MASTER

KrF LASER FUSION

PERFORM OPTICAL SYSTEMS STUDY OF DIRECT SHORT PULSE EXTRACTION

**FINAL TECHNICAL REPORT
FOR THE PERIOD
26 JUNE 1978 — 14 MARCH 1981**

R. S. Bradford, Jr., J. A. Betts, R. Aprahamian and P. M. Livingston

MARCH 1981

Work Performed Under Contract

DE-AC08-78DP40069

TRW

DEFENSE AND SPACE SYSTEMS GROUP

One Space Park, Redondo Beach, California 90278

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

DISCLAIMER

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, make, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Printed in the United States of America

Available from:

National Technical Information Service
U.S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22161

Price:	Printed Copy	\$9.00
	Microfiche	\$3.00

KrF LASER FUSION

PERFORM OPTICAL SYSTEMS STUDY OF DIRECT SHORT PULSE EXTRACTION

FINAL TECHNICAL REPORT

FOR THE PERIOD

26 JUNE 1978 - 14 MARCH 1981

R. S. Bradford, Jr., J. A. Betts, R. Aprahamian and P. M. Livingston

MARCH 1981

Prepared for the

U.S. Department of Energy
Nevada Operations Office
Las Vegas, Nevada 89114

Under Contract DE-AC08-78DP40069

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

TRW

DEFENSE AND SPACE SYSTEMS GROUP

One Space Park, Redondo Beach, California 90278

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

MGW

ACKNOWLEDGEMENTS

The authors would like to thank Robert Barth, Richard Boyd, John Godden, Michael Lainhart, Robert Margulies and Lee Frantz for their technical support. Special thanks goes to Carol Smith who typed the manuscript.

Physics International fabricated the Marx Generators and blade cathodes described in this report.

TABLE OF CONTENTS

	<u>PAGE</u>
ABSTRACT	1
1.0 INTRODUCTION AND SUMMARY	2
1.1 Modeling Summary	3
1.2 Experimental Effort Summary	4
2.0 KINETICS MODELING	5
2.1 KrF Kinetics Code	5
2.2 Modeling of Sandia Experiments	9
2.3 Modeling of TRW MOPA Experiments	14
3.0 AMPLIFIER MODELING	22
3.1 Amplifier and ASE Equations	22
3.2 PLSE Amplifier Codes	28
3.3 Comparison of the Code with Experimental Data	32
4.0 E-BEAM DEVICES	35
4.1 Marx Generator	37
4.2 Electron Guns	44
4.3 Magnetic Guide Fields	50
4.4 Laser Cavity	58
4.5 Optics	59
5.0 KrF EXPERIMENTS	60
5.1 KrF Oscillator Performance	60
5.2 ASE Clamping Tests	66
5.3 Gain Length Scaling Tests	78
6.0 REFERENCES	81
APPENDIX A - Kinetic Rate Processes	82
APPENDIX B	86
DISTRIBUTION LIST	109

FIGURES

	<u>PAGE</u>
2-1 Key KrF Kinetics Pathways	1
2-2 Comparison of Intensity Time Histories	10
2-3 Comparison of Code and Experiment	12
2-4 KrF Code Verification	13
2-5 Gain versus Pressure at Optimum F_2 Concentration	15
2-6 Gain/Loss versus Pressure at Optimum F_2	16
2-7 Gain versus Percentage F_2	17
2-8 Gain/loss versus Percentage F_2	18
2-9 Predicted Two-Port Oscillator Performance	19
2-10 Predicted One-Port Oscillator Intensity versus Pressure	21
3-1 Coordinate Transformation	23
3-2 Maximum and Mean Half-Angle	26
3-3 Fraction of ASE Radiation Escaping the Cavity	27
3-4 Three-Dimensional plot of Amplified Intensity	29
3-5 Three-Dimensional plot of Forward ASE	30
3-6 Three-Dimensional plot of Backward ASE	31
3-7 Sandia Data Modeling	33
4-1 Diode Chamber and Laser Cavity	36
4-2 Photograph of Marx Generator	38
4-3 Marx Generator Schematic	39
4-4 E-Beam Circuit Model	40
4-5 Current Waveform of Diode Short Circuit Test	42
4-6 MOPA E-Beam Block Diagram	43
4-7 Densitometer Trace of an X-ray Photogtaph	46
4-8 E-Beam Total Current	47
4-9 E-Beam Voltage Waveform	48
4-10 Transmitted Total E-Beam Current	49
4-11 E-Beam Deposition Profile	51
4-12 Average Energy Deposition	52
4-13 Magnetic Field Distribution Cross-Section	55
4-14 Measured Magnetic Field Distribution	56
4-15 Magnetic Field Distribution	57

FIGURES

	<u>PAGE</u>
5-1 KrF Oscillator	62
5-2 KrF Oscillator Intensity	63
5-3 KrF Oscillator Performance	64
5-4 KrF Code Experiment Comparison	65
5-5 KrF Maser Oscillator-Power Amplifier	67
5-6 Experimental Arrangement for Gain Clamping Single Pass	68
5-7 KrF Burn Patterns	69
5-8 MOPA Amplification	72
5-9 Predicted Amplifier Intensity	73
5-10 Predicted Amplifier Forward ASE	74
5-11 Predicted Amplifier Backward ASE	75
5-12 Forward and Backward ASE	76
5-13 ASE Clamping and Normalized Input Intensity	77
5-14 Experimental Arrangement for Extraction Efficiency Tests	79
5-15 Predicted Amplifier Performance	80

ABSTRACT

KrF is a prime candidate for a laser inertial confinement fusion (ICF) driver, however, there are still scaling issues to be addressed. The purpose of this program is to determine KrF performance in a regime of ICF interest both experimentally and analytically.

Using a master oscillator-power amplifier (MOPA) configuration, 100J has been obtained from a five liter optical volume KrF amplifier pumped at 300 kW/cm^3 peak at the end of a 500 nsec pulse. A normalized input flux of 0.4 was required to reduce amplified spontaneous emission (ASE) to one half the free running value. A peak extraction efficiency of 0.46 was obtained under these conditions at 1 atm total pressure.

The TRW kinetics model has been refined to give excellent agreement with high and low pump regimes. An integrated kinetics and pulse propagation code has also been developed and agrees well with experiment.

1.0 INTRODUCTION AND SUMMARY

The Department of Energy is considering several candidate drivers for inertial confinement fusion (ICF) where the driver system must be capable of delivering ~ 3 MJ to a deuterated-tritium pellet in a 10 nsec time period so that sufficiently high pressures and densities are produced to cause thermo-nuclear burn. The minimum driver overall electrical efficiency must be on the order of 5%, with significant advantages for higher efficiency operation. Among the mainline drivers for the implosion of deuterated-tritium pellets in ICF are the short wavelength advanced lasers, the light ion beams and heavy ion beams.

Among the candidate laser systems for ICF drivers, the rare gas halide (RGH) lasers are the most promising due to their high intrinsic electrical efficiencies. The characteristics of these e-beam pumped systems are such that pulse lengths range from 50 to 500 nsec for efficient operation. The pulse is then shortened by geometric optical angular multiplexing and/or Raman pulse compression in high pressure gas cells. The most notable RGH molecule, KrF, has been well characterized in the long pulse regime (> 500 nsec) at low e-beam pump rates (< 200 kW/cm³) and in the short pulse regime (< 50 nsec) at high e-beam pump rates (~ 7 MW/cm³). The overall objective of this program is to characterize the performance of KrF oscillator/amplifier lasers in a regime of ICF interest. Specific objectives are: 1) to determine the required optical input energy into a KrF amplifier medium to suppress amplified spontaneous emission (ASE), 2) to determine gain length scaling and 3) to develop and verify an integrated kinetics/amplifier code to assist in scaling to devices of ICF interest. Data have been obtained using two identical five liter optical volume e-beam pumped KrF devices arranged in a master oscillator-power amplifier configuration. In support of the experimental effort a series of analytical tasks have been performed. The objective of these analytical tasks was to produce a computer model which describes the growth of ASE and of an injected optical pulse by a KrF gain medium. The models have considered both one and three dimensions. The TRW kinetics code has been

refined by comparison with existing data at high and low pump rates as well as data generated in this program.

Data and analysis of KrF lasers generated under this program has helped to clarify scaling issues in determining optimum design ranges for ICF drivers. A progress summary for the experimental and analytical efforts is given below.

1.1 MODELING SUMMARY

The TRW kinetics code has been refined by extensive comparison with data from other laboratories as well as that taken during the program. Direct experimental data was given to TRW by Sandia National Laboratory [1-3] and this was input into the code. The major modification was the inclusion of superelastic electron quenching reactions into the kinetics. Excellent agreement between code and experiment was achieved for the superelastic quenching of KrF by electrons of $4 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$. The same rate package was also able to successfully model AVCO [4] experiments which had been performed under much lower electron beam pumping conditions. The PLSE amplifier code was improved by addition of a kinetics package which calculates the time history of the gain and absorption in the amplifier medium. The kinetics (including the time-dependent electron beam pulse) and the time history of the intensity of the pulse to be amplified are the primary inputs to the program. The output is a detailed calculation of the space and time history of the amplifier pulse and the amplified spontaneous emission. In order to incorporate these changes, the basic 1-D code was rewritten using a new analytical technique. The code has been able to predict amplifier performance with excellent agreement to experiment.

Kinetics modeling results are discussed in Section 2 and amplifier modeling is discussed in Section 3. Appendix A lists all rate constants used in the TRW kinetics code. Appendix B presents the development of the pulse propagation code formalism. The references are found in Section 6.

1.2 EXPERIMENTAL EFFORT SUMMARY

Two e-beam pumped KrF systems have been designed and fabricated. An oscillator output of 54J has been obtained from the five liter volume at 1 atm operating pressure. Using the MOPA configuration, 100J has been obtained from the amplifier for an input of 17.7J. Gain-length products, $g_0 L$, of approximately three were observed at the end of a 500 nsec pulse at a pump rate of 300 kW/cm^3 . A normalized input intensity (I/I_{sat}) of 0.4 was found to be necessary to reduce ASE to one half of the no input value. This resulted in an extraction efficiency of ~ 0.46 . Lesser input values may be used and higher gain values occur, however, the extraction efficiency is greatly reduced. It was also shown that optimum operation, in term of efficiency, occurs at a total pressure near 1 atm.

The operating parameters and characteristics of the e-beam devices are discussed in Section 4. Section 5 reports on KrF performance of MOPA experiments.

2.0 KINETICS MODELING

It is necessary to understand the basic chemistry and photon processes associated with KrF lasers to predict the performance of pulsed KrF fusion drivers. This is best done by developing computer codes that successfully model small scale KrF experiments over a wide range of operating conditions. At the beginning of the current program, most documented KrF experiments has been performed at low pump intensities ($\sim 0.1 \text{ MW/cm}^3$) for long times ($\sim 1 \text{ } \mu\text{sec}$). However, Sandia Laboratories^[1-3] had begin short pulse (50 nsec), high pump rate (7 MW/cm^3) experiments. An agreement was reached whereby TRW's KrF laser code would be used to model these experiments. This provided an opportunity to test TRW's KrF laser model over a wide range of pump regimes and to interface with Sandia to provide support in the area of laser modeling. The verified code was then used to perform a parametric study over a range of experimental conditions possible with TRW's e-beam devices.

This section contains a description of TRW's kinetic code, results of code verification by comparison with Sandia experiments and results of the parametric study.

2.1 KrF KINETICS CODE

The TRW KrF code is a time dependent kinetics and extraction code. The code is designed to directly interface with experimental data. The electron beam deposition is input as a function of time with the total amount of energy deposited input directly from experimental measurements. The code provides for partitioning of this deposited energy into ions and excited states of argon and krypton. Since laser gas mixtures for KrF consist mainly of argon (< 90 percent), most of the energy goes into making argon ions and excited states. The energy partitioning is calculated separately from the kinetics code and is part of the code input.

The main kinetic paths used in the TRW code are summarized in Figure 2-1. A detailed set of reactions with associated rates is contained in Appendix A. The electron beam ionizes the gas and also produces some excited states. Secondary electrons produced in the primary ionization are attached to fluorine to produce F^- ions, i.e.,

The primary Ar^+ ions can undergo a series of reactions to form Kr^+ and Kr_2^+ ions

The positive ions can then combine with F^- ions to form rare gas fluorides by the processes

KrF^* can also be formed from reaction of Kr^* atoms with F_2

and by transfer from ArF^*

FIGURE 2-1. Key KrF Kinetics Pathways

The excited states can be quenched by rare gases, fluorine, or electrons. The code includes quenching of ArF^* and KrF^* by argon and krypton to form Ar_2F^* and Kr_2F^* , i.e.,

The Ar_2F^* and Kr_2F^* either radiate or are quenched. Reactions involving Ar_2F^* and Kr_2F^* are needed to model the fluorine burnup rate [5]. The code also includes superelastic quenching reactions of excited states by electrons such as

These quenching steps are needed in order to model high pump rate experiments. The kinetics code was used to calculate the small signal gain and absorption used in the amplifier model discussed in Section 3. The small signal gain is just the product of the stimulated emission cross section and the KrF^* concentration. The absorption, β , can be expressed as

$$\beta = \sum_n \sigma_{an} [A_n]$$

where $[A_n]$'s are concentrations of absorbing species and σ_{an} are absorption cross sections for each species. The summation is over all absorbing species. The principal absorbers are F_2 , F^- , Kr_2F^* , Ar_2^+ , Kr_2^+ , Ar^{**} and Kr^{**} . The absorption cross sections used for these species are included in the Table in Appendix A.

For oscillator modeling the optical extraction model in the code uses a Fabry Perot cavity and models laser buildup from spontaneous emission. The equation for intensity buildup is

$$\frac{dI}{dt} = c \alpha I + S$$

where α is the gain and S is the driver term from spontaneous emission. The overall gain, α , is expressed as

$$\alpha = \frac{\ell_p}{\ell_c} \sigma_s \cdot [\text{KrF}] - \frac{\ell_p}{\ell_c} \cdot \beta + \frac{\ln R_1 R_2}{2 \ell_c}$$

where ℓ_p is the plasma length, ℓ_c is the cavity length, σ_s is the stimulated emission cross section and R_1 and R_2 are mirror reflectivities. A term, β , was also added to account for loss due to unreacted fluorine in the optical cavity.

