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CONF-840611S--1

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UCRL--90711

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GENERALIZED LASER FILAMENTATION INSTABILITY COUPLED TO COOLING INSTABILITY

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This paper was prepared for summittal to
AIAA 17th Fluid Dynamics, Plasmadynamics
and Laser Conference, Snowmass, Colorado,
June 25-27, 1984.

April 24, 1984

Lawrence
Livermore
National
Laboratory

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GENERALIZED LASER FILAMENTATION INSTABILITY COUPLED TO COOLING INSTABILITY

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Abstract

We consider the propagation of laser light in an initially slightly nonuniform plasma. The classical dispersion relation for the laser filamentation growth rate (see e.g., B. Langdon, in the 1980 Lawrence Livermore National Laboratory Laser Program Annual Report, pp. 3-56, UCRL-50021-80, 1981) can be generalized to include other acoustical effects. For example, we find that the inclusion of potential imbalances in the heating and cooling rates of the ambient medium due to density and temperature perturbations can cause the laser filamentation mode to bifurcate into a cooling instability mode at long acoustic wavelengths. We also attempt to study semi-analytically the nonlinear evolution of this and related instabilities. These results have wide applications to a variety of chemical gas lasers and phenomena related to laser-target interactions (e.g., jet-like behavior).

1. Introduction

When a laser beam propagates in a slightly nonuniform plasma, or when the beam has slight nonuniformity at its wavefront, this nonuniformity will tend to grow with time, ultimately leading to the filamentation (or even self-focusing) of the beam and the plasma. This well-known filamentation (or self-focusing) instability is due to the density response of the plasma to the enhanced laser ponderomotive pressure and the enhanced heating by inverse-bremsstrahlung absorption of the laser energy. Its linear growth rate has been studied extensively by a number of authors^{1,2,3,4} in recent years. In this paper we generalize it to the case when the overall heating and cooling rates of the plasma itself are dynamically important and sensitive to density and temperature variations. Imbalance of heating and cooling caused by density perturbations could then lead to a cooling instability by itself. The coupling of the filamentation instability (FI) and the cooling instability (CI) leads to new dispersion relation for the growth rate. We first investigate the linear growth rates of these instabilities. Their dependence on the perturbation wavelength can be described in terms of a generalized fifth order dispersion relation. We then investigate the nonlinear growth of the perturbations using the Hunter-Keller method,⁵ assuming that a certain small dimensionless parameter exists in the problem. Finally we mention some sample applications of the current results to laser propagations in the subcritical (local plasma frequency < laser frequency) blowoffs of laser-heated targets.

11. Linear Growth Rates

The coupled hydro-laser propagation equations are:

a) mass conservation:

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (1)$$

b) momentum conservation:

$$\dot{\mathbf{v}} + \mathbf{v} \cdot \nabla \mathbf{v} = - \frac{1}{\rho} \nabla (p + p_L) \quad (2)$$

c) energy conservation:

$$\dot{E} + \rho \mathbf{v} \cdot \nabla E + \rho \mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot (\kappa \nabla T) + \rho \kappa_{ib} I + H - C \quad (3)$$

d) laser propagation (geometric optics approximation):

$$\frac{d^2}{dt^2} I = I c^2 \frac{\nabla^2 \omega}{\omega_0} \quad (4)$$

Here ρ =density; $\mathbf{v}$ =fluid velocity; p =gas pressure; E = internal energy; c =light speed; ω =laser frequency in plasma; ω_0 =laser frequency in vacuum; ω_p = plasma frequency; I =laser intensity; $p_L \equiv$

$\left(\frac{\omega_p}{\omega}\right)^2 \frac{I}{c}$ is laser ponderomotive pressure; κ =thermal conductivity; κ_{ib} =inverse bremsstrahlung opacity; H =intrinsic heating rate; C =intrinsic cooling rate. d/dt in Eq. (4) is the convective derivative along laser rays R_L :

$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + c \hat{k}_L \cdot \nabla$. Perturbing the above equations from a static uniform background solution in which $\rho_0 \kappa_{ib} I_0 + H_0 =$

C_0 ($\rho_0 = \rho_0 + \delta \rho$; $\mathbf{v} = \delta \mathbf{v}$; $p = p_0 + \frac{\kappa}{m} (\rho_0 \delta T + T_0 \delta \rho)$, etc.)

