2
JUNE 1978

PPPL-1454

UC-20g

NUMERICAL SIMULATION OF TRAPPED
ELECTRON INSTABILITIES IN
TOROIDAL GEOMETRY

BY

C. Z. CHENG AND H. OKUDA

**PLASMA PHYSICS
LABORATORY**

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

**PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY**

This work was supported by the U. S. Department of Energy
Contract No. EY-76-C-02-3073. Reproduction, translation,
publication, use and disposal, in whole or in part, by or
for the United States Government is permitted.

Numerical Simulation of Trapped Electron Instabilities
in Toroidal Geometry

C. Z. Cheng and H. Okuda

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08540

ABSTRACT

Dissipative trapped particle instabilities are studied by means of three-dimensional particle simulations in toroidal geometry. In the linear stage growth rate of the unstable modes, radial and poloidal ballooning mode structures observed in the simulation agree reasonably well with linear theory. In the nonlinear stage, the ballooning effect diminishes due to the generation of strong plasma turbulence resulting in the broad frequency spectrum and convective cells consistent with experimental observations.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees nor any of their contractors, subcontractors, or their employees makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

1
MASTER

104

The importance of low-frequency drift-type instabilities has long been recognized in relation with the anomalous plasma transport in toroidal confinement devices.¹ At present, a fairly complete linear theory for such instabilities in toroidal systems has been developed.² However, nonlinear behavior of these instabilities and the resultant anomalous particle and energy diffusion have not been well understood. Recently, it has been demonstrated that three-dimensional particle simulations³ are useful for understanding the strong plasma turbulence and the anomalous diffusion due to collisionless drift instabilities.⁴ In particular, it was found that the collisionless drift instabilities can generate strong turbulence and convective cells which in turn enhance particle diffusion.

In this paper, we report our recent simulation studies on the dissipative trapped electron instabilities using a three dimensional toroidal plasma model.³ It will be shown that, in the earlier stage of the instabilities, the simulation results agree reasonably well with the linear theory. In particular, we observe the ballooning effect on the poloidal mode structure of the unstable modes. In the nonlinear stage, however, the ballooning effects diminish due to the generation of strong plasma turbulence and broad spectrum consistent with the experimental observations.¹

The simulation model is a torus in an external magnetic field given by $\vec{B} = B_\phi \hat{e}_\phi + B_\theta \hat{e}_\theta$ where $B_\phi = B_\phi^{(0)} / (1 + \epsilon \cos\theta)$

is the toroidal field and $B_\theta = (\epsilon/q)B_\phi^{(0)}$ is the poloidal field. $\epsilon \equiv r/R_0$ is the inverse aspect ratio, ϕ is the toroidal angle, θ is the poloidal angle measured from outside toward inside of a torus, and r and R_0 are the minor radius and major radius of the torus. Full Lorentz force is employed for the ion dynamics to keep the ion inertia and finite gyroradius effects while a guiding center drift approximation is used for the motion of electrons.³ Conducting walls are assumed for the electrostatic field and the particles are elastically reflected back at the wall. The simulation parameters are the following: aspect ratio $R_0/a = 3$, safety factor $q = 1.1 (1 + 2.5 r^2/a^2)$ which corresponds to a current profile of $j_\phi \sim (1 + 2.5 r^2/a^2)^{-2}$. Initially, electrons and ions have local Maxwellian velocity distribution with the density profile taken to be $n_e(r) = n_i(r) = n_0 \exp(-4r^2/a^2)$. The initial electron temperature profile is chosen similarly to provide $\eta_e \equiv d(\ln T_e)/d(\ln n_e) = 1$ everywhere. The initial ion temperature is taken to be uniform. The average density and temperature are specified by choosing $\Omega_e/\langle\omega_{pe}\rangle = 5$, $m_i/m_e = 100$, $\langle T_e \rangle/T_i = 4$, $\langle \lambda_e \rangle = 2$ where Ω_e is evaluated at the magnetic axis. These parameters may correspond to low density tokamak discharges in laboratory experiments. In the cross section, a $64 \times 64 (L^2)$ spatial grid is used for numerical computation with its physical length $L/\rho_i = 32$, $\rho_i = (2T_i/m_i)^{1/2}/\Omega_i$. Seven Fourier modes $n = 0, \pm 1, \pm 2, \pm 3$ are kept in the toroidal direction. The electron-ion pitch

