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By

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MARCH 1983

PLASMA
PHYSICS
LABORATORY

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MASTER

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CNO-3073.

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ABSTRACT

A reduced set of resistive MHD equations is solved numerically in three dimensions to study the stability of tokamak plasmas. Toroidal effects are included self-consistently to leading and next order in inverse aspect ratio, ϵ . The equations satisfy an energy integral. In addition, the momentum equation yields the Grad-Shafranov equation correct to all orders in ϵ . Low beta plasmas are studied using several different q-profiles. In all cases, the linear growth rates are reduced by finite toroidicity. Excellent agreement with resistive PEST is obtained. In some cases, toroidal effects lead to complete stabilization of the mode. Nonlinear results show smaller saturated island widths for finite aspect ratio compared to the cylindrical limit. If the current channel is wide enough so as to produce steep gradients towards the outside of the plasma, both the finite aspect ratio cases and cylindrical cases disrupt.

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I. INTRODUCTION

The equations of magnetohydrodynamics can be expanded in powers of the inverse aspect ratio, ϵ . The resulting reduced equations, to leading order in ϵ , [$O(\epsilon^2)$], have been used extensively to study the linear and nonlinear resistive and/or ideal stability of finite beta [$\beta \sim O(\epsilon)$] plasmas.¹⁻⁴ To this order in ϵ , toroidicity enters only through finite beta effects. The initial value code, HIB, numerically advances these equations in three spatial dimensions and time. Previous work showed reasonable agreement between HIB computations and experimental results from PDX.² Encouraged by this agreement, we have attempted to improve our model by extending the reduced equations to next order in ϵ [$O(\epsilon^3)$]. In addition, we have taken $\beta \sim O(\epsilon^2)$, a more realistic ordering for present day tokamaks. With this scaling for β , pressure independent toroidal effects enter at the same order as ballooning or pressure-curvature effects. Therefore, in addition to the usual nonlinear and finite beta mode coupling we can also study toroidal mode coupling. When beta is set to be of order ϵ in these third order equations, they go over uniformly to the high beta equations used previously. Hence we use the name HILO for the initial value code which advances these third order equations in time.

The third order equations satisfy an energy integral similar to the original high beta equations. The third order momentum equation also yields the Grad-Shafranov equation correct to all orders in ϵ . A similar set of equations has been used elsewhere⁵ to study toroidal effects on resistive tearing modes. Our equations differ in the manner in which the toroidal field is treated. Previously, B_T was modeled as a vacuum field with the usual I_0/R dependence where I_0 is a constant and R is the major radius coordinate. By including third order terms self-consistently, we find $B_T \sim (I_0 + \tilde{I})/R$ where $\tilde{I} \sim \epsilon^2 I_0$ varies with both position and time and describes the paramagnetism or

diamagnetism of the plasma. This effect leads us to different conclusions from Ref. 5 regarding the effects of toroidicity.

The third order equations have been applied to zero beta plasmas using several different q -profiles. During the linear phase there is a strong coupling of the unstable mode to nearby m -numbers. (We use m and n as the poloidal and toroidal mode numbers, respectively.) The toroidal coupling yields a reduction in the linear growth rates of the 2/1 ($m=2$, $n=1$) tearing mode for each of the q -profiles studied. For slowly growing modes, toroidal effects lead to complete stabilization. The results from the initial value code have been compared with the resistive PEST code which solves the linearized resistive MHD equations in toroidal geometry as a boundary layer problem.⁶ Excellent agreement of computed growth rates at large values of S , the ratio of the resistive time to the poloidal Alfvén time, is obtained using the two different codes.

A nonlinear analysis has been performed for q -profiles with $q(0) = 1.1$ and $q(a) = 3.9$. The width of the current channel is varied. In following the nonlinear evolution we have obtained saturated states with islands of multiple helicities present, as well as disruptive plasmas. The latter are characterized by strong nonlinear interactions of all modes and overlapping of islands. The effect of toroidicity in the nonlinear calculations is such as to decrease the saturated 2/1 island width for nondisruptive cases. For cases which disrupt in the cylindrical limit, the overall effect of toroidicity is a drastic slowing down of the growth of the mode but very little change in the equilibrium current profiles that can be operated without disruptions.