and Fourier analyzing the perturbed quantities:

$\delta \rho = \delta \rho_0 e^{i\Gamma t + i\mathbf{k} \cdot \mathbf{x}}$; $\delta T = \delta T_0 e^{i\Gamma t + i\mathbf{k} \cdot \mathbf{x}}$ etc., $k_L \hat{k}_L^0$

we obtain a 5th order dispersion relation for $\Gamma(k)$:

$$\Gamma^5 + \Gamma^4 \left\{ \frac{2}{3} \chi k^2 + \frac{1}{2} \frac{v_0^2}{e i \sqrt{2}} \frac{2}{3} \alpha R_0 \right\} + \Gamma^3 c_s^2 k^2 \quad (5)$$

$$+ \Gamma^2 \left[\chi k^2 + \frac{7}{4} \frac{v_0^2}{e i \sqrt{2}} (\alpha - 1) R_0 \right] \frac{2}{5} c_s^2 k^2$$

$$- \frac{3}{40} \Gamma c_s^2 k^4 \left(\frac{\omega_p}{\omega_0} \right)^2 \left(\frac{v_0}{v_e} \right)^2 - \frac{1}{40} c_s^2 k^4$$

$$\left(\frac{\omega_p}{\omega_0} \right)^2 \left(\frac{v_0}{v_e} \right)^2 \left[v_{ei} + 2 \chi k^2 + \frac{3}{2} \frac{v_0^2}{e i \sqrt{2}} \alpha R_0 \right] = 0$$

where we have assumed that H and C have power-law dependences on T and ρ :

$$H = H_0 \left(\frac{\rho}{\rho_0} \right)^{\beta} \left(\frac{T}{T_0} \right)^{\eta}; \quad C = C_0 \left(\frac{\rho}{\rho_0} \right)^{\delta} \left(\frac{T}{T_0} \right)^{\sigma} \quad (6)$$

$$\alpha \equiv \left(\frac{\eta - \sigma}{\beta - \delta} \right); \quad R_0 \equiv \frac{| \delta - \beta | H_0}{P_0}$$

and $\chi = c_s / c_p \theta$; ν_{ei} = electron-ion collision frequency; c_s = sound speed;

$$v_s = \sqrt{\frac{\chi T_s}{m_s}}; v_o = \left(\frac{a}{m_o}\right) \sqrt{\frac{B_0 I}{c}}.$$

In the absence of heating and cooling imbalance ($R_0=0$) Eq. (5) reduces to the well known laser filamentation dispersion relation (see e.g. Langdon¹). On the other hand, when $\alpha \gg 1$ and $I_0=0$ we get a cubic relation

$$\Gamma^3 + \Gamma^2 \left[\frac{2}{3} \chi k^2 + \frac{2}{3} \alpha \frac{R_0}{c_s} \right] + \Gamma c_s^2 k^2 + \frac{2}{3} c_s^2 k^2 [\chi k^2 + (\alpha-1) R_0] = 0 \quad (7)$$

For $R_0 > 0$ Eq. (7) says that all roots are stable ($\text{Re} \Gamma < 0$) for $\alpha \geq 1$. However, for $\alpha < 1$, in principle there could be one unstable ($\text{Re} \Gamma > 0$) root if $-3/2 < \alpha$ and three unstable roots if $\alpha < -3/2$. However, thermal conductivity (i.e. the χ terms) tends to act against these unstable modes, and whether the above modes actually become unstable depends on the value of thermal conductivity. To see the effects of thermal conduction in a more transparent way, it is convenient to scale all quantities by appropriate combinations of χ and c_s to make them dimensionless:

$$\bar{\Gamma} \equiv \frac{\Gamma \chi}{c_s^2}; \bar{k} \equiv \frac{k \chi}{c_s}; \bar{R}_0 = R_0 \chi / c_s^2 \text{ etc.} \quad (8)$$

Figure 1 illustrates the typical behavior of the unstable root (for $\alpha > -3/2$) of Eq. (7) in the absence of a laser. It is clear that the instabilities are strongly inhibited in the limit of large $\bar{k}$ (short wavelengths or high conductivity). From Eq. (7) it is also clear that in the limit $\chi \rightarrow 0$, the maximum growth rate is attained at $\bar{\Gamma}_{\text{MAX}} = \frac{2}{5} (1-\alpha) \bar{R}_0$, i.e. same order as the heating-cooling rate imbalance.

