**The Supersymmetric Generalized Modified KdV Hierarchy
and
Odd Minimal Superconformal Field Theories
Coupled to 2D Supergravity. II ***

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We further study the universal equations of the supersymmetric modified KdV (MKdV) hierarchy in its generalized form. We show that these equations describe the dynamical quantum equations of the odd series of $N=1$ minimal (p,q) superconformal field theory coupled to $N=1$ supergravity in particular those unitary series with $p=2k+3$, and $q=2k+1$. The string susceptibility of these models is $\gamma_{str}^{(0)} = -2/2k+1$. We demonstrate explicitly the cases $k=2$; and $k=3$.

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I. Introduction

Non-critical (bosonic) fermionic strings can be of potential interest for understanding the solubility of certain three dimensional models like the Ising model, and equally important gauge theories like QCD in $d=4$ space-time. So far we know that these string models are a consistent coupled system of 2-d quantum (super)gravity and (super)matter in space-time dimensions $d < 1$; $d > 25$ for bosonic strings and $d < 1$; $d > 9$ for the fermionic strings [1]. The multimatrix model [2] (c.f. also [9]) offers a scheme to understand the continuum theory of bosonic strings in $d < 1$ in terms of a set of differential polynomial equations which govern the behavior of the free energy in terms of the coupling constants of the theory. Not only that but these eqs., which fall into the generalized KdV hierarchy, give the critical points of the coupled system. In [2] it is proposed that the matter part of this coupled system is a minimal conformal field theory (CFT). Another proposal was given in [3] which considers the coupled system as a higher spin gravity, W-gravity, theory. A more precise picture is conjectured in [4] in which this W-gravity theory must be topological (c.f [10]). The problem of triangulating superRiemann surfaces makes it unclear how to proceed from the multisupermatrix model to get the universal equations of fermionic strings, and in particular spinning strings in dimensions less than one, however there is some progress along these directions [8]. The generalized (Modified) KdV structure inherited from the multimatrix model can serve as a good starting point for understanding 2-d $N=1$ supergravity coupled to $N=1$ supermatter. In particular, when the supermatter is the minimal (p,q) superconformal field theory (SCFT), we will identify such coupled system as (p,q) spinning string theory.

In a recent paper, part I [5] we proposed the supersymmetric (Generalized) modified KdV (MKdV) hierarchy as the dynamical quantum eqs. of a certain class of spinning strings in $d < 1$ in which $p+q=4k+4$, and the string susceptibility $\gamma_{str}^{(0)} = \frac{-2}{2k+1}$. In particular we showed that the universal eqs. of the MKdV hierarchy describe a Neveu-

Schwarz-Ramond (NSR) string with $p=4k+2$; $q=2$. The Ramond sector comes from the supersymmetry of the MKdV hierarchy. We argued that such a type of spinning string model can only make sense in the presence of quantum gravity and two dimensional local supersymmetry. On the other hand it has been conjectured [6],[7] that the critical points of the even series of $N=1$ (p,q) minimal superconformal field theory (SCFT) coupled to $N=1$ supergravity, in the NS sector, are given by the universal eqs. of the supersymmetric generalized KdV hierarchy. However the universal eqs. of the odd series are still missing and in fact they cannot be obtained from the generalized SKdV hierarchy.

In this paper we will show that the critical points of the odd series are given by the universal eqs. of the (r,n) generalized supersymmetric MKdV (SMKdV) hierarchy in which $r+n = 2k+3$. In particular, we will show that the $(2k+3, 2k+1)$ unitary series correspond to $(r,n) = (k+2, k+1)$. As an illustration, we consider in detail the universal eqs. of the third and the fourth SMKdV hierarchy and demonstrate that they describe the N.S sector of the minimal $(7,5)$ and $(9,7)$ theories respectively.

