction can not be taken for granted. The statistical error in CE is not negligible ($\pm 0.23\%$). It contributes an error in R equal to $\pm 0.33\%$.

As the efficiency correction is an integral quantity, which averages out the differential distribution over the whole active Δx_f region, there is an error introduced when it is applied to a restricted region within Δx_f . Moreover, as the x_f distributions of the events corresponding to the angular gates used in the analysis are not exactly the same for ^3H and ^3He a corresponding error in the ratio can be introduced. An estimate of this source of error can be obtained with the information from the Monte Carlo simulation. As mentioned before, the program reproduces rather accurately the distribution of events relative to the coordinates

at the front plane. The x_f distribution, obtained under the assumption of uniform efficiency of the detectors within their geometrical boundaries, agrees rather well with the experimental one in the central region ($-10.$ to $+10\text{cm}$). However it exhibits a slower fall-off at the two edges of the vertical acceptance, This effect can be attributed to non-uniform efficiency of the drift chambers or to a less efficient transmission of peripheral rays (for instance, because of scattering at the walls of the spectrometer vacuum chamber). Fig. 3.24 shows the x_f distribution for the events corresponding to ^3H and ^3He detection for the gates $\Delta\theta_1$ and $\Delta\theta_2$ respectively. It can be seen that the ^3H and ^3He spectra have a different distribution and therefore a relative error can be introduced in applying only an integral chamber efficiency correction to them. In order to get an estimate of this error, we used the differential efficiency distribution obtained by comparison between the experimental x_f spectrum and the corresponding one generated by the Monte Carlo simulation for the continuum run $d(p,p')X$. In each region the ratio between the area under the experimental Δx_f bin and the corresponding Monte Carlo simulation bin has been taken ($\epsilon_c=1$ in the central region). The efficiency in the central region has been obtained by inverting the relationship:

$$CE = \frac{1}{6} \sum_{n=1}^6 \frac{Y_i^{sim}}{Y_i^{exp}} \cdot \epsilon_c \quad (3.26)$$

where:

CE is the experimental chamber efficiency correction

Y_i^{sim} is the partial yield in the i 'th x_f interval for the spectrum generated by the Monte Carlo simulation.

Y_i^{exp} is the corresponding partial yield for the experimental spectrum,

ϵ_c is the efficiency in the central region.

Then the relative error between the corrected yield calculated by weighting the partial yields in Δx_i with their own chamber efficiency coefficient and the

corrected yields calculated by applying the overall CE correction has been evaluated for the two reactions studied in our experiment. This calculation has given a 0.61% error in R for gate $\Delta\theta_1$ and 0.84% in R for gate $\Delta\theta_2$. This source of error has no effect on $\Delta A_{\nu 0}$, as this observable does not depend upon the absolute value of the efficiency correction.

7. Background Subtraction.

A major source of error is the uncertainty in the procedure of background subtraction. In order to evaluate the sensitivity of the R and $\Delta A_{\nu 0}$ to this source of systematic error, background distributions increased by $\pm 10\%$ were generated and the corresponding corrected yields were used in calculating $\Delta A_{\nu 0}$ and R .

The effect of the background subtraction uncertainty on R is found to be $\pm 0.68\%$ for the angular gate $\Delta\theta_1$ and $\pm 0.61\%$ for the angular gate $\Delta\theta_2$.

The corresponding uncertainty in $\Delta A_{\nu 0}$ is found to be $\pm 1.8 \times 10^{-3}$ for both gates.

8. Summary.

Table 3.4 summarizes the various sources of error mentioned before. The errors have been combined in quadrature to determine the total systematic error attributed to the observable R . The error in $\Delta A_{\nu 0}$ is essentially the one produced by background subtraction, i.e. $\pm 1.8 \times 10^{-3}$.

Table 3.4: Summary of error analysis.

Source of error	% in R		$\Delta A_{\nu 0}$
	$\Delta \theta_1$	$\Delta \theta_2$	$\Delta \theta_1$ and $\Delta \theta_2$
Beam energy variation	<0.15		0.06×10^{-3}
Beam position variation	<0.1		
Error in beam polarization			
Error in beam integration	<0.15		
Target Density	≈ 0		
Chamber efficiency (statistical accuracy)	0.33	0.33	1.8×10^{-3}
Chamber efficiency (differential)	0.61	0.84	
Background subtraction	0.68	0.61	
Total	0.97	1.09	1.8×10^{-3}

Figure 3.24: x_f distribution of events for ${}^3\text{H}$ and ${}^3\text{He}$ within the gates $\Delta\theta_1$ and $\Delta\theta_2$.

3.8 Discussion of Experimental Results.

Tables 3.5 and 3.6 summarize the results for the difference in analyzing power and the ratio between differential cross sections. The average values of θ_{π}^{cm} and the range $\Delta\theta_{\pi}^{cm}$ over which the integration is performed are given both for ^3H and ^3He . The average values of θ_{π}^{cm} differ by less than .05% in the two reactions compared. This suggests that no difference in kinematical conditions affects these results. It should be noted that the angular regions of integration are slightly different (by $\approx 0.5\%$) for ^3H and ^3He . The ratio R has been corrected for this effect by normalizing the cross sections to the width of the angular integration region.

Recently some claims have been made that the correct variable at which to perform the comparison is the 4-momentum transfer t between the proton and the pion. The average values of t which are given in Table 3.5 correspond to a difference of less than .5% between the two reactions. Since the two angular gates have approximately 1% difference in average t , we can easily see that the 0.5% difference in t between the two reactions produces differences less than the statistical errors.

It is worthwhile to compare the results of the present experiment with the earlier results in published literature. The results for the ratio of cross sections, R have already been summarized in Table 1.7. We have also given the critique for individual experiments in Chapter 1. As stated earlier, most of the experiments quote only statistical errors. Little or no discussion of systematic errors is given. As far as statistical errors alone are concerned, our result

$$R = 2.193 \pm 0.007 \pm (0.024)$$

has only $\pm 0.3\%$ errors as compared to $\pm 2.5\%$ (Harting et al.⁷¹) to $\pm 17\%$ errors (Silverman et al.⁷³) associated with other experiments. Even after our estimate of $\pm 1.1\%$ systematic errors is included, our result for R represents a factor five to

ten improvement over all earlier experiments.

There are no earlier measurements of $\Delta A_{\nu 0}$ for the reactions studied by us. The precision of our experimental result

$$-\Delta A_{\nu 0} = 0.0065 \pm 0.0040 \pm (0.0018)$$

can only be compared to a completely different experiment, i.e., the most recent experiment to test charge symmetry in np elastic scattering⁶³. In this extremely elaborate experiment

$$\Delta A_{\nu 0} = 0.0034 \pm 0.0017 \pm (0.0008)$$

was measured. It is extremely gratifying that we have reached almost an equal level of precision in our experiment.

Table 3.5: Summary of Experimental results(I). The errors are statistical only.

Gate	$d(p, {}^3\text{H})\pi^+$				$d(p, {}^3\text{He})\pi^0$			
	$\Delta\theta^\circ$	$\langle\theta_\pi^{cm}\rangle$	$-\langle t \rangle$ (GeV/c) ²	$A_{\nu 0}(\theta)$	$\Delta\theta^\circ$	$\langle\theta_\pi^{cm}\rangle$	$-\langle t \rangle$ (GeV/c) ²	$A_{\nu 0}(\theta)$
1.	6.46	130.02	0.559	$0.3923 \pm .0023$	6.43	130.08	0.563	$0.4005 \pm .0028$
2.	4.29	130.51	0.563	$0.3937 \pm .0027$	4.27	130.52	0.567	$0.3986 \pm .0033$
	average			$0.3930 \pm .0025$	average			$0.3996 \pm .0031$

Table 3.6: Summary of experimental results (II). On each quantity, the first errors are statistical, and the second errors (in parentheses) are systematic.

Gate	$-\langle t \rangle ({}^3\text{H})$ (GeV/c) ²	$-\langle t \rangle ({}^3\text{He})$ (GeV/c) ²	R	$-\Delta A_{\nu 0}$	$-100 \cdot \Delta A_{\nu 0} / \langle A_{\nu 0} \rangle$
1.	0.559	0.563	$2.195 \pm .006 \pm (.021)$	$0.0082 \pm .0036 \pm (.0018)$	$2.05 \pm .900 \pm (.45)$
2.	0.563	0.567	$2.190 \pm .007 \pm (.024)$	$0.0049 \pm .0044 \pm (.0018)$	$1.23 \pm 1.10 \pm (.46)$
Av.	0.563		$2.193 \pm .007 \pm (.024)$	$0.0065 \pm .0040 \pm (.0018)$	$1.63 \pm 1.00 \pm (.46)$

Chapter 4

Theoretical analysis and conclusions.

As we mentioned in chapter 1, a proper interpretation of our experimental results can only be done if one has a successful theory of pion production in pd collisions. We will describe several attempts to develop such a theory below.

In absence of a theory of pion production one is forced to take a more limited approach to the interpretation of our results. Such approaches are based on the premise that although one may not be able to explain the course of differential cross sections for either of our reactions, one can still make reasonable calculations of the differences or ratios of the cross sections for the two reactions. This approach is obviously more valid for the geometrical and kinematical aspects of the two reactions. We begin with the description of one such calculation.

