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THREE-DIMENSIONAL RESISTIVITY INVERSION USING ALPHA CENTERS

By

W. R. PETRICK JR., W. R. SILL and S. H. WARD

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Department of Geology and Geophysics

**University of Utah
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ALPHA CENTERS

Wm. R. Petrick, Jr.*

Wm. R. Sill*

S. H. Ward*

*University of Utah, Salt Lake City, UT 84112

ABSTRACT

The alpha center formulation leads to a forward routine which is fast enough to make three-dimensional (3-D) resistivity inversion practical.

The method of alpha centers represents a class of solutions to the D.C. conduction equation based on certain nonlinear substitutions for electric potential and earth conductivity. The inversion routine results in a conductivity distribution defined by α centers which simultaneously fits the data from several parallel or perpendicular dipole-dipole profiles.

To illustrate the characteristics of this modeling approach we apply the inversion algorithm to three theoretical and four field data sets. The four field data sets represent samples from massive sulfide and geothermal environments. The technique gives fairly good estimates of the positions of conductive inhomogeneities but poor estimates of their actual conductivities.

The entire inversion algorithm requires less than 15000 words of computer memory thus making it tractable for small computers. We envision two major applications. The first is for in-field data interpretation to site drilling locations or guide further exploration. The second is for obtaining a good initial guess for more sophisticated and costly multi-dimensional inversion schemes.

INTRODUCTION

Solutions of the D.C. conduction equation using α centers were completely derived by Stefanescu and Stefanescu (1974). In that paper references to the α center method are given which date back to 1950. Most recently alpha media (the term "alpha media" refers to the conductivity distribution defined by α centers) have been used to calculate magnetometric resistivity responses (Edwards et al., 1978).

The method makes use of nonlinear substitutions for media conductivity and electric potential, followed by a straight-forward application of image theory. This results in an economical solution which is exact for alpha media.

Due to the speed and ease of parameterization of this forward algorithm it seemed only natural to implement it in an inverse routine. The ridge regression inversion algorithm was chosen because previous experience with the method has proven it to be stable and simple to implement in a wide variety of inverse applications (Inman, 1975; Rijo et al, 1977; Petrick et al., 1977; Pelton et al., 1978).

We don't maintain that this modeling scheme is ideal but it seems to be successful in locating the positions of conductive inhomogeneities in the presence of substantial geologic noise. It is also flexible enough to handle multi-body problems and problems of mixed dimension. Typical run times for ten iterations (convergence is usually attained by five iterations) is about 30 seconds on the University of Utah Univac 1108.

THE ALPHA CENTER FORWARD SOLUTION

This development follows, and uses the same notation as, Stefanescu and Stefanescu (1974).

We start with the general D.C. conduction equation in inhomogeneous isotropic media

$$(1) \quad \nabla \sigma \cdot \nabla \phi + \sigma \nabla^2 \phi = 0.$$

Where: σ = media conductivity

ϕ = electric potential

Through use of the transformations $\phi = \psi/\alpha$ and $\sigma = \alpha^2$ equation (1) becomes

$$(2) \quad \alpha \nabla^2 \psi - \psi \nabla^2 \alpha = 0.$$

Upon separating variables in (2) we obtain

$$(3) \quad \frac{\nabla^2 \psi}{\psi} = \frac{\nabla^2 \alpha}{\alpha} = f(m)$$

where m is any point in a whole space.

At this point $f(m)$ is arbitrary and can be chosen so that closed form solutions to (3) are obtained. In choosing $f(m)$ we determine the functional form of α which in turn fixes the form of the conductivity distribution since $\sigma = \alpha^2$.

One possible choice for $f(m)$ is $f(m) = k^2$ where k is a real positive constant. This choice of $f(m)$ was the subject of the 1974 Stefanescu and Stefanescu paper. It leads to a formulation having α media described by

$$\alpha(m) = \sum_i C_i \frac{e^{-KR_i}}{R_i} + C_{ii} \frac{e^{+KR_i}}{R_i}$$

where R_i is the distance from point m to the position of the i th α center. We first tested an inversion routine utilizing α media of this type. In all test cases on data generated with prismatic bodies k was forced to zero resulting in α being a sum of C_i/R_i terms.

Another choice for $f(m)$ which results in (3) having a closed form solution is $f(m) = 0$. This results in

$$(4) \quad \nabla^2 \psi = 0, \quad \nabla^2 \alpha = 0.$$

Choosing solutions to $\nabla^2 \alpha = 0$ of the form

$$(5) \quad \alpha(m) = B + \sum_i C_i / R_i$$

allows us now to easily specify a background conductivity (B^2). Because of this ease of specifying a background conductivity we chose the $f(m) = 0$ solution over the $f(m) = k^2$ solution for the final forward problem formulation, also as noted above the inversion routine often forced k to zero.

The spatially decaying solution to $\nabla^2 \psi = 0$ is

$$(6) \quad \psi(m) = \psi_0 + \sum_i D_i / R_i.$$

ψ_0 may be determined by noting that near a current electrode in a whole-space the primary electric potential ϕ_0 must behave as $I/4\pi r \alpha_0$. Substituting this in $\psi = \phi \alpha$ we have

$$\psi_0 = I/4\pi r \alpha_0$$

Where α_0 is the value of α at the current electrode.

At this point we have the following relationships;

$$\begin{aligned}
 \phi &= \psi/\alpha \\
 \sigma &= \alpha^2 \\
 (7) \quad \alpha &= B + \sum_i C_i/R_i \\
 \psi &= I/4\pi r\alpha_0 + \sum_i D_i/R_i
 \end{aligned}$$

Where: B, C_i are chosen constants

α_0 = value of α at the current electrode

r = distance from current electrode to the point at which the potential is measured

R_i = distance from i th alpha center to the point at which the potential is measured

D_i = constants to be determined

The earth conductivity is determined by choosing B, C_i , and the positions of the α centers. Once we determine ψ we may obtain the electric potential.

