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VISAR ANALYSIS AT THE LAWRENCE LIVERMORE LABORATORY WITH THE FOUR-DETECTOR SYSTEM

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Norval L. Parker

June 22, 1979

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ABSTRACT

We present a detailed description of the analysis of velocity-vs-time data taken with LLL's four-phototube VISAR system. Separate sections deal with a general analysis of on-axis rays when only one frequency of reflected laser light is present, a simplified analysis where two distinct frequencies are present and the rays are still on axis, and a discussion of the effects of off-axis rays. The description is given in such a manner that a VISAR user could employ the formulae in this report to directly analyze data. Unlike previous literature on the VISAR, this analysis does not assume that the mirrors are set at the "best fringe contrast" position.

I. INTRODUCTION

The usefulness of using laser Doppler shift techniques to measure velocities for shock-wave physics studies has been recognized for many years. A great deal of the early development work on the VISAR interferometer was done by Barker and Hollenbach¹ using a three-phototube system consisting of two signal detectors and one intensity monitor. As pointed out by Hemsing,² the three-phototube system does not use at least half of the available signal power, but a four-phototube system

uses all of the laser light. Hemsing² presents a discussion of several VISAR systems. The purpose of our report is to describe explicitly how actual data are converted to velocities at LLL with the four-detector system. The report also provides introductory definitions and results necessary as a starting point in the discussions of off-axis effects and double-frequency effects that occur in real experiments.

II. ON-AXIS SIGNALS IN THE VISAR

A. GENERAL CONSIDERATIONS

Figure 1 shows the four-phototube VISAR system used at LLL. Monochromatic laser light of angular frequency ω_0 is reflected by a surface moving with velocity V , and diverted by a mirror with a hole in it to a beam-reducing telescope. The reflected light has frequency ω . It is filtered and split into equally intense s and p polarization components by polarizer A. That light is split at point X on beamsplitter A and diverted two directions. The $\lambda/8$ plate in front of mirror B causes the p polarization

to be delayed in time by an amount $\lambda/4c$ with respect to the s-polarized component. The glass cylinder provides most of the necessary optical time delay between the two paths for proper operation of the VISAR. The two beams leaving point Y are then separated according to their polarizations and detected by phototubes D_1 through D_4 . The position of mirror B is usually¹ set so that for off-axis rays the apparent position of mirrors A and B coincide. Barker's analysis¹ assumes these two positions do coincide, but experience at LLL has shown that the best fringe contrast often results when these

FIG. 1. The four-phototube VISAR system. Light entering beamsplitter A makes angle of about 0.03 rad with the normal to the beamsplitter. In the three-phototube system, where detectors D_3 and D_4 are not present, light may enter normal to the glass cylinder.

mirrors are one or two millimeters from coincidence. To include the effects of this situation, the usual VISAR formulae^{1,3} are rederived to include this additional degree of freedom.

Figure 2 shows a close-up of the interferometer part of the VISAR. The index of refraction of the glass is indicated to be a function of wavelength because, as Barker has pointed out,³ this very slight dependence of the index n on the surface velocity V (through the wavelength λ) does materially affect the velocity inferred in the analysis. Over the very narrow range of wavelengths involved, it is assumed that the wavelength dependence of the index of refraction is adequately represented by

$$n(\lambda) = n_0 + \frac{dn}{d\lambda} \bigg|_{\lambda_0} (\lambda - \lambda_0), \quad n_0 \equiv n(\lambda_0), \quad (1)$$

where λ_0 is the wavelength during the setup where no Doppler shift has yet occurred, and λ is the wavelength at any time.

The notation of Fig. 2 follows that of Barker,³ except that the parameter e has been introduced to include the possibility of e being nonzero. Barker³ shows that when $L_R = L_L + L_E(1 - 1/n_0)$, the apparent position of mirrors A and B coincide and one can expect to get best fringe contrast. Thus a natural definition of the offset e is by the relation

$$L_R \equiv L_L + L_E \left(1 - \frac{1}{n_0}\right) - e. \quad (2)$$

The time delay between the two optical paths shown in Fig. 2 also depends on the wavelength and the polarization, and is for the p polarization

$$\tau \equiv \tau_p = \frac{2L_E}{c} \left(n - \frac{1}{n_0}\right) - \frac{2e}{c} \quad (3)$$

and

$$\tau_0 \equiv \tau_{p0} = \frac{2L_E}{c} \left(n_0 - \frac{1}{n_0}\right) - \frac{2e}{c} \\ = \frac{2}{c} [(n_0 - 1)L_E + L_R - L_L], \quad (4)$$

where Eq. (2) has been used to eliminate L_R , and the symbol L_E is the difference in the sum of the lengths of the glass components of the right leg of the interferometer relative to the sum of the lengths of all the glass in the left leg. For the s polarization the time delays are decreased by an amount $\lambda_0/4c$, so that

$$\tau_s = \tau_p - \frac{\lambda_0}{4c} \quad (5)$$

and

$$\tau_{s0} = \tau_{p0} - \frac{\lambda_0}{4c}. \quad (6)$$

The wavelength changes with velocity according to

$$\lambda = \lambda_0 \left(1 - \frac{2V}{c}\right), \quad (7)$$

FIG. 2. The interferometer section of the VISAR. This figure defines the mirror-offset parameter e . This parameter is assumed to be zero in other treatments of the operation of the interferometer. The symbol L_E means the sum of all of the lengths of the glass elements on the right side minus the corresponding sum on the left side.

so that a simple calculation for the ratio of τ/τ_0 results in

$$\frac{\tau}{\tau_0} = 1 + \frac{2V\delta_1}{c} \left(1 + \frac{2e}{c\tau_0} \right), \quad (8)$$

where the symbol δ_1 is the same¹ as Barker's δ , namely

$$\delta_1 = \frac{-\lambda_0 \frac{dn}{d\lambda} \Big|_{\lambda_0}}{n_0 - \frac{1}{n_0}} \doteq 0.034 \text{ for quartz at } 5145 \text{ \AA}. \quad (9)$$

For velocities of a few km/s, τ and τ_0 differ by less than one part in 10^6 , but it is this small difference that causes a correction of about 3% to the velocity per fringe as first pointed out in Ref. 3, using a different method of analysis. The small difference in the time delays for the s and p polarizations causes some of the VISAR signals to be 90° out of phase with each other, as will be illustrated.