4.1 The calculation of Köhler.

Köhler⁹¹ has made a detailed analysis of the corrections to the ratio of cross sections

$$R = \sigma(\theta)[^3\text{H} + \pi^+]/\sigma(\theta)[^3\text{He} + \pi^0]$$

which result from the kinematical differences between the two reactions (originating from π^+ , π^0 mass differences) and the external electromagnetic corrections (wave function differences and Coulomb distortions).

While Köhler makes no attempt to calculate cross sections in his paper, his

Figure 4.1: The Impulse approximation for the $pd \rightarrow t\pi^+$ reaction (from Ref. 94).

model for the reaction essentially follows the impulse approximation of Ruderman⁶⁷ and Bludman.⁹²

The basic premise of the impulse approximation is that the incident proton interacts essentially with one of the nucleons of the deuteron. A $pp \rightarrow d\pi^+$ or $pn \rightarrow d\pi^0$ reaction takes place, and the resulting deuteron forms ^3H or ^3He with the spectator nucleon of the deuteron. This is illustrated in Fig. 4.1. In this model the differential cross sections are given by

$$\frac{d\sigma}{d\Omega}(\theta) = K \cdot g \cdot |F(\Delta)|^2 \cdot \frac{d\sigma}{d\Omega}(pN \rightarrow d\pi, \langle\theta\rangle, \langle E\rangle) \quad (4.1)$$

Here K is a kinematical factor, g is a numerical factor arising from proper inclusion of spin, isospin and antisymmetrization effects, $F(\Delta)$ is a form factor evaluated at a momentum transfer Δ , and the $pp \rightarrow d\pi$ cross section is evaluated at an average two body energy and angle.

1. Corrections in R due to wavefunction differences.

The ratio of the cross sections

$$R \equiv \frac{d\sigma(pd \rightarrow ^3\text{H}\pi^+)}{d\sigma(pd \rightarrow ^3\text{He}\pi^0)} = \frac{K^+ g^+ |F(\Delta)|^2 \sigma(pp \rightarrow d\pi^+)}{K^0 g^0 |F^0(\Delta)|^2 \sigma(pn \rightarrow d\pi^0)} \quad (4.2)$$

If, for the moment, we assume that the π^+ and π^0 masses are equal, then $K^+ = K^0$

and $g^+ = g^0$. Since $\sigma(pp \rightarrow d\pi^+) = 2\sigma(pn \rightarrow d\pi^0)$, the simple prediction $R=2$ acquires a correction factor $=|F^+|^2/|F^0|^2$. It is this correction factor due to the wave function differences which Köhler finds to be most important.

Köhler defines the form factor $F(\Delta)$ as

$$F(\Delta) = \int \psi_3(rxx)\psi_d(x)\exp(i\vec{x} \cdot \vec{\Delta})d\vec{x} \quad (4.3)$$

where ψ_3 is the three body (${}^3\text{H}$ or ${}^3\text{He}$) wavefunction, and the momentum transfer in the elementary $pp \rightarrow d\pi$ reaction is

$$\vec{\Delta} = (\vec{k}/2) + (\vec{q}/3) \quad (4.4)$$

where $\vec{k}$ and $\vec{q}$ are center of mass momenta of the incident proton and the outgoing pion. For relatively simplified choices of wave functions for the three-body systems and the deuteron, Köhler is able to obtain an analytic expression for $F(\Delta)$

$$F(\Delta) = \left(\frac{1}{(\beta + \alpha)^2 + \Delta^2} - \frac{1}{(\gamma + \alpha)^2 + \Delta^2} \right) \cdot N_3. \quad (4.5)$$

Here $\beta = 0.229 \text{ fm}^{-1}$ and $\gamma = 1.371 \text{ fm}^{-1}$ are parameters of the Hulthén wave function for the deuteron, the α 's are related to the separation energy of a neutron from ${}^3\text{H}$ and that of a proton for ${}^3\text{He}$ ($\alpha^+ = 0.907 \text{ fm}^{-1}$, $\alpha^0 = 0.890 \text{ fm}^{-1}$), and N_3 are the overall normalization constants for the ${}^3\text{H}$ and ${}^3\text{He}$ wave functions. According to Köhler $[N({}^3\text{H})/N({}^3\text{He})]^2 \equiv [N^+/N^0]^2 = 1.09$.

Köhler evaluated the correction factor for $T_p = 600 \text{ MeV}$ and found it to be weakly angle-dependent, ranging from +5.9% to +8.3%. We have repeated Köhler's calculation for our kinematical conditions, i.e., $T_p = 730 \text{ MeV}$, $\theta_{cm} = 130^\circ$, and find it to be equal to +5.3%.

Köhler estimated an additional 0.6% correction due to $T=3/2$ admixtures in the ${}^3\text{He}$ wavefunction and 1% correction due to the inclusion of the hard core in the nucleon-nucleon interaction. Thus using Köhler estimates for these corrections, we get the net correction factor for our experiment due to wave function differences to be +6.9%.

2. Phase space differences.

Due to the π^+ , π^0 mass difference the relativistic phase space for the two reactions is different.

$$PS(\pi^+) = q_{\pi^+} \frac{E_H E_{\pi^+}}{(E_{\pi^+} + E_H)} \quad (4.32)$$

$$PS(\pi^0) = q_{\pi^0} \frac{E_H E_{\pi^0}}{(E_{\pi^0} + E_H)} \quad (4.33)$$

where q_π refers to the pion momentum and E represents the total energy of particle identified by the corresponding subscript.

The ratio $PS(\pi^+)/PS(\pi^0)=0.995$ in our case, i.e. the correction is -0.5%.

3. Correction due to q_π and θ_π differences.

If the two reactions are measured at the same laboratory angle, because of the differences in π^+ and π^0 masses both the momentum q_π and $\theta_{cm}(\pi)$ are different for the two reactions. In order to correct for the differences one has to know the variation of the differential cross sections both with respect to incident energy and outgoing pion angle. Köhler estimated a +3.58% correction due to these effects for the experiment of Harting et al. In our experiment we have made the angular cuts in such a manner as to minimize both the differences in q_π and in $\theta_{cm}(\pi)$ to $\leq 1.0\%$. As mentioned earlier (in section 1.8), we estimate that these corrections lead to entirely negligible corrections in our experiment ($< .1\%$).

4. Corrections due Coulomb distortion and Impulse approximation.

Köhler argues that both are negligible ($\leq 1\%$).

To summarize, the 'external' corrections considered by Köhler amount to $\approx 6.4\%$ in our experiment. In other words, with these external corrections only, we expect $R=2.13$.

It is worth noting the since Köhler's original calculation (1960) great progress has been made in the study of few-nucleon systems. In particular, excellent wave functions are now available⁹³ for the deuteron, ^3H and ^3He . The three-body wave

functions are now available⁹³ for the deuteron, ^3H and ^3He . The three-body wave functions are based on Faddeev calculations including Coulomb forces. However, generally the greatest attention has been paid to asymptotic aspects of these wave functions. In our reactions large momentum transfers are involved and the wave function problem is much more complex. Nevertheless, it would be very instructive to make a calculation similar to Köhler's with the best wave functions available today. Hopefully, with such a calculation one would not have to resort to hand-waving arguments about effects of hard-core, and the d-state effects (neglected by Köhler) would be included in a natural manner. One would also hope that Coulomb distortion effects can also be taken into account in a more satisfactory manner. Until such a calculation is available we should consider the above estimate of the overall external correction factor as essentially qualitative.

We note that Köhler has considered no 'internal' electromagnetic corrections. These arise due to the differences in the masses of the different charge pions which are exchanged and the different charge Δ 's (Δ_{33}) which may occur as intermediate states. To shed some light on these corrections we have to consider more sophisticated theoretical calculations.

4.2 Theories of pion production.

1. Calculation of Fearing.⁹⁴

Fearing has improved on the impulse approximation theory in many ways to calculate the differential cross sections for the reaction $p\,d \rightarrow {}^3\text{H}\,\pi$.

1. Proton and pion distortions are included via an eikonal approximation which requires as input the nuclear shape and the elementary πN and $N\,N$ scattering cross sections, thus making the model into a distorted wave impulse approximation (DWIA).
2. A number of technical improvements involving spin, antisymmetrization and better kinematics have been made.
3. Better wavefunctions, which reproduce electromagnetic form factors have been used. D-state component of the deuteron wave function is included.
4. Better energy choice for $pp \rightarrow \pi\,d$ amplitude is made.

These improvements lead to mixed results. Below $T_p=500$ MeV the shape of the experimental differential cross sections is well reproduced, although the predicted absolute values are approximately factor two too small at all angles. With increasing energy the discrepancy with experimental data becomes worse, especially at back angles. By $T_p=800$ MeV the forward angle cross sections are almost a factor 3 too small and the back-angle cross sections are predicted to be more than a factor 20 too small. This is illustrated in Fig. 4.2 where the experimental data from Ref. 90 are also shown.