The continuity equation

$$\nabla \cdot \bar{J} + \frac{\partial \rho}{\partial t} = 0.$$

reduces to $\nabla \cdot \bar{J} = 0$ in the D.C. case. To determine ψ we enforce $\nabla \cdot \bar{J} = 0$ at each α center. This leads to a set of linear equations to be solved for the coefficients D_i .

From the constitutive relation $\bar{J} = \sigma \bar{E}$ (assuming isotropic media), $E = -\nabla \phi$, and the relationships given by equations (7) the expression for current density is

$$(8) \quad \bar{J} = \psi \nabla \alpha - \alpha \nabla \psi$$

Figure 1 shows the relationships between the components summarized in equations (7) for two alpha centers.

Expanding (8) and evaluating the expression at point m we have

$$(9) \quad \bar{J}(m) = \left(\frac{I}{4\pi r_{\alpha_0}} + \sum_i \frac{D_i}{R_{im}} \right) \left(-\sum_i C_i \frac{R_{im}}{R_{im}^2} \right) \\ - \left(B + \sum_i \frac{C_i}{R_{im}} \right) \left[\left(-\frac{I}{4\pi \alpha_0} \right) \left(\frac{\nabla r_{\alpha_0 m}}{r_{\alpha_0 m}^2} \right) - \sum_i \frac{D_i}{R_{im}^2} \right]$$

After constructing a sphere of radius R_{jm} about the j th α center we have

$$\oint_{S_j} \bar{J} \cdot \hat{n} ds_j = \int_{\Omega} (\bar{J} \cdot \nabla R) R_{jm}^2 d\Omega = 0.$$

$$\lim_{R_{jm} \rightarrow 0} [\bar{J}_{\text{radial}, m} R_{jm}^2] = 0 \quad \text{therefore}$$

$$-\left[\frac{I}{4\pi \alpha_0 r_{\alpha_0 m}} + \sum_{i \neq j} \frac{D_i}{R_{im}} + \frac{D_j}{R_{jm}} \right] C_j + \left[B + \sum_{i \neq j} \frac{C_i}{R_{im}} + \frac{C_j}{R_{jm}} \right] = 0.$$

Solving for the D's we obtain

$$(10) \quad D_j \left[B + \sum_{i \neq j} \frac{C_i}{R_{ij}} \right] - C_j \sum_{i \neq j} \frac{D_i}{R_{ij}} = C_j \frac{I}{4\pi \alpha_0 r_{\alpha_0 j}}$$

Equation (10) is valid for a point current source in a whole space. To account for a conductivity distribution in a half space ($z > 0$) and two current electrodes consider image alpha centers as shown in Figure 2. The image components are indicated by asterisks. Carrying out the same analysis as that leading from equation (9) to equation (10) except now

considering current input, extraction, and the image centers we have

$$(11) \quad D_j [B + \sum_{i \neq j} C_i (\frac{1}{R_{ij}} + \frac{1}{R_{ij}^*})] - C_j [\sum_{i \neq j} D_i (\frac{1}{R_{ij}} + \frac{1}{R_{ij}^*})] = C_j \frac{I}{2\pi} [\frac{1}{\alpha_p} r_{pj} - \frac{1}{\alpha_q} r_{qj}]$$

The system of equations defined by (11) may now be solved for the D's. Two important characteristics of this system are the coefficients defining the matrix are source independent, therefore the matrix only need be inverted once for a particular alpha center distribution, and the matrix is guaranteed to be non-singular for positive coefficients B and C_i .

The function ψ at point m on the plane $z = 0$ due to current injection and extraction on the same plane at points p and q is

$$(12) \quad \psi_{m,p-q} = (I/2\pi) [1/\alpha_p r_{pm} - 1/\alpha_q r_{qm}] + 2 \sum_i D_i / R_{im}$$

The expression for α at point m, where now m may be anywhere in the half space $z > 0$ is

$$(13) \quad \alpha_m = B + \sum_i C_i (\frac{1}{R_{im}} + \frac{1}{R_{im}^*})$$

The associated electric potential at point m on $z = 0$ is then

$$(14) \quad \phi_m = \psi_{m,p-q} / \alpha_m$$

From (14) the apparent resistivity for any electrode array on the plane $z = 0$ may be calculated. Figure 3 shows a comparison of the apparent resistivities calculated using the alpha center forward solution and a 2-D finite element program (Rijo, 1977) for the dipole-dipole array. The agreement is everywhere better than 10%. Alpha centers were placed every dipole length/3 normal to the plane of the profile in order to represent a 2-D body. The center of the string of α centers is 1.5 dipole lengths

below electrode 7 and have coefficients $B = .001$, $C = 2$. This particular alpha center body appears 2-D when its strike extent is 11 to 12 dipole-lengths.

Figure 4 is an example of the type of conductivity distribution that can be expected with alpha center modeling. This conductivity distribution is due to two centers located at $r_1 = 500\text{m}$ and $r_2 = 600\text{m}$. The three different curves indicate how the conductivity changes by varying the coefficients of the centers with the background conductivity being given by B^2 on Figure 4. The conductivity has the form $\sigma = B^2 + 2BC/R + C^2/R^2$ showing a rapid $1/R^2$ fall-off near the α center at $R = 0$ with a slower $1/R$ behavior at greater distance. The smaller the C coefficient the more rapid the conductivity decrease.

We can gain insight into an α center response by comparing it with an infinitely conductive sphere response. If we define r_{cs} as the distance from a single α center at which the conductivity contrast saturates, that being twenty times the background conductivity (i.e. $20B^2$), then in terms of α center parameters B and C the expression for r_{cs} is

$$(15) \quad r_{cs} = C/B (\sqrt{20}-1).$$

One might, therefore, (at least to a first approximation) expect to see a similar response from an infinitely conductive sphere with radius r_{cs} as that from an α center with parameters B and C . In the following development we show that this is not the case.

