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TWO-PHASE FLOW IN GEOTHERMAL ENERGY
SOURCES

Final Technical Report

July 1981

Work Performed Under Contract No. AS02-76ET28359

University of Denver
Denver Research Institute
Denver, Colorado

and

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Lakewood, Colorado

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U. S. DEPARTMENT OF ENERGY
Geothermal Energy

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Two-Phase Flow In Geothermal Energy Sources

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R.E. Williams
Editor

ABSTRACT

The majority of geothermal deposits consist of pressurized liquid rather than vapor. The presently exploited wells in Mexico and New Zealand are very hot, and therefore production of hot steam at the wellhead has been relatively simple.

The majority of pressurized liquid deposits, however, are expected to be at moderate temperatures, e.g., between 350° and 500°. These will be exploited by natural flash vaporization in the well. The result will be a two-phase flow regime up the well, in which the continuous steam phase entrains the liquid to the surface, the liquid being present as discrete droplets in the steam phase, as a continuous film on the wall of the well, or as both.

Two-phase flow cannot be analyzed in the same way as a single phase system consisting of liquid or gas alone. For example, in a single phase system, increasing the well diameter will reduce friction losses and increase the flow rate. The opposite may be true in two-phase flow, and too large a well diameter will cause the well to cease producing entirely. In addition, stimulation techniques to induce a new well to initiate production, or troubleshooting diagnoses to correct production problems, must be conducted in the light of the principles of two-phase flow.

A geothermal well consisting of single and two-phase flow sections was modeled in order to explore the variables important to the process. For this purpose a computer program was developed in a versatile form in order to be able to incorporate a variety of two phase flow void fraction and friction correlations.

A parametric study indicated that the most significant variables controlling the production rate are: hydrostatic pressure drop or void fraction in the two-phase mixture; and, heat transfer from the wellbore to the surrounding earth.

Downhole instrumentation was developed and applied in two flowing wells to provide experimental data for the computer program. The wells (East Mesa 8-1, and a private well) behaved differently. Well 8-1 did not flash and numerous "shakedown" problems in the probe were encountered. The private well did flash and the instrumentation detected the onset of flashing.

A Users Manual was developed and presented in a workshop held in conjunction with the Geothermal Resources Council. User feedback to the manual resulted in an improved document that will allow engineers to rapidly evaluate several parametric design considerations.

The laboratory analyses, the computer program, well testing, and the Users Manual will be refined in the next phase of the research.

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Appendix I. University of Houston Final Report

I. INTRODUCTION

A. General

The effective use of liquid dominated geothermal deposits can hinge on a seemingly minor link in the chain of producing equipment. The well itself, when properly designed, can often mean the difference between a successful or an unsuccessful geothermal development. The well design must be made in accordance with the productivity of the producing formation and with the type of process to be used on the surface. If the well is too large in diameter, for example, even for only a portion of its depth, production may not occur at all since the continuous steam phase would not be able to exert sufficient shear stress on the liquid to pull it out of the well.

The relationship of well design to the balance of the system occurs because of certain characteristics of geothermal deposits. Liquid dominated systems (unless highly over-pressurized) will be produced as a two phase flow of steam and liquid as flash vaporization occurs in the well. The pumping of an all-liquid stream up the well is a possibility, but the high temperatures and often the salinity of the brine make this an uncertain alternative. Research and development on down-hole pumping has been progressing, but we feel it is not sufficient to rely completely on the success of these efforts.

In addition, when a choice of producing horizons is available for an existing well, the best choice may not be the more permeable horizon. Because of the characteristics of two phase-flow, a small increase in the bottom hole pressure can lead to higher production rates and higher wellhead pressures that a large increase in permeability. This is especially important because the efficiency of the Rankine cycles to be used to convert geothermal heat to electricity are strongly dependent on wellhead temperatures. By analysis of the two phase flow--pressure relationships up the well, the best producing horizon can often be chosen. This is particularly critical for moderate temperature geothermal

deposits (350°F to 500°F) where the flow rate is especially sensitive to small changes. It is conversely less critical to higher temperature deposits, such as those in Mexico (600°F).

Initiating flow in geothermal wells is oft-times difficult, moreso as the well becomes deeper, the permeability lower, and the liquid temperature lower. A case-in-point is the first well drilled by the Bureau of Reclamation at East Mesa in the Imperial Valley of California where the brine temperature is almost 400°F at a depth of about 8000 feet, and the permeability is on the order of 10 millidarcies. Initiation of flow would often require several days, but was achieved immediately by the injection of nitrogen gas that stimulated early formation two phase flow. For routine operation, however, nitrogen injection is expensive and can be dangerous if the well is shocked too rapidly so as to produce high speed liquid slugs within the well. Two-phase flow design procedures, on the other hand, will permit the development of design criteria for initiation of flow in such cases by easily operated, less expensive, and less hazardous stimulation techniques.

Production rates can often be increased while maintaining higher wellhead pressures by continuously injecting gas into the well which forces the development of a more efficient two-phase flow regime. This is completely analagous to gas-lift operations in the oil industry. This approach must be carefully considered to assure that the cost of gas injection does not exceed the value of benefits derived from the increased production rates and increased pressure.

As a corollary to the continuous gas injection technique, the injection of CO₂ gas into the geothermal well can also prevent scaling and plugging due to the brine chemistry. When the brine is high in calcium carbonate, the release of CO₂ that occurs when the brine flashes will increase the pH and cause the precipitation of the carbonate. By increasing the partial pressure of CO₂ in the vapor stream, the pH will tend to remain stable and keep the carbonate in solution as bicarbonate.

B. Objectives

The objective of the proposed research program was to provide a Design Manual that would aid drillers of geothermal wells and designers of process plants for utilizing geothermal fluids to determine:

- The preferred producing zone in which to complete the geothermal well;
- The best well design, including the diameter of the producing stringer and allowable changes of diameter up the well;
- Production techniques, whether by natural flashing two phase flow, stimulated two phase flow by injection of liquids or gases, or suppressed flashing flow by pumping or artesian methods;
- Optimum wellhead temperature and pressure upon which to base the design of the energy conversion system on the surface;
- Scaling potential due to CO_2 release and calcium carbonate deposition on the well casing, and the potential effectiveness of scale control by gas injection.

The methods outlined in the users Design Manual permit rapid parametric studies to be made of specific geothermal field discoveries as related to specific energy conversion systems. The parametric study permits the best design to be made in terms of the long range potential of the field and allow the forecasting of when new wells will have to be drilled to maintain production levels into the conversion plant.

In summary, the fledgling geothermal industry lacks sufficient information on which to predicate its future. Much of the missing needed information is concerned with well design. This study will make its impact felt through:

- Improving production efficiency;
- Decision-making on best completion horizons;
- More effective stimulation techniques.

The key points, when implemented, can turn a "disappointing" well into a good producer and possibly an entirely new geothermal field. Therefore, the objective of the research program was to provide a Design Manual, based upon the well design factors mentioned above, which would guide geothermal producers in selection of operating parameters.

C. Attachments

The laboratory research performed by the University of Houston resulted in an extensive report that could not adequately be incorporated into the body of this report, thus is attached as an appendix.

II. RESEARCH PLAN

The nature of two-phase flow is presented in this section to create a foundation of knowledge that will permit the following sections to be seen in their proper perspective. The factors that affect flow rate and pressure drop in a two-phase flow system are first presented and then current state of design and theoretical knowledge are evaluated. These characteristics of two-phase flow are then related to geothermal production wells.

For example, certain minimum flow rates are required for the well to operate at all, in contrast to single phase situations where any flow, no matter how small, can be sustained. The discussion also presents the approach whereby the wellhead pressure and temperature of the liquid-vapor mixture can be calculated as a function of the flow rate, downhole pressure, temperature and brine chemistry, well depth and well diameter profile.

The calculation procedures discussed here apply to the well alone for a given bottom hole pressure. The actual downhole pressure for any particular case is determined by coupling the flow equations in the well with the flow equations for the reservoir. The results of this study will permit this coupling to be accomplished, with inclusion by reservoir engineers of the properties of the subject reservoir.

The technical description begins with the steady-state mechanical energy balances over the phases in question. The experimentally measured pressure drop, dP , in any conduit represents the combined effects of friction, acceleration and changes in potential energy due to change in elevation relative to the earth's datum.

A. Single Phase Flow

For the flow of a single phase, these terms are readily related through the mechanical energy balance.

$$-dP = dP_f + \frac{g}{g_c} \rho dh + \frac{\rho V}{g_c} dV \quad (1)$$

Here dP_f is the pressure drop due to friction, h is the vertical elevation and V the velocity. For a compressible fluid, the density and thus the velocity, the friction coefficient and the pressure gradient vary along the length of the pipe. For a single fluid phase, an equation of state relates the local pressure and density of the fluid when the flow is adiabatic. For a single phase, the following expressions are available

$$dP_f = \frac{2fG^2}{g_c \rho D} dz \quad (2a)$$

$$dh = \sin \theta dz \quad (2b)$$

$$\rho V = G \quad (2c)$$

$$P = \gamma RT\rho \quad (2d)$$

where G is the mass velocity, ρ the density, θ the angle the conduit makes with a horizontal plane, f the friction factor, z the axial distance along the pipe and γ the compressibility factor. Substitution of these quantities into Equation (1) and integrating over the pipe length, z_ℓ gives

$$-\int_{P_0}^{P_\ell} dP = -(P_\ell - P_0) = \frac{G^2}{g_c} \int_{\rho_{\ell 0}}^{\rho} d(\ell/\rho) + \frac{2G^2}{g_c D} \int_0^{z_\ell} \frac{f dz}{\rho} + \frac{g}{g_c} \int_0^{z_\ell} \sin \theta \frac{P}{\gamma RT} dz. \quad (3)$$

Equation (3) provides the desired relationship between the pressure drop, $(P_\ell - P_0)$, the mass flow rate, G , the length of tube, z_ℓ , and the tube diameter, D . Functionally, this equation is of the form

$$\phi [(P_\ell - P_0), G, z_\ell, D, T_0] = 0. \quad (4)$$

The additional term which appears in the equation, such as the compressibility or the density, is calculable from physical property correlations for the particular substance which is flowing. The result is not easily obtainable analytically since, in an adiabatic system, as the fluid moves along the pipe and the pressure drops the compressibility, γ , the temperature, T , the viscosity and the Reynolds number and, thus the friction factor f , all vary with z . However, the relationship can be obtained by numerical integration of Equation (3). This provides, for single phase flow, a direct and very accurate way to calculate the relationship between flow rate, tube diameter and length and inclination angle once the available pressure drop, $(P_L - P_O)$, and inlet temperature, T_O , are known and so long as the flow remains subsonic.

B. Two Phase Flow

The same differential mechanical energy balance as in Equation (1) accurately describes the situation in two phase flow. However, because of the absence of an equation of state, such as Equation (2d), and because of the complex nature of the pressure drop due to friction, the integration of this equation is a much more complicated process for two-phase flow. The reasons for this may be understood from the following considerations:

1. At any position along the tube, the mixture density ρ , is related to the density of the individual phases by

$$\rho = \rho_L R_L + \rho_G (1 - R_L)$$

where R_L is the local volume fraction of liquid in the mixture. The local density, ρ , is needed to determine the hydrostatic contribution to pressure drop, the acceleration term and the friction term. However, to determine ρ , it is necessary to know not only the local density of both the vapor and liquid phases, but the local volume fraction of liquid as well. This is a problem in fluid mechanics, not in phase behavior or thermodynamics.

2. R_L , the local volume fraction or holdup, depends strongly on the flow pattern or configurations which exist in the pipe. Experience has shown that a variety of patterns can be expected. Figure 1 is an example of flow patterns which have been observed in horizontal tubes while Figure 2 shows typical patterns for vertical systems. At high liquid loadings and small amounts of vapor, bubble flow takes place.

This is the situation in a geothermal well when the hot brine, during its flow up the well, just begins to vaporize, having reached the height in the well where the local pressure equals the saturation pressure. An increase in the vapor flow rate (which occurs in a geothermal well as flashing continues up the well) results necessarily in the successive flow patterns of stratified, slug, and annular flow. As the vapor rate increases still more, the liquid film is gradually destroyed and an annular/dispersed flow takes place. In vertical tubes, such as geothermal wells, the stratified flow regions will not occur. It should be clear that in order to predict the phase density, ρ , a knowledge of the phase distribution across the pipe is necessary. This in turn requires information on the flow pattern or the conditions for existence of one of the observed flow pattern types.

3. An additional complication is encountered in the two-phase flow systems when vaporization takes place along the tube due to a drop in pressure. In this case, the relative volumetric flow rates of liquid and vapor as well as the vapor density and liquid viscosity vary along the bore. Because the flow pattern which exists depends on these same factors, the flow pattern can also be expected to vary. The situation is described schematically in Figure 3. It is seen that water enters at the bottom of the pipe. As vaporization takes place, bubble or churn flow is observed first. As more vapor is formed, the mixture passes through regions of slug flow, annular flow and

FIGURE 1. Flow Regimes in Horizontal Flow

A BUBBLE

B SLUG

C ANNULAR

D FOG OR DISPERSED

E SPRAY ANNULAR

FIGURE 2 . Flow Regimes in Vertical Flow

FIGURE 3 . Regime Transitions During Vaporization

dispersed flow. In principle, as the static pressure continues to fall, complete vaporization can take place and dry steam is produced. In geothermal wells, such an extent of conversion is unlikely and the situation will normally be similar to that in gas/oil wells where substantial quantities of both gas and liquid are discharged at the surface. It is expected that most naturally producing geothermal brine systems will arrive at the well-head in a spray flow regime or the liquid flow regime, although other flow regimes will exist at the same time at various depths within the well.

4. The flow pattern controls the holdup, given known amounts of liquid and vapor flowing. For example, consider the flow regime of liquid which is fully dispersed in the gas phase and is thus moving at the vapor velocity with little or no slip (as actually takes place in dispersed flow). If V represented the phase velocity, Q the volumetric flow rate, and A the pipe flow area, while the subscripts L and G designate liquid and gas respectively:

$$Q_L = V_L R_L A \quad (5a)$$

$$Q_G = V_G (1-R_L) A \quad (5b)$$

In this equation ($R_L A$) is assumed to be the effective cross-sectional area for liquid flow. That is, it is assumed that the volumetric holdup is the same as the 2-dimensional, or cross-sectional holdup. The same applies to $(1-R_L)A$.

If $V_L = V_G$, as discussed above, then

$$\frac{R_L}{1-R_L} = \frac{Q_L}{Q_G} \quad (6)$$

and the holdup is readily calculated from the known flow rates of gas and liquid, Q_G and Q_L . But if the liquid flows as a

thin annular film, then the velocity of the gas and liquid are not equal and

$$\frac{R_L}{1-R_L} = \frac{Q_L V_G}{Q_G V_L} \quad (7)$$

However, V_G is much greater than V_L when annular film flow takes place. Thus, for the same gas and liquid flow rates R_L will be substantially larger.

5. The pressure drop due to friction can be expected to depend not only on the flow rates, phase properties and holdup, but on the flow regime that exists. This is seen from the relation between wall shear and frictional pressure drop.

$$\tau_W = \left(\frac{dP}{dz} \right) f \frac{D}{4} \quad (8)$$

$$\text{where } \tau_W = [(u + \epsilon\rho)] \frac{du}{dy} \bigg|_{y=0} \quad (9)$$

where ϵ is the eddy viscosity, u is the local fluid velocity and y is the coordinate measured normal to the wall. Again, the situation can be made clear by a comparison of dispersed and annular flow. In dispersed flow, the wall shear depends on the viscosity and density of the gas, and is proportional to the velocity gradient of the gas at the wall. If the same volumetric liquid flow rates were confined as a film on the wall, the τ_W would depend on the properties of the liquid even if the velocity gradient were the same. Since the ratio of liquid to vapor viscosity can be one order of magnitude, much higher wall shear and pressure drop can be expected in annular flow. Thus, the prediction of the local pressure drop at any position along the pipe depends on the flow regime which exists there. However, the flow regime is controlled by the local rates and properties of the phases, and thus depends on the inlet flow rate and conditions and on the cumulative vaporization and pressure drop which has taken place downstream of the inlet position.

Based on the discussion of items (1) through (5) above, it is seen that integration of Equation (1) for this case of two phase flow is a task of considerably greater complexity than for single phase flow. It requires information for the prediction of flow patterns, correlations for pressure drop and holdup associated with each of the flow patterns, and a well designed numerical technique for integrating up a tube where properties, velocities and flow patterns may be changing. In a flashing system, there is also a need for reliable thermodynamic data in the two phase domain over the full region of interest. One numerical technique for accomplishing this numerical integration is presented later in this section.

C. Variation of Local Pressure Gradients Along the Pipe

The variation of local pressure drop as a result of flow regime variations can be seen from Figure 4. These curves were made available in a private communication to Dr. A. E. Dukler from Mr. N.C.J. Ross (Royal Dutch Shell Exploration and Production Company, the Netherlands). Shown is the result of experimental measurements of dimensionless pressure gradient, G and R_L , holdup from upflow in a vertical pipe of about three inches in diameter. A light oil and natural gas were the fluids and the coordinates are defined as

$$G = \frac{g_c}{\rho_L g_L}$$

$$V_{SG} = Q_G / A$$

$$V_{SL} = Q_L / A$$

where A is the pipe cross-sectional area. It is important to note several features of the graph. At high liquid rates (high V_{SL}) the liquid and gas are well mixed as in froth or bubble flow. The pressure gradient and R_L remain about constant and independent of the amount of gas present, over a wide range of gas flow rates (V_{SG}). But at lower

FIGURE 4. Holdup and Gradients from Producing Wells (Note: Coordinates are Defined in Text)

liquid rates, (the condition which would exist as soon as appreciable vaporization of water would take place in a geothermal well) the pressure gradient changes drastically with the gas rate. As gas rate is increased the pressure gradient first decreases rapidly. Normally one expects pressure gradient to increase with increasing flow--but the decrease takes place here because the pressure drop due to hydrostatic level decreases more than the pressure drop due to friction increases. The sum of these two reflect a decrease in measured drop. Eventually, with still more gas, the incremental change in density as a result of further decreases in R_L becomes small and, as seen, the friction then begins to control and the pressure gradient then begins to rise again. Experiments show that this takes place during conditions of annular flow. From the graph, the potential usefulness of gas injection, as used in the oil fields becomes evident. If the flow conditions are such that operation is at high gradients, then a suitably designed injection system can move the operating point to a position of lower gradient. Then more total flow will take place within the limits of the total pressure drop available. However, such conditions must be carefully designed to be sure that the power required to provide the inert gas does not exceed the cost of producing at a different set of operating conditions without inert gas.

D. The Problem of Minimum Lift Velocity

When a well is "shut in" the difference between reservoir pressure and well head pressure exactly balances the hydrostatic level of the fluid within the well. If the surface valve is opened and the pressure on the fluid lowered at that point, upward flow takes place. Once a significant amount of liquid is vaporized because of the reduced pressure, the vapor becomes the continuous phase in the flow system because of the large density difference between water and steam.

At that point, liquid can continue to flow upward only if sufficient energy is transferred to the liquid by the gas to drag the liquid along.

This transfer of energy takes place by means of shear stresses exerted on the liquid droplets or film at the gas-liquid interface. The upward force so exerted must be greater than the force of gravity and that due to wall resistance acting on the liquid. If this is not the case, then the gas may continue to flow out the well but the liquid will fall back and accumulate until it chokes off the well. Sometimes, when liquid accumulation takes place, the resulting behavior is like that of a Cotrell pump with large slugs of liquid pumped to the surface intermittently. During this period, the entire surface collection system can be flooded out with large volumes of flashing liquid. In other cases the well simply ceases to function.

An analysis of such a situation reveals that there are two possible conditions under which high enough liquid flow rates can be sustained to insure continuous removal of liquid from the well. The first condition occurs when the liquid flows as an annular film on the wall. In this flow regime, the condition of incipient flooding corresponds to the situation where the interfacial shear, τ_i , is just equal to the weight of the liquid in the film, since the wall shear stress is zero at this transition condition. Using well established equations for the velocity distribution in the film it is possible to relate the film thickness to the flow rate. The criterion for transition is then

$$\frac{\tau_i g_c}{m \rho_L g_L} = 1.0 \quad (10)$$

where m is the film thickness. Since such film thicknesses are usually small compared to the pipe diameter, D . Equation (8) can be used to give

$$\frac{g_c D}{4 m \rho_L g_L} \left(\frac{dP}{dz} \right) f = 1.0 \quad (11)$$

and the relation may be used to estimate the frictional gradients which are suitable for annular flow.

To insure that liquid which is moving in the form of entrained drops continues to be carried out the tube, a somewhat different approach is necessary. Here the criterion must be that the drag force between the gas and liquid must exceed the force of gravity, or it may be viewed as requiring a condition that the vapor velocity exceed the free fall velocity of the largest drops which exist. Thus to make this calculation, it is necessary to estimate the drop size. This can be done using the well established concept of a critical Weber number:

$$We = \frac{V_R^2 \rho_G D_D}{\sigma g_c} \quad (12)$$

where V_R is the relative velocity between the gas and drop and D_D is drop diameter. Both theory and experiment have shown that if the Weber number exceeds a critical value a liquid drop would shatter. Thus, the largest stable drop size under any condition of surface tension, density and relative flow rate can be obtained from Equation (12). For this discussion, it should be clear that calculations of design flow to be expected in vertical wells must include a secondary set of calculations to determine the minimum allowable flow conditions for operation under reduced load. This minimum flow will be a function of flow regime as discussed above, and will thus be different at different points in the well.

E. The Problem of Sonic Velocity

At high volumetric vapor rates, the vapor phase is continuous, as discussed above. This condition is to be expected even for small fractional vaporization of the feed. For example, for steam at 300 psia the specific volume of the vapor is 6.5 cubic feet per pound while that of the liquid is 0.017 in the same units. Thus, if 20% by weight of one pound of liquid water is vaporized, the volume fraction of vapor in the resulting mixture will be over 99%. The maximum rate at which it is possible to flow vapor through the pipe is then controlled largely by the velocity of propagation of sound through the vapor.

Up to velocities which approach 95% of this sonic velocity, the method discussed above can be used to calculate the relationship between flow rates and pressure drop in pipes of known size. However, when operating at or near the sonic velocity, the usual models are no longer valid. Under those conditions the propagation of the shock wave up the tube will cause atomization of the liquid film into very small droplets and the usual relationships for flow regime and pressure drop must be abandoned and new models created. There has been considerable study of the phenomena of two phase critical flow over the past two years. However, it is necessary to recognize that few experimental verifications are available and the models are substantially different than for subsonic flow.

F. The Analogy With Two Phase Flow Design in the Oil and Gas Production Industry

The plans to produce steam from geothermal wells and the long established practice of producing oil and gas are perfectly analogous for high API gravity crudes. Vaporization takes place up the wellstring in both cases. With crude oil, the lower molecular weight fractions vaporize so the composition varies along the pipe. In both cases, the design problem is to predict the expected rate of production given the fluid properties, wellhead pressure and well bore diameter and the task is to make economically attractive selections of line size and surface pressure. Field separation at the wellhead is commonly used for oil wells and the liquids from a large number of such separations are gathered and flow to a plant for further processing. In these gathering lines, the pressure drop causes further vaporization once the gathering line operates in two-phase flow. The analogy is perfect, and this makes it possible to apply to the case of the geothermal system the enormous amount of technical effort and large investment already made in developing design methods for the oil fields.

G. The Calculation Procedure

The following discussion presents one way in which the numerical

integration of Equation (1) may be accomplished to determine the pressure drop-flow rate-surface temperature relationship for two-phase flashing flow up a geothermal well. This example is based upon a specified bottom hole pressure. A complete solution to the problem requires, of course, that this bottom hole pressure at the well be calculated by consideration of the size and permeability of the geothermal reservoir.

Because the properties and flow regimes can vary along the flow string, a convenient way to make this calculation is to solve Equation (1) by increments.

$$\Delta P_i = \left(\frac{\Delta P_f}{\Delta z} + \frac{g}{g_c} \bar{\rho}_i \sin \theta_i + \frac{G_i^2}{g_c} \frac{\Delta i}{\Delta z} \left(\frac{1}{\rho} \right) \right) \Delta z_i \quad (13)$$

where the subscript, i , represents the conditions over the respective increment.

For each calculation, the following factors are provided as input data.

GIVEN: Desired production rate, bottom hole pressure and temperature, profile on diameter, profile on angle θ .

PROCEDURE:

- a. Select an assumed ΔP_i .
- b. Calculate the adiabatic flash flow rate of each phase, temperature, and local properties.
- c. Obtain linear average of flow rates, and properties over that increment.
- d. Estimate flow regime.
- e. Based on flow regime which exists, flow rates and properties, calculate the holdup R_L and the frictional pressure gradient, $\frac{\Delta P_f}{\Delta z}$.
- f. Calculate the mean density from $\bar{\rho} = \rho_L R_L + \rho_G (1-R_L)$

- g. Calculate Δz_i from Equation (13).
- h. If the value of Δz is too large because of an anticipated change in pipe diameter or angle θ_i in the increment, or if the flows change too drastically over this increment, return to (a) and reselect the value of ΔP_i . Otherwise, select the next increment ΔP_i and calculate the next value of Δz , while varying the diameter and angle as required, and introducing injection gas as desired.