II. The Odd Superdiscrete Series coupled to $N=1$ Supergravity In $d < 1$

We start with a brief review of the formalism developed in [5]. The Hamilton equations of the MKdV hierarchy are

$$\frac{\partial u_1}{\partial t_k} = \{\mathcal{H}_{2k+1}, u_1\} = \frac{\partial}{\partial x} R_{2k+1}(u_1) \quad (2.1)$$

where $u_1(x)$ is the MKdV flow, $\mathcal{H}_{2k+1}$ are conserved charges, and R_{2k+1} are the Gelfand-Dikii differential polynomials of the MKdV hierarchy [5]. The proposed universal spinning string equation is

$$\{H, u_1(x)\} = 1 \quad (2.2)$$

where

$$H = \sum_{k=0}^{\infty} \alpha_k \mathcal{H}_{2k+1} \quad (2.3)$$

is the full Hamiltonian. The general solution of (2.2) is given by

$$u_1(x) = x^{1/2k+1} \sum_{h=0}^{\infty} C_h x^{-4\{k+1/2k+1\}h} \quad k = 0, 1, 2, \dots \quad (2.4)$$

The specific heat $\theta(\Delta)$ ($\Delta = \lambda - \lambda_c$, is the cosmological constant) with the appropriate rescaling that gives the correct string perturbation expansion is determined as:

$$\begin{aligned} \theta(\Delta) &= N^{-1/2k+1} u_1^2(x) \\ x &= N^{\frac{2k+1}{2k+2}} \Delta; \quad \Delta = \lambda - \lambda_c \end{aligned} \quad (2.5)$$

Using (2.4) we obtain:

$$\theta(\Delta) = \Delta^{-(-2/2k+1)} \sum_{\substack{h=0 \\ N \rightarrow \infty}}^{\infty} A_h N^{-2h} \Delta^{-(2-(\frac{-2}{2k+1}))h} \quad (2.6a)$$

where $\frac{1}{N}$ is the string coupling constant and

$$A_h = \sum_{h'=0}^{\infty} C_{h'} C_{h-h'} \quad (2.6b)$$

From (2.6) it follows that

$$\gamma_{sstr}^{(0)} = \frac{-2}{2k+1} \quad k = 0, 1, 2, \dots \quad (2.7)$$

In [5] we have demonstrated that a string theory with the property (2.7) cannot correspond to a (p,q) minimal CFT coupled to quantum gravity, but instead it corresponds to minimal (p,q) N=1 SCFT coupled to N=1 supergravity whose

$$\gamma_{sstr}^{(0)} = \frac{-2}{\frac{p+q}{2} - 1} \quad (2.8)$$

Hence

$$p + q = 4k + 4 \quad k \in N \quad (2.9)$$

The correlation functions of the theory (2.1) as derivatives of $\theta(\Delta)$ with respect to the thermal parameters α_k were computed. We found that the field operators are the NS operators of the (4k+2,2) spinning string field theory in $d < 1$. Surprisingly, the Ramond

operators come from the supersymmetrization of (2.2). Even though the $(4k+2,2)$ minimal SCFT is inconsistent nevertheless when coupled to $N=1$ supergravity (SG) it does make sense due to the presence of spin $3/2$ in the SCFT multiplet which needs another spin $3/2$ in the SG multiplet for consistent coupling. This is reminiscent of $N=2$ SG in $D=4$ space-time.

II-a. The Lax-Pair Form of the Generalized MKdV Hierarchy

The Lax-pair form of the n^{th} generalized MKdV hierarchy is given by:

$$\frac{\partial L_{(n)}}{\partial t_r} = [L_+^{r/n}, L_{(n)}] \quad r > n \quad (2.10)$$

where L is an n^{th} order differential operator,

$$L_{(n)} = D^n + u_{n-1}D^{n-1} + \dots + u_0. \quad (2.11)$$

The operator $Q_+^r = L_+^{r/n}$ is defined as follows: we take the n^{th} root of (2.11) in the algebra of pseudo differential operator

$$Q = L^{1/n} = D + \frac{u_1}{2} + \sum_{j=1}^{\infty} (-1)^j q_j D^{-j} \quad (2.12)$$

raised to the power r , and project it in the sector of positive powers of the differential derivative operator D . The q_j are functions of u_j and their derivatives. It is straightforward to convince oneself that the MKdV hierarchy (2.1) is nothing but the second generalized MKdV hierarchy with $r=2k+1$. The universal eq. (2.2) is rewritten in the Lax-pair form as

$$\sum_{k=0}^{\infty} \alpha_k [L_+^{2k+1/2}, L_{(2)}] = D \quad n = 2 \quad (2.13)$$

which is drastically different from the KdV hierarchy (cf. M.Douglas in [2] and [3]). It is clear from the previous constructions [5] that the universal eqs. of the supersymmetric generalized MKdV hierarchy will describe an infinite new class of (p,q) minimal $N=1$ SCFT