Figure 1. Dispersion relation for pure cooling instability ($I=0$) for various values of $\bar{R}_0$ and α .

Figure 2. Dispersion relation for coupled cooling and filamentation instabilities. Numbers on right denote value of $(u_p/u_o)(v_o/v_s)$.

Figure 2 illustrates the modulation of the filamentation instability (FI) growth rates ($I_0 \neq 0$) by the heating and cooling imbalance ($\alpha \gg 1$). It is clear that at small $\bar{k}$ (long wavelengths or low conductivity), the FI and CI roots merge, and the CI root dominates. For large $\bar{k}$ the CI root disappears and only FI exists. Typically, there exists a "valley of stability" for $100 > \bar{k} > 1$ where the growth rate is a minimum. We also show the result of varying I_0 and R_0 . It is important to realize that the FI and CI roots merge at small $\bar{k}$ and there is always only one unstable root instead of two (for $\alpha > -3/2$).

III. Weakly Non-linear Perturbation Theory

The general Fourier analysis of the linearized perturbation equations typically yields solutions of the form

$$\delta p(t, x) = (\delta p)_0 e^{(\text{Re} \Gamma)t} e^{i[(\text{Im} \Gamma)t + kx]}, \quad \Gamma = \Gamma(k)$$

and $(\text{Im} \Gamma / \text{Re} \Gamma) \approx 0$ ($f \ll 1$), and $\frac{\delta p}{\rho_0} \approx 0$ ($g \ll 1$).

The small parameters f and g need not be related to each other in any way; and the linear analysis is strictly valid only for time $t \ll 0[(\text{Re} \Gamma)^{-1}]$.

Hunter and Keller (H-K) recently proposed⁵ a weakly non-linear perturbation theory for high frequency waves, that is capable of tracking the coherent secular growth over a much longer time duration $t < 1/f \ll 0[(\text{Re} \Gamma)^{-1}]$. However the theory is not completely general in that the perturbed amplitude $\delta p/p$ must be restricted to be of $O(f)$.

A. A Brief Sketch of the H-K Method

They consider a strictly hyperbolic system of quasi-linear first order partial differential equations

$$\sum_{i=0}^n A^{(i)}(x, u) \frac{\partial u}{\partial x_i} + B(x, u) = 0 \quad (9)$$

When specialized to the inviscid fluid equations without heat conductivity, $X = (t, \vec{x})$, $U = (\rho, v, \varepsilon)$ and $A(i)$ are 3×3 matrices and $B(X, U)$ represents the source term. The theory prescribes a small perturbation about an exact known solution $U^{(0)}$, satisfying Eq. (9) identically. Next the perturbed solution V is introduced having two quite different scales of variations, and is formally enforced as follows:

$$V = V(X, \theta(X)) \quad , \quad \theta(X) = \frac{\phi(X)}{f} \quad , \quad f \ll 1$$

$$O\left(\frac{\partial V}{\partial X_i}\right) \approx O\left(\frac{\partial V}{\partial \theta}\right) = O\left(\frac{\partial \phi}{\partial X_i}\right) \quad .$$

It is further imposed that the perturbation is of $O(f)$

$$U = U^{(0)}(X) + fV(X, \theta) \quad , \quad O(U^{(0)}) = O(V) \quad .$$

Now by the chain rule a variation of V with respect to X_i produces two terms of vastly different magnitudes since $f \ll 1$,

$$\frac{\partial}{\partial X_i} V(X, \theta(X)) = \frac{\partial V}{\partial X_i} + \frac{1}{f} \frac{\partial \phi}{\partial X_i} \frac{\partial V}{\partial \theta} \quad .$$