Several aspects of this calculational problem become clear from a study of this sequence:

First; the pressure drop is the dependent variable and is the product of such a calculation procedure. In actual practice, however, the pressure drop is the independent operating variable which is controlled. Thus, many such calculations as discussed above must be made and the results plotted and correlated to obtain useful general results.

Second; the calculations require a procedure for estimating flow regime, holdup and frictional gradient in each increment.

Third; the calculations must be done with relatively small increments and this calculation must be repeated many times for different conditions in order to map the regions of interest. In order to do this efficiently, it would be necessary to prepare a computer program to execute these steps. Such a program would automatically integrate, search for flow regime, make the necessary calculations and indicate the existence of undesirable conditions such as critical flow. The existence of such a program would make an efficient execution of the parametric study possible.

H. The Frictional Pressure Drop

There now exist almost 30 correlations and equations by which pressure drop in vertical two phase flow may be calculated. Some of

these can be eliminated because they are too restrictive in applications and have already been shown to be inefficient. Part of the effort of this study was to evaluate the best correlations for vertical flow for each of the flow regimes.

For this purpose there was available to the authors an extensive data bank on IBM tape which included both published and unpublished data on pressure drop in oil and gas wells. A full and complete evaluation like this has already been executed by one of the authors for the case of horizontal flow. No such thorough study for vertical flow was contemplated here, nor is it needed. However, the equations judged best by our experience were tested against actual field data before they were used in the correlation. The program was written with the pressure drop correlations as subroutines so that an improved program can be evolved as better correlations are obtained at a later date from additional field tests.

I. The Problem of Flow Regime

In recent years considerable new information on flow pattern transitions for high pressure steam has become available. Much of this research was stimulated by the growing use of the pressurized water and steam-water nuclear reactor cooling systems. Photographic data, analytical studies and correlations for steam-water at pressures up to 3500 psi have been developed and shown to be reliable for pipe sizes less than 4". Part of the effort of this study extended these correlations to larger line sizes.

J. The Question of Uncertainty

The prediction of pressure drop, holdup and flow regime is not a fully determined procedure at this time. Uncertainties exist in each step. However, as in most physical situations, integral results are sometimes insensitive to differential assumptions in a model. Thus, it

is of value to test the sensitivity of the computed results in a complex calculation to changes in the sub-calculational procedures, assumptions or correlations. Such a sensitivity analysis was executed and the results will help in the planning of test runs in the field and in the laboratory.

III. THEORETICAL BASIS

Of the several correlations analyzed, and described fully in the First Annual Report (1976), four frictional pressure loss and two holdup correlations were selected for further study. The friction equations were those of Orkiszewski, Dukler (Case 2), Wisman, and Martinelli, while the holdup correlations were those of Orkiszewski and Hughmark. In addition, the Orkiszewski correlation was modified so as to use the Dukler and Hughmark equations when in the annular flow regime. Based on several analyses of data that have already been made, including data presented by some well operators, it is believed that the correlation set of Dukler (Case 2) for friction losses and Hughmark for holdup is the best set presently available; this correlation set will be used for further calculations until a basis for modification of the correlations is developed.

The Orkiszewski correlation was studied primarily because it is so widely used in the petroleum industry for oil and gas wells. However, a careful analysis of this correlation shows that it can fail under conditions typical of geothermal wells, so as to give a negative pressure gradient at some points in the well, which is, of course, erroneous. This problem is compounded by the fact that the error is hidden within internal calculations, so that the casual user of the correlation would not be aware of the problem and would accept incorrect results. In addition, several test runs were made of cases for which a negative gradient did not appear, using the Orkiszewski method; however, the results invariably gave flow rates much higher than those experienced by the operators. For the above reasons, the Orkiszewski correlation will no longer be considered for geothermal wells.

The recent Wisman correlation was included in the preliminary evaluation because it compared well to the Dukler equations with respect to a set of data available to Wisman. However, Wisman used this same data set to develop the constants required for his correlation, in effect

curve fitting his equations to the data set he was analyzing. When the Wisman correlation is applied to geothermal data that is not part of his original data bank, it appears to give flow rates much lower than experienced.

The Martinelli and Dukler correlations were included because they gave the best results in a study published in AICHE J., Vol. 10, 1964 when compared to an extensive data bank for two-phase flow in horizontal lines. In addition, the Dukler correlation is based on a fundamental similitude analysis, which is essentially independent of flow direction. When applied to geothermal conditions, Martinelli also seems to give very low flow rates. The Dukler correlation, combined with Hughmark's holdup correlation, appears to give results in the right range of values.

Several other correlations could be included, and have been considered by other investigators of two-phase flow. In general, it was decided that other correlations are too empirical and would not lend much to our understanding of geothermal production wells. Particularly because of the empiricism involved, it is unlikely that these correlations could be extrapolated effectively beyond the range of parameters for which they were developed, and still maintain reasonable accuracy.

Among these other correlations, one of those proposed by Hagedorn and Brown merits further discussion, since it has been suggested by T.L. Gould in the Journal of Petroleum Technology, 833, 1974, that it closely predicts data from some geothermal wells. However, our own analysis shows that the Hagedorn-Brown equations give good predictions only after the assumption of an arbitrary rate of heat transfer from the well bore to the surrounding formation. Since this heat transfer rate is not actually known, this calculational procedure cannot be used to effectively evaluate the correlation for momentum transfer in two-phase flow.

Further work is planned with the correlation set of Dukler and Hughmark. As more data from well tests and laboratory tests become

available, these correlations will be modified if necessary, or new correlations will be developed.

B. Computer Program

A program has been developed that can be used either to design a geothermal well based on given downhole conditions, or to predict the wellhead pressure for a particular case and to compare the predicted value with existing data. Used in the design mode, the program will produce a set of parametric data to provide the basis for the Users' Manual which is discussed in a later section. Used in the analysis mode, the program will provide the means to evaluate various proposed correlations against the data bank.

The program can be used for either flashing water wells, or for two-phase, air-water flow. The latter capability is required because the University of Houston laboratory data was developed for air-water systems. Based on given bottomhole conditions, the program determines wellhead pressure and the steam-to-brine production ratio. The downhole conditions are provided as input data by the user, as a result of a separate analysis of reservoir hydraulics. Combined with a reservoir calculational system, the program will permit an analysis of the production characteristics of an entire field with respect to both space and time.

The program is applicable to cases for which flow into the well bore is either single phase or two-phase flow, and to cases for which the well diameter changes at various depths. It will determine the effect of several dissolved gases in the brine on the two-phase flow regime and pressure drop; the gases considered are CO_2 , H_2S , NH_3 , N_2 , and H_2 . Other gases could be added. At present, however, the equilibrium constants that determine the distribution of these gases between the steam and liquid phases were not available for this phase of the research.

The program also considers the effects of dissolved salts on the brine physical properties. The point at which calcium carbonate scale

can be expected to form is also predicted. These effects are only accurate for low salinity brines, and do not correct for the non-ideality encountered in solutions with high salt concentrations. It is estimated that reasonable accuracy is achieved with total dissolved salt contents of three percent by weight or less.

When the effects of dissolved salts and gases are taken into consideration, a double trial and error calculational procedure is required by the computer program. At present, a convergence problem is sometimes encountered during the calculations, and when this occurs no solution can be attained. The exact reason for occasional lack of convergence has been recently determined and corrective procedures are being developed.

IV. FIELD TESTING

When the research was initiated, three naive assumptions were made; 1) that adequate downhole pressure/temperature monitors existed, 2) that they could be obtained at reasonable cost, and 3) that sufficient two phase flow wells were available for testing. Very early into the program the first two assumptions were found to be invalid. About a year into the program, the third assumption was found to be invalid.

A. Development of Downhole Pressure/Temperature Probe

The originally proposed program of research envisioned that measurements of pressure and temperature would be performed in static wells, and in wells producing at various flow rates, as a routine part of the data-gathering program in support of the subsequent correlations and analysis.

By the fall of 1975 (i.e., 3 months after project initiation) it had become apparent, on the basis of assessment by the authors that the conventional techniques of pressure and temperature measurement in geothermal wells would not be sufficient to provide data of the accuracy required for the analysis anticipated by the research program.

These "conventional techniques" consist of mechanically driven pens that record a trace on a brass foil as a function of the relative expansion of a bourdon tube. The principle is used to record both pressure and temperature. The objection to the use of this type of measurement probe were:

- Real time measurements cannot be obtained, and therefore the actual hydrodynamic mode of the mixed fluid cannot be evaluated (e.g., slug flow, bubble flow, etc).
- Reproducibility is questionable, particularly in pressure measurements at the higher temperatures. Some users have found that a temperature of 500°F induces weakening of the drive springs within the mechanism, and performance is altered permanently.

- A time delay of several minutes is required to warm (or cool) the probe to each new temperature.

These considerations led the authors to analyze the present-day state of pressure and temperature measurement technology, in search of devices to measure temperatures up to 500°F and pressures up to 3,000 psia. It was soon apparent that the well-logging industry has an upper limit of about 300°F, due to the fact that their instruments, packers, and cables employ insulating and construction materials that will not withstand higher temperatures. Therefore it was decided to use probes to generate electrical signals downhole, and to transmit these signals to the surface via a cable where they would be conditioned and recorded.

In the use of such probes, it was recognized that calibration would be necessary. Therefore a calibrator assembly was constructed with its own temperature and pressure measurement capability.

1. Pressure and Temperature Probe

For measurement of pressure, the bourdon tube principle was believed to offer the best promise of reliability plus accuracy. Therefore a potentiometric type transducer was specified for service at conditions up to 500°F and 3,000 psia in a hydrogen sulfide-containing environment. Gulton Industries, Inc., was the only vendor among those contacted who was responsive. Therefore the order was placed for a Gulton Industries Model 2091 unit, consisting of an Inconel helical bourdon tube, with specifications of $\pm 1\%$ static error band (full range) and a temperature coefficient of 0.012% per degree fahrenheit of full range. The static error includes effects of nonlinearity, hysteresis, frictional contributions, resolution and repeatability. The temperature effect can be calibrated out, and the pressure transducer accuracy is therefore approximately the static error band ($\pm 1\%$ of full range).

Four transducers were ordered: two for service up to 500°F and 2,000 psia, and two stock transducers for service up to 350°F and 500 psia. The order was placed on December 29, 1975; delivery was taken on May 23, 1976.

For measurement of temperature, the platinum resistance principle was judged to offer the best combination of accuracy and simplicity. Four probes were ordered from the YSI Sostman Company, with specifications as follows: accuracy $\pm 1/3\%$, self heating coefficient approximately 20 milliwatts/ $^{\circ}\text{C}$ in H_2O at 3 ft/sec, response speed about 5 sec in water at 3 ft/sec.

The body of the containing probe is an alloy steel tube 2 inches in external diameter. Figure 5 shows a scale drawing of the probe with all elements in place. The shape of nose was suggested by the U.S. Geological Survey for minimum chance of hanging upon protrusions down the well bore. The cage was designed such that the outlet slot area exceeded the nose inlet port area, thus assuring flow-through under even low flows. In this drawing, the pressure transducer is shown at left, just ahead of the retaining wall. The cable head connector was purchased from Gearheart-Owen Industries. The probe is designed for use with a seven-conductor armored cable.

2. Calibrator Assembly

The calibrator body is shown in Figure 6. The assembly is designed to hold the probe in position such that its temperature probe is a fraction of an inch from a similar probe inserted through the flange at right. Pressure is fixed by an Ametek model T-30 dead weight tester, designed for use to a pressure of 3,000 psia in increments of 20 psia, with an accuracy error of $\pm 0.1\%$ of output pressure. The dead weight tester is filled with distilled water. The calibrator is heated by a wrap-around Iso Tape heater rated at 2200 watts; it employs 110-volt power for convenience in field use. Temperature is controlled by a West Controller, model 405-R with a range of 100°F to 800°F . The design temperature of the calibrator is 600°F . Temperature is read with a Doric Scientific digital temperature readout, model 410A-PT-4-F. The calibrator assembly is fully insulated and has a heat-up time of about 1 hour to achieve 500°F .

Figure 5.

PIPE: 3" SCHEDULE 80 SEAMLESS
 .312 WALL, 3.5" O.D. STEEL
 FLANGES: 600LB FORGED CARBON STEEL
 SEALS: 2 - 242 SILICONE "O" RINGS
 RETAINED IN MACHINED STEEL SPACERS

Figure 6.

3. Riser Assembly

When the probe is mounted on the cable, and any necessary sinker bars have been attached, the assembly is housed in a vertical section of pipe that is mounted on a tee that permits the probe to be inserted down the well. A valve is always required to isolate this vertical pipe, which is termed the "riser".

In consultation with Dr. J. K. Sullivan of the U.S. Geological Survey, a riser assembly was designed and purchased. It consists of the following parts:

- 3 10 ft sections of steel pipe, 6 inches in diameter, threaded at both ends
- 3 hammer unions to join the pipe sections
- 1 swadge nipple to reduce the diameter at top
- 1 adapter
- 1 wire line pressure control device ("lubricator") to permit entry of cable into well

In the field, since each well features its own particular size of tee, an adapter would have to be used to mate the riser assembly with the flange of the tee.

Technologies were identified which would permit the required measurements, but the capability for lowering the probes into geothermal wells was soon recognized as the limiting factor. The cost of a hoist or winch assembly and an armored cable with internal conducting wires was prohibitive for this study and furthermore, it was believed that such a hoist-cable assembly might duplicate the existing or planned capability of other DOE supported agencies.

Commercial well-logging companies were willing to assist the project on a contract basis. However, the projected cost was quite high, on the order of \$10,000 per week, and there was genuine doubt that the cables employed could withstand geothermal temperatures.

The authors then identified two organizations which would be capable of lowering probes into geothermal wells under flowing conditions, i.e.,

U.S. Dept. of Interior
Geological Survey
Water Resources Division
Borehole Geophysics Research
(W. Scott Keys, Research Hydrologist)

Sandia Laboratories
Drilling Research Division
(M. M. Newsom)

Both of these organizations are involved in measurements within geothermal wells, and both expressed interest in cooperating with the DOE program.

The Denver Research Institute and the U.S. Geological Survey reached agreement on mutual cooperation in research whereby the U.S.G.S. would employ the DRI-developed probe in conjunction with some of their geothermal measurement series.

Sandia Laboratories agreed to cooperate with the Denver Research Institute in the future, when Sandia's advanced capability for downhole geothermal measurements was fully developed.

B. Well Availability

After the first well test of the probe in the East Mesa 8-1, it became apparent that other tests would not be possible without the cooperation of private owner/operators. In preliminary discussions with the private companies it became a matter of prime concern to address liability for well damage and personal injury, fishing insurance, and the confidentiality of data. The problem of liability proved to be fairly simple to solve. The University of Denver's general coverage was adequate to protect the well, and personal liability could be covered if each party would assume responsibility for its personnel. Fishing insurance and the treatment of proprietary data, however, involved extensive work, not planned for in the original program plan. An agreement was finally created under

which DOE, the owner/operator, the U.S.G.S., DRI, and Coury Associates could function. A copy of this agreement follows and may well serve as a model for other programs operating under similar circumstances.

The U.S. Department of Energy approved project expenditures for fishing insurance and approved the confidentiality of data with the rationale that the data was to be used to support the computer model.

C. Well Tests

The first operational test of the downhole probe occurred in late September 1976. The test was conducted on Well 8-1 in the East Mesa field with the U.S.G.S. providing the wireline. The second test occurred in July 1977 at private well in California, again with the U.S.G.S..

A. Well 8-1

As described in the Semi-Annual Report (June 1-Nov. 30, 1976), this first test was ultimately considered as a failure from the two-phase standpoint -- the well did not flash, but was considered a success from the probe "shakedown" aspect.

Problems in fluid leakage into transducers, poor cable head connections, wire failures, etc. made many of the results questionable. Field modification to the probe resulted finally, in satisfactory results. The diary of the first test is attached.

B. Private Well

As agreed by the owner/operator, DRI, Coury and Associates, U.S.G.S., and DOE; the results of this test are maintained in confidentiality. Several general observations however can be made.

- The well did flow in two phase.
- The downhole probe did delineate the on-set of flashing and gave good results.

- Computer simulations showed that the Hughmark correlation for holdup and Dukler (Case II) for friction gave the most accurate match to measured well profiles.
- A sudden, unexpected temperature decrease at depth was attributed by the owner/operator to a leak in the casing permitting the influx of cooler liquid.
- The U.S.G.S. used a high temperature cable for this test and it performed excellently, however there were problems with the cablehead termination.

AGREEMENT

THIS AGREEMENT is entered into this _____ day of _____, 1980, between Denver Research Institute of the University of Denver, (Colorado Seminary), a Colorado corporation ("DRI"), and Republic Geothermal, Inc., a California corporation ("RGI").

WHEREAS, DRI has requested permission from RGI to perform downhole testing, using various logging tools and related equipment, in a geothermal well owned and operated by RGI; and,

WHEREAS, RGI is willing to grant such permission on the terms and conditions hereinafter set forth;

NOW, THEREFORE, in consideration of the premises and the mutual covenants hereinafter set forth, the parties agree as follows:

1. Right to Conduct Tests. Subject to the terms and conditions hereinafter set forth, DRI shall have the right to cause its personnel to enter upon the site of a geothermal well to be designated by RGI and, using logging probes and equipment to be supplied by DRI, to conduct downhole testing in such well as required of DRI to comply with its subcontract no. 4XP0-6302P-1 with the Los Alamos Scientific Laboratories (LASL). It is anticipated that such testing will commence on or about 1 December, and that it will be completed on or before 31 December 1980. Testing using the DRI logging probe is to be started and completed within five days, not necessarily consecutive, during the above testing period. The exact commencement date and duration of the testing shall be determined by mutual agreement of DRI and RGI provided, however, that RGI may, in its sole discretion, at any time it deems necessary, cancel

or postpone the well tests. All physical operations on the well and test facility equipment shall be under the control and direction of RGI or its subcontractors at all times.

2. Lost or Damaged Equipment. DRI shall assume full responsibility for any of its equipment, including cable, which is lost, destroyed, or damaged in the well, at the well site, or while being transported by DRI, and RGI shall have no responsibility therefor, unless such loss, destruction or damage was caused solely by the negligence or willful misconduct of RGI, its officers, employees, subcontractors or agents, in which case the party causing the loss, destruction or damage shall be liable therefor.

3. Fishing.

a. In the event it becomes necessary for RGI or any of its subcontractors to fish for any DRI equipment lost in the well while tests are being made using the DRI probe, and if such loss was not caused solely by the negligence or willful misconduct of RGI, its officers, employees, subcontractors or agents, DRI shall be financially responsible to RGI for the cost of such fishing operations to the extent of \$25,000 for each such occurrence. RGI shall have the right either to conduct such equipment fishing operations in-house with its own equipment and personnel and to charge DRI therefor at RGI's regularly established rates for such work to the extent of \$25,000 or to engage a subcontractor for such work and to charge DRI therefor at actual cost to the extent of \$25,000.

b. In the event RGI or its subcontractor should be unable, within the \$25,000 cost limitation specified in the above sub-paragraph a, to recover from the well any DRI equipment lost in the well while

tests were being made using the DRI probe, RGI may, at its own expense, take whatever further action it deems necessary toward recovery of such equipment. If RGI does not wish to recover DRI equipment from the well, DRI agrees to abandon such equipment, and RGI shall not be liable or responsible for the cost of such equipment, unless the loss of such equipment in the well was caused by the negligence or willful misconduct of RGI its officers, employees, subcontractors, or agents.

4. Indemnification and Liability.

a. Indemnification. DRI agrees to save, hold, indemnify, and defend RGI its officers, employees, agents, and subcontractors free and harmless from and against any and all claims, demands, suits, actions, and causes of action for bodily injury, death, and loss of, damage to, or loss of use of property, and financial loss of any kind, sustained by any person (including, but not limited to, employees of DRI, RGI, or their subcontractors) arising out of, or alleged to arise out of, or in any way connected with the work to be undertaken by DRI and RGI hereunder, or out of any defect, malfunction, or deficiency of any equipment supplied by DRI or RGI or their subcontractors (including well site, wellhead hardware and facilities and well bore) unless such bodily injury, death, loss or damage of property, loss of use of property, or financial loss is caused solely by the negligence or willful misconduct of RGI, its officers, employees, subcontractors or agents, in which case the party which caused the loss shall be liable therefor.

b. Liability for Damage. DRI shall be liable to RGI for damage to or destruction of the well site provided by RGI, the wellhead hardware and facilities, or the wellbore, as a result of the work to

be undertaken by DRI pursuant to this Agreement, unless such damage or destruction is caused solely by the negligence or willful misconduct of RGI. its officers, employees, subcontractors or agents, in which case the party causing the damage or destruction shall be liable therefor.

c. Limitation of Liability. The liability of DRI under this Section 4 shall be limited to amounts recovered by or paid on behalf of DRI from insurance maintained by DRI as evidenced by Certificate of Insurance to be furnished to RGI by DRI prior to the date upon which testing by DRI is to commence.

5. Information and Data Disclosure. All information furnished by RGI to DRI relating to the geothermal well in which tests are made, the field in which it is situated, or the geothermal reservoir is confidential and is not to be disclosed to any outside party without the prior written consent of RGI. Such information may be used by DRI for "in-house" research and development purposes only. DRI shall be responsible for taking reasonable measures to assure that its employees having access to such information are informed of the foregoing confidentiality requirement. DRI may disclose to DOE in writing well testing procedures followed in using DRI pressure/temperature tool.

6. Information to be Furnished to RGI. All information derived from the use of the DRI logging tools, and associated experimentation, shall be furnished to RGI by DRI. Information required to be furnished to RGI pursuant hereto shall include resultant raw data only. Interpretations, opinion, comments, or access to resultant computer programs or models shall be included only at the sole discretion of DRI.

7. Assignments. This agreement may not be assigned in whole or in part by any party hereto.

8. Applicable Law. This agreement shall be governed by and construed in accordance with the laws of the State of California.

IN WITNESS WHEREOF, the parties have executed this agreement on the date first above written.

UNIVERSITY OF DENVER (Colorado Seminary)

By _____

Title _____

REPUBLIC GEOTHERMAL, INC.

By _____

Title _____

IV. UTILIZATION PLAN

It has been the intent of this plan to disseminate the research results of the program to industry. Our goal has been to develop a Users Manual and to test this in a series of workshops.

A. Users Manual

The user should be in a position to evaluate rapidly several parametric design conditions. The results of this preliminary analysis would be an approximate envelope of costs for the various design conditions. Individual data items provided by the Users' Manual, which serve as the basis for the cost envelope, include productivity of a single well as a function of wellhead pressure and temperature, when several field and well characteristics are taken into consideration. These characteristics include the formation transmissivity, temperature, and static pressure, and the well depth and diameter profile.

The calculation procedure requires the synthesis of two sets of calculations that are separated for the sake of convenience and generality: these two sets are made up of calculations for the single-phase flow zone in the well and calculations for the two-phase flow zone in the well. The single-phase flow zone would be handled by standard equations quickly run on a pocket calculator. The calculations for the two-phase zone take place on graphical reductions of data poured out by the computer program. The computer program is being modified slightly to provide this data in a format convenient for construction of graphs. The finalized version of the Manual will be produced during the projected next phase of the research.

B. Workshop

An all-day workshop was conducted by Coury and Associates as a special feature of the Geothermal Resources Council Annual Meeting in

May 1977. Two half-day workshops were conducted, each attended by ten to twelve people. The sessions were highly successful. Unfortunately, many other people signed up for the workshop, but did not attend; these were apparently people with only a marginal interest who signed up because the workshop was free. As a result, the workshop was widely advertised as being over-subscribed, and many who wanted to attend were excluded.

A valuable two-way information exchange occurred at the workshop. Industry representatives learned the "what" and the "why" of the research effort, and they learned how to use the computer program and to interpret the results. Many learned the basic principals of two-phase flow and developed an improved intuition for what to expect from their wells. Almost all participants submitted data of particular interest to them and received the computer output for their own examination.

In a complementary sense, we learned what many of the specific problems and concerns of industry are, and subsequently were able to make minor program modifications to accommodate them. In this vein is the added ability of the program to deal with changes of well bore diameter at various depths. In addition, several suggestions were made as to what would be valuable additions to the computer program. These suggestions were written up by workshop participants and sent to the University of Denver for review. The major suggestions were:

- Calculate in the reverse order, to obtain downhole conditions when wellhead conditions are given.
- Include heat transfer from the wellbore to the surrounding formation, to be applicable to wells in the first few days after startup, to assist in the evaluation of data taken during field tests;
- Provide a more precise calculation procedure for non-ideal, high salt concentration brines;
- Predict the scaling tendencies of silicates and heavy metal sulfides in non-ideal, high salt concentration brines;
- Provide for metric units in the program input and output;

- Include the pressure drop versus flow characteristics of the horizontal and sloped surface gathering network, through which geothermal fluids flow from several wells to a central processing location;
- Adapt the program to be applicable to geopressured wells.

