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PROPERTIES OF UNDULATOR RADIATION*

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Properties of undulator radiation

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Abstract

Properties of the radiation emitted by a plane sinusoidal undulator are calculated in the far field approximation. Software has been developed to calculate the spectral distribution and polarization of the radiated intensity $I(E)$ at a point on (or integrated over) a cross sectional observation plane of the photon beam. Spatial distribution of monochromatic radiation and power density contours are also calculated. Spectral broadening caused by an electron beam of finite spatial distribution is considered. Dispersive properties of the photon beam, including the dependence on deflection parameter, are analyzed. It is shown that reasonably constant intensity distribution $I(E)$ can be obtained by properly shaping the beam acceptance aperture.

I. Introduction

In general, the spectral properties of radiation from plane sinusoidal undulators are well understood. (We refer to undulators of constant pole gap and periodicity with magnetic field components $B_x = 0$, $B_z = 0$, and $B_y = B_0 \sin 2\pi z/\lambda_0$ where B_0 is the peak field and λ_0 is the undulator period.) However, radiation is emitted in a complex pattern, with spectral content (energy dependence), intensities, and polarizations varying dramatically over the emitted photon beam cross section. Spectral properties may be further and dramatically altered by undulator design changes that alter the undulator wavelength, number of periods, or magnetic field strength. Thus it is necessary, for the user of undulator radiation, to have access to versatile software that permits ready computation of a wide range of spectral characteristics. This becomes a particularly pressing need when tunable, variable gap undulators are used. In this paper we report the development of software for computing spectral properties of undulator radiation. A variety of spectral characteristics of the radiation in a cross section of the photon beam are considered. The spectral properties are calculated for the far field region using the formalism presented by Hirai, et al.² Software capabilities include computation of:

- A. Intensity as a function of photon energy
 1. At a point (x,y)
 - a. From an ideal (point cross section) electron source
 - b. From a distributed electron source
 2. Integrated over the photon beam cross section (ideal electron source)
- B. Intensity at a constant photon energy as a function of position in the beam cross section (ideal source)
- C. Polarization of $I(E)$ at a point (x,y)
- D. Power density over the beam cross section (ideal electron source).

We demonstrate these spectral characteristics with calculations for an undulator utilizing a 6 GeV electron beam with 100 mA beam current. For illustrative purposes we consider an undulator with 50 periods of 6 cm wavelength. The photon beam will be examined at a distance of 50 m from the insertion device (ID) center. At an observation angle of $\theta = 1/\gamma$ (0.085 mrad), the beam radius is 0.426 cm.

Figure 1 shows the coordinate system used here. An electron moves along the z-axis, undulating in the horizontal (x-z) plane as a consequence of traversing a sinusoidally periodic magnetic field in the (y-z) plane. Photons are emitted at angles θ from the z axis and are analyzed at P, a point in the beam cross section. ϕ designates the angle to P measured from the intersection of the central beam spot (z-axis) and the horizontal plane. We define the deflection parameter K proportional to B_0 , the peak field. That is, $K = 0.934 B_0 \lambda_0$ with B_0 in tesla and λ_0 in cm.

II. Beam power distribution

Figure 2 shows contours of constant total power (in W/sq cm) within a cross section of the photon beam. As indicated in Fig. 2, the deflection parameters are 0.2, 1.0, and 1.7. Each beam cross section displayed in Fig. 2 is 2 cm on a side. At small K, the beam is significantly squeezed along the x-axis with the highly peaked central spot being somewhat elongated in the vertical direction. A minimum in the power density appears near $x = 0.4$ cm and a weak secondary peak occurs near $x = 0.6$ cm. As K increases, the power

distribution becomes more elongated in the x-direction and no secondary maxima appear.

For $K = 0.2$, the central contours are separated by 2 W/sq cm intervals. The outer contours (starting at contour 2) are separated by 0.1 W/sq cm intervals. The power density on axis is 43.4 W/sq cm. For $K = 1.0$, the central contours are separated by 20 W/sq cm increments while the outer contours are separated by increments of 1 W/sq cm. The power density on axis is 481 W/sq cm. For $K = 1.7$, the central contours are separated by 40 W/sq cm and outer contours are separated by 5 W/sq cm. The on-axis power density is 847 W/sq cm.

The total integrated power traversing the 4 sq cm beam cross sections represented in Fig. 2 is 8.6, 215, and 614 W. These quantities compare closely with the corresponding quantities 8.7, 218, and 629 W obtained from the closed-form expression $P = 7.25 I E_R^2 N K^2 / \lambda_0$, where E_R is the electron energy in GeV, I is the current in amps and N is the number of undulator periods. The radiated power is so highly directed that, in each case, more than 97% of the radiated power traverses the 4 sq cm beam cross section. As K decreases, an increasingly larger fraction of the total radiated power passes through the beam cross section.

