APPLICATION OF THE STELLARATOR EXPANSION FOR
PLASMA-STABILITY STUDIES in STELLARATORS

G. Anania^a and J.L. Johnson^b

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08544

ABSTRACT

A numerical code, which utilizes the stellarator expansion, is developed and tested. It is used to investigate the magnetohydrodynamic stability properties of several stellarator configurations, including Heliotron E, Wendelstein VII-A, a modular-coil device, and ATF-1.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

^aAt Pp02 Allen & Hamilton, Management Consultants

^bOn loan from Westinghouse Research and Development Center

I. INTRODUCTION

The recent revived interest in stellarators¹ has led to renewed efforts to understand the magnetohydrodynamic properties of this type of configuration. The stellarator expansion,²⁻⁵ in which a large-aspect-ratio expansion reduces the problem to two dimensions, provides a useful approach. It leads to a simple model where much of the physics can be understood. A program to modify an axisymmetric numerical equilibrium code⁶ to incorporate this expansion with application to several stellarator configurations has been reported.⁷ In this paper we describe the continuation of this work to modify the corresponding stability codes.^{8,9}

In the next section we discuss the mathematical formulation of the problem. We describe the code modification and validation studies in Sec. III. Application is made to several devices in Sec. IV, and a discussion of the results is given in Sec. V. This work is closely related to that of Ref. 7. We make no attempt to repeat any of that material.

II. FORMALISM

The stellarator expansion³ orders the magnetic field

$$\mathbf{B} = \mathbf{B}_0 + (\mathbf{B}_\delta) + (\mathbf{B}_\kappa + \mathbf{B}_\beta + \mathbf{B}_\sigma + \mathbf{B}_{\delta\delta}) + \dots, \quad (1)$$

with $\mathbf{B}_0$ a uniform toroidal field,

$$\mathbf{B}_\delta = \nabla \int_{\ell, h} \mathbf{B}_0 \frac{\epsilon_{\ell, h}}{h} I_\ell(hr) \sin(\ell\theta - hr),$$

$$\mathbf{B}_\kappa = -\epsilon_z \mathbf{B}_0 \frac{r}{R} \cos \theta,$$

$$E_{\theta} = -e_z p(\Psi)/B_0 ,$$

$$\vec{B}_0 = \frac{e_z}{r} \times \nabla A(r, \theta) ,$$

so that

$$\left(\frac{B_{\theta}}{B_0}\right)^2 \sim \frac{B_K}{B_0} \sim \frac{B_{\theta}}{B_0} \sim \frac{B_{\sigma}}{B_0} \sim \frac{B_{\delta\delta}}{B_0} \sim \frac{a}{R} \ll 1.$$

Here we work in an orthogonal (r, θ, z) coordinate system with metric elements $h_1 = 1$, $h_2 = r$, $h_3 = (1+r/R \cos \theta)$; where R is the major radius, a is a measure of the plasma radius, r is a minor radius coordinate, $z = R\phi$ is a measure of the distance along the axis, and Ψ is a flux function which is constant on a magnetic surface. The helical fields are assumed to have a wave length the same order as the plasma radius, $ka \sim 1$. This expansion reduces the equilibrium problem to

$$\Psi_0 = \frac{2\pi R}{B_0} \overline{B_{\theta r}} \int B_{\theta\theta} dz + 2\pi R A , \quad (2)$$

$$\nabla^2 A = 4\pi J_0 = -8\pi^2 R p'(\Psi_0)(Q - \langle Q \rangle) + \bar{G}(\Psi_0) , \quad (3)$$

$$Q(r, \theta) \equiv \frac{\overline{B_{\theta}^2}}{B_0^2} + \frac{2r}{R} \cos \theta , \quad (4)$$

with the bar indicating an average over z and the angular brackets denoting an average over the flux surface. The equilibrium has zero net toroidal current on every flux surface if $\bar{G}(\Psi_0) = 0$. A numerical code has been modified to incorporate these equilibrium equations.⁷