B. ON-AXIS RAYS CONSISTING OF ONLY ONE FREQUENCY AND NO INCOHERENT LIGHT

Referring to Fig. 1, the amplitude of the light entering the beamsplitter at point X is by definition $2A(t')$. Since a phase change of π occurs on rare to dense reflections, the amplitudes of the light leaving point Y towards phototubes D_1 , D_2 , D_3 , and D_4 are, assuming a 50% beamsplitter B and no optical losses in VISAR components,

$$\begin{aligned} A_1(t) &= +A(t) - A(t - \tau_p) \\ A_2(t) &= +A(t) - A(t - \tau_s) \\ A_3(t) &= -A(t) - A(t - \tau_p) \\ A_4(t) &= -A(t) - A(t - \tau_s), \end{aligned} \quad (10)$$

where

$$A(t) \equiv \cos \omega t = \cos \left[\omega_0 \left(1 + \frac{2V}{c} \right) t \right]. \quad (11)$$

The distinction between the times t' and t is due to the optical path from point X to mirror A to point

Y. The phototubes record the time-averaged squares of each amplitude, which are

$$\begin{aligned} D_1 &= \overline{A_1^2} = \overline{[\cos \omega t - \cos \omega(t - \tau_p)]^2} \\ &= \overline{\cos^2 \omega t + \cos^2 \omega(t - \tau_p)} \\ &\quad - \overline{[\cos(2\omega t - \omega \tau_p) + \cos \omega \tau_p]} \\ &= 1 - \cos \omega \tau_p. \end{aligned}$$

Similarly, the expressions for the other phototube signal intensities are

$$\begin{aligned} D_2 &= 1 - \cos \omega \tau_s = 1 - \sin \omega \tau_p \\ D_3 &= 1 + \cos \omega \tau_p \\ D_4 &= 1 + \sin \omega \tau_p. \end{aligned} \quad (12)$$

When the intensity of a phototube signal undergoes a complete oscillation from maximum to maximum due to changes in velocity, this signal variation represents one fringe by definition. One fringe results when $\omega \tau_p$ changes by 2π due to the velocity changes. But both ω and τ_p change when the velocity changes. These dependences are described by Eqs. (11) and (8), respectively. Using both Eqs. (8) and (11) results in the explicit velocity dependence of the product $\omega \tau_p$

$$\omega \tau_p = \omega_0 \tau_0 \left(1 + \frac{2V}{c} \right) \left[1 + \frac{2V\delta_1}{c} \left(1 + \frac{2e}{c\tau_0} \right) \right]. \quad (13)$$

Expanding and neglecting terms in V^2/c^2 gives

$$\omega \tau_p \doteq \omega_0 \tau_0 \left\{ 1 + \frac{2V}{c} \left[1 + \delta_1 \left(1 + \frac{2e}{c\tau_0} \right) \right] \right\}. \quad (14)$$

The velocity increment per fringe, denoted K , is obtained by the relation

$$\omega(V = K) \cdot \tau_p(V = K) - \omega_0 \tau_0 = 2\pi. \quad (15)$$

Substituting Eq. (14) into Eq. (15) and solving for K results in

K = velocity per fringe

$$= \frac{\lambda_0}{2\tau_0 \left[1 + \delta_1 \left(1 + \frac{2e}{\tau_0 c} \right) \right]} \quad (16)$$

This result is similar to Barker's¹ except that Eq. (16) includes the mirror offset term e both explicitly and implicitly in τ_0 , which is the *actual* delay instead of the usual delay, which is the value τ_0 has when e is zero. Equation (4) illustrates this dependence.

Using the four signals D_1 through D_4 expressed in Eq. (12) and using Eq. (14), the result for the velocity in general can be written as

$$V(t) = \frac{K}{2\pi} \left[\tan^{-1} \left(\frac{D_2 - D_4}{D_1 - D_3} \right) \right]_{t=0}^t + \frac{mK}{2} \quad (17)$$

where the velocity per fringe constant K is given by Eq. (16). In Eq. (17) the symbol m is an unknown integer. It is included in Eq. (17) to represent the situation where the velocity changes by more than one fringe so fast that the recording system of phototubes and oscilloscopes cannot follow the change; only the final signal intensities D_i are recorded, and an unknown number of fringes has been missed by the recording system. Note that the velocity ambiguity in this situation is an integer number of half fringes $K/2$. The time $t = 0$ in Eq. (17) is anytime such that the velocity was zero.

The result, Eq. (17), is similar to that of Hemsing,² except that the expression here for K is more general.

