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The Optics of Mass Separator I

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Issued: July 1981

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Silvio J. Balestrini

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THE OPTICS OF MASS SEPARATOR I

by

Silvio J. Balestrini

ABSTRACT

The ion optics of an existing mass separator are documented. The electrostatic and magnetic stages are analyzed theoretically, both separately and in combination, by paying particular attention to the ion trajectories, the linear and angular magnifications, and the dispersion. The possibility of converting the magnet into a tunable unit by means of current-carrying elements in the gap is demonstrated. The feasibility of correction coils constructed from printed circuit board is shown.

I. INTRODUCTION

This report is an inquiry into the optics of mass separator I, the first of two existing mass separators, located in Group CNC-11 at Technical Area 48 of the Los Alamos National Laboratory. It was manufactured in 1964 by K/B C. Mileikowski & Co., Vasteras, Sweden. The magnet is a 1.6-m radius, 90° sector preceded by an electrostatic lens system. The optics of the present configuration are analyzed to understand what performance should be expected. We then ask to what extent can the theoretical performance be affected by changing the magnetic field index. Finally, we propose a printed circuit board design to make the magnetic field index a controllable parameter.

II. ELECTROSTATIC LENS OPTICS

This section concerns the effects on ion trajectories by the draw-out cone and einzel lens of mass separator I before the ions enter the magnetic field. We find the linear and angular magnifications produced by the voltages applied to the lens elements, and we also calculate the displacement of the virtual ion source from the actual source position.

A. The Paraxial Ray Equation

The dimensions of the lens optics layout are given in Fig. 1. The geometry has cylindrical symmetry about the center-line. We adopt cylindrical coordinates in which an ion proceeds from left to right in the z -direction, and the coordinate normal to the center-line is ρ . Voltages are applied to the draw-out cone and lens elements. An ion in this region will execute a trajectory governed by the paraxial ray equation¹

$$\rho'' + \frac{V'}{2V} \rho' + \frac{V''}{4V} \rho = 0, \quad (1)$$

where V is a function of z and equal to the kinetic energy of an ion traveling along the z -axis. It can be related in a simple way to the axial profile of the electrical potential. The primes denote the derivatives with respect to z . If $V(z)$ is known and the initial conditions for the trajectory at S are given, i.e., $\rho = \rho_1$ and $\rho' = \rho'_1$ at $z = 0$, then one can solve Eq. 1 for ρ and ρ' as functions of z up to $z = z_2$, where the trajectory becomes subject to the magnetic field. Having obtained two independent solutions in this way, any other solution can be constructed as a linear combination of these two (see Ref. 1, p. 79).

Fig. 1. Electrostatic lens layout for mass separator I. The configuration has cylindrical symmetry up to the magnet. The linear dimensions are in centimeters.

Let two independent solutions be represented by $f(z)$ and $g(z)$. They are obtained with the following initial conditions placed at $z = 0$:

$$\begin{aligned} f(0) &= 1 \text{ and } f'(0) = 0, \\ g(0) &= 0 \text{ and } g'(0) = 1. \end{aligned} \quad (2)$$

Any solution can be constructed from these, given the value and slope at $z = 0$:

$$\rho(z) = \rho_1 f(z) + \rho'_1 g(z), \quad (3)$$

and the derivative is

$$\rho'(z) = \rho_1 f'(z) + \rho'_1 g'(z). \quad (4)$$

In Fig. 2, two trajectories leave a point P at $z = 0$, where $\rho = \rho_1$, and when they pass through $z = L$, they appear to have come from a point P' . The angular magnification at $z = L$ is the ratio of the angular separation between the two trajectories at $z = L$ to their angular separation at $z = 0$, or the derivative of Eq. (4) with respect to ρ'_1 at $z = L$:

$$M_{\text{ang}} = g'(L). \quad (5)$$

Point P' is located at $z = z_v$, where

$$z_v = L - \frac{g(L)}{g'(L)}, \quad (6)$$

and $\rho = \rho_v$, where

$$\begin{aligned} \rho_v &= \rho(L) - (L - z_v)\rho'(L) \\ &= \rho_1 [f(L) - g(L)f'(L)/g'(L)]. \end{aligned} \quad (7)$$

The linear magnification is

$$\begin{aligned} M_{\text{lin}} &= \rho_v / \rho_1 \\ &= f(L) - (L - z_v)f'(L) \\ &= f(L) - g(L)f'(L)/g'(L). \end{aligned} \quad (8)$$

We note the important result that Eqs. (5), (6), and (8) do not depend upon the initial conditions ρ_1 and ρ' . In a region away from lens elements, where electric fields are absent, V' and V'' are zero, and all solutions here become straight lines. One can show that in this case, z_v , M_{lin} , and M_{ang} are independent also of the value of L .