2.2 MODELING OF SANDIA EXPERIMENTS

For modeling of Sandia [1-3] experiments, normalized e-beam current versus time curves obtained from Gary Tisone were input into the program. Measured deposited energy was also input to calibrate these curves. Both lasing and non-lasing computer runs were performed. Non-lasing runs were used to determine small signal gain g_0 , absorption α and the saturation intensity, I_s . These quantities can be compared with those derived from a Rigrod^[6] analysis of Sandia experiments performed by Jim Rice. Lasing runs were also performed to compare laser energies, efficiencies and pulse shapes.

Figure 2-2 shows normalized intensities versus time for experiment and code for both sidelight fluorescence and lasing. Excellent agreement was obtained.

FIGURE 2-2: Comparison of Intensity Time Histories - Sandia Data/TRW Code
(see [1-3])

Comparison of gains calculated by the code with experiment indicated that the observed gain was very dependent on quenching rates of KrF*. The neutral quenching rates used in the code were quite well established and modeled low current data well. Consequently, electron quenching rates were varied to obtain the best fit between the high current data and the code. The values of small signal gain times length, $g_0 \ell$, absorption times length, $\alpha \ell$, and saturation intensity, I_{SAT} , are shown in Figure 2-3 for several values of electron quenching rate, k_{qe} . Note that the best agreement between experiment and theory was obtained for high electron quenching rates of $4 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$. This is a factor of 2 higher than the quenching rates favored by AVCO of $2 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$. The value of $4 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$ depends on a fit to experimental data keeping other rates constant and thus contains a large uncertainty as an absolute number. Figure 2-4 compares code and experiment for AVCO^[4] experiments at low current densities and for Sandia^[1-3] experiments at two pressures at the higher current densities. The code provided good comparison with experiment under all pump conditions. The code had difficulty reproducing the experimental Rigrod^[6] curve, especially at the high outcoupling fractions. This indicates that there are still areas where the optical laser model could be improved.

In conclusion, the TRW KrF kinetics and laser code was able to model KrF* experiments performed at Sandia if a rate for electron quenching of $4 \times 10^{-7} \text{ cm}^3/\text{sec}$ was included in the model. The critical parameter for comparison was the small signal gain. The total predicted laser output was not as sensitive to electron quenching rates as was the gain.

	$g_0 l$	αl	I_{SAT} (MW/cm ²)
EXPERIMENT, 2500 TORR	7.0 ± 1	0.4 - 0.6	7 - 9
CODE 2500 TORR $k_{Qe} = 0$	18	0.45	3.6
CODE 2500 TORR $k_{Qe} = 1 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$	16	0.45	4.3
CODE 2500 TORR $k_{Qe} = 4 \times 10^{-7}$	10	0.45	6.0
EXPERIMENT, 1000 TORR	6.5 ± 1	0.2 - 0.6	1.9 - 3
CODE 1000 TORR $k_{Qe} = 0$	15	0.29	1.1
CODE 1000 TORR $k_{Qe} = 1 \times 10^{-7}$	12	0.29	1.3
CODE 1000 TORR $k_{Qe} = 4 \times 10^{-7}$	6.0	0.29	2.0

FIGURE 2-3. Comparison of Code and Experiment, Determination of Electron Quenching Rates (see [1-3])

DATA		EFFICIENCY (%)	OUTPUT (J/l)	$g_0 l$	αl
AVCO p = 1.7 ATM J _{eb} = 11.7 A/CM ²	CODE EXPERIMENT	9.2 9.0	12.4 12.0	3.9 3.7	0.34 0.35
SANDIA p = 1.3 ATM J _{eb} = 300 A/CM ²	CODE EXPERIMENT	11.4 12.8	10.1 11.3	6.0 6.5 ± 1	0.29 0.2–0.6
SANDIA p = 3.3 ATM J _{eb} = 300 A/CM ²	CODE EXPERIMENT	8.0 9.0	27.1 30.4	10.0 7.0 ± 1	0.45 0.4–0.6

FIGURE 2-4. KrF Code Verification (see [1-4])

2.3 MODELING OF TRW MOPA EXPERIMENTS

Having verified the KrF kinetics code accuracy by comparing predicts with predicted performance data, computations were made to predict the behavior of experiments planned for this program. Code runs were made using the measured transmitted current density waveform (see Figure 4-10) as a function of the parameters described below.

Figure 2-5 and 2-6 show predicted histories of net gain and gain/loss, respectively, for total pressures of 1.0, 1.5, 2.0, 2.5 and 3.0 atm with F_2 percentages of 0.53, 0.45, 0.40, 0.35 and 0.35, respectively. The percentage of Kr was 10% for 1.0, 1.5 and 2.0 atm while for 2.5 and 3.0 atm, the value was 5%. These mixture ratios were found to be optimum for the pump density (300 kW/cm^3 peak), time duration and temporal profile. Although the gain increases from 1 to 1.5 atm, it decreases from 1.5 to 3 atm. Notice, however, that gain/loss constantly decreases with pressure. The optimum pressure in terms of both gain and gain/loss is 1 atm.

Figure 2-7 and 2-8 display predicted histories of net gain and gain/loss, respectively, for F_2 percentages of 0.2, 0.4, 0.6, 0.8 and 1.0. The percentage of Kr was 10% and the peak pump density was 300 kW/cm^3 with a total pressure of 1 atm. Note that for the lowest concentration of F_2 , the gain rise is fastest but the fluorine is quickly burned-up. At higher concentrations, the gain is suppressed early in the pulse due to fluorine quenching. The optimum F_2 concentration was found to be 0.53% but it is not shown on this plot.

Figure 2-9 is a plot of two-port oscillator intensity versus time. Early experiments damaged the total reflector as well as AR coatings on the windows. To continue on with experiments, the oscillator was operated as a two-port device with two 50% reflectivity mirrors and two uncoated windows. The windows were aligned perpendicular to the cavity so that the net reflectivity was about 0.58 for each port. Calculations for this figure were performed with F_2 concentrations of 1, 2, 4, and 8 torr with a

FIGURE 2-5. Gain Versus Pressure at Optimum F_2 Concentration as a Function of Time

FIGURE 2-6. Gain/loss Versus Pressure at Optimum F_2 Concentration as a Function of Time

FIGURE 2-7. Gain Versus Percentage F₂ as a Function of Time

FIGURE 2-8. Gain/loss Versus Percentage F_2 as a Function of Time

FIGURE 2-9. Predicted Two-port Oscillator Performance as a Function of F_2 Concentration

peak pump density of 240 kW/cm^3 at a total pressure of 1 atm with 10% Kr. Note that this is the output intensity from one aperture. This corresponds to 27.4 J from one aperture at the optimum F_2 partial pressure of 4 torr. These curves compare well with experimental data shown in Figure 5-2.

Figure 2-10 is a plot of one-port oscillator intensity versus pressure as a function of time for optimum output coupling, optimum $[F_2]$, and optimum $[Kr]$. The fluorine and krypton partial pressures are the same as for that for Figure 2-7 and 2-8. The peak pumping density was 300 kW/cm^3 . Although the output increases with pressure, the extraction efficiency decreases; $\eta_{\text{extr}} = 0.448, 0.402, 0.362, 0.332$ and 0.303 for 1.0, 1.5, 2.0, 2.5 and 3.0 atm, respectively.

It is clear from these code simulations that the optimum operating pressure is 1 atm with 10% Kr and 0.53% F_2 . Oscillator optimization experiments, described in Section 5, verified these code predictions. For large volume KrF devices, pump densities of 300 kW/cm^3 are quite feasible and experimental results of this program are applicable in scaling. To obtain the highest overall kinetic/optical efficiency, large scale devices should be operated at pressures near 1 atm.

FIGURE 2-10. Predicted One-Port Oscillator Intensity Versus Pressure as a Function of Time at Optimum F_2 and Output Coupling

3.0 AMPLIFIER MODELING

3.1 AMPLIFIER AND ASE EQUATIONS

A general time-dependent formulation of an amplifier medium was developed under this contract. The derivation of the detailed amplifier equations using a time-dependent geometric optics formalization is presented in Appendix B. The complete set of equations in three dimensions is very complex. To simplify solution, a first generation amplifier model was formulated. The assumptions inherent in this model are: one dimensional propagation in both signal and ASE, quasi-steady state gain, amplification at line center, and uniform gain as a function of x . In addition, the small signal gain and absorption are input as functions of time. The limitations of this model are that the amplified spontaneous emission (ASE) is overestimated, the excited state kinetics are not represented and non-uniform gain-induced aberrations are not accounted for. With these assumptions, the equation for pulse amplification is

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) I_p(x,t) = g(x,t) I_p(x,t) - \alpha I_p(x,t)$$

and the equations for forward ASE (I^+) and backward ASE (I^-) are:

$$\left(\frac{1}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial x}\right) I^+(x,t) = g(x,t) I^+(x,t) - \alpha I^+(x,t) + \beta I_s g(x,t)$$

$$\left(\frac{1}{c} \frac{\partial}{\partial t} - \frac{\partial}{\partial x}\right) I^-(x,t) = g(x,t) I^-(x,t) - \alpha I^-(x,t) + \beta I_s g(x,t)$$

where $g(x,t)$ is the gain in cm^{-1} , α is the absorption in cm^{-1} , β is the ratio of excited state lifetime to radiative lifetime, and I_s is the saturation intensity. In the quasi steady-state gain model

$$g(x,t) = \frac{g_0(t)}{1 + (I_p + I^+ + I^-)/I_s}$$

where $g_0(t)$ is the small signal gain. Transforming coordinates (see Figure 3-1), These equations can be written in dimensionless form as:

FIGURE 3-1. Coordinate Transformation for Propagation Code

$$\frac{\partial \psi_p}{\partial v} = 1/2(\phi\eta - \gamma) \psi_p$$

$$\frac{\partial \psi^+}{\partial v} = 1/2(\phi\eta - \gamma) (\psi^+ + \beta) + \frac{\psi\beta}{2}$$

$$\frac{\partial \psi^-}{\partial w} = 1/2(\phi\eta - \gamma) (\psi^- + \beta) + \frac{\psi\beta}{2}$$

where $\phi = g_0(t)\ell$, $\gamma = \alpha(t)\ell$, $\psi = I/I_s$, $\eta = 1/(1 + \psi^+ + \psi^- + \psi_p)$, $w = (ct-x)/\ell$, $v = (ct+x)/\ell$, β is a source term for ASE production, and ℓ is the amplifier length. The formal solutions to these equations are written as functions of the unknown path integrals ζ and ξ as follows.

$$\text{Pulse: } \psi_p(v,w) = \psi_p^{(0)} \theta(w_2 - w_1) \theta(w - w_1) e^\xi$$

$$+\text{ASE } \psi^+(v,w) = \beta(e^\xi - 1 + 1/2\gamma e^\xi \int_w^v e^{-\xi} dv')$$

$$-\text{ASE } \psi^-(v,w) = \beta(e^\zeta - 1 + 1/2\gamma e^\zeta \int_{v-2}^w e^{-\zeta} dw')$$

where θ is the step function

$$\theta(y) = \begin{cases} 1 & y > 0 \\ 0 & y < 0 \end{cases}$$

The solutions are the reduced fluxes with saturation flux serving as the reduction parameter. Finally, the path integrals $\xi(v,w)$, $\zeta(v,w)$ are defined as the effective integrated gain.

$$\xi(v,w) = \int_w^v 1/2(\phi\eta - \gamma) dv'$$

$$\zeta(v,w) = \int_{v-2}^w 1/2(\phi\eta - \gamma) dw'$$

These integrals are then evaluated numerically. This solution has the advantage that it automatically takes into account gain growth and satisfies boundary conditions.

An additional issue is the determination of β which depends on the amount of ASE radiation emerging from the cavity and the range of angles at which it is emitted. An ASE radiation vignetting model was developed to handle this issue. This model assumes isotropic ASE production, a linear ASE growth profile and black absorbing walls in the cavity. The ASE vignetting is expressed as a weighted average pipe cross-section through which light can emerge at an angle θ to the optical axis. The emergent fraction of radiation is a function of the aspect ratio, z^* (the ratio of the amplifier diameter to length), and the angle θ . This fraction can be expressed mathematically as

$$\bar{f}(z^*, \theta) = 1 - \frac{8}{3\pi z^*} \tan \theta + \frac{1}{2\pi z^{*2}} \tan^2 \theta$$

No energy is emitted into an angle greater than θ_m where

$$\theta_m = \tan^{-1} 1.7567624 z^*$$

The mean radiation half-angle is then

$$\begin{aligned} \phi &= 2\pi \int_0^{\theta_m} \bar{f}(z^*, \theta) \sin \theta d\theta \\ &= [(1 - \cos \theta_m) \left(1 - \frac{1}{2\pi z^{*2}} + \frac{1}{2\pi z^*} \frac{1}{\cos \theta_m}\right) + \frac{8}{3\pi z^*} \sin \theta_m - \frac{8}{3\pi z^*} \ln \left[\tan \frac{\pi}{4} + \frac{\theta_m}{2}\right)] \end{aligned}$$

The maximum, θ_m , and mean half-angle, ϕ , are plotted versus aspect ratio in Figure 3-2 for ASE emission from an amplifier. The fraction of ASE radiation escaping a cavity versus aspect ratio is plotted in Figure 3-3. This term is incorporated into β to correct for the actual amount of ASE radiation that would be observed.

FIGURE 3-2. Maximum and Mean Half-Angle for ASE Emission from an Amplifier versus Aspect Ratio z^*

FIGURE 3-3. The Fraction of ASE Radiation Escaping the Cavity Versus Aspect Ratio

3.2 PLSE AMPLIFIER CODES

A computer code was written to solve the above set of equations. The input to the code includes a time dependent gain profile, the saturation intensity for the system of interest, the amplifier length, and the absorption length profile. The code generates a set of three dimensional plots of intensity of the primary pulse and backward and forward ASE versus length and time.