coupled to N=1 SG and one would naively expect that such eqs. are given by (2.13) for arbitrary r and n . However this is not true in fact the correct answer is

$$\sum_{r=1}^{\infty} \alpha_r [L_+^{r/n}, L_{(n)}] = Q_+ \quad n > 2 \quad (2.14)$$

where Q_+ is defined from (2.12). Eq. (2.14) is the natural choice because a priori there is no reason not to expect $u_1/2$ (being of dimensions one) appearing on the r.h.s. of eq. (2.13). However it happened that the $n=2$ is a special case. It is in this respect that the $(4k+2,2)$ spinning string model is a particularly special model with the properties (2.7) and (2.9). Therefore the supersymmetric extension of (2.14) (which we will discuss shortly) must describe either of the two only remaining possibilities. The even series of N=1 SCFT coupled to N=1 SG, or the odd series. The even series correspond to $p+q = 2(p' + q')$ where p' and q' are relatively prime integers. Inserting this into (2.8) we find that the string susceptibility is nothing but that of (p', q') minimal CFT coupled to quantum gravity whose critical points are given by the universal eqs. of the generalized KdV hierarchy (c.f Douglas in [2] and [3]). Consequently the critical points of the even series of the N=1 SCFT coupled to N=1 SG are described, at least in the NS sector, by the universal eqs. of the generalized supersymmetric KdV hierarchy, this is easy to demonstrate by explicit calculations of the correlation functions. We note that the even series $(2k+2, 2k)$ are immediately ruled out as they do not satisfy (2.9), and (2.7). This brings us to the unavoidable conclusion that the odd series (both p and q are odd) of the N=1 SCFT coupled to N=1 SG must be described by the universal eqs. of the (r,n) generalized SMKdV hierarchy. In particular, we will show that the unitary series

$$p = 2k + 3, \quad q = 2k + 1 \quad n > 2. \quad (2.15)$$

correspond to $(r,n) = (k+2, k+1)$.

The general solution of (2.14) is of the same structure as (2.4) with the asymptotic

behavior:

$$u_1(x) \simeq x^{\frac{1}{r+n-2}} + \dots \quad (2.16)$$

Therefore the specific heat θ behaves as $\Delta \rightarrow \infty$:

$$\theta(\Delta) \sim \Delta^{\frac{2}{r+n-2}} + \dots \quad (2.17)$$

Hence

$$\gamma_{sstr}^{(0)} = \frac{-2}{r+n-2} \quad (2.18)$$

comparing with (2.8) we deduce that the model (2.14) is conditioned by:

$$p+q=4k+4=2(r+n-1) \quad n > 2 \quad (2.19a)$$

i.e

$$r+n=2k+3 \quad n > 2. \quad (2.19b)$$

The expansion of the l.h.s of (2.14) has the structure:

$$\frac{\partial}{\partial x} (G_{(r,n)}^{(r+n-1)} - \frac{x}{2} u_1) + \frac{\partial}{\partial x} R_{(r,n)}^{(r+n-2)} D + \frac{u_1}{2} + \dots \quad (2.20)$$

Comparing with the r.h.s of (2.14) we obtain:

$$\begin{aligned} \sum_{r=1}^{\infty} \alpha_r R_{(r,n)}^{(r+n-2)} &= x \\ \sum_{r=1}^{\infty} \alpha_r (G_{(r,n)}^{(r+n-1)} - \frac{x u_1}{2}) &= \alpha_0 \\ \sum_{r=1}^{\infty} \alpha_r G_{(r,n)}^{(r+n-3)} &= \beta_0 \\ &\dots \\ &\dots \end{aligned} \quad (2.21)$$

..Etc..

where R , and G are differential polynomials of $\{u_i\}$ $i=1, \dots, n-1$, (The superscript on R and G denotes their scaling dimensions) and α_0 , β_0 are constants of integration and Etc.. denotes all the lower dimensional differential polynomial functions G .

