LASER/PLASMA THEORY FOR MICROWAVE MODELING EXPERIMENTS

FINAL REPORT

J.J. THOMSON

W.F. DIVERGILIO

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During the last year, we have carried out theoretical investigations of microwave-plasma interactions in support of both the UCLA program, headed by N. Luhmann, and the TRW program, headed by W.F. DiVergilio. The UCLA program concentrated on experimental studies of Stimulated Brillouin Scattering (SBS). We derived a theory which successfully explained the basic features of their experiment. This collaboration resulted in a series of three technical papers submitted for publication,^{1,2,3} and several conference presentations of the results.

The TRW program was originally conceived of as an investigation of electron heating and thermal transport; however, in response to interest from our customers, Lawrence Livermore Laboratory, the subject was changed to the interaction of SBS and self focusing. The experimental program has not yet started; however, we have developed a theoretical description of the expected interaction.

We discuss in the following the various tasks performed under this contract.

TASK A: INTERPRETATION OF UCLA EXPERIMENTS ON BRILLOUIN SCATTERING

The aim of the UCLA experiments was to perform the first unequivocal microwave experiments on SBS, verifying growth rates, investigating saturation levels, and checking the damping effect of finite bandwidth. The experiments were performed in an unmagnetized plasma with dimensions .75 m diameter and 2 m length produced by a multifilament discharge with surface multipole confinement (Figure 1). Typical operating parameters were: gas filling pressure $1-3 \times 10^{-4}$ T, electron density $n_e = 10^{10}-10^{11} \text{ cm}^{-3}$, electron temperature $T_e \approx 10-12$. Various gasses were used, including helium and argon. The microwave pump frequency was 5.3-16 GHz, power $P \leq 1$ MW and pulse width $\tau = .1-20$ usec. The wave was launched along the chamber axis by a high gain gridded horn. The plasma density was maintained at about .1 critical. The effective interaction length, set by the horn pattern was about 100 cm. Although efforts were made to reduce the chamber reflectivity as much as possible, about 1% residual reflectivity remained. The backscattered waves were fed into a spectrum analyzer, so the frequency shift and magnitude of the SBS scattered wave could be measured. The associated ion waves were measured using movable Langmuir probes, just after cessation of the incident RF.

The initial experiments showed a very fast rise of a red shifted component and an apparent saturation at $\tau \approx \omega_s^{-1}$. Measurements of the ion density fluctuations showed the expected k-matching, $k = 2 k_0$. The frequency shift of the backscattered light was $\Delta\omega \approx \omega_s$. The original interpretation by the UCLA group was that this was in fact SBS, with the fast temporal growthrate given by Forslund, et.al.⁴ However, we pointed out that a further discussion in Forslund, et.al, shows that after a time sufficiently long that a scattered wave has time to traverse the system (on the order of nanoseconds), the relevant growth rate is the spatial growthrate, less than the temporal growthrate by the small factor $(c_s/c)^{1/2}$:

$$\begin{aligned} \gamma_s &= 2 \sqrt{c_s/c} \gamma_t \\ &= \omega_0 \frac{c_s}{c} \left(n/n_c v_o^2/v_e^2 \right)^{1/2} \end{aligned} \tag{1}$$

FIGURE 1. SCHEMATIC DIAGRAM OF EXPERIMENTAL APPARATUS

where V_o is the oscillatory velocity, V_e is the electron thermal velocity, c_s is the sound speed, ω_o is the microwave frequency. We suggested that the small amount of chamber reflectivity set up a standing ion wave due to the ponderomotive force. The component of the standing wave which was backward traveling at a speed c_s was responsible for the initial large reflectivity. We suggested that looking at a longer time scale would show the more slowly growing SBS mode. This was undertaken experimentally, and the long time SBS growth rate was found to agree well with theoretical predictions.

In the next section, we discuss our theoretical model for the ponderomotive force driven initiation of SBS. Then the UCLA results are compared, showing excellent agreement. The third section discusses saturation of SBS and comparison with experiment. Finally, we discuss the effects of finite bandwidth.

A.1 Ponderomotive Force Driven Density Fluctuations and Their Effect on SBS.

When a plane electromagnetic wave propagating through an underdense plasma is partially reflected from a boundary, a partially standing wave is set up, which then causes ion density striations due to the ponderomotive force. It is perhaps surprising that even the small ($r \sim 1\%$) reflectivity in the UCLA experiment can give rise to significant ripples in the standing wave. To see this, consider the ratio of the maximum power in the standing wave:

$$\frac{(E_B + E_R)^2}{(E_o - E_R)^2} = \frac{(1 + r^{1/2})^2}{(1 - r^{1/2})^2} = 1.49 \quad (2)$$

Thus, just 1% reflectivity causes almost a 50% power ripple.