An example of the type of plot generated for the primary pulse is shown in Figure 3-4. Figures 3-5 and 3-6 display the forward and backward ASE, respectively, for the Sandia experimental conditions.^[1-3]

The PLSE code was later joined to the TRW KrF code to allow the time dependent gain and absorption to be calculated from the kinetics. In the original code, ψ^+ and ψ^- were solved for algebraically since $\phi(t)$ was a prescribed function. When kinetics are incorporated, ϕ is influenced by both the amplifying pulse and the kinetics and thus must be determined by iteration. The code was therefore rewritten to numerically evaluate ξ and ζ by a loop process. The steps are as follows:

Step One

For $w = -v$ or $t = 0$, the fluxes, ψ_p , $\psi^\pm$ are zero. Hence $\eta = 1$. Advance time to Δt , compute $\phi(\Delta t)$.

Step Two

Advance v or w according to

$$\begin{aligned}\Delta v &= -w_0 + \frac{2c}{\ell} \Delta t \\ \text{or} \quad \Delta w &= -v_0 + \frac{2c}{\ell} \Delta t\end{aligned}$$

(where v_0 , w_0 are coordinates on the $t = 0$ line). Compute fluxes $\psi^\pm$ (and ψ_p if present) still assuming $\eta = 1$. Compute η . If $\eta \leq 1 - \epsilon$ for any set of v , w along the Δt line, and ϵ is a prescribed small number,

RP

FIGURE 3-4. Three-Dimensional Plot of Amplified Intensity versus Time and Distance for the Sandia Case (see [1-3])

RF

FIGURE 3-5. Three-Dimensional Plot of Forward ASE versus Time and Distance

RB

FIGURE 3-6. Three-Dimensional Plot of Backward ASE versus Time and Distance

repeat the calculation of ψ^+ , ψ_p and re-enter into y until such time as

$$(|n_k - n_k - 1|)_{av} < \delta; \quad \delta \ll 1$$

represents the convergence of the k -th y estimate.

Step Three

Advance time by Δt . Recompute ϕ (or call it from a prepared table of ϕ versus t), and repeat Step Two.

Loop closure with respect to the kinetics package is as follows: a table of $\phi(t)$ values for the entire pump history is produced. At certain intervals dependent on the KrF* rise time, a new table of $\phi(t)$ is produced for succeeding times which has been modified by introduction of the space averaged total flux to adjust the KrF* population through flux induced loss mechanisms. Current values of the two tables are compared. If they deviate by a fixed, prescribed amount, the old is replaced by the new; β and the saturation parameter are updated, and the computation carried on until the next comparison time is called out.

3.3 COMPARISON OF THE CODE WITH EXPERIMENTAL DATA

The only runs of the original PLSE code with no kinetics were under idealized conditions of a 50 nsec pulse with a $g_0 l$ of 70. Thus, these runs did not model any realistic experimental data. However, the modified PLSE code with kinetics was run to model Sandia Corporation amplifier^[1-3] experiments. The set of kinetic rates that best modeled the gain and loss in the Sandia oscillator experiments was used.

Runs with the PLSE code indicated that the source term β was indeed critical in predicting the amount of ASE produced. If β was assumed equal to the spontaneous emission rate divided by 2, the amount of ASE produced was greatly overpredicted. When the ASE radiation vignetting model was included in β , much better agreement was achieved. Best results were obtained when β was adjusted to provide the best fit to experiment. A comparison of the output of the PLSE code with experiment is shown in Figure 3-7. An input pulse to be amplified was obtained from experiment.

FIGURE 3-7. Sandia Data Modeling (see [1-3])

The agreement in output pulse is only fair, especially for early time. The code predicted ASE started at a later time but was somewhat greater than experiment. Decreasing β makes the agreement in magnitude of ASE better but does still not show the early part of the pulse observed experimentally.

The runs performed above illustrated the difficulties in using a 1-dimensional model to predict a 3-dimensional experiment. Geometric factors related to the experiment to be modeled were found to be critical in obtaining agreement with experiment.

In summary, a 1-dimensional PLSE amplifier code including kinetics was developed and used to model experimental data. The formalism required for a 3-dimensional model was developed as shown in Appendix B. However, the model was not developed into a code, mainly due to the complexities of the partial differential equations involved. The 1-dimensional model is satisfactory as long as proper geometrical corrections are made to model the ASE.

4.0 E-BEAM DEVICES

E-beam pumped KrF lasers have become quite attractive candidates for laser ICF drivers. There are still a number of issues to be answered in terms of scaling to large size devices. A large scale system would use two opposing e-beams for optimum e-beam deposition uniformity and energy utilization. For single-sided devices, e-beam energy utilization will be poor if uniform volume deposition is required.

In designing the e-beam devices for these experiments, it is important to address the issue of uniformity. For this reason, an accelerating potential of 400 kV is chosen. At pressures near 1 atm, energy deposition uniformity is within $\pm 10\%$ for 10 cm path (see Figure 4-11). To assist in deposition uniformity and to help counteract scattering in the laser gas, magnetic guide fields are desirable. To reduce the electron back scatter and increased energy deposition near the walls, the opposing side of the laser cavity is terminated with a 50 μm titanium foil.

The e-beam pumped oscillator/amplifier devices each consist of a Marx generator, e-gun, magnetic guide field, a gas excitation chamber or laser gain volume and optics. A cross-section of one of the devices is shown in Figure 4-1. These devices have been designed to have the following characteristics.

E-beam aperture	10 cm x 50 cm
Current density (J_{eb})	$\sim 50 \text{ A/cm}^2$
Pulse duration	100 - 500 nsec
Transmitted beam energy	$\sim 400 \text{ keV}$
Laser volume	10 cm x 10 cm x 50 cm

The individual components of the devices are discussed in the following sections.

FIGURE 4-1. Diode Chamber and Laser Cavity

4.1 MARX GENERATOR

A cold cathode e-beam system requires a source of high energy pulsed power. This is provided by a Marx generator in which an array of capacitors are charged in parallel then rapidly discharged in series through triggered spark gas switches. The bank consists of 5 dual capacitors (0.925 μ F, 120 kV) and five series switches with a total storage capacity of 33 kJ. A photograph of one of the Marx banks is shown in Figure 4-2. The insulating oil required for operation has been removed for clarity. An electrical schematic of the system is shown in Figure 4-3. A 2Ω series resistor in conjunction with a high voltage diverter switch permit bank operation in a variable pulse width mode. After the command has been given to initiate the Marx bank, a second signal (delayed) is routed to the diverter switch which then conducts, removing the voltage from the cathode and causing the balance of the energy to be dissipated in the 2Ω series resistor. The value of this resistor was chosen to critically dump the capacitor and prevent large voltage reversals. A simple model of the circuit is shown in Figure 4-4.

To test the total circuit inductance of the Marx bank-cathode system, the bank was fired with the cathode shorted to the anode plate. From the frequency of the current oscillations, the total circuit inductance can be determined. Consider an LRC circuit where the total current is given by:

$$I = \frac{V_0}{\omega L} e^{-(R/2L)t} \sin \omega t \quad (4-1)$$

where
$$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

and L is the inductance to be determined, $R = 2.25\Omega$ (see Figure 4-4), $C = 0.185 \mu$ F and ω is found from the short circuit tests. Solving for L at the zero crossings of the current where $\omega\tau = n\pi$ and τ is the time period between crossing, Equation (4-1) becomes

FIGURE 4-2. Photograph of Marx Generator

FIGURE 4-3. Marx Generator Schematic

FIGURE 4-4. E-Beam Circuit Model

$$L = \frac{1/C \pm [1/C^2 - (n\pi R/\tau)^2]^{1/2}}{2(n\pi/\tau)^2} \quad (4-2)$$

Experimental waveforms for the short circuit tests were taken. A typical oscilloscope trace is shown in Figure 4-5. Using values obtained from this figure in Equation (4-2), the total inductance was found to be 1.25 μ H.

In the master oscillator - power amplifier (MOPA) configuration, two identical e-beam devices are used. Their electrical operation must be carefully controlled. The electrical block diagram for the system is shown in Figure 4-6. A common ± 60 kV DC power supply charges both Marx banks. The fire signal (start) initiates a common trigger generator which sends signals to both Marx banks. The cables to amplifier are 40 nsec longer to ensure that the oscillator is already over threshold before the amplifier starts. After the chosen time delay, the e-beam pulses are terminated by the initiation of the diverter trigger and subsequently the diverter switches. The practical pulse duration is 100 to 500 nsec. Pulses longer than 500 nsec may result in unstable diverter control due to the decreasing voltage on the Marx bank. If the diverter fails to fire, the foil (see later sections) will not survive since e-beam energy deposition in the foil is quite high when the voltage falls to less than 150 kV. Subsequently, most of the stored energy will be deposited in the foil causing it to rupture.

The Marx bank provides energy to the cathode whose operation is discussed in the following section.

FIGURE 4-5. Current Waveform of Diode Short Circuit Test

FIGURE 4-6. MOPA E-Beam Block Diagram

4.2 ELECTRON GUNS

The e-gun consists of a cathode, cathode enclosure, anode foil, drift space, pressure foil and support structure. The various parts and operation are described.

The e-beam cathode consists of a vertical array of 40 thin blade stainless steel emitters with an inter-blade spacing of 12.7 mm. The length of each blade is 12 cm and the blade height is 1 cm above the mount. The whole assembly is surrounded by a tubular corona shield to assist in field shaping. A magnetic guide field, which has been provided to counteract scattering in the gas, helps to increase the current density due to the converging field lines. The cathode assembly is enclosed in a vacuum box (40 cm x 60 cm x 100 cm) evacuated to less than 10^{-4} torr by an 8 inch oil diffusion pump.

Referring back to Figure 4-1, the e-beam operation is as follows. When the high voltage pulse is applied to the cathode, the local field at the tip of each blade is so high that a micro-portion of the blade is blown off and a plasma is formed. Electrons in the plasma are accelerated towards the anode (F_1) which is a 25 μ m titanium foil. The initial anode-cathode spacing is 4.4 cm. The current density available from the cathode plasma is described by the Child law for a plane parallel diode

$$J = 2.336 \times 10^{-6} \frac{V^{3/2}}{(d - v_0 t)^2}$$

where V is the applied voltage in volts, d is the initial anode-cathode spacing in cm, v_0 is the closure velocity in cm/sec, t is time in seconds and J is current density in A/cm^2 . The closure velocity is due to the motion of the cathode plasma towards the anode and has a value of ~ 2 cm/ μ sec. The closure causes the impedance of the diode to collapse and the current density to increase with time for a constant voltage driver.

In the case of a real Marx bank, the voltage drops with time due to finite energy storage. This can be observed in the figures of a following section.

The applied magnetic field (~ 2 kGauss) causes the emission sites at the blades to be imaged virtually 1:1 onto the anode. This effect is observed by a wrinkled pattern formed on the anode foil. The pattern is skewed by about five degrees due to net beam rotation caused by the applied magnetic field. After passing through the anode foil, the beam drifts about 10 cm to the pressure foil, F_2 . This foil is held in place by a "hibachi" foil support structure fabricated from 304 stainless steel. The structure has 2.54 mm ribs with an inter-rib open space of 22.9 mm yielding a 90% open area. It has been designed to handle pressures of 3 atm static and 6 atm dynamic in conjunction with a 25 μ m titanium foil.

Due to the space charge effects of propagating an unneutralized beam in a vacuum, some of the beam is lost to the walls. However, the cathode image originally formed at the anode foil is completely smeared out by the time it reaches the pressure foil. The transmitted beam uniformity has been determined using x-ray photography and was found to be within $\pm 10\%$ over the 250 cm² area. A typical densitometer trace of the resultant negative is shown in Figure 4-7.

E-beam performance was measured with large area (10 cm x 50 cm) Faraday cup. Using a Marx charging voltage of 120 kV, a current density of 44 A/cm² was obtained at the end of a 500 nsec pulse. Current, voltage and transmitted current wave forms are shown in Figures 4-8, 4-9, and 4-10, respectively. The constantly increasing current is due to the characteristic impedance collapse of cold cathode diode.

FIGURE 4-7. Densitometer Trace of an X-ray Photograph of E-beam Transmitted Current

FIGURE 4-8. E-beam Total Current

FIGURE 4-9. E-Beam Voltage Waveform

FIGURE 4-10. Transmitted Total E-beam Current

In determining the pumping power density available from these e-beam devices, consider a mixture of Ar and Kr with a mixture ratio of 9:1. Using the scattering formalism developed by Spencer ^[7], the energy deposition of electrons emerging from a foil can be computed as a function of distance. These computations have been performed as a function of pressure for 400 keV electrons emerging from a 25 μm titanium foil and are shown graphically in Figure 4-11. Figure 4-12 lists deposition rate for current density versus pressure. In the current device experiments, it can be seen that less than 20% of the e-beam is utilized over the 10 cm path. However, the uniformity is better than $\pm 10\%$ for a total pressure of 1 atm. In a scaled-up device, two opposing e-guns would be used separated by approximately one residual range. This would allow almost complete utilization of the e-beam with excellent deposition uniformity.

4.3 MAGNETIC GUIDE FIELDS

In large area, high current electron beams, the magnetic field generated by the current flow can cause the beam to pinch. The self-magnetic field at the edge of a rectangular beam is given by

$$B_{\text{self}} = \frac{\mu_0 J_{\text{eb}} wh}{2 (w + h)}$$

where μ_0 is the permeability of free space ($4\pi \times 10^{-7}$), J_{eb} is the current density in A/m^2 , w is the width in meters and h is the height in meter. For $J_{\text{eb}} = 50 \text{ A/cm}^2$, $w = 50 \text{ cm}$ and $h = 10 \text{ cm}$, the self magnetic field is found to be

$$B_{\text{self}} = 26.2 \times 10^{-3} \text{ Webers/m}^2 = 262 \text{ Gauss}$$

FIGURE 4-11. E-Beam Deposition Profile

J_b (A/cm ²)	PRESSURE (ATM)				
	1.0	1.5	2.0	2.5	3.0
10	69	107	145	180	212
20	137	214	289	359	424
30	206	321	434	539	636
40	274	428	579	719	847
50	343	535	723	898	1059
75	514	802	1085	1347	1589
100	686	1070	1447	1796	2119

10 CM INTEGRATED PATH, 400 kV

FIGURE 4-12. Average Energy Deposition (kW/cm³)
0.90 Ar - 0.10 Kr

Electron scattering in the foil and high pressure gas can cause nonuniform volumetric energy deposition. By applying a guide magnetic field (B_z) along the direction of e-beam propagation, these problems can be minimized. The required B_z field should be sufficiently strong so that the electrons will spiral along the field lines in the transverse direction. This condition can be satisfied with $B_z \geq 5 B_{\text{self}}$. Hence, the applied field should be greater than 1300 Gauss.