II.b. Global Supersymmetry:

We will describe the techniques of Supersymmetry briefly here and we refer to [5] for further details. The superpartner eqs. of (2.21) take the form

$$\begin{aligned}
\sum_{r=1}^{\infty} \alpha_r S_{(r,n)}^{(r+n-5/2)} &= \tau_1 \\
\sum_{r=1}^{\infty} \alpha_r (J_{(r,n)}^{(r+n-3/2)} - \frac{x}{2} \chi) &= 0. \\
\sum_{r=1}^{\infty} \alpha_r (J_{(r,n)}^{(r+n-1/2)} &= \tau_0 \\
&\dots \\
&\text{.Etc..}
\end{aligned} \tag{2.22}$$

The functions $S_{(r,n)}$ and $J_{(r,n)}$ are differential polynomials defined through the SUSY transformations

$$\begin{aligned}
\delta u_i &= \eta_i \chi_i \quad i = 1, \dots, n-1. \\
\delta \chi_i &= \eta_i \partial u_i
\end{aligned} \tag{2.23}$$

and the τ s are anticommuting constants of integrations, however with τ_1 having a non-trivial SUSY transformation:

$$\delta \tau_1 = \eta_1 \tag{2.24}$$

It is of utmost importance to realize that it is the supersymmetry associated with θ (2.5), and not that of the MKdV flow u_1 that is of physical relevance. Let ψ be the superpartner of θ we have

$$\begin{aligned}
\delta \theta &= \eta_1 \psi \\
\delta \psi &= \eta_1 \partial \theta
\end{aligned} \tag{2.25}$$

hence

$$\psi = 2u_1 \chi. \tag{2.26}$$

Since the functions $S_{(r,n)}$ and $J_{(r,n)}$ expressed in terms of θ and ψ are linear in ψ and their derivatives we can simply solve (2.22) for ψ in terms of τ_1 by the following ansatz:

$$\psi(x, \alpha_1, \dots, \alpha_r) = e^{\Theta(x, \alpha_1, \dots, \alpha_r)} \tau_1 \tag{2.27}$$

In [5] we showed that the correlation function of the R field operators of the $(4k+2,2)$ spinning string are determined by the derivatives of Θ with respect to α_r . In this paper we will show that Θ provides some of the missing N.S operators in the odd series of minimal $N=1$ SCFT coupled to $N=1$ SG.

III. The $n=3$ Supersymmetric MKdV Hierarchy

In this subsection we will explore the contents of (2.21), and (2.22) by calculating the next non-trivial example of $n=3$; with $r=4$. From (2.19) we get $p+q=12$ which gives $(p,q)=(10,2)$ and $(p,q)=(7,5)$. The $(10,2)$ model corresponds to the $n=2$ SMKdV hierarchy (the standard hierarchy (2.1)) at the fifth level ($r=5$) as shown in [5]. Therefore *the $(7,5)$ minimal model must correspond to the $n=3$, $r=4$ hierarchy*. It remains to demonstrate this.

The $L_{(3)}$ operator is

$$L = D^3 + \frac{3u_1}{2}D^2 + \frac{3}{2}\left(u_2 + \frac{u_1^2}{4} + \frac{u_1'}{2}\right)D + \frac{3}{4}(u_2' + u_2u_1) + \frac{1}{8}(u_1'' - \frac{u_1^3}{2}) + u_0 \quad (3.1)$$

A tedious but straightforward computations determine the universal eqs. (2.21):

$$R_{(4,3)}^{(5)} + \alpha_1 R_{(2,3)}^{(3)} + \alpha_2 R_{(1,3)}^{(2)} = x \quad (3.2a)$$

$$G_{(4,3)}^{(6)} + \alpha_1 G_{(2,3)}^{(4)} + \alpha_2 G_{(1,3)}^{(3)} = \frac{1}{2}xu_1(1 + \alpha_2) \quad (3.2b)$$

The expressions for the Rs and Gs are given in the appendix. We will compute correlation functions on S^2 thus we delete all derivative terms in (3.2). The normalized correlation function of an operator is by definition:

$$\langle O_r \rangle := \frac{1}{Z} \frac{\partial Z}{\partial \alpha_r} \quad (3.3a)$$

where Z is the partition function and $F = \ln Z$ is the free energy.