In Sec. II we derive the third order reduced equations and discuss the associated numerical techniques. Results from the initial value linear calculations are presented, with comparisons to resistive PEST in Sec. III.

The nonlinear evolution of tearing modes is described in Sec. IV, and our conclusions are stated in Sec. V.

II. DERIVATION OF THIRD ORDER EQUATIONS AND NUMERICAL METHODS

Our derivation is essentially the same as Ref. 7 with minor differences. Here we sketch the procedure. We begin with the resistive MHD equation in rationalized electromagnetic units.

$$\frac{dp}{dt} = -\rho \nabla \cdot \vec{v} , \quad (1)$$

$$\rho \frac{d\vec{v}}{dt} = -\nabla p + \vec{j} \times \vec{B} , \quad (2)$$

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} , \quad (3)$$

$$\frac{dP}{dt} = -\gamma P \nabla \cdot \vec{v} , \quad (4)$$

$$\vec{E} = \eta \vec{j} - \vec{v} \times \vec{B} , \quad (5)$$

$$\vec{j} = \nabla \times \vec{B} . \quad (6)$$

Here ρ is the mass density, $\vec{v}$ is the fluid velocity, $\vec{j}$ is the plasma current, $\vec{B}$ is the magnetic field, p is the thermodynamic scalar pressure, γ is the ratio of specific heats, and η is the plasma resistivity. The usual cylindrical coordinate system, (R, θ, Z) , is used (see Fig. 1). As an expansion parameter we choose the inverse aspect ratio, $\epsilon \sim a/R_0 \ll 1$, where R_0 is the major radius of the device. Following a low beta tokamak ordering we write

$$\rho, B_\phi, v_\perp \sim O(\epsilon^0),$$

$$\vec{v}_\perp, \vec{B}_\perp, \frac{\partial}{\partial t}, \vec{B} \cdot \nabla, 1/R \sim O(\epsilon^1),$$

$$P, v_\phi, 1/R \nabla (RB_\phi), \nabla \cdot \vec{v}_\perp \sim O(\epsilon^2).$$

The subscript "1" denotes components perpendicular to ϕ . The toroidal field is written as

$$B_\phi = \frac{I_0 + \tilde{I}}{R},$$

with $I_0 = \text{constant}$ and $\tilde{I} \sim \epsilon^2 I_0$. Combining this form of B_ϕ with $\nabla \cdot \vec{B} = 0$ yields

$$\vec{B}_\perp = R_0 \nabla \psi \times \nabla \phi + \vec{B}_{3\perp}$$

where ψ is a stream function and $\vec{B}_{3\perp}$ satisfies

$$\nabla_\perp \cdot \vec{B}_{3\perp} = -\frac{1}{R^2} \frac{\partial}{\partial \phi} \tilde{I}.$$

Since $\vec{B}_{3\perp}$ never enters the equations to third order, it suffices to take

$$\vec{B} = R_0 \nabla \psi \times \nabla \phi + (I_0 + \tilde{I}) \nabla \phi + O(\epsilon^3). \quad (7)$$

Combining the ϕ component of Faraday's law with the above expression for $\vec{B}$ yields a relation for the poloidal component of velocity,

$$R^2 \vec{v}_\perp \cdot \left(-\frac{\vec{v}_\perp}{R^2} \right) \sim O(\epsilon^3) .$$

Again using a stream function, we have

$$\vec{v}_\perp = \frac{R^2}{R_0} \nabla U \times \nabla \phi + O(\epsilon^3) . \quad (8)$$

This is the familiar result that, to leading order, the fluid velocity perpendicular to the magnetic field is simply the $\vec{E} \times \vec{B}$ drift. In this manner the velocity stream function U can be related to an electrostatic potential. Substituting for $\vec{B}$ and $\vec{v}_\perp$ in Faraday's law and considering only the poloidal components, we obtain

$$\frac{\partial \phi}{\partial t} = \frac{R}{R_0} \left\{ (\nabla \phi \times \hat{\phi} \cdot \nabla) U + \frac{B_0}{R} \frac{\partial U}{\partial \phi} - \eta J_\phi \right\} + O(\epsilon^4) . \quad (9)$$

