DECOUPLING WITH STABILITY FOR A CLASS OF RIGHT INVERTIBLE SYSTEMS

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I. INTRODUCTION

In this paper, we extend the result of [1] to ensure the stability of the decoupled systems. Furthermore, we derive a design algorithm of static state feedback decoupling with arbitrary pole assignment.

Consider a system described by a $q \times p$ proper rational transfer function matrix $G(s) = N(s)D^{-1}(s)$, where $N(s)$ and $D(s)$ are right coprime polynomial matrices, and $D(s)$ is column reduced with column degree μ_i , for $i = 1, 2, \dots, p$. Let x be the corresponding state vector of a minimum realization (A, B, C, E) of the system. Let the constant matrices (H, K) be the static state feedback compensator pair. Then the overall system transfer function matrix is

$$G_f(s) = (C + EK) [sI - A - BK]^{-1} BH + EH,$$

and the corresponding open loop compensator $T(s)$ can be written as

$$T(s) = [I - K(sI - A)^{-1}B]^{-1}H.$$

The notations used in this article are the same as defined in [1].

II. STATIC STATE FEEDBACK DECOUPLING WITH INTERNAL AND EXTERNAL STABILITY

In the internal stability of the decoupled system, it is clear that the only pole zero cancellation involved in the design is the zeros of $N_Q(s)$, i.e., the roots of $\det N_Q(s)$. Since the partition of $N(s)$ is often not unique, therefore, if $G(s)$ satisfies the conditions in Theorem 1 [1], and there exists a partition such that $N_Q(s)$ has no nonminimum phase zeros, then by using this partition in the Design Algorithm 1 [1], the static state feedback decoupling with internal stability can be achieved.

The study of external stability is preceded by a lemma which shows some properties of the interactors of the original and mapped systems.

Lemma 1

Let $\hat{s} = s + a$. Then $G(s) = N(s)D^{-1}(s)$ can be written as $\hat{G}(\hat{s}) = \hat{N}(\hat{s})\hat{D}^{-1}(\hat{s})$, where $\hat{N}(\hat{s}) = N(s)|_{s=\hat{s}}$ and $\hat{D}(\hat{s}) = D(s)|_{s=\hat{s}}$, and a is a finite positive real number. The standard interactor $\xi(s)$ of $G(s)$ and the standard interactor $\hat{\xi}(\hat{s})$ of $\hat{G}(\hat{s})$ have the same set of column degree indices $\{\mu_i\}$, also $U(s)$ and $\hat{U}(\hat{s})$ have the same column degree indices $\{\gamma_i\}$.

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** Note that $\hat{\xi}(\hat{s})$ is computed in $\hat{s}$ plane, which satisfying the definition of standard interactor. $\hat{\xi}(\hat{s})$ is not defined as $\xi(s)|_{s=\hat{s}}$.

We remark that $\hat{\xi}(\hat{s})$ is computed in $\hat{s}$ plane, and it is in general not the same function as $\xi(s)$. Therefore, it can not be replaced by $\xi(\hat{s})$. Some properties of $\hat{\xi}(\hat{s})$ may look similar to those of the generalized interactor studied in [3], [4], however they are not necessarily the same.

Theorem 2

If $G(s)$ satisfies the conditions in Theorem 1 [1], then the static state feedback decoupling with external stability is always achievable.

Proof

Let $\hat{s} = s + a$, with a being a finite positive real number, which maps $G(s)$ into $\hat{G}(\hat{s})$. Let $\hat{\xi}(\hat{s})$ be the standard interactor of $\hat{G}(\hat{s})$. Applying the decoupling Design Algorithm 1 [1] in $\hat{s}$ plane, the decoupled system $\hat{G}_f(\hat{s}) = \hat{G}_f(s + a)$ has all the poles located at $-a$. Thus the decoupled system is externally stable. The open loop compensator is

$$\hat{T}(\hat{s}) = \hat{D}(\hat{s})\hat{D}_T^{-1}(\hat{s})T = D(s)\hat{D}_T^{-1}(s + a)T$$

with $D(s)\hat{D}_T^{-1}(s + a)$ being biproper. Therefore, the open loop compensator is static state feedback implementable.

III. STATIC STATE FEEDBACK DECOUPLING WITH ARBITRARY POLE ASSIGNMENT

In this section, the static state feedback decoupling with arbitrary pole assignment will be derived. The algorithm uses both polynomial and state space representations.

Theorem 3

If $G(s)$ satisfies the conditions in Theorem 1 [1], then the static state feedback decoupling with arbitrary pole assignment is achievable.

Proof

Let the open loop compensator be chosen as

$$\begin{aligned} T(s) &= D(s)\hat{D}_T^{-1}(s + a)T \\ &= D(s)[\hat{M}(s + a)N_A(s)]^{-1}T, \end{aligned} \quad (3-1)$$

where

$$\hat{M}(s + a) = \begin{bmatrix} \hat{\xi}(s + a) & \hat{Q}(s + a) \\ 0 & \Lambda_d(s + a) \end{bmatrix},$$

and a is a finite real number that is not located at the zeros of $N_Q(s)$. Then the decoupled system matrix is

$$G_f(s) = G(s)T(s) = \Lambda_f^{-1}(s + a), \quad (3-2)$$

where $\Lambda_f^{-1}(s + a)$ is the resulting system after pole-zero cancellation

Let $\{A, B, C, E\}$ be the controllable canonical form realization of $G(s)$, and H_D, K_D be the static state

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feedback gain matrices associated with $T(s)$. Then the decoupled system $G_f(s)$ has the state space realization $\{\bar{A}, \bar{B}, \bar{C}, \bar{D}\}$

Considering that the eigenvalues of $\bar{A}$ are the roots of $\det \bar{D}_T(s+a)$, and

$$\begin{aligned} \det \bar{D}_T(s+a) &= \det \bar{M}(s+a) \det N_A(s) \\ &= \det \Lambda_f(s+a) \det N_Q(s), \end{aligned} \quad (3-3)$$

the eigenvalues of $\bar{A}$ are the zeros of $\Lambda_f(s+a)$ and the zeros of $N_Q(s)$.