C. ANALYSIS WHEN INCOHERENT LIGHT, VELOCITY DISPERSION, AND CHANGING REFLECTED LASER LIGHT MAY OCCUR

In many real experimental situations, the intensity of the reflected laser light changes with time. In addition, incoherent light with an unknown time-dependent intensity may be present. Also, different parts of the illuminated object may move at slightly different velocities, causing a continuum of reflected

frequencies and phases. This continuum of frequencies has been shown by Asay⁴ to cause a contrast degradation in the fringe oscillations, so that the detector signals no longer oscillate between zero and maximum intensity, but have a nonzero minimum intensity whose value increases as the velocity dispersion increases. Asay's proof assumed⁴ that the VISAR detector intensities caused by a mixture of frequencies can be calculated by incoherently adding the intensities caused by each frequency component separately. By using an analysis of amplitudes similar to that used in Sec. B for the case of a mixture of frequencies, and then taking the time-averaged square of the result, the contrast degradation factor is the same as that of Ref. 4, provided there is a fine continuum of frequencies with random phases in the return laser light. A mixture of a few discrete frequencies will, however, cause interference. This case is treated in Sec. D.

The eighth-wave plate may in real situations not provide a phase shift of exactly 90° between signals D_1 and D_2 . The present analysis allows that phase angle denoted by δ to be arbitrary.

The following formalism is that presently used in the LLL four-phototube VISAR system. It involves taking a dry run before the actual experiment, the purpose of which is to measure the phase angle δ and part of the contrast degradation factor. The assumptions preceding Eq. (10) regarding the 50% beamsplitter and no optical losses can be removed by absorbing real losses into the contrast factor, as will be described in detail.

Definitions and assumptions:

$L(t)$ —return laser light intensity (unknown)

$B(t)$ —intensity of unpolarized incoherent background light (unknown)

$D_i(t)$, ($i = 1, 4$)—photodetector signal intensities (measured)

$C_i(t)$, ($i = 1, 4$)—contrast factors for detectors i due to laser light only, and not due to the addition of the incoherent light B (unknown)

$V(t)$ —test object velocity (to be determined)

t_0 —time just before motion starts and the velocity is still 0.

t —anytime

t_D —anytime during the taking of the dry run, where $V = 0$

λ_0 —laser wavelength at time t_0

τ_0 —measured optical delay [see Eq. (4)]. It is assumed constant during the dry run and during the actual experiment, but it is allowed to change between the dry run and the experiment due to slow

thermal drifting of the separation between mirrors A and B of Fig. 1. This assumption allows the dry run to precede the experiment by several minutes, if necessary.

δ_1 —defined by Eq. (9) and is about 0.034

ω_0 —initial laser angular frequency at time t_0 . It is assumed constant during the dry run and the experiment, but it may change slightly between the two times. Once motion starts, all light of this frequency is assumed to disappear.

e —measured mirror offset defined by Fig. 2 and Eq. (2)

K —velocity-per-fringe constant given by Eq. (16) and depending upon e , λ_0 , τ_0 and δ_1 .

$\bar{V}$ —defined as $2\pi V/K$

δ —phase shift between the s and p polarized light that is close to $\pm 90^\circ$ in most situations, and is measured by the dry run. In Eq. (12) it was $+90^\circ$.

G_i ($i = 1, 4$) factors representing the gains of the four detection systems. They are assumed independent of time and their relative sizes are measured as explained later during the dry run. The analysis allows all four gain factors to change by the same amount between the dry run and the experiment, but their relative sizes must not change.

The results for the on-axis rays are used in this analysis. This means that, strictly speaking, only the center of the VISAR interference pattern is being viewed by the phototubes. The discussion of off-axis rays and the application of the results of this section to the case where the phototubes view the entire interference pattern are given in Sec. III.

In analogy with Eq. (12), the expressions for the signals D_i are

$$\begin{aligned} D_1 &= G_1 \{ L [1 - C_1 \cos(\omega_0 \tau_0 + \bar{V})] + B \} \\ D_2 &= G_2 \{ L [1 - C_2 \cos(\omega_0 \tau_0 + \bar{V} - \delta)] + B \} \\ D_3 &= G_3 \{ L [1 + C_3 \cos(\omega_0 \tau_0 + \bar{V})] + B \} \\ D_4 &= G_4 \{ L [1 + C_4 \cos(\omega_0 \tau_0 + \bar{V} - \delta)] + B \}. \end{aligned} \quad (18)$$

The background light B is assumed to have a short coherence time compared with τ_0 , so that it adds incoherently to the laser light. The phase angle δ is $+90^\circ$ if D_1 leads D_2 by 90° upon acceleration of the test object, or, in the case of a dry run where fringes are produced artificially by motion of either mirror

A or mirror B by piezoelectric transducers, when the mirror motion is such that τ_0 increases with time. Either situation causes the phases of the cosines in Eq. (18) to increase with time. The contrast factors are time-dependent, but in a good experimental setup should all be nearly equal at a given time. They are assumed⁵ to be a product of a time-dependent part $C_b(t)$, which is the same for all four detectors, and a time-independent part $C_a(i)$, which is dependent slightly on the photodetector it applies to. That is,

$$C_i(t) = C_a(i) C_b(t) \quad 0 \leq C_i \leq 1, \quad (19)$$

where C_b is the contrast degradation due to velocity dispersion and C_a is the contrast degradation due to everything else, including small constant amounts of incoherent light that the phototubes view during the dry run where B is assumed to be 0. As explained in Sec. III, C_a includes contrast losses due to spatial integration of the entire interference pattern by the photodetector, if the VISAR is being run in that mode. For off-axis rays the interference pattern may not simply be a field of constant brightness, but usually is, for example, a bright large central region surrounded by one or more concentric dark rings, as in a bull's-eye pattern. If the detector sees only the central part of the pattern, then the $C_a(i)$ should be very close to unity. If, however, the phototube views the entire bull's-eye pattern in order to use more of the reflected laser light, then the $C_a(i)$ will be somewhat less than unity. In addition, if the beamsplitter A and mirrors A and B and the other optical components do not produce equally intense interfering beams at point Y, then the $C_a(i)$ will also be less than unity. (See Hemsing² for explicit results.) Whatever the causes for the contrast degradation, the dry-run measures the $C_a(i)$.