To see the basic effect, we first show a simplified analysis of the ponderomotive-driven ion waves. The incident and reflected wave field is expressed as:

$$\begin{aligned} E_{in} &= E_o \sin(\omega t - k_o x) \\ E_r &= r^{1/2} E_o \sin(\omega t + k_o x) \end{aligned} \quad (3)$$

The standing wave pattern is set up in nanoseconds, much shorter than the ion response time. The ion acoustic wave equation can then be written

$$\frac{\partial^2 n}{\partial t^2} - c_s^2 \frac{\partial^2 n}{\partial x^2} = k_0^2 c_s^2 n_0 r^{1/2} \frac{v_o^2}{v_e^2} \cos 2 k_0 x \quad (4)$$

with the initial conditions

$$n(x, 0) = 0$$

$$\frac{\partial n}{\partial t}(x, 0) = 0$$

The solution is

$$\frac{n(x, t)}{n_0} = r^{1/2} \frac{v_{os}^2}{v_e^2} (1 - \cos \omega_s t) \cos 2 k_0 x. \quad (5)$$

Note that there are three components in Equation 5: a zero-frequency wave, a left-traveling wave and a right traveling wave. The right traveling wave (away from the microwave horn) will scatter waves in the manner of a grid (Bragg scattering) which are red shifted by ω_s . The left traveling wave will give a blue shifted component. The backscattered wave satisfies the wave equation:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - c^2 \frac{\partial^2}{\partial x^2} \right) E_s = \omega_{pe}^2 \frac{n}{2n_0} E_{in} \quad (6)$$

Letting

$$E_s = A_s \sin [(\omega - \omega_s) t + k_0 x]$$

we find

$$2\omega_o c \frac{\partial}{\partial x} A_s = - \omega_{pe}^2 \frac{\tilde{n}}{2n_0} E_o \quad (7)$$

where $\tilde{n}$ is the right traveling ion wave amplitude

$$\tilde{n} = \frac{r^{1/2}}{2} \frac{v_o^2}{v_e^2} \quad (8)$$

Solving Equation (7) with the boundary conditions $A_s(L) = 0$, we find

$$A_s = \frac{r^{1/2}}{8} \frac{n}{n_c} \frac{v_o^2}{v_e^2} E_o k_o (L - x) \quad (9)$$

The short time reflectivity is thus

$$R = \frac{A_s^2}{E_o^2} = r \left(\frac{n}{n_c} \frac{v_o^2}{v_e^2} \frac{k_o L}{8} \right)^2 \quad (10)$$

We have thus seen that the residual chamber reflectivity will produce a traveling wave with the right k ($= 2 k_o$) and frequency ($= \omega_s$) to interact with SBS. After a time $\omega_s \approx \pi/2$, the wave reaches the amplitude given by Equation (9) and then serves as an enhanced noise level from which SBS will grow. Typical values of the initial reflectivity from Equation (10) are .2 - 1%, much higher than expected noise levels, which might be 10^{-4} - 10^{-5} .

We next discuss a more complete theory, which includes damping and plasma flow, and handles SBS and the initial Bragg scattering on an equal footing.

For ease in calculation, we change our notation somewhat (see Figure 2). In the plasma frame:

$$\frac{\partial^2 n}{\partial t^2} + v \frac{\partial n}{\partial t} - c_s^2 \frac{\partial^2 n}{\partial x^2} = \frac{e^2 n_o}{4mM\omega_o^2} \frac{\partial^2 |E|^2}{\partial x^2} \quad (11)$$

$$\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} + \omega_{po}^2 E = - \omega_{po}^2 \frac{n}{n_o} E \quad (12)$$

Let:

$$E = E_+ e^{i(kx - \omega_o t)} + E_- e^{-i(kx + \omega_o t)}$$

$$n = \text{Re}(\tilde{n} e^{-2ikx})$$

where E_+ , E_- , $\tilde{n}$ are slowly varying envelopes and $k = k_o \sqrt{\epsilon} = k_o (1 - n/n_c)^{1/2}$.

Then we have:

$$\frac{\partial E_+}{\partial t} + c\sqrt{\epsilon} \frac{\partial E_+}{\partial x} = - \frac{i\omega_o}{4n_c} \tilde{n}^* E_- \quad (13a)$$

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BOUNDARY REFLECTIVITY = f^2 PUMP E^- REFLECTED
SCATTERED E^+ n_0 v_0

PLASMA

FIGURE 2. GEOMETRY OF PLASMA EXPERIMENT

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$$\frac{\partial E_-}{\partial t} + c\sqrt{\epsilon} \frac{\partial E_-}{\partial x} = - \frac{i\omega_0}{4nc} \tilde{n} E_+ \quad (13b)$$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} + v \frac{\partial \tilde{n}}{\partial t} + \omega_s^2 \tilde{n} + 2 i \omega_s c_s \frac{\partial \tilde{n}}{\partial x} = - \frac{2e^2 n_0 k^2}{m M \omega^2} E_+ E_- \quad (13c)$$