In order for the magnetic field to penetrate the aluminum chamber it is desirable to use direct current electromagnets since even 60 Hz field generate eddy currents sufficiently large to effectively cancel the applied field. In designing the coils it was convenient to choose a typical DC welding supply with a current capability of 1000 A at 44 volts as the power source. Different coil configurations were examined with respect to field uniformity and compatibility with the welding supply. A pair of rectangular coils (dimensions 30 cm high x 70 cm long) were chosen since they could provide the desired field uniformity as well as a converging magnetic field from the e-beam cathode to the anode aperture.

To meet the 44 m Ω resistance requirement (44 v/1000 A) as well as the field strength requirement, the coils were fabricated from 36 turns of 5.7 cm wide by 0.16 cm thick copper strip. Each coil was designed to have a resistance of 22 m Ω . The measured resistance was 15 m Ω while the measured inductance was 1.2 mH per coil. The optimum coil separation of 20 cm (center to center) was determined by iteratively computing the spatial field uniformity as a function of separation. The results of the computation for the optimum separation are shown in Figure 4-13, where the magnetic field was computed to be 1.48 ± 0.02 kGauss/kA over the entire cavity region. The computations include the effects of the strap width as well as a dimensional allowance for insulating materials. The coordinate system shown in the figure is referenced to the center of the coil. Code calculations have been empirically verified to within $\pm 5\%$ with a Hall-effect gaussmeter using the actual electromagnets. A field of 2.1 kGauss (at 1460 A) is obtained routinely at the center of the cavity.

The measured magnetic field maps are shown in Figures 4-14 and 4-15. The coils are energized just a few seconds before the e-beam is fired and then are shut off immediately. With this short time duration, there is little coil heating. Experiments have shown that there is full field penetration into the laser cavity ~ 30 msec after reaching steady state current flow in the coils. The power supplies are able to deliver 1460 A at 47.5 V into the coils for the amplifier and 1320 A at 48.5 V into the coils for the oscillator. The difference is due to the extra cable length resistance in connecting the power supply to the oscillator coils. These currents correspond to 2.1 kGauss. and 1.9 kGauss for the amplifier and oscillator, respectively, which is greater than $7 B_{\text{self}}$.

It should also be noted that there is a net beam rotation of ~ 5 degrees due to the applied B_z field. This was displayed as a rotated wrinkle pattern of the discrete images of the vertical cathode blade emitters on the anode foil. The applied field also assists in propagating the e-beam through the drift region. Experiments were performed looking at transmitted current density with and without applied magnetic field. There was a factor of two increase in transmitted current when the applied field was used.

		COIL					CAVITY					DRIFT REGION				
		5	4	3	2	1	0	1	2	3	4	5				
COIL	15	1.327	1.311	1.298	1.290	1.285	1.283	1.285	1.290	1.298	1.311	1.327				
	14	1.357	1.339	1.325	1.316	1.310	1.308	1.310	1.316	1.325	1.339	1.357				
	13	1.384	1.365	1.350	1.340	1.334	1.332	1.334	1.340	1.350	1.365	1.384				
	12	1.410	1.389	1.374	1.363	1.357	1.355	1.357	1.363	1.374	1.389	1.410				
	11	1.432	1.411	1.395	1.384	1.377	1.375	1.377	1.384	1.395	1.411	1.432				
	10	1.452	1.430	1.413	1.402	1.395	1.393	1.395	1.402	1.413	1.430	1.452				
	9	1.469	1.447	1.430	1.418	1.412	1.409	1.412	1.418	1.430	1.447	1.469				
	8	1.483	1.461	1.444	1.433	1.426	1.424	1.426	1.433	1.444	1.461	1.483				
	7	1.494	1.472	1.456	1.445	1.438	1.436	1.438	1.445	1.456	1.472	1.494				
	6	1.502	1.481	1.466	1.455	1.448	1.446	1.448	1.455	1.466	1.481	1.502				
	5	1.509	1.489	1.474	1.463	1.457	1.455	1.457	1.463	1.474	1.489	1.509				
	4	1.513	1.494	1.480	1.469	1.463	1.461	1.463	1.469	1.480	1.494	1.513				
	3	1.517	1.498	1.484	1.474	1.468	1.466	1.468	1.474	1.484	1.498	1.517				
	2	1.519	1.501	1.487	1.478	1.472	1.470	1.472	1.478	1.487	1.501	1.519				
	1	1.520	1.502	1.489	1.480	1.474	1.472	1.474	1.480	1.489	1.502	1.520				
CAVITY	0	1.520	1.503	1.490	1.480	1.475	1.473	1.475	1.480	1.490	1.503	1.520				
	1	1.520	1.502	1.489	1.480	1.474	1.472	1.474	1.480	1.489	1.502	1.520				
	2	1.519	1.501	1.487	1.478	1.472	1.470	1.472	1.478	1.487	1.501	1.519				
	3	1.517	1.498	1.484	1.474	1.468	1.466	1.468	1.474	1.484	1.498	1.517				
	4	1.513	1.494	1.480	1.469	1.463	1.461	1.463	1.469	1.480	1.494	1.513				
	5	1.509	1.489	1.474	1.463	1.457	1.455	1.457	1.463	1.474	1.489	1.509				
	6	1.502	1.481	1.466	1.455	1.448	1.446	1.448	1.455	1.466	1.481	1.502				
	7	1.494	1.472	1.456	1.445	1.438	1.436	1.438	1.445	1.456	1.472	1.494				
	8	1.483	1.461	1.444	1.433	1.426	1.424	1.426	1.433	1.444	1.461	1.483				
	9	1.469	1.447	1.430	1.418	1.412	1.409	1.412	1.418	1.430	1.447	1.469				
	10	1.452	1.430	1.413	1.402	1.395	1.393	1.395	1.402	1.413	1.430	1.452				
	11	1.432	1.411	1.395	1.384	1.377	1.375	1.377	1.384	1.395	1.411	1.432				
	12	1.410	1.389	1.374	1.363	1.357	1.355	1.357	1.363	1.374	1.389	1.410				
	13	1.384	1.365	1.350	1.340	1.334	1.332	1.334	1.340	1.350	1.365	1.384				
	14	1.357	1.339	1.325	1.316	1.310	1.308	1.310	1.316	1.325	1.339	1.357				
DRIFT REGION	15	1.327	1.311	1.298	1.290	1.285	1.283	1.285	1.290	1.298	1.311	1.327				
	16	1.296	1.281	1.270	1.262	1.258	1.256	1.258	1.262	1.270	1.281	1.296				
	17	1.263	1.250	1.241	1.234	1.230	1.228	1.230	1.234	1.241	1.250	1.263				
	18	1.230	1.219	1.210	1.204	1.201	1.200	1.201	1.204	1.210	1.219	1.230				
	19	1.196	1.187	1.179	1.174	1.171	1.170	1.171	1.174	1.179	1.187	1.196				
	20	1.162	1.154	1.148	1.144	1.141	1.140	1.141	1.144	1.148	1.154	1.162				

FIGURE 4-13. Magnetic Field Distribution Cross-Section in the Cavity and E-gun Regions (in kGauss/kA)

Coordinates are in centimeters and are referenced to the center of the coils. Coil positions in the vertical direction are located at $x = \pm 15$ cm and have been exaggerated in this drawing

FIGURE 4-14. Measured Magnetic Field Distribution in the Laser Cavity in kGauss/kA. These measurements were made in planes at five longitudinal positions along the optical axis.

CATHODE MAGNETIC FIELD DISTRIBUTION								
Y								
5	0.82	0.82	0.82	0.82	0.83	0.82	0.77	0.54
4	0.90	0.91	0.92	0.93	0.93	0.93	0.90	0.70
3	0.93	0.93	0.93	0.93	0.93	0.93	0.88	0.68
2	0.95	0.95	0.95	0.95	0.96	0.95	0.92	0.72
1	0.97	0.97	0.97	0.97	0.98	0.98	0.93	0.75
0	0.97	0.97	0.97	0.97	0.97	0.98	0.93	0.75
	0	2	4	6	8	10	12	14
	X

FIGURE 4-15. Magnetic Field Distribution for One Quadrant in the Plane of the Cathode Emitters in kGauss/kA.

4.4 LASER CAVITY

The laser cavity has been formed from a machined solid block of aluminum. The ends of the cavity are closed by fused silica (Dynasil UV 1000) windows (12 mm thick). Initially the windows were anti-reflection and protected from F_2 by a MgF_2 overcoat. These coatings were quickly destroyed by the combined action of KrF photons and F_2 . The rest of the experiments were performed with uncoated windows.

The back wall of the gas cavity consists of a foil sandwich between two foil support structures. Electrons traversing through the cavity will pass this foil then into the air thereby reducing electron back scatter into the gas. The double support structure is necessary for the second foil since the cavity pressure may be greater than or less than atmospheric pressure causing outward or inward foil motion, respectively.

The entire gas cavity is surrounded with lead to attenuate x-rays generated by e-beam interaction with the gas and cavity walls. Even with this shielding, the radiation dose at the cavity is 10 mR per shot.

4.5 OPTICS

The laser resonator consists of Fabry-Perot plane parallel optics. In one configuration, a maximum reflectivity dielectric coated fused silica mirror is used at one end while a 50% dielectric coated fused silica output coupler is used at the other end. The opposing surface of the output coupler is anti-reflection coated. In other configuration, 50% reflectivity mirrors are used at both ends of the cavity.

In the first few oscillator extraction shots, the maximum reflectivity mirror was badly damaged due to high intra-cavity flux. For most of the remaining experiments, the second configuration (with two 50% mirrors) was used.

The optical setup of the master oscillator-power amplifier (MOPA) configuration will be described in detail in the experimental results section.

5.0 KrF EXPERIMENTS

The overall objective of the experimental part of the program is to characterize the performance of pulsed KrF lasers as related to laser fusion driver applications. Specific objectives are: 1) to determine the required optical input energy into a KrF amplifier medium to suppress amplified spontaneous emission (ASE) and 2) to determine gain length scaling. These data have been obtained experimentally using e-beam pumped KrF devices arranged in a maser oscillator-power amplifier (MOPA) configuration. Details of these experiments and the resulting analyses are described in the following sections.

5.1 KrF OSCILLATOR PERFORMANCE

In scaling KrF devices to large sizes, it was shown in Section 2 that the most efficient operation would be obtained by using gas mixtures at a total pressure less than two atmospheres. The optimum mixture ratio (Ar:Kr:F_2) depends on the pump rate and run time due to fluorine burn-up. Oscillator experiments were performed at a pump rate of 240 kw/cm^3 (33A/cm^2) as a function of fluorine concentration. Early experiments had damaged AR window coatings as well as the maximum reflector. To continue with experiments, the fused silica windows were used uncoated and the beam was coupled out both ends by utilizing two 50% output couplers. The opposing side of each mirror was AR coated. The windows were aligned to the optical cavity so that the net reflectivity of each output port was approximately 58%. The output of the oscillator was directed to a Scientech 8 inch volume integrating calorimeter. The temporal intensity response was observed from a beam splitter with an S-5 Hamamatsu bi-planar photodiode in conjunction with appropriate uv neutral density filters and a fast oscilloscope (HP 1740A). Burn patterns, obtained using exposed and developed lithograph film, indicate that the output is extremely uniform over the $10 \text{ cm} \times 10 \text{ cm}$ area. This would imply that the e-beam deposition uniformity is quite good. During these experiments, the total e-beam current and voltage were monitored with a TEK 7844 dual-beam scope. E-beam operating conditions remained

quite constant during these experiments. The oscillator e-beam was usually operated at a peak current density of 33A/cm^2 (240 kW/cm^3) at the end of a 550 nsec pulse. The magnetic guide field intensity was 1.9 kGauss. A photograph of the oscillator is shown in Figure 5-1. The cavity is surrounded by lead to attenuate x-rays.

Oscillator performance as a function of F_2 is shown in Figure 5-2 where fluorine partial pressures were varied from 1 torr to 8 torr in steps of 1 torr. The total pressure was fixed at 760 torr and the Kr partial pressure was fixed at 76 torr. The intensity was computed from the measured energy and is for the output from one end. The highest output was at 4 torr F_2 and corresponded to a total energy of 48J. A tabulation of intensity and output energy is shown in Figure 5-3. Note that at the lowest F_2 concentration, the rate of change of intensity is the highest but the F_2 is depleted causing termination of the oscillator at 300 nsec. At 8 torr F_2 , the intensity is still increasing when the e-beam shuts off but the overall level is less than in lower pressure cases due to F_2 quenching. The early termination of the pulse in the 8 torr case is due to premature diverter switch operation in the e-beam.

To determine performance in terms of Kr concentration, the oscillator was run with 4 torr F_2 at both 30 torr and 56 torr. In both cases, only 36J was attained at a peak intensity of 0.53 MW/cm^2 .

The kinetics code was run to predict two-port oscillator operation. Figure 5-4 demonstrates the excellent agreement between code and experiment. This comparison was made at 4 torr F_2 , 76 torr Kr and 680 torr Ar at 240 kW/cm^3 . Similar agreement is obtained for the oscillator versus F_2 experiments described earlier.