The fourth order terms involve $\vec{v}_\phi$ and $\tilde{I}$. The toroidal current is given by Ampere's law as

$$J_\phi = \frac{-R_0}{R} \Delta^* \phi \quad (10)$$

where

$$\Delta^* \equiv R \frac{\partial}{\partial R} \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2} .$$

To follow the time dependence of the velocity stream function to third order self-consistently, one must include $\tilde{I}$ -terms. However, by using an appropriate annihilation operator, we remove any explicit dependence on $\tilde{I}$. In

particular, we take the ϕ -component of the curl of the momentum equation multiplied by R^2 ,

$$\begin{aligned} \hat{\phi} \cdot \nabla \times R^2 \rho \frac{d\vec{v}}{dt} &= \hat{\phi} \cdot \nabla \times R^2 (-\nabla p + \vec{j} \times \vec{B}) , \\ \rho_0 \left[\frac{D}{dt} v^2 u - \frac{2}{R_0} \frac{\partial U}{\partial z} v^2 u \right] &= \frac{2 \nabla R \times \nabla p}{R_0} \cdot \hat{\phi} \\ &+ \vec{B} \cdot \nabla \Delta^* \phi + O(\epsilon^4) , \end{aligned} \quad (11)$$

where

$$\begin{aligned} \frac{D}{dt} &= \frac{\partial}{\partial t} + \vec{v}_\perp \cdot \nabla , \\ v^2 &= \frac{1}{R} \frac{\partial}{\partial R} R \frac{\partial}{\partial R} + \frac{\partial^2}{\partial z^2} . \end{aligned}$$

Once again, the fourth order terms involve v_ϕ and $\vec{I}$. Here we take $\rho R^2 = \rho_0 R_0^2 = \text{constant}$ for convenience. With such a choice for ρ and $\vec{v}_\perp$ as in Eq. (8), it is simple to show that to third order the continuity equation is satisfied trivially.

The pressure equation is

$$\frac{Dp}{dt} = O(\epsilon^4) \quad (12)$$

with v_ϕ entering only in fourth order. In this work, since we consider purely resistive tearing modes, the pressure is set to zero. Finite beta results will be reported in the near future. This leaves us with two coupled, partial differential equations for two unknowns ψ and U . We repeat the equations with pressure omitted;

$$\rho_0 \left\{ \frac{D}{dt} \nabla^2 U - \frac{2}{R_0} \frac{\partial U}{\partial z} \nabla^2 U \right\} = \frac{R_0}{R} \{ \nabla \psi \times \hat{\phi} + B_0 \hat{\phi} \} \cdot \nabla \Delta^* \psi, \quad (13)$$

$$\frac{\partial \psi}{\partial t} = \frac{R}{R_0} \{ (\nabla \psi \times \hat{\phi} + B_0 \hat{\phi}) \cdot \nabla U \} + \eta \Delta^* \psi. \quad (14)$$

Toroidal effects enter through R/R_0 , R_0/R and the ∇^2 and Δ^* operators. The nonlinear term which appears on the lefthand side of Eq. (13) is needed for energy conservation. Note that at equilibrium the momentum equation yields the Grad-Shafranov equation to all orders in ϵ . The boundary conditions are for a rigid conducting wall, i.e., $U=0$, $\psi = \text{constant}$.

An energy integral is obtained by multiplying Eq. (11) by U and combining with Eqs. (9) and (12). After rearrangement of terms we get

$$\begin{aligned} U \frac{\partial}{\partial t} \nabla^2 U &= \nabla \cdot \{ U \Delta^* \psi \vec{B} - U \nabla^2 U \vec{V} \} - \Delta^* \psi \left(\frac{R_0}{R} \right)^2 \frac{\partial \psi}{\partial t} \\ &+ \nabla \psi \cdot \left\{ \nabla \times \frac{U R^2}{R_0} \nabla P \right\} - \frac{\partial P}{\partial t}. \end{aligned}$$

Integrating over the toroidal volume, invoking Gauss' and Stokes' theorems in conjunction with the boundary condition for ψ and U , we obtain

$$\frac{\partial}{\partial t} \int \frac{1}{2} \left\{ (\nabla_{\perp} U)^2 + \left(\frac{R_0}{R} \nabla_{\perp} \psi \right)^2 - 2P \right\} d\tau \leq 0,$$

where the inequality is valid for the resistive equations and the equality holds for the ideal equations.