In the plasma frame, the incident light is upshifted by $k_0 v_0$ and the light reflected from the surface is downshifted by $k_0 v_0$. A third component is down shifted by ω_s . We thus make the following separation:

$$E_- = E_0 e^{-ik_0 v_0 t}$$

$$E_+ = E_R e^{ik_0 v_0 t} + E_S e^{i(\omega_s - k_0 v_0)t}$$

$$\tilde{n} = n_+ e^{i\omega_s t} + n_- e^{-i\omega_s t} + n_D e^{-2ik_0 v_0 t}$$

Substituting into the set of equations (13) we obtain:

$$\frac{\partial n_+}{\partial t} + \frac{v}{2} n_+ + c_s \frac{\partial n_+}{\partial x} = 0 \quad (14a)$$

$$\frac{\partial n_-}{\partial t} + \frac{v}{2} n_- - c_s \frac{\partial n_-}{\partial x} = - i \frac{e^2 n_0 k^2}{m M \omega_0 \omega_s} E_0 E_S^* \quad (14b)$$

$$\frac{\partial^2 n_D}{\partial t^2} + v \frac{\partial n_D}{\partial t} + \omega_s^2 n_D + 2 i \omega_s c_s \frac{\partial n_D}{\partial x} = - \frac{2e^2 n_0 k^2}{m M \omega_0^2} E_0 E_R^+ \quad (14c)$$

for the ion waves, and

$$\frac{\partial E_R}{\partial t} + ik_0 v_0 E_R + c\sqrt{\epsilon} \frac{\partial E_R}{\partial x} = - \frac{i\omega_0}{4n_c} n_D^* E_0 \quad (15a)$$

$$\frac{\partial E_S}{\partial t} + i(\omega_s - k_0 v_0) E_S + c\sqrt{\epsilon} \frac{\partial E_S}{\partial x} = - \frac{i\omega_0}{4n_c} n_-^* E_0 \quad (15b)$$

$$\frac{\partial E_0}{\partial t} - ik_0 v_0 E_0 - c\sqrt{\epsilon} \frac{\partial E_0}{\partial x} = - \frac{i\omega_0}{4n_c} (n_- E_S + n_D E_R) \quad (15c)$$

for the fields, where we have neglected all non-resonant or multiple scattering terms. As discussed in Forslund, et.al., after a short time, the

convective terms are dominant in the field equations. In the UCLA experiment, the maximum reflectivity is about 10%, thus pump depletion is not important. We may then decouple our equations into a set describing SBS:

$$\frac{\partial n_-}{\partial t} + v/2 n_- - c_s \frac{\partial n}{\partial x} = -i \frac{e^2 n_0 k^2}{m M \omega_0^2 \omega_s} E_0 E_s^* \quad (16)$$

$$c\sqrt{\epsilon} \frac{\partial E_s}{\partial x} = -i \frac{\omega_0}{4n_c} n_-^+ E_0$$

and Bragg scattering:

$$\frac{\partial n_D}{\partial t^2} + v \frac{\partial n_D}{\partial t} + \omega_s^2 n_D = - \frac{2e^2 k^2 n_0}{m M \omega_0^2} E_0 E_R^* \quad (17)$$

$$c\sqrt{\epsilon} \frac{\partial E_R}{\partial x} = - \frac{i\omega_0}{4n_e} n_D^+ E_0$$

with the auxiliary equation:

$$\frac{\partial n_+}{\partial t} + v/2 n_+ = 0 \quad (18)$$

Let us first consider the Bragg equations (17). Since we have taken out the sinusoidal dependence on time, n_D can at most be a power series in time. Substituting, we find that

$$n_D = -2 \frac{e^2 k^2 n_0}{m M \omega_0^2 \omega_s^2} E_0 E_R^* \quad (19)$$

and

$$c\sqrt{\epsilon} \frac{\partial E_R}{\partial x} = \frac{i\omega_0}{4n_c} \left(\frac{2e^2 k^2 n_0}{m M \omega_0^2 \omega_s^2} \right) |E_0|^2 E_R = i \frac{\omega_0}{2} \frac{n_0}{n_c} \frac{v_0^2}{v_e^2} E_R$$

with the solution

$$E_R = f E_0 e^{i \frac{1}{8} \frac{n_0}{n_c} v_0^2 / v_e^2 \frac{R_0 X}{\sqrt{\epsilon}}} \quad (20)$$

Note that "Bragg scattering" doesn't give rise to scattering at all, but a phase shift which depends on X .