Application of the stellarator expansion to reduce the stability

problem to a manageable one was originally made using a variational approach.⁴ The same algebraic operations can be performed, starting with a Lagrangian, so as to obtain an eigenvalue formulation for all possibly unstable modes with the proper kinetic energy normalization. It is straightforward to start with the standard magnetohydrodynamic equations in the plasma and obtain the Lagrangian

$$L = \omega^2 T - \delta W, \quad (5)$$

$$2T = \int_P \rho |\xi|^2 d\tau,$$

$$\begin{aligned} 8\pi\delta W = \int_P [|Q_\perp|^2 + B^2 |\nabla \cdot \xi_\perp|^2 + 2\xi_\perp \cdot \kappa|^2 + 4\pi\mu_P |\nabla \cdot \xi|^2 + 4\pi\mu_\perp \times \xi_\perp^* \cdot Q_\perp \\ - 8\pi\xi_\perp \cdot \nabla_P \xi_\perp^* \cdot \kappa] d\tau + \int_V |\xi_B|^2 d\tau, \end{aligned}$$

where $\xi = \xi(r) \exp i\omega t$ is a small displacement from the equilibrium position, $Q = \nabla \times (\xi \times B)$ is the perturbed magnetic field, and κ is the local curvature of the magnetic field lines. The condition that the perturbed field be continuous at the plasma-vacuum interface must be imposed; the pressure balance constraint is automatically satisfied when $\delta L = 0$.

To apply the expansion, we assume that $\xi = \xi(r, \theta) \exp i(\omega t + s\theta + nz/R)$ with s and n finite integers. In the lowest order we obtain the dispersion relation for fast magnetosonic waves and for short-wave-length shear Alfvén waves in a uniform magnetic field B_0 . Since these waves can never become unstable, we introduce the constraint

$$\xi_\perp^{(0)} = \xi_\perp \times \nabla \eta(r, \theta, z/R) \quad (6)$$

(corresponding to $s = 0$), and restrict consideration to modes with $\rho^{1/2} \omega a/B_0 \sim (a/R)^{1/2}$. In the next significant order we again obtain only stable modes: The fast magnetosonic and short-wave-length, shear-Alfven waves can again be eliminated by picking $\xi_1^{(1)}$ to make $Q_1^{(1)} = \nabla \cdot \xi_1^{(1)} = 0$. The acoustic mode is associated strictly with flow along the field lines. It is removed by making $\xi_z^{(0)} = \xi_z^{(0)}(r, \theta, z/R)$. Then $\rho^{1/2} \omega a/B_0 \sim a/R$.

In the fourth order of the expansion we again obtain all three components of the magnetohydrodynamic spectrum, with distinct separation of the different branches. All but the long-wave-length, shear-Alfven modes, which are associated primarily with $\xi_1^{(0)}$, are definitely stable. Continuing our approach of considering only those modes which could become unstable, we pick $\xi_1^{(2)}$ to eliminate the fast magnetosonic and short-wave-length shear-Alfven waves, and $\xi_z^{(1)}$ to remove the acoustic mode. This reduces the Lagrangian to

$$2L = 2(\omega^2 K - 4\pi \delta W)$$

$$= \omega^2 \int_V \rho |\xi_1^{(0)}|^2 d\tau - \int_V \left[|Q_1^{(2)}|^2 + 4\pi \mu_{\parallel} \times \xi_1^{*(2)} \cdot Q_1^{(2)} + 4\pi \xi_1^{(2)} \cdot \nabla_P \xi_1^{*} \cdot \nabla Q \right] d\tau - \int_V |\delta B|^2 d\tau \quad (7)$$

with $\xi_1^{(0)}$ given by Eq. (6) and

$$Q_1^{(2)} = \xi_z \times \nabla \left[\frac{B}{R} \frac{\partial}{\partial z} + \xi_z \times \nabla \cdot \nabla \right] \eta. \quad (8)$$

It should be noted that the eigenfunctions of Eq. (7) represent the only parts of the magnetohydrodynamic spectrum that have the possibility of

becoming unstable in systems satisfying the stellarator expansion. The terms in the potential energy are well-known. The vacuum-field contribution and the first term from the plasma region represent the energy associated with bending the magnetic field lines, and have a stabilizing effect. The second term is the energy that can be obtained through the interaction of the perturbed magnetic field with the force-free current, and is responsible for driving the well-known kink and tearing instabilities. The third represents the energy obtained from plasma expansion, and drives interchange and ballooning modes. Much work has been done where one or the other of the second and third terms has been ignored. By looking at localized modes, for which the second term does not contribute, the Mercier criterion^{4,5} and ballooning mode restrictions¹⁰ can be obtained. Similarly, by restricting consideration to force-free configurations with concentric magnetic surfaces, kink modes can be studied.¹¹ The code developed here can provide an understanding of the nature of instabilities where the contributions from both of these terms are important.