The x component of electric field due to an infinitely conductive sphere subject to a uniform x -directed electric field E_{ox} in a whole space is

$$(16) \quad E_s(x_p, y_p, z_p) \hat{i} = E_{ox} \left[1 - \frac{a^3}{R^3} \left(1 - \frac{3\Delta x^2}{R^2} \right) \right] \hat{i},$$

where: a = radius of the sphere,

$$R = [\Delta x^2 + \Delta y^2 + \Delta z^2]^{1/2}, \quad (R \geq a)$$

$$\Delta x = (x_p - x_c),$$

$$\Delta y = (y_p - y_c),$$

$$\Delta z = (z_p - z_c),$$

and the subscripts p and c refer to the position of the field point and the center of the sphere respectively.

The same field component due to a single α center subject to an x -directed uniform electric field E_{ox} in a whole space is

$$(17) \quad E_\alpha(x_p, y_p, z_p) \hat{i} = E_{ox} \left\{ B + \frac{C}{R} \left[1 + \frac{\Delta x^2}{R^2} \right] \right\} B / \left[B + \frac{C}{R} \right]^2 \hat{i}$$

$$\text{where: } R = [\Delta x^2 + \Delta y^2 + \Delta z^2]^{1/2},$$

$$\Delta x = (x_p - x_\alpha),$$

$$\Delta y = (y_p - y_\alpha),$$

$$\Delta z = (z_p - z_\alpha),$$

B and C are the usual α center parameters, and the subscripts p and α refer to the position of the field point and the α center respectively.

If we consider the responses at the point directly above the sphere and α center such that $\Delta x = 0$ and $\Delta y = 0$ in both (16) and (17), then equate (16) and (17) and solve for the radius of the sphere with $\Delta z = (z_p - z_c) = (z_p - z_\alpha)$ we find

$$(18) \quad a_{eq} = |\Delta z| \left[1 - \frac{B}{B + \frac{C}{|\Delta z|}} \right]^{1/3}$$

Therefore an infinitely conductive sphere of radius a_{eq} defined by (18) results in exactly the same response over its center as an α center with parameters B and C. Conversely C/B in terms of a_{eq} is

$$\frac{C}{B} = \frac{a_{eq} |\Delta z|}{|\Delta z|^3 - a_{eq}^3}, \quad |\Delta z| > a_{eq}.$$

The ratio C/B is defined at a particular Δz in terms of the single parameter a_{eq} since the α center conductivity is given by

$$\sigma = \alpha^2 = B^2 + \frac{2BC}{R} + \frac{C^2}{R^2}$$

whose fall-off depends not only on C but on B as well.

Figure 5 shows the single α center and similar infinitely conductive sphere responses for two different Δz values. The sphere radius a_{eq} is obtained from (18) using values of $B = .1$ and $C = 1$. The conductivity profile for two such α centers is shown as one of the curves in Figure 4. Using these values of B and C in equation (15) we obtain a radius of only 2.9m, while (18) results in a_{eq} of 27.5m for $\Delta z = 50$ m and $a_{eq} = 72.5$ m for $\Delta z = 200$ m. From this analysis we see that the radius r_{CS} provides a very misleading indication of the size of the conductive body.

From Figure 5 and equation 17 we see that the secondary alpha electric field does not change sign, resulting in E_x/E_{ox} always less than 1, therefore, an alpha center response is not a dipolar response. If we

substitute $\Delta x = n\Delta z$ in equations (16) and (17) we find that the α center electric field response approaches the background value as $1/n^2$ while dipolar behavior is $1/n^3$.

We can gain further understanding of the α center response by calculating the charge distribution due to an α center in a whole space subject to an incident uniform electric field E_{0x} .

$$(19) \quad \rho/\epsilon_0 = \nabla \sigma \cdot \nabla \phi / \sigma = \frac{2}{\alpha^2} \nabla \alpha \cdot [\nabla \psi - \psi \nabla^2 / \alpha].$$

Therefore the charge density at point p is

$$(20) \quad \rho(p)/\epsilon_0 = \frac{2E_{0x} BC\Delta x [BR + 2C]}{(BR+C)^3 R},$$

where the variables in (20) are the same as those defined for (17).

Figure 6 shows a plot of the charge density and the secondary electric field pattern on the plane $\Delta y = 0$. The response from this diffuse cloud of charge may be understood by integrating (20) over the volume defined by $\Delta x > 0$. The total charge in the right half space is given by

$$(21) \quad Q = 2E_0 CB \pi \epsilon_0 \int_0^\infty \frac{R [BR+2C]}{[BR+C]^3} dR.$$

If we pick an arbitrary hemispherical surface at the point R_C we can calculate the total charge in the regions $R < R_C$ and $R > R_C$. From (21) we have

$$(22) \quad Q = 2E_0 CB \pi \epsilon_0 \left[B \int_0^{R_C} \frac{R^3}{[BR+C]^3} dR + \int_{R_C}^\infty \frac{R^3}{[BR+C]^3} dR \right. \\ \left. + 2C \left(\int_0^{R_C} \frac{R^2}{[BR+C]^3} dR + \int_{R_C}^\infty \frac{R^2}{[BR+C]^3} dR \right) \right]$$

All but the second of the integrals in (22) converge. For large R the integrand approaches a constant indicating that no matter how far away from the α center you take R_C the component of total charge due to the charge density remaining in the region beyond R_C is infinite. This forces the secondary electric field to never change sign and so can never exactly duplicate a true dipolar field. The response is similar to that which would be experienced off to the side of a dipole. Note that the total charge in the volume is zero since the component of total charge in $\Delta x < 0$ is opposite in sign from that in $\Delta x > 0$.

THE INVERSION ALGORITHM

Now that we are aware of the idiosyncracies of responses to α defined conductivity distributions it is easier to understand how a few α centers can simulate the effect of large conductive bodies. In practice, we have found that by locking together several centers we can fairly successfully represent responses due to conductive layers, dikes, podiform masses, cylinders or any combination of these. Alpha centers are locked together by fixing intercenter distances while allowing the entire group to move as a whole.