SUMMARY HIGHLIGHTS

- The computer program is essentially complete and will be finished during the next phase.
- The Users Manual will be completed next phase.
- Final laboratory data and well test data will provide a more explicit basis for selection of preferred correlations for calculating the two-phase flow parameters.
- From the success of the first workshop, we anticipate conducting a second when the project attains the next higher level of accomplishment.
- The downhole probe offers the opportunity to provide real-time data in a flowing two-phase well.

Chronology of First Geothermal Well Tests

Date of Testing: September 27 through October 1, 1976.

Location: Well 8-1 at the Bureau of Reclamation East Mesa
Geothermal Test Facility, Holtville, California.

Participants: Coury and Associates, Inc.
Denver Research Institute
U.S. Geological Survey

Assisted by: Bureau of Reclamation
Bechtel Co.

Monday, September 27

Calibrator unloaded into laboratory trailer and partially assembled. Probe partially assembled.

Logging truck unloaded. Riser assembly (two sections) and lubricator assembled. Fence surrounding well rolled back to provide free section for cable passage. Crane and pulleys stationed. Survey temperature probe installed. Riser assembly lifted into place using logging cable, with Survey probe tight against lubricator.

Large guard valve (250 turns) was opened. Eight-inch gate valve above wellhead assembly was opened.

A temperature log was made to the bottom of the well, to about 6,000 feet. No log made as tool was pulled out.

Tuesday, September 28

Riser assembly removed and laid on ground. Survey temperature probe was replaced with gamma radiation probe. A log to bottom hole was made with this tool.

High pressure pipe fittings required to install relief valves on calibrator were obtained from the Yuma Desalting Test Facility.

Wednesday, September 29

DRI electronics specialist arrived. Resoldered wiring within DRI temperature and pressure probe (T/P probe) to repair break and to correct circuit. Probe

was assembled, and tested for general temperature element response by holding under exhaust pipe from truck.

Riser assembly removed and laid on ground. Survey probe was removed, and replaced by T/P probe. Three sinker bars were installed on cable and third riser section was put in place.

Electrical hookup of calibrator completed by DRI electronics specialist.

T/P log within well begun. Appeared OK, and descent of tool was initiated. Shortly thereafter, temperature reading was anomalous. Superficial trouble shooting indicated that a break in one of the leads occurred. Decision to continue run, to check out pressure sensor and probe assembly at high pressure, was made. Descent continued until maximum reading of pressure sensor was attained, at about 4600 feet.

During descent, temperature sensor appeared to be working again. Toward end of descent, temperature readings became erratic, indicating a loose connection.

Maximum pressure reading attained (about 1050 ohms) was less than expected maximum of over 1100 ohms that had been attained in DRI laboratory.

During rapid ascent, several stops were made in attempt to learn time constant for the sensors, as they approach temperature equilibrium.

After T/P probe was returned to wellhead, riser assembly was dismantled and probe was removed. After probe cooled, it was dismantled for inspection. A considerable amount of water was found within the probe body. One of the pressure sensor leads had almost worked loose. One of the temperature sensor leads was found broken within its sheath, but not loose.

It appears that high resistance short circuits due to water, and broken but loosely touching leads could explain the problems encountered.

Thursday, September 30

Repairs were made to T/P probe. The body and O-ring assemblies were cleaned and regreased. O-rings appeared OK. Male pipe fittings were removed and cleaned. They were wrapped with teflon tape and replaced securely.

Excess lead wire from sensors was removed. Individual wires were exposed; this proved to be very difficult because of the manner in which the insulation on the temperature sensor lead wires alternately tore and stuck.

Lead wires were soldered directly to the cable head connector pins at the top of the T/P probe. They were insulated by means of heat shrink insulation tubing.

T/P probe was attached to cable again and riser assembly was replaced on

wellhead. Another logging run was initiated.

Shortly thereafter, anomalous readings were attained. The Dana voltmeter seemed to be drifting. Because of previous problems, the T/P probe itself was suspected. However, extensive trouble shooting could not isolate the problem, and it was decided to try another voltmeter. The Cimron voltmeter was obtained from the calibrator unit and installed.

Reasonably stable readings were obtained, and it was thought that the problem was resolved. It was later discovered, however, that the Cimron output changed when the scale setting was changed from ohms to kilo-ohms, or from kilo-ohms to megohms.

Since it was already late in the day, it was decided to complete the logging run.

At the end of the run, the riser assembly was dismantled and laid on the ground. The T/P probe was removed and opened for inspection while still hot. As the head was being gingerly removed, it was suddenly shot out of the probe by internal pressure.

The interior of the T/P probe appeared dry and in order. The pressure was believed due to the heating of air inside the probe body. The probe was reassembled.

Tests were made on the cable system. It appeared that there was leakage between one of the pins and the ground.

Friday, October 1

The T/P probe was disassembled again, having cooled off since assembly. This time, it was under vacuum, verifying that it was now leakproof.

The cable was tested thoroughly, and leakage was found. The entire cable head assembly was dismantled and resoldered.

Testing of the Cimron voltmeter with fixed resistors showed definitively that its reading depended on the scale. Its readings are relatively stable, however, in the face of high-resistance leakage.

Testing of the Dana voltmeter with fixed resistors showed it to be stable and in good order. It is unstable, however, in the face of high-resistance leakage.

Testing of the Dana voltmeter with fixed resistors showed it to be stable and in good order. It is unstable, however, in the face of high-resistance leakage.

The T/P probe was replaced on the cable, and the riser assembly was put back in place. Logging to 2000 feet was initiated. The system appeared to be in good order.

The well was started up in single phase flow. Flow rate of liquid downstream of the silencer, over the V-notch weir, was about 220 gpm at about 212°F. Wellhead temperature and pressure were about 220°F and 76 psig, respectively.

A logging run was made to test the sensors and to determine the source of heat loss to the surroundings. The heat loss in the horizontal pipe could not be determined, because only short stem thermometers were available, while the thermowell at the silencer required a long stem. Accordingly, this temperature measurement was not accurate.

The eight-inch gate valve at the silencer was opened wide, in parallel with the open three-inch globe valve, to reduce the pressure restriction in the horizontal line, in an attempt to induce two-phase flow in the well. However, the increased flow rate in the horizontal line resulted in an increased frictional pressure drop, and maintained a high back pressure at the wellhead.

A large temperature loss from the geothermal fluid within the wellbore to the surrounding formation kept the wellhead temperature below the flashing point.

Well temperatures were monitored to permit an estimation of the rate of heating within the well. This proved to be excessive, and the run was terminated.

The riser assembly was dismantled and all equipment was stowed away. The three riser sections and associated hammer unions were stored within the fence with Bureau permission. The fence was left in the rolled-back position, again with Bureau permission.

The eight- and three-inch valves at the silencer and the two eight-inch valves at the wellhead were closed. The chain and lock on the side eight-inch valve was to be secured on Monday by the Bureau. The large guard valve was closed by a Bechtel employee.

The calibrator and the T/P probe were stowed on the Survey truck for return to Denver.

The crane operators invoices were signed by Glenn Coury for the Denver Research Institute.

APPENDIX I

TWO PHASE FLOW IN GEOTHERMAL ENERGY SOURCES

TWO PHASE FLOW IN GEOTHERMAL ENERGY SOURCES

FINAL REPORT
FOR
DENVER RESEARCH INSTITUTE
SUBCONTRACT TO DOE CONTRACT NO. E(11-15-2729)

UNIVERSITY OF HOUSTON
CHEMICAL ENGINEERING DEPARTMENT

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AN OVERVIEW

The flow rate in a geothermal well which is not pumped will adjust itself until the integral of the pressure gradients along the length of the well equal the difference between the bottom hole and wellhead pressure. Thus the task of predicting the flow rate to be expected, given the bottom hole condition, reduces to the need to predict the pressure gradient at each point up the well. Such gradients are the sum of the gradients due to hydrostatic and frictional contributions. For those portions of the bore experiencing single phase flow the problem comes down to knowing the fluid properties. For all possible flow rates both the hydrostatic and frictional contributions to the gradient can be calculated unambiguously.

However, for those sections of the well bore in gas-liquid flows, the problem is a much more difficult one. The pressure gradient due to friction depends in a complex way on the flow rates and fluid properties. Furthermore, the dependence changes as the relative flow rates change. In this case the hydrostatically caused pressure gradient no longer depends only on the density of each phase but the amount and geometry of the mixture. These problems are explored in this report. The various sections of the report are free standing and consider different aspects of the problems as follows.

Part I - An Analysis of Two Phase Flow in Geothermal Wells: The role of two phase flow in geothermal wells is explored. The results of a computer simulation of a well are discussed. It is shown that the two phase hydrostatic contribution to the gradient is of central importance to prediction of the flow rate. Therefore, it is important to be able to predict the two phase cross-sectional average void fraction or holdup. Various correlations for

void fraction are tested and discussed. It is concluded that no existing correlation has a satisfactory basis in physical reality largely because they ignore the fact that a variety of flow patterns exists up the well. It is suggested that flow pattern prediction must be the first step followed by modelling of the voids for each pattern.

Part II - Two Phase Flow Patterns in Vertical Tubes: Newly developed correlations for predicting flow patterns are presented here. The correlations are developed from realistic physical mechanisms which are suggested as controlling the transitions.

Part III - Flow Pattern Based Models for Voids and Friction in Vertical Tubes: For each flow pattern momentum and mass conservation equations are used to model the average void fraction. In several cases closed form solutions are possible. In others, method for estimating unknown quantities are presented. In a similar way, models for frictional loss for each flow pattern are developed.

Part IV - Experimental Studies of Voids in a Vertical Tube: A test loop was constructed and measurements of holdup executed for an air-water system. The experimental equipment is described, methods of measurement detailed and experimental results presented. Comparison is made with the models presented in Part III.

Part V - A Technique for Dynamic Measurement of Cross-Sectional Average Void Fraction: An important flow pattern is that of churn flow. This condition is characterized by inherently unsteady variations of voids with time. Some understanding of this time variation was considered necessary in order to model the voids and friction for this pattern. The results of an effort to develop an instrument to measure the time varying voids is presented.

PART I
AN ANALYSIS OF TWO PHASE FLOW
IN GEOTHERMAL WELLS

-3-

I. AN ANALYSIS OF TWO PHASE FLOW IN GEOTHERMAL WELLS

SUMMARY

A geothermal well consisting of single and two phase flow sections was modelled in order to explore the variables important to the process. For this purpose a computer program was written in versatile form in order to be able to incorporate a variety of two phase flow void fraction and friction correlation.

A parametric study was executed which indicated that the most significant variables controlling the production rate of a geothermal well are

- . hydrostatic pressure drop or void fraction in the two phase mixture
- . heat transfer from the well bore to the surrounding earth.

Under most operating conditions frictional losses are not dominant in fixing the production capability of the well. Therefore, the pressure profile which will be established depends largely on the density of the two phase mixture. The flow rate which will take place depends in turn on the pressure profile, since the amount of flow will automatically adjust so that the pressure drop across the well is exactly equal to the difference between the bottom hole and wellhead pressure. The void fraction which determines the hydrostatic head depends on the relative amount of gas and liquid flowing and for this reason the heat loss term is important.

It thus becomes clear that the void fraction plays a central role in predicting the performance of a well. For that reason a variety of published correlations for voids were tested in the computer program to determine the sensitivity of the results to the particular correlation selected. The results

showed that a wide variety of predicted profiles would be expected. Analysis of the correlations showed that no published relationship was based on physically sound concepts. Largely this was because all correlations neglected the fact that the flow pattern of the two phase system varied as the relative rates of gas and liquid changed up the borehole. No single existing correlation was structured to predict the holdup for all of these patterns.

Thus, the objective of the rest of the research program was directed to modelling the flow pattern and holdup in vertical tubes and of testing these models with experiments.

INTRODUCTION

The majority of geothermal deposits consists of pressurized, hence subcooled liquid (brine). These deposits are produced as a vapor-liquid mixture by geothermal wells operating at characteristic bottom hole pressure and temperature.

Under normal operating conditions the pressurized hot brine flows up the well and undergoes a pressure drop due to gravitational and frictional effects. At a certain well depth the local pressure attains its saturation value at the local liquid temperature, resulting in flash vaporization of the liquid. Flashing continues up the well due to further decrease of pressure and the vapor flow rate increases gradually. The two-phase mixture reaches the wellhead at a total mass flow rate determined by the total pressure drop along the well. The geothermal well therefore consists of two sections: the single phase region and the two-phase region.

The flow in the two-phase region, unlike the one in the single phase region, is complex. Because the pressure decreases, vapor is generated up the well and the volume fraction vapor or void fraction increases. This results in a variety of flow patterns that appear successively. In addition, the void fraction affects both the mixture density and viscosity and hence the local gravitational, frictional and, at high vapor flow rates, the accelerational pressure gradients.

It is thus apparent that the total production rate at the wellhead and, in particular, the relative mass flow rates of liquid and vapor, or flow quality, are a function of a fairly large number of variables. These include: bottom hole flowing pressure and temperature, wellhead pressure, well diameter profile, well depth and heat losses to the surrounding formation.

The ability to predict the relationship between the vapor and liquid flow rates at the wellhead, operating conditions and well characteristics is of prime importance for optimal design and reliable operation of geothermal production wells that contain both liquid and vapor. For this reason a detailed parametric study of the major variables that affect the pressure profile in the geothermal well, and hence its performance, was conducted in order to identify and define the pertinent problems that require further experimental and theoretical investigation.

The well and, in particular, its two-phase section were modeled on the basis of sound two-phase flow principles. A computer algorithm was developed for pressure profile calculations and simulation of well performance and written in a versatile form that allows incorporation of any desired pertinent correlation.

This algorithm was used to study the effects of total mass flow rate, heat losses to the surrounding formation and void fraction on the pressure profile and the vapor and liquid production rates. In addition, several existing void fraction and frictional pressure drop correlations were evaluated in order to determine their applicability to geothermal systems.

WELL MODELLING

Simulation of the pressure profile in the entire well and the evaluation of the effects of the operating variables and well characteristics on its performance require consideration of both the single phase and the two-phase sections of the well casing.

A. Two-Phase Section

The two-phase flashing flow is modeled by considering a vapor-liquid mixture that flows through the section in steady one-dimensional diabatic flow, with unequal phase velocities, namely, in slip flow. The phases are assumed to be in thermodynamic equilibrium.

The governing equations for evaluating the two-phase pressure drop were derived by numerical integration of the appropriate conservation equations for the two-phase mixture over a well segment i of length Δz_i . The length of the section is segmented since the flow rate of each phase varies along the well string.

The resulting momentum equation is given by

$$\Delta P_i = \frac{g}{g_c} \bar{\rho}_{TP_i} \Delta z_i + \left(\frac{d\bar{P}_f}{dz} \right)_i \Delta z_i + \frac{G^2}{g_c} \Delta_i \left[\frac{(1-X)^2 v_L}{1-\alpha} + \frac{X^2 v_G}{\alpha} \right] \quad (1)$$

where subscript i represents the conditions over the increment. Equation (1) relates the total pressure drop over the increment, ΔP_i , to the gravitational pressure drop, frictional pressure drop and the net rate of momentum efflux from the increment, or accelerational pressure drop. The mixture density, defined by

$$\rho_{TP} = \alpha \rho_G + (1-\alpha) \rho_L \quad (2)$$

and the friction pressure gradient, dP_f/dz , are averaged over the increment.

The integrated energy equation, given by

$$-\frac{\bar{q}_i}{W_T} = \Delta_i [h_L + X h_{fg}] + \frac{g}{g_c} \frac{\Delta z_i}{J} + \frac{G^2}{2g_c J} \Delta_i \left[\frac{(1-X)^3 v_L^2}{(1-\alpha)^2} + \frac{X^3 v_G^2}{\alpha^2} \right] \quad (3)$$

relates the changes of enthalpy, potential energy and kinetic energy of the flowing mixture to the wellbore heat losses per unit mass. The kinetic energy term is usually very small and may be neglected.

The average heat transfer rate to the surrounding formation was approximated by

$$\bar{q}_i = U_{TP} \pi D \Delta z_i (\bar{T}_i - \bar{T}_{R_i}) \quad (4)$$

where U_{TP} is the overall heat transfer coefficient and $\bar{T}_i$ and $\bar{T}_{R_i}$ are the average temperatures of the mixture and the formation, respectively.

In principal, equation (1) may be solved for the pressure drop once the length increment is specified. The required entrance and exit flow qualities are obtained from equation (3). However, since the exit flow quality depends upon the specific enthalpy of the phases, and hence, on the exit pressure, which is not known apriori, it is more advantageous to specify ΔP_i and compute the corresponding length of the increment.

It should be noted that equation (3) cannot be solved independently for the increment exit quality since it requires knowledge of Δz_i and, hence, is coupled to equation (1). Consequently, the pressure profile is obtained by a simultaneous interative numerical solution of the two equations, recast into the following form:

$$\Delta z_i = \frac{\Delta P_i - \frac{G^2}{g_c} \Delta_i \left[\frac{(1-X) v_L}{1-\alpha} + \frac{X^2 v_G}{\alpha} \right]}{\frac{g}{g_c} \bar{\rho}_{TP_i} + \left(\frac{dP}{dz} \right)_i} \quad (5)$$

$$X_i = \frac{1}{h_{fg_i}} \left[(h_{L_{i-1}} - h_{L_i}) + X_{i-1} h_{fg_{i-1}} - \frac{\bar{q}_i}{w_T} - \frac{g}{g_c} \frac{\Delta z_i}{J} \right] \quad (6)$$

where subscripts $i-1$ and i represent the inlet and outlet conditions.

The solution procedure requires knowledge of both the local void fraction and frictional pressure gradient. These must be obtained from a correlation or model provided as input to the computation.

B. Single Phase Section

The pressure profile in this region is obtained by numerical integration of the mechanical energy balance and the energy balance over successive increments as the properties of the pressurized liquid vary along the flow path. The governing equations are

$$\Delta P_i = \left(\frac{g}{g_c} \bar{\rho}_{L_i} + \frac{2\bar{f}_i G^2}{g_c \bar{\rho}_{L_i} D} \right) \Delta z_i \quad (7)$$

$$T_i = T_{i-1} - \frac{1}{\bar{C}_{P_{L_i}}} \left[\frac{\bar{q}_i}{w_T} + \frac{g}{g_c} \frac{\Delta z_i}{J} \right] \quad (8)$$

$$\bar{q}_i = U_{SP} \pi D \Delta z_i (\bar{T}_i - \bar{T}_{R_i}) \quad (9)$$

COMPUTER PROGRAM

Simulation of a geothermal well and parametric study of its performance were carried out on a digital computer. For this specific purpose an algorithm was written to execute the numerical solution of the appropriate governing equations iteratively. The algorithm consists of two subprograms, one for the single phase region and one for the two-phase region of the well. The subprograms are coupled at the flash point depth. Since the wellhead for quality X_{top} is not always available, the calculations start at the bottom of the hole, where pressure and temperature are specified.

The single phase subprogram is straightforward. It uses the following input: bottom hole pressure and temperature, total mass flow rate, production casing I. D., relative pipe roughness, overall heat transfer coefficient and the formation temperature profile.

The pressure profile of the pressurized liquid is evaluated by stepwise calculations up the well using equations (7)-(9). For a prescribed well length increment the subprogram computes the gravitational and frictional components of the pressure drop as well as the drop in temperature. An iterative procedure is used as the liquid exit temperature is not known at the outset. As both the temperature and pressure vary along the well, the liquid density is evaluated at the local pressure and temperature, neglecting the brine salinity. The Fanning friction factor is computed using an expression proposed by Dukler (1961).

As a final stage of the calculations in this region the program determines the position in the well where the subcooled liquid attains its saturation pressure, and thereby begins to flash, using the iterative Regula-Falsi method.

The subprogram output includes: pressure and temperature profiles, flash point pressure and temperature and length of the single phase region.

The two-phase subprogram is by far much more complex. It evaluates the pressure profile of the two-phase mixture that flows under flashing and slip conditions by a stepwise solution of equations (4)-(6). The program can incorporate any desired void fraction correlation and frictional pressure drop correlation as subroutines. The required input is: wellhead pressure, total mass flow rate, casing I. D., well depth, pipe roughness, overall heat transfer coefficient and formation temperature profile.

The subprogram takes over at the flash point. It computes the length of successive well increments, corresponding to a specified ΔP_i step, by evaluating the average gravitational and frictional pressure gradients and the accelerational pressure drop, equation (5). The frictional pressure gradient is determined from a selected correlation.

The pressure components depends upon the void fraction and the flow quality. However, only the latter needs to be considered directly as the α -X relationship is established by a void fraction correlation. As the flow quality varies along the increment due to continuous flashing, both its entrance and exit values are required. The exit quality is computed from equations (6) and (4). As the length of the increment is not known apriori, an initial value of Δz_i is estimated from equation (6) by neglecting the frictional and accelerational terms. Based on the computed exit quality, a new Δz_i value is computed from equation (5). A successive approximation procedure is used to affect convergence.

The manner in which the calculations are completed depends upon the available input data. In the case where the bottom hole pressure, P_{Bot} , is available the calculations are completed when the local pressure in the well equals the wellhead pressure. If, on the other hand, P_{Bot} is not know, its initial value has to be assumed. New values are obtained using a Regula-Falsi procedure and

the program terminates when the total sum of the Δz_i increments converges to the given well depth.

The subprogram output includes: wellhead flow quality, pressure, temperature, quality and void fraction profiles, overall gravitational, frictional and accelerational pressure drops and the length of the two-phase section of the well.

The program uses steam table data for evaluation of specific enthalpy and specific volume of the phases. The required physical properties of the two phases as a function of temperature are entered as tables. Data interpolation is performed by appropriate function subprograms.

The present form of the program was evolved for a specific purpose, namely, a parametric study of the important variables that affect the geothermal well performance and behavior. However, once the program is proven by reliable field data, it can easily be adapted to field usage as well as for optimum design of new wells.

FIELD DATA

Two sets of field data were used in this investigation. Both sets came from U. S. Bureau of Reclamation test wells in California.

Holtville Well. The parametric study of well behavior was conducted on the basis of the following preliminary data from a geothermal test well near Holtville, California, as supplied by Dr. Glen Coury:

Well depth: 6000 ft.

Casing I. D.: 8 7/8"

Bottom hole temperature: 370°F

No-flow down hole pressure: 3000 psia

Wellhead pressures: corresponding to temperature range of 240-270°F

Total flow rate: 200-300 gpm equivalent liquid

Brine salinity: 2%

The ranges of the wellhead pressure and total flow rate were narrowed down to the following data base:

$$W_T = 150,000 \text{ lb/hr}$$

$$T_{Top} = 260^\circ\text{F}, \text{ corresponding to a saturation pressure of } 35.43 \text{ psia}$$

It should be noted that although the data could be used as a preliminary basis for the proposed parametric study, it was incomplete and therefore insufficient for testing any desired geothermal well model since vital input data were unavailable, namely, bottom hole flow pressure, wellhead vapor and liquid flow rates and the temperature profile of the formation.

The formation temperature profile was approximated by a linear temperature-depth relationship with the following end conditions:

$$(T_R)_{Bot} = 370^\circ\text{F} @ \text{ depth} = 6000 \text{ ft.}$$

$$(T_R)_{Top} = 100^\circ\text{F} @ \text{ ground surface}$$

Mesa 6-1 Well. For the purpose of testing the applicability of several existing void fraction correlations, the more complete data from the Mesa 6-1 test well in Imperial Valley, California (Lundberg, 1973) were chosen, as follows:

Bottom hole depth: 7100 ft.

Casing I. D.: 8.625 in.

Total mass flow rate: 50,000 lb/hr

Bottom hole flow pressure: 1330 psia

Bottom hole temperature: 388°F

Wellhead pressure: 18 psia

The quasi-equilibrium formation temperature profile was taken from Figures 2 and 5 of the Lundberg report and approximated as two straight lines connecting at a depth of 2500 ft. The ground surface temperature was estimated at 110°F.

RESULTS AND DISCUSSION

The present investigation was conducted in two stages. The first part was devoted to a parametric study of the effect of several major variables on the well's pressure profile and performance. The second part was devoted to an assessment of the applicability of some published void fraction correlations to geothermal wells.

A. Parametric Study

As the total number of variables affecting well performance is by no means small, it was decided to analyze the effect of three important parameters, namely, heat losses, total mass flow rate and void fraction. Frictional effects were also evaluated.

The void fraction of the flashing mixture was determined using the empirical correlations of Hughmark (1962), Hughmark and Pressburg (1961) and Hagedorn and Brown (1970). The two-phase frictional pressure drop was evaluated by the correlations of Dukler, et al (Case II, 1964) and Wisman (1975).

