To be presented at the Sixth International Heat Transfer Conference-
Toronto, Ontario Canada - August 7-11, 1978

BNL-23054

COUPLED CONDUCTION-TURBULENT CONVECTION IN A CIRCULAR TUBE

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MASTERABSTRACT

An analysis for hydrodynamically and thermally fully developed heat transfer in a circular tube coupled with heat conduction in the tube wall is presented. The energy equations for the fluid and solid body are solved simultaneously under the conditions of continuity in the heat flux and temperature at the interface. Circumferential as well as radial temperature gradients are permitted in both energy equations along with internal heat generation in the tube wall. Solution of the equations are found by the method of separation-of-variables along with superposition of particular solutions. As an application, cooling of the first wall in a fusion reactor is considered. Comparisons of temperature profiles in the tube wall are made for high (aluminum) and low (stainless steel) thermal conductivity materials. Comparisons are also made under the assumption that radial temperature gradients are negligible.

NOMENCLATURE

$A_n, a_n, b_n, c_n, d_n, e_n, f_n$ = Fourier Coefficients

A_i = inside tube area, πR_i^2 , m^2

A_w = solid tube area, $\pi (R_o^2 - R_i^2)$, m^2

c_p = specific heat at constant pressure, J/kgK

E = diffusivity function, $(1+K/K)$

K = eddy thermal diffusivity, m^2/s

L = tube length, m

L_s = operator, $\lambda_s \left[\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right]$

L_f = operator, $\kappa \left[\frac{1}{r} \frac{\partial}{\partial r} (r E_r \frac{\partial}{\partial r}) + \frac{1}{r^2} \frac{\partial}{\partial \phi} (E_\phi \frac{\partial}{\partial \phi}) \right]$

L_f^* = dimensionless operator, $\frac{1}{\xi} \frac{\partial}{\partial \xi} (\xi E \frac{\partial}{\partial \xi}) +$

$\frac{1}{\xi^2} \frac{\partial}{\partial \phi} (E_\phi \frac{\partial}{\partial \phi})$

Nu = Nusselt number

$P = (P_o q_o + A_w Q) / 2\pi$, W/m

P_o = outside tube circumference, $2\pi R_o$, m

Pr = Prandtl number

q = applied surface heat flux, W/m^2

Q = average volumetric heating, W/m^3

r = radial co-ordinate, m

R = tube radius, m

R_n = radial temperature function

Re = Reynolds number

S = dimensionless temperature function

u = axial velocity, m/s

U = dimensionless velocity, u/U_m

x = axial co-ordinate, m

α = heat transfer coefficient, W/m^2K

δ = tube-wall thickness, m

$\Delta\theta_m$ = fluid temperature difference between inlet and outlet, $^{\circ}C$

η = dynamic viscosity, kg/ms

θ = temperature, $^{\circ}C$

κ = thermal diffusivity, m^2/s

λ = thermal conductivity, W/mK

ξ = dimensionless radius, r/R_i

ρ = mass density, kg/m^3

Subscripts

f = fluid

i = inside

m = mixed mean

o = outside

s = solid

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INTRODUCTION

In the development of fusion reactors the ability to cool the first wall, i.e., the first material surface interfacing the plasma looms as the critical region of concern since all of the emissions from a reacting plasma may intersect this wall. These include ions and neutral particles, primary neutron, X-rays (bremsstrahlung) and cyclotron radiation. In addition, scattered neutrons and gamma radiation generated in the blanket regions exterior to this first wall are also imposed on it. Finally, D-T gas surrounding the plasma may react chemically with the first wall materials. One result of these many interactions is the generation of appreciable heat which can be up to 20% of the plasma output. In most analyses the above effects are usually lumped together as a radiant flux. While the incident fluxes are absorbed in a small depth of any first wall material, the usual assumption is to treat the lumped radiant flux as if it were deposited on the first wall surface. In heat transfer analyses it is an applied heat flux boundary condition. A number of ideas have been set forth to accommodate the high heat fluxes and neutron + gamma heat generation in the first wall as well as high energy fluxes which may also arise from neutral beam injectors. A feature common to several of the ideas is an array of tubes or tube bank forming a first wall shield.

The attendant heat transfer problem for single phase fluids is that of turbulent heat transfer in a circular tube with variable circumferential heat flux and volumetric neutron-gamma heat generation in the tube wall. Theoretical work allowing for a nonuniform heat flux distribution round the circumference of a circular tube for both fully developed laminar and turbulent flow has been considered by Reynolds (1,2,3), Sparrow and Liu (4), Rapier (5) and most recently by Gartner et al (6). Reynolds (3) also allowed for circumferential wall heat conduction.