1. Dry Run

During measurement of the dry-run data, the piezoelectric transducer causes a linear motion of either mirror A or B, producing an approximately sinusoidal signal at each photodetector. The four signals D_i are recorded on oscilloscopes, and the peak and valley intensities P_i and V_i are determined. Figure 3 illustrates the dry-run data for the usual case of negative signals. By assumption both the

velocity, V , and the experiment-induced background light, B , are zero during the dry run. From Fig. 3 and Eq. (18), the dry-run signals can be written as

$$D_1(t_D) = G_1 L(t_D) [1 - C_1(t_D) \cos \omega_0 \tau_0] \\ = \frac{P_1 + V_1}{2} \left(1 - \frac{P_1 - V_1}{P_1 + V_1} \cos \omega_0 \tau_0 \right). \quad (20)$$

Similarly,

$$L_2 = \frac{P_2 + V_2}{2} \left[1 - \frac{P_2 - V_2}{P_2 + V_2} \cos(\omega_0 \tau_0 - \delta) \right]. \quad (21)$$

Hence,

$$G_1 L(t_D) = \frac{P_1 + V_1}{2}, \quad (22)$$

and

$$C_1(t_D) = \frac{P_1 - V_1}{P_1 + V_1}. \quad (23)$$

FIG. 3. The time-dependence of signals from detectors 1 and 2 during the dry run, showing the definition of V_i , P_i , and the time $\bar{t}$.

In Eq. (20), t_D indicates anytime during the dry run, and $D_1(t_D)$ is time-dependent because the optical delay τ_0 in this equation is changing linearly with time.

To determine the unknown phase angle δ , pick a time, $\bar{t}$, such that

$$D_2(\bar{t}) = P_2 \quad (\text{see Fig. 3}). \quad (24)$$

It then follows from Eqs. (20) and (21) that the solution for δ is

$$\delta = \cos^{-1} \left[\frac{2D_1(\bar{t}) - P_1 - V_1}{P_1 - V_1} \right]. \quad (25)$$

This gives two solutions differing in sign. The sign is positive if D_1 leads to D_2 when τ_0 increases with time, or if D_2 leads D_1 when τ_0 decreases with time. The time delay τ_0 changes with time because of the piezoelectrically driven mirror, but it is not always obvious which way the mirror is moving. This direction can be determined by using a slow ramp voltage on the driver such that the VISAR interference bull's-eye pattern appears to either converge towards the center or to diverge away from the center, as viewed by the eye. If it converges, for example, the driver is then turned off and the same mirror is physically pushed in a direction to cause the rings to converge. Thus one can tell whether the time delay is increasing or decreasing with time.

2. Velocity-Measuring Experiment

During the time between the dry run and the actual experiment, several minutes may have elapsed. Consequently, the laser output may have changed and the time delay τ_0 will have probably drifted thermally enough to affect the product significantly (several radians). This thermal drift is assumed to be negligible during the duration of the experiment, about 10^{-5} s. Define the initial starting phase at $t = 0$ to be $\omega_0 \tau_0 \equiv \theta_0$. Then Eq. (22) shows that

$$\frac{G_i}{P_i + V_i} = \frac{1}{2L(t_D)}, \quad (26)$$

which is the same value for any detector. Now, using Eq. (18), except that $\omega_0 \tau_0$ is now replaced by the new phase θ_0 due to thermal drift, it follows that

$$R(t) \equiv \frac{\frac{D_2}{P_2 + V_2} - \frac{D_4}{P_4 + V_4}}{\frac{D_1}{P_1 + V_1} - \frac{D_3}{P_3 + V_3}} \quad (27)$$

$$= \frac{C_2 + C_4}{C_1 + C_3} [\cos \delta + \sin \delta \tan(\theta_0 + \bar{V})] .$$

Because of the assumption expressed by Eq. (19), the ratio of contrast factors in Eq. (27) has the same value at time t as it did during the dry run, so that

$$C \equiv \frac{C_2 + C_4}{C_1 + C_3} = \frac{\frac{P_2 - V_2}{P_2 + V_2} + \frac{P_4 - V_4}{P_4 + V_4}}{\frac{P_1 - V_1}{P_1 + V_1} + \frac{P_3 - V_3}{P_3 + V_3}} \approx 1 . \quad (28)$$

Using Eqs. (28) and (27) and the definition of $\bar{V}$ to solve for V results in

$$V(t) = \frac{K}{2\pi} \left[\tan^{-1} \left(\frac{\frac{R(t)}{C} - \cos \delta}{\sin \delta} \right) \right]_{t=t_0}^t + \frac{mK}{2} . \quad (29)$$

Again the integer m is unknown and represents possible skipped half-fringes, if the velocity changes too rapidly for the recording medium to follow. Again the velocity at time t_0 is assumed to be 0. This result is similar to Eq. (29) of Ref. 2, except the constant K here has a more general definition and the fringe ambiguity integer m is mentioned explicitly.

Note that several unknown time-dependent quantities are not involved in the result for the velocity. They include the incoherent background light, the changing laser intensity, and velocity dispersion.

Several quantities must be measured in order to determine the velocity. The quantities L_R , L_L , and L_E of Fig. 2 are first measured. The determination of L_E may result from several measurements, since L_E really represents the difference between the sum of all the glass lengths in one arm of the interferometer and the sum of all the glass lengths in the other arm. Then the mirror offset e and the time

delay τ_0 are calculated by Eqs. (2) and (4), respectively. K is then calculated from Eq. (16). The eight quantities, P_i and V_i ($i = 1, 4$) and the time $\bar{t}$ are measured during the dry run, so that the phase angle δ and the C factor may be calculated by Eqs. (25) and (28), respectively. The four detector signals, D_i , are measured as a function of time, allowing $R(t)$ to be calculated from Eq. (27). Then the velocity is calculated from Eq. (29).

