A MINUS-I QUADRUPOLE SYSTEM FOR CONTAINING
ABERRATION-CORRECTION OCTUPOLES

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Abstract

Octupoles may be used to correct the third order spherical aberration of quadrupole transport systems. Crosstalk in the coupling of an octupole placed at a given point causes it to add a term with the wrong sign in the y-channel if it has the right sign in the x-channel, thus severely reducing efficiency. It is often convenient to utilize a special correcting section insertion which is seen as a +I transfer matrix by the first order focussing. Within point-to-point thin lens optics we give two-parameter systems with 16 magnets having locations with large S_x where $S_y = 0$ and vice versa for octupole placement.

The problem is to set up a -I quadrupole system which contains two special locations. At the first, S_x is large and S_y is zero; while at the second the reverse is true. Octupoles are placed at these locations and make corrections free of channel crosstalk. The layout is:

-I System

Section 1

subsection 1A Note: Part B is the same as A but with the order of the elements reversed. Subsection 2 is the same as 1 but with the polarities of the quadrupoles reversed. Section 1 is $X = -I$, $Y = +I$. Section 2 is $X = +I$, $Y = -I$.

part 1Aa
part 1Ab
subsection 1B
part 1Ba
part 1Bb

Section 2

subsection 2A
part 2Aa
part 2Ab
subsection 2B
part 2Ba
part 2Bb

The basic module for the 1Aa part is a five element OFODO sequence of drift spaces and thin lenses and part 1Aa consists of two such modules notated in English and Greek respectively as

1Aa: $\underbrace{\ell P m Q n}_{\text{one module 1Aa}} \quad \underbrace{\lambda \pi \nu \tau \nu}_{\text{one module 1Ab}}$

For example, the second element (P) after the source has transfer matrix

$$\begin{pmatrix} 1 & 0 \\ -P & 1 \end{pmatrix}$$

in the X-channel. Each module is one of the modes:

PP = point-to-point $= \begin{pmatrix} x & 0 \\ x & x \end{pmatrix} R_{12} = 0$
 LP = parallel-to-point $= \begin{pmatrix} 0 & x \\ x & x \end{pmatrix} R_{11} = 0$
 PL = point-to-parallel $= \begin{pmatrix} x & x \\ x & 0 \end{pmatrix} R_{22} = 0$
 LL = parallel-to-parallel $= \begin{pmatrix} x & x \\ 0 & x \end{pmatrix} R_{21} = 0$

For the present model we have chosen

X_{1A}		PL	and	LP
Y_{1A}		PP	and	LL
X_{1Aa}	LL	Y_{1Aa}	LP	and PL
X_{1Ab}	LP	Y_{1Ab}	LP	and PL

This decomposition makes the problem tractable because the one LL zero of X_{1Aa} and the two (PL and LP) zeroes of Y_{1Aa} are easy to solve. By matrix multiplication

$$(LP \text{ and } PL) \cdot (LP \text{ and } PL) = PP \text{ and } LL$$

$$Y_{1Aa} \cdot Y_{1Aa} = Y_{1A}$$

$$LP \cdot LL = LP$$

$$X_{1Ab} \cdot X_{1Aa} = X_{1A}$$

Thus Y_{1A} is automatically PP and LL, while X_{1A} needs to have one condition satisfied ($(X_{1A})_{22} = 0$) to become LP and PL. We should also like to specify, at the end of subsection 1A, the value of $S_x (= [X_{1A}]_{12})$. A count shows that $3+3+1+1 = 8$ restrictions have been put on the 10 parameters in the two modules of 1Aa leaving two (mq and np) free. One finds, with notation