Not only is the forward solution exceptionally fast but parameterization in α center inversion is conveniently accomplished. The inversion results are not discrete blocks or sharp contacts but a continuous resistivity distribution which, in some cases, may more nearly reflect the true geologic situation. The method is, however, not without its shortcomings and we have attempted to illustrate these as well as its advantages in several examples to be presented later.

The actual inversion technique employed is weighted least squares with ridge regression. The application to electrical methods has been described in many papers (Inman, 1975; Rijo et al., 1977; Petrick et al., 1977; Pelton et al., 1978) so in the interest of conciseness we refer the reader to these.

After having found a model which best fits the observed data it is useful to have some idea concerning how well each of our model parameters is determined. We may obtain some feeling for this by examining the parameter covariance matrix, which for weighted least squares is given by

$$(23) \quad \text{cov}(p) = \frac{\chi^2}{\nu} (A^T W T W A)^{-1},$$

where p is an inversion parameter (in this case B , C_i , and R_i), χ^2_{ν} is the reduced chi-square value defined as the weighted least squared error divided by $\nu = N-M$, the problem degrees of freedom, A is a matrix of derivatives and W is a data weight matrix.

Taking the square root of the diagonal elements of $\text{cov}(p)$ gives us an estimate of the parameter standard deviation while normalizing (23) as follows;

$$(24) \quad \text{cor } p_{ij} = \frac{\text{cov } p_{ij}}{[\text{cov } p_{ii} \cdot \text{cov } p_{jj}]^{1/2}}$$

results in parameter correlation coefficients. If the correlation coefficient is near +1.0 only the product of two parameters is determined. If near -1.0 only the quotient is determined.

To construct the A matrix it is necessary to take derivatives of apparent resistivity with respect to parameters of the conductivity distribution, essentially α in this case. Repeating (13)

$$\alpha_m = \sqrt{\sigma_m} = B + \sum_i C_i \left[\frac{1}{R_{im}} + \frac{1}{R_{im}^*} \right]$$

we see that the conductivity distribution depends on B , C_i , and the x , y , z coordinates of the α centers. Since B , C_i , and z_i can never be negative we can most simply assure this by inverting with respect to $\log B$, $\log C_i$ and $\log z_i$. Derivatives with respect to x_i and y_i are taken with respect to the normal Euclidean space parameters. When several α centers are locked together the derivative is taken with respect to the component of α in

which movement of the entire group is desired. The derivatives are approximated using the first forward difference.

We actually fit log apparent resistivity data so the reduced chi-square data variance estimate actually refers to a percent data error. Similarly the parameter standard deviation estimates are in terms of percent standard deviation for B , C_j , and z_j while the estimates are in units of meters for the x, y coordinates. By percent standard deviation we refer to a measure that is normally distributed about the parameter estimate in log space. For example, if we determine the parameter B to within $\pm 60\%$ the upper value in Euclidean space is given by $B \times 1.6$ while the lower value is $B/1.6$.

Originally a program was used which calculated the percent standard deviations of the inverted earth model resistivities from the α center statistics. Experience has, however, shown that the direct α center statistics provide more useful information concerning possible reparameterizations. The reason for this is the direct α parameter statistics provide a breakdown of exactly what aspects of the model are poorly determined, while resistivity confidence limits represent a combination of all direct α parameter statistics. The one standard deviation error bars on the plots to follow refer to the direct α parameters B , C_j , x_j , y_j , z_j .

INVERSE TEST CASES

During the normal course of program development we, of course, assured ourselves that the algorithm was capable of inverting to the correct α conductivity distributions for data generated by α media. A more important practical question concerns the ability of the alpha inversion to determine the location and conductivities of more general conductive inhomogeneities. To test this ability we used theoretical data from 3-D conductive prisms. Two data sets were obtained from finite difference calculations (Dey and Morrison; 1979). The models chosen were a 3-D conductive prism in a half space (Model 9-3D) and Dey and Morrison's basin and range geothermal model. A third data set was obtained from Gerry Hohmann (Personal communication) and represents an integral equation solution of two finite length vertical dikes in a half space.

Test case 1: Model 9-3D

The theoretical data shown in Figure 7 is from a profile bisecting the prism perpendicular to its strike. The strike extent is two units.

These data were inverted several times with the different parameterizations shown in Table 1. Since theoretical data was available for only one profile we constrained the lateral positions of four α centers at ± 0.5 units with respect to the profile. The rather poor initial guess for the positions of these centers are indicated by the open circles in the model shown in Figure 7. The α centers are actually constrained to lie at ± 0.5 units off of the profile to mimic the strike extent of the prism.

Table 1 shows that all parameterizations were capable of fitting the theoretical data to within about 20%. This is probably about as good a fit to data from prismatic models as we can expect from an α center approximation. Note that the parameterization for inverse four in Table 1 includes dip as an inversion parameter. In this modeling approach dip is approximated by allowing the lower two α centers to move along the profile independently from the top two centers. We found that the positions along the profile as well as the depth of these deeper α centers were very poorly determined parameters supporting the observation that dip is not well determined by dipole-dipole surveys.

The inversion results for parameterization 3 are shown in Figure 7. The final positions of the α centers are shown by the solid circles. The one standard deviation error bars indicate that the depths to the top α centers and their positions along the profile are fairly well determined ($\pm 63\%$ and ± 0.13 unit respectively) while the depth extent, as implied by the

position of the lower α centers, is very poorly determined. Table 2 shows the parameter statistics and correlation coefficients. High correlations between alpha center coefficients C and background value B were virtually always obtained. The reason for this is easily explained by Figure 4, noting that the $1/R$ component of conductivity fall-off is controlled by the product of B and C. The correlation is reduced by data containing more samples distant from the inhomogeneity. The remainder of the large correlation coefficients are all in some way associated with the very poorly determined depth to the lower α centers.