In order to control the numerical convergence of the perturbation series expansion with the assumed form of V , the following ansatz,

$$U = \sum_{n=0}^{\infty} f^n U^{(n)} \quad , \quad B = \sum_{n=0}^{\infty} f^n B^{(n)} \quad , \quad A(i) = \sum_{n=0}^{\infty} f^n A_n^{(i)}$$

$$\frac{\partial}{\partial X_i} + \frac{\partial}{\partial X_i} + \frac{1}{f} \left(\frac{\partial \phi}{\partial X_i} \right) \frac{\partial}{\partial \theta}$$

is put back into the Eq. (9) and coefficients of like powers in f are separately set equal to zero. The f^{-1} -eq.,

$$\left(\sum_i \frac{\partial \phi}{\partial X_i} A_0^{(i)} \right) U^{(0)} \equiv M \partial_\theta U^{(0)} = 0 \quad ,$$

is trivially satisfied since $U^{(0)}$ is assumed to be independent of θ . The f^0 -eq., $M \partial_\theta U^{(1)} = 0$ implies that $U^{(1)} \neq 0$ if and only if the sikonal equation $\det M = 0$. In the one-dimensional inviscid fluid case, $\det M = 0$ gives rise to three modes (characteristics)

$$(\partial_\theta \phi) [(\partial_\theta \phi)^2 - \varepsilon_0 (\partial_\theta \phi)^2] = 0 \quad (v^{(0)} \neq 0)$$

of "Fast variation" or "propagation". For each mode j distinct left and right null-eigenvectors $L^{(j)}$ and $R^{(j)}$, $L^{(j)} M(\phi^{(j)}) = M(\phi^{(j)}) R^{(j)} = 0$, are constructed with normalization $L^{(j)} \cdot R^{(j)} \equiv 1$. The solution $U^{(1)}$ is generally written as

$$U^{(1)} = a^{(j)}(X, \theta) R^{(j)}(X) \quad .$$

The function $a^{(j)}$ is determined by the next order f^1 -eq.. By focusing on one particular j mode, the f^1 -eq. yields a non-linear transport equation for $a^{(j)}$

$$\partial_\theta a + La + M \partial_\theta a = 0$$

$$L(s) = (L^T A_0 \partial_\theta X) + \left(\frac{L^T A_1 \partial_\theta U^{(0)}}{a} \right) + \left(\frac{L^T B^{(1)}}{a} \right)$$

$$N(s) = \frac{\partial \phi}{\partial X} \left(\frac{L^T A_1 R}{a} \right)$$

where L and M are independent of a . ∂_θ designates integration along the corresponding characteristic. The quadratic term $M \partial_\theta a$ is the weakly non-linear perturbation correction to the usual geometric acoustic equation $\partial_\theta a + La = 0$.

B. A Choice of f for the Hydro-Laser Equations

For the sake of definiteness, let us ask how the heating-cooling term $W(\rho, T) \equiv H-C$ of Eq. (1) drives the instability of the hydro-laser system. Therefore a characteristic rate R_0 should be abstracted from $W(\rho, T)$ by

$$R_0 \approx O\left(\frac{W(\rho_0, T_0)}{\rho_0 c_0}\right) \quad .$$

Introducing dimensionless time and space variables: $\tau = t R_0$, $r = x R_0 / C_s$, and scaling ρ by $\rho = \rho_0 \tilde{\rho}$, $v = C_s \tilde{v}$, etc., we obtain a dimensionless set of hydro-laser equations

$$(a) \quad \partial_\tau \tilde{\rho} + \tilde{v} \partial_r \tilde{\rho} + \tilde{\rho} \partial_\tau \tilde{v} = 0 \quad (10)$$

$$(b) \quad \partial_\tau \tilde{v} + \tilde{v} \partial_r \tilde{v} + \frac{1}{\gamma} \frac{1}{\tilde{\rho}} \partial_r (\tilde{\rho} \tilde{\varepsilon}) + \tilde{P}_{10} \frac{1}{\tilde{\rho}} \partial_r \tilde{P}_I = 0$$

$$(c) \quad \partial_\tau \tilde{\varepsilon} + \tilde{v} \partial_r \tilde{\varepsilon} + (\gamma - 1) \tilde{\varepsilon} \partial_r \tilde{v} - \tilde{X} \frac{1}{\tilde{\rho}} \partial_r \tilde{\varepsilon}^2$$

$$- \tilde{\beta} \tilde{I} - \tilde{W}(\tilde{\rho}, \tilde{\varepsilon}) = 0$$

$$(d) \quad \partial_\tau \tilde{I}^2 - \tilde{I}^2 \partial_r \tilde{P}_I = 0$$

where γ is ratio of specific heats and

$$\tilde{P}_{10} = \frac{P_{10}}{\rho_0 C_s^2} \quad , \quad \tilde{X} = \frac{\gamma X R_0}{C_s^2}$$

