MASTER

Neutrino Mixing in a Grand Unified Theory

K. Milton† and K. Tanaka
Department of Physics
The Ohio State University
Columbus, Ohio 43210

Abstract

We have investigated neutrino mixing in a grand unified theory in which the neutrino mass matrix is determined by the Gell-Mann-Ramond-Slansky mechanism. With an arbitrary real right-handed Majorana mass matrix which incorporates three neutrino mass scales, we find that the effects of the up-quark mass matrix are dominant and as a result no significant mixing of ν_e occurs, while $\nu_\mu - \nu_\tau$ mixing can be substantial.

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

EAB

†Permanent address: Department of Physics, University of California
Los Angeles, California 90024

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Recently, experimental indications¹ have appeared that the electron neutrino may be massive and that oscillations^{2,3} between different neutrino types may occur. The question is, can these phenomena be accommodated in a grand unified field theory (GUT) such as $SO(10)$,⁴ with neutrino masses generated by the Gell-Mann-Ramond-Slansky (GM-R-S) mechanism⁵. Then the neutrino mass matrix is expressible in block form with the left-handed Majorana neutrino masses and CP-violating phases set equal to zero:

$$M^\nu = \begin{pmatrix} 0 & \nu \\ \nu & M \end{pmatrix}, \quad (1)$$

which refers to the neutrino field written as $\Psi = (\nu_e, \nu_\mu, \nu_\tau, \nu_e^c, \nu_\mu^c, \nu_\tau^c)$ where c means charge conjugate. The Dirac neutrino mass matrix ν , which is related to the up-quark mass matrix, is given in the standard $SO(10)$ model by⁶

$$\nu = \begin{pmatrix} 0 & a & 0 \\ a & 0 & m \\ 0 & m & b \end{pmatrix} \quad \text{where } a \ll m, b. \quad (2)$$

In the following we will take the form of ν to be given by (2), since that gives satisfactory charged-lepton and quark mass ratios as well as reasonable quark mixing angles, and concentrate our attention on the totally unknown Majorana mass matrix M . When the CP violating phases are set equal to zero, M can be written as

$$M = \begin{pmatrix} E & A & F \\ A & C & D \\ F & D & B \end{pmatrix}. \quad (3)$$

We note that because of the large disparity between the Majorana mass scale and the quark mass scale one needs to diagonalize only the 3×3 matrix $-\nu M^{-1} \nu$, this being the matrix generalization of the GM-R-S mechanism.⁷ We obtain from (2) and (3), assuming M^{-1} exists,

$$m_{ij} \equiv -(\nu M^{-1} \nu)_{ij}, \quad (4)$$

with $M_{11} = a^2 \alpha$, $M_{21} = M_{12} = -a(m\beta + a\delta)$, $M_{31} = M_{13} = a(m\alpha - b\beta)$,
 $M_{32} = M_{23} = m(b\gamma - m\beta) + a(b\delta - m\delta)$, $M_{22} = m^2 \gamma + 2am\delta + a^2 \epsilon$,
 $M_{33} = m^2 \alpha - 2mb\beta + b^2 \gamma$, where

$$\alpha = EB - F^2, \beta = ED - AF, \gamma = EC - A^2, \delta = AB - DF, \delta = AD - CF, \text{ and } \epsilon = CB - D^2. \quad (5)$$

The factor $(-\det M)^{-1}$ has been suppressed on the right hand side of the matrix elements M_{ij} . Although M involves 6 real parameters, it is not the most general 3×3 symmetric matrix, for the following inequalities hold:

$$\begin{aligned} M_{11} << M_{13} << M_{33} & \text{ if } \alpha \neq 0, \\ M_{12} << M_{23} & \text{ if } \beta \neq 0 \text{ or if } \beta = 0, \delta \neq 0. \end{aligned} \quad (6)$$

These inequalities of themselves are nearly sufficient to prove the absence of ν_e oscillations. [We will see below that the restrictions $\alpha \neq 0$, $\beta \neq 0$ can be lifted.]

To proceed systematically, we now diagonalize M by finding its eigenvalues λ_a and its eigenvectors x_j^a :

$$(M_{ij} - \lambda_a \delta_{ij}) x_j^a = 0. \quad (7)$$

The mixing matrix U is determined by

$$U^+ M U = \lambda, \quad (8)$$

where λ is the diagonal neutrino mass matrix, and where U is given by the eigenvectors, found from $(i, j, k=1, 2, 3)$

$$x_i^a = [m_{ik} m_{ij} + m_{jk} (\lambda^a - m_{ii})]^{-1}, i \neq j \neq k. \quad (9)$$

To limit the number of possibilities we suppose there are only three mass scales as indicated in (6), M_{11} , M_{13} , and M_{33} , and then it turns out that only two distinct cases emerge:

$$\begin{aligned} A: m_{23} &\sim m_{13}, \\ B: m_{23} &\sim m_{33}, \end{aligned} \quad (10)$$

where in either case m_{22} can be at any scale. The corresponding eigenvalues and mixing matrices are⁸ (we are free to permute columns in U)

$$A: \lambda_1 \sim m_{11}, \lambda_2 = m_{22}, \lambda_3 = m_{33}; \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B: \lambda_1 \sim m_{11}, \lambda_{2,3} \sim m_{33}; \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}.$$

Here, generally in the mixing matrix "1" and "0" mean $O(1)$ and $O(\frac{a}{m})$, respectively. We have assumed here that no "accidental" cancellations between parameters occurs, that is, for example, $\text{Tr} m \ll \max m_{ii}$ does not occur.

