Chiral Perturbation Theory with Nucleons

DOE/ER/40561--28

DE92 004383

Ulf G. Meissner

*Universität Bern, Institut für Theoretische Physik
Sidlerstrasse 5, CH-3012 Bern, Switzerland*

and

*Institute for Nuclear Theory, University of Washington
HN-12, Seattle, Washington 98195, USA*

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY
UNDER GRANT DE-FG06-90ER40561

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, mark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

CHIRAL PERTURBATION THEORY WITH NUCLEONS * **

Ulf-G. Meißner^{† †}

*Universität Bern
Institut für Theoretische Physik
Sidlerstr. 5
CH-3012 Bern, Switzerland
and*

*Institute for Nuclear Theory
University of Washington
Seattle, WA 98195, USA*

ABSTRACT

I review the constraints posed on the interactions of pions, nucleons and photons by the spontaneously broken chiral symmetry of QCD. The framework to perform these calculations, chiral perturbation theory, is briefly discussed in the meson sector. The method is a simultaneous expansion of the Greens functions in powers of external momenta and quark masses around the massless case, the chiral limit. To perform this expansion, use is made of a phenomenological Lagrangian which encodes the Ward-identities and pertinent symmetries of QCD. The concept of chiral power counting is introduced. The main part of the lectures consists in describing how to include baryons (nucleons) and how the chiral structure is modified by the fact that the nucleon mass in the chiral limit does not vanish. Particular emphasis is put on working out applications to show the strengths and limitations of the method. Some processes which are discussed are threshold photopion production, low-energy Compton scattering off nucleons, πN scattering and the σ -term. The implications of the broken chiral symmetry on the nuclear forces are briefly described. An alternative approach, in which the baryons are treated as very heavy fields, is touched upon.

BUTP-91/32

September 1991

DOE/ER/40561-14-INT91-05-06

---

* Lectures delivered at the Summer School on Nucleons and Nucleon Structure, Institute for Nuclear Theory, University of Washington, Seattle, USA.

** Work supported in part by Deutsche Forschungsgemeinschaft and by Schweizerischer Nationalfonds.

[†] Heisenberg Fellow

^{††} M.J. Murdock Fellow

0. PRELUDE: WHAT TO EXPECT (OR NOT)

The aim of these lectures is to provide the reader with a flavor of how chiral perturbation theory can be used and to give some survey on what has been done so far. Particular emphasis is put on applications involving baryons (mostly nucleons to be more precise) and the meson sector is only covered rudimentary – some basic facts from the meson sector are discussed to set the stage for the more complex processes involving nucleons. The reader totally unfamiliar with the concepts might find it useful to first consult the lectures by Gasser¹, by Leutwyler² and by Donoghue³. As it will become clear (I hope), the topic of chiral perturbation theory (CHPT) with nucleons is by far a closed textbook chapter. I will address the potential problems lurking at various places. This should be sufficient stimulus for the interested reader to dig into the original literature and try to apply and/or improve the method. In any case, space forbids to discuss technical details. Nevertheless, I outline some simple calculations in various appendices. Obviously, these lectures can only serve as an appetizer and I hope that the reader will follow with a complete meal by solving some of the still open problems.

1. OUTLINE OF CHPT: MOSTLY MESONS

1.1 CHIRAL SYMMETRY AND ITS SPONTANEOUS BREAKDOWN

QCD is the theory of strong interactions. It is a local gauge theory of fermions (quarks) interacting via the exchange of colored vector bosons (gluons). The quarks come in various species, and I will only be concerned with the light ones (u, d, s). Loosely spoken, by light I mean a quark whose mass is significantly smaller than a typical hadronic scale, *e.g.* the proton mass (in the case of the strange mass a bit more care has to be taken). It is therefore a good starting point to consider the QCD Lagrangian with all quark masses set to zero, $\mathcal{L}_{QCD}^0$. This Lagrangian is invariant under global flavor transformations with an $SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_V$ symmetry. The $U(1)_A$ is eventually broken by instantons effects and will not be discussed in what follows. So we are left with a chiral symmetry (left and right-handed quarks don't mix) of $16 = 2(N_f^2 - 1)$ conserved Noether currents of vector and axial-vector type (with N_f the number of quark flavors). The associated charges generate a Lie algebra characterized by the well-known structure constants f^{abc} (see any textbook). One might now ask the question whether the ground state (g.s.) shares the same symmetry. In fact, it does not. This can be most simply be seen from the fact that the particle spectrum does not show partners of the same mass but of opposite parity. One calls this phenomenon spontaneous breakdown of the symmetry. Only the vector charges annihilate the vacuum – that is good old flavor $SU(3)$. Of course, the information about the axial charges is not lost. Goldstone's theorem⁴ tells us that with any broken generator a massless excitation of the vacuum appears. The reader should convince her/himself that in the QCD case these excitations are of pseudoscalar nature. The empirical fact that the lowest lying hadrons are indeed pseudoscalar mesons (π, K, η) neatly confirms this picture. But wait – these particles

have finite masses? This is related to the explicit symmetry breaking from the quark masses, and these are small leading to the small pseudoscalar masses. To be specific, in terms of the scale $\Lambda_\chi = 4\pi F_\pi$ (with $F_\pi \approx 93$ MeV the pion decay constant) we have,

$$(M_\pi/\Lambda_\chi)^2 \approx 0.014, (M_K/\Lambda_\chi)^2 \approx 0.18, (M_\eta/\Lambda_\chi)^2 \approx 0.22 \quad (1.1)$$

In the limit of vanishing quark masses (the chiral limit), one has $M_\pi = M_K = M_\eta = 0$ – and that has important consequences as we will see.

1.2 THE USE OF EFFECTIVE LAGRANGIANS

A major step to make use of the constraints given by chiral symmetry on the low energy structure was the work of Dashen and Weinstein.⁵ They showed the equivalence between chiral symmetry, current algebra and PCAC on one side and the use of phenomenological (effective) Lagrangians on the other. Indeed the celebrated current algebra theorems arise as tree level results from the most general phenomenological Lagrangian which incorporates the fundamental properties of chirality, unitarity, analyticity, parity and so on. In this framework, quark masses (or explicit symmetry breaking at that time) can be treated as a perturbation of the chirally symmetric theory. This is the origin of the name *chiral perturbation theory*. It was, however, quickly realized that virtual pions generate infrared singularities,⁶ i.e. the perturbations get enhanced by vanishing energy denominators, the state consisting of “something + n pions” has the same energy as the one “something + (n + 1) pions” (in the chiral limit). Therefore, the quark mass expansion of most observables is not simply a Taylor series, but includes non-analytic terms like $\sqrt{m_q}$ or $m_q \ln m_q$, with m_q the current quark mass, i.e.

$$\mathcal{H}_{QCD} = \mathcal{H}_{QCD}^0 + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s \quad (1.2)$$

for the three light flavors. The rules to play this game (calculating tree-level diagrams and the leading non-analytic pieces) were laid out by Langacker and Pagels.⁷ The excellent review by Pagels⁸ contains more references and gives a better historical perspective.

The next milestone was Weinberg’s⁹ seminal paper in 1979, where he showed how to systematise the calculation of corrections to tree-level results in a controllable fashion. I strongly urge you to work through this paper. In fact, the notion of a systematic expansion in external momenta was spelled out. Finally, in a series of papers, Gasser and Leutwyler^{10,11} worked out the complete generating functional to one-loop order and determined the low-energy constants at next-to-leading order for the two as well as three flavor case.

The framework to calculate low-energy QCD Greens functions is therefore based on constructing an effective Lagrangian which just has the same basic properties as QCD and to work out tree-level, one loop, ... contributions in a systematic fashion.

This is detailed very nicely in Leutwyler's lectures² and I will only give a short summary. The basic properties of the theory and its corresponding effective Lagrangian are:

- $\mathcal{L}_{eff}$ is symmetric under some Lie group G , here $G = SU(3)_L \times SU(3)_R$.
- The g.s. $|0\rangle$ is asymmetric, the symmetry is broken down to the subgroup $H \subset G$ (here, H is the diagonal $SU(3)_V$). In turn, Goldstone bosons appear (here, the pions).
- The Goldstones have a finite probability of coupling to the vacuum via the axial current

$$\langle 0 | A_\mu^i(0) | \pi^k(p) \rangle = i p_\mu \delta^{ik} F \neq 0 \quad (1.3)$$

F is a basic scale of strong interactions at low energies and it is related to (but it is not exactly) the pion decay constant $F_\pi \approx 93$ MeV.

- There are no other (strongly interacting) massless particles.

Equipped with this, let us consider the interactions of the Goldstones. At low energies, their interactions are weak. In fact, at zero energy, chiral symmetry forbids any interaction (otherwise states with different numbers of pions could not be degenerate). The prime example is the scattering amplitude for the process $\pi^+(p_1)\pi^0(p_2) \rightarrow \pi^+(p_3)\pi^0(p_4)$ first given by Weinberg:¹²

$$T_{\pi^+\pi^0} = \frac{t}{F_\pi^2} \quad (1.4)$$

Here, $\sqrt{t}$ is the invariant four-momentum of the pions. Eq.(1.4) just smells like the first term in a systematic expansion in powers of p^2 . Of course, in the real world (1.4) is not quite correct ($m_{quark} \neq 0$). The smallness of the quark masses allows to treat them as a perturbation. This will lead us to familiar concepts like PCAC, written on the quark level as $\partial_\mu(\bar{u}\gamma^\mu\gamma^5 d) = i(m_u + m_d)\bar{u}\gamma^5 d$.

1.3 EFFECTIVE THEORY AT LOWEST ORDER (TREE LEVEL)

Let me now turn to the lowest order effective theory of weakly strong interacting pions. From here on, I will restrict myself to flavor $SU(2)$ and work in the isospin limit $m_u = m_d = \hat{m}$ (I will come back to the isospin breaking case later on). It is most convenient to parametrize the pions in a non-linear fashion. This ensures that the interactions between the pions will be of derivative type and thus vanish at zero energy. One suitable parametrization is (see also appendix A)

$$U(x) = \exp\{i\pi(x)/F\} ; \quad \pi = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix} \quad (1.5)$$

Of course, any representation will do (as long as it is in harmony with the underlying symmetries). This is discussed by Weinberg¹³ and in the elegant paper by CCWZ.¹⁴ To lowest order, our effective theory is nothing but a non-linear σ -model,

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \text{Tr} [\partial_\mu U \partial^\mu U^\dagger + M^2(U + U^\dagger)] \quad (1.6)$$

We are dealing with a term quadratic in momenta – a derivative is booked as $\mathcal{O}(p)$. Parity forbids odd powers in p and a constant term has been discarded. F is the pion decay constant in the chiral limit, $F_\pi = F[1 + \mathcal{O}(\mathcal{M})]$ with $\mathcal{M} = \hat{m} \text{diag}(1, 1)$ the quark mass matrix. M^2 is the leading term of the quark mass expansion of the pion mass squared, $M^2 = 2\hat{m}B = M_\pi^2[1 + \mathcal{O}(\mathcal{M})]$. The constant B is related to the vacuum expectation value of the scalar quark density, $B = -\langle 0|\bar{u}u|0 \rangle / F^2$. This quark condensate is the order parameter of the spontaneously broken chiral symmetry. As a matter of fact, we never need the explicit knowledge of the quark masses $\hat{m}$ or the constant B^* since they always appear together. That is why (1.6) contains M^2 , not $\hat{m}$ or B . So at low energies, strong interactions physics is determined by two parameters.

Naturally, tree level calculations with (1.6) lead to current algebra results. For that, consider the case of elastic $\pi\pi$ -scattering. Expanding $U(x)$ in powers of $\pi(x)$ and working out terms up to order π^4 , we have

$$\mathcal{L}^{(2)} = \frac{1}{4} \text{Tr} [\partial_\mu \pi \partial^\mu \pi - M^2 \pi^2] + \frac{1}{48F^2} \text{Tr} ([\partial_\mu \pi, \pi] [\partial^\mu \pi, \pi] + M^2 \pi^4) + \dots \quad (1.7)$$

with properly normalized kinetic and mass terms. I leave it as an exercise for the reader to recover Weinberg's famous result $A(s, t, u) = (s - M_\pi^2)/F_\pi^2$ from the second term in (1.7) (at this order you are allowed to identify F with F_π and M with M_π). Clearly, $A(s, t, u)$ is real and cannot abide to unitarity constraints. To include these in a perturbative fashion, one has to go to the next order, *i.e.* $\mathcal{O}(p^4)$.

1.4 EFFECTIVE THEORY AT NEXT-TO-LEADING ORDER (ONE-LOOP)

Let me discuss the effective theory at next-to-leading order. The first important observation is that the lowest order interaction generates loops. Before discussing how to deal with them, we need a book keeping method to sort out all terms of a given order (chiral counting rules). We had already seen that tree-level graphs count as $\mathcal{O}(p^2)$ (I explicitly consider four-point functions). Let us now patch together two of them (again use the example of $\pi\pi$ scattering) and work out the powers in p^2 . There are two vertices, *viz.* p^4 , two propagators, *viz.* p^{-4} , and the four-dimensional momentum integration to close the loop. Altogether, we end up with two powers of p^2 . Weinberg⁹ tells us how to do this in general: Diagrams with l loops are suppressed by powers of p^{2l} . The general structure of the chiral expansion is shown in Fig.1. Therefore, at next-to-leading order, we have to consider

- Tree-level and one-loop graphs generated by vertices from $\mathcal{L}^{(2)}$.
- Tree-level graphs from $\mathcal{L}^{(4)}$ (which count as $\mathcal{O}(p^4)$).