$$\tilde{\beta} = \frac{\kappa_{ib} I_0}{\varepsilon_0 R_0} \quad , \quad \tilde{\varepsilon}^2 = \frac{1}{2} \left(\frac{C}{C_s} \right)^2 \left(\frac{W}{W_0} \right)^2 \quad ,$$

$$\tilde{W}(\tilde{\rho}, \tilde{\varepsilon}) = \frac{1}{\rho_0 \varepsilon_0 R_0} \frac{W(\rho_0 \tilde{\rho}, T_0 \tilde{\varepsilon})}{\tilde{\rho}}$$

Here we have five competing processes with four dimensionless parameters (scaled by R_0) $\tilde{P}_{10}$, $\tilde{X}$, $\tilde{\beta}$ and $\tilde{\varepsilon}^2$. However the radiation-pressure $\tilde{P}_{10}$ will be set equal to zero to reduce somewhat the complexity of the ensuing discussion. The linear small perturbation, $\delta \rho \approx (\delta \rho)_0 e^{kz\tau + ikr}$, etc., analysis of Eq. (10) yields a 5th order dispersion relation analogous to Eq. (5) (z is the dimensionless growth rate):

$$p_5(z) = z^5 + az^4 + z^3 + bz^2 - c = 0 \quad (11)$$

$$a = k\tilde{X} - \frac{1}{k} \frac{\partial \tilde{W}}{\partial \varepsilon} \Big|_\rho$$

$$b = \frac{1}{\gamma} \left\{ k\tilde{X} + \frac{1}{k} \left(\frac{\partial \tilde{W}}{\partial \rho} \Big|_\varepsilon - \frac{\partial \tilde{W}}{\partial \varepsilon} \Big|_\rho \right) \right\}$$

$$c = \frac{1}{\gamma} \frac{\tilde{\beta}}{k} \tilde{\varepsilon}^2 \quad .$$

The small dimensionless parameter f can be introduced as

$$\frac{1}{k} \frac{\partial \tilde{u}}{\partial \rho} = O\left(\frac{1}{k}\right) \propto O(f) .$$

then the heat conduction term $\tilde{u}k$ in the coefficients a and b will compete with the heating-cooling term $\tilde{u}$ only if we set $k \propto O(f)$, or $k \propto O(f^2)$.

Furthermore if $\tilde{u}$ is chosen to be of $O(1)$ and $\tilde{u} \propto O(f^2)$, then the 5th order polynomial will have five approximate roots,

$$z_0 = -b \propto O(f) , \quad (\text{non-propagating})$$

$$z_{\pm} \approx \pm i - \left(\frac{a-b}{2}\right) \approx \pm i + O(f), \quad (\text{propagating})$$

$$z_s \approx \pm \sqrt{\frac{\tilde{u}^2}{kb}} \approx O(f) ,$$

satisfying $p_j(z)$ to $O(f^3)$.

The above choice for f , $\tilde{u}$ and $\tilde{u}^2$ leads to the existence of at least one growth mode, z_0 or z_s , being positive, for either sign of b . Putting (e.g.) the z_s root back in the perturbed quantity $\delta\rho$, then

$$\begin{aligned} \delta\rho &\approx (\delta\rho)_0 e^{k \propto O(f)\tau} e^{ik(\tau+\tau)} \\ &= (\delta\rho)_0 e^{O(1)\tau} e^{\frac{i(\tau+\tau)}{f}} \end{aligned}$$

which is precisely the form required for the H-K method to work. The magnitude of the various parameters can be explicitly displayed by defining $\tilde{u} = f^2 \chi_0$, $\tilde{u} = \beta_0$ and $\tilde{u}^2 = f^2 \epsilon_0^2$, where $\chi_0 \approx \beta_0 \approx \epsilon_0 \propto O(1)$ and $k \propto O\left(\frac{1}{f}\right)$, in Eq. (10).