Fig. 2. Mechanism producing angular and linear magnification and a virtual image at P' from an object at P.

B. The Immersion Lens

We consider an immersion lens composed of two coaxial, semi-infinite conducting cylinders of like radius R , separated by a distance S , and maintained at different potentials. Let z equal z_0 midway between them. Then

$$V(z) = A \cdot U(z, z_0) + B, \quad (9)$$

where $U(z, z_0)$ is the axial profile of a solution to the Laplacian, and the constants A and B are chosen to fit the boundary conditions. When the separation is zero, a very good approximation is^{2,3}

$$U(z, z_0) = \tanh\left[1.315 \frac{(z-z_0)}{R}\right], \quad S = 0. \quad (10)$$

This approximation differs by less than one percent from the exact solution given by Bertram (Ref. 3), and is much simpler to use. If the separation S is taken into account (see Ref. 3),

$$U(z, z_0) = \frac{R}{1.135 \cdot S} \ln \frac{\cosh\left[\frac{1.315}{R}(z-z_0 + \frac{S}{2})\right]}{\cosh\left[\frac{1.315}{R}(z-z_0 - \frac{S}{2})\right]}, \quad (11)$$

which reduces to Eq. (10) when $S = 0$. We can approximate Eq. (11) with an expression similar to Eq. (10), or

$$U(z, z_0) = \tanh\left[w \frac{(z-z_0)}{R}\right], \quad S > 0, \quad (12)$$

if we set

$$w = \frac{2R}{S} \tanh\left[1.315 \frac{S}{2R}\right] \quad (13)$$

Equation (13) equates the slopes of Eqs. (11) and (12) at $z = z_0$. The largest difference between Eqs. (11) and (12) is less than 0.5% of the range when S is as large as 75% of R . The relative simplicity of Eq. (12) makes it worth using in place of Eq. (11).

C. The Electrostatic Lens System

Equation (9) is the axial profile of a solution to the Laplacian equation for two semi-infinite cylinders whose potentials have constant values at a distance R from the rotational axis of symmetry. The sum of any number of solutions for gaps S_i centered at $z_0 = z_1, z_2$, etc., also satisfies the Laplacian, and a solution can be constructed, by summing, to furnish an axial profile for the draw-out cone and lens elements of Fig. 1. Because the range of the function of Eq. (12) extends from -1 to +1, each term of the summation must be multiplied by half the voltage difference across the gap that it represents. A final additive constant is then selected to adjust for zero energy. The final expression for the axial profile for the configuration in Fig. 1 is

$$V(z) = A_0 + \sum_{i=1}^3 A_i \cdot \tanh\left[w_i \frac{(z-z_i)}{R}\right] \quad (14)$$

where

$$A_0 = (E + V_C)/2,$$

$$A_1 = (E - V_C)/2,$$

$$A_2 = V_L/2,$$

$$A_3 = -A_2,$$

E = final beam energy,

V_C = meter voltage on the draw-out cone, measured from the source,

V_L = meter voltage on the lens element, measured from ground,

$z_1, z_2, z_3 = 12.05, 25.4, 38.4$ cm, respectively,

$R = 5.75$ cm,

and w_1 is determined from Eq. (13) and the respective gap value for S_1 .

D. Ray Tracing by Computer

If the values for ρ and its derivatives are known at one point, where $z = z_n$, their values can be estimated at a nearby point, where $z_{n+1} = z_n + \Delta z$, by the recursion formulae

$$\rho_n'' = -\left(\frac{V'}{2V} \rho_n' + \frac{V''}{4V} \rho_n\right), \quad (15)$$

$$\rho_{n+1}' = \rho_n' + \rho_n'' \Delta z, \quad (16)$$

and

$$\rho_{n+1} = \rho_n + \rho_n' \Delta z. \quad (17)$$

Equation (15) follows from Eq. (1), and the other two are obvious. Starting with the initial conditions $\rho = \rho_1$ and $\rho' = \rho_1'$ at $z = 0$, and the axial kinetic energy profile from Eq. (14), one can trace a trajectory through the lens as accurately as he wishes for any given values of V_C and V_L by making the increment Δz as small as necessary.