Fearing has discussed several possible reasons for the failure of these calculations at $T_p > 500$ MeV. The most important of these is the neglect of all but $\pi\,d$ intermediate states. In the elementary pion production process the $pp \rightarrow \pi d$ channel certainly dominates at threshold (≈ 288 MeV). However at 500 MeV the three body channel $pp \rightarrow \pi^+pn$ becomes of equal strength, and at 800 MeV it

Figure 4.2: Differential cross sections for the $pd \rightarrow t\pi^+$ reaction at 800 MeV. The experimental data are from Ref. 90. The theoretical curves are from Refs. 94 and 98.

accounts for more than 90% of the single pion production cross section. Certainly the neglect of that part of the $pn\pi^+$ production in which the pn pair emerges in the nearly bound 1S_0 state is not justified.

The second important assumption is the factorization represented by Eq. 4.26, where the $pp \rightarrow d\pi$ cross section is factored out of the form factor integral $F(\Delta)$. This is only justified when the $pp \rightarrow d\pi$ amplitude does not vary rapidly. At the resonance, $T_p \approx 600$ MeV, this may not be a good approximation, as has been pointed out by Green et al.⁹⁵

To summarize, although Fearing's work represents definitive improvement over the earlier work of Ruderman,⁶⁷ Bludman⁹² and Köhler⁹¹, it fails badly in our domain of interest. Further, Fearing does not study the reaction $pd \rightarrow ^3\text{He}\pi^0$ and is therefore not able to help us with Coulomb differences in R . Fearing makes no calculation for $A_{\nu 0}(\theta)$.

2. Calculations of Green and collaborators.⁹⁵⁻⁹⁷

Green and Niskanen⁹⁶ achieved considerable success in describing the elementary reaction

in a coupled-channel isobar model. Following this success, Green and collaborators⁹⁵ attempted to describe the $pd \rightarrow t\pi^+$ reaction in terms of these same configurations, as illustrated in Fig. 4.3.

Early attempts with this model, in which one relates $pp \rightarrow d\pi^+$ amplitudes to $pd \rightarrow t\pi^+$ amplitudes (not cross sections, as is done in the approach of Fearing), had poor success. However the most recent calculations⁹⁷ show considerably better success with the use of Faddeev wave functions and several technical improvements. As shown in Fig. 4.4 differential cross sections at all angles are reproduced quite well at $T_p=400$ Mev. Above 400 MeV the agreement is reasonably good at forward angles and quite poor at back angles. By 600 MeV the predicted back-angle cross sections are almost a factor 50 too small. No cross

Figure 4.3: The basic pion production mechanism considered by Green and collaborators. The bubble in the diagram on the left is “expanded” on the right to show all the contributions.

Figure 4.4: $pd \rightarrow t\pi^+$ differential cross section predictions from the calculations of Green and collaborators (from Ref. 97).

section calculations have been reported for 800 MeV. No $A_{y0}(\theta)$ calculations have been made.

In summary, the model of Green and collaborators is as successful as that of Fearing at $T_p \leq 400$ MeV. At higher energies it suffers from the same malady as the Fearing model, only somewhat worse. Its cross sections fall far too rapidly at the back angles. It is therefore little consolation that this model is a microscopic one while the model of Fearing is essentially a phenomenological one. In any case, this model also promises little help in solving the problem posed by our experiment.

3. Calculation of Laget and LeColley.⁹⁸

This is the most recent calculation. It makes substantial departures from previous efforts and will therefore be described in some detail.

At intermediate energies, the formation of the Δ_{33} resonance in the s-channel is considered to be the dominant contribution to the cross section for the reactions $p + p \rightarrow d + \pi^+$ and $p + d \rightarrow {}^3\text{H}({}^3\text{He}) + \pi^+(\pi^0)$. Other processes play a significant role, particularly in polarization observables which are sensitive to interference terms between different amplitudes contributing to the reaction studied.

The amplitudes calculated in this model are illustrated in Fig. 4.5.

For clarity sake, the discussion of the various amplitudes will refer to the reaction $p + d \rightarrow {}^3\text{H} + \pi^+$. The changes made to calculate the cross section and analyzing power for $p + d \rightarrow {}^3\text{He} + \pi^0$ will be outlined at the end of this section.

The first diagram (a) corresponds to pion production mediated by the exchange of one nucleon in the t-channel. The vertex function which enters in the calculation of this amplitude is the overlap Ndt between one nucleon distributions in the triton and the deuteron. This amplitude is calculated for the deuteron and triton wave functions in momentum space.(the symbol N designates a nucleon). This diagram contributes mainly at forward angles.

The second diagram (b) corresponds to a two nucleon mechanism for pion

Figure 4.5: Diagrams for the amplitudes associated with the reaction $p + d \rightarrow {}^3\text{He} + \pi^+$ according to Laget and LeColley (Ref. 94).

production, by interaction with the deuteron cluster. In the calculation of this diagram the vertex function involved are $dNN\pi$ and $t(NN)N$. In this amplitude the active pair in the triton is constrained to be in an isospin state $T=1$ in order to conserve isospin at the vertex involving the deuteron. This diagram contributes both at forward and back angles with a minimum at 90° .

The third diagram (c) represents the formation for Δ_{33} resonance in the s-channel. The amplitude for this process involves the combination of elementary matrix elements $\sqrt{2}T_{\pi^+p \rightarrow \pi^+p}$ (Δ^{++} intermediate state) and $-T_{\pi^+n \rightarrow \pi^0p}$ (Δ^+ intermediate state). In this diagram the active pair in the triton can be either in the $T=0$ or in the $T=1$ isospin state. Both the contributions have been calculated, but the process turns out to be dominated by the Δ formation in the $T=0$ (pn) pair.

The last diagram (d) corresponds to non-resonant contribution to pion rescattering. The S,P,D partial waves are taken into account. They are calculated according to the phase shift fit by Rowe et al.⁹⁹.

The triton wavefunction used in the calculation of the vertex functions discussed above has been obtained¹⁰⁰ by solving the Faddeev equations for the Reid soft-core potential.¹⁰¹ This potential is also used in the Schrödinger equation which gives the deuteron wavefunction.

In the calculations performed for the final state ${}^3\text{He} + \pi^0$, the mass effects associated with the mesons and baryons involved in the intermediate and final states are included. The correct values of the masses of the π^+ and π^0 have been used, as well as the correct masses for ${}^3\text{H}$ and ${}^3\text{He}$. In the diagrams involving the formation of the Δ resonance, the Δ^{++} and Δ^+ intermediate states are involved in the case of ${}^3\text{H}$ final state, whereas Δ^+ and Δ^0 are involved in the case of ${}^3\text{He}$. The values of the isobar masses adopted in the present calculation are respectively $M_\Delta = M_{\Delta^{++}} \approx M_{\Delta^+} = 1230$ MeV for the ${}^3\text{H}$ case and $M_\Delta = \frac{1}{2}(M_{\Delta^+} + M_{\Delta^0}) = M_{\Delta^{++}} + 1.4$ MeV = 1231.4 MeV for the ${}^3\text{He}$ case. The same wave functions are used for ${}^3\text{H}$ and ${}^3\text{He}$, i.e., no Coulomb induced wave function differences have

Figure 4.6: Analyzing power for the $pd \rightarrow t\pi^+$ reaction at 800 MeV. The experimental data are from Ref. 90. The theoretical curve is from Ref. 98.

been considered.

Laget and LeColley have made $\sigma(\theta)$ and $A_{y0}(\theta)$ calculations for us at several energies. Since complete data is available at $T_p = 800$ MeV⁹⁰, we discuss the 800 MeV results first.

In Fig. 4.3 we show the results of these calculations for the differential cross sections at 800 MeV. We note that although the agreement of these calculations with the data is much better at forward angles than that obtained by Fearing, almost no improvement is observed at back angles. Fig. 4.6 shows the data and the predictions for $A_{y0}(\theta)$ at 800 MeV. The features of the data at forward angles are 'qualitatively' reproduced, but at the back angles there is little similarity between the data and the predictions. Even the sign of $A_{y0}(\theta)$ is not predicted correctly for $\theta \geq 120^\circ$.

In view of the above discouraging results for back-angle $\sigma(\theta)$ as well as $A_{y0}(\theta)$, it would appear that little confidence can be had in the results for R and $\Delta A_{y0}(\theta)$

from this model. However, in absence of anything better, this is what we do below.

In Figs. 4.7 and 4.8 we show R and $100\Delta A_{\nu 0}/A_{\nu 0}$ as calculated by Laget and LeColley for $T_p = 730$ MeV. From these calculations we obtain, for $\theta = 130^\circ$:

$$R' = 1.895 \quad (4.8)$$

$$[100 \cdot \Delta A_{\nu 0}/\langle A_{\nu 0} \rangle]_{th} = -0.308 \quad (4.9)$$

As noted earlier, since Laget and LeColley did not include the external Coulomb corrections, we need to raise the ratio R' by the external correction factor $= 1.064$ as determined in Sec. 4.1. This leads to

$$R_{th} = 1.064 R' = 2.014 \quad (4.10)$$

These 'theoretical predictions' (Eqns. 4.9 and 4.8) should be compared to our experimental results

$$R = 2.193 \pm 0.007 \pm (0.024) \quad (4.11)$$

$$100 \cdot \Delta A_{\nu 0}/\langle A_{\nu 0} \rangle = -1.65 \pm 1.10 \pm (0.46) \quad (4.12)$$

We note that our result for R is significantly larger than the theoretical prediction. Perhaps this discrepancy indicates nothing more than the uncertainty in the theoretical prediction. We have already dwelt at length on the problems of the theoretical calculations. On the other hand, it is possible that the discrepancy is truly indicative of a charge independence breaking effect. Only better theoretical calculations can resolve this tantalizing question.