---

* In principle, B could be zero and render these lectures obsolete. However, nature and lattice calculations gives us strong indications that $B \neq 0$ (a view not shared by everybody).

Fig. 1: Chiral expansion of the $\pi\pi$ scattering amplitude. N loop graphs count as order $\mathcal{O}(p^{2N+2})$ as indicated. $N = 0$ denotes the tree level. Some typical diagrams are shown in the insets.

What is $\mathcal{L}^{(4)}$? It is the most general effective chiral Lagrangian with p derivatives and q quark mass matrices such that $p+2q = 4$. Notice that $m_q = \hat{m} = \mathcal{O}(p^2)$, chiefly because $M_\pi^2 \sim \hat{m}$ and a pion propagator goes as $(p^2 - M_\pi^2)^{-1}$. So the building blocks of this polynomial Lagrangian of order four are U , $\partial_\mu U$ and $\chi = 2B\mathcal{M}$ (as promised, we don't need the quark masses). In $SU(2)$, $\mathcal{L}$ reads*

$$\mathcal{L}_4 = \sum_{i=1}^7 l_i P_i[U, \partial_\mu U, \mathcal{M}] \quad (1.8)$$

$$P_1 = \text{Tr} [(\nabla_\mu U \nabla^\mu U^\dagger)]^2, \quad P_2 = \text{Tr} (\chi^T U)^2, \dots$$

* In the three-flavor case, there is also the Wess-Zumino term.

and the complete list can be found in Ref.(10) (exercise: work it out by yourself). The constants l_i are not fixed by symmetry requirements, but could in principle be calculated in terms of Λ_{QCD} and the heavy quark masses. At present, one fixes them phenomenologically (consult GL for the details). The covariant derivative $\nabla_\mu U$ (which I should have already used in (1.6)) tells us how to couple external fields like *e.g.* the photon. The details of this procedure are spelled out in Ref.(10). Back to the loops: You can easily convince yourself that they are divergent, but their structure is exactly of the same type as introduced in $\mathcal{L}^{(4)}$, Eq.(1.8), and that is the reason why one has to consider *all* polynomial terms of a given order. Therefore, most of the low-energy constants l_i are in fact infinite – they are used to absorb the divergencies from the loops ($l_i \rightarrow l_i^r$, find out which of these are renormalized). This has to be done in harmony with the underlying symmetries and one uses dimensional regularization. The following basic features of CHPT emerge:

- CHPT is a systematic expansion in powers of momenta (p^2) and quark masses $\mathcal{M}$ (keeping the ratio $\mathcal{M}/p^2$ fixed).
- It is a renormalizable scheme (order-by-order).
- It involves a fauna of low-energy constants: $F, B, l_1^r, \dots, l_7^r$ (to one-loop order, at higher orders yet more polynomial terms appear).

A technical remark: To perform explicit calculations, one uses an external field technique and includes *e.g.* the quark mass matrix in a scalar source $s(x) = \mathcal{M} + \dots$. This is most convenient since any N-point function of quark currents of various types (vector, axial-vector, ...) can simply be found by expanding the generating functional

$$\exp\{i\mathcal{Z}\} = e^{i \int d^4x \mathcal{L}^{(4)}} \int d\mu[U] e^{i \int d^4x \mathcal{L}^{(2)}} \quad (1.9)$$

in appropriate powers of these external sources. The full machinery is outlined in Refs.10,11.

Let me come back to the example of $\pi\pi$ -scattering. The scattering amplitude $A^{(4)}(s, t, u)$ looks more messy (see *e.g.* Refs.10,15), but unitarity is perturbatively restored and one finds a reasonable description of the data in the low-energy region. I want to draw your attention to one particular phenomenon which is the appearance of chiral logarithms (*chiral logs*). To be specific, consider the one-loop prediction for the *S*-wave, isospin zero scattering length a_0^0 in the leading log approximation

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2} \left\{ 1 - \frac{9M_\pi^2}{32\pi F_\pi^4} \ln\left(\frac{M_\pi^2}{\mu^2}\right) + \mathcal{O}(M_\pi^2) \right\} \quad (1.10)$$

which clearly shows a non-analytic term $M_\pi^2 \ln M_\pi^2 \sim \hat{m} \ln \hat{m}^*$. These terms (together with large numerical coefficients) break the rule of thumb that chiral $SU(2) \times SU(2)$

---

* Don't get fooled here. The scale μ drops out when one writes down the full expression including the terms of order M_π^2 proportional to the low energy constants $l_1, \dots, l_4$. Choosing $\mu = 1$ GeV, one recovers the result given in the text.

is good within a few percent ($\hat{m}/M_{\text{proton}} \approx 0.01$). In fact, one has $a_0^0 = 0.20 \pm 0.01$ compared to the tree level result $a_0^0 = 0.16$ – a 25% increase. This is a very general phenomenon which we will encounter at various places.

I finish this hors d'oeuvre with two remarks. First, as already stated, the low-energy constants are determined phenomenologically. Can one, nevertheless, understand their values from fundamental principles? A partial answer has been given in Ref.16 where it was shown that to a high degree of accuracy the numerical values of these constants can be obtained from heavy resonance exchange (the QCD version of the old vector meson dominance principle also coined “QCD duality”). Such a Lagrangian of pseudoscalar and resonance fields also leads to a better description of $\pi\pi$ and πK scattering.¹⁷ Second, in some circumstances pions exhibit strong final state interactions, i.e. the one-loop approximation breaks down at surprisingly low energies. A method to deal with these final state interactions based on a modified Omnès representation has been proposed in Ref.18 and successfully applied to the scalar form factor of the pion, i.e. the matrix element $\langle \pi^a(p) \pi^b(p') | \hat{m}(\bar{u}u + \bar{d}d) | 0 \rangle$. The ambitious reader might want to generalize this method to arbitrary N-point functions (possibly involving weak interactions).

2. INCLUDING BARYONS

2.1 THE PLAYERS AND THEIR TRANSFORMATION PROPERTIES

Let me now come to the central topic, namely the inclusion of baryon fields. Most of the time, I will restrict myself to the two flavor case and essentially consider the pion-nucleon (πN) system. To set the stage, we have to establish the field content of the effective theory and discuss the pertinent transformation properties under the chiral $SU(2) \times SU(2)$ symmetry*. Our basic objects are

- The pions: These are parametrized in terms of a unimodular unitary matrix U which transforms as $U'(x) = g_L U(x) g_R^\dagger$ with $g_{L,R}$ elements of $SU(2)_{L,R}$. The pions are related to U either by the exponential form (1.5) or at some point I will also use the so-called “ σ -model gauge”,

$$U = \sigma + i\pi/F, \quad \sigma^2 + \pi^2/F^2 = 1 \quad (2.1)$$

with $\pi = \pi^a \tau_a$. Of course, S -matrix elements (i.e. physics) do not depend on the choice of how U is parametrized.

- The nucleons: These are the proton (p) and the neutron (n) combined in the field Ψ ,

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad (2.2)$$

---

* For more details, please consult the articles by Gasser, Sainio and Švarc¹⁹ (from here on referred to as GSS) and by Krause.²⁰

Again, we want to realize the nucleon field in a non-linear fashion so that meson-baryon interactions are of derivative type and thus vanish at zero energy. This is mostly a psychological advantage since there is any way the nucleon mass term (i.e. we have a meaningful constant). The way how to do this has been formulated most elegantly by Weinberg and CCWZ. We introduce a matrix-valued function K , which depends on the elements of $SU(2)_{L,R}$ and the pion field in a highly non-linear way, $K = K(g_L, g_R, U)$. Note that U depends on the space-time coordinate x and thus K implicitly depends on x . The type of chiral transformations related to K are therefore local. To be more specific, define K via

$$g_L \xi = \xi' K, \quad \xi g_R^\dagger = K^\dagger \xi' \quad (2.3)$$

with $\xi^2(x) = U(x)$, i.e. ξ is the square root of U and $\xi' = g_L \xi g_R^\dagger$. The transformation properties of the pion field therefore induce a well-defined transformation of ξ under $SU(2) \times SU(2)$ and this defines K as a nonlinear function of g_L , g_R and π . This somewhat messy object is discussed in more detail in Appendix A. K is a realization of $SU(2) \times SU(2)$ and the nucleon field transforms as*

$$\Psi \rightarrow \Psi' = K(g_L, g_R, U) \Psi \quad (2.4)$$

In fact, if one works out K more explicitly (for infinitesimal transformations), one realizes that for a chiral transformation, the nucleon field is multiplied by a function of the pion field – chiral transformation are related to the absorption or emission of pions²¹ (which is e.g. the main theme in Sakurai's book²²).

• The externals: For the applications I will discuss, we only need a scalar source to incorporate the quark masses,

$$s(x) = \hat{m} \quad (2.5)$$

and a vector source which gives the coupling to the electromagnetic (em) field,

$$v_\mu = \frac{e}{2}(1 + \tau_3) \mathcal{A}_\mu \quad (2.6)$$

Here, $\mathcal{A}_\mu$ is the photon field and I have included the appropriate charges of the proton and the neutron.

I should stress that the particles introduced here are structureless (pointlike), they have a certain charge and a certain mass as well as some specific internal quantum numbers. This appears queer since we know that both the nucleon and the pion are extended objects. The resolution of this puzzle is simple. All non-trivial structure of these particles is generated by loops and the associated counterterms. For

---

* Would we consider the full baryon octet in flavor $SU(3)$, (2.4) would read $\Psi \rightarrow K \Psi K^\dagger$.

example, the nucleon is surrounded by a cloud of pions which generates its charge radius and anomalous magnetic moment (the latter is discussed in detail in section 2.7). This is somewhat reminiscent of ancient physics (50ties) when it was tried to calculate baryon magnetic moments (or other static properties) from virtual meson clouds. At that time it was an unsuccessful approach since one did not know how to handle the high-energy behaviour (large momentum components) in the loops. Only in the presence of a consistent renormalization scheme this problem can be overcome. Our lack of knowledge about the high-energy components is hidden in the a priori unknown constants multiplying the various counterterms.

2.2 EFFECTIVE LAGRANGIAN OF MESONS AND BARYONS

Before discussing the general structure of the effective theory to describe the πN - or $\pi N\gamma$ -system, let us see what we have as building blocks at our disposal. From the meson sector, there are the field U and the associated covariant derivative (the most general form is given by GL),

$$\nabla_\mu U = \partial_\mu U - ie\mathcal{A}_\mu[Q, U] \quad (2.7)$$

and $Q = \text{diag}(1, 0)$ is the charge matrix of the nucleons. The covariant derivative associated with the nucleons takes the form

$$D_\mu \psi = \partial_\mu \psi + \Gamma_\mu \psi$$

$$\Gamma_\mu = \frac{1}{2} \left\{ \xi^\dagger (\partial_\mu - ie\mathcal{A}_\mu Q) \xi + \xi (\partial_\mu - ie\mathcal{A}_\mu Q) \xi^\dagger \right\} \quad (2.8)$$

It is easy to convince yourself that D_μ transforms homogenously under the local chiral transformations given by K . In fact, let us take a closer look at the connection Γ_μ . Suppose we switch off the external em field, then Γ_μ is nothing but a vector field

$$\Gamma_\mu = V_\mu = \frac{1}{2} \left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right) \quad (2.9)$$

This is a gauge field for the local transformations, $V_\mu \rightarrow K V_\mu K^\dagger + K \partial_\mu K^\dagger$. There is another object of axial-vector type which transforms homogenously,

$$u_\mu = \frac{i}{2} \left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right) = \frac{i}{2} \{ \xi^\dagger, \partial_\mu \xi \} \quad (2.10)$$

with $u'_\mu = K u_\mu K^\dagger$. Now we have all the tools to construct the tree-level Lagrangian. But let us first take a closer look at the effective pion-nucleon Lagrangian in its most general form. It can be written as an infinite string of terms with an even number of external nucleons, $n_{ext} = 0, 2, 4, \dots$. The term with $n_{ext} = 0$ obviously

corresponds to the meson Lagrangian discussed before, $\mathcal{L}_{\pi\pi}$. The general structure of $\mathcal{L}_{\pi N}$ therefore reads

$$\mathcal{L}_{eff}[\pi, \Psi] = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\bar{\Psi}\Psi} + \mathcal{L}_{\bar{\Psi}\Psi\Psi\Psi} + \dots \quad (2.11)$$

and some typical processes are pion-pion, pion-nucleon or nucleon-nucleon scattering. For the most parts of these lectures, I will only consider reactions with one incoming and one outgoing nucleon (for an exception, see section 5.1) such as pion-nucleon scattering, pion photoproduction or the electromagnetic structure of the nucleon. For this, it is convenient to factor out the nucleon fields and introduce a differential operator $\mathcal{D}$ such that

$$\mathcal{L}_{\bar{\Psi}\Psi} = \mathcal{L}_{\pi N} = -\bar{\Psi}(x)\mathcal{D}\Psi(x) \quad (2.12)$$

The explicit form of $\mathcal{D}$ will be discussed below.