III. CODE DEVELOPMENT AND VERIFICATION

The numerical problem of extremizing the Lagrangian of Eq. (7) is quite similar to that of PEST-II⁹ in which the tokamak stability problem is formally reduced to one involving a function of one dependent variable. We have thus been able to use this code with only simple changes made in evaluating the matrix elements.

Since application will be made to high-beta systems where the shift of the magnetic axis may be large, it is useful to adopt a general (Ψ, ζ, μ) coordinate system in which the magnetic field lines are straight and the Jacobian, $J(\Psi, \zeta)$ can be specified arbitrarily. Here by Ψ we mean Ψ_0 , and we

define

$$\mu = z/R - q(\Psi)\delta(\Psi, \zeta) \quad (9)$$

with ζ our azimuthal angle, $q \equiv 1/\chi$ the inverse of the rotational transform, and δ a measure of the departure of the toroidal surfaces from planar. We choose δ to straighten the magnetic field lines associated with the effective B_z , which appears in the square bracket of Eq. (8), so that

$$B_{\text{eff}} = q(\Psi)\nabla\Psi \times \nabla\zeta + \nabla\mu \times \nabla\Psi. \quad (10)$$

Then

$$\mu = \frac{z}{R} + q\zeta - \frac{B_0}{R} \int_0^\zeta f(\zeta) d\zeta. \quad (11)$$

The simplest choice of the Jacobian is $J = Rq(\Psi)/B_0$, which is independent of ζ .

As in the formulation of the PEST stability program,^{8,9} we introduce a Galerkin procedure and express the stream function η in terms of a finite set of expansion functions

$$\eta(\Psi, \zeta, \mu) = \sum_{m, l, n} \eta_{m, l, n} \Phi_m(\Psi) \exp i(l\zeta - n\mu). \quad (12)$$

The $\Phi_m(\Psi)$'s are a set of test functions which are linear piecewise-continuous functions. Modes with different values of n decouple. It should be noted that the use of a stream function to represent the displacement vector has introduced a possible extraneous mode to the configuration. The addition to

the $l = 0$ component of η of a constant term, independent of Ψ , would not alter ξ . The freedom to introduce such a term can allow both the kinetic and the potential energy matrices to be singular. As in PEST-II,⁹ we can add a projection operator to the kinetic energy matrix K to shift this mode to $\omega = 0$. Alternatively, it is possible to choose the value of the $l = 0$ component of η at any one radius.

With these considerations we have been able to replace the evaluation of the matrix elements in PEST-II⁹ with those associated with Eqs. (6) - (8) and (12). We make the calculation of δW in the vacuum region consistent with the stellarator expansion by changing the Green's function from a half-order Legendre Polynomial (toroidal sources) to a natural logarithm (cylindrical sources).

Several studies have been made to validate the code. Our first approach was to note that the only equilibrium parameters passed from the mapping code⁷ to the stability routines are $r(\Psi_0, \zeta)$, $\theta(\Psi_0, \zeta)$, and $\Omega(\Psi_0, \zeta)$. Thus a meaningful direct comparison of the matrix elements in δW for the stellarator code and the original PEST-II code can be obtained for systems with no contributions from the multipolar fields. This comparison can be made only for large-aspect-ratio systems with small poloidal fields because the field-line curvature associated with the poloidal field is a high-order effect that is neglected in the stellarator expansion. This introduces problems for PEST-II where, at these large aspect ratios, evaluation of the matrix elements requires the cancellation of large terms to about five significant figures. By calculating the terms omitted in the stellarator version, we have obtained good agreement. This provides confidence in most of the terms, including the ones containing $(dp/d\Psi)(\partial\Omega/\partial\Psi)$ and $(dp/d\Psi)(\partial\Omega/\partial\zeta)$ which are primarily responsible for the interchange modes observed.