Examination of the three cases $\alpha=0, \beta \neq 0$; $\alpha \neq 0, \beta = \delta = 0$; and $\alpha = \beta = 0$ shows that only two possibilities occur:

a) If $m_{22} \gg m_{23}$ or $m_{33} \gg m_{23}$, then $U = 1$.

b) Otherwise, $U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$,

coinciding with the results of cases A and B above. We recognize that the case $\alpha = \beta = 0$ includes the model of M previously studied where $C = D = E = F = 0$. Indeed detailed calculation there yielded significant $\nu_\mu - \nu_\tau$ mixing and no ν_e mixing⁷.

The mixing angles corresponding to the coupling of the ν mass eigenstates to the charged lepton mass eigenstates are given by $\Gamma = U^\dagger V$ where U is given by (8) and V is the matrix that diagonalizes the charged lepton mass matrix. The unitary matrix V depends on the small ratio $(m_e/m_\mu)^{\frac{1}{2}}$ so it does not change the conclusions on neutrino mixing.⁹ We assume that consistent with phenomenology the charged lepton mass matrix is nearly diagonal.

To summarize, only two cases occur. In case A no mixing of any neutrino type occurs, while in case B, large $\nu_\mu - \nu_\tau$ mixing appears, but ν_e is unmixed.

The ν mass matrix is representative of those that follows from the up-quark mass matrix M^U in GUTS. We used a generalization of GM-R-S mechanism for 3 neutrinos and allowed various mass scales in the right-handed Majorana mass matrix M . It is surprising that essentially only two types of mixing matrices U emerged, one with no mixing at all and the other with $\nu_\mu - \nu_\tau$ mixing only.¹⁰ The latter type includes the special case obtained previously. If we relax the GM-R-S mechanism, for example, set the left-handed Majorana mass $\neq 0$, then the present predictions are not applicable. If the experimental data confirms the existence of $\nu_e - \nu_\tau$ mixing, then the present method of mass generation in GUT's is suspect. Therefore, it may be of interest to examine alternative schemes of mass generation that have been suggested^{11,12}.

Acknowledgements

One of us (KM) thanks the Aspen Center for Physics for hospitality during an early stage of this work, and P. Ramond and P. Vogel for discussions. The other author (KT) would like to thank G. F. Chew and M. Suzuki for hospitality at Lawrence Berkeley Laboratory. This work was supported in part by the U. S. Department of Energy under Contract No. EY-76-C-02-1545 *000.

REFERENCES

1. F. Reines, H. W. Sobel, and E. Pasierb, University of California, Phys. Rev. Lett. 45, 1307 (1980); V. A. Lyubimov, E. G. Novikov, V. Z. Nozik E. F. Tret'yakov, V. S. Kosik, Phys. Lett. 94B, 268 (1980); J. N. Bahcall et al., Phys. Rev. Lett. 45, 945 (1980).
2. Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962); B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys. JETP 26, 984 (1968)]; S. M. Bilenky and B. Pontecorvo, Phys. Reports 41, 225 (1978).
3. A. De Rujula, M. Lusignoli, L. Maiani, S. T. Petcov, and R. Petronzio, Nucl. Phys. B168, 54 (1980); V. Barger, K. Whisnant, and R. J. N. Phillips, University of Wisconsin preprint DOE-ER/00881-151, 1980.
4. H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) 93, 193 (1975); H. Georgi, Particles and Fields, 1974 (APS/DPF Williamsburg) ed. C. E. Carlson (AIP, New York, 1975).
5. M. Gell-Mann, P. Ramond, and R. Slansky, Supergravity, eds. P. van Nieuwenhuizen and D. Z. Freedman (North Holland Publishing Company, 1979), p. 315.
6. J. A. Harvey, P. Ramond, and D. B. Reiss, Phys. Lett. 92B, 309 (1980)
7. S. Hama, K. Milton, S. Nandi, and K. Tanaka, Phys. Lett, in press.
8. There is an interesting case with $m_{23} \sim m_{13}$ and $m_{22} \sim m_{11}$ where large $\nu - \nu$ mixing appears to occur. In fact, however, from the detailed structure of m given in Eq.(4), we see that here m is singular, and hence $\lambda_1 \approx 0$. This in turn implies $U \approx 1$, i.e. no mixing.
9. P. Ramond, Caltech preprint CALT-68-772.
10. There is some evidence that ν oscillations do not occur. See E. Bellotti et al., Lett. Nuovo Cimento 17, 553 (1976); J. Blietschau et al., Nucl. Phys. B133, 205 (1978); A. M. Cnops et al., Phys. Rev. Lett 40, 144 (1978); H. Wachmuth, Proc. 1979 International Symposium on Lepton and Photon Interactions at High Energies, eds. T. B. W. Kirk and H. D. I. Abárbanel, Fermilab (1979), p. 541.
11. E. Witten, Phys. Lett. 91B, 81 (1980); R. Barbieri and D. V. Nanopoulos, Phys. Lett 95B, 43 (1980).
12. In so far as the Witten mechanism (Ref.11) simply gives rise to the Majorana mass matrix M , it is covered by the analysis presented here.