From (3.2) one computes the unrenormalized correlation function:

$$\langle O_r \rangle_c := \frac{\partial \theta}{\partial \alpha_r} \Big|_{\alpha_r=0} \quad (3.3b)$$

and this is related to (3.3a) by noting that the specific heat θ is $\frac{\partial^2 F}{\partial \Delta^2}$. (Δ is the cosmological constant) one finds

$$\langle O_r \rangle \sim \Delta^{\gamma_{str}^{(0)}} \langle O_r \rangle_c. \quad (3.3c)$$

Thus one obtains

$$\langle \epsilon \epsilon \rangle \sim \Delta^{-4/5} \quad (3.4a)$$

$$\langle \sigma \sigma \rangle \sim \Delta^{-6/5} \quad (3.4b)$$

where (3.4a) (resp. (3.4b)) is obtained by differentiating twice θ w.r.t α_1 (resp. α_2). From (3.4) we deduce that the scaling dimensions of the operators ϵ and σ are:

$$\Delta_\epsilon = 3/5; \quad \Delta_\sigma = 2/5. \quad (3.5a)$$

The supersymmetric extension of (3.2) as defined in (2.22) produces through Θ in (2.27) a new extra field operator whose correlation function is

$$\langle \rho \rangle_c := \frac{\partial \Theta}{\partial \alpha_2} \Big|_{\alpha_2, \alpha_1=0}, \quad (3.5b)$$

and hence

$$\langle \rho \rho \rangle \simeq \Delta^{-8/5}. \quad (3.5c)$$

which gives

$$\Delta_\rho = \frac{1}{5}. \quad (3.5d)$$

Therefore the operator field content of the supersymmetric (4,3) MKdV hierarchy constitutes of the field operators $(I, \rho, \sigma, \epsilon)$ with scaling dimensions $(0, 1/5, 2/5, 3/5)$. On the other hand, the scaling dimension of the field operators $\phi_{n,n'}$ of the continuum minimal (p,q) N=1 SCFT coupled to N=1 SG is given by [6]:

$$\Delta_{n,n'} = 1 - \frac{(p+q - |np - n'q|)}{p+q-2} \quad (3.6a)$$

$$1 \leq n < q, 1 \leq n' < p. \quad (3.6b)$$

In the N.S sector (resp. R sector) $n - n'$ is even (resp. odd). Since $\Delta_{n,n'} = \Delta_{q-n,p-n'}$ the field operators $\phi_{n,n'}$ and $\phi_{q-n,p-n'}$ represent the same primary field ϕ_Δ . It is straightforward to show that the field operators of the (4,3) SMKdV hierarchy are precisely the N.S operators $\{\phi_{(1,1)}; \phi_{(3,5)}; \phi_{(3,3)}; \phi_{(1,3)}\}$ of the (7,5) minimal model. The NS operators $\phi_{(1,5)}$ and $\phi_{(3,1)}$ with dimensions $8/5$ and $7/5$ respectively are irrelevant operators since their dimension is greater than one which means that they do not contribute for short distance behaviors in the SCFT. This proves that the minimal odd series $(2k+3, 2k+1)$ is indeed described by the universal eqs. of $(k+2, k+1)$ generalized SMKdV hierarchy for $k=2$.

IV. The $n=4$ SMKdV Hierarchy

Now we come to a crucial test for the credibility of the proposal we made above (namely, that the quantum eqs. of $(2k+3, 2k+1)$ unitary $N=1$ SCFT coupled to $N=1$ S.G are described by the universal eqs. of the $(k+2, k+1)$ generalized SMKdV hierarchy). The reason is that the first level of the $n=4$ ($k=3$) SMKdV hierarchy, the $r=5$, gives from (2.19) $p+q=16$. Apart from the solution $(p,q)=(14,2)$ which corresponds to the standard $n=2$ SMKdV at level 7 [5] there remains three more solutions

$$(p, q) = (13, 3) \quad (4.1a)$$

$$(p, q) = (11, 5) \quad (4.1b)$$

$$(p, q) = (9, 7) \quad (4.1c)$$

and one of them is the candidate for the (5,4) SMKdV hierarchy. Of course for our proposal to be true, eq. (4.1c) must be the correct answer. We will skip the technical details as they are unnecessary and very complicated (we give the expression for $L_{(4)}$ in the appendix). Instead we give the results for the computation of the correlation functions on S^2 . Our

calculations are simplified considerably by noting the following behaviors

$$\frac{\partial^2 \theta}{\partial \alpha_i^2} \Big|_{\substack{\alpha_j=0 \\ j \leq i}} \simeq \left(\frac{\partial u_1}{\partial \alpha_i} \right)^2 \Big|_{\substack{\alpha_j=0 \\ j \leq i}} \simeq u_1 \frac{\partial^2 u_1}{\partial \alpha_i^2} \Big|_{\substack{\alpha_j=0 \\ j \leq i}} \quad (4.2)$$