The code, HILO, is written in terms of the toroidal coordinate system (r, θ, ϕ) . The equations are advanced by finite differencing along the minor radius r and Fourier expanding in the poloidal and toroidal angles θ, ϕ . Three calculations are most frequently performed by the code. It is used to

compute an ideal axisymmetric equilibrium by repeatedly removing kinetic energy from the system until a minimum energy state is achieved. Both resistive and ideal linear stability analyses are performed by finding the fastest growing mode. Finally, the complete set of nonlinear resistive equations can be advanced. The code is capable of including a vacuum region around the plasma, thereby operating in a free boundary mode. The work presented here is for fixed boundary plasmas. In order to keep the equilibrium from resistively decaying during the nonlinear phase, the plasma resistivity is prescribed as $\eta(r, \theta) = C/\Delta^* \phi(r, \theta)$ where $\Delta^* \phi(r, \theta)$ is the original equilibrium current.

Typically, 100-200 radial grid points are used. For linear calculations, six to nine poloidal modes suffice. Nonlinear runs require a minimum of sixty modes (12 poloidal modes for each of five toroidal modes) to compute saturated states. For cases where there is a strong nonlinear interaction between modes of different helicities, as many as 100 modes are used. Convoluting variables is the most time consuming part of the calculation. Finite aspect ratio computations require ~20% more computer time than the cylindrical version of the code. While many additional convolutions are required for $\epsilon \neq 0$, they are all of a simple nature involving one variable with all (m, n) and another with only $(m, 0)$ components.

Since the computational grid in our calculation is a fixed toroidal system, the plasma equilibrium is shifted with respect to this grid. The only wave contained in the reduced equations is the shear Alfvén wave. Due to toroidal shift of the magnetic surfaces, a component of the wave can propagate across a radial grid. This restricts our time step. The larger the shift, the tighter the restriction. Typically, $1/(\max |m|) \gg \Delta t$ where Δt is in poloidal Alfvén units.

III. LINEAR STABILITY ANALYSES

We have studied the linear stability of resistive tearing modes and the effects of finite toroidicity. Values of the inverse aspect ratio up to 0.2 were considered during the linear studies. The safety factor profile, $q(\psi)$, was held fixed for each sequence of aspect ratios. Three different q -profiles were considered spanning a broad spectrum of parameter space. We varied the current channel width, q on axis, and q at the plasma edge. Table 1 summarizes this information. Throughout this paper, numerical values of S are computed as the ratio of the resistive time to the poloidal Alfvén time at the singular surface.

In Fig. 2 we plot the linear growth rate for the $n=1$ mode in units of the poloidal Alfvén time as a function of the inverse aspect ratio for cases 2 and 3. This plot illustrates the general result for the linear runs, that independent of the details of the q -profile, the principle effect of finite toroidicity is to reduce the growth rate of the resistive tearing mode.

In our attempts to compute growth rates as accurately as possible, convergence studies were performed in relation to the number of modes and radial grid points. Empirically, we found that when the smallest (m,n) component of the toroidal current has a maximum value two orders of magnitude smaller than the maximum value of the largest (m,n) component, enough modes have been included. Radially, seven to ten grid points across the current perturbation were found to be sufficient (Fig. 3).