Several solutions are plotted in Fig. 3 for an $E = 50$ beam passing through the lens optics in Fig. 1 while the elements are set at $V_C = 5$ and $V_L = 20$. The locations of the gap centers are also shown. As expected, the trajectories become straight lines in the field-free region, where $z = 50$ cm or more, and the magnifications and virtual object position become constants of the electrical parameters.

Tables I and II list the angular and linear magnifications, respectively, obtained at $z = 114$ cm (the equivalent pole-face edge) for an $E = 50$ beam versus V_C and V_L . Table III lists the distances from the virtual object to the equivalent pole-face edge, or $114 - z_v$ cm.

Fig. 3. Trajectories of ions passing through the lens optics of Fig. 1; $E = 50$, $V_C = 5$, and $V_L = 20$.

TABLE I
ANGULAR MAGNIFICATION, M_{ang}

V_C	V_L						
	0	5	10	15	20	25	30
0	-0.091	-0.091	-0.090	-0.089	-0.087	-0.085	-0.081
1	-0.030	-0.030	-0.032	-0.037	-0.044	-0.057	-0.077
2	0.047	0.046	0.042	0.033	0.017	-0.008	-0.046
3	0.116	0.115	0.109	0.096	0.074	0.038	-0.016
4	0.178	0.177	0.169	0.153	0.125	0.081	0.013
5	0.233	0.232	0.223	0.204	0.171	0.119	0.040
6	0.284	0.282	0.272	0.250	0.213	0.154	0.065
8	0.371	0.369	0.356	0.330	0.286	0.216	0.108
10	0.446	0.443	0.428	0.399	0.348	0.268	0.146
12	0.510	0.506	0.490	0.457	0.401	0.313	0.178
15	0.591	0.587	0.569	0.532	0.469	0.370	0.219
20	0.698	0.693	0.672	0.630	0.558	0.445	0.273
30	0.845	0.838	0.813	0.763	0.678	0.545	0.344
50	1.000	0.991	0.960	0.900	0.800	0.644	0.409

TABLE II
LINEAR MAGNIFICATION, M_{lin}

V_C	V_L						
	0	5	10	15	20	25	30
0	-0.78	-0.78	-0.79	-0.80	-0.81	-0.83	-0.87
1	-9.13	-5.27	-4.96	-4.35	-3.58	-2.79	-2.08
2	4.58	4.63	5.08	6.49	12.48	-26.43	-4.59
3	2.21	2.22	2.35	2.66	3.46	6.66	-16.40
4	1.64	1.65	1.73	1.91	2.34	3.62	21.92
5	1.39	1.40	1.46	1.59	1.90	2.72	8.10
6	1.25	1.26	1.30	1.41	1.66	2.30	5.48
8	1.09	1.10	1.14	1.23	1.42	1.89	3.76
10	1.02	1.02	1.06	1.13	1.30	1.69	3.11
12	0.97	0.98	1.01	1.08	1.23	1.58	2.78
15	0.93	0.94	0.97	1.04	1.18	1.49	2.52
20	0.91	0.92	0.94	1.01	1.14	1.43	2.33
30	0.92	0.93	0.95	1.02	1.15	1.43	2.26
50	1.00	1.01	1.04	1.11	1.25	1.55	2.15

III. MAGNETIC FIELD MAPPING AND TRAJECTORIES IN THE MAGNET

A. Mapping

The field was mapped in the gap and at the entrance and exit ends. The measurements are described with the aid of Fig. 4. The magnet dimensions are also included.

Forty-nine measurements were made with a Hall probe in the median plane at azimuthal angles ranging from 3° to 87° and at radial values ranging from -15 cm to +15 cm from the central radius. The field was set at 0.26 Tesla and monitored with a NMR unit at station 8 to correct for drift. The deviations are listed in Table IV in parts per ten-thousand from the average along the center-line. Their accuracy is estimated at ± 0.3 . A variation of one mil (English) in the gap width of 10 cm would produce a variation in the magnetic field of 2 parts in 10^4 .