Our result for the difference in analyzing powers has the same sign as the theoretical prediction, but it is larger. However, the difference is not statistically too significant. A factor two to three improvement in experimental errors would be of great help.

Figure 4.7: Laget and LeColley's prediction for the ratio $\sigma(^3\text{He} + \pi^+)/\sigma(^3\text{He} + \pi^0)$ at $T_p = 730$ MeV. These predictions include no corrections for external Coulomb effects.

Figure 4.8: Laget and LeColley's prediction for the difference in analyzing power between $\bar{p} + d \rightarrow ^3\text{He} + \pi^+$ and $\bar{p} + d \rightarrow ^3\text{He} + \pi^0$ reactions at $T_p = 730$ MeV.

4.3 Conclusions.

The goal of this experiment was to make high precision measurements of the ratio, R between the differential cross sections and the difference ΔA_{y0} between analyzing powers for the two reactions $\bar{p}d \rightarrow {}^3H\pi^+$ and $\bar{p}d \rightarrow {}^3He\pi^0$. We believe that we have satisfactorily realized this goal. Our new result for R is almost an order of magnitude more precise than previous measurements, and we have made the first measurement of ΔA_{y0} ever, with errors which are less than ± 0.005 , or $\approx 1\%$.

The theoretical interpretation of our results largely eludes us, at least so far. It is true that no theoretical explanations of charge symmetry or charge independence tests ever calculate the absolute values of the observables. Only the differences [e.g., scattering length differences $\Delta a = a_{nn} - a_{np}$, or analyzing power differences $\Delta A = A(\bar{n}p) - A(n\bar{p})$], or ratios [e.g., for the $pn \rightarrow d\pi$ reaction $\sigma(\theta \leq 90^\circ)/\sigma(\theta \geq 90^\circ)$] are calculated. However, in most cases the general theoretical model for the observable is reasonably well understood and accepted. Unfortunately this is not the case with our reactions. As we have shown, no satisfactory models for single pion production in pd collisions exist so far. The theory is beset with serious problems and there are indications (from the perennial difficulties of all the models at back angles) that some major piece of essential physics is missing in all the models extant. It is possible that despite the great progress made in few-nucleon physics in the last decade, the high momentum components in these systems are still not well understood¹⁰². It is possible that Δ -components in the two- and three-body systems play important role and need to be investigated seriously. Perhaps there are serious flaws in the use of impulse approximation. We are obviously not equipped to answer these questions. All we can hope is that our precision measurements will induce the theorists to pay serious attention to this important problem in few-body physics, in which study of ground state properties has so far been the preoccupation.

In absence of reliable theoretical predictions we can not draw any hard conclusions from our results. The large value of R observed can not be really interpreted unless at least the effect produced by Coulomb induced wave function differences is calculated with the best wave functions available today. We hope that this will happen soon. These wave function differences, in any case, should not affect analyzing powers, which are ratios of cross sections. Similarly many other uncertainties, e.g., distortions, should largely cancel out in A_{y0} . This makes us bold enough to suggest that the $-1.3\% \pm 1\%$ difference in analyzing powers which we have measured does, in itself, indicate that charge independence breaking is at the level less than $\approx 2\%$. This is a significant result because no measurements of charge independence breaking in this energy and momentum-transfer range have been made ever before.

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Appendix A

The linear accelerator.

An overall view of the Los Alamos Meson Physics Facility has been given in Fig. 2.1. The different experimental areas shown in this figure employ a beam which is accelerated in a two-staged linear accelerator. This LINAC is able to accelerate simultaneously particles from two different sources. It has three independent injectors. The first one produces protons (H^+), the second produces unpolarized negative ions (H^-), and the third produces polarized negative ions (P^-). Some details on the polarized proton source are given in the next appendix.

The first stage has an Alvarez drift tube structure operating at 201.25 MHz. It consists of acceleration regions and drift regions of increasing length to match the increasing particle velocity. In the drift tubes quadrupole lenses are used for focusing. The particles are accelerated to 100 MeV in this stage and are then fed to the main accelerator. The frequency of this section is 805 MHz. It is capable of producing a beam with final energy which can be varied step-wise between 100 and 800 MeV. The H^+ and H^- (or P^-) beams are separated magnetically at the switchyard shown in Fig. 2.1. The H^+ beam goes to the production targets of line A, from which several meson beams are generated.

The H^- beam travels along line X up to a section where some stripper foils transform part of this beam into a H^+ beam by stripping the electrons in the negative ions. The unstripped portion of the beam goes to line B. The protons are magnetically separated to go to line C, which is the beam line associated with the HRS spectrometer. A plane view of line C has been given in Fig. 2.2. This

beam line consists of a separation section, a dispersion section and a matching and twisting section.

In the separation section the beam is bent by three consecutive small angles equal to 1° , 1° , and 4° respectively. Subsequently the beam is deflected by two angles of approximately 57° in the dispersion section. Up to this point the dispersion of the beam is along the horizontal plane. As the HRS spectrometer operates in VHV mode (vertical dispersion, horizontal scattering, vertical momentum analysis), it is necessary to rotate the dispersion plane of the beam before it reaches the interaction region. This purpose is achieved by a set of quadrupole magnets which constitute the 'twister section'. Finally another set of quadrupole magnets provides the 'dispersion matching' function needed for the operation of the spectrometer in energy loss mode. In other words, they allow a tuning of the beam on target to optimize the energy resolution of the spectrometer. The design concept of 'energy loss mode' for high resolution spectrometers will be discussed in Appendix 3. More details on line C can be found in Ref. 103.

Appendix B

The polarized proton source.

The method employed to produce polarized protons was first proposed by Zavoiski ¹⁰⁴ and Madanski and Owen ¹⁰⁵. It takes advantage of the unique properties of the metastable $2s_{1/2}$ level of the hydrogen atom.

Fig. B.1 shows the energies of the $2s_{1/2}$ and $2p_{1/2}$ levels as function of an axial magnetic field applied to them.

Figure B.1: Energy-level diagram of the hydrogen atom for principal quantum number $n=2$ at an excitation energy of 10.2 eV. (from Ref. 107).

The splitting between the $2s_{1/2}$ and $2p_{1/2}$ in absence of magnetic field is produced by Lamb shift and the degeneracy between levels having the same electron spin

projection is removed by the hyperfine interaction with the proton magnetic moment. Fig B.1 shows clearly the principle on which this method is based. Atoms are pumped to the metastable state and a selective quenching of the levels α_- and β_+ is performed. Under general conditions, a longitudinal electric field couples by the α and e levels Stark mixing, whereas a transverse electric field couples the β and e levels. Besides, there is an interesting effect which was observed for the first time by Lamb and Rethford.¹⁰⁶ It affects the three levels, α , β , e , when a magnetic field $B_z \approx 574$ G is applied (this field corresponds to the neighbourhood of the crossing point shown in Fig. B.1) together with a static transverse electric field E_y and a radiofrequency longitudinal electric field E_z . E_y couples the levels β to the levels e and E_z couples α to e . In absence of the levels β , α would have a broad dip in the intensity when the radiofrequency corresponds to its separation from the levels e (which are rather broad). However, in the physical situation described above there are three sets of levels which are coupled, α , β , and e . The presence of the level β has the effect of 'sharpening' the quenching width, thus allowing selective quenching of the α_+ or the α_- levels depending upon the magnitude of B_z (the two fields correspond to the points A and B of Fig. B.1).

The actual polarized source consists of three main sections.

In the first stage protons extracted from a duplasmatron are passed through a cloud of Cs atoms. Hydrogen atoms are formed through the charge exchange reaction:

Cs is chosen because it gives the most favourable ratio between the formation of hydrogen atoms in the 2s state and in the ground state.

Next, there is a 'spin-filter' section, where a magnetic field $B_z=540$ G, a transverse static electric field $E_y=20$ V/cm and an axial RF field $E_z=20$ V/cm are applied.

Finally the hydrogen atoms are sent to an argon cell to produce H^- ions. Again the choice of the gas is determined by the goal of selective ionization of

atoms in the metastable state.

The negative hydrogen atoms (conventionally identified as P^-) are at this point injected into the main accelerator. The polarization achieved by this method is fairly good (typically ≈ 0.8).

Appendix C

The HRS Spectrometer.

The main design characteristics of the HRS magnetic spectrometer can be summarized as follows⁸⁵:

- VHV (Vertical dispersion – Horizontal Scattering – Vertical Analysis) Energy loss configuration.
- Quadrupole–dipole–dipole arrangement.
- Mean radius of curvature 3.5 m.
- Solid angle acceptance 3.6 msr.
- Momentum resolution 2.44×10^{-5}
- Range of scattering angles 2.5° – 10° with reduced beam intensity and 10° – 170° with full beam current available.
- Angular resolution 0.8 mr.
- Dispersion $\langle \delta x | \delta p \rangle$ 18.244 cm/%.