2.3 TREE LEVEL LAGRANGIAN

We are now at the point to discuss the most general lowest order Lagrangian consistent with chiral symmetry, Lorentz invariance, the discrete symmetries (P, C, T) and which has the least number of derivatives. It reads

$$\mathcal{L}_1 = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} \quad (2.13)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}(i\gamma^\mu D_\mu - \hat{m} + i\frac{\hat{g}_A}{2}\gamma^\mu\gamma_5 u_\mu)\Psi \quad (2.14)$$

The lowest order meson Lagrangian, $\mathcal{L}_{\pi\pi}^{(2)}$, is already given by (1.6) (make the substitution $\partial_\mu \rightarrow \nabla_\mu$). The lowest-order πN Lagrangian, $\mathcal{L}_{\pi N}^{(1)}$, contains (as advertised), the covariant derivative D_μ and the axial field u_μ as well as the nucleon mass (in the chiral limit) – this is just related to the term with no derivatives (nucleon mass term). It is important to notice that at this stage the chiral Lagrangian involves four parameters. Two appear in the meson Lagrangian (F and B) and the two new ones are the nucleon mass and the axial-vector coupling constant in the chiral limit, viz

$$\begin{aligned} m &= \hat{m}[1 + \mathcal{O}(\hat{m})] \\ g_A &= \hat{g}_A[1 + \mathcal{O}(\hat{m})] \end{aligned} \quad (2.15)$$

Here, m denotes the physical nucleon mass ($\simeq 939$ MeV) and g_A the axial-vector strength measured in neutron β -decay, $n \rightarrow pe^+\bar{\nu}_e$, $g_A \simeq 1.26$. Quantities in the chiral limit are denoted by the " on top. The appearance of the nucleon mass is all but surprising (we are dealing with massive fermions), the actual value of $\hat{m}$ has been subject to much recent debate. I will discuss its value in the context of pion-nucleon scattering. The occurrence of the axial-vector coupling is related to the

renormalization of the axial-vector current – the unbroken vector al (flavor) $SU(2)$ is, of course, protected at zero momentum transfer. There is no such constraint for the axial current. Something is still left unexplained about $\mathcal{L}_1$ (2.13), namely the superscripts 1 and 2 on $\mathcal{L}_{\pi N}$ and $\mathcal{L}_{\pi\pi}$, respectively. These are, of course, the corresponding low-energy dimensions. So before proceeding with the discussion of $\mathcal{L}_1$ and loops and so on, we have to consider the chiral counting in the presence of nucleons (more book-keeping – it might seem boring, but it is very helpful!).

2.4 CHIRAL POWER COUNTING

To classify the various terms appearing in the effective Lagrangian, it is mandatory to have a counting scheme. When discussing the meson sector, we had already introduced this notion. The pertinent fields are counted as follows:

$$U, \xi = \mathcal{O}(1) ; v_\mu = \mathcal{O}(p) ; s = \hat{m} = \mathcal{O}(p^2) \quad (2.16)$$

where p denotes a generic meson four-momentum. v_μ counts as order $\mathcal{O}(p)$ since it comes together with ∂_μ in the covariant derivative ∇_μ . In the case of the baryons, this counting scheme is somewhat more complicated. Before going into the details, let me summarize the pertinent results:

$$\begin{aligned} \hat{m} &= \mathcal{O}(1), \Psi = \mathcal{O}(1), D_\mu \Psi = \mathcal{O}(1) \\ (i\not{D} - \hat{m})\Psi &= \mathcal{O}(p) \end{aligned} \quad (2.17)$$

Here, p denotes a generic nucleon three-momentum. As a matter of fact, the nucleon mass in the chiral limit $\hat{m}$ is not small, but a constant, $\hat{m} = \mathcal{O}(1)$. Therefore, baryon four-momenta cannot be counted as small, only the three momenta. Clearly, the covariant derivative D_μ counts (to leading order) as $\mathcal{O}(1)$ since it contains the nucleon mass. The argument why $(i\not{D} - \hat{m})$ counts as order p goes as follows:²⁰ Consider the free Dirac equation (the interaction terms carry powers of p) and decompose the nucleon field into upper and lower components, $N^T = (\phi, \chi)$. This yields

$$\begin{aligned} i\partial_t \phi &= \vec{\sigma} \cdot \vec{p} \chi + \hat{m} \phi \\ i\partial_t \chi &= \vec{\sigma} \cdot \vec{p} \phi - \hat{m} \chi \end{aligned} \quad (2.18)$$

with $\vec{p}$ the nucleon three-momentum. Considering now non-relativistic (slow) nucleons, we can split off a factor $\exp\{-i\hat{m}t\}$ and expand the remaining functions $\tilde{\phi}, \tilde{\chi}$ in inverse powers of $\hat{m}$. One observes that the lower component is suppressed by one order in $\vec{p}$ compared to the upper one,

$$\tilde{\chi} = \frac{\vec{\sigma} \cdot \vec{p}}{2\hat{m}} \tilde{\phi} + \mathcal{O}\left(\frac{1}{\hat{m}^2}\right) \quad (2.19)$$

and this leads to

$$\begin{aligned} (i\partial_t - \hat{m})\phi &= \vec{\sigma} \cdot \vec{p} \chi = \mathcal{O}(\vec{p}^2) + \dots \\ (i\partial_t - \hat{m})\chi &= -2\hat{m}\chi + \mathcal{O}(\vec{p}) = \mathcal{O}(\vec{p}) + \dots \end{aligned} \quad (2.20)$$

so that to leading order $(i\partial_t - \hat{m})N = \mathcal{O}(\vec{p})$. This completes the proof.

Fig. 2: Feynman rules in the σ -model gauge. Nucleons, pions and photons are denoted by solid, dashed and wiggly lines, respectively.

2.5 FEYNMAN RULES IN THE σ -MODEL GAUGE

To perform explicit calculations, it will be instructive to consider the Feynman rules originating from the lowest order effective Lagrangian, $\mathcal{L}_1$. I will choose the σ -model gauge and only consider the first terms in the expansion in powers of the pion field π (at most three pions for our purposes) and to leading order in the em field. Expanding U and ξ in powers of π and v_μ , we find

$$\begin{aligned} i\Gamma_\mu &= v_\mu + \frac{i}{8F^2}[\pi, \partial_\mu \pi] - \frac{1}{8F^2}[\pi, [\pi, v_\mu]] + \dots \\ \frac{1}{2}u_\mu &= -\frac{1}{2F}\partial_\mu \pi + \frac{i}{2F}[v_\mu, \pi] - \frac{1}{16F^3}\{\pi, \{\pi, \partial_\mu \pi\}\} + \dots \end{aligned} \quad (2.21)$$

The corresponding meson-nucleon, nucleon-photon, ... vertices are shown in Fig.2. The meson momenta are labelled q and a, b, c are isospin indices. Lorentz indices are given in terms of greek letters ($\mu, \nu, \dots$). We are now at the position to calculate the tree-level diagrams related to processes such as $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$ or $\gamma N \rightarrow \gamma N$. We need, however, first an extension of our method to include loops in the meson-baryon system.

2.6 LOOPS AND THE CHIRAL STRUCTURE

Fig. 3: Nucleon self-energy to one-loop order (represented by a one-loop diagram and one counterterm).

The fact that the nucleon mass does not vanish in the chiral limit complicates life a lot. To see this, let us work out the nucleon self-energy to one-loop order (consider only pion loops and use the Feynman rules derived before). The inverse nucleon propagator has the form $\tilde{S}(p) = [\not{p} - \not{m} - \Sigma_N(p)]^{-1}$ and the nucleon self-energy $\Sigma_N(p)$ can be decomposed as

$$\Sigma_N(p) = \not{m}\sigma_1(s) + (\not{p} - \not{m})\sigma_2(s) \quad (2.22)$$

with $s = p^2$. The one-loop graph shown in Fig.3 leads to a mass correction,

$$m = \hat{m} - \frac{3\hat{g}_A^2}{F^2} \left[L + \frac{1}{32\pi} \ln \left(\frac{\hat{m}^2}{\mu^2} \right) \right] + \mathcal{O}(d-4) \quad (2.23)$$

$$L = \frac{\mu^{d-4}}{16\pi^2} \left\{ \frac{1}{d-4} - \frac{1}{2} \left[\ln(4\pi) + \Gamma'(1) + 1 \right] \right\}$$

with μ the scale of dimensional regularization*, and d the number of space-time dimensions. We see that the parameter $\hat{m}$ must be renormalized – the loop contributions shift the original position of the pole (by an infinite amount). A similar effect appears when one calculates $\hat{g}_A$ to one-loop order. Therefore, we have to introduce counterterms that ensure the values of $\hat{m}$ and $\hat{g}_A$ no to be shifted, *viz*

$$\Delta \mathcal{L}_{\pi N}^{(0)} = \Delta \hat{m} \left(\frac{\hat{m}}{F} \right)^2 \bar{\Psi} \Psi, \quad \Delta \mathcal{L}_{\pi N}^{(1)} = \Delta \hat{g}_A \left(\frac{\hat{m}}{F} \right)^2 \frac{1}{2} \bar{\Psi} i \gamma^\mu \gamma_5 u_\mu \Psi \quad (2.24)$$

This is entirely different from the meson sector, the constants B and F do not get renormalized by loops.

Instead of plunging into further technicalities, let me take up the results of GSS and discuss the general structure for the case of the πN scattering amplitude (*cf.* the discussion of $T_{\pi\pi}$ in section 1). A pictorial is shown in Fig.4. The low-energy structure of the πN scattering amplitude can be summarized as follows: If one considers only pion (no fermion) loops and denotes by p a generic three-momentum, pion four-momentum or pion mass, the following results emerge: Tree graphs start at order p , but there is a whole string of terms of higher orders, $p^2, p^3, \dots$. One-loop graphs start at order p^2 (after appropriate mass and coupling constant renormalization) and are non-analytic in the external momenta at order p^3 (remember, we are still in the chiral limit $\hat{m} = 0$). Higher order loops start out at p^2 and are analytic in $\mathcal{O}(p^2, p^3)$. This again means that the low-energy constants from the terms of order p^2 and p^3 get renormalized. Evaluation of one-loop graphs associated with $\mathcal{L}_1$ therefore produces all non-analytic terms from the external momenta of order p^3 like *e.g.* leading threshold or branch point singularities. Let us now consider the case $\hat{m} \neq 0$. Obviously, the πN amplitude also contains terms which are non-analytic in the quark masses. A prime example is the Adler-Weisberger relation in its differential form – it contains a factor F_π^{-2} and the quark mass expansion of F_π^{-2} contains a piece $\hat{m} \ln \hat{m}$. In the meson sector, one-loop graphs generate all leading infrared singularities, in particular the ones in the quark masses. The same has not been proven

---

* Since we do not consider closed fermion loops and anomalies, the occurrence of γ_5 's does not lead to problems. All integrands can be brought into the form (Dirac structure) $\times$ (Lorentz structure) and then integration over internal momenta can be performed using standard methods.

Fig. 4: Chiral expansion for the πN scattering amplitude, $T_{\pi N} \sim p^N$. Tree graphs contribute at $N = 1, 2, 3 \dots$, n -loop graphs at $N = 2, 3, \dots$ (after mass and coupling constant renormalization). The contributions from $2, 3, \dots$ loops are analytic in the external momenta at order p^3 (here, p is a pion four-, nucleon three-momentum or the pion mass)

for the meson-baryon system due to the complicated low-energy structure. It is, however, highly plausible and all further calculations are based on this assumption (non-trivial exercise: prove it!)

The loops associated with $\mathcal{L}_1$ (no fermion loops) are given in terms of the generating functional

$$\exp\{i\mathcal{Z}\} = N \int [dU] \exp\left\{i \int dx \mathcal{L}_{\pi\pi}^{(2)} + i \int dx \bar{\eta} S^{(1)} \eta\right\} \quad (2.25)$$

$$\mathcal{D}^{(1)ac} S^{(1)cb} = \delta^{ab} \delta^{(4)}(x - y)$$

with $\eta, \bar{\eta}$ external Grassmannian sources and $S^{(1)}$ is the inverse nucleon propagator related to $\mathcal{L}_{\pi N}^{(1)}$. This generating functional can now be treated by standard methods, for all the messy details I refer to GSS. Instead, I will try to clarify the loop expansion by working out some specific examples. Of course, the loops associated with $\mathcal{L}_1$ are

divergent, so we have to add counterterms to cure these. This means:

$$\begin{aligned}\mathcal{L}_1 &\rightarrow \mathcal{L}_1 + \mathcal{L}_2 \\ \mathcal{L}_2 &= \Delta\mathcal{L}_{\pi N}^{(0)} + \Delta\mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi\pi}^{(4)}\end{aligned}\tag{2.26}$$

The first two terms of $\mathcal{L}_2$ we have already discussed. $\mathcal{L}_{\pi N}^{(2,3)}$ is the most general set of counterterms at order p^2 , p^3 , respectively. A list can be found in Krause's paper, in total $\mathcal{L}_{\pi N}^{(2,3)}$ consists of 12, 60 independent terms (in $SU(3)$). This sounds like very bad news since each term carries an undetermined low-energy constant, but in fact, most of these are not needed in calculations of specific processes. For the case of $\pi N \rightarrow \pi N$ one *e.g.* needs 5 and 8 terms from $\mathcal{L}_{\pi N}^{(2)}$ and $\mathcal{L}_{\pi N}^{(3)}$, respectively. Life is even more simple for some electromagnetic processes, we will encounter one marvelous example in which no polynomial terms of order p^2 and p^3 appear (in the calculation of the nucleon's polarizabilities). In fact, at present, nobody has pinned down the values of all the low-energy constants. I will leave the formal discussion here and turn towards some specific examples.