These inversion results point out two serious defects in this modeling scheme. The first is that, although the location of the top α center is fairly well determined, we are given no clear cut indication of the actual depth to the top of the conductor. We may, however, be fairly confident that the α center position determined by inversion represents a maximum depth. The second defect is that we have no way of deducing the actual conductivity of the causative body from the α centers. This has been observed by practitioners of two-dimensional inversion using prismatic models who find that, without narrow constraints on prism boundaries, the conductivity distribution is very poorly determined from surface data (Oristaglio and Worthington, 1979).

Ten iterations for parameterization three required 18 seconds on the University of Utah Univac 1108 and cost 7 cents. Figure 8 shows the convergence.

Test Case 2: Dey and Morrison Basin and Range Geothermal Model

This model is shown in Figure 9. Data from line 1 and line 4 were simultaneously inverted. The initial guess is indicated in Figure 9 by the open circles. The small circles near the surface represent a distribution of α centers sufficient to account for the near surface conductive layer and the more conductive material to the left of the vertical contact. The positions of these were kept constant but their coefficients were included as inversion parameters. Four α centers were used to approximate the deeper conductive prism. Their intercenter geometry was specified by the position of the array was included as another inversion parameter. This is tantamount to assuming that one knew approximately the size of the reservoir but not its location.

The theoretical data from lines 1 and 4 are shown in Figure 10. Initially the data from line 1 were inverted alone with the off-profile position of the reservoir specified. All attempts failed to locate the reservoir, instead the reaction was to simply remove the 3-D inhomogeneity. This is not particularly surprising when, as Dey and Morrison point out, the difference between data along line 1 with the prism and data along line 1 without the prism is slight. The inclusion of the line 4 profile greatly improved the situation. Since we were simultaneously inverting data from orthogonal traverses there was no further need to specify the lateral position of the prism with respect to line 1. The simultaneous inversion placed all four prism α centers within the location of the hypothetical reservoir.

The resultant position of the centers and their one standard deviation

error bars are shown in plan and profile in Figures 9 and 10 by the solid circles. Figure 10 shows the resultant best fit pseudosections. These represent a data fit of approximately 30% the convergence to which is shown in Figure 11.

The parameter statistics, Table 3, indicate that this combination of parameters and data sets results in a well determined model with no highly correlated parameters.

Five iterations for this group of parameters, simultaneously inverting to both data sets, took 5 minutes and cost \$4.20.

Test Case 3: Parallel Dikes

This model is shown in Figure 12. The profile is perpendicular to, and bisects, the dikes, each of which is eight dipole lengths in strike extent.

The difficult aspect of inverting this data is that there is a rather attractive local chi-square minimum corresponding to a single large, deep conductor located at the center of the profile. The α center difference in data fit between the local and the "true" minima is only three percent. The situation is aggravated by the low contrast ($\rho_2/\rho_1 = .2$) bodies involved but the major difficulty is due to the inability of the α response to duplicate the high apparent resistivities between electrodes 7 and 9 at $n=1$.

Three parameterizations were attempted. The first two involved starting with α centers locked together to form two separate vertical dikes with inversion variables being; the background conductivity (B), coefficient for the left dike (C_{left}), coefficient for the right dike (C_{right}), position of the left dike along the profile (x_{left}), and the position of the right dike along the profile (x_{right}). The strike position of the dikes was held constant since we have only one profile. The difference between these two parameterizations was that one included as variables the depth (z_{left} and z_{right}) of each dike while for the other the depth was specified and held constant.

In the cases where the depth was a variable the inversion process converged to the local minimum. Only when the depth was specified were the correct positions along the profile determined. This is certainly a

serious constraint since rarely in practice would one have a better idea of the depth to the top of a body than of its position along the profile. Perhaps a better initial guess would remedy the problem but it is probably safe to say that the false minimum is a broad one capable of engulfing all but the best of initial guesses.

The third parameterization represents a more practical application of inversion in which the position along the profile of one body is known. The addition of this information deprives the solution of the single central body local minimum resulting in a very stable inverse. Included as inversion parameters were the background (B), the position along the profile (x_{left}) and depth (z_{left}) of the left dike, the coefficients of both dikes (C_{left} and C_{right}), and the depth of the right dike (z_{right}).

The initial guess α center positions are shown in Figure 12 as the open circles while the inverted α center positions are the solid circles. Figure 12 also shows the resultant best fit pseudosection. This represents a data fit of approximately 19%. The convergence to this fit is shown in Figure 13. Parameter statistics are listed in Table 4.

Thirteen iterations required 5.5 minutes and cost \$7.40.

FIELD EXAMPLES

To test the ability of a center inversion to locate conductive bodies in the presence of geologic noise we chose data from three massive sulfide environments and a geothermal area. The only available data for each of these test cases is from a single profile; therefore the off-profile geometry of the conductor is held fixed. In the massive sulfide test cases the off-profile geometry was known from drill hole information.

Field Example 1: Que River, N.W. Tasmania

The field data from Line 7350N and simplified geologic cross section are shown in Figure 14 (Webster and Skey, 1977). Figure 15 is a plan view of the massive sulfide ore outline.

An effective approach to invert real field data is to initially fix the depth to the top of the conductive zones and invert to background and conductor coefficients and positions along the profile. Once a minimum is obtained for this parameterization, the depth to the conductive zone is added as a new inversion variable. This approach was used on the Que River data allowing both P and S conductor parameters to vary independently. The result was a correct placement of the large, shallow S conductor beneath 5274E with the P conductor located at approximately 5250E. S and P conductor parameters were highly correlated so we decided to fix the position of the S conductor and search only for the position (x_p , z_p) of the P conductor and coefficients C_s and C_p . This parameterization consistently (regardless of any reasonable initial guess) resulted in a conductor being placed deeply below the electrode at 5250E. One might conclude that within the data fit error (approximately 120%) that there is no D.C. resistivity information about a P conductor beneath 5150E.