The results from HILO were compared with the linear eigenmode results of resistive PEST.⁶ In the resistive boundary layer, resistive PEST integrates the equations of Glasser, Greene, and Johnson⁹ which are derived at finite beta. A re-examination of the derivation of these layer equations showed that

the equations remain valid for zero beta plasmas, except that the coefficient M needs to be modified. Good agreement is obtained when the modified layer equations are integrated. One expects the agreement to improve with increasing S since the layer equations are an expansion in $(1/S)^{1/3}$. Figure 4 shows the very good agreement we have obtained in the linear growth rates computed from the two methods. The growth rates are normalized to the value at $\epsilon=0$. Figure 5 shows the absolute convergence of the PEST and HILO results as a function of S for ϵ equal to 0.0 and 0.05. The PEST results show that for $\epsilon \sim 0.32$, the tearing mode in case 1 is completely stabilized. As the aspect ratio is decreased, the increase of the toroidal coupling requires the inclusion of a greater number of modes. In addition, the equilibrium shift increases with ϵ . These factors lead to stringent restrictions on the allowable time step. Coupled with the fact that resistive modes grow as $\gamma \sim S^{-5/3}$, we have found that $S \gtrsim 5 \times 10^5$ calculations are not practical. The good agreement obtained between PEST and HILO supports our contention that we have treated the perturbed part of B_ϕ properly.

IV. NONLINEAR RESULTS

In the nonlinear calculations, a more general q -profile is considered. For comparison, our q -profiles are parameterized as in Ref. 5. For $\epsilon=0.0$ we take

$$q(r) = q(0) \{1.0 + r^{2\lambda} [(\frac{q(a)}{q(0)})^\lambda - 1.0]\}^{1/\lambda}.$$

and calculate $q(\psi)$. This fundamental form is fixed as ϵ is varied. There are three free parameters, $q(0)$, $q(a)$, and λ . Increasing λ corresponds to a

flattening of the q-profile and a wider current channel. It can be easily shown that $\lambda = 1$ and $\lambda = 4$ are the peaked and flat profiles used above in the linear calculations. We have considered profiles with $q(0)$ and $q(a)$ fixed at 1.1 and 3.9 respectively, while varying λ . Three types of discharges have been considered with the following results for the cylindrical limit ($\epsilon=0$). The first ($\lambda = 1.5$) is one in which the 2/1 island grows to a finite width in a well-defined saturated state where there is no island overlap. The second ($\lambda = 2.5$) is one in which the 2/1, 3/2, and 5/3 islands overlap, resulting in a wide region of stochastic fields. The core of the plasma and outside edge contain good magnetic surfaces. The nonlinear interaction between different helicities is much stronger than the $\lambda=1.5$ case. The final discharge analyzed has $\lambda=3$ and ends in a hard disruption. Islands overlap more rapidly and result in an extremely strong nonlinear interaction and a destruction of most of the magnetic surfaces.

Each of the three discharges has been run with an aspect ratio of four yielding the following results. In all cases, the nonlinear growth of the 2/1 island is drastically slowed by finite toroidicity. The 2/1 island width is plotted versus time in Fig. 6 for $\lambda = 1.5$. Toroidal effects decreased the size of the saturated island by ~ 12% as shown in Fig. 7. Toroidal effects produce a wider island on the inside of the tokamak (smaller major radius) since the toroidal shift results in a smaller value of $\partial^2\psi/\partial r^2$ on the inside. When the q-profile is flattened ($\lambda=2.5$), the nonlinear evolution is characterized by island overlap and a stochastic magnetic field pattern. While toroidicity slows the overlapping process, the final state is similar to that obtained for $\epsilon=0$ (Fig. 8).

For $\lambda = 3$, finite aspect ratio also results in islands overlapping; however, the evolution to disruption is quite different from the $\epsilon=0$ case. For

infinite aspect ratio, the 3/2 island appears at $t=100$ and growth of the 2/1 and 3/2 occur for the next 200 poloidal Alfvén times (Fig. 9). At this point the islands start to overlap and nonlinearly drive other helicities quite strongly, resulting in almost a complete loss of good magnetic surfaces (Fig. 10). When the aspect ratio is reduced to four, the 3/2 islands once again appear after 100 poloidal Alfvén times. However, both the 2/1 and 3/2 grow at a much slower rate than in the cylindrical limit. Indeed, the 2/1 island appears to be saturating after 500-700 τ_{Ap} . Eventually the 3/2 and 2/1 do overlap and the same explosive growth is observed (Fig. 11).

V. CONCLUSIONS

For all types of equilibria studied, toroidal effects reduce the linear growth rate of resistive tearing modes in contrast to the results of Ref. 5. In cases that are marginally unstable, toroidicity can completely stabilize the mode. Results from the initial value code give good agreement with resistive PEST in the limit of large S .