2.7 AN EXAMPLE: THE ANOMALOUS MAGNETIC MOMENT

The alert reader might already have noticed that the photon-nucleon vertex discussed before does only involve the charge of the nucleon. What happened to the anomalous magnetic moment? It comes from loops (and counterterms) and I think it is instructive to understand this in some more detail. To be specific, consider the nucleon matrix element of the vector current $V_\mu^a = \bar{q}\gamma_\mu(\tau^a/2)q$,

$$\langle p' | V_\mu^a | p \rangle = \bar{u}(p') \left[\gamma_\mu F_1^V(t) + \frac{i\sigma_{\mu\nu}k^\nu}{2m} F_2^V(t) \right] \frac{\tau^a}{2} u(p) \tag{2.27}$$

Fig. 5: Loop and counterterm graphs for calculating the isovector anomalous magnetic moment.

with $k = p' - p$ and $t = k^2$ the invariant four-momentum transfer. Let me concentrate on the Pauli form factor $F_2^V(t)$ (the Dirac form factor is, of course, discussed by GSS). The pertinent Feynman graphs are shown in Fig.5 (I only consider the ones necessary for calculating the anomalous magnetic moment, after appropriate renormalizations). There are two loop and one counterterm graph. The latter is generated by one counterterm from $\mathcal{L}_{\pi N}^{(2)}$ which reads

$$\mathcal{L}_{\pi N}^{(2)} = c_6 \frac{\hat{m}}{F^2} \bar{\Psi} \sigma^{\mu\nu} f_{\mu\nu}^+ \Psi + \dots \quad (2.28)$$

with $f_{\mu\nu}^+ = \vec{\tau} \cdot \vec{F}_{\mu\nu}$, $F_{\mu\nu}$ the em field strength tensor and the ellipsis denotes all other terms of $\mathcal{L}_{\pi N}^{(2)}$ which are of no relevance here. Performing the algebra, we end up with

$$F_2^V(t) = \frac{8\hat{g}_A^2}{F^2} \hat{m}^4 \left[4\gamma_4(t) + \Gamma_4(t) \right] + \frac{8c_6 \hat{m}^2}{F^2} \quad (2.29)$$

The so-called loop functions $\gamma_4(t)$ and $\Gamma_4(t)$ are defined in appendix B. It is now instructive to consider the quark mass expansion of $F_2^V(t)$. We set $c_6 = 0$ (see below). It is an easy exercise to work out $\Gamma_4(0)$ and $\gamma_4(0)$ in the chiral limit, $\Gamma_4(0) = \gamma_4(0) = (128\pi^2 \hat{m}^2)^{-1}$, so that (using $\hat{g}_A \simeq g_A$, $\hat{m} \simeq m$ and $F \simeq F_\pi$):

$$\begin{aligned} \kappa_v &= 5 \frac{\hat{g}_A^2 \hat{m}^2}{16\pi^2 F^2} + \mathcal{O}(\sqrt{\hat{m}}) \\ &\simeq 5 - 4\pi \left(\frac{M}{\hat{m}} \right) + \dots \simeq 3.1 \end{aligned} \quad (2.30)$$

In the second line, I have given the first correction to the leading result. One observes that it is big ($\sim 37\%$) and, more important, non-analytic, *i.e.* it scales as $\sqrt{\hat{m}}$ since $M^2 \sim \hat{m}$. You can convince yourself that the series goes like

$$\kappa_v = c_0 + c_1 \sqrt{\hat{m}} + c_2 \hat{m} \ln \hat{m} + c_3 \hat{m} + \dots \quad (2.31)$$

Evaluating now the full expression (2.29), one finds

$$\kappa_v = 3.73 \quad (2.32)$$

This shows that the two leading terms in the quark mass expansion gives the correct result to within 16%. From this example it should be clear that the one-loop approximation does more than just summing up the leading terms in a small parameter like *e.g.* (M_π/m) or alike. Of course, the result (2.29) will be altered by higher loops, but there will also be new counterterms. It is nevertheless comforting to know that to one-loop order, $F_2^V(0)$ is well described (for $c_6 = 0$). Now why is it a good idea to set $c_6 = 0$? I can offer the following handwaving argument: Think of estimating c_6

from resonance exchange, *i.e.* the photon couples to a ρ^0 and this ρ^0 to the nucleon. If you work out the $\gamma\rho^0$ -vertex, you will find that this coupling starts with a term proportional to t and thus vanishes at $t = 0$. So for strict vector meson dominance, $c_6 = 0$ could be justified (assuming we know how to formulate the ρN coupling).

Finally, two additional remarks on the form factors $F_{1,2}^V(t)$. First, let us consider the slope of $F_2^V(t)$ at $t = 0$,

$$\frac{d}{dt}F_2^V(0) = \frac{\bar{g}_A^2}{48\pi F^2} \frac{\bar{m}}{M} + \mathcal{O}(\ln M) \quad (2.33)$$

which clearly explodes as $M_\pi \rightarrow 0$. This also holds for $F_1^V(0)$ and was first observed by Bég and Zepeda.⁴² It is not difficult to understand the origin of this effect. The nucleon is surrounded by a cloud of virtual pions and as the pion mass tends to zero, the Yukawa suppression factor $\exp\{-M_\pi r\}$ turns into a long range $1/r$ term. Such a long range distribution has, of course, an infinite radius.

The second interesting phenomenon is related to the absorptive parts of the Pauli and Dirac form factors. In a dispersion theoretical analysis of the em form factors of the nucleon and the pion in 1975, Höhler and Pietarinen²³ observed a strong enhancement of $\text{Im}F_{1,2}^V(t)$ close to the threshold $t = 4M_\pi^2$ (see Fig.6). This shoulder could be traced back to the fact that there is a branch point singularity on the second Riemann sheet, in close proximity to the threshold (at $t = 4M_\pi^2 - M_\pi^4/m^2$). For the case of the imaginary part of $F_2^V(t)$ you can check that explicitly from the dispersion representation of $\bar{\gamma}_4(t)$ given in Appendix B. In the chiral limit, this singularity coalesces with the threshold – therefore it becomes obvious why there is this strong shoulder at threshold. This clearly underlines the importance of pion loops in describing the electromagnetic structure of the nucleon at low energies. More to come.

3. ELECTROMAGNETIC PROCESSES

In this lecture, I will first consider the photoproduction of neutral pions close to threshold and then the electromagnetic polarizabilities of the proton and the neutron. These topics are still under investigation, but I hope to convince you that in fact CHPT is a very useful tool to unravel the low-energy structure of the nucleon and its dynamics.

3.1 NEUTRAL PION PHOTOPRODUCTION AT THRESHOLD

Fig. 6: Spectral functions for the pion form factor $\text{Im}F_\pi/t^2$ (dashed line) and the nucleon Dirac form factor $2\text{Im}F_1^V/t^2$ (solid line).²³

Let me consider the reactions $\gamma(k) + p(p_1) \rightarrow \pi^0(q) + p(p_2)$ and $\gamma(k) + n(p_1) \rightarrow \pi^0(q) + n(p_2)$. First, I concentrate on the proton case. The threshold is defined when the photon energy is just sufficient to produce a pion at rest. In the conventional Mandelstam variables, this means for the cm energy $\sqrt{s} = m + M_\pi$, or in the photon lab frame

$$E_{\gamma,\text{thr}} = M_{\pi^0} \left(1 + \frac{M_{\pi^0}}{2m_p} \right) = 144.7 \text{ MeV} \quad (3.1)$$

In the threshold region, the produced pions will only be in relative S - or P -waves and one can analyse the data in terms of electric dipole and magnetic and electric quadrupole amplitudes. However, I want to issue a warning here which both experimenters and theorists should take seriously. Close to threshold, the total cross section dies out very fast, and one has to use also measurements of the differential X-section at energies not too far above threshold to eventually unravel the relative importance of these multipoles. This is a very non-trivial undertaking, and 30 years of analyzing πN scattering data (phase shifts) should have taught us that only by strictly adhering to analyticity constraints one gets proper information in the threshold region. At present, such a machinery exists for πN (the ‘‘Höhler analyticity cruncher’’) ²⁴ but has not been developed in full detail for the photoproduction case. I will pick up this theme later on. Now what is all the fuzz concerning this threshold region about? In

fact, as the pion momentum $\vec{q}$ tends to zero, we have

$$\left. \frac{|\vec{k}|}{|\vec{q}|} \frac{d\sigma}{d\Omega} \right|_{\vec{q} \rightarrow 0} = (E_{0+})^2 \quad (3.2)$$

This means that at threshold the differential cross section is entirely given in terms of the electric dipole amplitude (the subscript '+' refers to the total pion angular momentum $j = l + 1/2$). Furthermore, a so-called "low-energy theorem" (LET) has been derived 20 years ago. It states that to second order in the mass ratio $\mu = M_\pi/m$, the amplitude E_{0+} is given by²⁵

$$\begin{Bmatrix} E_{0+}^{\pi^0 p} \\ E_{0+}^{\pi^0 n} \end{Bmatrix} = -\frac{eg_{\pi N}}{8\pi m} \mu \begin{Bmatrix} 1 - \frac{1}{2}(\kappa_p + 3)\mu \\ \frac{1}{2}\kappa_n \mu \end{Bmatrix} + \mathcal{O}(\mu^3) \quad (3.3)$$

As an exercise, use the pseudovector pion-nucleon coupling and include the magnetic moment of the proton (neutron) in the photon-nucleon vertex, then (3.3) follows by calculating the associated Born diagrams and expanding the result in powers of μ . (But: This is not a proper CHPT calculation.) The first term proportional to μ is safely based on PCAC, the only assumption being that the non-Born parts are finite in the chiral limit. The μ^2 -term is more interesting, there exist various proofs in the literature that this term is also entirely a consequence of PCAC (combined with some seemingly harmless extra assumptions, see below). In the units used in photo-nuclear reactions, one finds

$$\begin{Bmatrix} E_{0+}^{\pi^0 p} \\ E_{0+}^{\pi^0 n} \end{Bmatrix} = \begin{Bmatrix} -2.26 \\ 0.48 \end{Bmatrix} \cdot 10^{-3}/M_{\pi^+} \quad (3.4)$$

Before 1986, the best experimental determination gave $(-1.8 \pm 0.6)^*$ for $\gamma p \rightarrow \pi^0 p$, no big deal. Then, however, came the measurements at Saclay and Mainz.²⁶ The value of E_{0+} published by the Saclay people was (-0.5 ± 0.3) . The Mainz group found two solutions, one giving $\text{Re} E_{0+}(146.8 \text{ MeV}) = -0.25 \pm 0.19$ and the other -1.61 ± 0.16 . These results seem to indicate a substantial deviation from the LET, and hectic activity set in trying to explain them. It was then, however, realized that the Saclay group had subtracted a particular rescattering correction of approximately -1.0 and the Mainz group had not taken into account the constraints from the total X-section.²⁷ If one does that, one finds that the "LET" is ok at π^0 threshold, but that there is a rapid variation in $\text{Im} E_{0+}$ due to the coupling to the channel $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ which opens at $E_\gamma = 149.9 \text{ MeV}$, i.e. approximately 5 MeV above the $\pi^0 p$ threshold. In view of this somewhat confused situation, it seems worthwhile to perform a complete one-loop calculation and to see whether the "LET" really is a direct consequence of chiral symmetry. Before I discuss this calculation in detail,

---

* I will drop the factor $10^{-3}/M_{\pi^+}$ as long as no confusion can occur

Fig. 7: Representative diagrams for neutral pion photoproduction. Shown are one diagram of classes I, II and III, in order. The class I diagram is the triangle graph discussed in the text and the class II graph obviously contributes to the anomalous magnetic moment.

let me tell you that it is done in the isospin limit $m_u = m_d = \hat{m}$ and one therefore has only *one* threshold. Thus one has to be careful in comparing the results of the calculation directly with the data. Despite this (and it remains an open problem how to include isospin breaking) one can make some firm statements in the form of the “LET”, as I will describe now.