There have been few experimental investigations, the most recent by Chan et al (7) indicating that the predictions from Reynolds' and Gartner's analyses for water ($Pr=6.0$) appear to overestimate the circumferential variation in wall temperature when a nonuniform circumferential heat flux is imposed. The experimental results also indicated that the local heat transfer coefficient was less sensitive to the variation in the circumferential thermal boundary conditions than current theories predict. They suggest that further theoretical as well as experimental work are required, in particular, theoretical work associated with the turbulence modelling.

For thicker wall structures, the current work allows for coupling of the energy equations for the fluid and solid body. Circumferential as well as radial temperature gradients in the wall are permitted. For definiteness in the problem formulation an eddy diffusivity model is assumed and in the computations, Gartner et al's model is applied.

FORMULATION OF BASIC EQUATIONS

We consider hydrodynamically fully developed tube flow, either laminar or turbulent, of a fluid with constant thermal properties. The fluid temperature field is also assumed to be fully developed. For an applied heat flux with arbitrary circumferential variation at the outer wall of the tube (see Figure 1), but invariant in the flow direction, the temperature distributions in the tube wall and fluid are to be determined. Both radial and circumferential temperature variations are permitted,

Figure 1 Heat Deposition in Tube

and the governing energy equations for the tube wall and fluid are solved simultaneously subject to the conditions of continuity in the heat flux and temperature at the interface. Internal heat generation is permitted in the tube wall, but it is assumed constant, i.e., averaged over the tube thickness.

The governing conduction equation for the tube is

$$L_s(\theta_s) + Q = 0 \quad (1)$$

subject to the boundary conditions

$$\lambda_s \frac{\partial \theta_s}{\partial r} \bigg|_{R_o} = \dot{q}(\phi) \quad (2a)$$

$$-\lambda_s \frac{\partial \theta_s}{\partial r} \bigg|_{R_i} = -\lambda_f \frac{\partial \theta_f}{\partial r} \bigg|_{R_i} \quad (2b)$$

$$\theta_s(R_i, \phi) = \theta_f(R_i, x, \phi) \quad (2c)$$

in addition to

$$\theta_s(r, \phi) = \theta_s(r, \phi + 2\pi) \quad (3a)$$

$$\frac{1}{r} \frac{\partial \theta_s}{\partial \phi}(r, \phi) = \frac{1}{r} \frac{\partial \theta_s}{\partial \phi}(r, \phi + 2\pi) \quad (3b)$$

The first boundary condition, Eq. (2a), is the applied heat flux and is prescribed to vary in an

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arbitrary manner at the outer wall of the tube. It is represented by

$$\dot{q}(\dot{\tau}) = \dot{q}_0 + F(\dot{\tau}) \quad (4a)$$

where

$$\int_0^{2\pi} F(\dot{\tau}) d\dot{\tau} = 0 \quad (4b)$$

Equation (2b) matches the heat flux at the boundary between the solid and fluid and allows for the coupling between the governing equations in the tube wall and fluid. Eq. (2c) assures continuity of temperatures at the interface; Eq. (3a) and (3c) satisfy the condition that the temperature in the tube wall is single-valued.

The mean thermal energy equation governing the fluid is

$$L_s(\dot{\theta}_f) - u \frac{\partial \dot{\theta}_f}{\partial x} = 0 \quad (5)$$

The formulation of the method of solution to Eq. (5) is identical to that originally due to Reynolds (2) and later extended by Gartner et al (6) for anisotropic turbulent energy transport. Results from those works may be directly applied in this analysis.

Subject to the fully developed temperature assumption, the temperature may be split into a mean plus fully developed temperature function, i.e.,

$$\dot{\theta}_f(\xi, x, \dot{\tau}) = \dot{\theta}_{fm}(x) + P/\lambda_f S_f(\xi, \dot{\tau}) \quad (6)$$

consequently, Eq. (5) becomes

$$L_f^*(S_f) - 2U = 0 \quad (7)$$

In turn, $S(r, \theta)$ is split into two terms

$$S_f(\xi, \dot{\tau}) = g_0(\xi) + g(\xi, \dot{\tau}) \quad (8)$$

where

$$L_f^*(g_0) - 2U = 0 \quad (9a)$$

and

$$L_f^*(g) = 0 \quad (9b)$$

The mixed mean temperature, $\dot{\theta}_{fm}$, is defined as

$$\dot{\theta}_{fm} = \frac{1}{A_i U_m} \int_{A_i} u \dot{\theta}_f dA \quad (10)$$