The effects of toroidicity on the nonlinear evolution are as follows. Finite aspect ratio effects do slow the nonlinear growth of islands and reduce the saturated island widths. However, if rational surfaces are very close ($\lambda > 2.5$) the islands of different helicity do eventually overlap and cause explosive growth. Thus, we conclude that finite aspect ratio causes a minimal change in the flatness of the current channel that can be tolerated without a disruption.

ACKNOWLEDGMENTS

The authors wish to thank their colleagues at PPL, in particular M. Okabayashi and M. Reusch, for many enlightening discussions. This work supported by U.S. Department of Energy Contract DE-AC02-76-CHO-3073.

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FIGURE CAPTIONS

- FIG. 1. Toroidal coordinate system (r, θ, ϕ) used in the initial value code HILO.
- FIG. 2. Linear growth rates for the $n=1$ eigenmode from HILO for equilibrium cases 2 and 3 as described in Table 1.
- FIG. 3. Resolution of the $(m=2, n=1)$ current perturbation along the radial coordinate.
- FIG. 4. Comparison of linear growth rates for the $n=1$ eigenmode obtained from the initial value code, HILO, and resistive PEST for equilibrium cases 1 and 2 as described in Table 1. The growth rates are normalized to the $\epsilon=0.0$ value.
- FIG. 5. Absolute comparison of linear growth rates for the $n=1$ eigenmode computed by resistive PEST and HILO for a) $\epsilon = 0.0$ and b) $\epsilon = 0.05$ as a function of S . The equilibrium is case 1 as described in Table 1.
- FIG. 6. Nonlinear growth of the $(m=2, n=1)$ island for $q(0)=1.1$, $q(a)=3.9$ and $\lambda=1.5$ with $S=10^{-4}$ at the singular surface. The cylindrical result saturates at $W_{2/1} = 0.17a$, while a slower growth and smaller island, $W_{2/1} = 0.15a$, is obtained for $\epsilon=0.25$.
- FIG. 7. Magnetic field pattern for nonlinear saturated state for the same case as Fig. 6. a) $\epsilon=0.0$, b) $\epsilon=0.25$.
- FIG. 8. Final state of equilibrium characterized by $q(0)=1.1$, $q(a)=3.9$, and $\lambda=2.5$ showing wide region of stochastic fields due to strong nonlinear interaction of islands of different helicities for a) $\epsilon=0.0$, b) $\epsilon=0.25$.

FIG. 9 Nonlinear growth of the ($m=2$, $n=1$) island for $q(0)=1.1$, $q(a)=3.9$, and $\lambda=3.0$. The cylindrical result (dashed line) shows a rapid growth of the 2/1 island and immediate island overlap. For an aspect ratio of four (solid line), the 2/1 island grows much more slowly but there is an eventual overlap of islands.

FIG. 10. Magnetic field pattern at three different times for the cylindrical case of Fig. 9. Once the 2/1 and 3/2 islands come in contact ($t=300 \tau_{Ap}$), the destruction of the magnetic surfaces occurs within $300 \tau_{Ap}$.

FIG. 11. Magnetic field pattern at four different times for the same equilibrium as Fig. 9 with $\epsilon=0.25$. The growth of the 2/1 island is drastically slowed by finite aspect ratio effects. The 3/2 and 2/1 islands do not begin to overlap until $t=555 \tau_{Ap}$.

TABLE 1

PARAMETERS USED IN LINEAR ANALYSIS

	<u>CASE 1</u>	<u>CASE 2</u>	<u>CASE 3</u>
$q(0)$	1.1	1.1	1.6
$q(a)$	2.9	3.9	4.1
q -profile	parabolic $\sim \phi^2$	flat(8)	peaked(8)
ϵ	0.0; 0.05; 0.2	0.0; 0.1; 0.2	0.0; 0.1; 0.2
S	10^4 - 10^6	2×10^4	10^5

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

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Time=100

Time=300

Time=600

Fig. 10

Time = 100

Time = 485

Time = 555

Time = 890

Fig. 11

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