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STUDY OF STELLARATOR DEVICES BY EXPANSION TECHNIQUES*

by

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Abstract

Expansion techniques have led to new tools for designing stellarators and interpreting experimental results. The stellarator expansion is coupled with Hamiltonian techniques to investigate the effect of plasma pressure on magnetic surface structure. An expansion for nonplanar-axis systems is described. A reduced-equation initial-value formalism is used to create three-dimensional heliac equilibria. A low- β expansion with an auxiliary expansion about the magnetic axis makes possible the determination of island widths in heliacs and allows analytic determination of the effect of equilibrium modifications associated with plasma pressure on radial transport.

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1. THE STANDARD STELLARATOR EXPANSION

Modeling of the Heliotron E device with the stellarator expansion [1] leads to good agreement between the theoretical predictions of instability if $\beta(0)$ is greater than 2.4%, and the value $\beta(0) \sim 2.5\%$ where instabilities are observed experimentally. Modification of the externally imposed field as β is increased, for example by introducing an axisymmetric pusher coil on the outside of the plasma and adjusting the vertical field to distort the lowest-order magnetic surfaces into a backward D, can modify the equilibrium properties and partially stabilize the instabilities.

Application of the code to the Wendelstein VII-A stellarator parameters shows that the fields associated with secondary currents change the shapes of the magnetic surfaces and modify the equilibrium properties. Poincaré plots of the magnetic surfaces for cases with $\beta(0) = 1\%$ and $\beta(0) = 2\%$ are shown in Fig. 1. The Shafranov shift is large even at these low values of β , and at $\beta(0) = 2\%$ bad island structure can be observed. In the $\beta(0) = 1\%$ case, Fourier decomposition of $\vec{B} \cdot \nabla \psi$, where ψ is an averaged axisymmetric magnetic surface label, shows an exponential decrease with the poloidal mode number m before numerical scatter causes a leveling off (Fig. 2). Interpolation between the different surfaces indicates island structure with $\Delta/a \sim 0.002$ [2,3].

2. HELICAL AXIS STELLARATOR EXPANSION

An expansion, applicable to systems with nonplanar magnetic axes, is being developed [4]. We consider a vacuum field

$$\vec{B} = a \nabla [\zeta + \gamma \Delta (1 + \gamma^2 \rho^2 / 8) \sin(\theta - \gamma \zeta) + \alpha \theta + (\gamma \epsilon \Delta / 4) \rho^2 \sin \gamma \zeta + \dots]$$

in a quasi-toroidal coordinate system with the ordering $\gamma \sim \alpha^{1/2} \sim \epsilon^{1/2} \ll 1$, $\beta \sim \epsilon^2$, $\Delta \sim 1$. Here $\gamma \equiv ha$ measures the wave length of an $\ell = 1$ helical field that is responsible for the nonplanarity of the magnetic axis, Δ the helical radius of the magnetic axis due to that field, a the field associated with a current in the central core, and ϵ the inverse aspect ratio; $\zeta = z/a$, $\rho = r/a$, and θ are normalized coordinates centered on the core of the helical system. By transforming to characteristic coordinates

$$C_1 \equiv \rho \cos \theta - \Delta \cos t, \quad C_2 \equiv \rho \sin \theta - \Delta \sin t, \quad t \equiv \gamma \zeta,$$

we can reduce the equilibrium problem to the simple formulation

$$\psi^{(0)} = \left(C_1 + \frac{5\epsilon}{2\gamma^2} \right)^2 + C_2^2 + \int_0^{2\pi} dt \left[\frac{\partial \Xi(C_1, C_2, t)}{\partial C_1} \int_0^t dt' \frac{\partial \Xi(C_1, C_2, t')}{\partial C_2} \right] + \frac{4A^2}{\gamma^3 \Delta^2},$$

$$\partial^2 A^2 / \partial C_1^2 + \partial^2 A^2 / \partial C_2^2 = -2\epsilon p'(\psi^{(0)}) C_1 / \gamma,$$

$$\Xi(C_1, C_2, t) \equiv (\alpha / \gamma^2 \Delta) \ln [C_1^2 + C_2^2 + 2\Delta(C_1 \cos t + C_2 \sin t)].$$

The boundary conditions are the same as in the ordinary stellarator expansion model [1]. A typical result is given in Fig. 3. The magnetic surfaces in an Asperator NP-4 model with $\gamma = 0.5$, $\Delta = 2.0$, $\epsilon = 0.06$, $\alpha = 0$, and $\langle \beta \rangle = 33\%$ show a Shafranov shift and modification of surface shape due to plasma pressure. This affects the equilibrium properties.