1. Heat Losses

The effect of heat losses was studied using the Holtville well data as a basis and assuming a formation temperature profile. The heat transfer rate from the bore to the surroundings was estimated by equations (4) and (9) using a single overall heat transfer coefficient for the entire well. The heat transfer coefficient was

varied in the range $0-10 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ at a constant total mass flow rate of $150,000 \text{ lb/hr}$ and constant wellhead pressure of 35.43 psia . The variation of the pressure, flow quality and void fraction profiles with heat transfer coefficient is summarized in Figures 1-4. The void fraction was evaluated by the Hughmark correlation and two-phase frictional pressure drop by the Dukler correlation.

Figures 1 and 2 indicate that at any constant U , the single phase pressure gradient is practically constant while in the two-phase region it decreases rather sharply. The flash point location penetrates deeper into the well as the heat loss rate becomes smaller.

It is interesting to note that at a fixed wellhead pressure, the flowing downhole pressure increases with heat loss rate. As a result larger pressure drops are required to deliver the geothermal fluid to the wellhead at the desired production rate. The reason becomes apparent from the following considerations. The pressure drop at the selected flow rate is due mainly to gravitation since the frictional pressure drop in this case constitutes only 0.7% of the total pressure drop. Consequently, as the flash point location approaches the wellhead the length of the all-liquid region becomes larger, resulting in higher pressure drops.

Figure 2 describes the pressure profile in the two-phase region. It illustrates the marked effect of the heat transfer coefficient on both the flash point depth and on the local pressure gradient. Based on the assumed temperature profile of the formation, which might not be too realistic, the model predicts that for $U > 10 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ the well will produce only liquid, which is obviously undesirable.

Figure 3 illustrates the pronounced effect of the heat loss on the flow quality profile in the well and, in particular, on the relative mass flow rates of vapor and liquid at the wellhead. The wellhead quality decreases markedly as the heat loss becomes significant since flashing takes place over progressively shorter sections of the well.

The predicted void fraction profiles are shown in Figure 4. Here, too, the effect of heat losses on the profile per se is marked, but unlike the quality the wellhead void fraction varies over a relatively narrow range. Hughmark void fraction correlation indicates that at any given value of U the void fraction increases sharply from 0 up to about 0.5 over a very short length of the well. As the bubbly flow regime rarely extends over $\alpha = 0.35$, it seems to be unimportant in geothermal wells, at least under the conditions studied. Various flow regime transition criteria indicate that in all cases the well fluids reach the wellhead in an annular-dispersed flow regime, except for $U = 10 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ where the mixture is in churn flow.

The important conclusion from this phase of the study is that heat losses have a pronounced effect on the pressure drop over the well and, in particular, on the relative flow rates of the fluids at the wellhead, which is of prime importance from the practical point of view. Consequently, any model that is based on the assumption that the well operates under adiabatic conditions is totally unrealistic, unless substantiated by field data.

A practical corollary to the above conclusion is that in the case of significant heat losses the wellhead pressure would have to be

lowered in order to obtain reasonable values of X_{Top} . However, the trade-off of increased steam production could be rather dubious as the low pressure steam could be unsuitable for direct power production.

2. Mass Flow Rate

The Holtville data base was restricted to a single flow rate. This restraint was relaxed by varying the total mass flow rate in the range 25,000-450,000 lb/hr while keeping P_{Top} and U constant at 35.43 psia and 1 Btu/hr-ft²-°F, respectively. The Hughmark void fraction correlation and the Dukler frictional pressure drop correlation were retained.

Figure 5 displays a plot of the total mass flow rate vs. bottom hole flowing pressure, which is directly proportional to total pressure drop over the well. The curve demonstrates the interesting result that once the bottom hole and well pressures are fixed, two mass flow rates may satisfy the total pressure drop in the well.

In principle, the explanation lies in the two-phase region and particularly in the interplay between the gravitational and frictional pressure drops. For the sake of simplicity, consider the case where the liquid flow rate is fixed while the rate of the vapor is increased. At low vapor rates the gravitational pressure gradient decreases more than the accompanying increase of the frictional pressure gradient, with the net result that the total pressure gradient decreases as the total flow is increased. At high vapor rates, however, the frictional gradient begins to control and the total pressure gradient rises. Such a behavior was argued by Gould (1974) in the case of a vapor dominated

geothermal well and demonstrated experimentally by Ros (1961) in the case of gas-liquid flow in pipe.

A detailed examination of the extensive computer output indicated that the average total pressure gradient does indeed exhibit a minimum with respect to W_T . However, the total two-phase pressure drop, which is a product of the pressure gradient and the length of the section, increased with the total flow rate whereas the total single pressure drop exhibited a minimum with respect to the total flow. Consequently, it is quite clear that the form of the curve in Figure 5 is dictated by the relative lengths of the two well sections rather than by the respective pressure gradients.

The length of the single phase region for $U = 1 \text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$ is plotted against W_T in Figure 6 and it can be seen that it exhibits a minimum at $W_T = 175,000 \text{ lb/hr}$, which corresponds to the minimum total pressure drop shown in Figure 5. Conversely, the length of the two-phase region reaches a maximum at this flow rate. The question still to be answered is why does the length of the two-phase region attain a maximum?

A possible suspect is the position of flash point in the well that depends on the temperature profile in the single phase region and hence on the flow rate. In order to clarify this point the program was run under adiabatic conditions. No significant change was observed, as seen from Figures 5 and 6.

As the length of the two-phase region depends upon the void fraction, the effect of this parameter was investigated next.

3. Void Fraction

The effect of the void fraction-flow quality relationship was examined by checking and comparing the following void fraction correlations: Hughmark (1962), Hughmark and Pressburg (1961) and Hagedorn and Brown (1970). The α -X relationship of the homogeneous flow model was also included for the sake of comparison.

The Hughmark-Pressburg correlation was soon abandoned as it was restricted to flow rates up to 180,000 lb/hr.

Attention was focused on the two-phase region and adiabatic flow was assumed. This condition was most suitable for comparing the selected correlations since both the total pressure drop and the quality range were constant and thus independent of W_T . The inlet pressure corresponded to the adiabatic flash point pressure of 173.37 psia and the wellhead pressure was fixed at 35.43 psia.

Hughmark Correlation. The results are shown in Figures 7-9. As seen from Figure 7 the length of the two-phase region reaches a maximum value at $W_T = 125,000$ lb/hr, which corresponds to the minimum total pressure drop shown in Figure 5 for the adiabatic case. Figure 8 displays pressure profiles over a wide range of flow rates. The pressure at a given distance from the flash point does not vary with W_T in an orderly fashion since the length of the two-phase region exhibits a maximum. At high flow rates the pressure near the wellhead begins to level off. The relationship between the three pressure drop components and W_T is shown in Figure 9. At low flow rates the gravitational pressure drop dominates. As W_T increases frictional effects become significant so that the attendant contribution of the gravitational pressure

drop diminishes. The acceleration pressure drop becomes a factor only at high flow rates, as expected.

Hagedorn-Brown Correlation. The dependence of the two-phase region length upon W_T is displayed in Figure 7. Unlike the case of the Hughmark correlation, the predicted length decreases monotonically as W_T increases in complete analogy to single phase flow. The pressure profiles at various flow rates, shown in Figure 8, vary in a regular fashion. The predicted dependence of the pressure drop components on the total flow rate is shown in Figure 9. Frictional effects begin to be felt at low flow rates and the gravitational pressure drop decreases more pronouncedly. The contribution of the accelerational pressure drop is rather small over the entire flow rate range.

Homogeneous Flow Model. The trend of the results, as shown in Figures 7 and 9, is similar to that of the Hagedorn-Brown correlation. In particular, the predicted length of the section decreases as the flow rate is increased. It should be noted that at $W_T = 25,000$ lb/hr the predicted length exceeds the well depth of 6000 ft. Figure 9 indicates that the contribution of the frictional pressure drop is significant over most of the W_T range. It exhibits a maximum at $W_T = 375,000$ lb/hr for reasons that are not yet understood. The accelerational pressure drop becomes important at high flow rates and at $W_T = 450,000$ lb/hr its relative value is roughly 40% of the gravitational pressure drop and 50% of the frictional pressure drop.

Comparison of Void Fraction Correlation. The comparison is summarized in Figures 7-12. Figure 7 indicates that the section length

based on the homogeneous model void fraction and Hagedorn-Brown correlation decreases monotonically whereas the one computed on the basis of Hughmark correlation reaches a maximum as noted earlier. In the flow rate range of 25,000-150,000 lb/hr the predicted length varies significantly from one model to another, the Hughmark correlation predicting the lowest values. At high flow rates the difference diminish. These trends are reflected by the pressure profiles, as seen in Figures 11 and 12.

The most illuminating result of the comparison is shown in Figure 10, where the wellhead void fraction, α_{Top} , is plotted against the flow rate. The curves predicted by the Hughmark and Hagedorn-Brown correlations exhibit opposite trends with respect to W_T .

The reason why the Hughmark correlation predicts progressively larger void fractions becomes apparent upon examination of its structure:

$$\alpha = \frac{K(Z)}{1 - \frac{v_L}{v_G} \left(1 - \frac{1}{X}\right)} \quad (10)$$

The void fraction depends upon the so-called Bankoff factor that is correlated empirically with a Z factor which, in turn, is a function of the mixture Reynolds and Froude numbers and the input liquid volume fraction:

$$Z = \frac{Re^{1/6} Fr^{1/8}}{C_L^{1/4}} \quad (11)$$

As W_T increases so do the Reynolds and Froude numbers and hence the Z factor. Consequently, K increases with the flow rate. Under adiabatic flow conditions both X , v_G , and v_L at the wellhead are independent of W_T and hence are constant. It follows that α_{Top} should increase with the flow rate, as seen in Figure 10.

It should be noted that in the W_T range of 350,000-450,000 lb/hr, α_{Top} is constant. This stems from the fact that the Z values computed in this range exceeded the maximum value specified by Hughmark. As a result the computer program freezes its value at the maximum value and hence α_{Top} remains constant.

The Hagedorn-Brown correlation is derived on a different basis altogether. α is defined as an "effective" in situ average void fraction, which may not be the same as the "true" in situ average void fraction. In addition, the correlation is not based on experimentally measured values of α but rather on values computed from total pressure drop data obtained from a test oil well. As a result it is hard to find a rational for the $\alpha_{Top}-W_T$ relationship shown in Figure 10.

Figure 10 supplies all the necessary clues for interpreting the results of Figure 7. Consider first the homogeneous model. As there is no slip between the phases the void fraction at any given pressure and hence α_{Top} are constant, regardless of the flow rate. At the lowest value of W_T the gravitational pressure drop is dominating resulting in the highest value of the two-phase region length. As the flow rate increases the frictional pressure drop rises sharply (Figure 9) resulting in a further decrease of the section length. At still higher values of W_T the accelerational pressure drop begins to pick up, causing a leveling off of the length.

The same line of reasoning holds for the case of the Hagedorn-Brown correlation all the more so since α_{Top} decreases as the flow rate is increased.

Lastly, consider the Hughmark correlation where α_{Top} increases monotonically with the flow rate. At low flow rates the gravitational pressure is dominating and hence the section length increases first. It reaches a maximum due to the interplay between ΔP_G and ΔP_f and then falls off as ΔP_f and ΔP_A become more and more effective.

The important conclusion from the foregoing is that the void fraction profile has a profound effect on the length of the two-phase region and on the pressure drop components. Thus, it is essential that void fraction correlations be available that are soundly based in physical reality.

4. Frictional Pressure Correlations

The predictions of Dukler (1964) and Wisman (1975) correlations were compared at $W_T = 150,000$ lb/hr for the values of U shown in Figure 1. The differences between the two correlations were rather small. The Dukler correlation predicted slightly higher pressure drops, the maximum difference being 7%. Either one of the correlations could therefore be used.

B. Model Testing

The data for the Mesa 6-1 test well were used in order to evaluate the applicability of the selected void fraction and frictional pressure drop correlations as they included well pressure and temperature profiles under flow conditions, flash point depth and formation temperature profile. The data did not include, however, the relative flow rates at the wellhead

which, aside from its practical importance, is vital for testing any given model.

The predictions of the models, using the Dukler correlation and separate heat transfer coefficients for the sections of the well, are summarized and compared with the data in Table 1.

The predicted profiles are compared with the data in Figure 13. The model based on the Hagedorn-Brown correlation agrees very well with the field data. The total well length is predicted within 86 ft., the flash point depth within 20 ft., the flash point pressure within 6 psia and flash point temperature within 3°F. The model based on Hughmark correlation is totally unsatisfactory. It fails to predict the total well length even under adiabatic conditions. This results from the fact that at any given pressure in the two-phase section the correlation predicts a lower void fraction than that predicted by the Hagedorn-Brown correlation. Consequently, that length of the well increment corresponding to a ΔP step is shorter.

The predicted temperature profile agrees reasonably well with the measured profile, as may be seen from Figure 13. The transition to the two-phase section is sharper than the one indicated by the data. The predicted quality profile indicated that a slight condensation of the steam phase took place as it approached the well exit. The predicted wellhead quality appears to be quite low.

TABLE 1

Comparison of Mesa 6-1 Data with Model Predictions

Parameter	Field Data	Predicted	
		Hagedorn-Brown	Hughmark
Well length, ft.*	7100	7186	5196
Distance from bottom hole to flash point, ft.**	2965	2986	
Flash pressure, psia**	190	196	
Flash temperature, °F**	377	380	
Exit quality	—	0.064	0.182
Exit void fraction	—	0.960	0.952

* Based on a chosen bottom hole reference depth

**Based on single phase flow calculations

CONCLUSIONS

It is essential that reliable void fraction correlations be developed because prediction of the pressure profile and flow rates in a geothermal well are critically dependent on this prediction. None of the correlations in the literature appear to be based on physically sound concepts and none recognize the existence of different flow patterns within which different variables control.

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NOMENCLATURE

C_L	Input liquid volume fraction
C_p	Heat capacity, Btu/lb-°F
D	Casing inside diamters, ft.
f	Fanning friction factor
Fr	Froude number
G	Mass flux, lb/ft ² -sec.
g	Gravitational acceleration, ft/sec ²
g_c	Conversion factor, 32.2 lb _m -ft/lb _f -sec ²
h	Specific enthalpy, Btu/lb.
h_{fg}	Latent heat of vaporization, Btu/lb.
J	Mechanical equivalent of heat, 778 ft-lb _f /Btu
K	Bankoff factor, Equation (10).
P	Local pressure, psia
q	Heat transfer rate, Btu/hr.
Re	Reynolds number
T	Temperature, °F
U	Overall heat transfer coefficient, Btu/hr-ft ² -°F
v	Specific volume, ft ³ /lb.
W	Mass flow rate, lb/hr.
X	Flow quality = W_G/W_T
z	Vertical coordiante
Z	Correlating factor, Equation (11).
α	In situ void fraction = volume fraction of gas
Δ	Change over well increment
ρ	Density, lb/ft ³

Subscripts

A	Due to acceleration
Bot	At bottom hole
f	Due to friction
G	Gas, due to gravity
i-1	Entering well increment i
i	Leaving well increment i, pertaining to increment i
L	Liquid
R	Rock formation
SP	Single phase
T	Total
TP	Two-phase
Top	At wellhead

Superscript

—	Average
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PART II

TWO PHASE FLOW PATTERNS
IN VERTICAL TUBES

-45-

II. TWO PHASE FLOW PATTERNS IN VERTICAL TUBES

SUMMARY

Models are developed to predict the transition boundaries between the four basic flow patterns for gas-liquid flow in vertical tubes: bubble, slug, churn and dispersed-annular. It is suggested that churn flow is the development region for the slug patterns and that bubble flow can exist in small pipes only at high liquid rates where turbulent dispersion forces are high. Each transition is shown to depend on the flow rate pair, fluid properties and pipe size but the nature of the dependence is different for each transition because differing mechanisms control. The theoretical predictions are in reasonably good agreement with a variety of published flow maps based on data.

INTRODUCTION

When gas-liquid mixtures flow in a conduit the two phases may distribute in a variety of patterns. The particular pattern one observes depends on the flow rates, fluid properties and tube size. Figure 1 shows the patterns to be expected for a 5.0 cm diameter pipe flowing water and air at low pressure. A central task is to predict which flow pattern will exist under any set of operating conditions as well as the flow rate pair at which transition between flow patterns will take place.

Many two phase flow pattern maps have been proposed in the past. Most of these have been based primarily on experiments and thus were limited in use to the conditions near those of the measurements. Maps prepared from data taken for one pipe size and fluid properties are not necessarily valid for other sizes or properties. Part of the problem arises from lack of agreement in the description and classification of the flow patterns and the subjectivity of the observer. Hubbard and Dukler (1966) proposed a method for fingerprinting flow regimes using spectral analysis of wall pressure fluctuations but this method has not generally been adopted.

In this work the main flow patterns are first described. In order to correctly interpret and predict the conditions for transition, it is essential to understand the mechanism by which the transition from one flow pattern to another takes place. Physical models which describe such transitions are presented and are used to develop theoretically-based transition equations which can be used to construct maps. The attack here is similar in principle to that presented by Taitel and Dukler (1976) for horizontal flow systems. Since the maps presented are constructed on a physical basis, it is expected that they can be utilized over a wider range of flow conditions and fluid properties. Finally, the newly developed predictions are compared with those of recent maps which coordinate experimental data.

FLOW DESCRIPTION

When gas liquid mixtures flow upward in a vertical tube the two phases may distribute in a number of patterns, each characterizing the radial and/or axial distribution of liquid and gas. The flow is usually quite chaotic and these phase distributions are difficult to describe. We will follow the lead of Hewitt and Hall-Taylor (1970) who designate four basic patterns for upflow as follows:

1. Bubble Flow: The gas phase is approximately uniformly distributed in the form of discrete bubbles in a continuous liquid phase.
2. Slug Flow: Most of the gas is located in large bullet shaped bubbles which have a diameter almost equal to the pipe diameter and which move uniformly upward and are sometimes designated as "Taylor bubbles." The Taylor bubbles are separated by slugs of continuous liquid which bridge the pipe and contain small gas bubbles. Between the Taylor bubbles and the pipe wall liquid flows downward in the form of a thin falling film. (This pattern has been designated by others as plug or piston flow at low rates where the gas liquid boundaries are well defined and as slug flow at higher rates where the boundaries are less clear.)
3. Churn Flow: Churn flow is somewhat similar to slug flow. It is, however, much more chaotic, frothy and disordered. The bullet shaped Taylor bubble becomes narrow and its shape is distorted. The continuity of the liquid in the slug between successive Taylor bubbles is repeatedly destroyed by a high local gas concentration in the slug. As this happens the liquid slug falls. This liquid accumulates, forms a bridge and is again lifted by the gas. Typical of churn flow is this oscillatory or alternating direction of motion of the liquid. (Some observers refer

to a froth flow pattern for higher liquid and gas rates where the system appears more finely dispersed.)

4. Annular Flow: Annular flow is characterized by the continuity of the gas phase along the pipe in the core. The liquid phase moves upwards partly as wavy liquid film and partially in the form of drops entrained in the gas core. (Annular flow has been described as a wispy-annular pattern when the entrained phase is in the form of large lumps or "wisps." Froth, mist or semi-annular flow patterns have also been used to describe the churn and annular patterns.)

EXISTING FLOW PATTERN MAPS

A wide variety of flow pattern maps for vertical flow have been presented in the literature. It is important to understand that these maps propose transition boundaries in a two dimensional coordinate system as determined from experiment; these experiments in some cases being those of the author and in other cases coming from observations of others. The selection of coordinates for the published maps has been of two basic types:

- a. One group uses dimensional coordinates such as superficial velocities U_{GS} and U_{LS} (Sternling, 1965; Wallis, 1969) or superficial momentum flux, $\rho_G U_{GS}^2$ and $\rho_L U_{LS}^2$ (Hewitt and Roberts, 1969). Given any single pipe size and set of fluid properties, these coordinates will map the transitions but there is no reason to expect that the location of these transition curves will be unchanged for changes in these variables. Govier and Aziz (1972) attempt to modify these dimensional coordinates for systems other than air-water by considering property ratios between the fluids of interest and that of the air-water system. However, there is no basis in theory to suggest this generalizes the results in any way.

b. A second group represents the results by dimensionless coordinates in the hope ~~that~~ the result will apply to line sizes and fluid properties other than those of the data used to locate the curves. In the absence of a theoretical basis the use of dimensionless coordinates is no more general than the use of dimensional ones. Furthermore, it will be shown below that one pair of dimensionless groups does not characterize the variety of transition boundaries that exist. The dimensionless groups selected by Duns and Ros (1963) and also used by Gould (1974) seem arbitrary. Griffith and Wallis (1961) were the only investigators who attempted to involve theory to arrive at suitable coordinates. They were able to show that the dimensionless coordinates U_S^2/gD and U_{GS}/U_S controlled the transition from the slug to annular patterns. The theory was not completed sufficiently to provide an analytical expression for the transition curve and experimental data was used to provide for the unknown constants. As discussed, the use of these same coordinates for the other transitions is open to question.

Comparisons between these various maps reveal differences both as to absolute value and trend. The problem seems to arise from the fact that in almost all cases the transition boundaries are empirically located and do not rest in suitable physical models. Thus, it is important to place a theoretical bases under the transition relationships both to improve the generality of the prediction and for classifying the experimentally observed flow patterns.

TRANSITION MECHANISMS

In order to predict the condition under which transition between flow patterns will take place, it is essential to understand the physical mechanisms by which such transitions occur. In this way the influence of fluid properties and pipe

size as well as flow rates can be accounted for naturally in the equations which result and these can be expected to apply generally without the need for "scale-up" rules or procedures. The task of constructing flow transition boundary curves or maps from these equations is usually straightforward. There is considerable disagreement among authors as to the mechanism for these transitions. In the following each transition is analyzed, a physical mechanism proposed, and then the conditions for transition mathematically modelled.

The Bubble-Slug Transition:

Transition from the condition of dispersed bubbles observed at low gas rates to slug flow requires a process of agglomeration or coalescence. Only in this way can the discrete bubbles combine into the larger vapor spaces having a diameter nearly that of the tube with lengths of 1-2 diameters which are observed at the transition to slug flow. As the gas rate is increased, the bubble density increases. This closer bubble spacing results in an increase in the coalescence rate. However, as the liquid rate increases, the turbulent fluctuations associated with the flow can cause breakup of larger bubbles formed as a result of agglomeration. If this breakup is sufficiently intense to prevent recoalescence, then the dispersed bubble pattern can be maintained. Thus, in order to predict conditions for this transition, it is necessary to determine when each of these factors will dominate the process.

When gas is introduced at low flow rates into a large diameter vertical column of liquid which is flowing at low velocity, the gas phase is distributed into discrete bubbles. Many studies of bubble motion have demonstrated that if the bubbles are very small, they behave as rigid spheres rising vertically in rectilinear motion. However, above a critical size (about 0.15 cm for air-water at low pressure) the bubbles begin to deform and the upward motion is a zig-zag

path with considerable randomness. The bubbles randomly collide and coalesce forming a number of somewhat larger individual bubbles having a spherical cap similar to the Taylor bubbles of slug flow but with diameters smaller than the pipe. Thus, even at low gas and liquid flow rates bubble flow is characterized by an array of smaller bubbles moving in zig-zag motion and the occasional appearance of larger Taylor type bubbles. These Taylor bubbles are not large enough to occupy the cross section of the pipe so as to cause slug flow in the manner described above. Instead, they behave as free rising spherically capped voids in the manner originally described by Taylor. With increases in gas flow rate at these low liquid rates the bubble density increases and a point is reached where the dispersed bubbles become so closely packed that many collisions occur and the rate of agglomeration to larger bubbles increases sharply. This results in a transition to slug flow.

Experiments suggest that the bubble void fraction at which this happens is around 0.25 to 0.30 (Griffith and Synder, 1964). A semi-theoretical approach to this problem has been given by Radovicich and Moissis (1962) by considering a cubic lattice in which the individual bubble fluctuates. They postulated that the maximum void fraction is reached when the frequency of collision is very high and it was shown that this happens around void fraction of 0.30. An alternative approach is to consider this problem from the point of view of maximum allowable packing of the bubbles. If we consider the bubbles to have spherical shape and arranged in a cubic lattice, the void fraction of the gas can be, at most, 0.52. However, as a result of their deformation and random path, the rate of collision and coalescence will increase sharply at void fractions well below this lattice spacing at which they touch. Therefore, the closest distance between the bubbles before transition must be the one which permits some freedom of motion for each individual bubble. If the spacing between the bubbles at which coalescence begins

to increase sharply is assumed to be approximately half their radius, this corresponds to about 25% voids. While this approach is not a prediction of the void fraction at transition from first principles, it does provide a reasonable interpretation of the experimental data. Published data agree in that the void fraction in bubbly flow rarely exceeds 0.35 whereas for void fractions less than 0.20 coalescence is rarely observed (Griffith and Wallis, 1961). Thus, at liquid rates low enough so that bubble breakup due to turbulence is small, the criteria for transition from bubbly to slug flow is that the void fraction reaches 0.25.