Since $N_Q(s)$ is canceled out in the decoupled system, and the remaining part of $G_f(s)$ is $\Lambda_f^{-1}(s+a)$, the states related to eigenvalues $-a$ are all controllable and observable, and the states related to $N_Q(s)$ are at least unobservable.

Let $\{A_f, B_f, C_f, E_f\}$ be the Jordan form realization of $\{\bar{A}, \bar{B}, \bar{C}, \bar{E}\}$, and P_f be the similarity transformation matrix from $\{\bar{A}, \bar{B}, \bar{C}, \bar{E}\}$ to $\{A_f, B_f, C_f, E_f\}$. Without loss of generality, we have

$$A_f = \begin{bmatrix} A_{fco} & 0 \\ 0 & A_{fu} \end{bmatrix} \quad (3-4a)$$

$$B_f = \begin{bmatrix} B_{fco} \\ B_{fu} \end{bmatrix} \quad (3-4b)$$

$$C_f = [C_{fco} \quad C_{fu}] \quad (3-4c)$$

$$E_f = \bar{E} \quad (3-4d)$$

where A_{fco} denotes the controllable and observable part which consists of the diagonal blocks related to eigenvalues $-a$, and A_{fu} denotes the unobservable part. This form shows that the controllable and observable states of the decoupled system can be separated from the unobservable states.

Let $\{\bar{A}_{co}, \bar{B}_{co}, \bar{C}_{co}, \bar{E}_{co}\}$ be the controllable canonical form realization of $\Lambda_f^{-1}(s+a)$, then

$$\bar{A} = \begin{bmatrix} \bar{A}_{co} & 0 \\ 0 & A_{fu} \end{bmatrix} \quad (3-5a)$$

$$\bar{B} = \begin{bmatrix} \bar{B}_{co} \\ B_{fu} \end{bmatrix} \quad (3-5b)$$

$$\bar{C} = [\bar{C}_{co} \quad C_{fu}] \quad (3-5c)$$

$$\bar{E} = E_f \quad (3-5d)$$

is a realization of $G_f(s)$. It is not necessarily minimum, but is linearly equivalent to $\{A_f, B_f, C_f, E_f\}$ and $\{\bar{A}, \bar{B}, \bar{C}, \bar{E}\}$. The similarity transformation matrix $\bar{P}$ from $\{A_f, B_f, C_f, E_f\}$ to $\{\bar{A}, \bar{B}, \bar{C}, \bar{E}\}$ is

$$\bar{P} = \begin{bmatrix} \bar{P}_{co} & 0 \\ 0 & I_k \end{bmatrix} \quad (3-6)$$

where $\bar{P}_{co} = [\bar{U}_{co} \quad U_{fco}^T] [U_{fco} \quad U_{fu}^T]^{-1} \bar{U}_{fco}$ and U_{co} are the controllability matrices of $\{A_{fco}, B_{fco}\}$ and $\{\bar{A}_{co}, \bar{B}_{co}\}$, respectively. Then we have

$$\bar{A} = \bar{P} \bar{A} \bar{P}^{-1} = \bar{P} P_f A_f P_f^{-1} \bar{P}^{-1} = P A P^{-1} \quad (3-7)$$

where $P = \bar{P} P_f$.

Let $\Lambda_s(s)$ be a diagonal Hurwitz polynomial matrix, with $\delta, \Lambda_s(s) = \delta, \Lambda_f(s)$. The resulting system

$$G_o(s) = \Lambda_f^{-1}(s) T_s(s) = \Lambda_s^{-1}(s) \quad (3-8)$$

is decoupled with arbitrary pole assignment, where $T_s(s) = \Lambda_f(s) \Lambda_s^{-1}(s)$. Since $T_s(s)$ is biproper, it is implementable by static state feedback. The static state feedback gain matrices can be found from following equations:

$$H_s = \Lambda_{fkc} \Lambda_{skc}^{-1} \quad (3-9)$$

$$K_s = \Lambda_{fku} - H_s \Lambda_{sku} \quad (3-10)$$

Notice that the state feedback K_s are related to the controllable and observable states only, and the state feedback gain matrix K_{ss} related to all states is

$$K_{ss} = [K_s \quad 0] \quad (3-11)$$

The realization of the overall system $G_o(s)$ is $\{A_f, B_f, C_f, D_f\}$. The transfer function matrix is

$$\begin{aligned} G_o(s) &= C_f (sI - A_f)^{-1} B_f + E_f \\ &= (C + EK) [sI - (A + BK)]^{-1} B H + E H, \end{aligned} \quad (3-12)$$

where the static state feedback gain matrices $\{H, K\}$ are

$$H = H_D H_s \quad (3-13)$$

$$K = K_D + H K_{ss} P \quad (3-14)$$

Thus the system is static state feedback decoupled with arbitrary pole assignment. This completes the proof.

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