Substituting Equation (20) into Equation (19), we have

$$R_D = - \frac{f}{2} \frac{v_0^2}{v_e^2} r_0^{-i2\beta x} \quad (21)$$

where

$$\beta x = \frac{1}{16} \frac{n_0}{n_c} v_0^2 / v_0^2 \frac{k_0 x}{\sqrt{\epsilon}} \quad (22)$$

Our initial conditions are

$$\tilde{n}(t=0) = \frac{\partial \tilde{n}}{\partial t}(t=0) = 0$$

giving

$$\hat{n}_+ + n_- + n_D = 0$$

$$\omega_s (n_+ - n_-) - 2 k_0 v_0 n_D = 0$$

at $t = 0$. These give

$$n_-(t=0) = \frac{n_D}{2} \left(1 + \frac{2k_0 v_0}{\omega_s} \right).$$

The boundary condition on E_s is $E_s(x=0) = 0$. Define

$$\eta = \frac{n_-}{n_0} \left/ \left(\frac{v_0^2}{4v_e^2} \right) \right.$$

$$A_s = E_s / E_0$$

$$\tau = \omega_s t$$

$$\xi = k_0 x$$

$$M = 2k_0 v_0 / \omega_s.$$

The SBS equations become

$$\frac{\partial \eta}{\partial \tau} + \frac{v}{2\omega_s} \eta - \frac{\partial \eta}{\partial \xi} = -i A_s^* \quad (23)$$

$$\frac{\partial A_s^*}{\partial \xi} = i \beta' \eta = i \frac{\beta}{k_0} \eta \quad (24)$$

We solve by Laplace transforming η and A_s^* in time (p) and space (q)

$$A_{sq}^* = -i \frac{\beta' \eta_q}{q} \quad \eta_q(0) = \frac{f}{q}$$

$$\eta_{pq} \left[-p + \frac{v}{2\omega_s} + q \right] = -\frac{\beta' \eta_{pq}}{q} + \frac{f}{q} (1 + M) \quad (25)$$

Where we have made use of the initial condition $\eta_q(t=0) = f/q$ in Equation (25). Solving for η_{pq} and inverting:

$$\eta_{pq} = \frac{f(1+M)}{q \left[-p + \frac{v}{2\omega_s} + q + \frac{\beta'}{q} \right]}$$

$$\eta_q = \frac{1}{2\pi i} \int_c \frac{f(1+M) e^{-p^2} dp}{q \left[-p + \frac{v}{2\omega_s} + q + \frac{\beta'}{q} \right]}$$

$$= \frac{f}{q} (1+M) \exp \left[-\frac{v}{2\omega_s} \tau + \frac{\beta' \tau}{q} - q\tau \right]$$

$$\eta(\xi, \tau) = \frac{1}{2\pi i} \int_c f(1+M) \frac{\xi}{q} e^{-\frac{v\tau}{2\omega_s}} e^{-\frac{\beta' \tau}{q}} - q(\tau + \xi) dq \quad (26)$$

$$= f(1+M) e^{-\frac{v\tau}{2\omega_s}} I_0[2\sqrt{\beta' \tau(\tau + \xi)}]$$

Now we use Equation (23) to calculate A_s :

$$A_s = -i f(1+M) e^{-\frac{v\tau}{2\omega_s}} \sqrt{\frac{\beta'(\tau + \xi)}{\tau}} I_1[2\sqrt{\beta' \tau(\tau + \xi)}] \quad (27)$$

The SBS reflectivity is

$$\begin{aligned} R_{SBS} &= A_s^* A_s \\ &= f^2(1+M)^2 e^{-\frac{v\tau}{\omega_s}} \left[\frac{\beta(\tau + k_0 L)}{\tau} \right] I_1^2[2\sqrt{\beta \tau(\tau + k_0 L)}] \end{aligned} \quad (28)$$

For very long times $\tau \gg k_0 L$, this goes to

$$R_{SBS} = f^2(1+M)^2 e^{-\frac{v\tau}{\omega_s}} \beta' \frac{e^{4\sqrt{\beta' \tau}}}{4\beta \tau^2} \quad (29)$$

The growth rate is just that given in Forsland, et.al., (Equation 1 of this paper), and the effective noise level is:

$$R(0) = \frac{f^2(1+M)^2}{16} \frac{N_o}{N_c} \frac{V_o^2}{V_e^2 \sqrt{e}}$$

The total reflectivity is given by

$$R_T = \frac{|E_+|^2}{|E|^2} = \frac{|E_R|^2 + |E_S|^2 + 2\text{Re}(E_R E_S^*)}{|E_0|^2}$$

$$= f^2(1 + S^2 + 2 S \sin \omega_s t) \quad (30)$$

where

$$S = (1 + m) e^{-\nu t/2} \sqrt{\beta \left(1 + \frac{k_0 L}{\omega_s t}\right)} I_1 \left[2 \sqrt{\beta \left(1 + \frac{k_0 L}{\omega_s t}\right)} \omega_s t \right].$$

We have used the computer code AURUS, developed by Randall at LLL to more fully simulate the UCLA experiments. This one-dimensional code incorporates a hydrodynamical description of the plasma and a wave optics solution for the laser electric field. This code self-consistently handles the effect of realistic density, temperature, and flow profiles on SBS reflectivity. In general, the results agree with the theory discussed here and the experimental results discussed in the next section.