FIGURE 5-1. KrF Oscillator

FIGURE 5-2. KrF Oscillator Intensity versus F_2 (Experimental)

I_{PEAK} (MW/cm²)	t_{PEAK} (nsec)	[F₂] TORR	E* (J)
0.46	140	1	19
0.58	310	2	38
0.68	400	3	48
0.70	480	4	48
0.58	360**	5	36
0.58	400	6	32
0.60	400	7	29
0.58	430	8	27

***INTENSITY IS FROM ONE APERTURE, ENERGY IS TOTAL FROM BOTH APERTURES**

****SHORT SHOT**

FIGURE 5-3. KrF Oscillator Performance versus F₂ (Experimental)

FIGURE 5-4. KrF Code Experiment Comparison for Two-Port Oscillator

Although the oscillator was normally used as a two-port device due to early optics damage of the total reflector, some shots were made with operation as a single port device with the damaged mirror. By this time, there was trouble with photon assisted F_2 etching of the windows. To be able to continue with data taking, poorer quality fused silica windows were used in the oscillator as the others were being re-polished. The window transmission was 76% of which 16% was to bulk absorption impurities. In this configuration, the oscillator was able to develop a peak intensity of 0.93 MW/cm^2 (32J) from a single port compared with 24J from one port of the two-port arrangement. It must be emphasized that this was from a damaged mirror of undetermined reflectivity. The code predicts a peak output intensity of 1.9 MW/cm^2 (79J) for 300 kW/cm^3 pump rate at optimum output coupling (0.39) from this size device. The present limitation appears to be optics.

To calibrate the photodiode/uv neutral density filter combinations, temporal intensity information was digitized and integrated. The unknown intensity scale factor was obtained by equating the integrated photodiode voltage with the measured energy as determined by the calorimeter.

5.2 ASE CLAMPING TESTS

The purpose of the ASE clamping test is to determine the minimum input intensity required to reduce ASE to one-half of its no input intensity value. These tests were performed with the single-pass MOPA configuration where the output of the oscillator is directed into the amplifier. To minimize losses, the windows which enclose the laser medium were initially anti-reflection coated. However, the coatings were quickly destroyed by the high intensity and the presence of fluorine. After this point, the windows were carefully polished about every 10 shots. The normal incidence reflection loss for fused silica at the KrF wavelength is 0.04 per surface. A photograph of the experimental setup is shown in Figure 5-5. A diagram of the experimental arrangement is shown in Figure 5-6. Referring to this diagram, the following paragraphs will describe the operation of the MOPA system.

FIGURE 5-5. KrF Master Oscillator-Power Amplifier

FIGURE 5-6. Experimental Arrangement for Gain Clamping Single Pass

FIGURE 5-7. KrF* Burn Patterns from the Oscillator (Left) and Amplifier (Right), Respectively

The oscillator volume is enclosed by uncoated fused silica windows W1 and W2. The optical Fabry-Perot resonator is formed by mirrors OC1 and OC2. Mirror OC1 may be either a maximum reflector or 50% reflector while OC2 is a 50% reflector. The opposing side of the partial reflectors have been AR coated. The oscillator output is attenuated by large Corning CS 7-54 UV filters which have a transmission of 48.5% at 248.9 nm. The attenuated beam is directed through beamsplitter BS1 and into the amplifier volume which is enclosed by uncoated fused silica windows W3 and W4. The amplified output is directed to calorimeter C2 by beam splitter BS2 (which is 0.07 reflecting at this angle and wavelength). The input intensity is monitored by PD5, while the off-axis ASE is monitored by PD1 through PD4. These photodiodes are set to monitor the integrated diagonal path ASE in both the forward and backward directions. Each photodiode has a CS 7-54 filter (to pass only uv) along with appropriate uv neutral density filters which attenuate the signal to avoid diode saturation. The input photodiode was calibrated by placing the calorimeter directly behind BS1 and comparing the measured energy with the integrated photodiode signal. Films F1 and F2 were used to record burn patterns of the oscillator and amplifier, respectively. Typical burn patterns are shown in Figure 5-7.

The experiment was to determine what input intensity was required to reduce ASE by one half from its value when there was no input intensity from the oscillator. This was performed by operating the system with 0, 1, 2, 3, and 4 attenuators in front of the oscillator as well as with the oscillator blocked. The amplifier e-beam was operated at $44\text{A}/\text{cm}^2$ peak ($300\text{ kW}/\text{cm}^3$) for these runs. The mixture ratio was $\text{Ar}:\text{Kr}:\text{F}_2 = 680:76:4$ torr since this was optimum for the oscillator. For each run, the total amplifier e-beam current, total oscillator e-beam current, and photo diode signals were monitored as a function of time. The following paragraphs describe the analysis of that data.

A summary of the amplifier output versus input is shown in Figure 5-8. The amplifier gain is high for low input values but the extraction efficiency is extremely low. A peak extraction efficiency of 0.46 was attained at the peak of the amplifier gain with an input intensity of 0.66 MW/cm^2 . The average extraction values were computed from $E(\text{KrF}^*)/[E_0 - E_i]$ where $E(\text{KrF}^*)$ is the energy in KrF^* as predicted by the kinetics code.

Although the temporal history of the output was not directly measured, it can be computed using the amplifier-kinetics code with the actual temporal input intensity and e-beam pump rate. Three-D plots for the maximum input case are shown in Figures 5-9 through 5-11. The first figure shows the intensity growing through the cavity from the input pulse, while the other plots show the forward and backward ASE growing through the cavity. Computations have also been performed for the other input cases and the calculated output energies have been tabulated in Figure 5-8.

Some ASE suppression was observed for all cases of input intensity. The most striking case was for the one-port oscillator input with no attenuator. The forward and backward ASE with and without input is shown for this case in Figure 5-12. This data has been summarized in Figure 5-13 where ASE clamping intensity and normalized input intensity are plotted versus time. From this figure, it can be seen that for $\phi_i > 0.35$ the ASE is less than 60% of its free running value. As was previously noted, this regime gives rise to the highest extraction efficiencies ($\eta_{\text{extr}} > 0.4$). As discussed in Section 2, the peak gain from the amplifier has been computed to be $\sim 0.06 \text{ cm}^{-1}$ or $g_0 l \simeq 3.0$. It is clear from these data that lengths longer than 50 cm are desirable for best extraction efficiency.

ATTN	PEAK I_i (MW/cm ²)	E_i (J)	E_o (J)		E_o/E_i	$\bar{\eta}_{EXTR}$ CALC
			EXP	CALC		
0*	0.556	17.7	100.0	101	5.65	0.35
0**	0.494	15.5	59.4	61	3.85	0.19
1	0.213	7.8	42.9	40	5.50	0.15
2	0.075	2.7	22.3	21	8.17	0.08
3***	0.053	1.9	22.3	15	11.74	0.08
4	0.014	0.5	10.3	5	22.4	0.05

*ONE PORT OSCILLATOR, ALL OTHERS ARE TWO-PORT

** SHORT SHOT

***THIS POINT IS AN UNEXPLAINED DISCREPANCY

FIGURE 5-8. MOPA Amplification as a Function of Input Energy

RP

FIGURE 5-9. Predicted Amplifier Intensity as a Function of Time and Distance

RF

FIGURE 5-10. Predicted Amplifier Forward ASE as a Function of Time and Distance.

RB

FIGURE 5-11. Predicted Amplifier Backward ASE as a Function of Time and Distance

FIGURE 5-12. Forward and Backward ASE with and without Input Intensity

FIGURE 5-13. ASE Clamping and Normalized Input Intensity versus Time

5.3 GAIN LENGTH SCALING TESTS

The purpose of the gain length scaling test is to study intensity as a function of gain length. These tests are performed with the MOPA configuration where the amplifier is optically divided into four quadrants. The beam is sequentially routed through each quadrant with the intensity being determined at each quadrant output. The experimental arrangement is shown in Figure 5-14. The input beam is routed into the first quadrant and is amplified. The dielectric coated mirrors steer the output beam into the input aperture of the second quadrant. Although the mirrors are maximum reflectors at 248.9 nm, some of the beam is expected to leak through and be detected by PDI. Similar routing and detection occurs for each quadrant. It is assumed that there will not be a great deal of optical communication between quadrants.

In designing the experiment, the KrF kinetics and amplifier code was run to predict amplifier performance as a function of gain length and pressure for an e-beam current density of 44 A/cm^2 . The results of these calculations are shown in Figure 5-15. At a length of 50 cm, the highest output is attained at a pressure of two atmospheres. As the gain length is increased, the one atmosphere pressure case is superior due to higher value of γ (gain/loss) which leads to a better extraction efficiency. In scaling to large devices, operation at pressures near one atmosphere is optimum. It should be noted that the mixture was optimized for each total pressure to generate the values in this figure.

Experiments have not been completed at this time due to damage coatings on the beam steering optics. The continuation of this test is being considered for another program.

FIGURE 5-14. Experimental Arrangement for Extraction Efficiency Tests

PREDICTED AMPLIFIER PERFORMANCE

($J_{eb} = 44 \text{ A/CM}^2, 500 \text{ nsec}$)

PRESSURE (ATM)	PUMP RATE (kW/cm ³)	% MIXTURE Ar/Kr/F ₂	g_o (cm ⁻¹)	γ	I_s (MW/cm ²)	Φ_i	Φ_o AND η VS CAVITY LENGTH (cm)							
							50		100		150		200	
							Φ_o	η	Φ_o	η	Φ_o	η	Φ_o	η
1.0	300	89.47	0.0568	22.4	1.27	0.1	0.75	0.23	2.21	0.36	3.95	0.45	5.69	0.49
		10.00				0.5	1.80	0.46	3.51	0.54	5.26	0.56	6.92	0.57
		0.53				1.0	2.56	0.55	4.31	0.58	6.03	0.59	7.64	0.58
1.5	450	89.55	0.0601	18.5	1.70	0.1	0.80	0.23	2.33	0.37	4.09	0.44	5.78	0.47
		10.00				0.5	1.85	0.45	3.59	0.51	5.31	0.53	6.90	0.53
		0.45				1.0	2.60	0.53	4.36	0.56	6.03	0.56	7.54	0.54
2.0	600	89.60	0.0634	14.9	2.24	0.1	0.84	0.23	2.41	0.36	4.13	0.42	5.71	0.44
		10.00				0.5	1.89	0.44	3.61	0.49	5.25	0.50	6.68	0.49
		0.40				1.0	2.62	0.51	4.33	0.53	5.88	0.51	7.23	0.49
2.5	750	94.65	0.0620	12.9	2.61	0.1	0.80	0.22	2.25	0.35	3.84	0.40	5.28	0.42
		5.00				0.5	1.81	0.42	3.40	0.47	4.90	0.47	6.19	0.46
		0.35				1.0	2.52	0.49	4.10	0.50	5.50	0.48	6.69	0.46
3.0	900	94.65	0.0593	10.7	3.24	0.1	0.72	0.21	2.01	0.32	3.41	0.37	4.66	0.38
		5.00				0.5	1.68	0.40	3.10	0.44	4.39	0.44	5.48	0.42
		0.35				1.0	2.37	0.46	3.74	0.46	4.94	0.44	5.93	0.42

FIGURE 5-15. Predicted Amplifier Performance as a Function of Length and Pressure

6.0 REFERENCES

1. G.C. Tisone, E.L. Patterson and J.K. Rice, "Studies of an 80-J KrF Oscillator at Excitation Rates of 2-7 MW/cm³," Appl. Phys. Lett. 35, 437 (1979)
2. E.L. Patterson, J.K. Rice and G.C. Tisone, "A master-oscillator power amplifier experiment for the determination of the gain coefficient and saturation intensity for an electron-beam-excited KrF amplifier", Appl. Phys. Lett. 36, 188 (1980)
3. J.K. Rice, G.C. Tisone and E. Patterson, "Oscillator Performance and Energy Extraction from a KrF Laser Pumped by a High-Intensity Relativistic Electron Beam," IEEE J. Quantum Electron. QE-16, 1315 (1980)
4. M. Rokni, J.A. Mangano, J.H. Jacob, and J.C. Hsia, "Rare Gas Fluoride Lasers," IEEE J. Quantum Electron. QE-14, 464 (1978)
5. C. Duzy, J. Hsia, H. Hyman, J. Jacob, D. Klimek, J. Parks and D. Trainer, "Rare Gas Halide Lasers for Fusion," Final Report DOE/P40103-4, April 1980
6. W.W. Rigrod, "Saturation Effects in High-Gain Lasers," J. Appl. Phys. 36, 2487 (1965)
7. L.V. Spencer, "Energy Dissipation by Gas Electrons," National Bureau of Standards Monograph 1, (1959)

APPENDIX A - KINETIC RATE PROCESSES

REACTION	RATE CONSTANT**
Secondary Electron Processes	
$F_2 + e \rightarrow F^- + F$	$2.8 \times 10^{-9} *$
$Ar^* + e \rightarrow Ar^+ + 2e$	$1.8 \times 10^{-9} *$
$Kr^* + e \rightarrow Kr^+ + 2e$	$4.7 \times 10^{-9} *$
$Ar^{**} + e \rightarrow Ar^+ + 2e$	$2.5 \times 10^{-8} *$
$Kr^{**} + e \rightarrow Kr^+ + 2e$	$2.5 \times 10^{-8} *$
$Ar^* + e \rightarrow Ar^{**} + e$	$1 \times 10^{-7} *$
$Kr^* + e \rightarrow Kr^{**} + e$	$1 \times 10^{-7} *$
$Ar^* + e \rightarrow Ar + e$	$3.2 \times 10^{-10} *$
$Kr^* + e \rightarrow Kr + e$	$4.4 \times 10^{-9} *$
$Ar_2^+ + e \rightarrow Ar^{**} + Ar$	$7 \times 10^{-7} *$
$Kr_2^+ + e \rightarrow Kr^{**} + Kr$	$1.9 \times 10^{-7} *$
$ArKr^* + e \rightarrow Kr^* + Ar$	$1 \times 10^{-7} *$
Rare Gas Ion Processes	
$Ar^+ + 2Ar \rightarrow Ar_2^+ + Ar$	2×10^{-31}
$Kr^+ + Ar + Kr \rightarrow Kr_2^+ + Ar$	2.4×10^{-31}
$Ar^+ + Kr + Ar \rightarrow ArKr^+ + Ar$	1×10^{-31}
$Kr^+ + 2Ar \rightarrow ArKr^+ + Ar$	1×10^{-31}
$Ar^+ + Kr \rightarrow Ar + Kr^+$	3×10^{-11}
$Ar_2^+ + Kr \rightarrow ArKr^+ + Ar$	7.5×10^{-10}
$ArKr^+ + Kr \rightarrow Kr_2^+ + Ar$	3.2×10^{-10}