Four more sets of Hall probe measurements were taken along straight-line traverses in the central plane, shown as dotted lines and numbered 3 to 6 in

TABLE III
DISTANCE L_1 IN CENTIMETERS OF THE VIRTUAL OBJECT
FROM THE EQUIVALENT POLE-FACE EDGE

V_C	V_L						
	0	5	10	15	20	25	30
0	87.0	87.0	87.0	87.1	87.4	87.7	88.2
1	-8.6	-9.0	-3.7	6.6	19.8	33.6	46.0
2	196.8	198.2	210.1	246.0	398.6	-589.9	-34.9
3	146.6	147.2	151.1	160.5	184.6	280.8	-406.8
4	134.4	134.8	137.4	143.4	157.4	199.4	794.6
5	128.7	129.2	131.2	136.0	146.5	175.1	360.1
6	125.4	125.8	127.6	131.7	140.6	163.4	276.8
8	121.7	122.0	123.5	127.0	134.2	151.6	221.4
10	119.6	119.9	121.3	124.4	130.8	145.8	199.8
12	118.2	118.6	119.8	122.7	128.6	142.2	188.3
15	116.9	117.2	118.4	121.1	126.6	138.8	178.5
20	115.7	116.0	117.1	119.6	124.6	135.8	170.2
30	114.5	114.8	115.9	118.2	123.0	133.2	163.7
50	114.0	114.3	115.4	117.7	122.3	132.3	161.7

Fig. 4. Field mapping patterns for mass separator I relative to the pole face and lens box (drawn to scale).

TABLE IV
MASS SEPARATOR I MAGNETIC FIELD DEVIATIONS IN THE MEDIAN PLANE
IN PARTS PER 10^4 AT 0.26 TESLA (2600 GAUSS)

ΔR , cm from central rad.	Azimuth						
	3°	15°	30°	45°	60°	75°	87°
+15.0	-569.5	-554.5	-558.7	-562.5	-561.0	-556.7	-568.2
+10.0	-30.3	-28.4	-31.5	-32.2	-31.1	-26.5	-28.8
+5.0	-1.9	0.0	-3.1	-4.3	-1.7	+1.4	-2.5
0.0	+1.2	+1.2	-1.8	-2.2	-0.2	+2.9	-1.0
-5.0	+4.3	+1.2	-2.6	-3.3	-1.4	+2.5	-3.3
-10.0	-10.0	-14.3	-18.5	-18.5	-17.0	-12.0	-22.0
-15.0	-384.6	-371.9	-379.6	-388.4	-378.5	-370.0	-406.1

Fig. 4, and at distances ranging from -15 cm to +15 cm from the center-line. These are listed in Table V in percent of the gap value well inside the pole-face edges.

Traverse 7 was taken vertically at the center-line and one-half inch from the exit pole-face edge. The results are listed in Table VI versus probe distance from the bottom pole face and in percent of the value well inside the pole-face edges. This traverse is also plotted in Fig. 5. The dotted curve is a parabola fitted to all but the two outermost points of the data. Its center-line is off by 1.5 mm, reflecting the uncertainty of the vertical positioning of the probe, which was difficult to obtain in this traverse. A second-degree approximation offers a good description of the variation in the vertical component of the magnetic field over the useful space between the pole faces. Hence, having the value B_1 for the vertical component at the median plane and the value B_2 at a distance Y away,* one can interpolate reasonably well for the value B_y at any other distance, y , by

$$B_y = B_1 + (B_2 - B_1)(y/Y)^2 \quad (18)$$

* In order to adopt rectangular coordinates that are consistent with the cylindrical coordinate notation in Section II, the z -axis is tangent to the center-line in the median plane and points to the right (Fig. 4). The x -axis points up, and the y -axis points out of the paper. Both x and y are zero on the center-line.

TABLE V
FIELD STRENGTHS ALONG SEVERAL TRAVERSES IN THE MEDIAN PLANE BUT NOT
IN THE POLE-FACE AREA, IN PERCENT OF VALUE WELL INSIDE THE POLE FACE EDGES

<u>ΔR, cm from center-line</u>	<u>Traverse</u>			
	<u>3^a</u>	<u>4^b</u>	<u>5^c</u>	<u>6^d</u>
+15.0	39.0	41.2	22.3	77.6
+10.0	43.2	46.8	25.3	82.2
+5.0	44.9	47.9	26.7	81.7
0.0	45.2	48.4	27.4	80.7
-5.0	45.1	48.3	27.3	79.6
-10.0	45.9	48.0	27.7	78.1
-15.0	44.4	46.2	27.3	73.1

^aParallel to pole-face edge, one-half gap away, entrance end.