If one draws all the diagrams to one-loop order, one finds that there are approximatively 60. Life is made easier by the fact that these diagrams fall into three classes which are separately gauge invariant, $k \cdot T(\gamma p \rightarrow \pi^0 p) = 0$. Some representative diagrams are shown in Fig.7. Using the Feynman rules given in section 2.5, you can easily convince yourself that the diagrams of class I carry a common factor $g_A/F_\pi^3 \sim g_{\pi N}$, whereas class II scales as $g_A^3/F_\pi^3 \sim g_{\pi N}^3$ * and class III are the counterterms from $\mathcal{L}_{\pi N}^{(2,3)}$ – there is only one and it contributes at order $\mathcal{O}(\mu^3)$ to E_{0+} . Class II clearly contains the diagrams which generate the anomalous magnetic moment. Let me from now on concentrate on class I. One can express the electric dipole amplitude E_{0+} in terms of 4 Lorentz-invariant functions à la CGLN,²⁸

$$E_{0+} = \frac{em}{8\pi} \frac{\mu(2+\mu)}{(1+\mu)^{3/2}} \left\{ A_1 + \frac{\mu(1+\mu/2)}{1+\mu} m A_3 + \frac{\mu^2}{2} \frac{m}{1+\mu} A_4 \right\} \quad (3.5)$$

so we see that A_1 is potentially most dangerous since it carries the lowest power of μ . An explicit calculation of the diagrams of class I gives²⁹

$$\delta A_1(s, u) = \frac{2 g_{\pi N}}{F_\pi^2} \left\{ (s - m^2) [\gamma_6(s) - \gamma_4(s)] + (s \leftrightarrow u) \right\} \quad (3.6)$$

* In fact, the graphs $\sim g_{\pi N}^3$ fall into two separately gauge invariant subsets.

and analogous expressions for $\delta A_3, \delta A_4$ (they are not needed for the further discussion!). The functions γ_4 and γ_6 are defined in appendix B. Let us now expand δA_1 around threshold. We find

$$\begin{aligned}\delta A_1^{\pi^0 p}(s, u)|_{th} &= \delta A_1^{\pi^0 n}(s, u)|_{th} = \frac{eg_{\pi N}}{32F_\pi^2} \mu + \mathcal{O}(\mu^2) \\ &= c \cdot \sqrt{\hat{m}} + \mathcal{O}(\hat{m})\end{aligned}\tag{3.7}$$

There we have it – a non-analytic term in $\hat{m}$. Now it is obvious that the good-old “LET” is spoiled, in fact, it just is not correct. It receives a correction at order μ^2 from the graphs of class I (only from the triangle diagram and its crossed partner). The modified expression for E_{0+} reads

$$\begin{aligned}\left\{ \begin{array}{c} E_{0+}^{\pi^0 p} \\ E_{0+}^{\pi^0 n} \end{array} \right\} &= -\frac{eg_{\pi N}}{8\pi m} \mu \left\{ F_1 + \mu F_2 + \mathcal{O}(\mu^2) \right\} \\ F_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad F_2 = -\frac{1}{2} \begin{pmatrix} 3 + \kappa_p \\ \kappa_n \end{pmatrix} - \Delta \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \dots\end{aligned}\tag{3.8}$$

with $\Delta = (m/4F_\pi)^2$ and the ellipsis in (3.8) stands for terms from class II which we have not yet analyzed in detail. However, we have shown that the PCAC prediction for F_1 is correct – this should lay at rest all models in which the leading term has been modified. Where does this non-analytic term come from? It can be traced back to the triangle diagram in Fig.7 and the fact that the loop-function γ_6 contains a logarithmic singularity in the chiral limit. In fact, as M_π tends to zero, one has (this is a cute little exercise):

$$\begin{aligned}\gamma_6(s_{th}) &= \frac{1}{64\pi^2 m^2} \left\{ -\ln \mu + \frac{3\pi^2}{8} - \frac{1}{2} + \mathcal{O}(\mu^2) \right\} \\ \gamma_6(u_{th}) &= \frac{1}{64\pi^2 m^2} \left\{ -\ln \mu - \frac{\pi^2}{8} - \frac{1}{2} + \mathcal{O}(\mu^2) \right\}\end{aligned}\tag{3.9}$$

where $u_{th} = m^2[1 - (\mu^2 + 2\mu)/(1 + \mu)]$. This invalidates the usual assumption that δA_1 is of order μ^2 , based on the assumption that $\delta A_1(\nu)$ is a differentiable function under crossing $\nu \rightarrow -\nu$ (i.e. interchange of s and u since $\nu = (s - u)/4m$). Clearly, this is not the case for the function γ_6 ! Stated differently, when one expands in powers of ν and $\nu_B = (t - 4M_\pi^2)/m$, $\delta A_1 = a_{20}\nu^2 + a_{02}\nu_B + \dots$. Now ν^2 and ν_B are of order μ^2 and if the Taylor coefficients would be non-singular, the contribution of δA_1 to E_{0+} would start out at order μ^3 . However, due to the singularity in the function γ_6 , some of the Taylor coefficients are singular. In particular, one has $a_{20} = \mathcal{O}(\mu^{-1})$ so that one ends up with a contribution of order μ^2 .

A few remarks on the numerology are in order. If you plug in the numbers for the corrections displayed in (3.8), you will find that at order μ^2 , the term proportional

to $\Delta = (m/4F_\pi)^2$ is dominant and the prediction for E_{0+} would even change sign if taken face value. This is, however, an artefact of the expansion in μ . If one does the full one-loop calculation for the diagrams of class I (plus the tree level terms), one finds $E_{0+}^{\pi^0 p} = -0.8$ – a sizeable reduction from the “old LET” value but also different from the one given by the first two terms in the expansion (3.8). What remains to be done is the chiral expansion of the box diagrams ($\sim g_{\pi N}^3$) up to and including order μ^2 . These will produce an extra term $\tilde{\Delta}$, which is, however, algebraically independent from Δ since it carries a different power of $g_{\pi N}$. Clearly, (3.8) is still not the final result. To my opinion, the moral to be drawn from this calculation is that if one wants to make statements about the constraints from chiral symmetry at next-to-leading order, one has to consistently evaluate all one-loop graphs. We had already seen that the anomalous magnetic moment is generated by loops – why should that be the only such contribution? In fact, it would be rather mysterious if it were – but it is not! I should repeat that (3.3) is not a low-energy theorem, the terms of order μ^2 are not complete. There are numerous proofs in the literature that this old form is indeed correct. All these so-called proofs can be shown to be incorrect at various places, and people should (have to!) forget to talk about (3.3) as a LET!

At present, we are carrying out a full-scale investigation concerning not only the photo/electroproduction of neutral but also of charged ($\pi^\pm$) pions. For the latter case, the leading term is of order $\mathcal{O}(1)$ (Kroll–Ruderman term) and completely dominant. The loop-corrections are much more tame in that case. Let me, however, reiterate that in the future one has to build in isospin-breaking. This is a non-trivial and yet unresolved problem (exercise: go for it)!

3.2 ELECTROMAGNETIC POLARIZABILITIES

The electric charge radii of the neutron and the proton as well as their magnetic moments are known to a high accuracy. Low-energy Compton scattering reveals further important information about the internal structure of the nucleon, parametrized by the so-called electric and magnetic polarizabilities. These are the first non-trivial structure constants in two-photon observables. Let us place a neutron/proton into an external electric (magnetic) field $\vec{E}$ ($\vec{B}$). The particle will acquire an induced electric (magnetic) dipole moment,

$$\vec{d}_E = \alpha \vec{E} ; \quad \vec{d}_M = \beta \vec{B} \quad (3.10)$$

The electric (α) and magnetic (β) polarizabilities therefore characterize the ease with which a dipole moment can be induced in a composite system. This information is as fundamental as the one related to the one-photon processes (charge radii, magnetic moments).

Before discussing the CHPT calculation of the polarizabilities, let me give a brief overview about the experimental situation:

- The sum of the electric and the magnetic polarizabilities of the proton and the neutron can be determined rather precisely by use of a forward dispersion relation sum rule,

$$\bar{\alpha} + \bar{\beta} = \frac{1}{2\pi^2} \int_{E_{\gamma, \text{thr}}}^{\infty} d\omega \frac{\sigma_{\gamma}(\omega)}{\omega^2} \quad (3.11)$$

with $\sigma_{\gamma}(\omega)$ the total photoabsorption cross section of the proton or neutron, and $\omega = E_{\gamma}$ the photon energy in the lab frame. The threshold value $E_{\gamma, \text{thr}}$ is given in (3.1). The most commonly quoted values are $\bar{\alpha}_p + \bar{\beta}_p = (14.3 \pm 0.3) \cdot 10^{-4} \text{ fm}^3$ and $\bar{\alpha}_n + \bar{\beta}_n = (15.8 \pm 0.5) \cdot 10^{-4} \text{ fm}^3$.

- The separation into the individual contributions ($\bar{\alpha}_{p,n}$, $\bar{\beta}_{p,n}$) is afflicted with much larger uncertainties. However, there exists clear evidence that the proton and the neutron behave as electric dipoles, $\bar{\alpha}_{p,n} \gg \bar{\beta}_{p,n}$. Typical values quoted in the literature span the ranges $\bar{\alpha}_n \approx \bar{\alpha}_p \approx 8 \dots 12 \cdot 10^{-4} \text{ fm}^3$ and $\bar{\beta}_n \approx \bar{\beta}_p \approx -1 \dots 4 \cdot 10^{-4} \text{ fm}^3$. There is also some evidence that the intrinsic electric polarizability is larger for the neutron than for the proton.*

Fig. 8: Low-energy Compton scattering.

To perform a systematic one-loop calculation, we need an operational definition of the polarizabilities. Consider the low-energy expansion of the spin-independent Compton scattering amplitude depicted in Fig.8. In the rest frame it takes the form

$$T(\gamma N \rightarrow \gamma N) = T_0 + \bar{\alpha} \omega' \omega \vec{\epsilon}' \cdot \vec{\epsilon} + \bar{\beta} (\vec{\epsilon}' \times \vec{k}') \cdot (\vec{\epsilon} \times \vec{k}) + \mathcal{O}(\omega^4) \quad (3.12)$$

---

* The bar on α and β refers to the fact that we are dealing with the so-called Compton polarizabilities. In what follows, I will only consider these. They differ from the static polarizabilities (3.10) by recoil effects and alike. In the literature, one can find a variety of prescriptions how to relate these two definitions (mostly in terms of non-relativistic physics). Although it would be worthwhile to clarify this connection in more detail, lack of space forbids to do so.

with $(\omega, \vec{k}, \vec{\epsilon})$ and $(\omega', \vec{k}', \vec{\epsilon}')$ the frequencies, momenta and polarization vectors of the incoming and outgoing photon, respectively. The constant term T_0 is nothing but the Thomson scattering amplitude which carries only information about the charge and mass of the particle the photon scatters off (as its energy goes to zero). The internal structure is, therefore, embodied in the two structure constants $\bar{\alpha}$ and $\bar{\beta}$ which parametrize the corrections to T_0 at order ω^2 . In quantum field theory, the calculation of these structure constants (the polarizabilities) proceeds as follows. Obviously, we need the Fourier transformed matrix-element of two electromagnetic currents in a nucleon state,

$$T_{\mu\nu}(p, k) = \int d^4x e^{ik \cdot x} \langle N(p) | T J_\mu^{em}(x) J_\nu^{em}(0) | N(p) \rangle \quad (3.13)$$

with p and k the nucleon and photon four-momentum, respectively. From this, we can form the spin-averaged Compton tensor in forward direction,^{30,31}

$$\begin{aligned} \theta_{\mu\nu} &= \frac{1}{4} \text{Tr} \{ (\not{p} + m) T_{\mu\nu}(p, k) \} = e^2 \{ g_{\mu\nu} A(s) \\ &+ k_\mu k_\nu B(s) + (p_\mu k_\nu + p_\nu k_\mu) C(s) + p_\mu p_\nu D(s) \} \end{aligned} \quad (3.14)$$

with $e^2/4\pi = 1/137$ the fine structure constant. The kinematics is rather simple here. The Mandelstam variables are $s = (p + k)^2$, $t = 0$ and $u = (p - k)^2 = 2m^2 - s$ (in forward direction, $t = 0$ since $\cos\theta = 1$ and $t \sim (1 - \cos\theta)$). Gauge invariance ($k_\mu \theta^{\mu\nu} = 0$) and crossing symmetry reduce the number of independent scalar functions in (3.14) to two. If we choose to work with $A(s)$ and $B(s)$, the comparison with (3.12) leads to the following expressions for the polarizabilities (more details are given in Ref.31):

$$\bar{\alpha} + \bar{\beta} = -\frac{e^2 m}{2\pi} A''(m^2), \quad \bar{\beta} = -\frac{e^2}{4\pi m} B(m^2) \quad (3.15)$$

where the prime denotes differentiation with respect to s . The non-renormalization theorem of the electric charge leads to the additional constraint $A(m^2) = Z^2$.

What is most amazing about the CHPT calculation of $\bar{\alpha}_{p,n}$ and $\bar{\beta}_{p,n}$ to one-loop order is that there are no contributions from polynomial counterterms. This is essentially due to the fact that the kinematics is rather special and that there are no external pions. The complete discussion of the vanishing of possible counterterms from $\mathcal{L}_{\pi N}^{(0,1,2,3)}$ is given in Ref.31, here I just want to stress the similarity to the mesonic reactions $\gamma\gamma \rightarrow \pi^0\pi^0$,³² $K_L^0 \rightarrow \pi^0\gamma\gamma$ ³³ or $K_S^0 \rightarrow \gamma\gamma$.³⁴ They are also given purely by loops (to leading order) and therefore serve as a clean probe concerning the loop contributions. To my knowledge, the electric and magnetic polarizabilities are the first observables including baryons in which this intriguing phenomenon appears.

For the proton/neutron one has to evaluate 52/22 one-loop diagrams. This task is, however, tremendously simplified by the special kinematics discussed before. In

fact, one can perform all Feynman parameter integrations. The full expressions are given in Ref.30, here let me only discuss the chiral expansion of the polarizabilities. Since we are dealing with a forward amplitude, it does not come as a surprise that they diverge in the chiral limit. The most singular term reads

$$\begin{aligned}\bar{\alpha}_p &= \bar{\alpha}_n = \frac{5C}{2} \frac{1}{M_\pi} + \mathcal{O}(\ln M_\pi) \\ \bar{\beta}_p &= \bar{\beta}_n = \frac{C}{4} \frac{1}{M_\pi} + \mathcal{O}(\ln M_\pi)\end{aligned}\tag{3.16}$$

with $C = (e^2 g_{\pi N}^2 / 192 \pi^2 m^2)$. The first interesting observation from (3.16) is that the empirical result $\bar{\alpha}_p + \bar{\beta}_p \approx \bar{\alpha}_n + \bar{\beta}_n$ follows naturally from the leading term, in fact, we find $\bar{\alpha}_p + \bar{\beta}_p = \bar{\alpha}_n + \bar{\beta}_n = 15 \cdot 10^{-4} \text{fm}^3$, almost too close to the data. Furthermore, the coefficient of the term proportional to M_π^{-1} is ten times bigger for $\bar{\alpha}_{p,n}$ than for $\bar{\beta}_{p,n}$, in agreement with the empirical trends. The corrections to this leading term are discussed in detail in Refs.30,31. These have been determined *completely* to one-loop order, but important two-loop effects like the inclusion of the $\Delta(1232)$ have not yet been considered in a systematic fashion. There is, however, a “cheap way” of getting a handle on the two-loop effects for the sum $\bar{\alpha} + \bar{\beta}$ based on the dispersion sum rule (3.11). Once one has performed the one-loop calculation of the pion photoproduction amplitudes (neutral and charged pions) and tree level processes like $\gamma N \rightarrow \pi \pi N$, the dispersive integral can be evaluated to give $\bar{\alpha} + \bar{\beta}$ to two-loop order (in the isospin limit). This can be most easily understood if one considers a typical two-loop Feynman diagram for Compton scattering and cuts it in the interaction region (after/before the absorption/emission of the incoming/outgoing photon). Since these questions are presently under investigation, I will finish the discussion here. I only want to stress one more time that for the baryon sector the polarizabilities are a clean probe of loop effects and that the leading singular terms proportional to M_π^{-1} are generated by four Feynman diagrams (and their crossed partners). At present we do not know how to fish out these most singular diagrams without explicit calculations.