* Electron rates at 1 eV

** Rate constants in units of $cm^3/molecule \text{ sec}, sec^{-1}$, etc.

REACTIONRATE CONSTANT**

Ion-Ion Recombination

$\text{Ar}^+ + \text{F}^- \rightarrow \text{ArF}^*$	2.7×10^{-6}
$\text{Ar}_2^+ + \text{F}^- \rightarrow \text{ArF}^* + \text{Ar}$	1×10^{-6}
$\text{Kr}^+ + \text{F}^- \rightarrow \text{KrF}^*$	2.96×10^{-6}
$\text{Kr}_2^+ + \text{F}^- \rightarrow \text{KrF}^* + \text{Kr}$	1.0×10^{-6}
$\text{ArKr}^+ + \text{F}^- \rightarrow \text{KrF}^* + \text{Ar}$	1.0×10^{-6}

Neutral Rare Gas Processes

$\text{Ar}^{**} + 2\text{Ar} \rightarrow \text{Ar}_2^* + \text{Ar}$	1.4×10^{-32}
$\text{Ar}^* + \text{Kr} + \text{Ar} \rightarrow \text{ArKr}^* + \text{Ar}$	1×10^{-32}
$\text{Kr}^* + 2\text{Ar} \rightarrow \text{ArKr}^* + \text{Ar}$	1×10^{-32}
$\text{Kr}^* + \text{Kr} + \text{Ar} \rightarrow \text{Kr}_2^* + \text{Ar}$	5×10^{-32}
$\text{Ar}^* + \text{Kr} \rightarrow \text{Ar} + \text{Kr}^*$	6.2×10^{-12}
$\text{Ar}_2^* + \text{Kr} \rightarrow 2\text{Ar} + \text{Kr}^*$	8×10^{-11}
$\text{ArKr}^* + \text{Kr} \rightarrow \text{Kr}_2^* + \text{Ar}$	1×10^{-10}
$2\text{Kr}_2^* \rightarrow \text{Kr}_2^+ + 2\text{Kr} + \text{e}$	3×10^{-11}
$2\text{Ar}_2^* \rightarrow \text{Ar}_2^+ + 2\text{Ar} + \text{e}$	3×10^{-11}

Neutral Formation of Rare Gas Halide

$\text{Ar}^* + \text{F}_2 \rightarrow \text{ArF}^* + \text{F}$	8.5×10^{-10}
$\text{Kr}^* + \text{F}_2 \rightarrow \text{KrF}^* + \text{F}$	8.1×10^{-10}
$\text{Ar}^{**} + \text{F}_2 \rightarrow \text{ArF}^* + \text{Ar} + \text{F}$	8.5×10^{-10}
$\text{Kr}^{**} + \text{F}_2 \rightarrow \text{KrF}^* + \text{F}$	8.1×10^{-10}
$\text{Ar}_2^* + \text{F}_2 \rightarrow \text{ArF}^* + \text{Ar} + \text{F}$	3×10^{-10}
$\text{Kr}_2^* + \text{F}_2 \rightarrow \text{KrF}^* + \text{Kr} + \text{F}$	3×10^{-10}

REACTIONRATE CONSTANT**

$$6 \times 10^{-10}$$

$$1 \times 10^{-10}$$

Quenching Processes

$$1.8 \times 10^{-12}$$

$$6.5 \times 10^{-31}$$

$$2.9 \times 10^{-31}$$

$$7 \times 10^{-32}$$

$$7.8 \times 10^{-10}$$

$$4 \times 10^{-31}$$

$$1.9 \times 10^{-9}$$

$$1 \times 10^{-9}$$

$$1 \times 10^{-9}$$

$$1 \times 10^{-9}$$

$$1 \times 10^{-10}$$

Radiative Processes

$$1.5 \times 10^8$$

$$3.3 \times 10^7$$

$$1 \times 10^7$$

$$1 \times 10^7$$

$$2.5 \times 10^8$$

$$5 \times 10^6$$

$$7.8 \times 10^6$$

$$5.5 \times 10^6$$

REACTIONRATE CONSTANT**

Superelastic Reactions

$$4 \times 10^{-7}$$

$$4 \times 10^{-7}$$

$$1 \times 10^{-7}$$

$$1 \times 10^{-7}$$

$$1 \times 10^{-7}$$

$$1 \times 10^{-7}$$

Radiation Processes

$$2.6 \times 10^{-16} \text{ cm}^2$$

$$1.5 \times 10^{-20} \text{ cm}^2$$

$$5.6 \times 10^{-18} \text{ cm}^2$$

$$6 \times 10^{-18} \text{ cm}^2$$

$$1.3 \times 10^{-17} \text{ cm}^2$$

$$1.6 \times 10^{-18} \text{ cm}^2$$

$$6 \times 10^{-18} \text{ cm}^2$$

$$6 \times 10^{-18} \text{ cm}^2$$

APPENDIX B

TIME-DEPENDENT GEOMETRIC OPTICS IN AN AMPLIFYING MEDIUM

I. INTRODUCTION

It has been proposed that, in producing short, high energy pulses from a laser amplifier, angular multiplexing be used to maximize efficient use of the pump energy. In this process the input pulse is broken into segments passing through the gain region sequentially in time and at different angles. The individual segments each have the length of the desired final pulse; by means of relative time delays after they leave the amplifier they are superimposed at the target to provide one short, intense pulse.

The desired length of the final pulse is small compared to the required pumping time, so a single pulse passing through the amplifier would not utilize all the pump energy. If it entered at the beginning of the pump period, all the energy supplied after its passage would be wasted. If it entered late in the pump period, the super-radiance, or amplified spontaneous emission (ASE), would already have grown so intense as to monopolize the available energy. Thus, time sequencing the segments permits the initial one to enter at the onset of pumping and dominate the ASE, and the remaining ones to occupy the rest of the pump period and thereby acquire the rest of the energy. The use of different angles for the various segments allows them to be routed through different paths corresponding to appropriate time delays, so that they may be superimposed at the targets.

An analysis of angular multiplexing must describe the amplification of the input pulse segments, the competing amplified stimulated emission and, possibly, parasitic oscillations, so as to permit a calculation of the fraction of supplied energy going to the pulse, and of the pulse intensity needed to dominate or "clamp" the ASE. It must also describe the wavefront distortion due to refractive index variations, so that the beam quality may be evaluated. In addition it must determine the phase, magnitude and polarization of the propagating electromagnetic field, so that interference effects occurring when the pulse segments are superimposed at the target may be predicted.

In certain circumstances geometric optics constitutes an adequate description of these features. Geometric optics is the short wavelength limit of wave optics as given by Maxwell's equations. It is valid when there are no diffraction effects and no ray crossings, and there are only small fractional variations of the light intensity and refractive index over a wavelength or during an oscillation period of the light wave. Diffraction spreading is known to be ignorable for propagation distances much less than D^2/λ , where D is the transverse dimension of the light wave and λ is its wavelength. For the laser amplifiers under consideration D is about 30 cm and the wavelength is of the order of a micron, so that D^2/λ is nearly 100 km; since the pathlength involving the laser and associated optics is of the order of meters, it is evident that diffraction spreading is insignificant. There is to be no sharp focusing in the amplifier system, so there should be no ray crossings. The dynamics of the gain medium are such that there will be no significant variations in the refractive index over a wavelength, and the magnitude of the gain is less than 1 cm^{-1} , so there will be no significant fractional intensity variations over a wavelength. During pumping the gain will rise from zero to its maximum value over about ten nanoseconds, but the period of a wave oscillation is only slightly more than 10^{-15} seconds, so there will be no significant alterations in refractive index or intensity during such a period. Thus, it appears that geometric optics is appropriate for analysis of an angularly multiplexed laser amplifier.

Although geometric optics is often thought of as being identical to ray optics, in reality it is considerably more extensive. It leads to the ray paths by producing the (approximate) phase functions of the waves, the ray direction at any point being along the normal to the surface of constant phase. Thus, it permits a wave description of light propagation, albeit an approximate one, which does not contain diffraction effects, and becomes invalid at ray crossings or under conditions of very rapid change in refractive index or intensity. In a higher order of approximation than that which yields the phase functions, geometric optics even contains the spatiotemporal behaviors of the intensity and the polarization vector, thus allowing a quite full description of wave phenomena.

In this report the equations of geometric optics will be derived. Standard derivations are for time-independent situations and for media having neither gain nor loss. However, the gain of a laser amplifier is clearly a significant characteristic, and short pulse amplification is a highly time-dependent phenomenon. Therefore, for analysis of pulse amplification, the standard derivation is extended in this report to include both time dependence and gain.

The equations of geometric optics are obtained here from a perturbation expansion of the solution to Maxwell's equations for a quasi-monochromatic, propagating electromagnetic wave. The expansion is in powers of the wavelength, and is more valid at shorter wavelengths. In zero order the perturbation expansion yields a time dependent generalization of the eikonal equation, whose solution provides the ray paths and the phase functions. In first order an equation is obtained which describes the propagation of the electric field vector, exclusive of its oscillatory behavior, the latter being already determined by the phase function and the assumed quasi-monochromaticity. This equation assumes a prior knowledge of the ray paths, which are found from the eikonal equation. It may be resolved into an equivalent pair of equations, namely, a transport equation for the intensity, and a propagation equation for the polarization vector.

The intensity transport equation, being classical, does not account for the appearance of spontaneous emission, which is a quantum mechanical phenomenon. Since amplified spontaneous emission is an important characteristic of high gain lasers and must be accounted for, the geometric optics description is augmented by adding to the intensity equation an ad hoc source term for spontaneous emission.

II. DERIVATION OF GEOMETRIC OPTICS EQUATIONS

The starting point in developing geometric optics is formed by Maxwell's equations for a nonmagnetic medium ($\mu = 1$) with no free charge or current,

$$\nabla \times \vec{H} = \frac{1}{c} \frac{\partial \vec{D}}{\partial t}, \quad (1a)$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \quad (1b)$$

$$\nabla \cdot \vec{D} = 0, \quad (1c)$$

$$\nabla \cdot \vec{H} = 0, \quad (1d)$$

$$\vec{D} = \epsilon_c \vec{E}. \quad (1e)$$

The dielectric constant ϵ_c is complex, implying that the medium may be either attenuating or amplifying.

The $\vec{E}$ and $\vec{H}$ fields are assumed to have a form appropriate to an electromagnetic travelling wave, namely,

$$\vec{E}(\vec{r}, t) = \vec{e}(\vec{r}, t) e^{ikS(\vec{r}, t)} e^{-i\omega t}, \quad (2a)$$

$$\vec{H}(\vec{r}, t) = \vec{h}(\vec{r}, t) e^{ikS(\vec{r}, t)} e^{-i\omega t}. \quad (2b)$$

This form is actually completely general, but certain assumptions are implied that will become explicit and will be utilized in the development that follows.

The fields are assumed to be nearly monochromatic with frequency ω , implying that there are no hidden factors of the form $e^{i\omega t}$ in the terms $\vec{e} e^{ikS}$ or $\vec{h} e^{ikS}$. The vector amplitudes $\vec{e}(\vec{r}, t)$ and $\vec{h}(\vec{r}, t)$, and the phase factor $\exp [ikS(\vec{r}, t)]$ all vary with time for two reasons. First, the input fields are time dependent, and second, the dielectric constant $\epsilon_c(\vec{r}, t)$ changes with time, causing a temporal variation in the histories of the waves arriving at any

point, and therefore of their phases and amplitudes. However, these variations are assumed to be adiabatically slow compared to the basic oscillatory behavior $e^{-i\omega t}$.

The first mathematical statement of any of the above assumptions that will be used is

$$\left| \frac{\partial}{\partial t} k S(\vec{r}, t) \right| \ll \omega ,$$

or since the frequency and wave number in a dilute medium such as a gas are related approximately by $\omega = kc$,

$$\left| \frac{1}{c} \frac{\partial S}{\partial t} (\vec{r}, t) \right| \ll 1 . \quad (3)$$

Equation (3) states that in the phase factor $e^{i(kS - \omega t)}$ the principal time dependence is provided by $e^{-i\omega t}$, that is, that the wave is quasi-monochromatic.

The expressions (2) for the $\vec{E}$ and $\vec{H}$ fields may be inserted into Maxwell's equations (1). After some vector algebra utilizing the identities

$$\nabla \times (\phi \vec{A}) = \nabla \phi \times \vec{A} + \phi \nabla \times \vec{A} ,$$

and

$$\nabla \cdot (\phi \vec{A}) = \phi \nabla \cdot \vec{A} + \nabla \phi \cdot \vec{A} ,$$

the following equivalent set of equations may be obtained,

$$\left. \begin{aligned} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \epsilon_c \vec{e} + \nabla S \times \vec{h} &= \frac{1}{ik} \left[\frac{1}{c} \frac{\partial (\epsilon_c \vec{e})}{\partial t} - \nabla \times \vec{h} \right] , \\ \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \vec{h} - \nabla S \times \vec{e} &= \frac{1}{ik} \left[\frac{1}{c} \frac{\partial \vec{h}}{\partial t} + \nabla \times \vec{e} \right] , \\ \epsilon_c \vec{e} \cdot \nabla S &= - \frac{1}{ik} \left(\epsilon_c \nabla \cdot \vec{e} + \vec{e} \cdot \nabla \epsilon_c \right) , \\ \vec{h} \cdot \nabla S &= - \frac{1}{ik} \nabla \cdot \vec{h} \end{aligned} \right\} \quad (4)$$

Since the wave number k is related to the oscillation period T by

$$\frac{1}{kc} = \frac{1}{\omega} = \frac{T}{2\pi},$$

and to the wavelength λ by

$$\frac{1}{k} = \frac{\lambda}{2\pi},$$

the magnitudes of the terms on the right hand side of Eq (5) are of the order of either the changes in $\vec{e}$, $\vec{h}$, or ϵ_c during a time interval $T/2\pi$, or the changes in the same quantities in the distance λ .