Fig. 2.3 shows the HRS facility. The magnetic spectrometer is vacuum coupled to the scattering chamber through several fixed ports and bellows. The spectrometer consists of a quadrupole lens followed by two dipole magnets. The quadrupole focuses in the $\hat{x}$ direction (radial) and defocuses in the $\hat{y}$ direction. These optical characteristics were chosen in order to minimize the gap width.

The spectrometer provides point to point focusing in the $\hat{x}$ (dispersion) direction and parallel to point focusing in the $\hat{y}$ (scattering) direction. Therefore, up to 1st order terms in the spectrometer optics, the scattering angle and momentum analysis are decoupled.

In the dispersion direction the spectrometer is designed to operate in the ‘energy loss mode’. The principle of this design approach is the consideration that excellent energy resolution is needed in nuclear physics experiments in order to identify the state of the residual undetected particle. If the cross section has a slow energy dependence and the energy spread of the beam is not too wide, it is only necessary to measure the momentum difference between the beam particle and the corresponding detected particle ($p_3 - p_1$). This purpose is achieved by focusing all the particles which undergo the same momentum loss at the same coordinate in the dispersion direction. The variation of the missing mass MM corresponding to variations of the kinematical variables p_1, p_3, θ_{sc} , (θ_{sc} is the scattering angle) is given by:

$$\Delta MM = \frac{\partial MM}{\partial p_1} \cdot \Delta p_1 + \frac{\partial MM}{\partial p_3} \cdot \Delta p_3 + \frac{\partial MM}{\partial \theta_{sc}} \cdot \Delta \theta_{sc} \quad (C.1)$$

The first two terms can be rewritten as:

$$(\Delta MM)_p = \frac{\partial MM}{\partial p_3} \cdot p_3^0 \cdot \left[\delta_3 + \frac{p_1^0}{p_3^0} \frac{\partial MM / \partial p_3}{\partial MM / \partial p_3} \delta_1 \right] \quad (C.2)$$

where δ refers to the fractional deviation from central momentum, p_1^0 identifies the central momentum of the beam line, and p_3^0 identifies the central momentum of the spectrometer. The first order optics of the spectrometer gives:

$$x_f = D_s \delta_3 + R_{11} D_b \delta_1 \quad (C.3)$$

where x_f indicates the vertical coordinate at the spectrometer focal plane, D_b represents the beam dispersion, D_s represents the spectrometer dispersion and R_{11} corresponds to the vertical magnification factor of the spectrometer. Substituting δ_3 from C.2 in C.3, we obtain:

$$(\Delta MM)_p = \frac{\partial MM}{\partial p_3} \cdot \left[\frac{x_f}{D_s} + R_{11} \frac{D_b}{D_s} \delta_1 + \frac{p_1^0}{p_3^0} \frac{\partial MM / \partial p_1}{\partial p_3} \delta_1 \right]. \quad (C.4)$$

Therefore if:

$$R_{11} \frac{D_b}{D_s} + \frac{p_1^0}{p_3^0} \cdot \frac{\partial MM / \partial p_1}{\partial MM / \partial p_3} = 0 \quad (C.5)$$

the missing mass width is not affected, up to first order, by the momentum spread, ∂p_1 of the beam. Relationship C.5 defines the condition of ‘dispersion matching’ for optimum energy resolution. In other words, Eqn. C.5 gives the dispersion D_b to which the beam must be tuned in order that its energy spread does not affect the energy resolution of the spectrometer.

Appendix D

Listing of the Monte Carlo simulation program.

```

Program Hrssim
c
c -----
c
parameter j=120
parameter ma=80
parameter m=50
character*7 particle,target
character*30 title
byte h(120)
logical kflag,xflag,yflag,stopflag,test,cases
integer ioption,
1      moption
integer seed,events,eventmax
integer nxl,nyl,nx2,ny2,laxng
integer nwindow

integer ngen1,ngen2,nspec1,nspec2
integer nxs2,nxs4,nxs5
integer nys2,nys4,nys5
integer ovf_tout,ovf_theta,ovf_mml,ovf_mm2
integer ovf_th,ovf_thg,ovf_x
integer ovf_tsout,ovf_yf,ind_tsout

double precision xcoss_beam,yocoss_beam,zcoss_beam
double precision o,phi
double precision xcoss_scat,yocoss_scat,zcoss_scat
real m2,
1      m4,
1      mmin,mmax,
1      m4tot,
1      mtarg,
1      m4th
real lowth1,lowth2,highth1,highth2
real z_scint,a_scint,p_scint
real s_y(3),s_x(3),s_dz(3)

integer ovf_tht,ovf_pht,ovf_thf,ovf_delta
integer ovf_phf
integer ny(m),ngen(m),nspec(m),nphf(m)
integer nthf(ma),ntht(ma),npht(ma)
integer nmm1(j),nmm2(j),nx(j),ngentout(j),nspectout(j)
integer ndelta(j)

real deltax,xmax
real mmasmin,dmass
real mmasmax
real theam,tshift,ekin_beam,p1,e1,p3,e3,mm
real mpion,mrecoil
real zstep(10)
real sigmacof(3)

```

```

real ec(9)
real angtgt(ma)
real thfspec(ma)
real yfront(m),angdeg(m),angmr(m),eff(m)
real mmass(j),xf(j)
real ekin(j),deltaspec(j)

```

```

common seed
common/eloss/loss,ec,stopflag

```

```

data thspec/12./
data thfmax,dthf/80.,2./
data deltax/49.5/
data xmax/30./
data ymax/5./
data ec(4),ec(5),ec(7),ec(8),ec(9)/2.,1.,20.7,2*0./

```

```

data s thick/.98/
data z scint,a_scint,p_scint/3.65,8.23,62.6/
data s rad/43.8/
data s thick/.98/
data s dz/10.635,125.7,61./
data s x/29.1,38.,42.65/
data s_y/5.35,6.5,7.2/

```

```

parameter gamma=.5772156619
parameter convrad=.017453292          !conversion deg ->rad
parameter conv=.057295779            !conversion mr-->deg
parameter beam_mass=938.28
parameter beam_charge=1.
parameter tgtem=2.23
parameter deltamax=1.7
parameter deltamin=-1.3
parameter pl=3.141592654
parameter angmin=-40.

```

```

cases=.true.
dowhile(cases)

```

```

Read input information about the case to be processed.

```

```

read(1,*,end=30)title
read(1,*)seed,loption,eventmax,tbeam,tshift,ztarget
read(1,*)deltamax
read(1,*)particle,mrecoil,zrecoil,pcentral

```

```

read(1,*)target,mtarg,ztarg,atarg,titarg,t_rad,m2
read(1,*)m1,moption,mmin,mmax,m4th
read(1,*)sigmain
read(1,*)sigmacof(1),sigmacof(2),sigmacof(3)
read(1,*)sigpht,sigtht
read(1,*)phitmin,phitmax
read(1,*)thetatmin,thetatmax
read(1,*)thspdeg,thspar,thmin,thmax
read(1,*)lowth1,highth1,lowth2,highth2
read(1,*)deltapin,deltapout,ddeltapout
read(1,*)mmassmin,dmmass

```

c
c
c
c
c
c
c

Initialization step.

```

ixang=0
thspr=thspar*convrad
thsprtr=thspdeg*convrad
csctruc=cos(thsprtr)
ssctruc=sin(thsprtr)
sths=sin(thspr)
cths=cos(thspr)

```

c
c
c
c
c
c
c
c

Beam angular coordinates in the HRS event analyzer are (0,0,1) in the laboratory frame and (0,-sths,cths) in the spectrometer frame. (angular divergence supposed = 0)

```

zpa=0.
ypa=-sths
zpa=cths

```

c
c
c

```

ngen1=0
ngen2=0
rspec1=0
nspec2=0
k_ovf=0
ovf_x=0
ovf_mm=0
ovf_th=0
ovf_tout=0
ovf_theta=0
ndetected=0
ncounter=0
nwindow=0
nx1=0
nx2=0
ny1=0
ny2=0
nxs2=0
nxs4=0
nxs5=0