4. πN -SCATTERING AND THE σ -TERM

4.1 A FAMILIAR LOW-ENERGY THEOREM

You all know that chiral symmetry determines the pion-nucleon interaction at low energies. This is mostly clearly documented by the Golberger-Treiman relation (GTR)

$$g_{\pi N} = \frac{g_A m}{F_\pi}\tag{4.1}$$

The GTR relates the strong pion-nucleon coupling constant $g_{\pi N}$ to the basic scale F_π and the axial-vector (weak) coupling strength, g_A . In nature, the GTR is fulfilled within 5%, *i.e.* $13.4 = 1.26 \cdot 939 / 92.4 = 12.8$ (using the most recent values for the

various quantities). In the chiral limit ($m_u = m_d = 0$), the GTR becomes exact and constitutes a real low-energy theorem.

In what follows, I will concentrate on another LET related to the pion-nucleon scattering amplitude. The latter can be parametrized by four Lorentz-invariant functions, these are commonly called $A^\pm$, $B^\pm$ (the ' $\pm$ ' refers to the isospin even/odd combination). For our purpose, it will be more convenient to work with the amplitude $D^\pm = A^\pm + \nu B^\pm$, with $\nu = (s - u)/4m$ the conventional kinematical variable used in πN scattering analyses. D is useful since it is related to the total X-section by the optical theorem. Why is it useful to consider πN scattering in CHPT? The answer is rather obvious - it allows to unravel the amplitude in the unphysical region and clarify the situation concerning the so called σ -term. The full one-loop calculation of $T_{\pi N}$ has been performed by GSS¹⁹ and the σ -term puzzle was solved by Gasser, Leutwyler and Sainio (GLS).³⁵ I will heavily rely on their work, but don't forget: They deserve all the credit.

Fig. 9: Connection between the πN scattering amplitude, the σ -term and Σ at the Cheng-Dashen point. On the right side, the πN data are interpolated to the unphysical region and Σ at the CD point can be extracted (steps 3,4). On the left side, it is shown how σ is related to the baryon masses and the additional parameter y , and CHPT relates σ to Σ (steps 1,2). I am grateful to J. Gasser for providing me with this figure.

4.2 σ -TERM AND RELATION TO THE DATA

The σ -term is defined by

$$\sigma = \frac{\hat{m}}{2m} \langle p | \bar{u}u + \bar{d}d | p \rangle \quad (4.2)$$

for a proton state $|p\rangle$ with the normalization $\langle p' | p \rangle = 2(2\pi)^3 p^0 \delta(\vec{p} - \vec{p}')$. How is this quantity related to the πN scattering amplitude? For that, introduce the quantity

$$\Sigma = F_\pi^2 \bar{D}^+(\nu = 0, t = 2M_\pi^2) \quad (4.3)$$

where the bar on D^+ means that the pseudovector Born term has been subtracted, $\bar{D}^+ = D^+ - D_{pv}^+$. The clever choice of kinematics is from now on referred to as the Cheng-Dashen³⁹ point - you will see why it is clever. You notice that I have used different symbols in (4.2) and (4.3), but chiral symmetry relates the quantities σ and Σ as will be discussed below. σ itself can be calculated from the baryon mass spectrum with one additional parameter y , which is a measure of the strange quark content in the proton.

In fact, the relation between σ , Σ and the πN scattering data is a rather complex one, as visualized in Fig.9. I won't elaborate much on the right side (steps 3 and 4) but rather give you the flavor. First, one has to reconstruct $T_{\pi N}$ at $\nu = t = 0$ from data which are available for $E_{\text{kin}}^{\pi N} \geq 30 \text{ MeV}$, ($k_\pi \geq 100 \text{ MeV}$). Using dispersion relations, one can extrapolate the data to the unphysical region. Based on the Karlsruhe-Helsinki phase-shifts, it was found that $\Sigma = (64 \pm 8) \text{ MeV}$.³⁶ The error only reflects the error of the method. More recently, there has been much debate about the πN data at very low energies (some new measurements are in clear contradiction to the old KH phases, some agree³⁷). Despite this, GLS came to the conclusion that

$$\Sigma = 60 \text{ MeV} \quad (4.4)$$

in agreement with Koch's older result. It is very important to notice that Σ is a small effect, and that only dispersion theory is a meaningful tool to perform the unavoidable interpolations. The often used (and much simpler) effective range expansion fails. Let me now turn towards the determination of σ from the baryon spectrum and to the relation between σ and Σ .

4.3 σ FROM THE BARYON SPECTRUM

To get a lowest order estimate of σ , consider the QCD Hamiltonian

$$H_{QCD} = H_{QCD}^0 + \int d^3x \{m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s\} \quad (4.5)$$

$$\begin{aligned} &= H_{QCD}^0 + \frac{1}{3} \int d^3x (m_u + m_d + m_s) \{\bar{u}u + \bar{d}d + \bar{s}s\} \\ &\quad - \frac{1}{3} \int d^3x (m_s - \hat{m}) \{\bar{u}u + \bar{d}d - 2\bar{s}s\} \\ &\quad + \frac{1}{2} \int d^3x (m_u - m_d) \{\bar{u}u - \bar{d}d\} \end{aligned} \quad (4.5a)$$

where H_{QCD}^0 describes massless QCD. The second term in (4.5a) is a flavor-singlet, whereas the third term gives rise to $M_\Sigma > M_N$ and the last one the strong neutron-proton mass difference. To leading order in the quark masses, a simple exercise using flavor $SU(3)$ baryon wave functions leads to

$$\begin{aligned} \sigma &= \frac{\hat{m}}{m_s - \hat{m}} \frac{M_\Xi + M_\Sigma - 2M_N}{1 - y} + \mathcal{O}(\mathcal{M}^{3/2}) \\ y &= \frac{2 \langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle} \end{aligned} \quad (4.6)$$

in terms of one parameter, y , the strange quark content of the proton. y is a priori unknown, but the OZI rule suggests $y \sim 0$. The baryon masses M_Ξ , M_Σ and M_N are well-known, and we furthermore have $m_s/\hat{m} \sim 25$. The latter number can be produced on the back of an envelope. To leading order in the quark masses, we have $M_\pi^2 = 2\hat{m}B$ and $M_K^2 = (\hat{m} + m_s)B$, so that $(m_s/\hat{m}) \simeq 2(M_K/M_\pi)^2 \simeq 25$. This can be done much better, of course, but the result is essentially the same.³⁸ If you plug in the numbers, you will find that $\sigma = 26$ MeV (for $y = 0$), which is suspiciously different from Σ . But let us first see what the next-to-leading order corrections give.

At next-to-leading order, $M_\Sigma - M_N$ reads

$$M_\Sigma - M_N = c_1(m_s - \hat{m}) + c_2(m_s - \hat{m})^{3/2} + \dots \quad (4.7)$$

Now typical $SU(3)$ breaking effects are of order of 30%, so one better gets a handle on the terms of order $m_q^{3/2}$. The calculation including all effects of order $\mathcal{O}(m_q^{3/2})$ and an estimate of the ones of order $\mathcal{O}(m_q^2)$ has been performed by Gasser⁴⁰ ten years ago. The result including the estimate for the terms of order $\mathcal{O}(m_q^2)$ reads

$$\sigma = \frac{35 \pm 5 \text{ MeV}}{1 - y} \quad (4.8)$$

Another interesting result of this calculation was that the quark mass ratio $R = (\hat{m} - m_s)/(m_u - m_d) \approx 40$ for the baryon spectrum, a convincing consistency with R

determined from the meson spectrum (I strongly advice you to read Gasser's paper). But: We are facing a puzzle. For $\langle N|\bar{s}s|N \rangle \sim y = 0$, we have $\sigma \approx 35$ MeV, very different from $\Sigma = F_\pi^2 \bar{D}_{CD}^+ \approx 60$ MeV. This discrepancy can fairly easily be resolved if one embarks on one of the three following (rather) radical solutions:

- $y \neq 0$, *i.e.* the strange quark matrix element $\langle p|\bar{s}s|p \rangle$ is large. The problem with this is that the strange quarks would roughly contribute 400 MeV to the nucleon mass (*i.e.* the nucleon mass in the chiral limit would be very different from its physical value).
- $m_s/\hat{m}$ is much smaller than 25. The problem with this is that one would not understand why the GMO relation for the π , K , η works.
- $SU(3)$ is strongly broken in the baryon sector. As before, it would be hard to understand the success of the GMO relation for the N , Λ , Σ and Ξ .

So it appears natural to first take a closer look at the relation between σ and Σ .

4.4 RELATION BETWEEN σ AND Σ

To relate σ and Σ , we have to define the scalar form factor of the nucleon,

$$\begin{aligned} \langle p'|\hat{m}(\bar{u}u + \bar{d}d)|p \rangle &= \bar{u}'u\sigma(t) \\ \sigma(0) &= \sigma \end{aligned} \quad (4.9)$$

with $t = (p' - p)^2$ the invariant four-momentum transfer. Brown, Pagels and Pardee⁴¹ have derived a LET which relates $\sigma(t)$ and Σ based on the Ward-Takahashi identity for the pion-nucleon σ -term,

$$\bar{D}^+(\nu, t) = F_\pi^{-2}\sigma(t) + q'^\mu q^\nu r_{\mu\nu} \quad (4.10)$$

where the remainder $r_{\mu\nu}$ is not fixed by chiral symmetry. In general, one can not neglect this quantity since it is comparable to the other terms. At the Cheng-Dashen point, however, this correction is small (see below). The exact relation reads

$$\begin{aligned} \Sigma &= \sigma + \Delta_\sigma + \Delta_R \\ \Delta_\sigma &= \sigma(2M_\pi^2) - \sigma(0) \end{aligned} \quad (4.11)$$

and with Δ_R related to $r^{\mu\nu}$. Before discussing the numbers, let us make an estimate for $\sigma(t)$. Since $\sigma(t)$ is a form factor, we can parametrize it as $\sigma(t) = \sigma(0)\{1 + \langle r^2 \rangle_\sigma t/6 + \dots\}$, with $\langle r^2 \rangle_\sigma$ the scalar radius of the nucleon. Now assume that $\langle r^2 \rangle_\sigma \approx \langle r^2 \rangle_{em} \approx 0.8 \text{ fm}^2$. Then it follows that $\sigma(2M_\pi^2) - \sigma(0) = 3.5$ MeV. The one-loop calculation is given by GSS, for Δ_σ one finds

$$\begin{aligned} \sigma(2M_\pi^2) - \sigma(0) &= \frac{3g_A^2 M^3}{64\pi^2 F^2} + \mathcal{O}(M^4 \ln M^2) \\ &= 4.6 \text{ MeV} \end{aligned} \quad (4.12)$$

The correction Δ_R is small, GSS find $\Delta_R = 0.4 \text{ MeV}^*$, so altogether we have $\Sigma - \sigma = \Delta_\sigma + \Delta_R \approx 5 \text{ MeV}$. This leaves ample room for a large strangeness component in the nucleon – but it is not the whole story.

The crux is that in the S -waves, $\pi\pi$ and πN interactions are strong, so that the one-loop approximation is not sufficient (for a detailed account in the meson sector, see Ref.18). Therefore, GLS have used dispersion theory together with constraints from CHPT. Since the nucleon is a composite particle, the scalar form factor has to vanish as the momentum transfer becomes large, thus

$$\sigma(t) = \sigma(0) + \frac{t}{\pi} \int_{4M_\pi^2}^{\infty} \frac{\text{Im}\sigma(t')}{t' - t} dt' \quad (4.13)$$

where

$$\text{Im}\sigma(t) = \frac{3}{2} \frac{\Gamma_\pi^*(t) f_+^0(t)}{4m^2 - t} \sqrt{1 - \frac{4M_\pi^2}{t}} \quad (4.14)$$

with $\Gamma_\pi(t)$ the scalar form factor of the pion (see Ref.18) and $f_+^0(t)$ the conventional t -channel S -wave πN partial wave. The f_+^0 was determined by the Karlsruhe-Helsinki group in the space-like region. In the continuation to the time-like region, GLS modified the $\pi\pi$ phase-shift according to chiral symmetry constraints. The resulting numerical value is

$$\Delta_\sigma = (15 \pm 0.5) \text{ MeV} \quad (4.15)$$

a stummingly large correction to the one-loop calculation. The main contribution to the dispersion integral $\sigma(t)$ comes from the region $4M_\pi^2 \leq t \leq (500 \text{ MeV})^2$, and in this region the one-loop approximation is off by about a factor of two. The spectral decomposition of $\sigma(t)$ is therefore very different from the one related to the electromagnetic form factors (cf. Fig.6, here we have an even stronger enhancement at threshold and no plateau) and that leads to a huge scalar radius, $\langle r^2 \rangle_\sigma \approx 1.6 \text{ fm}^2$, *i.e.* twice as large as a typical $\langle r^2 \rangle_{\text{em}}$. The operator $\hat{m}(\bar{u}u + \bar{d}d)$ sees a much more extended nucleon than the em current, chiefly because it is more sensitive to the pion cloud. In the case of the pion, a similar enhancement of the scalar radius has been observed, one finds $\langle r^2 \rangle_\sigma / \langle r^2 \rangle_{\text{em}} = 0.6 \text{ fm}^2 / 0.44 \text{ fm}^2 \approx 1.4$.