If the gas bubbles rise at a velocity U_G , this velocity is related to the superficial gas velocity U_{GS} by

$$U_G = \frac{U_{GS}}{\alpha} \quad (1)$$

where α is the void fraction. Likewise, the average liquid velocity is given in terms of the liquid superficial velocity as

$$U_L = \frac{U_{LS}}{1-\alpha} \quad (2)$$

Designating U_0 as the rise velocity of the gas bubbles relative to the average liquid velocity, equations (1) and (2) yield:

$$U_{LS} = U_{GS} \frac{1-\alpha}{\alpha} - (1-\alpha)U_0 \quad (3)$$

The rise velocity U_0 of relatively large bubbles has been shown by Harmathy (1960) to be quite insensitive to the bubble size and given by the relation

$$U_0 = 1.53 \left[\frac{g(\rho_L - \rho_G)\sigma}{\rho_L^2} \right]^{1/4} \quad (4)$$

Substituting equation (4) into (3) and considering the transition to slug flow to occur when $\alpha = \alpha_T = 0.25$ results in an equation characterizing this transition for conditions where the dispersion forces are not dominant.

$$U_{LS} = 3.0 U_{GS} - 1.15 \left[\frac{g(\rho_L - \rho_G)\sigma}{\rho_L^2} \right]^{1/4} \quad (5)$$

Once fluid properties are designated the theoretical transition curve can be plotted on U_{LS} vs U_{GS} coordinates and will remain invariant with tube size. Such a curve is shown in Figure 2 for the water-air system at 25°C and 10 N/cm² where it is designated as curve A. At higher gas and liquid flow rates where the bubble rise velocity relative to the liquid velocity is negligible the theoretical transition curve is linear with a slope of unity in these log coordinates. On the other hand, at low liquid rates where liquid velocity is negligible the boundary of the bubble region is controlled by the free rise velocity of the bubbles and is essentially independent of liquid rate.

This method for predicting the bubble-slug transition is similar in principle to that of Griffith and Wallis (1961). They used $\alpha = 0.18$ as a criteria for maximum packing and set U_0 equal to a constant of 0.24 m/sec which is the rise velocity predicted from equation (4) for an air-water system. The results were, therefore, not general for fluids other than air-water.

At higher liquid flow rates turbulent forces act to break and disperse the gas phase into small bubbles even for vapor void fractions higher than 0.25. The theory of breakup of immiscible fluid phases by turbulent forces has been given by Hinze (1955) and recently confirmed by Sevik and Park (1973). Hinze determined that the characteristic size of the dispersion results from a balance between surface tension forces and those due to turbulent fluctuations. His study led to the following relationship for the maximum stable diameter of the dispersed phase, d_{max} .

$$d_{max} = k \left(\frac{\sigma}{\rho_L} \right)^{3/5} \left(\epsilon \right)^{-2/5} \quad (6)$$

where ϵ is the rate of energy dissipation per unit mass. Hinze's investigation

explored dispersion under non-coalescing conditions which can be realized only at very low concentrations of the dispersed phase. He applied his formula to the data of Clay (1940) for droplet breakup at low concentration of dispersed phase and found k to be equal to 0.725. Sevik and Park (1973) developed theoretical values of k by considering the natural frequency of a bubble or drop in its lowest order mode of vibration. Assuming this natural frequency to be the ratio of the rms velocity fluctuation in the turbulent stream to the diameter of the drop or bubble, they were able to prove that $k = 0.68$ for drops (density of dispersed phase $\gg$ density of continuous phase) and $k = 1.14$ for bubbles (density of dispersed phase $\ll$ density of the continuous phase). Experimental measurements of bubble breakup in a liquid jet showed remarkable experimental agreement with $k = 1.14$.

The rate of energy dissipation per unit mass for turbulent pipe flow, ϵ , is:

$$\epsilon = \left| \frac{dp}{dz} \right| \frac{U_M}{\rho_M} \quad (7)$$

where

$$\frac{dp}{dz} = \frac{2f}{D} \rho_M U_M^2 \quad (8)$$

Substitution of equation (8) into (7) shows that turbulent breakup of bubbles exists for all liquid rates high enough to cause turbulent flow. However, if the bubble size produced by the breakup process is large enough to permit deformation, then at values of α approaching 0.25 the large Taylor bubbles characteristic of slug flow again are formed by the process of coalescence. Thus, the turbulent breakup process can prevent agglomeration only if the bubble size produced is small enough to cause the bubbles to remain spherical. The bubble size at which this occurs is given by Brodkey (1967) as

$$d_{\text{crit}} = \left[\frac{0.4\sigma}{(\rho_L - \rho_G)g} \right]^{1/2} \quad (9)$$

For $d_{\text{max}} > d_{\text{crit}}$ the bubble rise velocity is almost independent of bubble size and is given by equation (4), but decreases very rapidly for bubble diameters below d_{crit} . Once turbulent fluctuations are vigorous enough to cause the bubbles to break into this small critical size, coalescence is suppressed and the dispersed bubble flow pattern must exist even for $\alpha > 0.25$. From equations (6), (7) and (9), the conditions for this transition can be found. Note that in this region of high flow rate the slip velocity can be neglected and the gas holdup calculated simply by

$$\alpha = \frac{U_{\text{GS}}}{U_{\text{GS}} + U_{\text{LS}}} \quad (10)$$

The friction factor needed in equation (8) can be predicted by the Blasius equation based on the superficial mixture velocity and the liquid kinematic viscosity, namely

$$f = C \left(\frac{U_M D}{\nu_L} \right)^{-n} \quad (11)$$

where C and n are taken as .046 and 0.2, respectively. Combining equation (7) to (11) results in a dimensionless expression relating the flow rates, properties and pipe size at which turbulent induced dispersion takes place.

$$U_{\text{LS}} + U_{\text{GS}} = 4.0 \left\{ \frac{D^{0.429} (\sigma/\rho_L)^{0.089}}{\nu_L^{0.072}} \left[\frac{g(\rho_L - \rho_G)}{\rho_L} \right]^{0.446} \right\} \quad (12)$$

Once the fluid properties and pipe size are set, equation (12) defines the relationship between the values of U_{GS} and U_{LS} above which slug flow cannot exist. For air-water at 25°C and 10 N/cm² pressure this result is shown in Figure 2 for a 5.0 cm diameter pipe and is designated by B. However, regardless of how much turbulent energy is available to disperse the mixture, bubble flow cannot exist

at packing densities above $\alpha = 0.52$. Thus, the B curve delimiting dispersed bubble flow must terminate at the curve C which relates U_{LS} and U_{GS} for $\alpha = 0.52$.

This approach neglects the effect of increasing holdup on the process of coalescence and on the resulting bubble size. Calderbank (1958) investigated the balance between bubble breakup and coalescence in determining bubble size in mechanically stirred gas-liquid columns. There he showed that the bubble size varied with the holdup to the one half power. However, the mechanism for such coalescence, namely, agitator causing eddy-like motion of the bubbles, is not present here. With flow induced radial motions the effect of holdup is likely to be less significant; however, its quantitative influence cannot be evaluated at this time.

Thus, the bubbly flow pattern can be seen to exist in the two zones of map I and II. In zone I to the left of curve A and below B, one predicts the presence of deformable bubbles which move upward with a zig-zag motion with Taylor type bubbles occasionally appearing in the liquid. In zone II above curve B and to the left of C, one observes a more finely dispersed bubble system without any Taylor bubbles. To the right of A and below B in zone III, one expects to see the slug pattern.

Still a different transition mechanism comes into play in the special case of tubes of small diameter. Consider zone I of Figure 2 where one observes discrete deformable bubbles rising in zig-zag paths and the occasional appearance of a Taylor bubble. The velocity of rise of the deformable bubbles relative to the liquid, U_0 , is given by equation (4) and depends only on the properties of the fluids. The rise velocity of the Taylor bubbles relative to the mean velocity of the liquid on the other hand is given by (Nicklin, et al, 1962).

$$U_G \approx 0.35\sqrt{gD} \quad (13)$$

and is property independent. Whenever $U_0 > U_G$ the rising bubbles approach the back of the Taylor bubble coalescing with it, increasing its size. Under

these conditions bubbly flow cannot exist in zone I. On the other hand, when $U_0 < U_G$ the Taylor bubble rises through the array of distributed bubbles and the relative motion of the liquid at the nose of the Taylor bubble sweeps the small bubbles around the larger one and coalescence does not take place. The properties of air-water at low pressure are such that $U_0 \approx U_G$ at $D \approx 5.0$ cm. Thus, for tubes smaller than 5 cm in diameter no bubbly flow can exist below the curve B and the entire zone I and III exist as the slug flow pattern. Only at high liquid rates, zone II, can bubbly flow exist for small tubes where dispersion occurs due to turbulence. The flow pattern map for 2.5 cm diameter tubes with the system air-water is shown in Figure 3 where zones I and III are combined. A system having a small diameter is one which satisfies the criterion below.

$$\left[\frac{\rho_L^2 g D^2}{(\rho_L - \rho_G) \sigma} \right]^{1/4} \leq 4.36 \quad (14)$$

It is of interest that the range of diameters used in most laboratory air-water experiments, i.e., 2-6 cm, spans this critical diameter of 5.0 cm. This accounts for the apparent differences in observations which have been reported. It also shows that for this particular transition, experimental data taken on small pipes cannot be scaled to larger diameters. It is only through an understanding of mechanisms, as discussed here, that rational size scaling can be accomplished.

Slug-Churn Transition

The slug flow pattern develops from a bubbly pattern when the gas flow rate increases to such an extent that it forces the bubbles to become closely packed and coalesce. At this point "Taylor" type bubbles are formed which, if the process of coalescence continues, occupy most of the pipe cross sectional area and are axially separated by a liquid slug in which small bubbles are dispersed.

The liquid confined between the bubble and the pipe wall flows around the bubble as a falling film.

As the gas flow rate is increased still more, a transition to churn flow occurs. There is considerable difficulty in accurately identifying the slug-churn transition because there is confusion as to the description of the churn flow itself. Some identify churn flow on the basis of the froth that appears within the gas region, and these investigators describe the pattern as frothy. Others associate churn flow with the instability of the liquid film adjacent to the Taylor bubble. We characterize the churn flow pattern as that condition where oscillatory motion of the liquid is observed. In slug flow the liquid between two Taylor bubbles moves at a constant velocity and its front as well as its tail have constant speed. In churn flow the liquid slug is too short to support a stable liquid bridge between two consecutive Taylor bubbles. The falling film around the bubble penetrates deeply into the liquid slug creating a highly agitated aerated mixture at which point the liquid slug is seen to disintegrate and to fall in a rather chaotic fashion. The liquid re-accumulates at a lower level at the next slug where liquid continuity is restored and the slug then resumes its upward motion. Thus, one observes an oscillatory motion of the liquid which we consider the characteristic identification of churn flow.

There have been several mechanisms proposed for transition to the churn pattern. Nicklin and Davidson (1962) suggested that the transition to churn flow occurs when the gas velocity relative to the falling liquid film around the Taylor bubble approaches the condition of flooding.

Moissis (1963) attributed this transition to Helmholtz instability of the liquid film bounding the Taylor bubble. The Helmholtz instability criterion was applied for an infinitely thick gas region and very thin liquid film using Feldman's (1957) result that the wave length is 10 times the film thickness.

The analysis is inconsistent since Feldman's theory is not based on Helmholtz type analysis. Furthermore, Feldman's calculation for the stability criterion is in error.

Griffith and Wallis (1961) suggested that transition from slug to annular flow occurs when the individual Taylor bubbles become very (infinitely) long. Their predicted transition to annular flow occurs at much lower gas rates than indicated by experiment which suggests that this mechanism might account for the slug/churn transition instead. No predictive model was presented and their transition curve was finally obtained solely from experimental data.

Careful and repeated observations of the slug-churn flow patterns on 2.5 and 5.0 cm diameter test sections in our laboratories suggest a rather different mechanism than any of the above. These observations indicate that the churn flow pattern is an entry region phenomenon associated with the existence of slug flow further along the pipe. That is, whenever one observes slug flow, the condition near the entry appears to be churning. Furthermore, the entry length, or the distance that such churning can be observed before stable slug flow takes place, depends on the flow rates and pipe size.

The process of developing a stable slug near the entrance section can be described as follows. At the inlet the gas and liquid which are introduced form short liquid slugs and Taylor bubbles. A short liquid slug is known to be unstable and it falls back and merges with the liquid slug coming from below causing it to approximately double its length. In this process the Taylor bubble following the liquid slug overtakes the leading Taylor bubble and coalesces with it as the slug between the two bubbles collapses. This process repeats itself and the length of the liquid slugs as well as the length of the Taylor bubbles increase as they move upwards until the liquid slug is long enough to be stable and form a competent bridge between two consecutive Taylor bubbles. Between the

inlet and the position at which a stable slug is formed, the liquid slug alternately rises and falls and this is precisely the condition of churn flow. As the gas rate increases, it is evident that the length of this entrance region increases to the extent that it can occupy the entire length of any test section. Thus, one should think of churn flow as an entrance phenomena. Since in practice all pipes are of a finite length, it would be useful to provide some estimates of the lengths over which churn flow is the predominant mode. With this objective we develop a method for calculating the entry length required to develop stable slug flow. The distance from the entrance to that length will be observed to be in the churn flow pattern.

Figure 4 shows a model of slug flow. Consecutive Taylor bubbles rise in a vertical pipe separated by regions of liquid containing small bubbles. The Taylor bubbles rise at a velocity designated by U_G . The liquid between the Taylor bubbles moves upwards at an average velocity U_L . The bubble cavity is at nearly constant pressure and the liquid film adjacent to the bubble flows downward as a free falling film at a velocity, U_f . Since the slug flow pattern is developed when $\alpha_T = 0.25$, the liquid slug between the Taylor bubbles is assumed to contain small bubbles at this bubble density. Furthermore, the dispersed bubbles are assumed to be confined to the region between the Taylor bubbles thus moving with the Taylor bubble velocity U_G . Although observations of slug flow show that some small bubbles are swept into the liquid film around the Taylor bubble, this assumption has little effect on the result.

The velocity of the Taylor bubble is given quite accurately by the relation (Nicklin, et al, 1962),

$$U_G = 1.2 U_L + 0.35\sqrt{gD} \quad (15)$$

In this equation the second term on the R.H.S. describes the rise velocity of a

large bubble in stagnant liquid and was derived theoretically by Davies and Taylor (1949) and by Dumitrescu (1943). The first term of the R.H.S. adds the liquid velocity at the centerline since 1.2 is approximately the ratio of centerline to average velocity in fully developed turbulent flow. The total volumetric flow rate, Q , is constant across any cross section. Therefore,

$$Q = U_M A = U_L A(1-\alpha_T) + U_G A \alpha_T = (U_{GS} + U_{LS}) A \quad (16)$$

The gas bubble velocity can be solved directly by eliminating U_L between (15) and (16) arriving at

$$U_G = \frac{1.2 \frac{U_M}{1-\alpha_T} + 0.35\sqrt{gD}}{1 + 1.2 \frac{\alpha_T}{1-\alpha_T}} \quad (17)$$

Using (16) the liquid velocity equals

$$U_L = \frac{U_M - U_G \alpha_T}{1 - \alpha_T} \quad (18)$$

Consider two consecutive Taylor bubbles (as shown in Figure 4). The first (top) bubble moves at a velocity given by equation (15). The second (lower) bubble will move at the same speed when the slug length ℓ_S is long enough so that the velocity profile in the liquid at the front of the second bubble will be the same as that at the front of the first bubble, namely, the average velocity is U_L and the centerline velocity is $1.2 U_L$. This is the situation to be expected when the slugs are long enough so that the turbulent velocity distribution in the liquid can be fully reestablished before the next Taylor bubble appears. Then the velocity of the two consecutive Taylor bubbles is the same,

the length of the liquid slug will remain constant with time and position in the direction of flow and stable slug flow exists.

However, since the liquid slugs are shorter in the developing region, the velocity distribution in the liquid can be severely distorted by the flow reversal near the wall as a result of the falling film. Consider the velocity distributions in the planes A-A and B-B behind the leading Taylor bubble shown in Figure 4. If the liquid slug is long, far enough behind the trailing edge of the bubble (plane B-B), the velocity becomes that typical of turbulent flow. However, at A-A the flow is downward near the wall as a result of the falling film around the bubble. In order to maintain mass continuity the velocity at the centerline must increase. Since the velocity of a Taylor bubble depends on the centerline velocity plus its rise velocity, it is clear that for liquid slugs too short to reestablish the turbulent velocity distribution the second bubble will overtake the first (Moissis and Griffith, 1962). As a result the two bubbles will coalesce, the liquid bridge between them will disintegrate and fall to a lower level creating churn flow.

Experimental observations for water-air systems suggest that the length of a stable slug relative to this diameter, ℓ_S/D is fairly constant and independent of gas and liquid flow rates (Govier and Aziz, 1972; Akagawa and Sakaguchi, 1966). The minimum value of ℓ_S/D reported was 8. Studies in our laboratory in very long 2.5 and 5.0 cm diameter tubes have shown stable slug lengths approaching 16D. The earlier observations can be considered the result of two slugs each not quite of stable length, $\ell_S/D = 8$, which approach each other so slowly that they would never coalesce except in a long tube. By use of the very approximate argument which follows, it is possible to show that this stable length, $\ell_S/D = 16$, observed for airwater should be essentially independent of fluid properties or pipe diameter.

The liquid film falling along the Taylor bubble has an average velocity U_f and velocity relative to the liquid at plane A-A behind the bubble of $(U_f + U_G)$. Consider this liquid sheet as a two dimensional jet which enters a stagnant pool of liquid (the slug) at a uniform velocity, $(U_f + U_G)$. The axial velocity, U , in the liquid induced by the jet will depend on the distance x in the direction of the jet and y , the normal distance from the jet centerline. Both experimental and theoretical studies have shown that the ratio of $U(x,y)$ to $U_{\max}(0,y)$ varies as

$$\frac{U}{U_{\max}} = 1 - \tanh^2\left(\gamma \frac{y}{x}\right) \quad (19)$$

where γ is a universal constant approximately equal to 7.67 (Schlichting, 1968). A stable slug is one which is long enough that the jet has been absorbed by the fluid and the velocities have slowed to that of the surroundings. In this case we explore the distance $x = \ell_S$ which at the centerline, $y = D/2$, the velocity is essentially flat, say $U/U_{\max} \leq 0.05$, and thus the normal turbulent distribution in the liquid slug is undistorted. Equation (19) shows that this takes place at $\ell_S/D \approx 16$. This is, of course, only an approximate argument because the falling film is a wall jet, not a free jet and the fluid is confined, not of infinite extent. However, Patel (1971) has shown that the velocity distributions in a wall jet on the side of the velocity maximum away from the wall can be estimated as in a free jet. Since the falling film is so thin, the approximations used above become quite reasonable.

Entry Length for Churn Flow: Designate ℓ_E as the entry length of pipe required to establish stable slug flow and therefore the region that one would observe churning. Consider a coordinate x pointing downward from the trailing edge of the leading Taylor bubble as in Figure 4. The velocity at the center

of the pipe, U_c , varies from U_G at $x = 0$ to $1.2 U_L$ at $x = \ell_S$. Assume exponential variation with x as follows

$$U_c = U_G e^{-\beta x / \ell_S} + 1.2 U_L (1 - e^{-\beta x / \ell_S}) \quad (20)$$

The constant β determines the decay rate and was chosen at $\beta = \ln 100 = 4.6$ so that at $x = \ell_S$ the decay will be 1%. The final results are not sensitive to the particular choice of β , or to the particular profile assumed as long as $U_c(x = 0) = U_G$ and $U_c(x = \ell_S) = 1.2 U_L$. Designating the leading bubble as the first and the trailing bubble as the second and using equation (15) for calculating the velocities of the two consecutive bubbles, we obtain an approach velocity between two bubbles, $-\dot{x}$, as

$$-\dot{x} = U_{G_2} - U_{G_1} = (U_G - 1.2 U_L) e^{-\beta x / \ell_S} = 0.35 \sqrt{gD} e^{-\beta x / \ell_S} \quad (21)$$

It is interesting to compare this result with that given by Moissis and Griffith (1962) who measured the velocity of a Taylor bubble that follows a leading bubble and correlated it empirically by the relationship

$$\frac{U_{G_2}}{U_{G_1}} = 1 + 8 e^{-1.06 x / D} \quad (22)$$

Using (21) and (17) our results for $\alpha_T = 0$ (as in Moissis and Griffith's case) can be expressed in the form

$$\frac{U_{G_2}}{U_{G_1}} = 1 + \frac{0.35}{1.2 U_M / \sqrt{gD} + 0.35} e^{-.2875 x / D} \quad (23)$$

While the form and constants in the second term of equation (23) seem quite different from Moissis and Griffith's equation (23), the agreement in U_{G_2} / U_{G_1}

is excellent for $x/D > 3$. For x/D less than three our prediction is poor, probably because for this condition the two bubbles are very close and are near collapse. However, for the purpose of calculating the entry length, knowing the approach velocity for bubbles separated by distances of the order of D is not important as will be seen below. The fact that this model derived without data agrees so well with the Moissis and Griffith data gives confidence for its use for other pipe sizes and fluid properties.

In calculating the entry length or length for churn flow, it is assumed that near the gas liquid inlet coalescence is instantaneous and short Taylor bubbles as well as short liquid slugs are formed. The merging of the Taylor bubbles to larger gas bubbles and larger liquid slugs takes place when the second bubble overtakes the leading bubble. When they combine the volume as well as the length of the newly created Taylor bubble is doubled. Likewise, the liquid slug behind the leading bubble falls and merges with the liquid slug behind the second bubble to create a slug twice as long. At approximately the same time the third and fourth bubbles will combine and again create a new bubble and new liquid slug of double length. This process will go on and pairs of bubbles will coalesce as they move upward, doubling their length each time until a stable liquid slug of length $\ell_S/D \approx 16$ is formed. ℓ_S designates the length of a stable slug while ℓ_L is the length of a slug formed during the period of coalescence. Because the last merger of two slugs of length $\ell_L = 8D$ is quite slow, we consider the entrance or churning length to exist up to the point where $\ell_L = 8D$ (or $\ell_L = \ell_S/2$), the region of churn flow.

Integrating (21) gives the time needed for each merger as a function the distance, ℓ_{Li} , between two consecutive bubbles

$$t_i = \frac{\ell_S}{0.35\beta\sqrt{gD}} \left[e^{\beta\ell_{Li}/\ell_S} - 1 \right] \quad (24)$$

where i takes successive values of 0, 1, 2, 3 Letting ℓ_{Li} take the sequence from 0 to $\ell_S/4$, namely, $\ell_{Li} = \ell_S/4, \ell_S/8, \ell_S/16 \dots$ 0 yields an infinite series for t_i whose sum multiplied by U_G yields the estimated entrance length ℓ_E .

$$\ell_E = \frac{\ell_S U_G}{0.35\beta\sqrt{gD}} \sum_{n=2}^{\infty} \left(e^{\frac{\beta}{2n}} - 1 \right) \quad (25)$$

or considering $\beta = 4.6$ and $\ell_S = 16D$ yields

$$\frac{\ell_E}{D} = 35.5 \frac{U_G}{\sqrt{gD}} \quad (26)$$

Substituting (17) into (26) for $\alpha_T = 0.25$ yields

$$\frac{\ell_E}{D} = 40.6 \left(\frac{U_M}{\sqrt{gD}} + 0.22 \right) \quad (27)$$

where $U_M = U_{GS} + U_{LS}$. This shows that the dimensionless entry length for churning depends on one parameter, namely, $U_M/\sqrt{gD}$. The solution to this equation for a low pressure air-water system at several values of ℓ_E/D is shown in Figures 2 and 3 for 5.0 and 2.5 cm diameter tubes where these curves are designated as "D."

Figure 2 shows typical trends for pipe diameters larger than that needed to satisfy equation (14), that is, pipe sizes where a bubbly flow pattern can be expected to exist. Note that the "D curves" delineating the transition between slug and churn flow terminate on the "A curve." This A transition boundary represents the locus of points in the $U_{LS} - U_{GS}$ plane where $\alpha = 0.25$ and for $\alpha < 0.25$ only bubble flow can exist.

On the other hand, for the smaller pipe sizes represented in Figure 3 only slug flow is observed, even for $\alpha < 0.25$. However, churning can be expected only

when $\alpha > 0.25$. Thus, the locus of $U_{LS} - U_{GS}$ values for $\alpha = 0.25$ is shown dotted on Figure 2 and still represents the terminus of the D curves.

It is now possible to predict if churn flow will be observed at any axial position along the pipe. Assume that the point of observation is 200 diameters above the entry. Over the flow rate region to the left of the $\ell_E/D = 200$ curve of Figure 2 and to the right of the A curve, slug flow will be observed. At flow rate pairs to the right of curve D churn flow will be seen, providing the rate is not high enough to cause the annular pattern. Although churning is visualized as an entry region phenomenon, conditions where churn flow would be observed over the entire length of a tube can be shown to exist. For example, consider a 5.0 cm diameter pipe whose length is somewhat less than $200D$. Values of U_{LS} and U_{GS} located just to the right of the $\ell_E/D = 200$ curve of Figure 2 would result in churn flow being observed at all positions along the pipe. In other words, the entire tube consists of entry length for purposes of developing slug flow.