angle scattering is simulated by Monte Carlo collision model⁵ where the collision frequency for an electron goes v^{-3} where v is the speed of the electron. For the bulk of the plasma, $v_e^* \equiv v_{eff}/\omega_{be}$, where $v_{eff} = v_{ei}/\epsilon$ is the effective collision frequency and $\omega_{be} = \sqrt{\epsilon} v_e/qR_0$ is the electron bounce frequency, varies from $v_e^* = 0.5$ to $v_e^* \approx 10$ suggesting the simulation parameters are for the banana and the plateau regimes close to high temperature tokamak experiments.

Figure 1(a) shows the time development of the electric field energy for different m modes with $n = 2$ (m, n are the poloidal and toroidal mode numbers), radial mode structure in (1b), and poloidal mode structure in (1c). The growth rate of the unstable modes is about $\gamma/\Omega_i = 0.03 \sim 0.07$ for the modes shown and this growth rate is within a factor of 1.5 from the prediction of the two-dimensional linear theory.² Since the aspect ratio is small ($R/a = 3$), toroidal coupling is strong for various m modes causing them to grow at about the same growth rate. These unstable modes grow about two orders of magnitude and saturate. The mechanism for saturation is partly because the energy is delivered nonlinearly to the stable modes resulting in the broad spectrum, strong plasma turbulence and convective cells. By the time of saturation, initial density and temperature have diffused considerably due to the generated strong turbulence.

Radial mode structures for $(m, n) = (4, 2)$ and $(3, 2)$ shown in (1b) are localized around the rational surface at $r/a \approx 0.5$

during the linear stage. Near saturation, the maximum amplitude reaches $e\phi_{mn}/T_e \approx 0.3$ and the mode extends near the entire plasma column. The peak of the amplitude shifts slightly outwards in the nonlinear stage.

Poloidal mode structure shown in (1c) at $r/a = 1/2$ and $n = 2$ indicates clearly the ballooning effect where the mode amplitude is larger at the outside ($\theta = 0$) than the inside ($\theta = \pi$) of a torus. When the instability enters into a nonlinear stage ($\omega_{pe} t = 1200$), however, Fig. (1c) indicates the ballooning effect almost disappeared and the amplitude is nearly constant along the field lines. We believe the reason for this is the plasma is already in strong turbulent state with many other modes excited by nonlinear mode-coupling processes as seen for the drift turbulence.⁶ Under such conditions, ballooning effects become less prominent since the nonlinearity generated broad spectrum (convective cells) do not require the presence of trapped particles.

Figure 2 indicates the frequency and wavenumber spectra for the turbulence generated. Time evolution of the electric field energy with respect to poloidal mode number $|E(m)|^2$ averaged over the plasma radius and toroidal angle is shown in (2a). We observe the spectral distribution is more or less monotonic with m number consistent with recent measurements in PLT tokamak.¹ Relatively large amplitude for $m = 1$ at early times is due to the ∇B drifts of the ions which can generate $(m, n) = (1, 0)$. At the time of saturation, not only the most unstable trapped-particle modes

with $m = 3 \sim 8$, but also $m = 1, 2$ modes grow to large amplitudes. This is the sign of broad spectrum of the frequency consistent with the generation of strong plasma turbulence and convective cells.

Frequency spectra of (4, 2) and (6, 2) modes are shown in (2b) and (2c). For (4, 2) mode the measurement was performed near its rational surface at $r/a = 0.47$. Observed frequency $\omega/\Omega_i = 0.07$ is close to the prediction from linear theory² and the spectrum is broad ($\Delta\omega \sim \omega$). For (6, 2) mode, the observation point is far away from its rational surface and in fact the observed frequency is peaked at zero although the amplitude is considerably smaller. This is one of the examples of strong plasma turbulence which can generate convective cells.