An alternative to measuring L_L , L_R , and L_E is to use white incoherent light in the VISAR and, with the glass cylinder removed, patiently move mirror A or B such that interference fringes are observed. If the remaining thickness of glass elements on either side of the beamsplitter are about equal, white light fringes will be observed at the zero time delay position, which can be determined very precisely in this way. The glass cylinder and the mirror are then repositioned to obtain good contrast fringes. The offset e can then be calculated easily.

In many real VISAR systems, the fringe contrast is not good when e is zero, for reasons still under study. The best contrast occurs for a finite value of e , and it is often important not to neglect the effects of e on the calibration constant K . The results for K used in Refs. 1 and 2 assumed $e = 0$.

D. THREE-PHOTOTUBE VISAR SYSTEM

In this system, detectors 1 and 2 are used in conjunction with an intensity monitor detector, M , which measures the sum of the incoherent and laser light, and is given by

$$M(t) = G_M [L(t) + B(t)] .$$

The result of a similar calculation under exactly the same assumptions as in Sec. C is

$$V(t) = \frac{K}{2\pi} \left\{ \tan^{-1} \left[\frac{\left(\frac{D_2(t) - \frac{(P_2 + V_2)M(t)}{2M(t_0)}}{(P_1 + V_1)M(t)} \right) \frac{P_1 - V_1}{P_2 - V_2} - \cos \delta}{\sin \delta} \right] \right\}_{t=t_0}^t + \frac{mK}{2} \quad (30)$$

This result is similar to Eq. (14) of Ref. 2. The three-phototube system uses one less detector, but uses only about half of the available signal utilized by the four-detector system.

E. TWO DISCRETE FREQUENCIES

In some experimental situations a small amount of un-Doppler shifted laser light may contaminate the Doppler-shifted light returning from the moving test object. This may arise due to reflections from windows or the use of optical fibers to transport the light. In this case the VISAR intensity is a coherent sum of the effects due to each frequency. For sufficiently low velocities, the interference between the two frequencies causes a significant error in the velocity inferred from either Eq. (29) or Eq. (30). The size of this error depends upon the time resolution of the recording system, as will be shown. This arises because at high velocities and therefore at high difference frequencies, the interference term is washed out by the finite-time resolution of the detection system.

The amplitude of the light entering point X of Fig. 1 is now represented as

$$2A(t') = 2A_1 \cos(\omega_0 t' + \psi) + 2A_2 \cos \omega t',$$

where ψ is an arbitrary phase difference at $t' = 0$. Using the same assumptions as used in Sec. B, Eq. (10) implies that

$$D_3 = [A_1 \cos(\omega_0 t + \psi) + A_2 \cos \omega t + A_1 \cos [\omega_0 (t - \tau_0) + \psi] + A_2 \cos \omega (t - \tau)]^2.$$

Here τ and τ_0 retain their distinction as before. The overhead bar denotes time averaging over a time interval, T , equal to the time resolution of the detection system. Specifically,

$$\bar{X} \text{ means } \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} X(t) dt.$$

Carrying out the indicated calculation results in

$$D_3 = A_1^2 (1 + \cos \omega_0 \tau_0) + A_2^2 (1 + \cos \omega \tau) + 8A_1 A_2 \cos \frac{\omega_0 \tau_0}{2} \cos \frac{\omega \tau}{2} \cos \frac{\omega \tau - \omega_0 \tau_0 + \psi}{2} \times \frac{\sin(\omega - \omega_0) \frac{T}{2}}{(\omega - \omega_0) T}. \quad (31)$$

The expression for D_1 has negative cosines in the A_1^2 terms and the first two cosines in the interference term become sines. The results for the quadrature signal D_4 is

$$D_4 = A_1^2 [1 + \cos(\omega_0 \tau_0 - \delta)] + A_2^2 [1 + \cos(\omega \tau - \delta)] + 8A_1 A_2 \cos \frac{\omega_0 \tau_0 - \delta}{2} \times \cos \frac{\omega \tau - \delta}{2} \cos \frac{\omega \tau - \omega_0 \tau_0 + \psi}{2} \times \frac{\sin(\omega - \omega_0) \frac{T}{2}}{(\omega - \omega_0) T}, \quad (32)$$

where $|\delta|$ is usually close to 90° . The expression for D_2 has negative cosines in the A_1^2 terms of Eq. (32) and the first two cosine terms in the interference expression become sines. The total intensity in either polarization is given by

$$D_1 + D_3 = D_2 + D_4.$$

In the special case where $\delta = 90^\circ$, one has

$$\frac{D_4 - D_2}{2} = A_1^2 \sin \omega_0 \tau_0 + A_2^2 \sin \omega \tau + 4A_1 A_2 \sin \frac{\omega_0 \tau_0 + \omega \tau}{2} \cos \frac{\omega \tau - \omega_0 \tau_0 + \psi}{2} \times \frac{\sin(\omega - \omega_0) \frac{T}{2}}{(\omega - \omega_0) T}. \quad (33)$$

The expression for $D_3 - D_1$ has cosines in the A_1^2 terms and the first sine term in the interference expression becomes a cosine.