The magnitude of ϵ_c is of the order of unity for a gas. The magnitude of ∇S is also of the order of unity, as can be seen by considering that the propagating fields may be roughly approximated by plane waves, whose principal spatial dependence is of the form $e^{ikS} = e^{i\vec{k} \cdot \vec{r}}$. Thus, for propagation along the z axis the phase function may be approximated roughly by $S = z$, so that in this approximation its gradient is $\nabla S = \partial z / \partial z = 1$. Therefore, the magnitudes of terms on the left hand side are of the order of $|\vec{e}|$ and $|\vec{h}|$, themselves. It follows that, if the fractional changes in $\vec{e}$, $\vec{h}$, and ϵ_c during a period T or in a distance λ are small compared to unity, then to a first approximation the terms on the right hand side of (5) may be neglected. If the dielectric constant ϵ_c is purely real, ignoring these terms leads immediately to a time dependent generalization of the eikonal equation, whose solution yields the ray trajectories. However, if the dielectric constant is complex, as it is in the amplifier gain medium somewhat more care must be taken. In order to handle the imaginary part of ϵ_c properly, and also to obtain the next order of approximation beyond the eikonal equation in a systematic

fashion, Eqs (5) will be solved by a perturbation treatment in which the vector amplitudes of the fields are expanded in powers of $(ik)^{-1}$,

$$\left. \begin{aligned} \vec{e} &= \sum_{n=0}^{\infty} \frac{e_n}{(ik)^n} \\ \vec{h} &= \sum_{n=0}^{\infty} \frac{\vec{h}_n}{(ik)^n} \end{aligned} \right\} \quad (5)$$

Before proceeding with the formal perturbation analysis it is necessary to examine the structure of the complex dielectric constant,

$$\epsilon_c = \epsilon + i\epsilon' \quad (6)$$

A plane wave propagating through the medium, whose complex refractive index is written as

$$n_c = n + i\kappa \quad (7)$$

has over a sufficiently small distance of travel z , the spatial dependence

$$e^{in_c kz} = e^{inkz} e^{-\kappa kz} \quad (8)$$

In a gas, the real part n of the refractive index is of the order of unity. The imaginary part κ is, according to (9), related to the intensity attenuation coefficient α by

$$2\kappa k = \alpha \quad ,$$

or

$$\kappa = \alpha/2k \quad (9)$$

Here α will be regarded as a generalized attenuation coefficient, meaning that if it is positive it characterizes a lossy medium, while if it is negative it characterizes an amplifying medium; in the latter case it is related to the gain g by

$$\alpha = -g \quad (10)$$

In this treatment consideration will be confined to media in which the gain or attenuation per wavelength is very small compared to unity. Specifically, in the laser amplifier to be studied the gain coefficient g is less than 1 cm^{-1} . Therefore, from (10) and (11) there follows the inequality

$$|\kappa| = \frac{g}{2k} = \frac{g\lambda}{4\pi} \ll 1 \quad (11)$$

The complex dielectric constant ϵ_c is related to the complex refractive index n_c by

$$\epsilon_c = n_c^2 ,$$

or

$$\epsilon + i\epsilon' = n^2 - \kappa^2 + 2in\kappa \quad (12)$$

Because of the orders of magnitude, $n \sim 1$ and $|\kappa| \ll 1$, the real part of the dielectric constant is, thus, given by

$$\epsilon = n^2 \quad (13)$$

to very good approximation. The imaginary part is

$$\epsilon' = 2n\kappa , \quad (14)$$

or, by virtue of (9).

$$\epsilon' = \frac{n\alpha}{k} \quad (14)$$

The complex dielectric constant, therefore, has the form

$$\epsilon_c = \epsilon + i\beta/k, \quad (15a)$$

where

$$\epsilon = n^2, \quad (15b)$$

and

$$\beta = n\alpha. \quad (15c)$$

The key point, now, is that the real part ϵ is of close to unity magnitude and is therefore of zero order in the perturbation coefficient $(ik)^{-1}$, while the imaginary part β/k is of first order.

The zero-order equations arising from the perturbation expansion of Equations (4) may now be obtained by inserting (5) and (15), and retaining only those terms proportional to the zeroth power of $(ik)^{-1}$. The result is

$$\left. \begin{aligned} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \epsilon \vec{e}_0 + \nabla S \times \vec{h}_0 &= 0 \\ \left(1 - \frac{1}{c} \frac{S}{t} \right) \vec{h}_0 - \nabla S \times \vec{e}_0 &= 0 \\ \vec{e}_0 \cdot \nabla S &= 0 \\ \vec{h}_0 \cdot \nabla S &= 0 \end{aligned} \right\} \quad (16)$$

Eliminating $\vec{h}_0$ between the first two equations, utilizing the vector identity

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} ,$$

and then introducing the third equation, one finds

$$\left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)^2 \epsilon - (\nabla S)^2 \right] \vec{e}_0 = 0 . \quad (17)$$

Since $\vec{e}_0$ is in general not zero, Eq (17) implies

$$(\nabla S)^2 = \epsilon \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)^2 , \quad (18a)$$

or, in view of (15b)

$$(\nabla S)^2 = n^2 \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)^2 . \quad (18b)$$

Eq (18) is the time dependent generalization of the eikonal equation. To case it in a more transparent form we define the unit vector

$$\hat{a} = \frac{\nabla S}{n \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)} . \quad (19)$$

Since S , as defined by (2), is the phase function of the $\vec{E}$ and $\vec{H}$ fields, surfaces of constant S are surfaces of constant phase of the propagating waves. The vector $\hat{a}$ is then normal to the surface of constant phase, and is, therefore, a definition parallel to the ray path. It is of unit length by virtue of (18b).

Next we define the differential path length $\partial\sigma$ along the ray, so that

$$\partial/\partial a = \hat{a} \cdot \nabla \quad (20)$$

is the derivative along the ray path. Now, taking the gradient of both sides of (18),

$$(\nabla S \cdot \nabla) \nabla S = n \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \nabla n - \frac{n}{c} \frac{\partial \nabla S}{\partial t} \right],$$

or, using (19) to eliminate ∇S , expanding the derivatives and reorganizing the terms,

$$\begin{aligned} \left(\frac{n}{c} \frac{\partial}{\partial t} + \hat{a} \cdot \nabla \right) \hat{a} &= \frac{\nabla n}{n} - \frac{a}{n} \left(\frac{n}{c} \frac{\partial}{\partial t} + \hat{a} \cdot \nabla \right) n \\ &+ \frac{\hat{a}}{c} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)^{-1} \left(\frac{n}{c} \frac{\partial}{\partial t} + \hat{a} \cdot \nabla \right) \frac{\partial S}{\partial t}. \end{aligned} \quad (21)$$

We then again use (19) to find

$$\begin{aligned} (\hat{a} \cdot \nabla) \frac{\partial S}{\partial t} &= \hat{a} \cdot \frac{\partial}{\partial t} \nabla S \\ &= \frac{\partial n}{\partial t} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) - \frac{n}{c} \frac{\partial^2 S}{\partial t^2} + n \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \hat{a} \cdot \frac{\partial \hat{a}}{\partial t}. \end{aligned} \quad (22)$$

The last term in (22) vanishes because $\hat{a}$, being a unit vector, has constant magnitude:

$$\hat{a} \cdot \frac{\partial \hat{a}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} |\hat{a}|^2 = 0.$$

When (22) is inserted into (21) and (20) is used to replace $\hat{a} \cdot \nabla$, the result is

$$\left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \hat{a} = \frac{\nabla_{\perp} n}{n}, \quad (23a)$$

where

$$\nabla_{\perp} = \nabla - \hat{a}(\hat{a} \cdot \nabla) \quad (23b)$$

is the component of the gradient normal to $\hat{a}$.

Eq (23) is an alternative form of the time dependent eikonal equation; it shows clearly that the rays bend in the direction of increasing refractive index. It also shows that the unit vector $\hat{a}$ propagates along the ray path with the local speed of light c/n , keeping pace with the wavefront.

The spatiotemporal development of the functions is found by rearranging (19) into

$$\nabla S = n(1 - \frac{1}{c} \frac{\partial S}{\partial t}) \hat{a},$$

and then taking the dot product with $\hat{a}$ and using (19) to obtain

$$\left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) S = n \quad (24)$$

To illustrate the behavior of S , we note that the solution to (24) for constant n is

$$S(\sigma, t) = S_0(t - n\sigma/c) + n\sigma, \quad (25)$$

where $S_0(t)$ is the value of $S(\sigma, t)$ at $\sigma = 0$. We see that, in a medium with constant refractive index points on a surface of constant S propagate along the ray paths with speed c/n , and the value of S increases linearly with distance σ .

In a time-independent situation, where the ray trajectories remain fixed, Eq (24) reduces to

$$\frac{\partial S}{\partial \sigma} = n; \quad (26)$$

this has the solution

$$S(\sigma) = S(\sigma_0) + \int_{\sigma_0}^{\sigma} n(\sigma') d\sigma', \quad (27)$$

where the integral is along the ray path. Since $k\Delta S$ is the phase difference between two points along a ray path, $\Delta S(\sigma) = S(\sigma) - S(\sigma_0)$ as given by (27) is defined to be the optical path difference between the two points.

The first order equations in the perturbation expansion of (4) are obtained by inserting (7) and (15), and retaining only the terms proportional to the first power of $(ik)^{-1}$. The result is

$$\left. \begin{aligned} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) (\epsilon \vec{e}_1 - \beta \vec{e}_0) + \nabla S \times \vec{h}_1 &= \frac{1}{c} \frac{\partial(\epsilon \vec{e}_0)}{\partial t} - \nabla \times \vec{h}_0 \\ \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \vec{h}_1 - \nabla S \times \vec{e}_1 &= \frac{1}{c} \frac{\partial \vec{h}_0}{\partial t} + \nabla \times \vec{e}_0 \\ \epsilon \vec{e}_1 \cdot \nabla S - \beta \vec{e}_0 \cdot \nabla S &= -\epsilon \nabla \cdot \vec{e}_0 - \vec{e}_0 \cdot \nabla \epsilon \\ \vec{h}_1 \cdot \nabla S &= -\nabla \cdot \vec{h}_0 \end{aligned} \right\} \quad (28)$$

As in the case of the zero order equations we proceed by eliminating the $\vec{H}$ field vector amplitudes. The vector algebra is complicated and will not be reproduced here in the text; it is outlined in the addendum. It is found that $\vec{e}_1$, the first order electric field vector amplitude, can be eliminated if the eikonal equation (18) is satisfied, leaving an equation for $\vec{e}_0$ alone,

$$\begin{aligned} &(\vec{e}_0 \cdot \nabla \ln \epsilon) \nabla S + \left[2 \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \frac{\epsilon}{c} \frac{\partial}{\partial t} + 2 (\nabla S \cdot \nabla) \right. \\ &+ \left. \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^2 \beta + \nabla^2 S - \frac{\epsilon}{c^2} \frac{\partial^2 S}{\partial t^2} \right. \\ &+ \left. 2 \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \frac{1}{c} \frac{\partial \epsilon}{\partial t} \right] \vec{e}_0 = 0. \end{aligned} \quad (29)$$

In (29) it is assumed that $\nabla S = n\hat{a}$ is already known from the solution of (23).

Equation (29) describes the spatiotemporal behavior of the electric field vector amplitude $\vec{e}_0$. From it one can derive two independent equations which provide the same total information. The first is a transport equation for the intensity I , which is related to $\vec{e}_0$ by

$$I = \frac{nc}{8\pi} |\vec{e}_0|^2 \quad (30)$$

The second is a transport equation for the unit polarization vector defined by

$$\hat{u} = \vec{e}_0 / |\vec{e}_0| \quad (30a)$$

To obtain the intensity equation we take the dot product of $\vec{e}_0^*$ with Eq (29) and add to the resulting equation its complex conjugate. After substituting for $\vec{e}_0^2$ from (30), and simplifying the means of (19) and (20), we obtain

$$\left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \frac{I}{n} = - \left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \frac{\beta}{n} + \frac{2}{c} \frac{\partial n}{\partial t} + \frac{1}{n} \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right)^{-1} \left(\nabla^2 S - \frac{n^2}{c^2} \frac{\partial^2 S}{\partial t^2} \right) \right] \frac{I}{n} = 0 \quad (31)$$

We then further simplify by finding an alternative form for the factor $\nabla^2 S - (n^2/c^2) \partial^2 S / \partial t^2$. First, we rewrite (19) as

$$\nabla S = n \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \hat{a} \quad (32)$$

Next, we take the gradient of (32):

$$\nabla^2 S = \left(1 - \frac{1}{c} \frac{\partial S}{\partial t} \right) \nabla \cdot (n\hat{a}) - \frac{n}{c} \hat{a} \cdot \frac{\partial \nabla S}{\partial t} \quad (33)$$

Then, substituting the expression (22) for $\hat{a} \cdot \partial \nabla S / \partial t$, we find

$$\nabla^2 S - \frac{n^2}{c^2} \frac{\partial^2 S}{\partial t^2} = \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \left[\nabla \cdot (n \hat{a}) - \frac{n \partial n}{c \partial t} \right] \quad (34)$$

Finally, combining (34) with (31), we obtain the intensity transport equation,

$$\left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \frac{I}{n} = - \left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \frac{\beta}{n} + \frac{1}{c} \frac{\partial n}{\partial t} + \frac{1}{n} \nabla \cdot (n \hat{a}) \right] \frac{I}{n} \quad (35)$$

We may now invoke the inequality (3) by dropping the term $(1/c) \partial S / \partial t$ relative to unity.

In a time-independent situation with no gain or loss ($\beta = 0$) Eq (35) reduces to

$$\frac{\partial}{\partial \sigma} \left(\frac{I}{n} \right) = - \frac{1}{n} \nabla \cdot (n \hat{a}) \quad (36)$$

Eq (36) is derived by Born and Wolf, who show that the term $(1/n) \nabla \cdot (n \hat{a})$ accounts for intensity variations due to ray convergence or divergence.

To determine the physical significance of the β/n term we consider a time independent situation with uniform refractive index n . From (23) it follows that ray directions remain unchanged, that is, ray paths are linear. Since $\hat{a}$ is a unit vector, not only its direction, but also its length is invariant, so that, according to (31) we have $(1/n) \nabla \cdot (n \hat{a}) = 0$. Then for propagation along the z -axis (35) reduces to

$$\frac{d}{dz} \left(\frac{I}{n} \right) = - \frac{\beta}{n} \left(\frac{I}{n} \right) \quad (37)$$

Evidently β/n is the generalized attenuation coefficient,

$$\alpha = \beta/n \quad . \quad (38)$$

This is the proper justification of (15c), which was obtained in a very heuristic fashion.