^bSame as (a), but at exit end.

^cParallel to pole-face edge, one full gap away, exit end.

^dAlong pole-face edge, exit end.

TABLE VI
VERTICAL TRAVERSE AT STATION 7; VALUE VERSUS
DISTANCE FROM LOWER POLE FACE

<u>Distance, cm</u>	<u>Value, %</u>
1	111.9
2	100.5
3	94.2
4	90.9
5	89.6
6	90.3
7	92.5
8	98.7
9	109.0

Fig. 5. Vertical traverse at station 7 versus distance from the lower pole face. The curve is a parabola fitted to all but the outer points.

A pair of continuous X-Y-plotter tracers was made by moving the Hall probe along traverse 1. One was taken in the median plane and the other on a parallel line 3.5 cm below (1.5 cm from the lower pole face). The two traces are superposed on one drawing for this report and shown in Fig. 6. Another pair, taken along traverse 2, is shown in Fig. 7. Figure 6 shows the location of the lens box relative to the pole-face edge and the equivalent edge of the pole face, computed at 51% of a gap width away from the actual edge due to the effect of the fringing magnetic field. Figure 7 shows the equivalent edge for the exit at 90% of a gap width past the pole-face edge due to the absence of

Fig. 6. Two Hall probe traces taken at traverse 1 and the computed equivalent pole edge relative to the magnet pole edge and the lens box position (drawn to scale). The magnetic field is in arbitrary units.

magnetic shunting. Because the exit trace extended only to 16 inches past the magnet, the field between this point and the collector (177 cm away) was estimated by assuming a suitable dipole field. The field strength at the collector is estimated to be 0.14% of the gap value.

B. Theorem and Corollaries on Fringing

First-Order Theorem: The areas under both curves in Fig. 6 are equal,
and the same is true for Fig. 7.

Two first-order corollaries are:

Fig. 7. Two Hall probe traces taken at traverse 2 and the computed equivalent pole edge relative to the magnet pole edge (drawn to scale). The magnetic field is in arbitrary units.

1. The equivalent pole-face edge is the same for all ion trajectories, whether in or out of the median plane.

2. The accumulated horizontal angular deflection after passing through a field-fringing region is the same for all ion trajectories, whether in or out of the median plane.

The theorem can be proved utilizing the fact that

$$\vec{\text{div}} \cdot \vec{B} = 0 \quad (19)$$

and Stoke's theorem

$$\int_V \vec{\text{div}} \cdot \vec{B} \, dv = \int_A \vec{B} \cdot d\vec{a} \quad (20)$$

The median plane is horizontal. One defines a volume of unit thickness in the x-direction, which is bounded by the median plane and a second plane parallel to it at a short distance, y, away. The volume is completed by two more planes perpendicular to the center-line, the first sufficiently inside the gap where the two traces coincide, and the second well outside the gap where the two traces again coincide. The right side of Eq. (20) equals the area under the trace taken off of the median plane minus that under the trace taken in the median plane. Eq. (19) indicates that the left side of Eq. (20) is zero, and therefore, the difference between the areas is zero for all values of y. We may encounter inaccuracies to the extent that the field might be curved at the vertical planes or that the trajectories are not all parallel to the median plane, but these inaccuracies are small.

The first corollary follows from the method of finding the equivalent edge, or the distance past the pole edge to where a uniform gap field must be extended to subtend an area equal to that under the actual field trace.

To prove the second corollary, we consider the trajectory of an ion entering the magnetic field described by the traces in Fig. 6. The trajectory proceeds along the z-axis from point A, chosen outside the gap, where the traces have equal value, to a point B (not to be confused with the magnetic field B) inside the gap where the traces are again equal in value. A differential impulse on an ion with velocity $\vec{v}$ is*

$$\vec{F} dt = m d\vec{v} \quad (21)$$

where $d\vec{v}$ is normal to $\vec{v}$. Because the force is $F = Bev$, and $v = dz/dt$, Eq. (21) can be reduced to

$$d\phi = dv/v = \frac{eB}{mv} dz, \quad (22)$$

where $d\phi$ is a differential deflection. Inside the magnet, the field is B_0 and uniform, and the trajectory radius is R. Equating the centripetal force $B_0 ev$ to mv^2/R , we have $B_0 R = mv/e$, and Eq. (22) becomes

$$d\phi = \frac{B}{B_0 R} dz. \quad (23)$$

* If the vector notation is omitted, it will be understood to mean the magnitude of the vector only.