The undesired frequency ω_0 has two effects on Eq. (33). If the intensity of the unshifted laser light is independent of time, then the first term in Eq. (33) is a constant. The interference expression depends on A_1 , V , T and the starting phase ψ . Since ψ will depend on the precise separation between the reflecting stationary and nonstationary objects at $t = 0$, the entire interference term will have a sign

which depends on wavelength-sized changes in that separation. The last term in the interference expression will guarantee the latter to be negligible provided that

$$(\omega - \omega_0)T > 100 ,$$

or that

$$V \geq \frac{8\lambda_0}{T} .$$

If V increases monotonically with time through several fringes, then the first term in Eq. (33) might

show up in the analysis as a constant baseline shift. However, for a fast jump in velocity, neither term involving A_1 would be evident.

As an example, let $\delta = 90^\circ$, $\psi = 0^\circ$, $\cos \omega_0 \tau_0 = 1$, $K = 1$ km/s, $\lambda_0 = 514$ nm, $T = 3$ ns and $A_1^2/A_2^2 = 0.10$. In this case, using the result Eq. (17), to calculate the velocity from the measured values of D_1 would result in the curves shown in Fig. 4, where the real velocity increases linearly with time. Here near $V = 0.2$ km/s, a 10% admixture of unshifted laser light causes a 10% error in deduced velocity.

FIG. 4. The actual and deduced velocity histories for the case where 10% of the reflected intensity is not Doppler-shifted. The deviation between the dotted and straight curves is due to interference between the two frequencies within the delay leg of the interferometer.

III. OFF-AXIS EFFECTS—ANGULAR DIVERGENCE

A. GENERAL DISCUSSION

All the foregoing results were derived assuming the laser beams within the interferometer had zero angular divergence. In reality, the beams have some convergence or divergence amounting to about 0.01 rad or less, relative to the center of the light beam. Even if the beam at $t = 0$ has very little angular divergence, once the test object has moved somewhat the divergence will increase by an amount depending upon the F number of the collecting lens of Fig. 1. Figure 5 shows a simplified situation where the back end of the glass cylinder and mirror B are coincident. The glass cylinder has length L_E and index n . The image plane of mirror A is situated a distance t' from the front of the glass cylinder. The so-called "best contrast" position corresponds to $t' = L_E/n$, or $e = 0$. Figure 5 is drawn for e negative. The entrance angle θ is small, about 2° or less.

There are two effects due to the finite entrance angle θ . They are the phase difference between beams A and B and the transverse displacement y between them. Hilliard and Shepherd⁶ have calculated the phase difference, but were not concerned with the displacement y because they were dealing with spatially coherent light, whereas we are concerned with spatially incoherent light. The resulting expression for the displacement y is

$$y = 2L_E \cos \theta \frac{\sin \theta}{n\sqrt{1 - \frac{\sin^2 \theta}{n^2}}} - 2t' \sin \theta, \quad (34)$$

expanding to third order in the angle θ gives

$$y = -2e\theta + \theta^3 \left[\frac{e}{3} - \frac{L_E}{n} \left(1 - \frac{1}{n^2} \right) \right] + \dots \quad (35)$$

Note that if $e = 0$, the displacement is zero to first and second order in θ . Thus the apparent position of mirrors A and B are the same for small angles θ . When the Doppler-shifted laser light arises from a partially diffuse reflecting surface of the test object, the light is spatially incoherent, and the return beam has the characteristic speckle pattern. If the mirror offset e is so large that the beam displacement y is

FIG. 5. Paths of rays in the delay leg entering the glass cylinder at an angle θ , and the image of the corresponding ray reflected from mirror A. This drawing is shown for the case of negative e .

larger than the characteristic size of the speckles, then good contrast interference between parallel beams A and B cannot be had regardless of the optical path difference (unless they are brought to interference at one plane by a lens). This is not true for specular test object surfaces and spatially coherent light, since a small transverse displacement has little effect on the interference provided it is small compared to the diameter of beam A or B.

The optical path difference between beams A and B is nonzero even for $\theta = 0$, since the VISAR requires a time delay to function properly. What is important is the difference between the B-A optical path at some angle θ and that same difference for $\theta = 0$. The result of such a calculation is

$$\Delta(\theta) - \Delta_0 = 2nL_E \left(\sqrt{1 - \frac{\sin^2 \theta}{n^2}} - 1 \right) - 2t'(\cos \theta - 1), \quad (36)$$

where Δ_0 is the B-A path difference at $\theta = 0$. Expanding to 4th order in θ gives

$$\Delta - \Delta_0 = e\theta^2 + \theta^4 \left[\frac{L_E}{4n} \left(1 - \frac{1}{n^2} \right) - \frac{e}{12} \right] \\ = \frac{\lambda}{2\pi} (\delta_2 - \delta_{20}) \quad (37)$$

where δ_2 and δ_{20} are phase differences in radians. The time delay is

$$\tau(\theta) = \frac{2L_E}{c} \left(n - \frac{1}{n_0} \right) - \frac{2e}{c} + \frac{\Delta(\theta) - \Delta_0}{c} \quad (38)$$

The analysis of Fig. 5 assumes that the mirror is in contact with the back surface of the glass. A similar analysis free of this assumption gives the same results, provided the additional air space is compensated for on the other side.

Table 1 lists calculated values of the phase difference relative to that at 0° and the transverse displacement for a range of offset e and angles commonly used in experiments. Specific values for L_E , n , and λ are shown in the table. The phase shifts can become large compared with 2π rad representing one complete bright and dark ring pair. The transverse displacements are to be compared to a typical speckle size of about 0.025 cm, in order to indicate whether severe loss of contrast will result.