Our second set of tests involved using the mapping code to generate exact cylindrical equilibria for which the growth rates are given by analytic formulas. The growth rates of modes in pressureless, constant-axial current equilibria with concentric elliptic cross sections have been determined.¹² We examined both stable and unstable modes in a circular case, and a mode with $n = 1$, $l = -2, 0, 2$, for a case with an ellipticity of two and $q = 1.5$. In all cases, we obtained agreement to four or five significant figures when we used 40 finite elements in Ψ , and exact agreement if we extrapolated to infinitesimal mesh size.

There are terms containing $dq/d\Psi$ in δW that are not checked by either of these studies. A verification that these are treated correctly was made by comparing the growth rates of the unstable modes in a finite-beta, finite-aspect-ratio Heliotron E equilibrium, which are obtained using two different Jacobians. The equations with $f = Rq(\Psi)/B_0$ are much simpler than those where f is also a function of the coordinate ζ . Therefore separate coding has been implemented. Thus, the observation that with 48 finite elements the growth rates obtained with different Jacobians agree to four significant figures, and that they extrapolate to the same number as the mesh size is decreased, provides a good demonstration that all the terms are being evaluated correctly.

IV. APPLICATION

Application of the code has been made to several stellarator configurations. Here we discuss some stability properties of two existing devices, Heliotron E and Wendelstein VII-A, a proposed device ATF-1, and one of the modular coil configurations of Ref. 7.

A. Heliotron E

Heliotron E is a high-shear, $l = 2$, torsatron with 19 field periods and an aspect ratio of 10. The helical distortion of the magnetic surfaces corresponds to $B_g/B_0 \approx 0.5$. In Ref. 13 we modeled this configuration as an 18-period device with $\varepsilon_2 = 0.74$ and a parabolic pressure profile, $p \propto \psi^2$, obtaining results that are in good agreement with those obtained from a three-dimensional code.¹⁴ This machine, with its large rotational transform, has a large equilibrium limit on β , with the shift of the magnetic axis with respect to the center of the plasma surface being half the radius at $\langle \beta \rangle \approx 5.8\%$. The brackets denote the volume average of β .

The growth rates for free-boundary unstable modes with azimuthal mode numbers $n = 1$ through $n = 4$ are given in Fig. 1. In the equilibrium sequences studied we have adjusted the vertical field to keep the plasma centered near $r = 0$ as represented by the black dots in the plot of the axis shift given in Fig. 1 of Ref. 13. These growth rates were determined by extrapolating the results obtained with up to 48 radial finite elements. Convergence studies show little effect as the number of finite elements is increased beyond this number. The $n = 0$ mode is always stable.

The nature of the $n = 1$ and $n = 4$ eigenfunctions is shown in Fig. 2, where the different Fourier components of the stream function η of Eq. (6) are given for a slightly unstable value of $\langle \beta \rangle$, and in Fig. 3, where the corresponding flow patterns of the displacement vectors are presented. It should be noted that, although many poloidal Fourier components were treated, only a few have a significant amplitude. The $m = 1$ component is dominant in the $n = 1$ case and the $m = 4$ component is largest for $n = 4$. These modes are moderately broad, but the region in which the potential energy is negative is fairly well localized near the $q = 1$ resonant surface on which the average

magnetic field line curvature is unfavorable. Different Fourier components are also driven at other resonant surfaces. As should be expected, the last term in the plasma-volume integral in Eq. (7) is primarily responsible for driving these unstable modes. Removing the term in δW which contains $J_{||}$ has little effect on the instabilities.