Transition to Annular Flow

For high gas flow rates the flow becomes annular. The liquid film flows upwards adjacent to the wall and gas flows in the center carrying entrained liquid droplets. The upward flow of the liquid film against gravity results from the forces exerted by the fast moving gas core. This film has a wavy interface and the waves tend to shatter and enter the gas core as entrained droplets. Thus, the liquid moves upwards due to both interfacial shear and form "drag" on the waves and drag on the droplets. Based on the idea by Turner, et al (1968) applied to gas lift operations, we suggest that annular flow cannot exist unless the gas velocity in the gas core is sufficient to lift the entrained droplets. When the gas rate is insufficient, the droplets fall back, accumulate, form a bridge and churn or slug flow takes place.

The minimum gas velocity required to suspend a drop is determined from the balance between the gravity and drag forces acting on the drop

$$\frac{1}{2} C_d (\pi d^2/4) \rho_G U_G^2 = (\pi d^3/6) g (\rho_L - \rho_G) \quad (28)$$

or

$$U_G = \frac{2}{\sqrt{3}} \left[\frac{g (\rho_L - \rho_G) d}{\rho_G C_d} \right]^{1/2} \quad (29)$$

The drop size is determined by the balance between the impact force of the gas that tends to shatter the drop and surface tension forces that hold the drop together. Hinze (1955) showed that the maximum stable drop size will be

$$d = \frac{K\sigma}{\rho_G U_G^2} \quad (30)$$

where K is the critical Weber number and takes a value between 20 and 30 for drops which are gradually accelerated.

Using (29) and (30) yields

$$U_G = \left(\frac{4K}{3C_d} \right)^{1/4} \frac{[g(\rho_L - \rho_G)]^{1/4}}{\rho_G^{1/2}} \quad (31)$$

As suggested by Turner, et al (1969) values of $K \approx 30$ and $C_d = 0.44$ were selected. Note that K and C_d appear in the power of 1/4. Thus, the result for U_G is quite insensitive to their exact values.

The gas velocity given by equation (31) will predict the minimum value below which stable annular flow will not exist. While this analysis is applied to the droplets within the gas core, the same treatment can be used for the crests of the waves on the rising film which are pictured as being supported by the gas stream in a manner similar to the support of the liquid droplets.

Characteristic of annular flow is that the film thickness is quite low even for relatively high liquid flow rates. As a result the true gas velocity U_G can be replaced by the superficial rate U_{GS} and the final transition boundary is given by

$$\frac{U_{GS} \rho_G^{1/2}}{[\sigma g(\rho_L - \rho_G)]^{1/4}} = 3.1 \quad (32)$$

This simple criterion shows the transition to the annular pattern is independent of liquid flow rate and pipe diameter. For water-air at 25°C, 10 N/cm² this velocity is calculated to be about 15 m/sec and the transition boundary is plotted as a vertical line in Figures 2 and 3 designated as curve E. The dimensionless group in equation (32) is recognized as the Kutataledze number.

Equation (32) is almost identical to the empirical results of Pushkin and Sorokin (1959) who determined the air velocity necessary to lift the liquid film for flooding experiments in tubes varying from 6 to 309 mm in diameter. They correlated their experimental results in terms of the Kutataledze number except the constant 3.1 is replaced by the constant 3.2. While Pushkin and Sorokin arrived at their result by dimensional analysis and experiment, the above development places a theoretical basis under the result.

Fluid properties and pipe diameter enter into the transition equations to differing degrees depending on the transition being considered. To illustrate this effect, the location of the transition boundaries are shown in Figure 5 for a crude oil-natural gas system flowing at 30°C and 670 N/sq.cm pressure in pipes with diameters of 5.0 and 30.0 cm. Oil density was set at 0.65 g/cu.cm with natural gas at 0.05 gr/cu.cm with oil and gas viscosities of 0.5 and 0.015 cp.

It is useful to observe that the flow pattern transition boundaries can be readily obtained by hand calculation from the algebraic equations presented here for any pipe size and fluid properties.

Criterion for existence of bubble flow: Eq. 14

Bubble to slug: Eq. 5

Bubble to dispersed bubble: Eq. 12

Slug to churn: Eq. 27

Annular: Eq. 32

DISCUSSION AND CONCLUSIONS

In Figures 6 through 8 each of the transitions predicted by this theory are compared with published transition boundaries. As discussed above these published transition boundaries are based on experimental data with little theoretical foundation. The wide discrepancy in the location of these curves emphasizes the role of observation and careful definition in the study of flow patterns. Location of curves in the published maps have been based on experiments in pipes ranging in size from 2.0-6.0 cm in diameter and with air-water at low pressure.

Transition A and B (Figure 6). The theory presented here identifies two bubble-slug transitions. Large Taylor type bubbles cannot exist above theoretical transition curve B due to turbulent dispersion. Transition to Taylor type bubbles or to churn flow takes place to the right of theoretical transition curve A. In the region between these two curves dispersed bubbles should appear for 5.0 cm and larger diameter pipes and Taylor bubbles should appear for 2.5 cm diameter pipes. "Bubble flow" has been used to describe the existence of both dispersed bubbles as well as Taylor bubbles that do not quite fill the pipe's cross section area. Note that except for the Govier and Aziz curve these two theoretical curves bound the range of the data. It thus includes all possible descriptions. If one

designates the slug flow pattern only for these cases where Taylor bubbles which nearly fill the pipe exist and rise with a velocity given by equation (15), then some of this ambiguity can be eliminated. Figures 9 and 10 show experimental data taken in our laboratories in 2.5 and 5.0 cm tubes which clearly demonstrate that for 2.5 cm tubes the A transition does not exist while for 5.0 cm tubes bubbly flow can be observed.

Transition D (Figure 7). Since churn flow is an entry region phenomenon, the location of the slug-churn transition boundary depends on the point of observation along the pipe as discussed above. Theoretically predicted transition curves are shown for $l_E = 50D$ in a 2.5 cm pipe and $l_E = 200D$ in a 5.0 cm pipe. Considering the fact that there is no information on the location of the observation point for the published transitions, the results must be considered in remarkably good agreement with all published curves except that of Hewitt and Roberts (1969). The new experimental data in Figures 9 and 10 show excellent agreement with the theory at the lower liquid rates. At higher velocities it is extremely difficult to visually discriminate between churn and slug flow.

Transition E (Figure 8). The vertical line representing transition E is compared with several published transitions in Figure 8. Except for the Griffith-Wallis curve the theory is an excellent compromise between the various empirical curves proposed. At high liquid rates there is increasing discrepancy both in trend and magnitude between the theory and data curves as well as between the various sets of data. In this region of high liquid rate the flow is extremely chaotic and visual observations are not very specific. Figures 9 and 10 show that the theoretical prediction of this transition is in excellent agreement with our data taken in 2.5 and 5.0 cm diameter tubes.

Theoretically predicted transitions for upflow have been compared with the recommendations of many earlier investigators whose results have been based

primarily on experimental measurements. There is considerable disagreement between the results of these various investigators. However, the theory presented here is in satisfactory agreement with the weight of the earlier experimental results. In addition, comparisons with new data taken in 2.5 and 5.0 cm tubes show good agreement, since the theory was developed without the use of experimental data.

NOMENCLATURE

A	Flow cross sectional area
C	Constant in the friction factor correlation
C_d	Drag coefficient
d	Bubble or drop diameter
D	Pipe diameter
f	Friction factor
g	Acceleration of gravity
k	Constant, equation (6)
K	Critical Weber number, equation (30)
l	Length
n	Exponent, equation (11)
P	Pressure
Q	Volumetric flow rate
U	Velocity
x	Axial coordinate
y	Coordinate perpendicular to flow direction
z	Coordinate in the flow direction

Greek Symbols

α	Void fraction
β	Constant, equation (20)
α_T	Void fraction at transition to slug flow
ϵ	Energy input per unit mass and time
ρ	Density
σ	Surface tension
τ	Shear stress

Subscripts

c	Centerline of pipe
E	Entrance
f	Falling film around Taylor bubble
G	Gas in Taylor bubble
GS	Superficial for gas flow alone in the pipe
i	Running index
L	Liquid
LS	Superficial for liquid flow alone in the pipe
M	Mixture of gas and liquid
o	Free rise
s	Slug
W	Pipe wall

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FIGURE 1: FLOW PATTERNS IN VERTICAL FLOW.

FIGURE 2
FLOW PATTERN MAP FOR VERTICAL TUBES
5.0 cm DIA.; AIR - WATER AT 25°C, 10 N/sq. cm.

NOTE CHURN FLOW EXISTS TO THE RIGHT
OF THE APPLICABLE D CURVE AND
TO THE LEFT OF E

FIGURE 3

FLOW PATTERN MAP FOR VERTICAL TUBES
2.5 CM DIA., AIR-WATER AT 25 °C, 10 N/CM²

NOTE: CHURN FLOW EXISTS TO THE RIGHT
OF THE APPLICABLE D CURVE AND
LEFT OF E.

FIGURE 4: SLUG FLOW GEOMETRY

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BUBBLE-SLUG TRANSITION
COMPARISON OF THEORY WITH OTHER MAPS

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SLUG TO CHURN TRANSITION
COMPARISON OF THEORY WITH OTHER MAPS
(SEE FIGURE 6 FOR LEGEND)

FIGURE 8
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COMPARISON WITH NEW DATA,
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PART III

FLOW PATTERN BASED MODELS FOR VOIDS AND FRICTION IN VERTICAL TUBES

III. FLOW PATTERN BASED MODELS FOR VOIDS AND FRICTION IN VERTICAL TUBES

SUMMARY

New models are developed for predicting void fraction and frictional drop based on a knowledge of the flow pattern.

INTRODUCTION

Two-phase flow in a pipe, as well as in a flashing geothermal well, is characterized by a variety of vapor-liquid configurations or flow patterns as discussed in Part II of this report. It is quite evident that the local vapor volume fraction, or void fraction, as well as the factors controlling friction depend strongly on the flow pattern that exists.

Published correlations for the average void fraction fall into two categories:

1. Gross empirical correlations that disregard the effect of flow patterns, or "black box-type" equations;
2. Specific correlations that apply to a given flow pattern.

Existing overall correlations have been found to be contradictory and inapplicable to a flashing flow in a geothermal well (see Part I of this report). It appears highly unlikely that such correlations can apply for all the flow patterns with the same degree of accuracy. Published specific correlations are all simplistic and disregard the dynamic characteristic of the flow patterns and hence void fraction. Furthermore, they provide no criteria on the conditions for applying the particular correlation.

In Part II of this report a new set of correlations are presented for predicting flow regimes in vertical upflow based on sound physical models. It is now possible to predict the flow regime at any point along the production string of a geothermal well. With this tool it is possible now to approach the prediction of the void fraction for each flow regime with a basis for knowing when each should be applied. Furthermore, each flow pattern or regime has a characteristic distribution of liquid and gas and there is then a physical basis for developing models for void fraction and frictional loss.

The basic flow patterns for upflow considered in this work are (Figure 1):

1. Bubbly flow. Here the gas phase is distributed in the form of discrete bubbles in a continuous liquid phase.
2. Slug flow. This particular form of gas-liquid distribution is characterized by a series of large bullet-shaped gas bubbles, or Taylor bubbles, occupying most of the pipe cross section. The successive Taylor bubbles are separated by slugs of continuous liquid that bridge the pipe and contain a swarm of small bubbles.
3. Churn flow. This pattern may be considered as disordered and more chaotic slug flow. The shape of the Taylor bubbles is distorted and they no longer occupy the entire pipe cross section. The continuity of the liquid slugs is oftentimes breached by high gas concentration causing the slugs to fall down. The liquid accumulates and is eventually lifted up by the gas. Thus, the liquid slug motion is oscillatory.
4. Annular flow. Here the continuous gas phase flows along the pipe core. The liquid phase moves upward partly as a wavy film adhering to the pipe wall and partly as droplets that are entrained in the gas core.

Void fraction in general is both space and time dependent. The dynamic nature of the local void fraction, which is often ignored, is most pronounced in slug and churn flow and is less important in bubbly and annular flows. Consequently, the evaluation of the average void fraction and frictional pressure drop both experimentally and theoretically, usually requires space averaging as well as time averaging procedures.

BUBBLY FLOW

At low gas rates bubbly flow can be expected to exist. The flow conditions under which this flow pattern will exist for low liquid rates is defined in Part II, equation (5). For high liquid rates the transition conditions are given in Part II by equation (12).

In this flow pattern a continuous swarm of bubbles flows upward with the liquid stream. When the pressure drop and turbulence levels are relatively low, the bubbly flow consists of a continuous swarm of bubbles in a liquid stream that flows upward in a vertical pipe. Due to buoyancy effect the gas "slips" past the liquid so that its velocity is higher than that of the liquid. The difference between the gas and liquid velocities or the relative velocity is simply the rise velocity of the bubbles relative to the flowing liquid.

Average Void Fraction

Due to bubble interaction attention is focused on the bubble bed relative velocity, namely on the rise velocity of a single bubble in a bubble swarm. This velocity is a function of the terminal velocity of a single bubble rising in an infinite medium and the local bubble concentration or local void fraction.

As the rise velocity of a single bubble in an infinite medium is known to depend strongly on its size, separate expressions have been proposed in the literature for various bubble equivalent diameter or Reynolds number regions. These equations were adapted to the case of bubble swarm in a straightforward manner that determines the exact relationship between the rise velocity and the local void fraction in the various regions. However, bubble sizes in two phase flow are known to be large enough so that the rise velocity becomes independent of diameter.

Several approaches to this problem have been developed. As an example, consider the case where the Reynolds numbers are high enough so that the velocity distribution is uniform across the pipe radius (the case to be expected at the flow rate in geothermal systems). Then the space average velocity of the gas, U_G , is equal to that of the liquid, U_L , plus the rise velocity, U_R , due to bubble buoyancy

$$U_G = U_L + U_R \quad (1)$$

Here U_G and U_L are the actual in situ average velocities. However, these are not readily calculable. From a balance, the known volumetric rate of gas flow, Q_G , and the fluid area for gas are related by

$$Q_G = U_G A_G \quad (2)$$

but

$$A_G = A\alpha \quad (3)$$

where α is the space average void fraction. Define a superficial gas velocity, U_{GS} , as the velocity of the gas if it flowed alone in the pipe of area A

$$U_{GS} A = Q_G \quad (4)$$

or

$$U_{GS} A = U_G A\alpha \quad (5)$$

Similarly for the liquid

$$U_{LS} A = U_L A(1-\alpha) = Q_L \quad (6)$$

Substituting in (1) for U_L and U_G

$$\frac{U_{GS}}{\alpha} = \frac{U_{LS}}{1-\alpha} + U_R \quad (7)$$

As indicated above the bubble rise velocity, in general, depends on the bubble size, or the bubble Reynolds number; but as the bubble size increases, U_R approaches an asymptotic value independent of Re . In two-phase flow these bubbles are always in the large size range and the following expression has been well accepted for a single bubble in a pipe (see discussion in Part II).

$$U_R = 1.53 \left[\frac{\sigma g (\rho_L - \rho_G)}{\rho_L^2} \right]^{1/4} \quad (8)$$

For a bubble rising in a swarm of surrounding bubbles this becomes

$$U_R = (1-\alpha)^{1/2} 1.53 \left[\frac{\sigma g (\rho_L - \rho_G)}{\rho_L^2} \right]^{1/4} \quad (9)$$

where σ is the interfacial tension and g is the gravitational acceleration.

Combining this with (7) gives

$$\frac{U_{GS}}{\alpha} = \frac{U_{LS}}{(1-\alpha)} + 1.53(1-\alpha)^{1/2} \left[\frac{\sigma g (\rho_L - \rho_G)}{\rho_L^2} \right]^{1/4} \quad (10)$$

Given the gas and liquid flow rate and the pipe diameter, it is now possible to calculate U_{LS} and U_{GS} from equations (4) and (6). With the physical properties known then equation (10) provides a basic equation for the space average void fraction, α , the quantity desired.

Frictional Pressure Gradient

An approach to correlating frictional pressure drop in two phase flow through similarity analysis has been discussed in detail by Dukler, et al (1969). This method was shown to be particularly useful under conditions where the holdup

was known and/or where near homogeneous flow existed. This is precisely the condition that exists in bubbly flow. The development shows that the frictional loss is expressed by a Fanning type equation

$$\left(\frac{dp}{dz}\right)_{TPF} = \frac{2f_{TP}\rho_{TP}(U_{GS}+U_{LS})^2}{D} \quad (11)$$

where

$$\rho_{TP} = \rho_G \frac{(1-\lambda)^2}{\alpha} + \rho_L \frac{\lambda^2}{1-\alpha} \quad (12)$$

$$\lambda = \frac{U_{LS}}{U_{LS}+U_{GS}} \quad (13)$$

Dukler, et al (1964) show that a normalized version of the two phase friction factor f_{TP}/f_o is uniquely related to X as shown in Figure 2. The algebraic expression for this curve is

$$\begin{aligned} \frac{f_{TP}}{f_o} &= 1 - \frac{\ln \lambda}{S} \\ S &= 1.218 + 0.478 \ln \lambda + 0.444 (\ln \lambda)^2 \\ &\quad + 0.094 (\ln \lambda)^3 + 0.00843 (\ln \lambda)^4 \end{aligned} \quad (14)$$

The normalizing friction factor is calculated from single phase friction factor correlations using a Reynolds number calculated as if both phases flowed as liquid.

$$Re_o = \frac{D(U_{LS}+U_{GS})\rho_L}{\mu_L} \quad (15)$$

SLUG FLOW

Slug flow is that flow pattern where there exists strong redistributions of the phases in the axial direction. An idealized sketch of the pattern appears in Figure 3. Large elongated bubbles of vapor, commonly called "Taylor" bubbles are separated by slugs of liquid which contain dispersed small bubbles. The Taylor bubbles nearly occupy the entire pipe cross-sectional area being separated from the wall by a thin film. The region inside the bubble has been shown to be a region of constant pressure. Therefore, there are no forces to support the liquid film and it falls under the influence of gravity. Thus, the bubbles rise through the moving liquid but at the bubble boundary the liquid film falls in the opposite direction. The falling film mixes with the liquid below to form a vortex just below the bubble.

Average Void Fraction

The liquid slug between successive bubbles occupies the entire cross-sectional area of the pipe. As discussed in Part II of this report, transition to bubbly flow takes place at a void fraction of about 0.25. Thus, when transition has taken place, bubbles continue to exist in the liquid phase at this void level.

Consider one slug unit being a single Taylor bubble of length l_G and one liquid slug of length l_L as shown in Figure 3. Designate α_{TB} and α_{LS} as the average voids in the Taylor bubble and liquid slug respectively. Then, α , the space average void fraction over one slug unit is

$$\alpha = \alpha_{TB} \frac{l_G}{l} + \alpha_{LS} \frac{l_S}{l} \quad (16)$$

In most cases the cap of the Taylor bubble is small compared to its length and α_{TB} can be estimated from

$$\alpha_{TB} = \frac{A - A_f}{A} = \frac{A_G}{A} \quad (17)$$

where A is the pipe area, A_f the area occupied by film and A_G is the cross-sectional area of the Taylor bubble. Since $l_s/l = 1 - l_G/l$, equation (16) can be rearranged as

$$\alpha = \left(\frac{A_G}{A} - \alpha_{LS} \right) \frac{l_G}{l} - \alpha_{LS} \quad (18)$$

The quantities l_G/l can be found from a gas phase balance. In one slug unit the gas content is

$$= A_G l_G + A(l - l_G)\alpha_{LS}$$

The time of passage of a slug unit is:

$$= \frac{l}{U_G}$$

Thus the flow rate of gas, $U_{GS}A$ becomes

$$U_{GS}A = \frac{A l_G + A(l - l_G)\alpha_{LS}}{l/U_G} \quad (19)$$

Some rearrangement produces

$$\frac{l_G}{l} = \frac{\frac{U_{GS}}{U_G} - \alpha_{LS}}{\frac{A_G}{A} - \alpha_{LS}} \quad (20)$$

Substitution of this expression into equation (18) gives

$$\alpha = \frac{U_{GS}}{U_G} \quad (21)$$

But, as discussed in Part II, there exists a well documented equation for the Taylor bubble velocity

$$U_G = 1.2 U_L + 0.35\sqrt{gD} \quad (22)$$

Thus, the problem reduces to finding U_L , the average liquid velocity in the slug. Consider a control volume formed by the volume between the inlet to the pipe and a plane cutting the flow in a liquid slug. Make a total flow balance

$$(U_{LS} + U_{GS})A = U_L A(1 - \alpha_{LS}) + U_{GL} A \alpha_{LS} \quad (23)$$

where U_{GL} is the velocity of gas bubbles in the liquid slug. But, from equation (1),

$$U_{GL} = U_L + U_R$$

$$U_L = (U_{LS} + U_{GS}) - U_R \alpha_{LS} \quad (24)$$

Thus, substituting in equation (22) using equation (9) for U_R and setting $\alpha_{LS} = 0.25$

$$U_G = 1.2(U_{LS} + U_{GS}) - 0.40 \left[\frac{\sigma g \Delta \rho}{\rho^2} \right]^{1/4} + 0.35 \sqrt{gD} \quad (25)$$

Now all quantities are known to find the average void fraction in the slug unit through equations (21) and (25) using $\alpha_{LS} = 0.25$.

Frictional Pressure Gradient

The liquid in the film along side the bubble is free falling, and the pressure drop across the bubble length, l_G , is zero. Thus, the average frictional pressure gradient over the slug unit is

$$\left(\frac{dP}{dz} \right)_{TPF} = \left(\frac{dP}{dz} \right)_{Slug} \cdot \frac{l_S}{l} \quad (26)$$

The liquid slug behaves as a system in essentially homogeneous bubbly flow. Thus, its pressure gradient can be calculated from the homogeneous flow equation

$$\left(\frac{dP}{dz}\right)_{\text{Slug}} = \frac{2f_M \rho_M (U_{LS} + U_{GS})^2}{D} \quad (27)$$

where

$$\rho_M = \rho_G \alpha_{LS} + \rho_L (1 - \alpha_{LS}) \quad (28)$$

and f_M is evaluated using single phase friction factor-Reynolds number curves where

$$Re_M = \frac{D(U_{LS} + U_{GS}) \rho_M}{\mu_M} \quad (29)$$

$$\mu_M = \mu_G \alpha_{LS} + \mu_L (1 - \alpha_{LS}) \quad (30)$$

The ratio of liquid slug length, ℓ_S , to the length of a slug unit, ℓ can be found from

$$\frac{\ell_S}{\ell} = 1 - \frac{\ell_G}{\ell} \quad (31)$$

where ℓ_G/ℓ is given by equation (20). In order to apply equation (20), it is necessary to find expressions for U_{GS}/U_G and A_G/A . The value of U_G can be solved from equations (22) and (23) by eliminating U_L to obtain

$$\frac{U_{LS} + U_{GS}}{U_G} = \frac{1 + 0.2 \alpha_{LS}}{1.2 + \frac{0.35(1 - \alpha_{LS})}{U_{GS}} \sqrt{gD}} \quad (32)$$

Thus, the first unknown, U_{GS}/U_G can be found from

$$\frac{U_{GS}}{U_G} = \frac{(1 + 0.2 \alpha_{LS})}{[1.2 + \frac{0.35(1 - \alpha_{LS})}{U_{GS}} \sqrt{gD}]} \frac{U_{GS}}{[U_{LS} + U_{GS}]} \quad (33)$$

Developing the expression for A_G/A is rather more involved. First, a continuity relationship can be developed by considering the liquid flow relative

to the nose of the bubble

$$A(1-\alpha_{LS})(U_G - U_L) = A_f(U_G + U_f) \quad (34)$$

where A_f is the area occupied by the falling film and the desired area ratio is

$$\frac{A_G}{A} = 1 - \frac{A_f}{A} \quad (35)$$

Expressions for U_G and U_L for substitution into equation (34) are given by equations (24) and (25). U_f can be found as follows. The flow of the liquid film is controlled by a balance between the force of gravity and the wall shear

$$\rho_L g A_f = \frac{f_f}{2} \rho_L U_f^2 \pi D \quad (36)$$

The film friction factor, f_f , is estimated from the Blasius equation

$$f_f = C_L \left(\frac{4A_f U_f}{\pi D v_L} \right)^{-n} \quad (37)$$

For turbulent flow, the case to be expected in large pipes, $C_L = 0.046$ and $n = 0.2$. For small pipes and low flow rates where the film Reynolds number is below 1000, $C_L = 16$ and $n = 1.0$. Substituting for f_f in equation (36) and solving for U_f give

$$U_f = K \left(\frac{A_f}{A} \right)^q \quad (38)$$

where

$$q = \frac{1+n}{2-n}$$

$$K = \left(\frac{D^{1+n} g}{2 C_L v_L^n} \right)^{\frac{1}{2-n}}$$

By substitution of equation (38) for U_f , (24) for U_L and (25) for U_G into equation (34), one obtains the relationship which permits the calculation of A_f/A

$$0.150 - \frac{0.066}{Ku} + \frac{0.262}{Fr} = \frac{A_f}{A} \left\{ 1.2 - \frac{0.40}{Ku} + \frac{0.35}{Fr} + \frac{K}{(U_{LS} + U_{GS})} \left(\frac{A_f}{A} \right)^n \right\} \quad (39)$$

where the dimensionless groups are

$$Ku = (U_{LS} + U_{GS}) \left[\frac{\rho_L^2}{\sigma g \Delta \rho} \right]^{1/2} \quad (40)$$

$$Fr = \frac{(U_{LS} + U_{GS})}{\sqrt{gD}} \quad (41)$$

Once Ku , Fr and K have been calculated from the known operating conditions, equation (39) can be used to find A_f/A by a trial solution and then A_G/A can be calculated.