```

```

nys2=0
nys4=C
nys5=0
dth=(thmax-thmin)/m
dx=2.*xmax/j
dy=2.*ymax/m
poutmin=pcentral*(1+.01*deltamin)
poutmax=pcentral*(1+.01*deltamax)
toutmin=sqrt(mrecoil**2+poutmin**2)-mrecoil
toutmax=sqrt(mrecoil**2+poutmax**2)-mrecoil
ddelta=(deltamax-deltamin)/j
dtout=(toutmax-toutmin)/j
angmax=-angmin
dang=2.*angmax/ma
iang=1
dowhile(iang.le.m)
    ny(iang)=0
    nphf(iang)=0
    yfront(iang)=-ymax+(iang-1.)*dy
    angmr(iang)=thmin+(iang-1.)*dth
    angdeg(iang)=thspdeg+angmr(iang)*conv
    ngon(iang)=0
    nspec(iang)=0
    eff(iang)=0.
    iang=iang+1
enddo
im=1
dowhile(im.le.ma)
    angtgt(im)=angmin+dang*(im-1.)
    nthf(im)=0
    ntht(im)=0
    nphf(im)=0
    thfspec(im)=-thfmax+(im-1.)*dthf
    im=im+1
enddo
mpar=1
jblk=1
dowhile(mpar.le.j)
    mmass(mpar)=mmassmin+(mpar-1.)*dmmass
    xf(mpar)=-xmax+(mpar-.5)*dx
    ekin(mpar)=toutmin+(mpar-1.)*dtout
    dltaspec(mpar)=deltamin+(mpar-1.)*ddelta
    nmml(mpar)=0
    nmml2(mpar)=0
    nx(mpar)=0
    ngentout(mpar)=0
    nspectout(mpar)=0
    ndelta(mpar)=0
    mpar=mpar+1
enddo
mmassmax=mmass(j)+dmmass
deltatmax=(tbeam+tshift+2.*beam_mass)*(tbeam+tshift)
1      /(tbeam+tshift+beam_mass)*deltapmax*.01

```

C
C
C
C
C
C

am12=0.5/mth


```

mpar=m
-----
events=1
-----
Generation of scattering events is now started.
-----
pl=sqrt(tbeam*(tbeam+2.*beam_mass))-deltapin
el=sqrt(pl**2+beam_mass**2)
ekin_beam=tbeam+tshift
dowhile(events.le.eventmax)
-----
Each event is characterized by (yt,phit,thetat,delta):
yt is generated according to a gaussian distribution,delta by
simulation of energy loss at the target
(straggling approximated by gaussian distribution
with sigma(E)).
-----

xflag=.true.
yflag=.true.
if(ioption.eq.0)then
  zbeam=random(0.,ztarget)
else if(ioption.eq.1)then
  zbeam=0.
else if(ioption.eq.2)then
  zbeam=ztarget
endif
yt=-.6+.24*gauss(dummy)
phit0=random(phitmin,phitmax)
thetat0=random(thetatmin,thetatmax)
-----

c tin0=random(-deltatmax,deltatmax)

if(zbeam.ne.0.)then
  sigma=sigmain*sqrt(zbeam)
  tin_stragl=sigma*gauss(dummy)
else
  tin_stragl=0.
endif
tin0=ekin_beam+deltatin
ec(1)=tin0
ec(2)=beam_mass
ec(3)=beam_charge
ec(6)=zbeam
ec(4)=atarg
ec(5)=ztarg
ec(7)=titarg
call alosst
tin=tin0-eloss-tin_stragl

```

```

-----
Multiple Coulomb scattering of beam up to the interaction point
is now calculated.
-----

```

```

tmid_beam=.5*(tin0+tin)
pmid_beam=sqrt(tmid_beam*(tmid_beam+2.*beam_mass))
call coul(pmid_beam,beam_mass,beam_charge,zbeam,t_rad,theta)
c=cos(theta*1.e-3)
s=sin(theta*1.e-3)
phi=random(0.,2.*pi)
xcos_beam=s*cos(phi)
ycos_beam=s*sin(phi)
zcos_beam=c
zcos_t=1.-.5e-6*(phit0**2+thetat0**2)
ycos_scat=csctrue*phit0*1.e-3+ssctrue*zcos_t
xcos_scat=thetat0*1.e-3
zcos_scat=ssctrue*phit0*1.e-3+csctrue*zcos_t
costh=xcos_beam*xcos_scat+ycos_beam*ycos_scat+zcos_beam*zcos_scat
thscat=(acos(costh)-thsprtr)*1.e3

```

```

      arg=1.-costh**2
      if(arg.gt.0.)then
        sinth=sqrt(arg)
      else
        sinth=0.
      endif

```

```

-----
Average energy loss of the beam particle in the thickness traversed
by it is now calculated.
-----

```

```

thick=ztarget-zbeam
if(moption.eq.1)then
  deltam=random(mmin,mmax)
  m4tot=m4+deltam
else
  m4tot=m4
endif
call kin(tin,costh,mtarg,mrecoil,m4tot,tout,kflag)
if(.not.kflag)then
  k_ovf=k_ovf+1
  go to 1934
endif
tout=tout

```

```

-----
Average energy loss of the detected particle from interaction point
to end of the target is now generated.

```

```

c -----
c
ec(1)=eout
ec(2)=mrecoil
ec(3)=zrecoil
ec(6)=thick
ec(4)=atarg
ec(5)=ztarg
ec(7)=titarg
call eloss
      if(thick.ne.0.)then
        sigma=sigmaef(1)*thick+sigmacof(2)*sqrt(t)+
1      sigmaef(3)*thick**2
        deltastrag1=sigma*gauss(dummy)
      else
        deltastrag1=0.
      endif
if(deltastrag1.lt. eloss)then
  deltastrag1=eloss
endif
deltae=eloss+deltastrag1

c -----
c
c
c tout=eout-deltae
c -----
c
if(tout.gt.0.)then
  pout=sqrt(tout*(tout+2.*mrecoil))
endif
tmid=0.5*(eout+tout)
pmid=sqrt(tmid*(tmid+2.*mrecoil))
call coul(pmid,mrecoil,zrecoil,thick,t_rad,theta)
c=cos(theta*1.e-3)
s=sin(theta*1.e-3)
phi=random(0.,2.*pi)

c
c
c
phit=phit0+sigpht*gauss(dummy)
thetat=thetat0+sigtht*gauss(dummy)
z_cos=1.-.5e-6*(phit**2+thetat**2)
ycos_scat=csctrue*phit*1.e-3+ssctrue*z_cos
xcos_scat=thetat*1.e-3
zcos_scat=-ssctrue*phit*1.e-3+csctrue*z_cos
call devi(xcos_scat,ycos_scat,zcos_scat,c,phi)

c -----
c
c
thetat=xcos_scat*1.e3
ycos_sp=csctrue*ycos_scat-ssctrue*zcos_scat
phit=ycos_sp*1.e3

c -----
c
c
ywindow=yt+50.*ycos_sp
call acceptance(ywindow,5.,0.,5.,xflag,yflag)
if(.not.xflag)then
  nwindow=nwindow+1

```

```

        go to 1934
    endif
    if(.not.yflag)yflag=.true.
o
o
o
-----
    call assign(m,indthg,ovf_theta,thmax,thmin,dth,thsoat,ngen)
    if(thscat.gt.highth1.or.thsoat.lt.lowth1)then
        go to 1953
    else if(thscat.gt.highth2.or.thsoat.lt.lowth2)then
        go to 1933
    endif
    ngen2-ngen2+1
    ngen1-ngen1+1
1933
o
c
c
-----
1953
xpo-thetat*1.e-3
ypo-yoos_sp
zpo-1.-0.5*(ypo**2+xpo**2)
cphe-1. 0.5*ypo*ypo
cosca-oths*cphe-sths*ypo
sinoa-sths*cphe+oths*ypo
c
c
c
-----
dot=ypp*ypo+zpa*zpo
if(dot.gt.0.9999999)dot=0.9999999
thso=acos(dot)
ang=-(thso-thspr)*1.e3
sths=sin(thso)
othso=zoos_soat
delta=(pout-poentral)/poentral*100.
xdelta-gauss(dummy)
if(xdelta.gt.0.)then
    deltavar=6.51e-3*xdelta
else
    deltavar=9.12e-3*xdelta
endif
delta=delta+deltavar
pout-poentral*(1.+0.01*delta)
p3=pout+deltapout+ddeltapout*delta
e3=sqrt(p3**2+mrecoll**2)
o
o
o
o
-----
    call assign(j,ind_tout,ovf_tout,toutmax,toutmin,
    1      dtout,tout,ngentout)
o
    call hrs(xf1,thetaf1,yf1,phif1,yt,phit,thetat,delta)
    call acceptanc(xf1,xmax,yf1,ymax,xflag,yflag)
    if(.not.xflag.and..not.yflag)then
        nx1-nx1+1
        ny1-ny1+1
        go to 1934
    endif
    if(.not.xflag)then
        nx1-nx1+1
        go to 1934