As β is increased, the rotational transform profile distorts, becoming double-valued beyond $\langle\beta\rangle \approx 4\%$. Two $q = 1$ surfaces occur just above this value, the inner one lying in a magnetic well and the outer one on a hill. When this occurs, the nature of the modes changes. The contribution to δW occurs over a wide region, spanning both $q = 1$ surfaces, although the maximum mode amplitude occurs at the outer $q = 1$ surface, and there is an increased coupling between the various harmonics which make up the mode. In the region where the rotational transform profile is flat and the field lines have nearly the same helicity, a nearly resonant perturbation can grow. Although, as in the lower- β cases, the interchange term is primarily responsible for destabilizing the modes, the contribution from the kink term could now drive the instability by itself.

As can be seen in Fig. 4, the growth rates of these modes saturate if a conducting shell surrounds the plasma, and a second stable regime apparently sets in when $\langle\beta\rangle$ is sufficiently high. Since the magnetic well is improved with an increase in $\langle\beta\rangle$, it could be hoped that similar behavior would also be seen for free-boundary modes. We have seen no evidence of this for $\langle\beta\rangle$ up to 7.5%.

B. Wendelstein VII-A

Wendelstein VII-A is a conventional, nearly shearless, $\ell = 2$, stellarator with five field periods and a relatively small helical distortion; $\epsilon_2 = 1.27$,

$R = 2.0$ m., $a = 0.1$ m, and $B_g/B_0 \sim 0.1$. The equilibrium properties of the configuration are shown in Fig. 7 of Ref. 7 for a parabolic pressure profile ($p \propto \psi^2$) and $\chi = 0.55$ at the plasma boundary. Even for low values of $\langle \beta \rangle$, the axis shift is sufficient to create a magnetic well.

In this nearly shearless configuration, unstable modes develop throughout the cross section of the plasma and exist even when no rational surface is present. Growth rates for modes with $n = 1, 2$, and 3 are shown in Fig. 5, and the stream function and perturbations for the $n=2$ mode are given in Fig. 6. A second stable region develops for the $n > 1$ modes. The instabilities are global with contributions arising from many harmonic components. As might be expected, these modes are strongly wall stabilized. Again, the $n = 0$ modes are stable.

Since all the unstable modes are resonant at $q = 2$, it seems worthwhile to decrease the rotational transform at the plasma edge to 0.45 and move the $q = 2$ surface away from the plasma. In this case, the $n = 1$ mode becomes unstable at $\langle \beta \rangle = 0.3\%$, $\beta(0) = 0.9\%$, which is close to the peak pressure observed experimentally, and the higher- n modes become stable. Flattening the pressure distribution reduces the shift of the magnetic axis for the same value of $\beta(0)$ and allows the higher- n modes to become unstable at successively lower values of $\langle \beta \rangle$.

C. Modular stellarator

Much of our first application of the equilibrium code⁷ was to modular coils with a winding law^{15,16}

$$z = z_i + \sum_j d_j \sin j(\theta - \theta_j), \quad z_i = 2\pi R_i/N, \quad \theta_i = 2\pi i/N, \quad i = 1, 2, \dots, N.$$

Here we consider the stability of a configuration with $\ell = 2$, $m = 6$, $N = 48$, $R/a = 6.6667$, $d_1/a = 0.3354$, $d_2/a = -0.05$, $d_3/a = 0.0354$, and $r/a = 0.4875$. The equilibrium properties are given in Fig. 4 of Ref. 15 and Fig. 5 of Ref. 7. At the time of this work, it was anticipated that the system would be quite stable because of the large magnetic well produced by the shift of the magnetic axis.

As can be seen in Figs. 7 and 8 this configuration has unstable modes whose characteristics are typical of low-shear stellarators such as Wendelstein VII-A. A very low critical value of $\langle\beta\rangle$ is obtained with most of the modes quite broad, extending over most of the plasma. Although no rational surface exists in the plasma, it is always possible to find a value of ℓ such that $\ell - nq$ is small and the stabilizing mechanism is weak. We see from Fig. 8 that the mode amplitude is large on the outside of the torus where the curvature is unfavorable, a behavior which is typical of ballooning modes. Both the kink and interchange terms in δW contribute to driving the instability.