Note that, in Fig. 2, where $K = 1$ and 1.7, respectively, the power density (for $\phi = 0$) falls to about half its maximum value at the angle $\theta = K/\gamma$. (Recall that, at 50 m, $1/\gamma$ corresponds to 0.426 cm from the beam center.) However, in Fig. 1, where $K = 0.2$, the power distribution approaches its limiting (small K) value. Further reduction in K does not lead to additional spatial sharpening. At small K , essentially all the radiated energy appears in the first harmonic. In the y-direction (vertical), the half height shows only a weak dependence on K .

III. Spectral properties

A. Integrated spectrum

In conjunction with the study of power distributions across the photon beam (Fig. 2), it is instructive to make a corresponding study of the spatial properties of the undulator radiation. We have seen that, at typical distances between the ID and the monochromator or work station, the beam cross section is approximately 1 sq cm. Before monochromatizing or focussing, it may be desirable to exploit spatial characteristics, selectively admitting radiation from a specific point or region of the beam cross section. Such practices may, for example, be helpful in limiting the heat loading on optical elements.

Figure 3 shows the integrated intensity contributed by the first 5 harmonics of the radiated beam when the deflection parameter $K = 1$. While this display gives a general indication of the spectral character of the radiation, it has limited usefulness to the spectroscopist or instrumentation designer because of the highly dispersive character and spatially variable polarization of the radiation. We remind the reader that spectral properties of undulator radiation are influenced by interference phenomena at all points in the photon beam cross section. For the ideal sinusoidal undulator, radiation through a pinhole located at any point in the beam cross section shows (harmonic) peaks in $I(E)$ that are modulated by a "grating" function.³ As the position of the observation point is changed, amplitudes of the peaks will modulate and the peaks will shift in energy. For an ID with a large number of periods, the "grating" modulation tends to be unimportant since the modulation rapidly smoothes as the pinhole becomes finite in size. Similarly, radiation generated by a synchrotron with a finite size electron beam will also show smoothed-out "grating" fine structure. In the subsequent discussion, we will, for simplicity, consider only spectra in which the "grating" modulation has been smoothed by convolution. A pinhole spectrum, for the point $(x,y) = (2 \text{ mm}, 2 \text{ mm})$ viewed through a 0.1 mm^2 pinhole, is shown in Fig. 4. The polarization of the pinhole radiation, expressed as the difference between horizontal and vertical polarizations divided by the sum, is also shown.

The manner in which radiation of the type displayed in Fig. 4, that occurs at different points of the beam cross section (with modulated peak intensities and energies), is combined to produce the integrated spectrum of Fig. 3 is a matter of considerable interest to the innovative user of the undulator radiation.

B. Dispersive properties - constant K

Figure 5 shows the flux from 5 harmonics passing through a line of pinholes (each 0.1 mm^2 cross section) positioned along the y-axis. Pinholes are separated by 0.5 mm in the vertical direction. Contributions to the total flux are added sequentially beginning at $y = 0$. On the z-axis, only odd harmonics appear (with the fundamental at 3799 eV). As the pinhole is moved from the spot center (increasing θ), even harmonics rapidly develop and the peaks move to lower energies. Of course, the energy of higher harmonic radiation moves more rapidly to lower values as the angle θ increases than does radiation from lower harmonics. Radiation from a 3 mm long slit along the y-axis would yield a smooth spectrum

with the envelopes indicated in Fig. 5 (dashed lines).

A similar summation of the flux from a series of pinholes located along the x-axis yields the spectrum of Fig. 6. Here we illustrate with a more closely spaced line of pinholes (0.2 mm separation) because, unlike Fig. 5, a strong modulation of the peak amplitudes now occurs along the line, a modulation that affects different harmonics differently. This behavior is reminiscent of the results shown in Fig. 2 where power densities are seen to behave monotonically in the y-direction but show a more complex x-dependence.

For some experiments, large and rapid changes in intensity with varying photon energy represent a severe experimental complication. It could, for example, be troublesome for EXAFS (extended x-ray absorption fine structure) experiments where extraneous structure might be introduced in the scanned spectrum. For EXAFS, it is desirable to have an extended spectral range (several KeV is desirable for heavy element studies) that is free of structure. Note in Fig. 6 that, if third (or higher) harmonic radiation were being utilized, amplitude changes of more than two orders of magnitude would be encountered in an energy scan of the spectrum obtained from a 3 mm slit oriented along the x-axis. If the slit length were further increased, some reduction in the variability of the third harmonic amplitude would occur since a substantial overlapping contribution would then come from fourth harmonic radiation originating at large distances from the spot center.