In the three-phototube VISAR system, it is possible to have the centroid of the light beam normally incident upon the surface of the glass delay cylinder of Fig. 1. Then the values in the first four columns of Table 1 are directly usable, since they represent the phase shifts relative to $\theta = 0$, which in this case is the centroid of the beam. Thus for a beam with a given angular divergence, the phase shifts and the number of bright and dark rings in the VISAR interference pattern may be inferred directly from Table 1, for this example. It is clear that as e increases, the number of those rings increases rapidly, unless the angular content of the beam is all concentrated very near 0° . For a light beam with very low emittance, such as might arise when the diffuse test object is at the focal plane of the collecting lens of Fig. 1, the visual contrast between dark and light rings as viewed by a white card placed just downstream of point Y of Fig. 1 will decrease as the ring diameter increases. This is because of the displacement-vs-angle relationship. If this beam is focused by a spherical lens, this con-

trast should be restored at the focal plane of the lens.

In the four-phototube VISAR, however, the beam centroid is necessarily at a small angle of about 0.03 rad with respect to the normal to the glass surface, as shown in Fig. 1. The angular divergence or convergence about this centroid is usually about 0.003 rad. Thus the situation can be described by a VISAR system that produces a phase difference and displacement which is a function only of the angle between the ray and the glass normal, illuminated by a light beam that has nonzero intensity only in a restricted range of angles about the beam centroid angle. Figure 6 illustrates this situation. In principle, then, the fringe pattern for

FIG. 6. The inherent ring pattern as a function of polar angle θ is illuminated in the four-phototube system with a beam whose centroid angle is θ_0 and whose angular divergence about this centroid is less than θ_0 . If a white card is placed just downstream of position Y in Fig. 1, the actual ring pattern in space will look like this figure, provided that a unique position on the card determines a unique angle θ . This requires that the laser beam be well-focused on the test object so that the reflected light appears to originate from a point, allowing the return beam to have minimum area in phase space. As the target moves, the size of the illuminated spot increases and the visual contrast between light and dark rings is degraded.

TABLE 1. Calculated phase differences and transverse displacements as a function of angle and offset for $n = 1.5208$, $\lambda = 514.5$ nm, and $L_E = 10$ cm.

Mirror offset c , cm	Entrance angle θ , rad	$\delta_2 - \delta_{20}$ rad	Transverse displacement, cm	Number of dark "rings" in pattern if $\theta_0 = 0.03$ rad, $\Delta\theta = \pm 0.003$ rad, for a bright center spot
0.0	0.020	0.0182	-0.00003	0.0
	0.027	0.060		
	0.030	0.092	-0.0001	
	0.033	0.135		
	0.040	0.291	-0.00024	
0.1	0.020	4.90	-0.0040	0.72
	0.027	8.96	-0.0055	
	0.030	11.08	-0.0061	
	0.033	13.43	-0.0067	
	0.040	19.83	-0.0082	
0.2	0.020	9.78	-0.0080	1.40
	0.027	17.86	-0.0109	
	0.030	22.07	-0.0121	
	0.033	26.73	-0.0133	
	0.040	39.14	-0.0162	
0.3	0.020	14.67	-0.0120	2.05
	0.027	26.77	-0.0163	
	0.030	33.06	-0.0181	
	0.033	40.03	-0.0199	
	0.040	58.90	-0.0242	
-0.1	0.020	-4.86	0.0038	0.68
	0.027	-8.84	0.0053	
	0.030	-10.90	0.0059	
	0.033	-13.16	0.0065	
	0.040	-19.24	0.0078	
-0.2	0.020	-9.75	0.0080	1.36
	0.027	-17.74	0.0107	
	0.030	-21.88	0.0119	
	0.033	-26.46	0.0131	
	0.040	-38.78	0.0158	

the four-phototube system should not be circular, but should be as shown in Fig. 6. Column 5 of Table 1 shows the variation of phase shift in this case. Experimentally, however, one can easily attain a circular ring pattern by a very slight tilt of either mirror.

There are two kinds of "contrast" to be distinguished in this discussion. The first is visual contrast, which is defined relative to the ratio of inten-

sities in the bright and dark regions of the ring pattern of Fig. 6, for a stationary test object. It is affected by interferometer alignment, equality of the intensity of the two beams leaving the beamsplitter at point Y (Fig. 1), the size of the transverse displacement y relative to the size of the speckle pattern in the case of spatially incoherent light, and by the size of the laser-illuminated spot on the test object.

The second kind of contrast is velocity contrast, which relates the maximum and minimum phototube signal intensities caused by a monotonically increasing velocity of the test object. It is affected by all the things that can affect visual contrast, as well as several new effects. These are velocity dispersion of the test object (different parts moving at different speeds), and just how the VISAR is operated. If the phototube views the entire ring pattern, then velocity contrast is lower than if the phototube views only the central part of the pattern. Zwick and Shepherd⁷ present a formula for calculating the contrast reduction as a function of e and maximum angle for the case of uniform light illumination out to a definite angle θ_m at the input to the interferometer. When the entire interference pattern consists of only one bright and one dark ring, the velocity contrast can be quite good. This mode of operation seems to be worth using because the intensity of the light at the photocathode is much higher than when the latter views only the central portion of the pattern. Some additional velocity contrast degradation occurs from the finite Doppler-shift variation due to the angular acceptance of the collecting lens of Fig. 1, but this effect is usually very small.

As the test object moves, increasing the angular divergence and the number of rings, the velocity contrast will decrease if the entire pattern is integrated by the photocathode. In addition, there is a small velocity error, which is difficult to calculate. The most practical approach is to use a device that physically moves the test object by the amount expected during the experiment, and to record the four detector signals over this range of distance (with essentially no Doppler-shift in frequency). This usually involves using different phototubes than those used for the actual velocity-measuring experiment, since the latter normally won't work in a dc mode. The device moving the test object must move it the full distance in a time small enough that thermal drift of the delay time τ_0 is negligible, since the analysis of Sec. C assumes that this drift is zero during the experiment. This velocity error caused by angular divergence changes should be essentially zero if the phototube views only the very central part of the interference pattern.