D. ATF-1.

The ATF-1 device has been proposed by the Oak Ridge National Laboratory. It is a low-aspect-ratio ($R/a \sim 8$) torsatron with $\ell = 2$ and twelve field periods. There is enough sideband harmonic content ($\ell = 1$ and $\ell = 3$) to shift the magnetic axis outward, creating a magnetic well even in a vacuum. The configuration has enough shear, since q varies between 0.30 and 0.95, that no large islands are formed. We have studied the stability properties of this device with a $p \propto \psi^2$ pressure distribution for $\langle\beta\rangle \leq 2.5\%$. We see no instabilities with $n \leq 4$. Unstable $\ell = 1$, $n = 1$ external kink modes are seen for flatter pressure distributions,

$p = \psi^{1.5}$, where instability sets in at $\langle \beta \rangle \gtrsim 1\%$. These favorable results are, at this time, the only study of free-boundary modes for this device.

V. DISCUSSION

The stellarator expansion provides a useful method for studying the equilibrium and stability properties of many configurations. It enables one to treat the three-dimensional effects analytically and to reduce the problem to a relatively simple two-dimensional one. Since the physics involved can be interpreted easily, this model should be useful for obtaining an understanding of the magnetohydrodynamic limitations of stellarator configurations. It is surprising that the numerical results agree well with results from other techniques even when the expansion parameter is not small.

The applications given here show that there is considerable difference between systems with large shear and those with nearly constant rotational transform. It appears that shear is best particularly when there is an adequate magnetic well.

The low critical β observed in Heliotron E is associated with the $q = 1$ surface having unfavorable magnetic field line curvature. The shear strongly localizes the perturbations. Introduction of an outwards-pushing vertical field causes the magnetic surfaces to distort so as to remove the $q = 1$ surface from the plasma. This raises the critical $\langle \beta \rangle$ to 2%. Even more favorable results were obtained by decreasing the magnitude of the helical field ($\epsilon_2 = 0.67$ rather than 0.74) so that the Pfirsch-Schlüter terms alter the transform profile and remove the $q = 1$ surface. In this case $\langle \beta \rangle_{\text{CRIT}} \approx 2.5\%$.

The good behavior of ATF-1 could have been anticipated from these Heliotron E studies. The only significant low-order resonant surface in

ATF-1, $q = 2$, occurs in a region of strong shear and favorable average curvature. We would expect that much higher values of $\langle \beta \rangle$ could be reached before instabilities are seen. More work in this direction would be useful.

In systems with little shear, such as Wendelstein VII-A and the modular coil device treated here, the unstable modes are broad in the radial direction and contain considerable harmonic content. Both the kinking term and the interchange term in δW contribute to the instabilities. Although the value of the rotational transform can be chosen not to be rational, unstable modes can occur whose helicities are close to those of the field lines.

Although the critical $\langle \beta \rangle$'s obtained for these configurations are quite small, they are encouraging. They agree with experiment and with estimates obtained by other techniques.¹⁷ As can be seen from the ATF-1 design, the understanding gained from this type of study can lead to the development of high-beta stellarators.

VI. ACKNOWLEDGMENTS

We are indebted to many of our colleagues for help and useful discussions. R. C. Grimm, J. Manickam, and G. Rewoldt made significant contributions to the work on code development. We appreciate K. E. Weimer's many suggestions and encouragement. J. Holmes, B. Carreras, and V. Lynch were most helpful in providing vacuum fields for our ATF-1 studies. This work was supported by U. S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