$$a = m Q$$

$$b = \pi \nu$$

the expressions

$$\ell = \frac{b(1-a^2)(2-b^2)S_x}{(1+b)(1-b^2)(1+a+2a^2)}$$

$$P = \frac{(1+a+2a^2)(1-b^2)(1+b)}{2ab(1+a)(2-b^2)S_x}$$

$$m = \frac{2a^2b(2-b^2)S_x}{(1+a+2a^2)(1-b^2)(1+b)}$$

$$Q = \frac{(1+a+2a^2)(1-b^2)(1+b)}{2ab(2-b^2)S_x}$$

$$n = \frac{b(2-b^2)(1+2a)S_x}{(1+a+2a^2)(1-b^2)(1+b)}$$

$$\lambda = \frac{(1-b-b^2)S_x}{(1+b)^2(1+a)}$$

$$\pi = \frac{(1+b)(1+a)}{bS_x}$$

$$\nu = \frac{b^2S_x}{(1+a)(1+b)}$$

$$\tau = \frac{(1+a)}{b(1-b)S_x}$$

$$v = \frac{(1-b) S}{1^2}$$

$$S_y = (Y_{1A})_{12} = \frac{b(1+a)(2-b^2) S_x}{(1-b^2)(1+a+2a^2)}$$

where S_x is the peak of the X-channel sinelike function. The system allows all a, b choices within the limits

$$0 < a < 1$$

$$0 < b < \frac{\sqrt{5}-1}{2} = 0.618034$$

to keep all drift distances positive.

For a trial system it was found

$$(S_x)_{\max} = (S_y)_{\max} = 30 \text{ m}$$

was adequate to keep the octupole strength near 1 T/m. As an example we have chosen

$$a = 0.85 \quad b = 0.20$$

in an attempt to get a small total length

$$L = 4(\lambda + \mu + \nu + \lambda + \mu + \nu).$$

We find (MKS units)

$$\lambda = 0.85973117 \quad \mu = 8.5585586$$

$$P = 0.10263132 \quad \tau = 0.37000000$$

$$m = 4.4767956 \quad \nu = 0.54054056$$

$$Q = 0.18986795 \quad \tau = 0.38541667$$

$$n = 8.36494947 \quad v = 12.978973$$

The resulting cosinlike and sinelike trajectories are listed below

s	C _x	S _x	C _y	S _y
0.0	1.0	0.0	1.0	0.0
0.85973117	1.0	0.8597311	1.0	0.85973117
5.3365268	0.54054056	4.9415152	1.459	5.7315382
13.701476	0.54054056	20.416672	0.0	5.7315382
22.260055	0.54054056	36.250005	-1.4932394	5.7315382
22.800575	0.43243243	30.000000	-1.8861971	6.8778458
35.773548	0.0	30.000000	-1.8861971	0.0

$$X_{1A} = \begin{pmatrix} 0 & 30 \\ -1 & 0 \\ 30 & 0 \end{pmatrix}$$

$$Y_{1A} = \begin{pmatrix} -1.8861971 & 0 \\ 0 & -0.53016733 \end{pmatrix}$$

Octupoles are placed at 1A and 2A. A similar type of problem was solved for a beam rotator.

Alternate solutions are possible. One needs to consider only subsection 1A, which is divided into parts 1Aa, 1Ab, and repetitions as specified above. A part a or b may contain two or three magnets. There are sixteen cases, based on the following matrix multiplication table:

PL·LP = LL	LL·PL = PL
PL·PP = PL	LL·LL = LL
LP·PL = PP	PP·LP = LP
LP·LL = LP	PP·PP = PP

In the list below, N indicates the number of magnets per module. The example₂ given above corresponds to the first line of this list.

X _{Ab}	X _{Aa}	Y _{Ab}	Y _{Aa}	N _{Ab}	N _{Aa}	X _A	Y _A
LP	LL	(PL & LP)		2	2	LP	
LL	PL			2	2	PL	
PL	PP			3	3	PL	
PP	LP			3	3	LP	
PL	PP	(LL & PP)		2	2	PL	
PP	LP			2	2	LP	
LP	LL			3	3	LP	
LL	PL			3	3	PL	
PP	LP	(PL & LP)		3	2	LP	(LL & PP)
LP	LL			3	2	LP	
LL	PL			2	3	PL	
PL	PP			2	3	PL	
LL	PL	(LL & PP)		3	2	PL	
PL	PP			3	2	PL	
PP	LP			2	3	LP	
LP	LL			2	3	LP	

References

1. S. Kowalski and H. Enge, "Beam Rotator", Proc. Fourth Int'l. Conf. on Magnet Technology, CONF-720908, Brookhaven, 1972.
2. Karl Brown has emphasized +I sections as sextupole sacs.