```

```

      else if(.not.yflag)then
        nyl~nyl+1
        go to 1934
      endif
      xcos=thetaf1*1.e-3
      ycos=phif1*1.e-3
      zcos=1.-0.5*(xcos**2+ycos**2)
      call transport(xf1,xcos,yf1,ycos,deltaz,xf2,yf2)
      call acceptance(xf2,xmax,yf2,ymax,xflag,yflag)
      if(.not.xflag.and..not.yflag)then
        nx2~nx2+1
        ny2~ny2+1
        go to 1934
      endif
      if(.not.xflag)then
        nx2~nx2+1
        go to 1934
      else if(.not.yflag)then
        ny2~ny2+1
        go to 1934
      endif
      call transport(xf2,xcos,yf2,ycos,s_dz(1),xs2,ys2)
      call acceptance(xs2,s_x(1),ys2,s_y(1),xflag,yflag)
      if(.not.xflag.and..not.yflag)then
        nxs2~nxs2+1
        nys2~nys2+1
        go to 1934
      endif
      if(.not.xflag)then
        nxs2~nxs2+1
        go to 1934
      else if(.not.yflag)then
        nys2~nys2+1
        go to 1934
      endif
      ec(1)~tout
      ec(2)~mrecoil
      ec(3)~zrecoil
      ec(4)~a_scint
      ec(5)~z_scint
      ec(6)~s_thick
      ec(7)~62.6
      call elosst
      t_s2=tout-eloss
      t_s2_ave=.5*(t_s2+tout)
      if(t_s2_ave.le.0.)then
        go to 1934
      else
        p_s2_ave=sqrt(t_s2_ave*(t_s2_ave+2.*mrecoil))
      endif
      call coul(p_s2_ave,mrecoil,zrecoil,s_thick,
        1      s_rad,theta)
      c=cos(1.e-3*theta)
      phi=random(0.,2.*pi)
      call dev1(xcos,ycos,zcos,c,phi)
      call transport(xs2,xcos,ys2,ycos,s_dz(2),xs4,ys4)
      call acceptance(xs4,s_x(2),ys4,s_y(2),xflag,yflag)

```

c
c
c

```

        if(.not.xflag.and..not.yflag)then
            nxs4=nxs4+1
            nys4=nys4+1
            go to 1934
        endif
        if(.not.xflag)then
            nxs4=nxs4+1
            go to 1934
        else if(.not.yflag)then
            nys4=nys4+1
            go to 1934
        endif
        eo(1)=t_s2
        eo(2)=mrecoil
        eo(3)=zrecoil
        eo(4)=a_scint
        eo(5)=z_scint
        eo(6)=s_thick
        eo(7)=20.7
        call eelosst
        t_s4=t_s2-eloss
        t_s4_ave=.5*(t_s2+t_s4)
        if(t_s4_ave.le.0.)then
            go to 1934
        else
            p_s4_ave=sqrt(t_s4_ave*(t_s2_ave+2.*mrecoil))
        endif
        call coul(p_s4_ave,mrecoil,zrecoil,s_thick,
1          s_rad,theta)
        c=cos(1.e-3*theta)
        phi=random(0.,2.*pi)
        call devi(xcos,ycos,zcos,c,phi)
c
        call transport(xs4,xcos,ys4,ycos,s_dz(3),xs5,ys5)
        call acceptance(xs5,s_x(3),ys5,s_y(3),xflag,yflag)
        if(.not.xflag.and..not.yflag)then
            nxs5=nxs5+1
            nys5=nys5+1
            go to 1934
        endif
        if(.not.xflag)then
            nxs5=nxs5+1
            go to 1934
        else if(.not.yflag)then
            nys5=nys5+1
            go to 1934
        endif
o
o
o
o
o
o
o
o
        call thout(xfl,thetafl,yfl,phifl,iflag,ixang)
        if(iflag.eq.0)go to 1934
        p4x=-p3*xpo
        p4y=-p3*sinca
        p4z=p1-p3*cosca
        p4sq=p4x**2+p4y**2+p4z**2
        de=e1-e3+m2-m4th

```

```

        q1=de*(de**2*p4sq)*amd12
        arg2=1.+2.*q1/m4th
        if(arg2.gt.0)then
            arg3=sqrt(arg2)
        else
            arg3=0.
        endif
        mm=(2.*q1)/(1.+arg3)
        if(ang.gt.highth1.or.ang.lt.lowth1)then
            go to 1853
        else if(ang.gt.highth2.or.ang.lt.lowth2)then
            go to 1833
        endif
1833    nspec2=nspec2+1
1853    nspec1=nspec1+1
        continue
        call assign(m,indyf,ovf_yf,ymax,ymax,dy,yf1,ny)
        call assign(m,indphf,ovf_phf,thmax,thmin,dth,phif1,nphf)
        call assign(ma,indthf,ovf_thf,thfmax,-thfmax,dthf,thetaf1,n
        call assign(ma,ind_tht,ovf_tht,angmax,angmin,
1      dang,thetat,ntht)
        call assign(ma,ind_pht,ovf_pht,angmax,angmin,
1      dang,phit,npht)
        call assign(m,indth,ovf_th,thmax,thmin,dth,ang,nspec)
        call assign(j,indx,ovf_x,xmax,-xmax,dx,xf1,nx)
        call assign(j,ind_tsout,ovf_tsout,toutmax,toutmin,
1      dtout,tout,nspectout)
        call assign(j,inddelta,ovf_delta,deltamax,deltamin,ddelta,
1      delta,ndelta)
        if(ang.gt.highth1.or.ang.lt.lowth1)then
            go to 1934
        else if(ang.gt.highth2.or.ang.lt.lowth2)then
            go to 2033
        endif
        call assign(j,indmm2,ovf_mm2,massmax,massmin,dmmass,mm,nmm2)
        call assign(j,indmm1,ovf_mm1,massmax,massmin,dmmass,mm,nmm1)
2033    events=events+1
1934    enddo
        k=1
        dowhile(.true.)
            unter=ncounter+ngen(k)
            ndetected=ndetected+nspec(k)
            xxx=ngen(k)
            if(xxx.ne.0.)then
                eff(k)=nspec(k)/xxx
            endif
            k=k+1
        enddo
        xxx=ngen1
        if(xxx.gt.0.)then
            eff1=nspec1/xxx
        else
            eff1=0.
        endif
        xxx=ngen2
        if(xxx.gt.0.)then
            eff2=nspec2/xxx
        else
            eff2=0
        endif

```

```

-----
Output information is written to file e795.out
-----

```

```

i=1
dowhile(1.1e.120)
  b(1)=lh-
  i=i+1
enddo
write(2,299)title
write(2,300)b
write(2,303)b
write(2,301)eventmax,
1      tbeam,
1      tshift,
1      deltatmax,
1      ztarget,
1      particle,
1      poentral
write(2,700)target
write(2,701)mtarg
write(2,702)m4
if(moption.eq.1)then
  write(2,703)mmin,mmax
endif
write(2,705)m2,m4th
write(2,605)thetatmin,thetatmax,sigtht,phitmin,phitmax,sigpht
write(2,706)thspdeg,thspar
write(2,707)deltapin,deltapout,ddeltapout
write(2,302)b,ncounter,ovf_theta,ndetected,ovf_th
write(2,304)ovf_delta,ovf_tsout,ovf_tout
write(2,502)nwindow,nx1,ny1,nx2,ny2
write(2,506)nx52,nys2,nx54,nys4,nx55,nys5,ixang
write(2,507)lowth1,highth1,nspec1
write(2,508)lowth1,highth1,ngen1
write(2,509)lowth1,highth1,eff1
write(2,510)lowth2,highth2,nspec2
write(2,511)lowth2,highth2,ngen2
write(2,512)lowth2,highth2,eff2

```

```

-----
Variables in the dispersion plane are now written to output file.
-----

```

```

write(2,503)b
write(2,504)(xf(k),nx(k),k-1,j)
write(9,504)(xf(k),nx(k),k-1,j)
write(2,503)b
write(2,501)(thfspec(k),nthf(k),k-1,ma)
write(10,501)(thfspec(k),nthf(k),k-1,ma)
write(2,502)b

```



```

1      5x,'number of overflows in scattering angle generated'  =',2x
1      5x,'Total number of particles detected'                =',2x
1      5x,'overflows in thetasc of particles detected'        =',2x
303    format(5x,'Input Parameters for Simulation:'/5x,120a1
1      //)
304    format(5x,'overflows in delta'                            =',2x,17/
1      5x,'overflows in tout(detected)'                      =',2x,17/
1      5x,'overflows in tout(generated)'                      =',2x,17/)
305    format(5x,(3x,f8.3,3x,18/))
306    format('1',5x,' ang(mrad)',
1      5x,' ang(deg)',
1      5x,'ngen(thsc)',
1      5x,'ndec(thsc)',
1      5x,' eff(thsc)')
310    format('1',5x,' Tout(MeV)'
1      5x,' N(Tout)')
311    format(5x,(5x,f10.3,5x,110/))
312    format(5x,120a1/)
401    format('1',5x,' MM(MeV)',5x,'N(M_Mass)G1',5x,'N(M_Mass)G2'/
1      120a1)
406    format(5x,(5x,f8.3,5x,110,5x,110/))
402    format(5x,(5x,f8.3,5x,110/))
403    format(5x,(5x,f8.3,6x,110/))
405    format('1',5x,'delta(%)',6x,'dn/ddelta')
404    format(5x,120a1)
501    format(5x,'overflows in kinematic parameters'          =',2x,17/
1      5x,'overflows in scattering angle'                    =',2x,17/
1      5x,'overflows in theta at front chamber plane'        =',2x,17/
1      5x,'overflows in missing mass range'                  =',2x,17/
502    format(5x,'particles rejected at spectrometer entrance port-',1x,
1      17/ 5x,'particles outside 1x acceptance'              =',2x,17/
1      5x,'particles outside 1y acceptance'                  =',2x,17/
1      5x,'particles outside 2x acceptance'                  =',2x,17/
1      5x,'particles outside 2y acceptance'                  =',2x,17/
503    format('1',5x,'xf reconstructed spectrum'//
1      5x,' xfl(cm)',4x,' n(xfl)')
1      5x,120a1/)
504    format(5x,(5x,f8.2,5x,18/))
506    format(5x,'particles outside s2x acceptance',2x,17/
1      5x,'particles outside s2y acceptance',2x,17/
1      5x,'particles outside s4x acceptance',2x,17/
1      5x,'particles outside s4y acceptance',2x,17/
1      5x,'particles outside s5x acceptance',2x,17/
1      5x,'particles outside s5y acceptance',2x,17/
1      5x,'particles rejected by xang cut',2x,17/)

c
c
c
507    format(5x,'particles detected in range',2x,f5.1,':',f5.1,'-',3x,17
508    format(5x,'particles generated in range',
1      2x,f5.1,':',f5.1,'-',3x,17)
509    format(5x,'efficiency in range',
1      2x,f5.1,':',f5.1,'-',3x,f6.3/)
510    format(5x,'particles detected in range',2x,f5.1,':',f5.1,'-',3x,17
511    format(5x,'particles generated in range',
1      2x,f5.1,':',f5.1,'-',3x,17)
512    format(5x,'efficiency in range',
1      2x,f5.1,':',f5.1,'-',3x,f6.3)

c
c