However, Figs. 5-6 suggest that attenuation of an appropriate portion of the beam can leave a spectrum that is generally well behaved over a substantial range in energy. For this purpose, it is sufficient to exclude contributions from the vicinity of the x-axis. To illustrate this point, we consider integrations over 1/4 of the beam cross section; first, in the vicinity of the x-axis and second, in the vicinity of the y-axis. Figure 7 shows the integrated spectrum encompassing 1/4 of the beam (a wedge) along the x-axis. Here, $-\pi/8 < \phi < \pi/8$ and $7\pi/8 < \phi < 9\pi/8$. Like Fig. 6, this spectrum contains undesirably large variations in the intensity as the energy is scanned. Figure 8 shows a wedge encompassing 1/4 of the beam cross section that is centered on the y-axis (i.e., $3\pi/8 < \phi < 5\pi/8$ and $11\pi/8 < \phi < 13\pi/8$). Compared to the integrated spectrum of Fig. 3, significant smoothing has occurred without appreciable loss of intensity. (Of course the position of the radiation shifts along the slit length as the photon energy is scanned.) This spectral "smoothing" scheme is particularly effective when higher harmonic ($k = 3$ or greater) radiation is utilized. Thus, reference to a total integrated spectrum as an exclusive guide to beamline and monochromator design, particularly if attenuating slits or pinholes are utilized, can lead to undesirable spectral characteristics in the photon source. Clearly, more detailed information on source characteristics is needed.

C. Dispersive properties - variation with K

Increasing the deflection parameter K (by increasing the magnetic field or undulator wavelength) results in a compression of the harmonic peaks to lower energy. Also, a greater fraction of the radiated energy is shifted to the higher harmonics. At high K, where harmonic peaks are closely spaced, it is expected that undulator aperiodicity errors (resulting from variations in the magnetic field or undulator period) will act to smooth the spectrum, particularly at the higher photon energies.⁴ Nonetheless, the radiation will still be significantly dispersed across the beam profile, a profile that expands in the x-direction as K increases (see Fig. 2).

Since the smearing caused by aperiodicity errors should be less severe for the low harmonics, it is instructive to examine, for the first few harmonics, the dispersive properties of the radiation under conditions of high and variable K. For this purpose we consider the (superimposed) radiation from a series of slits (concentric arcs 0.1 mm in width subtending a $\Delta\phi = 0.5$ rad angle) positioned along the y-axis. The slits are separated by 0.5 mm. The spectral makeup of the total radiation emerging from the slits, when $K = 0.25, 2,$ and 8 , respectively, is shown in Fig. 9. As K is varied, the number of slits is correspondingly varied such that, for a given harmonic, the same fractional bandpass of radiation is displayed. These figures show the dramatic dependence of the spectral dispersion on the deflection parameter K. At small K, the bandpass is exhausted at 2.5 mm from the beam center. When $K = 8$, however, the same fractional bandpass is contained in a window of about 15 mm radius. The high dispersion at high K is a feature that might be usefully exploited. Note that, for an ideal electron beam traversing an undulator with no aperiodicity errors, the on-axis spectral resolution of radiation passing through a pinhole is given by $dE/E = 1/kN$ where N is the number of undulator periods. In general, a spatially extended and diverging electron beam will reduce this resolution. When the dispersion is large, however, the influence of an extended electron beam in the synchrotron has minimal effect on this pinhole spectrum. On the other hand, for small K, where the dispersion is small, electron beam size effects become important.

Let us consider the third harmonic in the example of Fig. 9. For $K = 0.25$, the dispersion is sufficiently small so that slits separated by 0.5 mm produce peaks shifted by more

than 200 eV. Since the electron beam typically has an effective size (σ_x) close to 0.5 mm, broadening of the pinhole spectrum, by a comparable magnitude, will occur. In contrast, for the case when $K = 8$, the dispersion is so large that a 0.5 mm pinhole or slit separation produces an insignificant change in the transmitted radiation. At the same time, radiation from a source with a 0.5 mm cross sectional dimension will have no significant broadening effect on the pinhole radiation. Thus, a relatively high resolution capability ($dE/E \sim 1/kN$) is available from a pinhole (or arc slit) source without monochromatization if a large number of undulator periods are used (providing that aperiodicity errors are small). This point is quantitatively demonstrated in Fig. 10 where radiation through a pinhole on-axis is calculated for third harmonic radiation with $K = 0.25$. The solid line shows the radiation from an ideal electron source. The dashed line is a calculation of the pinhole flux when an extended (but nondivergent) electron beam with dimensions $\sigma_x = 0.43$ mm, $\sigma_y = 0.07$ mm, is considered. Significant broadening with accompanying reduction of peak intensity is apparent. Further broadening will be introduced when effects of electron beam divergence are included. In contrast, for the case when $K = 4$ (or greater), no discernable broadening of the ideal spectrum is encountered when the finite size of the electron beam is considered. Furthermore, e-beam divergence degrades the (pinhole) resolution when