REFERENCES

- ¹J. L. Johnson, Nucl. Technol./Fusion 2, 340 (1982).
- ²J. L. Johnson, C. R. Oberman, R. M. Kulsrud, and E. A. Frieman, Phys. Fluids 1, 281 (1958).
- ³J. M. Greene and J. L. Johnson, Phys. Fluids 4, 875 (1961).
- ⁴J. L. Johnson and J. M. Greene, Phys. Fluids 4, 1417 (1961).
- ⁵K. E. Weimer, J. M. Greene, and J. L. Johnson, Phys. Fluids 5, 815 (1962).
- ⁶J. L. Johnson, H. E. Dalhed, J. M. Greene, R. C. Grimm, Y. Y. Hsieh, S. C. Jardin, J. Manickam, M. Okabayashi, R. G. Storer, A. M. M. Todd, D. E. Voss, and K. E. Weiner, J. Comput. Phys. 32, 212 (1979).
- ⁷G. Anania, J. L. Johnson, and K. E. Weimer, Princeton Plasma Physics Laboratory Report No. PPPL-1945, 1982. Phys. Fluids (to be published).
- ⁸R. C. Grimm, J. M. Greene, and J. L. Johnson, in Methods in Computational Physics, edited by J. Killeen (Academic Press, New York, 1976), vol. 16, p. 253.
- ⁹R. C. Grimm, R. L. Dewar, and J. Manickam, J. Comput. Phys. 49, 94 (1983).
- ¹⁰M. Wakatani, IEEE Trans. Plasma Sci. PS-9, 243 (1981).
- ¹¹R. M. Sinclair, S. Yoshikawa, W. L. Harries, K. M. Young, K. E. Weimer, and J. L. Johnson, Phys. Fluids 8, 118 (1965).
- ¹²R. L. Dewar, R. C. Grimm, J. L. Johnson, E. A. Frieman, J. M. Greene, and P. H. Rutherford, Phys. Fluids 17, 930 (1974).
- ¹³G. Anania, R. C. Grimm, D. Ho, J. L. Johnson, R. M. Kulsrud, J. Manickam, K. E. Weimer, G. Berge, W. H. Choe, J. P. Freidberg, T. M. Noterdaeme, P. A. Politzer, P. Rosenau, D. Sherwell, O. Betancourt, M. Mondt, and H. Weitzner, in Plasma Physics and Controlled Nuclear Fusion Research, 1982 (International Atomic Energy Agency, Vienna, 1983), paper IAEA-CN-41/V10.

- ¹⁴F. Bauer, O. L. Betancourt, P. R. Garabedian, and J. L. Shohet, IEEE Trans. Plasma Sci. PS-9, 239 (1981).
- ¹⁵T. K. Chu, H. P. Furth, J. L. Johnson, C. Ludescher, and K. E. Weimer, IEEE Trans. Plasma Sci. PS-9, 228 (1981).
- ¹⁶T. K. Chu, H. P. Furth, J. L. Johnson, C. Ludescher, and K. E. Weimer, Nucl. Fusion 22, 871 (1982).
- ¹⁷V. D. Shafranov, Phys. Fluids 26, 357 (1983).

FIGURE CAPTIONS

FIG. 1. Free-boundary-mode growth rates for the Heliotron E model of Fig. 1 of Ref. 13. We use 18 field periods, $\lambda = 2$, $\epsilon_2 = 0.74$, $R/a = 10$, and $\chi(a) = 2.25$.

FIG. 2. Stream function $\eta_2(\Psi)$ of Eq. (12) for the $n = 1$ and $n = 4$ modes in Heliotron E just above the onset of instability, $\langle\beta\rangle = 1.55\%$. The dashed line indicates the Ψ -dependence of $q = 1/\chi$. The numbers denote the value of λ .

FIG. 3. Vector plots of the $n = 1$ and $n = 4$ perturbations of Fig. 2. The arrow lengths denote the magnitude of the displacement vector. The diagrams for $n = 1$ are at $\phi = z/R =$ (a) 0° , (b) 30° , (c) 90° , and (d) 120° . Those for $n = 4$ are at $\phi =$ (e) 0° and (f) 15° .

FIG. 4. Fixed-boundary-mode growth rates for Heliotron E.

FIG. 5. Free-boundary-mode growth rates for the Wendelstein VII-A model of Fig. 2 of Ref. 7. Here $\lambda = 2$, $\epsilon_2 = 1.27$, $ha = 0.25$, $R/a = 20$, and $\chi(a) = 0.55$.

FIG. 6. Stream function and vector plot for the $n = 2$ mode of Fig. 6 at $\langle\beta\rangle = 0.25\%$. The numbers denote the value of λ .

FIG. 7. Free-boundary-mode growth rates for the modular-coil configuration of Fig. 5 of Ref. 7.

FIG. 8. Stream functions and vector plots for the $n = 2$ mode of Fig. 8 at $\langle \beta \rangle = 1.1\%$ and for the $n = 4$ mode at $\langle \beta \rangle = 0.3\%$. The numbers denote the value of l .