while the superpartner of θ, ψ , expressed in terms of Θ (2.27) has the following behavior:

$$\frac{\partial^2 \Theta}{\partial \alpha_i^2} \Big|_{\substack{\alpha_j=0 \\ j \leq i}} \simeq u_1^{-1} \frac{\partial^2 u_1}{\partial \alpha_i^2} \Big|_{\substack{\alpha_j=0 \\ j \leq i}} \quad (4.3)$$

Therefore to compute the two-point functions on S^2 we only need to know $\frac{\partial u_1}{\partial \alpha_i} \Big|_{\substack{\alpha_j=0 \\ j \leq i}}$. The universal eqs. of the (5,4) SMKdV are:

$$\begin{aligned} R_{(5,4)}^{(7)} + \alpha_1 R_{(3,4)}^{(5)} + \alpha_2 R_{(1,4)}^{(3)} &= x \\ G_{(5,4)}^{(8)} + \alpha_1 G_{(3,4)}^{(6)} + \alpha_2 G_{(1,4)}^{(4)} &= \frac{1}{2} x u_1 (1 + \alpha_2 + \alpha_1) \\ G_{(5,4)}^{(6)} &= \alpha_3 \end{aligned} \quad (4.4)$$

with their superpartner eqs. defined as in (2.22). α_3 is an integration constant. Analyzing (4.4) on S^2 we arrive at the following results:

$$\frac{\partial u_1}{\partial \alpha_1} \Big|_{\alpha_1, \alpha_2, \alpha_3=0} \simeq u_1^{-1} \quad (4.5a)$$

$$\frac{\partial u_1}{\partial \alpha_2} \Big|_{\alpha_1, \alpha_2, \alpha_3=0} \simeq u_1^{-3} \quad (4.5b)$$

$$\frac{\partial u_1}{\partial \alpha_3} \Big|_{\alpha_1, \alpha_2, \alpha_3=0} \simeq u_1^{-5} \quad (4.5c)$$

Note that $u_1 \sim x^{1/7}$ as $x \rightarrow \infty$. The normalized two-point functions follow from the definition (3.3c) with $\gamma_{sstr}^{(0)} = -\frac{2}{7}$ and the second derivatives of θ and Θ with respect to α_i .

Equipped with (4.2), the second derivatives of θ w.r.t. α_i , $i=1,2,3$ give:

$$\langle \epsilon \epsilon \rangle_{\alpha_1} \simeq \Delta^{-4/7}; \quad \langle \sigma \sigma \rangle_{\alpha_2} \simeq \Delta^{-8/7}; \quad \langle \rho \rho \rangle_{\alpha_3} \simeq \Delta^{-12/7} \quad (4.6)$$

and from the Θ derivatives (4.3) we obtain

$$\langle \xi \xi \rangle_{\alpha_1} \simeq \Delta^{-6/7}; \quad \langle \nu \nu \rangle_{\alpha_2} \simeq \Delta^{-10/7}; \quad \langle \mu \mu \rangle \simeq \Delta^{-2}. \quad (4.7)$$

from which we deduce the following scaling conformal dimensions:

$$\Delta_\epsilon = 5/7; \quad \Delta_\sigma = 3/7; \quad \Delta_\xi = 4/7; \quad \Delta_\rho = 1/7; \quad \Delta_\mu = 0; \quad \Delta_\nu = 2/7 \quad (4.8)$$

Therefore the operator field content of the (5,4) SMKdV universal eqs. (4.4) is $(I, \rho, \nu, \sigma, \xi, \epsilon)$ with dimensions $(0; 1/7; 2/7; 3/7; 4/7; 5/7)$. By examining the continuum theory of the models (4.1), using eq. (3.6) we can easily determine that the above field operators as nothing but the N.S operators of the (9,7) minimal $N=1$ SCFT coupled to $N=1$ SG.