A.2 Experimental Results and Comparison with Theory

Figure 3 shows the time history of both the incident and reflected waves. Figure 3(a) shows a case below threshold. The reflectivity seen is due to the residual chamber reflectivity. Figure 3(b) shows a case above threshold. The ion acoustic beat frequency predicted in Equation (30) is easily visible here. Figure 4 shows the frequency spectrum of the back-reflected wave for the below-threshold and above-threshold cases, showing the expected red shifted component in the latter case.

Figure 5 shows the ion density fluctuations as a function of time, showing the expected fast growth on the time scale ω_s^{-1} , and the subsequent slower growth due to SBS. Figure 6 plots the observed time to the first density peak, for various powers and ion species. The good fit to the experimental curve $\omega_s t_p = \pi$ is also shown. At this time, the density fluctuation should be given by Equation 5:

$$\frac{\tilde{n}}{n_0} = 2r^{1/2} v_o^2/v_e^2 \quad (31)$$

Fig. 3. Time History of Incident Wave and Reflected Wave.
(a) $P_0 \approx 60$ kW, (b) $P_0 \approx 220$ kW.

Fig. 5. Frequency Spectra of Incident Wave and Reflected Wave.
 (a) $P_0 \approx 25$ kW, (b) $P_0 \approx 270$ kW.

Fig. 2. Normalized Density Fluctuations as a Function of Incident Pulse Width. (a) H Plasma, $P_0 \approx 300$ kW, (b) He Plasma, $P_0 \approx 87$ kW.

FIGURE 6. TIME TO FIRST DENSITY PEAK

Figure 7 shows $\tilde{n}/n_0$ versus v_0^2/v_e^2 . The best fit is obtained for $r = .6\%$, which is consistent with the estimated reflectivity of 1% obtained from power reflected back into the horn.

Figure 8 shows the time history of a typical experiment simulated by AURUS. The dotted line in the reflected power curve is taken from an oscilloscope trace such as in Figure 3. The agreement is good, except for some possible interference effects at early times, and an apparent larger damping at later times, possibly due to ion or electron heating. Figure 9 shows the frequency spectrum of the reflected wave, which may be compared with the experimental spectrum in Figure 10, showing good agreement.

In conclusion, experimental results show good agreement with the theoretical prediction of SBS initiation from a standing ion wave caused by the ponderomotive force. In the next section, we discuss growth rates and saturation levels.

A.3 Long Time Saturation of SBS

For times longer than $k_0 L/\omega_s$, Equation (28) shows that SBS reflectivity will grow as long as $\sqrt{\beta} > v/4$, the usual threshold condition. However, for times shorter than $k_0 L/\omega_s$, the reflectivity is given by:

$$R_{\text{SBS}} \sim e^{-v\tau/\omega_s} \frac{\beta k_0 L}{\tau} S_1^2 (2 \sqrt{\beta k_0 L \tau})$$

The damping can overcome the growth at a time given by

$$\frac{v}{2\omega_s} = \sqrt{\frac{\beta k_0 L}{\tau}} \frac{I_2}{I_1}$$

or approximately

$$\tau \cong 4\beta k_0 L (\omega_s/v)^{1/2}.$$

Using parameters of the UCLA experiment, $\omega_s t \approx 1000$, which is longer than the experiment length.

FIGURE 7. DENSITY AT FIRST PEAK VERSUS $(v_o/v_{te})^2$

Simulation: Time History of Microwave Power

1 time unit = 1.42 μ sec

FIGURE 1

FIGURE 9.1

Center Frequency: 3.3 GHz
Horizontal Scale: 100 kHz/div
Vertical Scale : 5 dB/div

FIGURE 10

However, there does seem to be a power dependent saturation and possible decrease in reflectivity. Figure 11 shows SBS reflectivity as a function of power. In order to model this, we take into account two effects: electron heating and ion tail acceleration. Figure 12 shows a time history of a typical backscatter trace at high powers. Note an increase of ion wave frequency, corresponding to $\sim 70\%$ increase in T_e . The electron heating can be simply estimated from inverse bremsstrahlung on the neutrals, also taking account of the plasma heat capacity:

$$\frac{d}{dt} (nT_e) = \nu_{iB} \frac{E_0^2}{8\pi} \frac{V_B}{V_p} \quad (32)$$

where V_B is the irradiated volume and V_p is the total plasma volume. Solving, and using the experimental parameters:

$$T_e(t) = T_e(0) \left[1 + 1.5 \times 10^{-6} P(\text{kw}) t(\mu\text{sec}) \right]^2 \quad (33)$$

Next, suppose the ion wave generates a super thermal ion tail. The magnitude of the ion tail density is obtained by balancing the energy flux deposited in the ion wave with the flux carried off in the tail (see Kruer and Estabrook)⁵:

$$R \frac{E_0^2 c}{8\pi} \frac{\omega_s}{\omega_0} = n_h T_h \sqrt{\frac{T_h}{M}} \quad (34)$$

where $T_h = M v_p^2 \approx M c_s^2$. Solving for n_h :

$$n_h = R \frac{V_0^2}{v_e^2} n_c \quad (35)$$

Then the damping is estimated by

$$\frac{\gamma_h}{\omega_s} \approx .8 \frac{n_h}{n_0} \quad (36)$$

To check Equation 35, note that experimentally about 5-6% of the ion density is observed for a 15 m-sec pulse at $P = 300 \text{ kW}$. Equation (35) using the reflectivity given from Figure 11 predicts 8%, in reasonable agreement.