The term $(1/c) \partial n / \partial t$ also describes a generalized attenuation. To see that a time varying dielectric constant in general leads to attenuation or gain we consider the first Maxwell equation in its complete form, including a real current density,

$$\nabla \times \vec{H} = \frac{4\pi}{c} \vec{j} + \frac{1}{c} \frac{\partial \vec{D}}{\partial t} \quad . \quad (39)$$

Writing $\vec{D} = \epsilon \vec{E}$ and $\vec{j} = \sigma_c \vec{E}$, where σ_c is the conductivity, we obtain

$$\nabla \times \vec{H} = \frac{1}{c} \left(4\pi\sigma_c + \frac{\partial \epsilon}{\partial t} \right) \vec{E} + \frac{\epsilon}{c} \frac{\partial \vec{E}}{\partial t} \quad (40)$$

The term $\partial \epsilon / \partial t$ appears in exactly the same form as the conductivity term $4\pi\sigma_c$. Therefore, since the conductivity leads to an attenuation of the electric field, the time varying dielectric constant must also.

The equation for the unit polarization vector $\hat{u}$ can be obtained by substituting $\vec{E}_0 = |\vec{E}_0| \hat{u}$ into (29). Assuming that the intensity equation (35) is satisfied leads to the dropping out of a number of terms with the result

$$\left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \hat{u} = - (\hat{u} \cdot \nabla \ln n) \hat{a} \quad . \quad (41)$$

Equations (23), (24), (35), and (41) provide the geometric optics description of time dependent light propagation in an amplifying medium. They are not, however, adequate for analysing the laser amplifier, because Eq (35) does not contain a description of spontaneous emission. The reason spontaneous emission is absent from this picture is that it is a quantum mechanical phenomenon, and we have started with Maxwell's equations, which

are strictly classical. To include spontaneous emission without resorting to a quantum mechanical analysis we must introduce an appropriate ad hoc term in (35).

To do so we first recall that the quantity I represents the intensity at any point $\vec{r}$ and time t propagating along a ray path having the direction $\hat{a}$, so that $I = I(\vec{r}, \hat{a}, t)$ is a function of the variables $\vec{r}$, $\hat{a}$ and t . As any infinitesimal amount of radiative energy progresses with time along a ray path, both its position $\vec{r}$ and its direction $\hat{a}$ vary accordingly. Thus, both $\vec{r}$ and $\hat{a}$ are implicit functions of time. With this in mind we demonstrate as follows that the left hand side of (35) is the product of n/c and the total time derivative of I/N . We first expand $d(I/N)/dt$ according to the chain rule,

$$\frac{d}{dt} \left(\frac{I}{n} \right) = \frac{\partial}{\partial t} \left(\frac{I}{n} \right) + \nabla \left(\frac{I}{n} \right) \cdot \frac{d\vec{r}}{dt} + \frac{\partial}{\partial \hat{a}} \left(\frac{I}{n} \right) \cdot \frac{d\hat{a}}{dt} \quad (42)$$

The quantity $d\vec{r}/dt$ has the magnitude c/n of the speed of light in the medium, and the direction $\hat{a}$ of the ray vector,

$$\frac{d\vec{r}}{dt} = \frac{c}{n} \hat{a} \quad (43)$$

The last term on the right hand side of (42) vanishes. To see this we note that, since $\hat{a}$ is a unit vector, it satisfies

$$\hat{a} \cdot \hat{a} = 1 \quad , \quad (44)$$

and the time derivative of Eq (44) is

$$\hat{a} \cdot \frac{d\hat{a}}{dt} = 0 \quad , \quad (45)$$

which states that $\hat{d}\mathbf{a}/dt$ is orthogonal to $\hat{\mathbf{a}}$. However, $\partial(I/N)/\partial\hat{\mathbf{a}}$ is parallel to $\hat{\mathbf{a}}$, so the dot product vanishes,

$$\frac{\partial}{\partial \hat{\mathbf{a}}} \left(\frac{I}{n} \right) \cdot \frac{d\hat{\mathbf{a}}}{dt} = 0 \quad (46)$$

Inserting (43) and (46) into (42) we then obtain

$$\frac{d}{dt} \left(\frac{I}{n} \right) = \frac{\partial}{\partial t} \left(\frac{I}{n} \right) + \frac{c}{n} \hat{\mathbf{a}} \cdot \nabla \left(\frac{I}{n} \right),$$

which, when combined with (20), gives

$$\frac{n}{c} \frac{d}{dt} \left(\frac{I}{n} \right) = \left(\frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \frac{I}{n} \quad (47)$$

This quantity is the rate of change of I/n per unit distance travelled, as the radiation traverses the medium with speed c/n .

The right hand side of (35), as discussed earlier, is the sum of sources of growth or attenuation of I/n . We wish to introduce an additional source term to account for growth due to spontaneous emission. The energy density spontaneously radiated per unit time is $Nh\nu/\tau$, where N is the number density of excited radiating molecules, h is Planck's constant, ν is the frequency of the spectral line under consideration, and τ is the lifetime for spontaneous emission. Although each molecule has a radiation pattern depending on its orientation, on the average they are randomly oriented, so the spontaneously radiated energy density per unit time per unit solid angle in any direction $\hat{\mathbf{a}}$ is $Nh\nu/4\pi\tau$. The spontaneously radiated intensity per unit time per unit solid angle is then the above quantity multiplied by the light speed c/n , that is, $(Nh\nu/4\pi\tau) (c/n)$.

The quantity $I(\vec{r}, \hat{a}, t)$ may be regarded as intensity per unit solid angle in the direction $\hat{a}$, so the contribution to $(n/c) d(I/n)/dt$ from spontaneous emission is

$$\begin{aligned} \frac{n}{c} \frac{d}{dt} \left(\frac{I}{n} \right) \Big|_{sp} &= \left(\frac{n}{c} \right) \left(\frac{Nh\nu}{4\pi\tau} \right) \left(\frac{c}{n} \right) \frac{1}{n} \\ &= \frac{Nh\nu}{4\pi\tau n} \end{aligned}$$

This is the source term that must be added to the right hand side of (35), yielding, finally,

$$\frac{n}{c} \left(\frac{\partial}{\partial t} + \frac{\partial}{\partial \sigma} \right) \frac{I}{n} = \left[g - \frac{1}{n} \nabla \cdot (na) - \frac{1}{c} \frac{\partial n}{\partial t} \right] \frac{I}{n} + \frac{Nh\nu}{4\pi\tau n}, \quad (48)$$

where we have neglected $(1/c) \partial S/\partial t$ relative to unity and have used (38) and (10) to write β in terms of the gain g .

Equations (23), (41) and (48) now constitute the augmented geometric optics description of the laser amplifier. In general the term $(1/c) (\partial n/\partial t)$ is small enough to be ignored, and in analyzing the amplification of the pulse the spontaneous emission term can be neglected. However, the latter term is crucial to the amplified spontaneous emission (ASE), and must be retained in analyzing it.

ADDENDUM

In this appendix we start with the first order perturbation expansion of Maxwell's equations, Equations (28), and from them derive Eq (29), the determining equation for the electric field vector amplitude $\vec{e}_0$.

First, eliminating h_1 between the first two of Eqs (28) gives

$$\begin{aligned} & \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) (\epsilon \vec{e}_1 - \beta \vec{e}_0) \\ & + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \nabla S \times \left(\nabla S \times \vec{e}_1 + \frac{1}{c} \frac{\partial h}{\partial t} + \nabla \times \vec{e}_0\right) \\ & = \frac{1}{c} \frac{\partial (\epsilon \vec{e}_0)}{\partial t} - \nabla \times \vec{h}_0 \end{aligned} \quad (A1)$$

Then, using the vector identity,

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}, \quad (A2)$$

we note

$$\begin{aligned} & \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \epsilon \vec{e}_1 + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \nabla S \times (\nabla S \times \vec{e}_1) \\ & = \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \left\{ \left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^2 \epsilon - (\nabla S)^2\right] \vec{e}_1 \right. \\ & \quad \left. + (\nabla S \cdot \vec{e}_1) \nabla S \right\}. \end{aligned} \quad (A3)$$

The eikonal equation (18) allows (A3) to be reduced to

$$\begin{aligned} & \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \epsilon \vec{e}_1 + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \nabla S \times (\nabla S \times \vec{e}_1) \\ &= \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} (\nabla S \cdot \vec{e}_1) \nabla S \end{aligned} \quad (A4)$$

The factor $\nabla S \cdot \vec{e}_1$ is then obtained from the third of Eqs (28), using the third of Eqs (16) to eliminate the term $\beta \vec{e}_0 \cdot \nabla S$; inserting the resulting expression for $\nabla S \cdot \vec{e}_1$ into (A4), then inserting (A4) into (A1), we obtain

$$\begin{aligned} & - \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} (\nabla \cdot \vec{e}_0 + \vec{e}_0 \cdot \nabla \ln \epsilon) \nabla S - \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \beta \vec{e}_0 \\ & + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \nabla S \times \left(\frac{1}{c} \frac{\partial h_0}{\partial t} + \nabla \times \vec{e}_0\right) \\ & = \frac{1}{c} \frac{\partial (\epsilon \vec{e}_0)}{\partial t} - \nabla \times \vec{h}_0 \end{aligned} \quad (A5)$$

Next we eliminate $\vec{h}_0$ from (A5) using the expression

$$\vec{h}_0 = \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) (\nabla S \times \vec{e}_0)$$

obtained from (16b). Then we use the vector identity

$$\nabla \times (\phi \vec{A}) = \phi \nabla \times \vec{A} - \vec{A} \times \nabla \phi$$

to expand the resulting expression, and we rearrange terms, eventually obtaining

$$\vec{F} + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^2 \beta \vec{e}_0 + \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \frac{1}{c} \frac{\partial (\epsilon \vec{e}_0)}{\partial t} - \vec{G} = 0, \quad (A6a)$$

where

$$\vec{F} = (\nabla \cdot \vec{e}_0 + \vec{e}_0 \cdot \nabla \ln \epsilon) \nabla S - \nabla S \times (\nabla \times \vec{e}_0) - \nabla \times (\nabla S \times \vec{e}_0) \quad (A6b)$$

and

$$\begin{aligned} \vec{G} = & \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \frac{1}{c} \frac{\partial \nabla S}{\partial t} \times (\nabla S \times \vec{e}_0) \\ & + \nabla S \times \frac{1}{c} \frac{\partial}{\partial t} \left[\left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right)^{-1} \nabla S \times \vec{e}_0 \right] \end{aligned} \quad (A6c)$$

Using the vector identity

$$\begin{aligned} \nabla S \times (\nabla \times \vec{e}_0) + \nabla \times (\nabla S \times \vec{e}_0) &= \nabla (\nabla S \cdot \vec{e}_0) \\ &+ (\nabla \cdot \vec{e}_0) \nabla S - 2(\nabla S \cdot \nabla) \vec{e}_0 - (\nabla^2 S) \vec{e}_0 \end{aligned}$$

and (16c), we convert $\vec{F}$ to

$$\vec{F} = (\vec{e}_0 \cdot \nabla \ln \epsilon) \nabla S + 2(\nabla S \cdot \nabla) \vec{e}_0 + (\nabla^2 S) \vec{e}_0 \quad (A7)$$

Then we observe that the two terms in $\vec{G}$ may be combined into

$$\vec{G} = \frac{\partial}{\partial t} \left[\frac{\nabla S \times (\nabla S \times \vec{e}_0)}{1 - \frac{1}{c} \frac{\partial S}{\partial t}} \right] \quad (A8)$$

However, the vector identity (A2), combined with (16c) and the eikonal equation (18), permits the transformation

$$\nabla S \times (\nabla S \times \vec{e}_0) = - \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \epsilon \vec{e}_0 \quad ,$$

which, when inserted into (A8), leads to

$$\vec{G} = \left(1 - \frac{1}{c} \frac{\partial S}{\partial t}\right) \frac{\partial(\epsilon \vec{e}_o)}{\partial t} + \frac{\epsilon}{c} \frac{\partial^2 S}{\partial t^2} \vec{e}_o \quad (A9)$$

Combining (A7) and (A9) with (A6) now leads directly to Eq (29).

DISTRIBUTION LIST

Joyce Lewis (6 copies)
Advanced Lasers Program Manager
Office of Laser Fusion, C-404
U.S. Department of Energy
Washington, DC 20545

Dr. L. D. Pleasance
Lawrence Livermore Laboratory
Mail Stop L-470
Post Office Box 5508
Livermore, CA 94550

Dr. James B. Gerardo
Sandia Laboratories
Post Office Box 5800
Albuquerque, NM 87115

Dr. Roger B. Perkins
L. Division Leader
Los Alamos Scientific Laboratory
Post Office Box 1663
Los Alamos, NM 87545

Dr. Joel Parks
AVCO Everett Research Laboratory
2385 Revere Beach Parkway
Everett, MA 02149

Dr. Robert Center
Mathematical Sciences Northwest, Inc.
Post Office Box 1887
Bellevue, WA 98009

Dr. Nicholas I. Djeu
Code 5540
Naval Research Laboratory
Washington, DC 20375

Dr. Paul L. Kelley
Massachusetts Institute of
Technology
Lincoln Laboratory
Lexington, MA 02173

Dr. Morris Krauss
National Bureau of Standards
U.S. Department of Commerce
Washington, DC 20234

Dr. Evan Chicklis
T. M. Sanders Associates, Inc.
Defensive Systems Division
95 Canal Street
Nashua, NH 03060

Dr. Ray Taylor
Physical Sciences, Inc.
30 Commerce Way
Woburn, MA 01801

Dr. Eddy Stappaerts
Northrop Research & Technology
Center
One Research Park
Palos Verdes Peninsula, CA 90274

Mr. R. B. Purcell
Contract Management Division
U.S. Department of Energy
Nevada Operations Office
Post Office Box 14100
Las Vegas, NV 89114