Integrating Eq. (5) over the cross sectional area of the tube yields

$$R_i \int_0^{2\pi} \lambda_f \left(\frac{\partial \dot{\theta}_f}{\partial r} \right)_{R_i} d\dot{\tau} = \rho c_p A_i U_m \frac{d\dot{\theta}_{fm}}{dx} \quad (11)$$

The integrated heat flux at the inner wall surface is determined by integrating Eq. (1) over the tube wall area and applying the matching condition, Eq. (2b), so that

$$P_o \dot{q}_0 + A_w Q = \rho c_p A_i U_m \frac{d\dot{\theta}_{fm}}{dx} \quad (12)$$

$\dot{\theta}_m(x)$ is found by integrating Eq. (13).

Radial boundary conditions for Eqs. (9a) and (9b) are fixed by the matching condition, Eq. (2b). For example it can be shown that g_0 satisfies the condition

$$\frac{dg_0}{d\xi} \bigg|_{\xi=1} = 1 \quad (13)$$

and is the temperature field associated with the average heat flux, $\dot{q}_0$, and internal heat generation, Q . The function $g(r, \dot{\tau})$ accounts for radial as well as circumferential variation in the fluid but does not contribute to bulk temperature rise, is independent of the velocity field, but is dependent on the eddy diffusivity distribution for turbulent flow.

Solution to Conduction Equation

The solution to Eq. (1) may be found by the superposition of two solutions,

$$\dot{\theta}_s(r, \dot{\tau}) = \dot{\theta}_s(r) + \psi_s(r, \dot{\tau}) \quad (14)$$

where

$$L_s(\dot{\theta}_s) + Q = 0 \quad (15a)$$

and

$$L_s(\psi_s) = 0 \quad (15b)$$

The solution to Eq. (15a) is simply

$$\dot{\theta}_s(r) = -Qr^2/4\lambda_s + C_1 \ln r + C_2 \quad (R_i \leq r \leq R_o) \quad (16)$$

Using the method of separation-of-variables solution to Eq. (15b) becomes

$$\begin{aligned} \psi_s(r, \dot{\tau}) = & a_0 + \sum_{n=1}^{\infty} [r^n (a_n \sin n\dot{\tau} + b_n \cos n\dot{\tau}) \\ & + r^{-n} (c_n \sin n\dot{\tau} + d_n \cos n\dot{\tau})] \quad (R_i \leq r \leq R_o) \end{aligned} \quad (17)$$

where conditions (3a) and (3b) have been employed. Combining Eqs. (16) and (17) we have

$$\begin{aligned} \dot{\theta}_s(r, \dot{\tau}) = & C_3 + C_1 \ln r - Qr^2/4\lambda_s + \sum_{n=1}^{\infty} \\ & [r^n (a_n \sin n\dot{\tau} + b_n \cos n\dot{\tau}) + r^{-n} (c_n \sin n\dot{\tau} \\ & + d_n \cos n\dot{\tau})] \end{aligned} \quad (18)$$

where $C_3 = a_0 + C_2$.

Solution to Thermal Energy Equation

The solution to Eq. (9a) is dependent on a particular form of the velocity profile, u , as well as eddy diffusivity. Solutions have been formulated

in (2) and (6) as functions of Reynolds and Prandtl numbers.

Eq. (9b) admits to a solution of the form

$$g(\xi, \eta) = \sum_{n=1}^{\infty} R_n(\xi) (e_n \sin n\eta + f_n \cos n\eta) \quad (19)$$

where the radial temperature function, R_n , satisfies the following differential equation (obtained by separation-of-variables)

$$\frac{d}{d\xi} \left(E_n(\xi) \xi \frac{dR_n}{d\xi} \right) - n^2 \frac{E_n(\xi)}{\xi} R_n = 0 \quad (20)$$

subject to boundary conditions

$$R_n(0) = 0 \text{ and } \frac{dR_n}{d\xi} \Big|_1 = 1 \quad (21)$$

The boundary condition at the inner tube wall is normalized to one. This explicitly decouples the R_n solution from the tube wall conduction equation.