At this point we should define the components of "data fit error" with reference to real field data. The first component is measurement error which, for modern equipment in conductive terrains, is perhaps of order 5%. The second component is geologic noise: the response of small (or possibly large) scale inhomogeneities not directly associated with the sought-after orebody. The third component is actually very difficult to separate from

geologic noise and that is the possibility, or inevitability, that the wrong forward problem is being done. Doing the wrong forward problem may be further subdivided into;

- a) the ability of a model conductivity distribution to fit what is actually in the subsurface,
- b) the adequacy of, or our interest in, modeling the "geologic noise" reflected in the field data.

We've found that using an α center model to fit theoretical data from conductive prisms usually results in a data fit error in the neighborhood of 20 to 30 percent.

To summarize, unless we really know the exact subsurface conductivity distribution it is impossible to separate the geologic noise component of data fit error from the component due to doing the wrong forward problem.

The model statistics and correlation coefficients for the last parameterization are shown in Table 5. Ten iterations required 1.5 minutes and cost \$1.90.

Field Example 2: Woodlawn, New South Wales, Australia

Field data from this massive sulfide prospect are shown in Figure 16 (Tyne et al., 1978). As shown in the plan view, Figure 17, the profile crossed the northern edge of the near vertical orebody at an angle of approximately 30° .

Initial parameterization took into account only variation in parameters B, C, and position along the profile. From an initial guess shown by open circles in Figures 16 and 17 the position of the α described body was moved horizontally along the profile to approximate the actual position of the conductive (approximately $1 \Omega m$) massive sulfide body.

At this point we reparameterized to include also the depth of the α centers, using as an initial guess the results from the first inversion. This resulted in an insignificant position adjustment along the profile and a slight increase in the depth to the top α center from 15m to 18m. The final position of the centers with one standard deviation error bars are shown by the solid circles in Figures 16 and 17. These results would certainly be accurate enough to locate a drill hole correctly along the profile.

Table 6 shows the parameter statistics and correlation coefficients for the final parameterization. The data fit was approximately 160%. Each inversion took from 40 to 50 seconds for 10 iterations.

Field Example 3: Elura, New South Wales, Australia

The area in the vicinity of the Elura prospect is flat with extensive alluvial cover and little outcrop (Ogilvy, R.D., 1976). There is no surface expression of mineralization. The massive sulfide orebody is a near-vertical pipe (McCracken, personal com.) below approximately 90m of overburden in a host of steeply dipping sediments. The dipole length is 30.48m.

The data and outline of the ore body are shown in Figure 18. Due to the abundant geologic noise and depth of burial the inversion scheme failed to locate the conductor. The approach was to attempt to fit the near surface response with a surface α center distribution, then hold their positions constant and invert to the position of an α center defined vertical cylinder. The inverted section in Figure 18 represents the α approximation to the near surface noise. All attempts to locate the vertical conductor failed. The inversion routine simply removed the cylindrical conductor from the area of interest.

The data fit is approximately 75%.

Field Example 4: Black Rock Desert, Utah

The data shown in Figure 19 is from the north portion of a 3km multifrequency dipole-dipole survey extending north for 33km from the north end of the Mineral Range. The pseudosection reflects the apparent resistivity data from the low frequency asymptote. A deep drill hole 3km west of the north end of the Mineral Range is the only local geologic control. The lithologic log reveals approximately 1km of pleistocene sediments overlying paleozoic sediments. The basalt and rhyolite flows shown in Figure 20 have been dated at 2.1 to 2.2 m.y.b.p. The cambrian sediments comprising the Cricket Range just to the west of the profile are also present in thrusts at the north end of the Mineral Range. Water was virtually always encountered in shallow drill holes which penetrated the basalt and rhyolite flows.

Since only one profile was available we made use of what meager geologic information we had in constraining the α centers. The inversion routine insisted on only shallow conductors so the off-profile centers played a negligible role in the interpretation. At this point all α centers were constrained to lie immediately beneath the profile.

The two different inversion results illustrate the trade-off between data fit and parameter resolution. When three α centers are used the data fit error is 97% and the α center parameters are fairly well determined (Table 7a). When five α centers are used the data fit improves to 73% at the expense of parameter resolution (Table 7b).

The first model with three α centers, defines, in a general fashion, the near surface conductive zone due to the sedimentary fill between the

Cricket Range and the Mineral Range. The two additional α centers improve the fit by defining a conductive zone between the rhyolite and basalt flows (stations 24-27). The location of the deepest α center is very poorly determined indicating that from this single profile not much can be said about the deeper conductive zone.

This example serves as an illustration of how the inversion algorithm could provide information to guide further exploration or a reasonable initial guess for more sophisticated and probably much more costly multidimensional forward modeling or inversion schemes. The cost of obtaining these results was 7 cents for ten iterations.

DISCUSSION AND CONCLUSIONS

Through the use of this very fast and simple forward solution the D.C. response of complex three dimensional bodies may be easily calculated. Inverting data from prismatic bodies shows that their response can usually be approximated to within 20 to 30 percent. Multiple bodies and problems of mixed dimension present no special complications although greatest efficiency is achieved for compact bodies. The most severe limitation inherent in the α center formulation is the inability to approximate resistive bodies. Another limitation is the difficulty in determining the actual conductivity of prismatic bodies.

The field examples illustrate the capability of the inversion algorithm to, in most cases, determine the positions of conductive bodies in the presence of abundant geologic noise. The program size, including both instruction and data storage, is currently less than 15000 words and could easily be decreased to 10000 words with judicious programming. These storage requirements bring the program within reach of small truck mounted computer systems such as those currently being employed in data acquisition systems. This application would make possible in-field data interpretation.

A practical approach to inverting actual field data was first to use any available geologic information to determine the gross conductor geometry (e.g. podiform mass, vertical dike, manto, etc.) and lock together several α centers to approximate this shape. Again using as much geologic information as possible, fix the depth to the body at some plausible figure and invert to the position along the profiles. Once obtaining a minimum

for this parameterization include the depth to the conductor as a variable and invert again using the initial inversion results as a starting model.