```

```

518 format('j',(5x,f10.3,5x,17//))
519 format('l',5x,'Spectrum of kinetic energy of detected particles'///
1      5x,'      Tout(MeV)',
1      5x,'      Ngen(Tout)',
1      5x,'Ndetected(tout)'/
1      120a1)
520 format('j',(5x,f14.3,5x,114,5x,114//))
c
c
601 format('ma',(5x,f7.2,5x,17//))
603 format('l',5x,'ang(mr)',5x,'dn/dthf'/
1      5x,120a1)
605 format( 5x,'min      th_at_tgt (mr)  =',1x,f6.2/
1      5x,'max      th_at_tgt (mr)  =',1x,f6.2/
1      5x,'sigma th_at_tgt (mr)  =',1x,f6.2/
1      5x,'min      ph_at_tgt (mr)  =',1x,f6.2/
1      5x,'max      ph_at_tgt (mr)  =',1x,f6.2/
1      5x,'sigma ph_at_tgt (mr)  =',1x,f6.2)
c
c
c
700 format(5x,'target nucleus(kinematics)              =',1x,a7)
701 format(5x,'m_target                                =',1x,f10.3,1x,'
702 format(5x,'mass of undetected particle(kinematics) =',
1      1x,f10.2,1x,'MeV')
703 format(5x,'Excitation energy of undetected particle=',
1      1x,f8.2,1x,'MeV',1x,'-',1x,f8.2,'MeV'//)
705 format(5x,'m_target (mm calculation)                =',1x,f10.3,1x,'
1      5x,'m of undetected particle(m calo.)          =',1x,f10.3,1x,'
706 format(5x,'true spectrometer angle                  =',1x,f8.3,1x,'deg
1      5x,'spectrometer angle in input kinematics =',1x,f8.3,1x,'deg
707 format(5x,'momentum loss(beam)                      =',1x,f8.3,1x,
1
1      5x,'momentum loss(scattered particle)=-',1x,f8.3,1x,
1
1      5x,'derivative ddeltapout/ddelta                =',1x,f8.6,1x,
1
1      'MeV/c/%')
704 format(5x,'Interaction point (zref-tgt in-window)=-',1x,f8.5,1x,'cm'
799 format('l',5x,'Deltatheta(mr)',
1      5x,' dn/ddeltatht',
1      5x,' dn/ddeltapht'//)
801 format(5x,120a1)
800 format('ma',(5x,f14.3,5x,114,5x,114//))
802 format('l',5x,'yfront(cm)',5x,'dN/dyfront'/120a1)
803 format('m',(5x,f10.3,5x,110//))
804 format('l',5x,'phif(mr)',5x,'dN/dphif'/120a1)
805 format('m',(5x,f8.1,5x,18//))
end
c
c
c
subroutine hrs(xfl,thetafl,yfl,phifl,yt,phit,thetat,delta)
c
c
c
c
c
c
xfl=-3.418+18.771*delta-0.89387e-02*thetat+
1 -0.11164e-02*yt-0.18458e-03*phit+

```

```

1  -.5326*delta**2+0.1275e-01*delta*thetat+
1  -0.178e-03*thetat**2+0.48159e-03*phit**2+
1  0.1426e-04*delta*phit-0.783e-05*thetat*phit

```

```

-----
thetaf1= 6.160+17.089*delta-1.2219*thetat+
1  -0.13567*yt-0.66019e-02*phit+
1  -0.14756*delta**2-0.079619*delta*thetat+
1  0.15891e-03*thetat**2+0.36757e-02*phit**2+
1  0.125e-03*delta*phit-0.1607e-04*thetat*phit

```

```

-----
ccc= 0.429e-1-0.261e-2*xfl
ddd= .1619+.1032e-2*xfl
aaa= 5.3439+0.0347*xfl+0.0011*thetaf1
bbb= 0.09598+0.00939*xfl-0.0215*thetaf1
det=aaa*ddd-bbb*ccc
if(det.eq.0.)then
    yfl=100.
    phifl=1000.
else
    yshift=yt-.0123+0.246e-2*xfl+0.139e-2*thetaf1
    phishift=phit+2.01-0.0116*xfl+0.0027*thetaf1
    yfl=(phishift*ddd-yshift*bbb)/det
    phifl=(aaa*yshift-ccc*phishift)/det
endif
return
end

```

```

-----
subroutine transport(xfl,thetaf1,yfl,phifl,deltaz,xf2,yf2)

```

```

-----
xf2=xf1+deltaz*thetaf1
yf2=yfl+deltaz*phifl
return
end

```

```

-----
subroutine acceptance(xf,xmax,yf,ymax,flagx,flagy)

```

```

-----
logical flagx,flagy
if(xf.gt.xmax.or.xf.lt.-xmax)then
    flagx=.false.
endif
if(yf.gt.ymax.or.yf.lt.-ymax)then
    flagy=.false.
endif
return
end

```

```

C -----
C
C
C Subroutine thcut(xfl,thetaf1,yfl,phif1,iflag,ixang)
C -----
C
C parameter thcutlow=-29.51
C parameter thcuthigh=29.29
C -----
C
C iflag=1
C thc= -0.515521E+01+
C 1 xfl*0.672947E+00+
C 1 thetaf1*-0.805446E+00+
C 1 yfl*.351555E-01+
C 1 phif1*0.129565E-01+
C 1 xfl**2*-0.487944E-02+
C 1 xfl*thetaf1*0.319828E-02+
C 1 thetaf1**2*0.513152E-04+
C 1 yfl**2*0.489751E-02+
C 1 yfl*phif1*-0.340292E-02+
C 1 phif1**2*0.257161E-03+
C 1 xfl*.26
C -----
C
C if(thc.lt.thcutlow.or.thc.gt.thcuthigh)then
C     iflag=0
C     ixang=ixang+1
C endif
C return
C end

```

```

Subroutine Coul(pout,mrecoil,zrecoil,thick,lrad theta)
integer seed
real lrad
real mrecoil,lratio
common seed
if(thick.ne.0.)then
  theta=0.
  return
endif
lratio=thick/lrad
pbeta=pout**2/sqrt(pout**2+mrecoil**2)
lratio=thick/lrad
sigmatheta=14.1e+3/pbeta*zrecoil*sqrt(lratio)*
1 (1.+0.111111111111*log10(lratio))
temp=gauss(dummy)
theta=sigmatheta*temp
return
end

```

```

C
C .....
C This file collects miscellaneous short subroutines and functions
C called by the program SPECTRUM.
C

```

```

FUNCTION RANDOM(RMIN,RMAX)

```

```

C
C .....
C This function generates a uniform distribution defined in an
C interval comprised between RMIN and RMAX.
C

```

```

C .....
C
C INTEGER SEED
C COMMON SEED
C rarg=ran(seed)
C rtemp=rarg*(RMAX-RMIN)+RMIN
C random=rtemp
C RETURN
C END

```

```

C
C .....
C
C FUNCTION GAUSS(DUMMY)

```

```

C
C .....
C This function generates a gaussian distribution of
C unit standard deviation.
C

```

```

C .....
C
C integer seed
C common seed
C PARAMETER PI=3.1415926
C TEMP1=SQRT(-2.*ALOG(RANDOM(.00001,1.)))
C TEMP2=COS(2.*PI*RANDOM(0.,1.))
C GAUSS=TEMP1*TEMP2
C RETURN
C END

```

```

Subroutine assign(m,index,overflow,xmax,xmin,dx,x,ngen)
c
c
c
integer ngen(m)
integer overflow
c
c
c
index=0
xlow=xmin
xhigh=xmin+dx
j=1
if(x.lt.xmin.or.x.gt.xmax)go to 200
dowhile(j.le.m)
  if(x.ge.xlow.and.x.lt.xhigh)then
    ngen(j)=ngen(j)+1
    index=j
    go to 200
  else
    j=j+1
    xlow=xhigh
    xhigh=xhigh+dx
  endif
enddo
200 if(index.eq.0)then
  overflow=overflow+1
endif
return
end

```

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