A summary of the sequence of calculations for frictional pressure gradient follows:

- . Find A_f/A from equation (39)
- . Find A_G/A from equation (35)
- . Find U_{GS}/U_G from equation (33)
- . Find l_G/l from equation (20) with $\alpha_{LS} = 0.25$
- . Find l_S/l from equation (31)
- . Find $(dP/dz)_{\text{Slug}}$ from equation (27)
- . Find $(dP/dz)_{\text{TPF}}$ for a slug unit from equation (26)

In order to calculate the pressure drop, it is necessary to know the length of a slug unit, ℓ . In Part II it is shown that liquid slugs have a near constant length in relation to diameter such that

$$\frac{\ell_S}{D} \approx 16 \quad (42)$$

Knowing $\frac{\ell_S}{\ell}$ this permits calculation of ℓ and the pressure drop along the slug.

ANNULAR FLOW

Annular flow is characterized by the flow of a thin liquid film around the wall with the gas flowing as a core. The gas phase carries entrained droplets.

Average Void Fraction

At any cross section the area occupied by liquid, A_L , consists of that occupied by liquid in the film of thickness, m , plus that of the entrained liquid.

$$A_L = A_{LF} + A_{LE} \quad (43)$$

These films are usually thus compared to the diameter and their area is given by $A_{LF} = \pi Dm$. Thus

$$1 - \alpha = \frac{4m}{D} + \frac{A_{LE}}{A} \quad (44)$$

It is thus necessary to find expressions for m and A_{LE} . If F_E represents the fraction of the volumetric liquid rate flowing, Q_L , which is entrained then

$$Q_{LE} = F_E Q_L \quad (45)$$

Let V_{LE} be the average velocity of the entrained drops.

$$Q_{LE} = U_{LE} A_{LE} \quad (46)$$

At the high velocities of interest in geothermal systems the slip between the drops and gas is small. Thus, $U_{LE} \approx U_G$.

$$\frac{A_{LE}}{A} = \frac{Q_{LE}}{U_{LE}} = \frac{F_E Q_L}{U_G A} = \frac{F_E U_{LS}}{U_G} \quad (47)$$

From a mass balance on the gas

$$(1-\alpha) U_G = U_{GS} \quad (48)$$

Thus

$$\frac{A_{LE}}{A} = Fe \frac{U_{LS}}{U_{GS}} (1-\alpha) \quad (49)$$

Substituting into equation (44) and rearranging provides the relationship for the average voids in annular flow

$$\alpha = 1 - \frac{4m/D}{1 - \frac{Fe U_{LS}}{U_{GS}}} \quad (50)$$

To find the voids one must have an independent expression for the film thickness, m/D and the fraction of liquid entrained, F_E .

Estimation of F_E : Wallis (1969) has proposed a correlation for entrainment which can be expressed by the following equation

$$F_E = 1 - \exp \left[-0.125(\phi - 1.5) \right] \quad (51)$$

where

$$\phi = 10^4 \alpha \frac{U_G \mu_G}{\sigma} \left(\frac{\rho_G}{\rho_L} \right)^{1/2} \quad (52)$$

Substituting for V_G from equation (48)

$$\phi = 10 \frac{\alpha}{1-\alpha} \frac{U_{GS} \mu_G}{\alpha} \left(\frac{\rho_G}{\rho_L} \right)^{1/2} \quad (53)$$

Estimation of m/D : The film thickness, m , can be estimated by the method originally presented by Dukler (1959) for falling liquid films in the presence of interfacial shear. In this instance it is necessary to apply the technique for upflow and the methodology for doing that has been presented by Hewitt (1961).

The method, which is not complex, is involved algebraically and will not be presented here. It is shown in the original work that a dimensionless film thickness can be related unambiguously to a dimensionless flow rate, namely, the Reynolds number and a dimensionless interfacial shear, β . Or,

$$m\Psi = \phi(\text{Re}_{\text{LFS}}\beta) \quad (54)$$

where

$$\Psi = \left(\frac{\rho_L^2 g}{\mu_L^2} \right)^{1/3} \quad (55)$$

$$\text{Re}_{\text{LF}} = \frac{4w_{\text{LF}}}{\pi S \mu_L} \quad (56)$$

$$\beta = \frac{(dP/dz)_{\text{TPF}}^D}{4\rho_L^{1/3} (g_L)^{2/3}} \quad (57)$$

Figure 4 gives the results of the integrations for upflow over a wide range of values of β and Re_{LF} . It becomes clear that in order to solve for the film thickness, it is necessary to know the frictional pressure drop.

Frictional Pressure Gradient

Wisman (1975) evaluated the Dukler similarities method (1964) for calculating the frictional gradient under conditions of upward annular flow and showed good agreement with experiment. The equations for this correlation have already been presented as 11-13.

It should be noted that the calculation of average voids and pressure gradient is an iterative process once given the flow rates, properties and pipe size. Equation (50) for voids, α , requires a knowledge of F_e and m . In order to find F_e (equation 51), one must know the voids (equation (53)). In addition, in order to find m , it is necessary to know β (equation 54). Knowing β requires the frictional pressure gradient (equation 57). As shown in equations 11 and 12 this requires the void fraction.

CHURN FLOW

It is shown in Part II of this report that churn flow represents a transitional flow pattern. At very high values of L/D , characteristic of a geothermal producing well, the length of well string experiencing the churn flow pattern will be small. As seen from Part II, Figure 3, for low pressure, at values of L/D above 500 the annular and slug boundaries approach each other. This is even more emphasized at higher gas densities as shown in Figure 5. Thus, for an 8 inch diameter well, the length in churn flow could be of the order of 500-750 feet.

Because of the transition nature of the flow definitive models are not available. It is suggested that voids be calculated by linear interpolation of the void values calculated at the slug flow and annular transitions. That is, let α_s and α_a be the voids at the slug transition boundaries where $(U_{GS})_s$ and $(U_{GS})_a$ are the superficial gas velocities predicted for transition. Then

$$(\alpha - \alpha_a) = \frac{U_{GS} - (U_{GS})_s}{(U_{GS})_G - (U_{GS})_s} (\alpha_s - \alpha_a) \quad (58)$$

The frictional gradient can be calculated using the similarity equations as in annular flow.

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NOMENCLATURE

A	Flow cross-sectional area
C	Constant in the friction factor correlation, equation (37)
D	Pipe diameter
Fr	Dimensionless group defined by equation (41)
Fe	Fraction liquid entrained
f	Friction factor
g	Acceleration of gravity
K	Constant of equation (38)
ku	Dimensionless group defined by equation (40)
l	Length
M	Film thickness
n	Exponent, equation (37)
P	Pressure
Q	Volumetric flow rate
Re	Reynolds number
S	Defined by equation (14)
U	Velocity
w	Mass flow rate
z	Axial coordinate direction

Greek Symbols

α	Void fraction
β	Dimensionless interfacial shear, equation (57)
λ	Flowing volume fraction liquid, equation(13)
μ	Viscosity

ρ	Density
σ	Surface tension
ϕ	Dimensionless group, equation (52)
ψ	Dimensionless film thickness, equation (55)

Subscripts

f	Falling film
G	Gas phase
GS	Superficial for gas flowing alone
L	Liquid
LE	Liquid entrained
LF	Liquid film
LS	Liquid flowing alone in the pipe, also refers to liquid slug when used with voids
m	Gas-liquid mixture
R	Relative
S	Slug
TP	Two phase
TPF	Two phase friction

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4. Film Thickness Correlation.

FIGURE 1 FLOW PATTERNS IN VERTICAL FLOW.

FIGURE 2
NORMALIZED FRICTION FACTOR CURVE

FIGURE 3 SLUG FLOW GEOMETRY

FIGURE 4 FILM THICKNESS CORRELATION

PART IV

EXPERIMENTAL STUDIES OF
LIQUID HOLDUP IN UPWARD CO-CURRENT
TWO PHASE FLOW IN VERTICAL TUBES

SUMMARY

An experimental apparatus was designed and constructed to measure flow rates, temperatures and pressures over a fairly wide range of gas and liquid flow rates pertinent to the bubbly, slug, churn and annular flow regimes. The fluids used were air and water.

In order to test the physical models for the liquid holdup - or, equivalently, the void fraction - a special Quick Closing Valves (QCV) System was designed and installed in the test section; several data points for the volume-average liquid holdup for all flow patterns were taken by means of the Quick Closing Valves System. These data were used to check the void fraction models for the bubbly, slug and churn flow regimes and provided a better understanding of the void fraction phenomenon in co-current gas-liquid two-phase flows in vertical pipes.

EXPERIMENTAL EQUIPMENT AND MEASUREMENT TECHNIQUE

Test Section and Flow Loop

The flow system was designed to measure the following parameters and variables:

- a) air and water flow rates
- b) temperatures of air and of water just before being mixed to form the two-phase flow
- c) pressure of the two-phase mixture within the test section, at the measuring stations
- d) volume-average liquid holdup of the two-phase flow, over a range of air and water flow rates.

A schematic diagram of the flow loop is shown in Figure 1. The vertical test section was made up of segments of Plexiglas pipe of inside diameter equal to $5.074 \times 10^{-2} \text{ m} \pm 8 \times 10^{-4} \text{ m}$ and with a nominal wall thickness of $6.35 \times 10^{-3} \text{ m}$. The experimental section length, from the beginning of the two-phase flow to the end of the vertical test section, was 10.084 meters. Air and water were mixed together in an injection nozzle, which is drawn schematically in Figure 2. Water flowed axially, while air was injected radially through a horizontal row of 80 cylindrical holes of diameter equal to $1.59 \times 10^{-3} \text{ m}$ and $5.0 \times 10^{-3} \text{ m}$ long. These holes were equally spaced on the internal circumference of the injection block which was made of Plexiglas.

Three sets of valves and pertinent actuators were installed at the locations shown in Figure 1. Each valve/actuator set weighed 40 kg approximately. Metallic supports were designed not only to hold these sets in position and aligned precisely with the vertical test section but also to avoid any propagation of vibrations to the Plexiglas structure, due to the mechanical shock at the end of the stroke caused by the fast simultaneous closure of the 3 valves.

Each measuring station consisted of a pair of adjacent valves. The distance between the injection nozzle and the first measuring station was made long enough such that entrance effects in the two-phase flow were essentially eliminated.

At the end of the test section there was an overhead sealed separator (0.76 m x 1.83 m galvanized steel cylindrical tank). The exit end of the test section penetrated the bottom of the separator tank at its center line, through a specially machined O-ring, sealed fitting, which allowed axial as well as some lateral movement of the test section to avoid breakage of the Plexiglas pipe.

The entire system was supported by a Unistrut structure and the air and water lines were connected to the test section by means of flexible rubber hoses to eliminate external vibrations. Care was taken to maintain alignment of the test section with the vertical.

Air was cycled in an open loop; the incoming air (at about $6.895 \times 10^5 \text{ N/m}^2$) was filtered, regulated to $3.447 \times 10^5 \text{ N/m}^2$ at most and measured through rotameters or orifice meters, then injected radially in the test section through four flexible polyethylene tubes connected to the injection nozzle. The air was separated from the two-phase mixture in the overhead separator tank and then vented to the atmosphere through a back pressure valve.

Tap water was injected axially at the bottom of the test section and was circulated in the system in a closed cycle by means of a centrifugal pump (48.768 m total discharge head).

Air flow rates in the test section were determined by four calibrated rotameters and one calibrated orifice meter with overlapping ranges. Water flow rate measurements were accomplished by two calibrated rotameters and one calibrated orifice meter with overlapping ranges.

The temperatures of the air and water were measured by thermometers which could provide readings down to 0.1°C , in the range -1°C to 51°C . These thermometers were located in the air and water lines close to the injection nozzle such that the temperatures of the fluids could be taken just before the air and water were mixed to form the two-phase mixture.

The static pressure of the two-phase mixture within each measuring station was determined by Bourdon-tube pressure gages, which were connected to the wall of the test section by means of a special acrylic-made purging device used to eliminate air bubbles eventually entrapped in the pressure gage line caused by the two-phase flow.

Experimental Technique

The liquid holdup measurements were accomplished by the Quick Closing Valves method. The system designed for this study consisted of three identical valve/actuator sets. The valves utilized were Jamesbury model 2" 6150, full ported ball valves 150# flanged all stainless steel with teflon seats. These 2 inch I.D. (5.08×10^{-2} m) flanged ball valves were required to be of full port design so that when they were open the two-phase flow was completely unobstructed, the valves being essentially a section of straight pipe. The actuators were Jamesbury Quadra-Powr B-60S spring-diaphragm actuators (air-to-open/spring-to-close), supplied with a Conoflow pressure regulator, an ASCO solenoid pilot valve and a Humphrey quick exhaust valve. The solenoid pilot valves were oversized and the diameter of the orifice of the quick exhaust valves was increased to insure a fast closing action of the ball valves. The period of time that each valve/actuator set takes to move from a fully open to a fully closed position was measured by an optical method. The average closing time interval was determined as being equal to 54 milliseconds.

All three actuators were connected to a common on-off electrical switch so that a simultaneous closure of the three ball valves could be accomplished.

The experimental procedure to obtain the volume average liquid holdup data was the following: a specific flow regime was established in the two-phase column and maintained long enough so that equilibrium conditions were attained. After the measurements of pressures, temperatures and superficial velocities of the fluids were taken, the Quick Closing Valves System was actuated entrapping a previously selected structure of the two-phase flow within one of the measuring stations; the water was allowed to flow completely (by gravity) to the lower region of the particular measuring station and the height of the interface air-water was determined by an appropriate tape measure attached to the system.

The measuring station No. 1 was made much shorter than the other to enable the determination of the liquid holdup of the liquid slug and the Taylor bubble in the slug flow regime.

Photographic Technique

In order to determine the average lengths of the Taylor bubble and the liquid slug as a function of the gas and liquid superficial velocities, the slug flow pattern was photographed by means of a Beaulieu R-76, 16 mm movie camera. A speed of 16 frames/second was selected to provide a representative picture of the two-phase flow under study.

EXPERIMENTAL RESULTS

Foreword: The Lump Phenomenon

During the data acquisition for the present work, a distinct phenomenon concerning the liquid holdup was observed visually and measured with the Quick Closing Valves System, both in the churn flow and in the annular flow patterns. We have called it the lumping phenomenon. This can be described by the sudden formation of a foamy wisp which bridges the pipe and in so doing is swept along the duct by the gas phase at a high velocity. After a certain period of time, ranging from 2 to 5 seconds depending on the flow regime and the superficial velocities, other lumps are formed in sequence at a nearly regular frequency. From the experimental observations it appears that at equilibrium conditions the lumps travel with a uniform speed equal to the mean velocity of the gas phase and essentially with no slip occurring within it.

Since the lumps carry a great quantity of liquid, they affect drastically the local liquid holdup at a given section of the pipe during their passage. Therefore, any mass, momentum or heat transfer process taking place in churn flow and in annular flow is bound to be affected by these wisps. It is clear that the two-phase flow literature has greatly underestimated this important hydrodynamic phenomenon.

Presently, we are conducting studies to model the lumps in order to predict their liquid holdups.

Tabulated Data

Due to the stochastic nature of the two-phase flow, a series of repeated measurements of the volume average liquid holdup, R_L , was performed (typically 10 determinations) for each given pair of superficial velocities, U_G^S and U_L^S . The mean liquid holdup, $\bar{R}_L$, was computed as well as the sample standard

deviation of the mean, $\sigma_{\bar{R}_L}$, and the percent standard deviation (ratio of the standard deviation to the mean expressed as percentage), $\sigma_{\bar{R}_L}/\bar{R}_L \times 100$, which is a measure of the uncertainty in the liquid holdup data points. All of these quantities are presented in the pertinent tables in addition to the average temperatures of the air and water (which were kept within $\pm 2^\circ\text{C}$ during the experiments).

In the churn and the annular flow regimes data points for the holdup were obtained both with one lump entrapped within the measuring station No. 2 and without any lump entrapped inside that station; this is pointed out at the bottom of the tables under "Observations." Liquid holdup data for bubble flow is presented in Table 1.

The results concerning the slug flow regime are presented in Tables 2-A and 2-B for the Taylor bubble and the liquid slug, respectively. Also, the average lengths of the Taylor bubble, $\bar{l}_{TB}$, and of the liquid slug, $\bar{l}_{LS}$, were determined from the photographic technique for 6 pairs of superficial velocities (additional data points are being analyzed from the photographs taken); they are shown in Table No. 3. These values, together with the mean liquid holdups of the Taylor bubble and the liquid slug presented in Tables 2-A and 2-B, enable the computation of the experimental volume average liquid holdup of the whole slug unit, $\bar{R}_{LSU}$, by the following formula:

$$\bar{R}_{LSU} = \frac{\bar{l}_{TB}}{\bar{l}_{SU}} \bar{R}_{LTB} + \frac{\bar{l}_{LS}}{\bar{l}_{SU}} \bar{R}_{LLS}$$

where $\bar{l}_{SU} = \bar{l}_{TB} + \bar{l}_{LS}$.

Since by definition $\alpha = 1 - R_L$, we can also compute the volume average void fraction over the entire slug unit, $\bar{\alpha}_{SU}$, either by

$$\bar{\alpha}_{SU} = 1 - \bar{R}_{LSU}$$

or by the formula

$$\bar{\alpha}_{SU} = \frac{\bar{l}_{TB}}{\bar{l}_{SU}} \bar{\alpha}_{TB} + \frac{\bar{l}_{LS}}{\bar{l}_{SU}} \bar{\alpha}_{LS}$$

These results are reported in Table No. 4.

Holdup data for churn flow are listed in Table 5-A, in the absence of a lump, and in Table 5-B with a lump in the measuring section. Similarly Table 6-A represents holdup without lumps present and 6-B in the presence of a lump in the annular flow regime.

Graphical Results

The experimental data points are represented graphically in Figures 3 to 10. These graphs locate the mean volume-average liquid holdup, $\bar{R}_L$, versus the gas superficial velocity, U_G^S , having U_L^S as the parameter.

The pairs of superficial velocities which characterize the liquid holdup data taken are plotted in the flow pattern transition map for upward gas-liquid flow in vertical tubes, developed by Taitel, et al (submitted to AIChE Journal for publication recently) in order to provide a comprehensive picture of the flow pattern regions explored in the experimental work. This is shown in Figure 11. A more complete set of data could not be taken due to practical limitations in the system (centrifugal pump, pressure of the incoming compressed air, etc.), in the quick closing valves technique and/or in the visual observations.

Theoretical Predictions

The physical models for the bubbly and for the slug flow regimes were solved, generating a set of theoretically calculated values for the volume-average liquid holdup. These are presented in Tables 7 and 8 for the bubble and slug flow patterns, respectively.

The comparison between the experimental and the theoretical holdup is represented graphically in Figures 12 and 13. We note that for slug flow only 6 points could be compared (namely, the ones shown in Table 4) because the analysis of the photographs made for obtaining the mean lengths of Taylor bubbles and liquid slugs has not been completed.

Observing Figure 12 we see that the physical model for the bubbly flow predicts fairly well the liquid holdup. Excluding the transition points, the maximum percent relative deviation (defined as $\Delta R_L \% = |R_{L_{Th}} - \bar{R}_{L_{exp}}| / \bar{R}_{L_{exp}} \times 100$) obtained was 3.52%, which is good. At higher values of holdup, the model's predictions are in a slightly better agreement with the experimental data. On the other hand, the holdup predictions for the bubbly-slug transition points are not good by this model, being off the corresponding experimental values up to 14.2% (in the worst case). These same transition points were fed to the slug flow model which, on the average, gave worse results for the holdup than the bubbly flow model. Therefore, at this stage we would recommend the use of the latter for predicting the holdup at the transition bubbly slug. It is possible that an eventual refinement of these models, or the application of a new comprehensive theory presently being developed in our laboratory will generate better results in the near future.

Figure 13 and the corresponding tables show that the slug flow model does an acceptable job of predicting liquid holdup, the error being within 10%.

Similarly, the slug flow model applied to the churn flow data is highly encouraging. Figure 14 shows the experimental data collected both with and without lumps. A horizontal line between two points connects data taken at corresponding flow conditions. Notice that, in most cases, the average of the two are in good agreement with theory.

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FIGURE 2

FIGURE NO. 3
 LIQUID HOLDUP VERSUS U_G^S , U_L^S
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TABLE 1 - LIQUID HOLDUP DATA

(1/3)

FLOW PATTERN: BUBBLY FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L} / \bar{R}_L \times 100(\%)$
	$\bar{T}_G (^{\circ}\text{C})$	$\bar{T}_L (^{\circ}\text{C})$	$U_G^S (\text{m/s})$	$U_L^S (\text{m/s})$			
	23.2	23.9	0.0093	0	96.0	0.11	0.11
	22.6	23.5	0.0161	0	93.0	0.20	0.22
	23.5	23.5	0.0203	0	91.9	0.26	0.28
	23.5	23.6	0.0226	0	90.3	0.20	0.22
Transition to slug flow	23.6	23.8	0.0273	0	89.8	0.28	0.32
	23.6	29.5	0.0161	0.0161	94.1	0.30	0.32
	23.7	30.0	0.0228	0.0161	92.2	0.53	0.58
Transition to slug flow	22.9	28.5	0.0281	0.0161	92.0	0.22	0.24
	23.8	29.1	0.0229	0.099	93.4	0.24	0.25
	23.7	30.1	0.0356	0.099	91.2	0.39	0.43

TABLE 1 (CONTINUED)

(2/3)

FLOW PATTERN: BUBBLY FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
Transition to slug flow	23.8	27.7	0.0431	0.099	89.9	0.32	0.36
Transition to slug flow	23.7	29.9	0.048	0.099	89.4	0.24	0.27
	23.2	28.5	0.0379	0.322	94.2	0.31	0.33
	23.8	29.7	0.099	0.323	86.0	0.40	0.47
	23.7	27.2	0.169	0.322	76.7	0.26	0.33
Transition to slug flow	24.0	30.8	0.195	0.323	77.6	0.67	0.87
	23.3	26.5	0.098	0.658	89.9	0.16	0.18
	23.6	28.0	0.228	0.658	80.4	0.13	0.16
Transition to slug flow	23.8	26.9	0.41	0.658	77.0	0.47	0.61
	24.0	28.6	0.35	1.01	77.7	0.12	0.15

TABLE 1 (CONTINUED)

(3/3)

FLOW PATTERN: BUBBLY FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G$ (°C)	$\bar{T}_L$ (°C)	U_G^S (m/s)	U_L^S (m/s)			
	23.9	30.0	0.42	1.01	74.9	0.30	0.40
Transition to slug flow	23.8	31.2	0.48	1.01	77.0	0.67	0.87
	23.2	27.4	0.38	1.93	84.6	0.23	0.27
	23.4	29.1	0.49	1.93	80.3	0.16	0.20
	23.5	30.3	0.63	1.93	77.3	0.21	0.27
Transition to slug flow	23.4	30.6	0.71	1.93	75.8	0.27	0.36
	23.3	26.8	0.46	2.90	86.3	0.13	0.15
	23.5	28.1	0.63	2.90	82.4	0.06	0.07
Transition to slug flow	23.4	30.1	0.89	2.91	77.9	0.15	0.19

TABLE 2-A - LIQUID HOLDUP DATA

(1/2)

FLOW PATTERN: SLUG FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_{L_{TB}}$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
	23.6	26.9	0.51	0	16.5	0.77	4.66
	23.3	27.0	0.99	0	15.2	0.93	6.15
	23.5	28.5	0.42	0.099	15.8	0.57	3.60
	23.5	28.6	0.51	0.121	16.0	0.69	4.32
	23.5	29.4	0.67	0.121	14.8	0.85	5.73
	23.8	28.9	0.83	0.121	16.0	0.54	3.38
Transition to churn flow	23.2	28.6	1.32	0.121	14.8	0.92	6.20
	24.7	30.0	0.68	0.222	13.9	0.61	4.36
	23.8	28.4	0.91	0.222	13.8	0.74	5.34
Transition to churn flow	23.5	28.7	1.40	0.222	15.4	0.49	3.16

TABLE 2-A (CONTINUED)

(2/2)

FLOW PATTERN: SLUG FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_{L_{TB}}$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
	23.1	28.1	0.76	0.322	14.8	0.92	6.22
	24.5	30.4	1.08	0.323	13.3	0.78	5.86
Transition to churn flow	23.4	28.8	1.41	0.322	12.8	0.50	3.87
	23.5	27.5	1.17	0.490	13.8	0.54	3.91
Transition to churn flow	23.7	30.3	1.50	0.490	13.6	0.59	4.36
	23.6	30.1	1.16	0.608	16.0	0.45	2.83