The wave is certainly strongly damped when $\gamma_h/\omega_s = 1$. This maximum reflectivity is given by

$$R = \frac{1}{.8 \frac{V_0^2}{V_e^2} \frac{R_c}{R_0}}$$

$$= \frac{1}{8 \frac{V_0^2}{V_{eo}^2}} (1 + 1.56 \times 10^{-6} P_t)^2$$

We have evaluated the reflectivity numerically using Equation 28 and 33 and limiting the reflectivity to a maximum given by Equation 37. We also used the damping rate determined experimentally, $\nu/\omega_s \approx .04$. These results are shown as the dashed line in Figure 11, again showing reasonable agreement.

We conclude that both electron heating and ion tail formation are important in the long time behavior of SBS reflectivity, but the latter is the stronger mechanism.

A.4 Finite Bandwidth Effects

In the UCLA experiment, the reflectivity is set for pulses of less than 5 μ sec by the ponderomotive density fluctuations, so it is of interest to determine the effect of bandwidth on them. Figure 13 shows an example of reflectivity versus percentage bandwidth for a random phase modulated pump. Suppose we are in the plasma at a point L from the reflecting wall. The incident field is

$$E_{in} = E_0 e^{i(kx - \omega t) - i\phi(t)}$$

The reflected field is

$$E_r = r^{1/2} E_0 e^{-i(kx + \omega t) - i\phi(t - 2L/c)}$$

The ponderomotive force is proportional to

$$E_0 E_r = r^{1/2} E_0^2 e^{2ikx + i[\phi(t - 2L/c) - \phi(t)]}$$

If the bandwidth is larger than the acoustic frequency, $\Delta\omega \gg \omega_s$, which is true, we may average the ponderomotive force over the correlation time $t_c = \Delta\omega^{-1}$, obtaining

Fig. 11

$t(\mu\text{sec})$	$f_s(\text{kHz})$	$\Delta T_i(\text{eV})$ inferred	$\Delta T_e(\text{eV})$ inferred
5	190		
10	213		
20	240		
		0.2	0.6
		0.45	1.4

Fig. 12

Plasma Reflectivity as a Function of Bandwidth of Random Phase Modulated Pump.

$$E_0 E_r = r^{1/2} E_0^2 e^{2ikx - 2 \frac{\Delta\omega L}{c}}$$

We thus expect the short time reflectivity to be proportional to $\exp(-2L/L_c)$, where $L_c = c/\Delta\omega$ is the correlation length. Using $L \approx 100$ cm, we find the e^{-1} point to correspond to $\Delta f/f = 7 \times 10^{-2}$, which is in good agreement with the experimental results.

B. SBS AND SELF-FOCUSING

The second major part of this theoretical program was to involve modeling of the TRW Microwave Interaction experiments. Originally, we expected the experiments to concentrate on electron thermal transport, and the original TRW proposal to LLL had both a section on transport and a section on a proposed experiment to investigate the interaction of SBS and self-focusing. In response to the expressed interest of LLL, we concentrated on the latter experiment. This experiment will be on line in October, 1980. We have thus limited our investigations to theoretical considerations of the interaction of SBS and self-focusing and performed numerical calculations to more closely define optimum operating parameters for the TRW experiments.

Consider a laser fusion experiment with long pulse lengths, relevant to Nova size targets. At first, the electron temperature is much higher than the ion temperature, and SBS is in the weakly damped regime. In this regime, it has a much larger growthrate than self-focusing. However, energy transfer to the ions will heat them up to a temperature $T_i \approx ZT_e$ (Z is the ion charge state) on a time scale of tens of picoseconds. Now SBS is in the heavily (Landau-damped) regime, and its growth length is comparable to self-focusing.

The growth of both instabilities are proportional to local light intensity. As the beam shrinks due to self-focusing, the SBS level increases, thus increasing the local intensity since the incident and reflected waves add. This in turn increases the growth rate of self-focusing. This picture leads us to expect two things: the self-focusing length is decreased by about a factor of 2, and after the first focus the beam is drastically depleted. This has important consequences for laser fusion experiments since it suggests that there is a maximum propagation distance for laser light, given by the distance to first focus.