Let me summarize these results and draw some conclusions. For the σ -term, we end up with

$$\begin{aligned} \Sigma &= 60 \text{ MeV}, & \sigma &= 45 \text{ MeV} \\ \sigma &= \sigma_0 / (1 - y), & \sigma_0 &= (35 \pm 5) \text{ MeV} \end{aligned} \quad (4.16)$$

and the errors are discussed in Ref.35. What does that mean for the strange quark component? To go from σ_0 to σ , the strange quarks contribute 10 MeV, or $y \approx 0.2$, which amounts to

$$m_s \langle p | \bar{s}s | p \rangle \approx \frac{m_s}{2\hat{m}} \cdot 10 \text{ MeV} \approx 130 \text{ MeV} \quad (4.17)$$

---

* At the CD-point, $\Delta_R = \mathcal{O}(M_\pi^4 \ln M_\pi^2)$. Now you see why $\nu = 0$, $t = 2M_\pi^2$ is a clever choice !

so that $\bar{m}/m \approx 0.9$. There is a strange contribution to the nucleon mass, but it is not tremendously large as indicated by the lowest order estimate. The most important effect which leads to this result is in fact the rapid rise of $\sigma(t)$ in the region $0 \leq t \leq 2M_\pi^2$. So this issue can be considered settled, and I only wish to make two final remarks. First, one would like to have more accurate and reliable low-energy πN data to further pin down the uncertainties, and, second, the proton is not strange!

5. FURTHER DEVELOPMENTS

Here, I want to sketch briefly two rather recent aspects. One is yet another application (Weinberg's calculation of nuclear forces) and the other methodological (the recent proposal by Jenkins and Manohar to treat baryons as very heavy fields). In both cases, I only give the flavor and leave it to the reader to dig deeper into the details.

5.1 CALCULATION OF NUCLEAR FORCES

On the semi-phenomenological level, nuclear forces are well understood in terms of meson exchange. There is, of course, the long-range pion component which dates back to Yukawa. The intermediate-range attraction between two nucleons can be understood in terms of correlated two-pion exchange (sometimes parametrized in terms of a fictitious scalar-isoscalar meson with $M_\sigma \approx 550$ MeV) and ω -exchange gives rise to part of the short-range repulsion. The ρ -meson features prominently in the isovector-tensor channel, where it cuts down most of the pion tensor potential. From a more fundamental level, our understanding is rudimentary (optimistically spoken), but as was pointed out by Weinberg,⁴³ the constraints from the spontaneously broken chiral symmetry can give us insight into the nature of the nuclear forces.

To be specific, let us consider the effective potential between N nucleons, *i.e.* no intermediate N -nucleon states are allowed, with all nucleons having small momenta of some scale Q . Since we will only consider pion-exchange, we have to choose $Q \ll m_\rho$, *i.e.* all heavy mesons, intermediate Δ 's, etc are integrated out. So we are seeking an expansion in Q and M_π as small parameters. Weinberg tells us how to do the power counting: Consider a vertex of type i with d_i derivatives and n_i nucleon fields. Then a graph with V_i vertices of type i and L loops scales with ν powers of Q or M_π ,

$$\nu = 2 - N + 2L + \sum_i V_i \left(d_i + \frac{n_i}{2} - 2 \right) \quad (5.1)$$

by use of some topological identities. For any interaction which is allowed by chiral symmetry (neglecting symmetry breaking), the coefficient of V_i is non-negative, *i.e.* the leading order diagrams are nothing but tree graphs ($L = 0$) with vertices which

have $d_i = 2, n_i = 0$ or $d_i = 1, n_i = 2$ or $d_i = 0, n_i = 4$ and any number of pion fields. The corresponding effective Lagrangian reads*

$$\begin{aligned} \mathcal{L}_{eff} = & -\frac{1}{2D^2}(\partial_\mu \vec{\pi})^2 - \frac{M_\pi^2}{2D} \vec{\pi}^2 - (\bar{\Psi} \Gamma_\alpha \Psi)(\bar{\Psi} \Gamma^\alpha \Psi) \\ & - \bar{\Psi} \left[i\partial_0 - m - \frac{g_A}{DF_\pi} \vec{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla}) \vec{\pi} - \frac{1}{2DF_\pi^2} \vec{\tau} \cdot (\vec{\pi} \times \partial_0 \vec{\pi}) \right] \Psi \end{aligned} \quad (5.2)$$

with $D = 1 + \vec{\pi}^2/4F_\pi^2$ and the matrices Γ_α are constrained by invariance principles. One can then derive an interaction Hamiltonian in the approximation of neglecting the nucleon kinetic energy and using zero-momentum Dirac spinors in nucleon matrix elements. It contains two new constants C_S and C_T :

$$\begin{aligned} H_{int} &= H_{int}^{\pi\pi} + H_{int}^{\pi N} + H_{int}^{NN} \\ H_{int}^{NN} &= C_S (\bar{\Psi} \Psi)(\bar{\Psi} \Psi) + C_T (\bar{\Psi} \vec{\sigma} \Psi) \cdot (\bar{\Psi} \vec{\sigma} \Psi) \end{aligned} \quad (5.3)$$

Deriving a two-nucleon potential from (5.3), one finds that C_S has to be large and C_T small to explain the intermediate range attraction (remember: these constants contain the information about all the stuff which was integrated out). In fact, one needs some fine-tuning to make the attraction sufficiently small (as it is in nature!). The most interesting observation in this approach concerns the forces between three nucleons. These fall into two classes, the one which has only two-body interactions and the other one with genuine three-body forces. The latter one only involves the non-linear pion interaction, *i.e.* emission/absorption of two pions at one vertex. If you sum up all these irreducible three-body diagrams, you find that they cancel each other. Thus, to leading order, the three-nucleon potential is entirely built from two-nucleon forces with no genuine three-body forces!

Clearly, this is a very interesting result, but if you have some knowledge about what has been going on in the few-body community, you might not be surprised. In case you are, here is some very intriguing piece of information. The Bochum group⁴⁴ developed a full machinery to treat the three-nucleon problem in the continuum. In the case of deuteron break-up just above threshold, *i.e.* a reaction in which the momenta of the involved nucleons are small, observables like *e.g.* the analyzing power A_γ or T_{11} (in nd or pd) can be nicely explained by using only realistic two-body forces (like *e.g.* the Bonn or Paris potential) with no need for a genuine three body force.[†] That smells very much like Weinberg's result. However, more work is needed to clarify this connection. In particular, the chiral people have to perform a one-loop calculation (as Weinberg points out) to check the self-consistency of the tree level results. For a first step, see Weinberg⁴⁸. Finally, we might get a more fundamental handle on the nature of the nuclear forces.

---

* Since we are not considering loops, we don't need to differentiate between quantities in the chiral limit and their physical values.

† However, some charge-symmetry breaking in the two-nucleon force has to be introduced (fine-tuning of some NN phase shifts).

5.2 BARYONS AS VERY HEAVY FERMIONS

In the discussion of the chiral power counting in the presence of baryons, we saw that $\hbar = \mathcal{O}(1)$ poses severe problems (complicates life a lot). One might therefore take a radical view and consider baryons (nucleons) in the limit that their mass becomes very large. Then, a non-relativistic treatment is appropriate and simplifies matters considerably. Such an approach has recently been proposed by Jenkins and Manohar⁴⁵, based on the effective field theory for heavy quarks as formulated by Georgi⁴⁶ and others⁴⁷.

The underlying idea of this approach is that the pions (or kaons) transfer only little momentum to the baryons and thus the baryon velocity is essentially conserved. This allows to write the baryon momentum as

$$P^\mu = m_B v^\mu \quad (5.4)$$

where m_B is the baryon mass and we have $v_\mu v^\mu = 1$ since $P^2 = m_B^2$. One can generalize (5.4) to allow for some off-shell momentum k^μ ,

$$P^\mu = m_B v'^\mu + k^\mu \quad (5.5)$$

with $v \cdot k \ll m_B$ measuring the off-shellness of the baryon. In the heavy mass limit, k^μ becomes irrelevant since it is assumed to be bounded ($v^\mu = v'^\mu$). Georgi calls this a superselection rule, since particles with different velocities do not talk to each other. Therefore, let us introduce a heavy field H_v with a given velocity v (thus the subscript 'v' on H). It is related to the conventional baryon field $B(x)$ via

$$H_v(x) = \exp\{im_B \not{v} \cdot x\} B(x) \quad (5.6)$$

The mass dependence is now entirely in the commutator of the momentum operator, $[P^\mu, H_v(x)] = (-m \not{v} v^\mu - i \partial^\mu) H_v(x)$. The free Lagrangian for the heavy field now reads (we consider particles with $\not{v} H_v = H_v$)

$$\mathcal{L}[H_v] = i \bar{H}_v \not{\partial} H_v \quad (5.7)$$

and H_v obeys a simple Dirac equation

$$\not{\partial} H_v = 0 \quad (5.8)$$

whereas the original field $B(x)$ is given, of course, by $(i \not{\partial} - m_B) B = 0$. In the interacting theory, there are of course corrections of the order $\mathcal{O}(1/m_B)$. So this means that derivatives are small since they produce powers of k^μ , rather than P^μ , and the baryon mass has been completely eliminated (to leading order in $1/m_B$).

The Diracology is considerably simplified in the basis of the heavy fields. To see that, introduce velocity projection and velocity-dependent spin operators,

$$\begin{aligned} P_v &= \frac{1}{2}(1 + \not{v}), & P_v H_v &= H_v \\ S_v^2 H_v &= -\frac{3}{4} H_v, & v \cdot S_v &= 0 \end{aligned} \quad (5.9)$$

If one now works out $\bar{H}_v \Gamma H_v$, with $\Gamma = 1, \gamma_5, \gamma_\mu \dots$, one finds *e.g.*

$$\bar{H}_v \gamma^\mu H_v = v^\mu \bar{H}_v H_v, \quad \bar{H}_v \gamma_5 H_v = 0, \quad (5.10)$$

and so on (the complete list is given in Ref.45). Furthermore, the baryon propagator is $i/(k \cdot v)$, and the pion-nucleon vertex is proportional to $k \cdot S_v$, so that the effective Lagrangian $\mathcal{L}[H_v, U]$ can be written in terms of v^μ and S_v^μ exclusively and without any γ -matrix. Furthermore, it is rather simple to include the spin-3/2 decuplet in this formalism. Naively, one would estimate the octet-decuplet splitting to be $\Delta m = \Sigma^* - p \approx 450$ MeV, *i.e.* comparable to the kaon mass, and a complicated decuplet propagator would be needed. Since one can only project out a common singlet mass for the octet and decuplet fields, one faces the problem of an expansion in M_K and Δm . Effectively, however, Δm is small and of the order of the intramultiplet splittings, $\Delta m = M_\Delta - M_\Sigma = 40$ MeV and can thus be treated perturbatively. This allows to use a very simple spin-3/2 propagator, $iP^{\mu\nu}/(v \cdot k)$, with $P^{\mu\nu}$ a projection operator expressed in terms of v^μ and S_v^α (the spin operator which only acts on the spinor portion of the Rarita-Schwinger field). Also, I should point out that the heavy mass approach leads to distinct ordering of operators according to the powers of m_B^{-1} they carry. For example, to calculate baryon magnetic moments one has to invoke higher dimension operators, which have not yet been considered. The authors of Ref.45 focus on the baryon masses and the axial current. The most prominent result concerning the latter is that the decuplet contributions to a large extent cancel the chiral loop contributions from the octet. For further details, you should consult these papers.

This approach, although promising, is still in its infant stage. In fact, a consistent renormalization scheme to include higher order polynomial terms has not yet been worked out. Therefore, the results still carry the spurious chiral scale $\mu \approx 1$ GeV in the logs. Ultimately, this scale dependence should drop out as discussed in sections 1 and 2. Also the baryon mass calculation is done in the limit $m_u = m_d = 0$ and therefore sheds no light on the quark mass ratios as determined by Gasser.⁴⁰ One would very much like to see such a calculation. Furthermore, many of the simplifications due to the fact that $\Delta m \ll M_K$ cannot be used when one approaches the chiral limit (which is necessary to understand the behaviour of certain operators like *e.g.* the condensates in the proton). Only after more work has been done in this framework, one will be able to judge how effective this approach really is. Conceptually, it remains an open problem whether the low-lying baryons can be considered so heavy that this formalism applies (remember: $m_B/(4\pi F_\pi) \approx 1$). It certainly can be used in B meson physics, but it might oversimplify the situation in the case of the baryons. Time will tell.