Let us denote the spinning string models associated with the universal equations of the n^{th} generalized SMKdV hierarchy by $\Xi_{(r,n)}$ eqs. (2.21-2.22). In conclusion, *We propose the universal equations of $\Xi_{(k+2,k+1)}$ as the dynamical quantum equations of the N.S sector of the unitary $(2k+3, 2k+1)$ minimal $N=1$ SCFT coupled to $N=1$ SG. This would be a special statment of the more general situation where the universal eqs. of $\Xi_{(r,n)}$ describe the quantum eqs. of the odd minimal SCFT coupled to 2d- SG.* We have demonstrated explicitly the cases $k=2$, and $k=3$. It is not clear how to obtain the Ramond sector of these theories, however a possible mechanism is twisting supersymmetry. We hope to come back to this issue in a future article. In [7] we give an alternative proposal of the critical points of $\Xi_{(r,n)}$ as being W-supergravity theories.

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Appendix

$$\begin{aligned} Q_+^4 := L_+^{\frac{4}{3}} = & D^4 + 2u_1 D^3 + (2u'_1 + 2u_2 + u_1^2) D^2 + (2u'_2 + u_1'' + u_1 u'_1 + 2u_2 u_1 + \frac{4}{3} u_0) D \\ & + \frac{5}{6} u_2'' + \frac{1}{12} u_1''' + \frac{1}{2} u_2 u'_1 + u'_2 u_1 + \frac{2}{3} (u_1 u_0 + u'_0) - \frac{1}{24} (u'_1)^2 \\ & - \frac{1}{32} u_1^4 + \frac{1}{2} u_2^2 + \frac{1}{4} u_1^2 u_2 - \frac{1}{8} u_1^2 u'_1 + \frac{1}{12} u_1 u_1'' \end{aligned} \quad (A.1)$$

$$\begin{aligned}
G_{(4,3)}^{(6)} = & \left[\frac{1}{24}u_1^{(V)} - \frac{1}{12}u_2^{(IV)} + \frac{1}{8}(2u_2u_1''' + 2u_2''u_1' + 4u_2'u_1'' + u_2'''u_1) \right. \\
& - \frac{1}{32}(3u_1^2u_1''' + 12u_1u_1'u_1'' + 4(u_1')^3) + \frac{1}{16}(6u_1u_1''u_2 + 6u_1u_1'u_2' - u_1^2u_2'') \\
& + \frac{1}{48}(2u_1u_1^{(IV)} - u_1'u_1'' + \frac{3}{2}(u_1'')^2) - \frac{3}{32}(u_1^3u_1'' + u_1^2(u_1')^2 + \frac{3}{64}u_1'u_1^4 \\
& - \frac{3}{8}(u_1^2u_1'u_2) + \frac{3}{4}(u_2^2u_1') - \frac{3}{4}(u_2u_2'' + \frac{1}{2}(u_2')^2) + \frac{1}{3}u_0''u_1 - \frac{1}{2}u_1u_1'u_0 \\
& \left. + \frac{1}{128}u_1^6 - \frac{1}{2}u_2^3 + \frac{2}{3}u_0^2 + \frac{3}{8}u_1^2u_2^2 - \frac{3}{32}u_1^4u_2 + u_1u_2u_0 - \frac{1}{4}u_0u_1^3 \right] \quad (A.2)
\end{aligned}$$

$$R_{(4,3)}^{(5)} = 2\left[\frac{1}{3}u_0'' - u_0\left(\frac{1}{2}u_1' + \frac{1}{4}u_1^2 - u_2\right)\right] \quad (A.3)$$

$$G_{(2,3)}^{(4)} = \frac{1}{4}[u_2'' - 3u_1'u_2 - \frac{1}{2}u_1u_1'' + \frac{1}{4}(u_1')^2 + 3u_2^2 - \frac{3}{2}u_1^2u_2 + \frac{3}{16}u_1^4] \quad (A.4)$$

$$R_{(2,3)}^{(3)} = \frac{1}{2}[u_1'' - u_1^3] \quad (A.5)$$

$$G_{(1,3)}^{(3)} = [u_0 - \frac{3}{16}u_1^3 + \frac{3}{4}u_1u_2 + \frac{3}{4}(u_2' - \frac{1}{2}u_1'' - u_1u_1')] \quad (A.6)$$

$$R_{(1,3)}^{(2)} = \frac{3}{2}(u_2 + \frac{1}{2}u_1' - \frac{1}{4}u_1^2) \quad (A.7)$$

$$L_{(4)} = Q_+^4 + u_4 \quad (A.8)$$

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