Assuming that the solutions to Eqs. (9a) and (20) are in hand, the unknown coefficients and constants can be determined from the matching condition, (2b) and (2c), as well as the boundary condition (2a). For $q(\cdot)$ in the form of a Fourier expansion where

$$F(\cdot) = \sum_{n=1}^{\infty} (A_n \sin n\eta + B_n \cos n\eta) \quad (22)$$

all of the coefficients $a_n, b_n, \dots$ can be expressed in terms of A_n and B_n , i.e.,

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \frac{R_1^{1-n}}{n^2} (1 + n^2 / \lambda_f R_n(1)) \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (23a)$$

$$\begin{pmatrix} c_n \\ d_n \end{pmatrix} = \frac{R_1^{n+1}}{n^2} (n^2 / \lambda_f R_n(1) - 1) \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (23b)$$

$$\begin{pmatrix} e_n \\ f_n \end{pmatrix} = \frac{2R_1}{P} \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (23c)$$

where

$$\begin{aligned} \zeta &= (1 + \delta/R_1)^{n+1} / [n \lambda_s / \lambda_f R_n(1) ((1 + \delta/R_1)^{2n-1}) \\ &+ 1 + (1 + \delta/R_1)^{2n}] \end{aligned} \quad (24)$$

The constant C_1 and C_3 are

$$C_1 = (R_0 \dot{q}_0 + QR_0^2/2) / \lambda_s \quad (24a)$$

and

$$C_3 + C_1 \ln R_1 = QR_1^2/4\lambda_s = \dot{q}_{fm} + Pg_0(1)/\lambda_s \quad (24b)$$

so that the temperature distribution in the solid becomes

$$\begin{aligned} \theta_s(r, \eta) - \dot{q}_{fm}(x) &= \{Pg_0(1) + P \ln r/R_1 + \\ &\frac{QR_1^2}{2} [\ln r/R_1 + 1/2 (1 - (r/R_1)^2)] / \lambda_s \} + \\ &\sum_{n=1}^{\infty} [r^n (a_n \sin n\eta + b_n \cos n\eta) + \\ &r^{-n} (c_n \sin n\eta + d_n \cos n\eta)] \end{aligned} \quad (25)$$

For $\delta/R_1 \ll 1$, the coefficients e_n and f_n become

$$\begin{pmatrix} e_n \\ f_n \end{pmatrix} = \frac{R_1}{\lambda_f} [1 + R_n(1)n^2(\lambda_s \delta / \lambda_f R_1)] \begin{pmatrix} A_n \\ B_n \end{pmatrix} \quad (26)$$

which are identical to the coefficients found in Eq. (6), Ref. [3] where circumferential variation in the wall temperature was permitted but no radial gradients.

Knowing the temperature distribution and heat flux at the interface between the solid and fluid allows determination of the local Nusselt number at any circumferential location, i.e.,

$$\alpha(\phi) = \frac{\dot{q}(1, \phi)}{\theta_f(1, \phi) - \theta_{fm}}, \quad Nu(\phi) = 2R_1 \alpha(\phi) / \lambda_f \quad (27)$$

where $\alpha(\phi)$ is the local heat transfer coefficient. Numerical computations are not discussed.

DISCUSSION OF RESULTS

Steady-state temperature distributions (temperature difference above the mean fluid temperature) given in Figures 2 and 3 show comparisons between aluminum (solid lines) and stainless steel (dashed lines) tubes for $\phi=0$, i.e., where maximum heat flux occurs. The temperature profiles in both figures are plotted as functions of tube wall radius for different Reynolds numbers. Temperature distributions are calculated for the parameters shown in Table I where the applied heat flux, $\dot{q}$, varies as $\dot{q} = \dot{q}_0 + B_1 \cos \phi$.

The choice of parameters corresponds to a fusion reactor design based on a wall loading = 1 MW(th)/m² where wall loading is defined as the maximum rated power output of the reactor divided by the torus surface area facing the plasma.

The Reynolds number is determined from an overall energy balance on the tube, i.e., integrating Eq. (12) over the length, L , of the tube, we have

$$Re = \frac{4LP}{\Delta T_{fm} R C_p}$$

The larger Reynolds number corresponds to either an increase in L or decrease in ΔT_{fm} or both since we've chosen to fix the inside pipe radius at 2 cm. These design parameters result in velocity and operating pressure compatible with HTGR technology.

Figure 2 Radial Temperature Distribution in Tube Wall

Figure 3 Radial Temperature Distribution in Tube Wall

In addition to plasma considerations the neutron (n) + gamma (g) heating is a function of the tube material. The difference, though, in n+g heating between stainless steel and aluminum should only be a few percent, so that assuming the same average value for both tubes should not result in appreciable differences in temperature. In a previous study (3) it was shown that the dominant effect on the first wall temperature was the surface heat flux.