If the field in the median plane is B_1 , the deflection ϕ_1 experienced by the trajectory in the median plane can be described as a function of z ; or, integrating from point A to point z , between points A and B,

$$\phi_1 = \frac{1}{B_0 R} \int_A^z B_1 dz \quad . \quad (24)$$

If ϕ_2 is the deflection in a plane parallel to the median plane, and the field there is B_2 , the difference between the deflections is

$$\begin{aligned} \Delta\phi = \phi_2 - \phi_1 &= \frac{1}{B_0 R} \int_A^z (B_2 - B_1) dz \\ &= Q(A, z) \quad . \end{aligned} \quad (25)$$

Eq. (25) defines the function Q . From the first-order theorem and the choice of locations for points A and B, $Q(A, B) = 0$, thus proving the second corollary.

C. Ray Tracing at the Magnet

If the distance between A and B is small compared to R , and we let x_1 and x_2 represent the departures of the trajectories from the original straight line due to the respective deflections, we get

$$x_1 = \int_A^z \phi_1 dz \quad (26)$$

and

$$\begin{aligned} \Delta x = x_2 - x_1 &= \int_A^z (\phi_2 - \phi_1) dz \\ &= \int_A^z Q(A, z) dz \quad . \end{aligned} \quad (27)$$

The same can be said for the exit end of the magnet.

1. The Central Ray. Two trajectory segments were computed through the fringing regions from the field strengths shown in Figs. 6 and 7, using Eqs. (24) through (27), and the results are listed in Tables VII and VIII for the entrance and exit, respectively, to the magnet. Each solution assumes $z = 0$ at the pole edge. A complete trajectory is shown qualitatively in Fig. 8. It leaves the virtual object at point A' with the initial conditions for a central

TABLE VII
MAGNETIC FIELD AND ITS EFFECTS ON TRAJECTORIES
DUE TO FRINGING, WHEN ENTERING THE MAGNET

z, cm	Relative Field		Azimuth, mrad		x, mm	Δx^b , mm
	y = 0	y = Y ^a	ϕ	$\Delta\phi^b$		
-40	0.004	0.004	0.0	0.0	0.0	0.0
-30	0.006	0.006	0.3	0.0	0.0	0.0
-20	0.012	0.012	0.7	0.0	-0.1	0.0
-15	0.064	0.053	1.7	-0.1	-0.1	0.0
-10	0.200	0.180	5.5	-0.5	-0.3	0.02
-8	0.273	0.241	8.3	-0.8	-0.4	0.03
-6	0.369	0.320	12.2	-1.3	-0.6	0.05
-4	0.499	0.424	17.4	-2.0	-0.9	0.08
-2	0.653	0.592	24.4	-2.9	-1.3	0.13
0	0.814	0.920	33.3	-2.8	-1.9	0.19
2	0.926	1.043	43.9	-1.1	-2.7	0.23
4	0.977	1.012	55.5	-0.3	-3.7	0.25
6	0.994	1.003	67.5	-0.1	-4.9	0.25
8	0.999	1.001	79.6	0.0	-6.4	0.25
10	1.000	1.000	91.7	0.0	-8.1	0.25

^aY = 3.5 cm.

^bEach difference is the value at y = Y minus the value at y = 0.

ray and arrives at the collector at point C on the center-line of the machine design. The trajectory was completed by connecting the two computer solution segments with an arc of radius R_T centered at O' , and allowing the exit solution to rotate about point C by an angle γ if necessary to match the solution segments at points F and G. The computations for the displacement of the trajectory from the center-line, in the last column of Table VIII, were possible from the values of x versus z after γ was known. The major parameters of the composite trajectory are listed in Table IX. The nonzero value for γ resulted from the nonsymmetrical fringing, which also increased the apparent sector angle to $\pi/2 + \gamma$. A second set of parameters is included for an assumed case where both entrance and exit are magnetically shunted.