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FIGURE CAPTIONS

- Fig. 1. Relationships between components summarized by equations (7) for two α centers (α_1 and α_2) in a whole space.
- Fig. 2. Relationships between components of a center solution now taking into account current input at point p, current extraction at point q, and image α distribution resulting in a half space solution. Image components are denoted by asterisks.
- Fig. 3. Comparison of 2-D finite element solution and α center solution for dipole-dipole array. Electrode spacing is a. The 2-D inhomogeneity is approximated by a string of α centers placed normal to the plane of the section spaced a distance $a/3$ apart at a depth of $1.5a$ below electrode seven.
- Fig. 4. Conductivity distribution due to two α centers. The three different curves show how the conductivity changes as the coefficients C_i of the α centers are changed. σ_{cs} is the level at which the conductivity is 20 times the background conductivity defined by B^2 .
- Fig. 5. Comparison between α center and infinitely conductive sphere responses to a uniform x-directed electric field in a whole space. The results are for two different Δz values. 1) α_1 and a_{eq1} at $\Delta z = 50$ m, 2) α_2 and a_{eq2} at $\Delta z = 200$ m. a_{eq} is obtained by substituting $B = .1$, $C = 1.$, and Δz in (18).
- Fig. 6. Plot of charge density and resultant secondary electric field pattern for one α center in a whole space subject to a uniform x-directed electric field E_{ox} . Plot is in plane $(x, 0, z)$ through an α center located at origin. Contours indicate values of $\rho/\epsilon_0 E_0$ and vectors are in terms of E_{sec}/E_0 .
- Fig. 7. Test case 1: Model 9-3D. Parameterization 3 inversion results using α centers to fit data from a prismatic model. The prismatic body which is bisected by the profile is two dipole lengths in strike and depth extent, has a thickness of one dipole length and is .5 dipole lengths to the top. The prism has a resistivity of $3\Omega m$ and is set in a $100\Omega m$ background.
- Fig. 8. Convergence for Model 9-3D, parameterization three. Reduced chi-square value vs. no. of iterations.
- Fig. 9. Plan and profile view of Dey and Morrison basin and range geothermal model. Small open circles indicate position of surface α center distribution sufficient to account for more conductive material to left of contact. Large open circles show initial guess α center representation of "reservoir". Solid circles indicate inverted position of reservoir with

FIGURE CAPTIONS (Cont.)

one standard deviation error bars on position.

- Fig. 10. Test case 2. Results of simultaneous inversion of data from orthogonal profiles (line 1 and line 4) of Dey and Morrison basin and range geothermal model. The final positions of the four α centers used to mimic the reservoir are shown as the solid circles projected on the appropriate plane. The one standard deviation error bars are also shown. For convenience a dipole length of 100 m was chosen.
- Fig. 11. Convergence for Dey and Morrison basin and range geothermal model. Reduced chi-square value vs. no. of iterations.
- Fig. 12. Test case 3: Two parallel dikes. Inversion results using α centers to fit data from prismatic model of two parallel, finite length vertical dikes. The dikes are bisected by the profile. Solid circles show projection of final α center positions with associated one standard deviation error bars. A dipole length of 100 m was used for the calculations.
- Fig. 13. Convergence for test case 3: two parallel vertical dikes. Reduced chi-square vs. no. of iterations.
- Fig. 14. Field example 1: Que River line 7350N. Inversion results showing field data, best fit pseudosection and alpha center model. Error bars represent one standard deviation. Note that although the S conductor has been very precisely located the inversion scheme failed to locate the deeper P conductor.
- Fig. 15. Plan view showing locations of P and S conductors with respect to line 7350N. Que River, N.W. Tasmania.
- Fig. 16. Field example 2: Woodlawn, N.S.W., Australia. Inversion results showing field data, best fit pseudosection and α center model. Initial guess α center positions are the open circles. Electrode six represents a local position along the profile of 243.8 m with a dipole length of 30.48 m.
- Fig. 17. Plan view showing position of massive sulfides at base of gossan with respect to survey line and electrode positions. Initial guess α center position are open circles while solid circles represent the inverted to position of the α -defined conductive inhomogeneity. Woodlawn, N.S.W., Australia.
- Fig. 18. Field example 3: Elura. Inversion results showing the inability of an α center model to determine the position of the deep vertical conductor in a conductive host. All α center parameters were undetermined. The dipole length is 100 m.

FIGURE CAPTIONS (Cont.)

- Fig. 19. Field example 4: Black Rock Desert, Utah. Two sets of inversion results illustrating the trade-off between data fit and parameter resolution when more α centers are used in the inverse solution. The plots to the left show the results when only three α centers are used to define the conductivity distribution while those on the right show the results for five centers. If any deep conductors exist in this region the α center inversion scheme failed to discriminate between their response and the near surface conductive areas. The dipole length is 3 km.
- Fig. 20. Area map showing the north end of the Black Rock Desert 3 km dipole-dipole survey line near the Roosevelt Hot Springs KGRA, Utah.

TABLE CAPTIONS

- Table 1. Different parameterizations used to invert prismatic Model 9-3D data. $\sigma = \alpha^2 = (B + \sum C_i/R_i)^2$. Note that data fit is virtually the same for all parameterizations. x denotes an inversion parameter.
- Table 2. Alpha center parameters, parameter statistics and correlation coefficients for Model 9-3D, parameterization three.
- Table 3. Alpha center parameters, parameter statistics and correlation coefficients for Dey and Morrison basin and range geothermal model.
- Table 4. Alpha center parameters, parameter statistics and correlation coefficients for test case 3, two parallel vertical dikes.
- Table 5. Alpha center parameters, parameter statistics and correlation coefficients for field example 1, Que River, N.W. Tasmania.
- Table 6. Alpha center parameters, parameter statistics and correlation coefficients for field example 2, Woodlawn, N.S.W., Australia.
- Table 7. Alpha center parameters, parameter statistics and correlation coefficients for two different parameterizations of the Roosevelt Hot Springs KGRA, a) three α centers, b) five α centers. Where depths are not shown they were constrained to be 10 m.