TABLE 2-B - LIQUID HOLDUP DATA

(1/2)

FLOW PATTERN: SLUG FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_{LS}$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G$ (°C)	$\bar{T}_L$ (°C)	U_G^S (m/s)	U_L^S (m/s)			
	23.4	26.8	0.51	0	72.7	0.29	0.39
	23.3	26.8	0.99	0	73.1	0.25	0.34
	23.8	29.9	0.42	0.099	72.3	0.60	0.83
	23.2	29.3	0.51	0.121	71.9	0.30	0.42
	23.4	30.3	0.67	0.121	71.3	0.36	0.50
	23.8	29.2	0.83	0.121	71.1	0.25	0.36
Transition to churn flow	23.2	28.1	1.32	0.121	72.3	0.41	0.56
	24.2	29.2	0.68	0.222	73.3	0.50	0.68
	23.9	27.5	0.91	0.222	71.9	0.54	0.75
Transition to churn flow	23.4	29.5	1.40	0.222	70.1	0.41	0.59

TABLE 2-B (CONTINUED)

(2/2)

FLOW PATTERN: SLUG FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_{L_{LS}}$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
	22.9	28.1	0.76	0.322	72.8	0.39	0.53
	24.3	30.9	1.08	0.323	71.3	0.53	0.74
Transition to churn flow	23.1	29.3	1.41	0.322	72.4	0.56	0.78
	24.7	31.0	1.17	0.491	73.4	0.59	0.80
Transition to churn flow	23.4	28.8	1.50	0.490	71.3	0.81	1.14
	23.6	28.4	1.16	0.608	73.7	0.52	0.71

TABLE 3
DATA ON AVERAGE LENGTHS OF TAYLOR BUBBLES AND LIQUID SLUGS

FLOW PATTERN: SLUG FLOW

PHOTOGRAPHIC TECHNIQUE

Superficial Velocities		Taylor Bubble			Liquid Slug		
U_G^S (m/s)	U_L^S (m/s)	Average Length $\bar{l}_{TB}$ (m)	Sample Standard Deviation $\sigma_{\bar{l}_{TB}}$ (m)	Percent Standard Deviation $\sigma_{\bar{l}}/\bar{l} \times 100(\%)$	Average Length $\bar{l}_{LS}$ (m)	Sample Standard Deviation $\sigma_{\bar{l}_{LS}}$ (m)	Percent Standard Deviation $\sigma_{\bar{l}}/\bar{l} \times 100(\%)$
0.42	0.099	0.8	0.10	12.1	1.3	0.10	7.9
0.51	0.121	1.0	0.13	12.4	0.97	0.09	9.6
0.67	0.121	1.3	0.15	11.9	1.06	0.07	6.7
0.83	0.121	1.5	0.18	12.1	1.2	0.12	9.9
0.68	0.222	0.94	0.09	9.7	1.01	0.07	6.7
0.91	0.222	1.7	0.30	17.7	0.93	0.06	6.0

TABLE 4 - LIQUID HOLDUP DATA

FLOW PATTERN: SLUG FLOW

Superficial Velocities		Average Lengths		Average Liquid Holdups		Overall Volume Average Liquid Holdup	Overall Volume Average Void Fraction
U_G^S (m/s)	U_L^S (m/s)	$\bar{l}_{TB}$ (m)	$\bar{l}_{LS}$ (m)	$\bar{R}_{LTB}$ (%)	$\bar{R}_{LLS}$ (%)	$\bar{R}_{LSU}$ (%)	$\bar{\alpha}_{SU}$ (%)
0.42	0.099	0.8	1.3	15.8	72.3	50.8	49.2
0.51	0.121	1.0	0.97	16.0	71.9	43.5	56.5
0.67	0.121	1.3	1.06	14.8	71.3	40.2	59.8
0.83	0.121	1.5	1.2	16.0	71.1	40.5	59.5
0.68	0.222	0.94	1.01	13.9	73.3	44.7	55.3
0.91	0.222	1.7	0.93	13.8	71.9	34.3	65.7

TABLE 5-A - LIQUID HOLDUP DATA

(1/2)

FLOW PATTERN: CHURN FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
	22.9	23.4	1.69	0	27	1.2	4.4
	22.7	23.1	3.16	0	18.3	0.74	4.04
	22.7	27.3	1.96	0.099	24	1.0	4.4
	23.0	27.8	3.25	0.099	19.7	0.50	2.52
Transition to annular flow	23.0	27.5	6.8	0.099	14.4	0.28	1.95
	23.1	27.5	3.33	0.322	20.2	0.51	2.51
	23.1	27.3	5.1	0.322	17.1	0.44	2.55
Transition to annular flow	23.0	27.6	7.5	0.322	14.3	0.54	3.73
	23.2	27.6	4.7	0.658	17.9	0.60	3.36
	22.9	27.9	5.9	0.658	16.8	0.45	2.66

Observations: These data were taken without any lump within the measuring station No. 2.

TABLE 5-A (CONTINUED)

(2/2)

FLOW PATTERN: CHURN FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G(^{\circ}\text{C})$	$\bar{T}_L(^{\circ}\text{C})$	$U_G^S(\text{m/s})$	$U_L^S(\text{m/s})$			
Transition to annular flow	22.9	26.4	8.1	0.658	13.4	0.48	3.55
	22.9	27.8	5.5	1.01	19.6	0.61	3.10
*	23.0	27.6	8.2	1.01	22	1.02	4.62
Transition to annular flow *	23.0	27.7	13.3	1.01	16	1.20	7.28
*	23.1	28.8	5.9	2.62	36.5	0.80	2.19
*	22.9	29.7	8.4	2.62	30	1.07	3.51

Observations: These data were taken without any lump within the measuring station No. 2.

* The points marked with an asterisk were obtained randomly with respect to the lumps due to limitations of the experimental method and/or in visual observations.

TABLE 5-B - LIQUID HOLDUP DATA

FLOW PATTERN: CHURN FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L}/\bar{R}_L \times 100(\%)$
	$\bar{T}_G (^{\circ}\text{C})$	$\bar{T}_L (^{\circ}\text{C})$	$U_G^S (\text{m/s})$	$U_L^S (\text{m/s})$			
	23.0	28.0	3.24	0.099	25.2	0.30	1.19
	23.2	27.7	3.33	0.322	27.3	0.83	3.05
	23.0	27.3	5.1	0.322	28.5	0.76	2.69
Transition to annular flow	22.9	26.3	7.4	0.322	22.2	0.44	2.00
	23.2	27.5	4.7	0.658	30	2.02	6.73
	22.8	27.5	5.9	0.658	24.6	0.56	2.26
Transition to annular flow	22.9	26.9	8.1	0.658	27	1.24	4.57
	22.9	27.7	5.5	1.01	29.4	0.77	2.61

Observations: The data points shown in this table were taken with one lump inside the measuring station No. 2.

TABLE 6-A - LIQUID HOLDUP DATA

(1/2)

FLOW PATTERN: ANNULAR FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L} / \bar{R}_L \times 100(\%)$
	$\bar{T}_G$ ($^{\circ}\text{C}$)	$\bar{T}_L$ ($^{\circ}\text{C}$)	U_G^S (m/s)	U_L^S (m/s)			
	23.4	25.4	11.9	0	6.0	0.18	2.99
	23.8	27.3	10.5	0.100	9.7	0.23	2.33
*	23.4	25.6	13.0	0.099	11.2	0.37	3.34
	23.4	27.0	18.0	0.100	6.5	0.13	1.99
	23.8	28.3	16.7	0.155	9.5	0.26	2.74
	24.6	28.2	19.0	0.220	10.1	0.16	1.57
*	23.7	26.0	15.0	0.322	13.3	0.58	4.34
	23.7	28.6	15.9	0.322	10.1	0.24	2.38
*	23.4	28.1	17.2	0.322	12.0	0.50	4.16
	24.0	27.1	16.5	0.658	11.6	0.45	3.85

Observations: These data points were taken without any lump within the measuring station No. 2.

* The points marked with an asterisk were taken randomly with respect to the lumps due to limitations in the system and/or in the visual observations.

TABLE 6-A (CONTINUED) (2/2)

FLOW PATTERN: ANNULAR FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L} / \bar{R}_L \times 100(\%)$
	$\bar{T}_G$ (°C)	$\bar{T}_L$ (°C)	U_G^S (m/s)	U_L^S (m/s)			
	23.2	27.9	14.5	1.01	12.4	0.46	3.73
	23.3	27.9	16.3	1.01	11.7	0.36	3.03
*	23.5	29.5	15.2	1.93	19.8	0.73	3.70

Observations: These data points were taken without any lump within measuring station No. 2.

* The points marked with an asterisk were taken randomly with respect to the lumps.

TABLE 6-B - LIQUID HOLDUP DATA

FLOW PATTERN: ANNULAR FLOW

Comments on the Flow Topology	Mean Temperature		Superficial Velocities		Mean Liquid Holdup $\bar{R}_L$ (%)	Standard Deviation $\sigma_{\bar{R}_L}$ (%)	Percent Relative Standard Deviation $\sigma_{\bar{R}_L} / \bar{R}_L \times 100(\%)$
	$\bar{T}_G (^{\circ}\text{C})$	$\bar{T}_L (^{\circ}\text{C})$	$U_G^S (\text{m/s})$	$U_L^S (\text{m/s})$			
	23.7	27.3	10.5	0.100	14.7	0.70	4.77
	23.6	28.0	15.9	0.322	16.0	0.86	5.38
	23.6	26.9	16.5	0.658	18.1	0.81	4.48
	22.9	27.7	14.5	1.01	20.7	0.95	4.59
	23.2	27.8	16.3	1.01	17.0	0.67	3.94

Observations: The data points presented in this table were taken with one lump entrapped within measuring station No. 2.

TABLE 7
LIQUID HOLDUP PREDICTIONS BY THE PHYSICAL MODEL

(1/2)

FLOW PATTERN: BUBBLY FLOW

Superficial Velocities		Volume Average Liquid Holdup (Theoretical) R_L (%)
U_G^S (m/s)	U_L^S (m/s)	
0.0093	0	96.2
0.0161	0	93.3
0.0203	0	91.5
0.0226	0	90.5
0.0273	0	88.4
0.0161	0.0161	93.8
0.0228	0.0161	91.1
0.0281	0.0161	89.0
0.0229	0.099	93.4
0.0356	0.099	89.7
0.0431	0.099	87.6
0.048	0.099	86.1
0.0379	0.322	93.5
0.099	0.323	83.9
0.169	0.322	74.0
0.195	0.323	70.7
0.098	0.658	89.9

TABLE 7 (CONTINUED)
LIQUID HOLDUP PREDICTIONS BY THE PHYSICAL MODEL

(2/2)

FLOW PATTERN: BUBBLY FLOW

Superficial Velocities		Volume Average Liquid Holdup (Theoretical) R_L (%)
U_G^S (m/s)	U_L^S (m/s)	
0.228	0.658	78.4
0.41	0.658	66.1
0.35	1.01	77.2
0.42	1.01	73.4
0.48	1.01	70.9
0.38	1.93	84.9
0.49	1.93	81.1
0.63	1.93	76.9
0.71	1.93	74.5
0.46	2.90	87.1
0.63	2.90	83.0
0.89	2.91	77.6

TABLE 8
LIQUID HOLDUP PREDICTIONS BY THE PHYSICAL MODEL

FLOW PATTERN: SLUG FLOW

Superficial Velocities		Volume Average Liquid Holdup for the Whole Slug Unit (Theoretical) $R_{L \text{ Slug Unit}} (\%)$
$U_G^S (m/s)$	$U_L^S (m/s)$	
0.51	0	32.8
0.99	0	24.3
0.42	0.099	45.4
0.51	0.121	43.2
0.67	0.121	38.0
0.83	0.121	34.3
1.32	0.121	27.6
0.68	0.222	43.7
0.91	0.222	38.1
1.40	0.222	31.0
0.76	0.322	46.2
1.08	0.323	39.3
1.41	0.322	34.6
1.17	0.490	43.6
1.50	0.490	38.8
1.16	0.608	47.2

TABLE 9
CHURN FLOW LIQUID HOLDUP PREDICTIONS BY
THE PHYSICAL MODEL FOR THE SLUG FLOW

FLOW PATTERN: CHURN FLOW

Superficial Velocities		Volume Average Liquid Holdup for the Churn Unit (Theoretical) $R_{L \text{ Churn Unit}} (\%)$
$U_G^S (m/s)$	$U_L^S (m/s)$	
1.69	0	19.8
3.16	0	16.6
1.96	0.099	22.5
3.25	0.099	18.8
6.8	0.099	15.6
3.33	0.322	23.4
5.1	0.322	20.0
7.5	0.322	17.7
4.7	0.658	25.4
5.9	0.658	23.1
8.1	0.658	20.5
5.5	1.01	27.8
8.2	1.01	23.4
13.3	1.01	19.5
5.9	2.62	40.5
8.4	2.62	34.2

PART V

A TECHNIQUE FOR DYNAMIC MEASUREMENT OF
CROSS-SECTIONAL AVERAGE VOID FRACTION

SUMMARY

A new method for making dynamic measurements of cross-sectional area void fraction is explored. The concept is based on the use of a microwave resonant cavity which is penetrated by the tube which carries the two phase mixture. As the void fraction changes the resonant frequency of a tuned circuit of which the cavity is part also changes. This shift can be detected by state of the art instrumentation. Results indicate a workable system but one which shows sensitivity to the holdup variation in the test column within inches on either side of the cavity. Good field solutions are being explored.

INTRODUCTION

This method for measuring dynamic void fraction uses a microwave resonant cavity built around a section of the pipe through which the two-phase flow is to be measured; the pipe walls of this section must be of non-electrical conductive material. Since the resonant frequency of a cavity is related to the magnitude and volume distribution of dielectric material within the cavity and since the liquid has a dielectric constant different from air, the resonant frequency of the cavity system will be, in general, a function of volume of the liquid "within" the cavity. The relationship between the volume of liquid in the cavity and the resonant frequency of the cavity system is extremely important. To be a useful measuring technique for two-phase flow, the resonant frequency of the cavity must be uniquely related only to the total volume of liquid within the cavity; it must not be volume distribution or volume geometry sensitive.

Theoretical Considerations

Cavity measurements are generally techniques which are based on rigorous solutions of the electromagnetic field problem. Such a solution is possible only when the dielectric volume geometry in context with the cavity geometry is sufficiently simple. Furthermore, the measurements are made under the assumption that the cavity resonates in a specific mode at the time of the measurement. While this latter criteria can be met, the volume geometry for two-phase flow measurement is unknown and indeed will be different at each instant of time and consequently a rigorous solution of the measurement problem is not possible.

However, there is a measurement method based upon valid approximations which appears to be applicable to the solution of the measurement problem.

This method is based on the cavity perturbation technique for the measurement of dielectric constants of arbitrary sample shapes introduced in a resonant cavity. This technique involves approximations in the formulation which lead to acceptable results only under very restricted conditions: The sample volume must be very small compared with the cavity itself so that a frequency shift which is small compared with the resonant frequency of the empty cavity is produced by the insertion of the sample. Also since the fields assumed in the sample are usually the solutions of static problems, certain sample dimensions must be small compared with the wavelength, or the unperturbed field in the sample region must be uniform (Sucher and Fox, 1963).

Cavity perturbation measurements can be highly accurate and the approach makes it unnecessary to account for many of the details of the cavity since these are the same with and without the sample and hence tend to cancel out, since the difference of the two cavity conditions is the significant result. Perturbation techniques also permit the measurement of dielectric samples of small size and of various shapes.

The basic cavity perturbation formula from which specific results can be derived is:

$$\frac{f_2 - f_1}{f_2} = \frac{\int_{v_c} [(\mu_2 - \mu_1)H_1 \cdot H_2 - (\epsilon_2 - \epsilon_1)E_1 \cdot E_2] dv}{\int_{v_c} (\epsilon_1 E_1 \cdot E_2 - \mu_1 H_1 \cdot H_2) dv}, \quad (1)$$

where f_1 = resonant frequency of the cavity without the sample

f_2 = resonant frequency of the cavity with the sample

v_c = volume of the cavity

$H_1, E_1, \mu_1, \epsilon_1$ = cavity parameters without the sample

$H_2, E_2, \mu_2, \epsilon_2$ = cavity parameters with the sample.

The case of interest is for cavity 1 to be empty so that $\epsilon_1 = \epsilon_0$, $\mu_1 = \mu_0$; the sample is homogeneous and non-magnetic so that ϵ_2 is constant and real, and $\mu_2 = \mu_0$; and the volume of the sample v_s is very small compared to v_c . For these conditions, the equation becomes

$$\frac{f_2 - f_1}{f_2} = \left(\frac{\epsilon_2}{\epsilon_0} - 1 \right) \frac{\int_{v_s} E_1 \cdot E_2 dv}{\int_{v_c} |E_1|^2 dv} \quad (2)$$

It should be noted that the field E_1 in the empty cavity is presumed known and only the perturbed field E_2 in the sample volume v_s remains unknown (Rzepecka, 1973).

In the preceding discussion, the technique assumed a single sample volume placed in the cavity. Obviously, this is not the case for the liquid volume that is to be measured. However, a number of investigators have considered the problem of obtaining an "average field approximation" for dielectrics containing a random concentration of randomly oriented identical shaped inclusions of another dielectric material. The results of these studies indicate that the dielectric property of the mixture can be considered in terms of an "effective dielectric constant" that is a linear function of the ratio of the volume of the particles to the volume of the host medium. This linear relationship is valid under the conditions that the particle volume concentration is small and that the largest dimension of the particle shape is very small compared to the wavelength of the electromagnetic field (Taylor, 1965).

EXPERIMENTAL SYSTEM DESIGN

It is seen from the above discussion that a rigorous analysis of the proposed measurement technique is not possible. It also appears that the perturbation technique could be used to obtain the void measurements provided the

required conditions can be met in the practical system. The major conditions are:

1. The maximum volume of liquid in the cavity must be small compared to the volume of the cavity. The important aspect of this volume limitation is that the change in the cavity resonant frequency due to this volume of liquid must be small, about 0.1%.
2. The electric field throughout the volume to be occupied by the liquid must be uniform.
3. The electric field in the volume to be occupied by the liquid should be the same with and without the liquid; that is, $E_1 = E_2$ in equation 2.

Note that for this condition, equation 2 becomes

$$\frac{f_2 - f_1}{f_2} = \left(\frac{\epsilon_2}{\epsilon_0} - 1 \right) \frac{\int_{V_s} |E_1|^2 dv}{\int_{V_c} |E_1|^2 dv} = \left(\frac{\epsilon_2}{\epsilon_0} - 1 \right) KV_s \quad (3)$$

where K is a constant that is a function of the cavity geometry.

The main question that must be answered is how closely must these conditions be met to provide a practical measuring system. The question can only be answered by essentially building an analog model of the proposed system and performing specific measurements relating to the two-phase flow conditions.

A cavity system was designed which was anticipated to satisfy to a reasonable degree the various requirements outlined previously. The system consists of a rectangular resonant cavity 24" x 24" x 0.5" wide with an entry hole in the center through which the pipe penetrated. The cavity is shown in Figure 1. The resonant frequency of the cavity is measured by making the cavity the frequency determining element of a microwave oscillator; hence, the oscillation frequency is the resonant frequency of the cavity. To function as a dynamic void measurement system, the oscillating frequency would be measured continuously

(or sampled at a sufficiently high rate), converted to an analog voltage proportional to the change in frequency and recorded as a function of time.

The rectangular cavity was selected because the fundamental resonant mode, H_{101} , can be readily coupled to the external oscillator circuitry, the higher modes of oscillation are sufficiently above the fundamental mode so that multimoding can be avoided and the electric field configuration within the cavity can be readily related to the required field condition. The cavity depth or thickness (see Figure 2) was kept small so that the effective cross-section of the flow that was "within the cavity" was small. It is obvious that the field across the center hole in the cavity will not be uniform and consequently the effect of this non-uniformity on the frequency relationship would have to be determined by experimental measurements. The cavity was made of aluminum of sufficient thickness to provide a mechanically rugged and stable cavity.

The oscillator circuit was built around the metal-ceramic triode 7462 electron tube. This tube provided sufficient signal output to drive directly the frequency counter used to measure the frequency of oscillation. The oscillator circuit diagram is shown in Figure 2. The shielded box containing the oscillator circuitry is shown in Figure 2. Also shown are the coupling loops which provide the electrical coupling between the cavity and the tube. The complete system used for obtaining the experimental data is shown in Figure 4a.

EXPERIMENTAL MEASUREMENTS

Several measurements were taken on the cavity before the oscillator circuit was designed. For these measurements the cavity resonant frequency was determined by measuring the frequency response curve of the cavity using a variable frequency signal generator and a voltmeter. While the accuracy of

this technique was limited, it did provide the important preliminary information that the electrical losses in the cavity due to the finite conductivity of the aluminum cavity walls were negligible, a reasonably linear relationship existed between the dielectric volume within the cavity and the change in resonant frequency, and that for measurements with the dielectric sample placed in the hole area the resonant frequency change appeared to depend upon the position of the sample within the area.

After the oscillator circuit was completed, measurements were made to verify the linear relationship between the volume of dielectric material placed within the cavity and the change in resonant frequency. Measurements were made with various shapes and sizes of Plexiglas ($\epsilon/\epsilon_0 \approx 3.4$) that were fitted within the cavity through the center hole. Solid Plexiglas pieces as well as thin Plexiglas walled discs and rings filled with water ($\epsilon/\epsilon_0 \approx 80$) were used. Some of the samples used are shown in Figure 4b. The shapes were made of uniform cross-section with a length equal to the cavity depth; consequently, the electric field relationship $E_1 = E_2$ was essentially satisfied for these measurements.

The results of these measurements indicate that the resonant frequency change of the cavity is dependent only on the total volume of dielectric material in the cavity; this volume can be made up of a single sample or several individual samples having a random geometrical relationship (see Figures 5 and 6).

Extensive measurements were made placing the dielectric samples in the hole area cross-section. The results of these measurements indicate the following:

1. The field strength across the hole area is not uniform and varies rapidly near the cavity edges (see Figure 7). The field was measured by using a voltmeter and probe and it was determined that the field in

the area near the center of the hole is essentially uniform but reduced in strength relative to the field inside the cavity; the reduction was approximately an order of magnitude.

2. For solid Plexiglas samples placed within the "uniform field" area of the hole, the relationship between the resonant frequency change and the volume of dielectric material is reasonably linear and essentially independent of the geometry.
3. For water samples placed within the same hole area as the Plexiglas the relationship between the change in resonant frequency and the volume showed some significant departure from linearity and was somewhat dependent upon the volume geometry.
4. Since the perturbation technique assumes that the difference between the cavity electromagnetic field configuration for the two conditions of measurement is only slightly changed, the value of the dielectric constant of the material as well as the volume must be considered. Specifically, in the case of water, which has a relative dielectric constant of 80, the frequency shift, $f_2 - f_1$, is more indicative of having met the perturbation requirements than the volume.
5. The change in resonant frequency is a function of the length of the dielectric column in the hole area (see Figure 8). This length extends several inches beyond the center-plane of the cavity and is a function of the dielectric constant of the material. The extensive-ness of this sensitivity was not anticipated.

SIGNIFICANT CONCLUSIONS

The results obtained to date indicate that theoretically the cavity perturbation technique can be used to measure two-phase flow but that the practical implementation of this technique is not immediately obvious at this time.

The failure of the proposed system to provide the solution to the measurement problem is due primarily to the sensitivity of the change in the cavity resonant frequency to the length of the dielectric column through the cavity.

Although no definitive solution to the column length sensitivity is known at this time or that a solution indeed exists, there are some possible approaches to the solution. One possibility is the use of an open-ended circular cavity; another would be the possibility of developing a "guard" field immediately above and below the cavity hole to effectively cancel the external field. Others may exist after additional study has been given to the problem.

In any case, it would appear that the basic simplicity of the cavity perturbation technique might warrant further investigation toward its use.

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Resonant Cavity with Flow Entry Hole

Figure 1

Oscillator Connector Arrangement Showing Cavity Cross-
Section, Oscillator Circuit and Oscillator Coupling Loops

Figure 2

FIGURE 3
CAVITY OSCILLATOR CIRCUIT DIAGRAM

(a) Experimental Cavity Measurement System

(b) Several of the Dielectric Samples Used
in Making Experimental Measurements

Figure 4

FIGURE 5
FREQUENCY SHIFT DUE TO PLEXIGLAS SAMPLE IN CAVITY.

FIGURE 6
MEASUREMENTS MADE ON WATER FILLED SAMPLE VOLUMES INSIDE CAVITY.
(DATA CORRECTED FOR PLEXIGLAS CONTAINER)

FIGURE 7
VARIATION OF FIELD STRENGTH
WITH POSITION IN CAVITY HOLE

FIGURE 8
VARIATION OF FREQUENCY SHIFT WITH LOCATION
OF DIELECTRIC OUTSIDE OF CAVITY WIDTH