C. An Illustrative Example

For illustrative purpose let us turn off the laser in Eq. (10) and the H-K method is applied to the resulting inviscid fluid equations with heat conduction present. Also the "tilde" decoration will be dropped, $U = (\rho, v, \epsilon) \rightarrow U = (\rho, v, \epsilon)$ and the truncated set of equation in matrix notation is,

$$\partial_t U + A \partial_x U + B + f^2 \tilde{X} \partial_x^2 U = 0$$

$$\begin{aligned} U &= \begin{bmatrix} \rho \\ v \\ \epsilon \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ -\tilde{u}(\rho, \epsilon) \end{bmatrix} \quad \tilde{u}(\rho, \epsilon) = \frac{1}{\rho_0 \epsilon_0 R_0} \frac{W(\rho_0, T_0, \epsilon)}{\rho} \\ A &= \begin{bmatrix} v & \rho & 0 \\ \frac{\epsilon}{\gamma \rho} & v & \frac{1}{\gamma} \\ 0 & \gamma_s v & v \end{bmatrix} \quad \tilde{X} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{\chi_0}{\rho} \end{bmatrix} \quad \gamma_s = \gamma - 1 . \end{aligned}$$

This is just the set of equations used in the sketch of the H-K method. We index the three modes by $j=0, \pm$ and assume a constant background solution $U(0) = (1, 0, 1)$,

$$M(j) = \begin{bmatrix} -j & 1 & 0 \\ \frac{1}{\gamma} & -j & \frac{1}{\gamma} \\ 0 & \gamma_s & -j \end{bmatrix} ,$$

$$L(0) = \frac{1}{\gamma} \begin{pmatrix} \gamma_s \\ 1 \\ -1 \end{pmatrix} , \quad R(0) = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad (\text{non-propagating})$$

$$L(\pm) = \frac{1}{2\gamma} \begin{pmatrix} 1 \\ \pm\gamma \end{pmatrix} , \quad R(\pm) = \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} , \quad (\text{propagating}).$$

The f^1 -eq. yields the non-linear transport equation

$$\begin{aligned} \partial_t a^{(j)} + L^{(j)} a^{(j)} + N^{(j)} a^{(j)} + \partial_x a^{(j)} + D^{(j)} a^{(j)} &= 0 \\ \partial_t a^{(j)} &= \partial_t a^{(j)} + j \partial_x a^{(j)} \end{aligned} \quad (12)$$

$$L(0) = \frac{\gamma_s}{\gamma} (1-\alpha) \operatorname{sgn}(\beta-\delta) ,$$

$$L(\pm) = -\frac{\gamma_s}{\gamma} \left[\frac{1+\gamma_s \alpha}{2} \right] \operatorname{sgn}(\beta-\delta)$$

$$N(0) = 0 , \quad N(j) = j \left(\frac{\gamma+1}{2} \right)$$

$$D(0) = -\left(\frac{\chi_0}{\gamma} \right) , \quad D(\pm) = -\left(\frac{\chi_0}{\gamma} \right) \frac{\gamma_s}{2\gamma} , \quad (\gamma_s = \gamma - 1) .$$

The model of Eq. (6) has been used for the evaluation above.

As it was shown in Ref. (6), ($D(j) \equiv 0$), the presence of the non-linear coefficient $N(j) \neq 0$ will decrease the linear exponential growth rate $L(j)$ by a factor of one-half. Since $N(0) = 0$ for the non-propagating mode, this mode is unchanged if it is already exponentially growing. For modes that are exponentially decreasing then the non-linear analysis has essentially no effect on the decay rates.

By a straight forward extension of the H-K method, the second order in spatial derivative heat-conduction term ($-k \nabla^2 \tau$) can be incorporated in the non-linear perturbation theory; and its competitive effects against (1) the steepening tendency of the non-linear term, and (2) the linear exponential growth behavior are clearly exhibited as an added diffusion term $D^2 a^2$ to the usual non-linear transport equation of the H-K theory. Some qualitative feature of this additional term can be obtained by setting $a = a L \delta b$ and $dz = a L \delta ds$, and Eq. (12) can be recast as ($L, D < 0$)

$$\partial_z b + b \partial_s b = \left(\frac{D}{Lz + \gamma} \right) \partial_s^2 b \equiv \text{R.H.S.} \quad (13)$$

For initial wave-packet centered about $k_0 \ll O(1/f)$, the R.H.S. of Eq. (13) is small, and we are back to the conclusions of the $D \equiv 0$ case. If $k_0 \ll O(1/f)$, then the R.H.S. is no longer small, the non-linear modification of the linear growth rate L becomes k_0 -dependent; and no equally general statement, as in the $D \equiv 0$ case, can be made at present.