Fig. 1

Fig. 2

2-D Finite Element Pseudosection (Ωm)

Model Resistivity Cross Section (Ωm)

Alpha Center Pseudosection (Ωm)

Fig. 3

Fig. 4

Fig. 5

$\frac{E_{\text{sec}}}{E_0}$ SCALE $\boxed{.1}$

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Table 1. Different parameterizations used to invert prismatic Model 9-3D data. $\sigma = \alpha^2 = (B + \sum_i C_i/R_i)^2$
 Note that data fit is virtually the same for all parameterizations.
 x denotes an inversion parameter.

Parameterization	B	$C_i = C$	Position Along Profile	Depth To Top α Center	Depth To Lower α Center	Dip	Data Fit
1	x	x	x				20%
2	x	x	x	x			21%
3	x	x	x	x	x		21%
4	x	x	x	x	x	x	21%

Table 2. Alpha center parameters, parameter statistics and correlation coefficient, for Model 9-3D, parameterization 3.

Parameters	Inverted Model	Parameter Statistics	Correlation Coefficients				
B	.071	$\pm 25\%$	1.0				
$C_i = C$	1.6	$\pm 104\%$	-.99	1.0			
$x_i = x$	748m	$\pm .14m$	.04	-.02	1.0		
z_{Lower}	334m	$\pm 225\%$	-.96	.99	-.01	1.0	
z_{Upper}	91m	$\pm 63\%$	-.95	.95	.04	.93	1.0

Table 3. Alpha center parameters, parameter statistics and correlation coefficients for Dey and Morrison basin and range geothermal model.

Parameters	Inverted Model	Parameter Statistics	Correlation Coefficients						
B	.024	±28%	1.0						
C _{Overburden}	.19	±20%	-.69	1.0					
C _{Reservoir}	1.25	±24%	-.14	-.58	1.0				
x _{Center Res.}	617m	±48m	-.61	.69	-.30	1.0			
y _{Center Res.}	699m	±35m	.0	-.01	.01	.0	1.0		
z _{Top. Res.}	184m	±21%	-.44	.11	.55	.15	-.01	1.0	

Table 4. Alpha center parameters, parameter statistics and correlation coefficients for test case 3, two parallel vertical dikes.

Parameters	Inverted Model	Parameter Statistics	Correlation Coefficients					
B	.06	±17%	1.0					
C _{Left}	.34	±35%	-.41	1.0				
x _{Left}	637m	±59m	.38	.37	1.0			
z _{Left}	120m	±109%	-.21	.78	.22	1.0		
C _{Right}	.37	±42%	-.72	-.28	-.67	-.21	1.0	
z _{Right}	125m	±85%	-.41	-.17	-.25	-.07	.73	1.0

Table 5. Alpha center parameters, parameter statistics and correlation coefficient for field example 1, Que River, N.W. Tasmania.

Parameters	Inverted Model	Parameter Statistics	Correlation Coefficients			
C_s	.313	$\pm 58\%$	1.0			
C_p	.23	$\pm 250\%$	-.98	1.0		
x_p	5248m	$\pm 94m$	-.91	+.91	1.0	
z_p	55m	$\pm 141\%$	.53	-.37	-.45	1.0
B	5×10^{-4}	poorly determined parameter - fixed from earlier inversion				
x_s	5274m	$\pm 8m$	fixed from earlier inversion			
z_s	fixed as shown in Figure 14					

Table 6. Alpha center parameters, parameter statistics and correlation coefficients for field example 2, Woodlawn orebody, Australia.

Parameters	Inverted Model	Parameter Statistics	Correlation Coefficients			
B	.02	$\pm 167\%$	1.0			
C	.37	$\pm 33\%$	-.89	1.0		
x	253m	$\pm 12\text{m}$	.14	-.08	1.0	
z	18m	$\pm 162\%$	-.58	.81	-.05	1.0

Table 7. Alpha center parameters, parameter statistics and correlation coefficients for two different parameterizations of Black Rock Desert data. Where depths are not shown they were constrained to be 10m.
a) three α centers b) five α centers

a										
Parameters	Model	Inverted Parameter Statistics	Correlation Coefficients							
B	.074.	$\pm 71\%$	1.0							
C ₁	321.	$\pm 35\%$	-.34	1.0						
x ₁	16.7km	$\pm 1.4\text{km}$	-.11	-.13	1.0					
C ₂	296.	$\pm 47\%$	-.47	-.33	.60	1.0				
x _L	24.4km	$\pm .9\text{km}$	.17	-.29	.05	.15	1.0			
C ₃	112.	$\pm 87\%$	-.24	-.13	-.57	-.31	-.14	1.0		
x ₃	13.2km	$\pm 2.2\text{km}$	-.20	.07	-.05	.13	-.02	.03	1.0	

b

Parameters Model Inverted Parameter Statistics

Correlation Coefficients

B	.024	undet.	1.0											
C ₁	72.6	undet.	-.34	1.0										
x ₁	26.9km	±8.6km	-.46	-.96	1.0									
C ₂	241.	±130%	-.64	.28	.34	1.0								
x ₂	16.0km	±2.8km	.62	-.23	-.29	-.75	1.0							
C ₃	260.	±120%	.57	-.29	-.34	-.94	.86	1.0						
x ₃	18.7km	±2.3km	.64	-.28	-.34	-.95	.87	.96	1.0					
C ₄	212.	undet.	.33	-1.0	-.95	-.28	.23	.29	.28	1.0				
x ₄	25.8km	±7km	-.32	.98	.92	.27	-.21	-.28	-.26	-.98	1.0			
z ₄	1.34km	undet.	-.49	.91	.97	.33	-.28	-.34	-.32	-.90	.89	1.0		
C ₅	90.2	±160%	-.59	.16	.24	.44	-.86	-.61	-.62	-.17	.14	.23	1.0	
x ₅	12.7	±1.3km	.44	-.18	-.23	-.51	.65	.55	.57	.19	-.17	-.21	-.49	1.0