The analysis of Sec. C assumed that these effects of the off-axis rays can be incorporated into the unknown time-dependent contrast factor. This seems reasonable, keeping in mind that with the

spatially integrating mode of recording, there is some small velocity error associated with target travel distance, which is denoted the depth-of-field error. This should be measured or a limit placed on its magnitude prior to the experiment.

B. ORDER OF INTERFERENCE AND THE CONVERGENCE OF THE RING PATTERN

The order of interference as a function of e and θ is given simply by

$$\text{Order} = \frac{c\tau}{\lambda} \quad (39)$$

where the time delay is given by Eq. (38). Thus

$$\begin{aligned} \text{Order} = \frac{2L_E}{\lambda} \left(n - \frac{1}{n_0} \right) - \frac{2e}{\lambda} + \frac{e\theta^2}{\lambda} + \frac{\theta^4}{\lambda} \\ \times \left[\frac{L_E}{4n} \left(1 - \frac{1}{n^2} \right) - \frac{e}{12} \right] \end{aligned} \quad (40)$$

Using typical values for $L_E = 5$ cm and $n = 1.52$, this becomes

$$\frac{8.62}{\lambda} - \frac{2e}{\lambda} + \frac{e\theta^2}{\lambda} + \frac{\theta^4}{\lambda} \left(0.466 - \frac{e}{12} \right) \quad (41)$$

The mirror offset e is usually such that its magnitude is less than 0.3 cm, and the angles θ being considered are less than 0.035 rad. With these restrictions, the order of interference increases as θ increases, provided that $e \geq 0$. If e is more negative than -0.0012 cm, then the order of interference decreases with increasing θ . This means that referring to Fig. 2, if one views the stationary interference ring pattern with a card and then pushes on either mirror A or B so as to increase e , different things will happen depending upon the sign of e . Increasing e (from a positive value to a more positive value or from a negative to a less negative value) causes the order of interference at any fixed angle to decrease, because the second term in Eq. (41) will always dominate. In more physical terms the time delay of the VISAR decreases as e increases, as is evident directly from Fig. 2, visualizing mirror B being pushed toward the beamsplitter. During this

motion, dark rings in the interference pattern that correspond to a *fixed* order will change their angle in whichever way tends to increase the order of interference, since the order at a fixed angle is decreasing. If $e > 0$, the rings will increase in angle, i.e., diverge from the center of the pattern. If e is less than -0.0012 cm, the rings will converge. Zwick and Shepherd⁷ illustrate a calculation of this result for small values of e and over a large range of angles.

The range of e between 0 and -0.0012 cm is one where the order of interference can either increase or decrease as θ increases, depending upon the value of θ . However, for angles between 0 and 0.035 rad, the order varies by no more than 0.002 fringes! Thus, for practical purposes there is no ring pattern observable for such a small value of e . The third term in Eq. (41) contains most of the dependence of the order upon θ , and it is essentially correct to estimate the maximum mirror offset e_m such that there is a variation of only one-half in the order of interference between $\theta = 0$ and $\theta = \theta_m$ by the relation

$$e_m = \frac{\lambda}{2\theta_m^2} \quad (42)$$

For $\theta_m = 0.035$ rad, this maximum offset becomes 0.02 cm. For the three-phototube VISAR, where the beam centroid may have $\theta = 0$, the maximum angle used may be only 0.02 rad. In this case, if the center of the pattern is a bright maximum, then the first dark ring will occur at $\theta = 0.02$ rad if the mirror offset is 0.064 cm. For smaller values of e , there will be no dark rings in the pattern. For the four-phototube system, where the beam centroid angle is not zero, an example of the number of rings for various values of e and θ is given in Table I.

C. REAL VISAR SYSTEMS

The preceding discussions describe what should happen to a VISAR constructed of perfect optical components. Several differences between

our systems and the description above have become apparent. The first is that the interference ring pattern usually contains more dark rings than the theory would predict. It does not seem to be possible to attain the ideal pattern where the entire interference pattern is of constant phase (all bright, for example) corresponding to the condition where e is very small. This is especially true with the four-phototube system. A striking difference becomes apparent when one tries to make the rings converge and diverge by physically pushing on one of the mirrors. For spatially coherent light, if one pushes on a mirror such as to decrease τ , the rings can be made to converge or diverge, depending upon the value of e . However, for spatially incoherent light, good contrast fringes can be had only for the case where decreasing τ causes the rings to converge. It is possible to change e such that decreasing τ causes the rings to diverge, but the fringe contrast for this configuration is so poor that the fringes are barely discernible. This effect has been observed on two completely independent VISAR systems. The theory of Sec. B is that as e passes through zero, rings that previously converged should now begin to diverge, so that $e = 0$ should effectively be a cross-over position. Our measurements, however, show that for certain optical delays, the cross-over point and the calculated $e = 0$ position differ by about 1 cm. The reason for this is not yet known.

Despite this puzzle, the velocity sensitivity of the VISAR is determined by the time delay τ , which is most easily determined experimentally. Using a white light and a green filter, one can find the temporally incoherent fringes (where τ must be very close to zero) and then insert the required glass delay cylinder. One mirror is then moved to get good contrast and the new value of τ is then easily calculated. Our measurements confirm to about 0.5 mm that the white-light fringe position does indeed correspond to $\tau = 0$. An uncertainty of 0.5 mm in e corresponds to an uncertainty of less than 1% in the velocity sensitivity K of Eq. (16) for delay cylinders longer than 6 cm.

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