IV. Illustrative Applications

Our results can obviously be applied to a variety of situations where the lasing or laser target medium is subjected to local cooling that is density sensitive. For example, it has recently been suggested that the jet-like behavior observed in some target blowoffs is in fact due to radiative cooling instabilities.⁷ As an application of the above analyses let us consider the propagation of a $0.25 \mu\text{m}$ wavelength ($\omega_0 = 7.5 \times 10^{15} \text{ rad}\cdot\text{s}^{-1}$) laser with $I = 10^{12} \text{ W cm}^{-2}$

in the subcritical ($n_e \ll n_{cr} = 1.6 \times 10^{22} \text{ cm}^{-3}$) regime of the heated blowoff. Let us further assume that the material is pure Si heated to 300eV and at a density $= 0.5 n_{cr}$. The Si would be stripped to the K-shell so Z=12. (We emphasize that these numbers are for illustration only and need not correspond to any specific experiment.) At these temperatures the dominant cooling is bound-free plus collisionally excited line radiation. There is no local heating other than that of the laser so $H=0$. A rough estimate of the cooling rate can be extrapolated from Cooper⁸:

$$C = 10^{-23} n_e^{2/3} T_{kev}^{-1/2} \text{ erg} \cdot \text{cm}^{-3} \cdot \text{s}^{-1}.$$

Hence $\delta = 2$, $\alpha = -1/2$. This gives $\alpha = -1/4$, $\Gamma_0 = 2 \times 10^{12} \text{ s}^{-1}$ [cf. Eq. (6) $\bar{E}_0 = 70$]. For comparison, the filamentation growth rate in the limit of large k and no local cooling ($C = 0$) can be obtained using Eq. (5):

$$\Gamma_f = \frac{1}{8} \left(\frac{c}{\lambda_e} \right) \left(\frac{v_p}{v_o} \right) \left(\frac{v_o}{v_e} \right).$$

Using $v_e = 1.33 \times 10^9 T_{kev}^{1/2} \text{ cm} \cdot \text{s}^{-1}$, $\lambda_e = 10^{-4} T^2 \text{ keycm}$; $v_o = 6.5 \times 10^6 I_{12}^{1/2} \text{ cm} \cdot \text{s}^{-1}$ where $I_{12} \equiv I/10^{12} \text{ W} \cdot \text{cm}^{-2}$, we get $\Gamma_f = 2.6 \times 10^{12} \text{ s}^{-1}$. Hence the cooling instability (CI) could compete with the filamentation instability (FI) at sufficiently long wavelengths (cf. Figs. 1, 2). From Fig. 1 we also see that typically the maximum CI growth occurs at $k \approx 5$. Using classical thermal conductivity⁹ this translates into an optimal wavelength of $\lambda \approx 10^{-3} \text{ cm}$. The e-folding growth time at this wavelength is ~ 100 pico-second.

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

References

1. B. Langdon, LLNL Laser Program Annual Report, p. 3-56, UCRL5002180 (1981).
2. C. E. Max, "Laser-Plasma Interaction", Les Houches Session XXXIV, p. 302; R. Balian and J. C. Adams, ed. (1982) and refs. therein.
3. J. F. Drake, et al., Phys. Fl. 17, 778 (1974).
4. L. M. Gorbunov, Sov. Phys. Usp. 16, 217 (1973) and refs. therein.
5. J. K. Hunter and J. B. Keller, Commun. Pure and Appl. Math. 36, 547 (1983).
6. R. C. Y. Chin, J. C. Garrison, C. D. Levermore and J. Wong, UCID-19960.
7. R. G. Evans, "Radiation Loss Driven Instabilities in Laser Heated Plasmas", Proc. Dense Plasma Conf. Sarasota, Florida (1984).
8. J. Cooper, Rep. Prog. Phys. 29, 35 (1966).
9. L. Spitzer, Jr., Physics of Fully Ionized Gases, (Interscience, N.Y. 1962).