Fig. 1

#83T0090

Fig. 2

Fig. 3

Fig. 4

Fig. 5

•83T0095

(b)

Fig. 6

Fig. 7

Fig. 8

EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

Plasma Res Lab, Austr Nat'l Univ, AUSTRALIA
 Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Prof. F. Cep, Inst Theo Phys, AUSTRIA
 Prof. Frank Verhaest, Inst theoretische, BELGIUM
 Dr. D. Palumbo, Dg XII Fusion Prog, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
 Dr. C.R. James, Univ of Alberta, CANADA
 Prof. J. Teichmann, Univ of Montreal, CANADA
 Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Hannes Bernard, Univ British Columbia, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Zhengyu Li, SW Inst Physics, CHINA
 Library, Tsing Hua University, CHINA
 Librarian, Institute of Physics, CHINA
 Inst Plasma Phys, SW Inst Physics, CHINA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Prof. Sonetzman, Observatoire de Nice, FRANCE
 J. Redet, CEN-BP6, FRANCE
 AM Dupas Library, AM Dupas Library, FRANCE
 Dr. Tom Mui, Academy Bibliographic, HONG KONG
 Preprint Library, Cent Res Inst Phys, HUNGARY
 Dr. A.K. Sundaram, Physical Research Lab, INDIA
 Dr. S.K. Trehan, Panjab University, INDIA
 Dr. Indra, Mohan Lal Das, Banaras Hindu Univ, INDIA
 Dr. L.K. Chavda, South Gujarat Univ, INDIA
 Dr. R.K. Chhajlani, Var Ruchi "org, INDIA
 P. Kew, Physical Research Lab, INDIA
 Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
 Prof. S. Cupernan, Tel Aviv University, ISRAEL
 Prof. G. Rostagni, Univ DI Padova, ITALY
 Librarian, Int'l Ctr Theo Phys, ITALY
 Miss Clelia De Palo, Assoc EURATOM-CNEN, ITALY
 Biblioteca, del CNR EURATOM, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Prof. M. Yoshikawa, JAERI, Tokai Res Est, JAPAN
 Prof. T. Uchida, University of Tokyo, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Prof. Sigeru Mori, JAERI, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Ichiro Kawakami, Nihon Univ, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Tech Info Division, Korea Atomic Energy, KOREA
 Dr. R. England, Ciudad Universitaria, MEXICO
 Bibliotheek, Fom-Inst Voor Plasma, NETHERLANDS
 Prof. B.S. Lilley, University of Waikato, NEW ZEALAND
 Dr. Suresh C. Sharma, Univ of Calabar, NIGERIA
 Prof. J.A.C. Cabral, Inst Superior Tech, PORTUGAL
 Dr. Octavian Petrus, ALI CUZA University, ROMANIA
 Dr. R. Jones, Nat'l Univ Singapore, SINGAPORE
 Prof. M.A. Hallberg, University of Natal, SO AFRICA
 Dr. Johan de Villiers, Atomic Energy Bd, SO AFRICA
 Dr. J.A. Tagle, JEN, SPAIN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Dr. Erik T. Karlson, Uppsala Universitet, SWEDEN
 Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
 Dr. W.L. Weise, Nat'l Bur Stand, USA
 Dr. W.M. Stacey, Georg Inst Tech, USA
 Dr. S.T. Wu, Univ Alabama, USA
 Prof. Norman L. Oleson, Univ S Florida, USA
 Dr. Benjamin Ma, Iowa State Univ, USA
 Prof. Magne Kristiansen, Texas Tech Univ, USA
 Dr. Raymond Askew, Auburn Univ, USA
 Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. M.S. Rabinovich, Lebedev Physical Inst, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
 Dr. T.J. Boyd, Univ College N Wales, WALES
 Dr. K. Schindler, Ruhr Universität, W. GERMANY
 Nuclear Res Estab, Julich Ltd, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Dr. H.J. Kaeppeler, University Stuttgart, W. GERMANY
 Bibliothek, Inst Plasmaforschung, W. GERMANY