6. BRIEF SUMMARY AND OUTLOOK

The constraints from the spontaneously broken chiral symmetry of QCD can be systematically included in an effective Lagrangian of the asymptotically observed fields, the Goldstones and the baryons. Once one has fixed the finite number of a priori unknown constants which appear at a given order in the chiral expansion, one can perform a huge variety of parameter-free calculation. In the presence of baryons there are, however, some yet unresolved problems. To my opinion, the most important one is the way how to deal with the fact that the nucleon mass in the chiral limit is not a small quantity. More work is needed to settle this issue. Furthermore, in some very interesting applications it seems mandatory to include the mass difference of the light quarks ($m_u \neq m_d$) and/or resonance exchange contributions. So far, this has not been addressed in a systematic fashion. Chiral perturbation theory with nucleons is a very active and interesting field and at present it is our only tool to perform systematic calculations in the regime where the strong interactions are really strong – at low energies. There are many challenging problems and beautiful applications to be worked out.

ACKNOWLEDGEMENTS

I am grateful to the organizers, in particular Jerry Miller, for providing a stimulating atmosphere and twisting my arms to give these lectures. Fruitful collaborations with Véronique Bernard, Jürg Gasser and Norbert Kaiser made this project feasible. I also thank the Institute for Nuclear Theory for its hospitality and the DOE for partial support during the completion of this work.

APPENDIX A: CHIRAL SYMMETRY – SOME BASIC FORMULAE

Neglecting color indices, the QCD Lagrangian in the chiral limit ($m_q = 0$) reads

$$\mathcal{L}_{QCD}^0 = -\frac{1}{2g^2} \text{Tr} [G_{\mu\nu} G^{\mu\nu}] + \bar{q} i \gamma^\mu (\partial_\mu - i A_\mu) q \quad (A.1)$$

with A_μ the gluon field and $G_{\mu\nu}$ the corresponding non-abelian field strength. The vector $q^T = (u, d, s)$ embodies the light quark fields. The chiral symmetry reveals itself most clearly in a basis of left- and right-handed quark fields

$$\begin{aligned} \mathcal{L}_{QCD}^0(G_{\mu\nu}, q', D_\mu q') &= \mathcal{L}_{QCD}^0(G_{\mu\nu}, q, D_\mu q) \\ q' &= g_R P_R q + g_L P_L q = g_R q_R + g_L q_L, \quad P_{R,L} = \frac{1}{2}(1 \pm \gamma_5) \\ g_L g_L^\dagger &= g_R g_R^\dagger = 1, \quad \det g_L = \det g_R = 1 \end{aligned} \quad (A.2)$$

with $g_{L,R}$ the generators of $SU(3)_{L,R}$. The associated currents are of axial-vector (L-R) and vector (L+R) type

$$V_\mu^i = \bar{q} \gamma_\mu \frac{\lambda^i}{2} q, \quad A_\mu^i = \bar{q} \gamma_\mu \gamma_5 \frac{\lambda^i}{2} q; \quad i = 1, \dots, 8 \quad (A.3)$$

The conserved charges build from these currents form an $SU(3)_L \times SU(3)_R$ algebra.

Let us now turn to the effective theory. The unitary matrix $U(x)$ which embodies the pseudoscalars transforms linearly under global $SU(3)_L \times SU(3)_R$

$$U' = g_L U g_R^\dagger, \quad U^\dagger U = 1 \quad (A.4)$$

so that the pion field $\pi(x)$ transforms in a highly non-linear fashion. We parametrize the right and left transformations as

$$g_L = e^{i\alpha} e^{i\beta}, \quad g_R = e^{-i\alpha} e^{i\beta} \quad (A.5)$$

so that for $\beta = 0$ we have pure chiral rotations and $\alpha = 0$ is a conventional vector transformation. I use $\alpha = \alpha_a \tau_a / 2$, $\beta = \beta_a \tau_a / 2$ with α_a , β_a real parameters. Under a pure chiral rotation, U transforms as $U' = e^{i\alpha} U e^{i\alpha}$.

Let me now turn towards an explicit construction of the matrix $K(g_L, g_R, U)$ for infinitesimal transformations. K is unitary and will be parametrized as $K = \exp(iv)$. Consider first the case $\beta = 0$ (chiral rotations) using (2.3):

$$\begin{aligned} 0 &= g_L \xi K^\dagger - K \xi g_R^\dagger \\ &= (1 + i\alpha) \xi (1 - iv) - (1 + iv) \xi (1 + i\alpha) + \mathcal{O}(\alpha^2, v^2) \\ &= i[\alpha, \xi] - i\{v, \xi\} + \alpha \xi v + v \xi \alpha + \mathcal{O}(\alpha^2, v^2) \end{aligned} \quad (A.6)$$

Now let α go to zero. It follows that to leading order v must be $\mathcal{O}(\alpha)$. Therefore

$$[\alpha, \xi] = \{v, \xi\} \quad (A.7)$$

Now expand $\xi = 1 + i\pi/F$ so that

$$i[\alpha, \pi]/F = 2v + i\{v, \pi\}/F \quad (A.8)$$

Finally, let π go to zero, which means that $v = \mathcal{O}(\pi)$. Thus

$$v = \frac{i}{2F} [\alpha, \pi] + \mathcal{O}(\pi^2) \quad (A.9)$$

A similar line of arguments in the case of vector rotations ($\alpha = 0$) leads to

$$v = \beta + \mathcal{O}(\pi^2) \quad (A.10)$$

I leave it as an exercise for the reader to consider the general case.

APPENDIX B: SOME LOOP FUNCTIONS

In sections 2 and 3, we encountered some loop functions, i.e. integrals of the type polynom in l (some pion propagators) (some nucleon propagators) with l the loop momentum. For general definitions see Refs.19,31, here I only want to give Feynman parameter representations (which allow you to work out quantities in the quark mass expansion). For the anomalous magnetic moment, we need

$$\gamma_4(t) = \frac{1}{64\pi^2} \int_0^1 dx \int_0^1 dy \frac{x^2 y^3}{M_\pi^2(1-xy) + m^2 x^2 y^2 + ty(y-1)(1-x)} \quad (B.1)$$

and

$$\Gamma_4(t) = \frac{1}{64\pi^2} \int_0^1 dx \int_0^1 dy \frac{y^3}{M_\pi^2(1-y) + m^2 y^2 + txy^2(x-1)} \quad (B.2)$$

In the case of pion photoproduction, the interesting physics is given by γ_6 ,

$$\gamma_6(s) = \frac{1}{64\pi^2} \int_0^1 dx \int_0^1 dy \frac{x(2-x)y^3}{M_\pi^2(1-xy) + xy^2(m^2 - s(1-x))} \quad (B.3)$$

Comparison of (B.1) and (B.3) shows that it depends on the problem what the best choice of the Feynman parameters is. As an exercise, you should derive $\gamma_4(0)$, $\Gamma_4(0)$ in the chiral limit and also $\gamma_6(s_{th})$, $\gamma_6(u_{th})$

REFERENCES

1. J. Gasser, in *Hadrons and Hadronic Matter*, eds. D. Vautherin *et al.* (Plenum, New York, 1990); "Chiral Dynamics", preprint BUTP-91/18, 1991, to appear in the proceedings of the DAFNE workshop, April 1991, Frascati, Italy.
2. H. Leutwyler, "Chiral Effective Lagrangians", preprint BUTP-91/28, 1991, to appear in the proceedings of the XXX Internationale Universitätswochen für Kernphysik, Schladming, Austria.
3. J. F. Donoghue, "Chiral Symmetry as an Experimental Science", preprint CERN-TH.5667/90, 1990, to appear in the proceedings of the School on Low-Energy Antiprotons, Erice, January 1990.
4. J. Goldstone, *Nuovo Cim.* **19** (1961) 154.
5. R. Dashen and M. Weinstein, *Phys. Rev.* **183** (1969) 1261; *Phys. Rev.* **188** (1969) 2330.
6. L.-F. Li and H. Pagels, *Phys. Rev. Lett.* **26** (1971) 1204.
7. P. Langacker and H. Pagels, *Phys. Rev.* **D8** (1973) 4595.
8. H. Pagels, *Phys. Rep.* **16** (1975) 219.

9. S. Weinberg, *Physica* **96A** (1979) 327.
10. J. Gasser and H. Leutwyler, *Ann. Phys. (N.Y.)* **158** (1984) 142.
11. J. Gasser and H. Leutwyler, *Nucl. Phys.* **B250** (1985) 465,517,539.
12. S. Weinberg, *Phys. Rev. Lett.* **17** (1966) 616.
13. S. Weinberg, *Phys. Rev.* **166** (1968) 1568.
14. C. G. Callan, S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2247;
S. Coleman, J. Wess and B. Zumino, *ibid*, 2239.
15. J. Gasser and Ulf-G. Meißner, *Phys. Lett.* **B258** (1991) 219.
16. G. Ecker, J. Gasser, A. Pich and E. de Rafael, *Nucl. Phys.* **B321** (1989) 311;
J. F. Donoghue, C. Ramirez and G. Valencia, *Phys. Rev.* **D39** (1989) 1947.
17. V. Bernard, N. Kaiser and Ulf-G. Meißner, *Nucl. Phys.* **B364** (1991) 283.
18. J. Gasser and Ulf-G. Meißner, *Nucl. Phys.* **B357** (1991) 90.
19. J. Gasser, M.E. Sainio and A. Švarc, *Nucl. Phys.* **B307** (1988) 779.
20. A. Krause, *Helv. Phys. Acta.* **63** (1990) 3.
21. H. Georgi, *Weak Interactions and Modern Particle Physics* (Benjamin / Cummings, Reading, MA, 1984).
22. J.J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, 1969).
23. G. Höhler and E. Pietarinen, *Phys. Lett.* **53B** (1975) 471.
24. G. Höhler, in Landolt-Börnstein, vol.9 b2, ed. H. Schopper (Springer, Berlin, 1983).
25. A. I. Vainshtein and V. I. Zakharov, *Nucl. Phys.* **B36** (1972) 589;
P. de Baenst, *Nucl. Phys.* **B24** (1970) 633.
26. E. Mazzucato *et al.*, *Phys. Rev. Lett.* **57B** (1986) 3144;
R. Beck *et al.*, *Phys. Rev. Lett.* **65B** (1990) 1841.
27. A.M. Bernstein and B.R. Holstein, *Comments Nucl. Part. Phys.* **20** (1991) 197.
28. G.F. Chew, M.L. Goldberger, F.E. Low and Y. Nambu, *Phys. Rev.* **106** (1957) 1345.
29. V. Bernard, J. Gasser, N. Kaiser and Ulf-G. Meißner, *Phys. Lett.* **B268** (1991) 291.
30. V. Bernard, N. Kaiser and Ulf-G. Meißner, *Phys. Rev. Lett.* **67** (1991) 1515.
31. V. Bernard, N. Kaiser and Ulf-G. Meißner, "Nucleons with Chiral Loops: Electromagnetic Polarizabilities", preprint BUTP-91/30, 1991.

32. J. Bijmens and F. Cornet, *Nucl. Phys.* **B296** (1988) 557.
33. G. Ecker, A. Pich and E. de Rafael, *Phys. Lett.* **189B** (1987) 363.
34. G. D'Ambrosio and D. Espriu, *Phys. Lett.* **175B** (1986) 237;
J. Goity, *Z. Phys.* **C34** (1987) 341.
35. J. Gasser, H. Leutwyler and M.E. Sainio, *Phys. Lett.* **253B** (1991) 252; 260.
36. R. Koch, *Z. Phys.* **C15** (1982) 161.
37. see πN Newsletter 2, eds. G. Höhler *et al*, 1990.
38. J. Gasser and H. Leutwyler, *Phys. Rep.* **87** (1982) 77.
H. Leutwyler, *Nucl. Phys.* **B337** (1990) 108.
39. T.P. Cheng and R. Dashen, *Phys. Rev. Lett.* **26** (1971) 594.
40. J. Gasser, *Ann. Phys.(N.Y.)* **136** (1981) 62.
41. L. S. Brown, W. J. Pardee and R. D. Peccei, *Phys. Rev.* **D4** (1971) 2801.
42. M.A.B. Bég and A. Zepeda, *Phys. Rev.* **D5** (1972) 3262.
43. S. Weinberg, *Phys. Lett.* **251B** (1990) 288.
44. H. Witala, W. Glöckle and Th. Cornelius, *Few Body Systems* **6** (1989) 79; *Nucl. Phys.* **A496** (1989) 446; *Phys. Rev.* **C39** (1989) 384;
I. Slaus, R. Machleidt, W. Tornow, W. Glöckle and H. Witala,
Comments Nucl. Part. Phys. **20** (1991) 85;
W. Glöckle, private communication.
45. E. Jenkins and A.V. Manohar, *Phys. Lett.* **255B** (1991) 558; **259B** (1991) 353;
E. Jenkins, preprint UCSD/PTH 91-12, 1991.
46. H. Georgi, *Phys. Lett.* **240B** (1990) 447.
47. M.B. Voloshin and M.A. Shifman, *Sov. J. Nucl. Phys.* **45B** (1987) 292;
N. Isgur and M.B. Wise, *Phys. Lett.* **B232** (1989) 113;
E. Eichten and B. Hill, *Phys. Lett.* **B234** (1990) 511.
48. S. Weinberg, *Nucl. Phys.* **B363** (1991) 3.

END

DATE
FILMED
01/30/92

I