The results in Figures 2 and 3 reflect the decreased temperature gradient as well as temperature for a high thermal conductivity material such as aluminum. While lower temperature gradients will occur for thinner walled tubes for the low conductivity materials, tube wall thickness limits will be set by radiation damage, thermal cycling, and operating pressure. Optimized wall thickness have not yet been established as a function of wall loading.

TABLE I: DESIGN PARAMETERS

$$\dot{q}_0 = 10^5 \text{ W/m}^2, B_1 = 10^5 \text{ W/m}^2, Q = 5 \times 10^6 \text{ W/m}^3$$

$$L = 4 \text{ m}, \Delta T_{fm} = 50^\circ \text{C}, \lambda_s(\text{Al}) = 175 \text{ W/mK}$$

$$\lambda_s(\text{ss}) = 17.2 \text{ W/mK}, \lambda_f(\text{He}) = 0.235 \text{ W/mK}$$

$$Pr = 0.7$$

TABLE II: TEMPERATURE DIFFERENCE ACROSS TUBE WALL

	Re = 3×10^5		Re = 1×10^6	
	Al	SS	Al	SS
r=2 cm	29.5	1.4	5.7	0
2.5	30.7	7.4	6.5	4.9
r=2 cm	59.4	21.8	15.7	6.5
3.0	63.5	47.1	18.9	28.8

Temperature results in Table II (wall temperature minus mean fluid temperature) are for $\theta = 180^\circ$, i.e., where minimum heat flux occurs. While stainless steel shows lower temperatures (except for $\delta/R_i = 0.5$, $Re = 1 \times 10^6$, $R_o = 3$ cm), when combined with the results from Figures 2 and 3 for a given radial position larger temperature variations with angle occurs. Variation in temperature across the tube wall is less for aluminum, decreasing with increase in Reynolds number.

Lastly, Table III gives some indication of the effect of neglecting radial conduction, but allowing for circumferential conduction as suggested by Reynolds (3). Neglecting radial conduction results in underestimating the wall temperature. The difference is attributed to the fact that the temperature is proportional to the average heat flux times the inner tube radius where as in the complete analysis it is the average heat flux times the outer tube radius.

Results differ approximately in proportion to the ratio of R_o/R_i . Neglect of radial conduction

TABLE III: COMPARISON BETWEEN MAXIMUM TEMPERATURE DIFFERENCE ($\Delta T=0$) WITH AND WITHOUT RADIAL GRADIENTS ($Q=0$)

	With $Re=3 \times 10^5$		Without	
	Al	SS	Al	SS
$t/R_1=0.25$	93.2	111.2	65.4	89.6
0.5	87.5	125.0	56.7	84.9

assumes that $t/R_1 \ll 1$ where $R_0/R_1 = t/R_1 + 1$. This also implies that the fluid temperature is underestimated when radial conduction is neglected (through e_0 and f_0). As tube wall thickness increases the coupling effect becomes more important.

ACKNOWLEDGMENT

This work was performed under the auspices of the U.S. Energy Research and Development Administration.

REFERENCES

1. Reynolds, W. C., Transactions ASME, Vol. 82, No. 2, p. 108-112, 1960.
2. Reynolds, W. C., International Journal of Heat and Mass Transfer, Vol. 6, p. 445-454, 1963.
3. Reynolds, W. C., International Journal of Heat and Mass Transfer, Vol. 6, p. 925, 1963.
4. Sparrow, E. M. and Liu, S. H., International Journal of Heat and Mass Transfer, Vol. 6, p. 866-867, 1963.
5. Rapiet, A. C., International Journal of Heat and Mass Transfer, Vol. 15, p. 527-538, 1972.
6. Gartner, D., Johannsen, K., and Ramm, H., International Journal of Heat and Mass Transfer, Vol. 17, p. 1003-1013, 1974.
7. Chan, A. L., Baughn, J. W., and Hoffman, M. A., Nonuniform Circumferential Heat Flux Experiments in a Circular Tube, ASME Winter Annual Meeting, Houston, Texas, Paper 75 - WA/MT-52, Nov. 1975.
8. Fillo, J. A., Powell, J., and Yu, W. S., Thermal-Hydraulic Analysis of Low Activity Fusion Blanket Designs, AIChE-ASME Heat Transfer Conference, Salt Lake City, Utah, Paper 77 - HT - 85, Aug. 1977.