3. INITIAL-VALUE PROGRAM FOR HELIAC EQUILIBRIA

Three-dimensional equilibrium configurations have been studied using a modified version of the HIB code [5]. This code is based on a large-aspect-ratio expansion which annihilates the fast waves. Because higher-order terms are retained, it can find exact equilibria. It is not limited to the assumption that magnetic surfaces exist or that the transform is frozen into the plasma. Solutions with no net current are determined by introducing resistivity and viscosity into the ideal MHD equations and by continuously averaging the pressure along field lines to ensure that $\vec{B} \cdot \nabla p = 0$. This pressure constraint also supplies energy to balance resistive losses. A model equilibrium for a reference-design heliac with $R = 1.5$ m, $a = 0.5$ m, $N = 3$, and $\langle \beta \rangle = 2\%$ is shown in Fig. 4. The pressure and current profiles are typical of zero-net-current symmetric systems that have recently been reported [6]. The rotational transform varies from 0.20 to 0.24. As can be seen in the puncture plots, magnetic islands exist only in the vacuum region. Other configurations with islands inside the plasma were no more difficult to produce.

4. ISLAND STRUCTURE STUDY USING EXPANSIONS IN β AND ρ/a

Magnetic island formation is driven by resonant pressure-driven current even if the vacuum field has good flux surfaces [7]. This current is associated with the variation of $\int dl/B$ on the rational surfaces, which is nonzero at finite β even if it vanishes for vacuum fields. This decreases the useful aperture of the plasma and provides an equilibrium limit.

The secondary currents can be determined by an expansion in β about the vacuum fields. For a three-period heliac with an aspect ratio of 5 and a rotational transform $\tau = 1.5$ at the axis and $\tau = 1.7$ at the edge, we calculate

an equilibrium β -limit of roughly 2%. The conventional limit where the axis shifts half way to the outer flux surface would be 15% for this case.

5. EFFECT ON TRANSPORT OF PRESSURE-INDUCED EQUILIBRIUM MODIFICATIONS

The precise harmonic decomposition of the magnetic field strength B is sensitive to the plasma β . Numerical [8] and analytic [9,10] work has shown that stellarator transport is strongly affected by this decomposition. This can be investigated by using an expansion to determine the harmonic content.

For a stellarator with parameters characteristic of planar-axis devices ($m = 2$, $N = 8$, $\iota = 0.3$, and $\epsilon_T = \epsilon_H = 0.1$ at $r/a = 0.5$), combining the analytic equilibrium and transport theories shows a factor of 5 increase in transport as β is changed from 0 to 1% (near the equilibrium β -limit of this configuration). For a similar device, but with the vacuum-field optimized by localizing the ripple towards the inboard side [8-10], the increase is twice as large.

The principal reason for this degradation in confinement is that the outward Shafranov shift redistributes the ripple strength toward the outboard side of the stellarator. Thus, helical axis devices, which have large transform per period, have smaller shifts and so have transport less affected by this mechanism. An external field applied to cancel the shift can eliminate this degradation on a single surface, and largely remove the effect on the neighboring surfaces.

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FIGURE CAPTIONS

- FIG. 1. Magnetic surfaces of Wendelstein VII-A constructed with puncture plots for (a) $\beta(0) = 1\%$ and (b) $\beta(0) = 2\%$.
- FIG. 2. Fourier decomposition of the field normal to an averaged flux surface for the $\beta(0) = 1\%$ case in Wendelstein VII-A, as a function of poloidal mode number.
- FIG. 3. Equilibrium properties of an Asperator NP-4 model with $\langle\beta\rangle = 33\%$; (a) magnetic surfaces, (b) rotational transform and, (c) $V'(\phi)$. The dashed curves are the vacuum-field properties.
- FIG. 4. Equilibrium profiles of a heliac with $\langle\beta\rangle = 2\%$. (a) pressure contours, (b) toroidal current contours, (c) magnetic field puncture plots, (d) rotational transform.

Fig. 1

Fig. 2

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Fig. 3

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Fig. 4

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