In order to treat this problem, we begin with the wave equation

$$\frac{1}{c^2} \frac{\partial^2}{\partial t^2} E - \nabla^2 E + \frac{\omega_p^2}{c^2} E = 0 \quad (38)$$

Let

$$\frac{\partial^2}{\partial t^2} = -\omega_0^2$$

$$\nabla^2 = -k^2 \pm ik \frac{\partial}{\partial z} + \nabla_1^2$$

and $n = n + \tilde{n}$, where $\tilde{n}$ is the fluctuating part of the total density.

We obtain

$$\pm ik \frac{\partial}{\partial z} E_{\pm} + \nabla_1^2 E_{\pm} + (k^2 - \omega_0^2 + \omega_{po}^2) E_{\pm} + \omega_{po}^2 (n/n_0 - 1) E_{\pm} \quad (39)$$

$$+ \omega_{po}^2 \tilde{n}_{\pm} E_{\pm} = 0$$

where E_+ is the incident wave and E_- is the reflected wave. Solving the ion density equation, we find

$$n/n_D = \exp \left[\frac{-e^2(|E_+|^2 + |E_-|^2)}{4m\omega_0^2 (T_e + T_i/z)} \right]$$

and the heavily damped ion fluctuations are

$$n_+ = in \frac{2Ze^2 k^2}{mM\omega_0^2 \nu \omega_s} E_+^* E_-$$

$$n_- = -in \frac{2Ze^2 k^2}{mM\omega_0^2 \nu \omega_s} E_+ E_-^*$$

Putting this all together, we have the coupled equations for the normalized amplitudes $a_0 = E_+/E_0$, $a_s = E_-/E_0$:

$$\frac{i}{k_0} \frac{\partial a_0}{\partial z} + \frac{1}{k_0^2} \nabla_1 a_0 - \left[\frac{\Gamma^2}{k_0^2 c^2} + \frac{\omega_{po}^2}{k_0^2 c^2} \left(1 - e^{-\beta(|a_0|^2 + |a_s|^2)} \right) \right] a_0$$

$$= -i \frac{\gamma_0^2}{\gamma k_0 c} a_0 |a_s|^2 e^{-\beta(|a_0|^2 + |a_s|^2)} \quad (40)$$

$$\begin{aligned} & - \frac{i}{k_0} \frac{\partial a_s}{\partial z} + \frac{1}{k_0^2} \nabla_{\perp}^2 a_s - \left[\frac{\Gamma^2}{k_0^2 c^2} + \frac{\omega_{po}^2}{k_0^2 c^2} \left(1 - e^{-\beta(|a_0|^2 + |a_s|^2)} \right) \right] a_s \\ & = i \frac{\gamma_0^2}{\gamma k_{oc}} a_s |a_0|^2 e^{-\beta(|a_0|^2 + |a_s|^2)} \end{aligned} \quad (41)$$

where

$$\Gamma^2 = k_0^2 c^2 - \omega_0^2 - \omega_{po}^2$$

$$\beta = v_0^2 / v_e^2$$

and γ_0 is the maximum SBS growth rate. If the right sides are set to zero, the usual self-focusing equations result, with the local field energy set to the sum of the incident and scattered waves. If the bracketed terms are set to zero, we have the SBS equations, with a reduction in growth rate due to density depression.

In general, these equations are difficult to solve. However, we can make some progress if we take the usual SBS relation:

$$|a_s|^2 = |a_0|^2 - T \quad (42)$$

where T is the total transmissivity. Actually, if Equations 40 and 41 are multiplied by a_0^* and a_s^* respectively and added, we find that

$$\begin{aligned} \frac{\partial}{\partial \xi} \left[|a_0|^2 - |a_s|^2 \right] &= 2 \left[\frac{\Gamma^2}{k_0^2 c^2} + \frac{\omega_{po}^2}{k_0^2 c^2} \left(1 - e^{-\beta(|a_0|^2 + |a_s|^2)} \right) \right] \\ &\quad \text{Im} \left(a_0^* \frac{\partial^2}{\partial \eta^2} a_0 + a_s^* \frac{\partial^2}{\partial \eta^2} a_s \right) \end{aligned} \quad (43)$$

where $\xi = kz$ and $\nabla_{\perp}^2 / k_0^2 = \partial^2 / \partial \eta^2$. We go over to the slab geometry representation for simplicity, where η represents the perpendicular component.

We can use Equation 42, which is valid if the right side of Equation 43 is small, and then check the validity a posteriori.