$$\sigma_{x',y'} > \frac{1}{Y} \left(\frac{1 + K^2/2}{2kN} \right)^{1/2}.$$

Thus, for $K = 4$, e-beam divergence degrades the pinhole resolution when $\sigma_{x',y'} > 0.015$ mrad which is approximately the expected σ_x divergence for the proposed 6 GeV machine. However, for $K = 0.25$, the pinhole resolution is degraded when $\sigma_{x',y'} > 0.005$ mrad. Clearly, in this case, e-beam divergence will add substantially to the loss of resolution already imposed by the finite beam size. The conclusion, then, is that high K operation, permitting exploitation of a highly dispersed beam, provides a pinhole radiation source whose resolution on axis is governed by $dE/E = 1/kN$ providing that aperiodicity errors are sufficiently small.

The high K radiation from the early harmonics necessarily occurs at low energy. This suggests that the 6 GeV machine, in addition to its obvious and well considered utility as a hard X-ray facility, might also provide an excellent radiation source for VUV and soft X-ray experiments. It would appear that VUV-soft X-ray operation might be performed in a parasitic mode, with a thin film serving as a beam splitter to separate the hard and soft radiation. At low energies, reflectivities can be high, even at relatively high reflection angles while hard radiation is transmitted. A beam splitter capable of withstanding the heat loading resulting from residual absorption is needed, however. A special feature of the resulting VUV source is that it is highly dispersed; in the example of Fig. 9 ($K = 8$), by about 15 mm for a 25% band pass.

IV. Spatial distribution of monochromatic radiation

For the ideal undulator, radiation of a particular energy appears in a series of concentric circles centered on the beam axis. Figure 11 shows the intensity as a function of position along the x-axis for the $K = 1$ undulator at 3758 eV photon energy. (At this energy, the first harmonic peak occurs approximately 1 mm off axis -- see Fig. 5.) Peaks occur in Fig. 11 when the concentric circles of monochromatic radiation intersect the x-axis. When considering the spatial distribution of monochromatic radiation, it is instructive to re-examine Fig. 5. Recall that, as pinholes are located at increasingly larger distance from the beam center, the spectra shift to lower energies. At appropriate radial distances, radiation from higher harmonics will occur at the specified energy (3758 eV). These distances are, of course, designated by the series of peaks in Fig. 11. Monochromatic radiation of lower energy will appear in a series of (spatially) concentric circles with larger radii. That is, the pattern of nested circles will expand inward or outward as the selected energy is varied. Of course, intensities vary dramatically around the circumferences of the monoenergetic circles.

Acknowledgment

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Fig. 1. The observation plane of the photon beam perpendicular to the z-axis. Electrons undulate in the x-z plane in response to a magnetic field in the y-direction. A point P in the observation plane is specified by the angles θ and ϕ .

Fig. 3. Integrated intensity as a function of photon energy for an ideal electron beam.

Fig. 2. Contours of constant power density for three values of the deflection parameter K. Contours are separated by constant intervals except at the contours labelled (a) where the contour interval scale changes (see text).

Fig. 4. The spectrum $I(E)$ seen through a pinhole at the point $(x,y) = (2 \text{ mm}, 2 \text{ mm})$ for an ideal electron beam. Polarization is also shown.

Fig. 5. Intensity $I(E)$ through pinholes positioned along the y-axis. The pinhole intensities are sequentially summed starting from $(x,y) = (0,0)$. Peak positions move as the pinhole position is changed.

Fig. 6. Intensity $I(E)$ through pinholes positioned along the x-axis. Again intensities are summed sequentially. Note strong intensity modulations that do not appear in Fig. 5.

Fig. 7. Intensity $I(E)$ integrated in a wedge, one-fourth of the observation plane, that includes the x-axis. Spectra are integrated over a 2.5 mm radius. [The photon beam is schematically represented by the power density contour (a) of Fig. 2.] Severe intensity modulations are observed in $I(E)$.

Fig. 8. Intensity $I(E)$ integrated over one-fourth of the observation plane. Integration is performed over a wedge that includes the y-axis. Within a given harmonic, $I(E)$ varies smoothly.

Fig. 9. Summed intensities $I(E)$ from a series of slits (concentric arcs) positioned on the y-axis for undulators with $K = 0.25$, 2 and 8. In all cases, slits are separated by 0.5 mm. (On-axis radiation is not included.) Spectra are displayed so that the same fractional bandpass is shown for the three undulators. Note that the dispersion is much greater for high K operation.

Fig. 10. Solid line - $I(E)$ for third harmonic radiation through a pinhole on-axis with an ideal electron beam. Dotted line - $I(E)$ when finite beam size is considered (see text).

Fig. 11. Intensity of monochromatic radiation versus position along the horizontal direction (x). Monochromatic radiation appears in circles (constant θ) with intensity variations around the circles.