Figure 3 indicates the gross particle diffusion (3a) and electron heat transfer (3b) associated with the instabilities. Relatively small diffusion, observed in the linear stage of the instability, is enhanced rapidly near saturation which leads to the stabilization of the system along with the generation of strong plasma turbulence. The observed particle diffusion is about $D_{\perp} \approx 2 \times 10^{-2} \rho_i^2 \omega_{pe}$ close to the theoretical estimate $D_{\perp} \approx (cT_e/eB)(\rho_i/L_n)$ based on the strong plasma turbulence and convective cells for drift wave turbulence.⁴

Electron temperature diffusion perpendicular and parallel to the confining magnetic field is shown in (3b) where the diffusion of $T_{\perp e}$ is larger than the density diffusion by a

factor of 3 ~ 4. The parallel temperature diffusion is, on the other hand, much smaller. This is because the high energy electrons trapped in a magnetic mirror [$v_{\perp} > (\epsilon)^{-1/2} v_{\parallel}$] preferentially diffuse out due to plasma turbulence resulting in large $T_{\perp e}$ diffusion. Heating of the parallel temperature at the outer side of the torus ($r/a \gtrsim 0.5$) is partly due to the anomalous diffusion and partly due to electron pitch angle scattering which leads the electron velocity distribution isotropic. The total running time of the simulation is, however, only one or two collision time for the electron temperature relaxation and the complete isotropization has not yet been reached.

We have also studied TEM by varying the initial electron temperature profile so that $\eta_e = 0, -1$. Simulation results show that for $\eta_e = 0$ the instabilities become less unstable with growth rate about a factor 3 smaller than the case for $\eta_e = 1$. For $\eta_e = -1$, we found the system becomes completely stable and no instabilities were observed, consistent with the prediction of linear theory.

In conclusion, dissipative trapped-electron instabilities have been studied successfully using a three-dimensional toroidal particle simulation model. Anomalous diffusion of heat and particles are observed along with the generation of strong plasma turbulence consistent with the measurements from laboratory experiments¹ and mode-coupling calculations.⁶

Ballooning structure, which is the characteristic feature of the trapped-particle mode, is found to diminish at the strongly turbulent nonlinear stage.

We acknowledge many useful discussions with Drs. G. Rewoldt and P. H. Rutherford.

This work was supported by United States Department of Energy Contract No. EY-76-C-02-3073.

REFERENCES

- ¹E. Mazzucato, Phys. Rev. Lett. 36, 792 (1976); C. M. Surko and R. E. Slusher, Phys. Rev. Lett. 37 1747 (1976); M. Okabayashi and V. Arunasalam, Nucl. Fusion 17, 497 (1977).
- ²B. B. Kadomtsev and O. P. Pogutse, Nucl. Fusion 11, 67 (1971); G. Rewoldt, W. M. Tang, and E. A. Frieman, Proceedings of the Annual Controlled Fusion Theory Meeting, Paper E9, Gatlinburg, TN, April 26-28, 1978.
- ³C. Z. Cheng and H. Okuda, J. Comp. Phys. 25, 133 (1977).
- ⁴C. Z. Cheng and H. Okuda, Phys. Rev. Lett. 38, 708 (1977) and Nucl. Fusion 18, 587 (1978).
- ⁵R. Shanny, J. M. Dawson, and J. M. Greene, Phys. Fluids 10, 1281 (1967).
- ⁶C. Z. Cheng and H. Okuda, Proceedings of the Annual Controlled Fusion Theory Meeting, Paper OB4, Gatlinburg, TN, April 26-28, 1978.

FIGURE CAPTIONS

Fig. 1(a) Temporal evolution of the trapped-electron instability for several modes with $n = 2$.

Fig. 1(b) Radial mode structure for $(4, 2)$ and $(3, 2)$ modes at different times.

Fig. 1(c) Poloidal mode structure at $r/a = 0.5$ and $n = 2$ at different times.

Fig. 2(a) Temporal evolution of the spectral distribution with $k_{\perp}$.

Fig. 2(b) Frequency spectrum for $(4, 2)$ and $(6, 2)$ modes at $r/a = 0.47$.

Fig. 3(a) Particle diffusion for the ions and electrons.

Fig. 3(b) Electron and ion temperature diffusion, perpendicular and parallel to the magnetic field.

Fig. 1. 782149

Fig. 2. 782148

Fig. 3. 782147