Substituting (42) into (40); and making the small v_0^2/v_e^2 approximation, we have

$$i \frac{\partial a_0}{\partial \xi} + \frac{\partial^2 a_0}{\partial \eta^2} - \left[\frac{\Gamma^2}{k_0^2 c^2} + \frac{\omega_{po}^2}{k_0^2 c^2} \beta (2|a_0|^2 - T) \right] a_0 = -i \frac{\gamma_0^2}{v r_0 c} a_0 (|a_0|^2 - T) \quad (44)$$

Define

$$\frac{\Gamma^2}{k_0^2 c^2} \equiv \Delta^2$$

$$\frac{\omega_{po}^2}{k_0^2 c^2} \beta \equiv B^2$$

$$\frac{\gamma_0^2}{v k_0 c} \equiv k_0$$

Equation 44 may be written

$$i \frac{\partial a_0}{\partial \xi} + \frac{\partial^2 a_0}{\partial \eta^2} - \left[\Delta^2 - B^2(T - ik_0) \right] a_0 - (2B^2 - ik_0) |a_0|^2 a_0 = 0 \quad (45)$$

Note that if $T = 0$, the quantity B^2 , which is a measure of the self-focusing strength, is multiplied by a factor of 2. Now let

$$a_0 = \alpha(\eta) e^{k\xi},$$

giving

$$\alpha'' - [\Delta^2 - ik - B^2T + ik_0T] \alpha - (2B^2 - ik_0) \alpha^3 = 0 \quad (46)$$

multiply by α' and integrate once:

$$\frac{\alpha'^2}{2} - [\Delta^2 - ik - B^2T + ik_0T] \frac{\alpha^2}{2} - (2B^2 - ik_0) \frac{\alpha^4}{4} = C \quad (47)$$

To evaluate the constant of integration, note that at $\eta \rightarrow \infty$, $\alpha' \rightarrow 0$, $\alpha \rightarrow 0$. Therefore $C = 0$. Now at $\eta = 0$, $\alpha' = 0$, $\alpha = \alpha_0$, giving

$$\Delta^2 - ik - B^2T + ik_0T = - (2B^2 - ik_0) \frac{\alpha_0^2}{2} \quad (48)$$

Equating real and imaginary parts

$$\Delta^2 = - B^2(\alpha_0^2 - T) \quad (49)$$

$$k = - k_0(\alpha_0^2/2 - T^2) \quad (50)$$

Letting $2B^2 - ik_0 = c^2$, Equation 47 can be written

$$\alpha'^2 - c^2 \alpha^2 (1 - \alpha^2) = 0$$

or

$$\int_0^{\alpha(\xi)} \frac{d\alpha}{\alpha \sqrt{1 - \alpha^2}} = \pm c \xi \eta$$

$$= \frac{1}{2} \ln \left[\frac{1 + \sqrt{1 - \alpha^2}}{1 - \sqrt{1 - \alpha^2}} \right] \Big|_{\alpha_0}^{\alpha}$$

with the solution

$$1 - \alpha^2 = \left[\frac{(1 + \sqrt{1 - \alpha_0^2}) e^{2c\eta} - (1 - \sqrt{1 - \alpha_0^2})}{(1 + \sqrt{1 - \alpha_0^2}) e^{2c\eta} + (1 - \sqrt{1 - \alpha_0^2})} \right]^2 \quad (51)$$

This is similar to the usual calculation. The right side of Equation 51 becomes $\tanh^2(c\eta)$ if $\alpha_0 = 1$, leading to $\alpha \sim \text{sech}(c\eta)$ as usual. One consequence of keeping the SBS terms is that now c is complex

$$\begin{aligned}
c &= [2B^2 - ik_0]^{1/2} \\
&= [4B^4 + k_0^2]^{1/4} e^{-i \frac{1}{2} \tan^{-1} (k_0/2B^2)} \\
&= C_R + iC_i
\end{aligned}$$

Suppose $\alpha_0 \approx 1$, so that

$$\alpha \approx \frac{1}{\cosh c\eta}$$

The shape of the beam power becomes

$$\alpha^* \alpha \approx \frac{2}{\cosh^2(2C_R \eta) + \cos^2(2C_i \eta)} \quad (53)$$

Thus SBS, in this limit affects the beam shape sinusoidally, with the most pronounced dip in the beam center, and a decreasing ripple as η gets large.

Summarizing these calculations, we have found a quasi-steady beam profile, in which the depletion effects of SBS just balance the accumulation effects of self-focusing. We have not yet explained the conditions required for the existence of this profile, but it must depend on the parameters $2B^2$ and k_0 , measuring the strength of the focusing and SBS scattering, respectively. The ratio is

$$\frac{2B^2}{k_0} = \frac{\nu \nu}{\omega_s}$$

For very strong damping, $\nu \approx \omega_s$, and $2B^2 \approx k_0$. Another parameter that enters is the initial beam power $a_0^2(\xi = 0)$. Our picture is that there are three possible states: 1) SBS dominated, where the beam is dissipated too quickly for focusing to occur, 2) focusing dominated, where a catastrophic focus occurs, with essentially 100% backscatter, and 3) a stationary state where the two effects are balanced, described by the above theory.

In attempting to design an experiment to investigate these effects, we came upon the problem that the microwave pulse (~ 10 -20 usec) is too short to allow significant ion heating and the subsequent evolution of

the beam possible. We therefore hit on the idea of suppressing SBS with a finite bandwidth pump. This will allow us to see pure self-focusing. Then, by reducing the bandwidth, we can control the amount of SBS introduced, hopefully allowing a careful investigation of the interaction.

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