TABLE VIII
MAGNETIC FIELD AND ITS EFFECTS ON TRAJECTORIES
DUE TO FRINGING, WHEN LEAVING THE MAGNET

z, cm	Relative Field		Azimuth mrad		x, mm	Δx^b mm	x_D^c mm
	y = 0	y = y ^a	ϕ	$\Delta\phi^b$			
-10	1.000	1.000	-115.2	0.0	-19.0	0.0	7.4
-8	0.999	1.001	-103.0	0.0	-16.8	0.0	9.3
-6	0.994	1.003	-91.0	0.1	-14.8	0.0	10.9
-4	0.979	1.011	-79.0	0.3	-13.1	0.0	12.4
-2	0.932	1.036	-67.3	1.0	-11.7	-0.02	13.5
0	0.830	0.959	-56.6	2.7	-10.4	-0.05	14.5
2	0.678	0.621	-47.4	3.0	-9.4	-0.11	15.3
4	0.530	0.455	-40.1	2.1	-8.5	-0.17	15.8
6	0.413	0.361	-34.4	1.3	-7.8	-0.20	16.3
8	0.328	0.298	-30.0	0.8	-7.1	-0.22	16.7
10	0.268	0.247	-26.4	0.5	-6.6	-0.23	16.9
15	0.171	0.165	-19.9	0.2	-5.4	-0.25	17.4
20	0.120	0.119	-15.6	0.1	-4.5	-0.25	16.6
30	0.065	0.065	-10.2	0.0	-3.3	-0.26	17.4
40	0.040	0.040	-7.1	0.0	-2.4	-0.26	16.9
60	0.017	0.017	-3.9	0.0	-1.4	-0.26	15.1
100	0.006	0.006	-1.4	0.0	-0.4	-0.26	10.4
177	0.001	0.001	0.0	0.0	0.0	-0.26	0.0

^ay = 3.5 cm.

^bEach difference is the value at y = Y minus the value at y = 0.

^cDisplacement of the trajectory from the center-line.

The largest separation, D, between the trajectory and the center-line, also in Table IX, is colinear with the points 0 and 0', and its value is obtained by a simple accounting of distances.

From Table VII, we see that the accumulated shift at entry is

$$\Delta x_{\text{entry}} = 0.25 \text{ mm} \quad (28)$$

for a trajectory 3.5 cm away from the median plane. This separation is equivalent to a displacement between the centers of curvature of the trajectories inside the magnet gap, but the two trajectories will coincide again at the exit because the sector angle is $\sim 90^\circ$. However, a second separation is produced on exiting that will persist to the collector. From Table VIII, we see that this amounts to

$$\Delta x_{\text{exit}} = -0.26 \text{ mm} \quad . \quad (29)$$

A treatment similar to that leading to Eq. (18) yields an estimate of

$$\Delta x_{\text{collector}} = -0.26 \left(\frac{y}{3.5}\right)^2 \text{ mm} \quad (30)$$

for a trajectory leaving the magnet at a distance y cm from the median plane.

2. Trajectory Shifts. The first part of a trajectory leaving point A' in Fig. 8 at a small angle α with respect to the center-line can be generated by rotating that portion of the solution between points A' and G by the angle α about point A'. To first order, the trajectory is completed by adjusting γ to a new value obtained by adding $(R_T/L_2)\alpha$. It can be shown in this way that the separation, D , is altered by

$$\begin{aligned} \Delta D &= \frac{uL_1 + vR_T}{(u^2 + v^2)^{1/2}} \cdot \alpha \\ &= 2.3 \cdot 10^2 \cdot \alpha \text{ cm} \quad . \end{aligned} \quad (31)$$

Table X lists the trajectory shifts produced by $\alpha \neq 0$ midway through the magnet and at the exit. The art of matching trajectory segments can be extended to compute the shifts for trajectories focused over the 30 cm span of the collector. These shifts are also listed in Table X. The shift for a combination of causes is the algebraic sum of the separate shifts.

IV. EXISTING CONFIGURATION OPTICS

This section studies the complete mass separator optics as the configuration now exists. This magnet is somewhat shunted at the entrance by the iron lens box, but it is not shunted at the exit. See Figs. 1, 4, 6, 7, 9.

TABLE X
TRAJECTORY SHIFTS PRODUCED MIDWAY THROUGH
THE MAGNET AND AT THE EXIT

<u>Cause</u>	<u>Midway</u>	<u>At Exit</u>
D from Fringing		
- Exit is shunted	-2.9 cm	-0.19 cm
- Exit not shunted	---	+1.45 cm
$\alpha \neq 0$	230 cm/rad	160 cm/rad
30-cm-wide collector	+2.2 cm	+7.5 cm
	-2.1 cm	-7.2 cm

An ion beam originates from a small area normal to the center-line at point S (Fig. 9). It is bounded by a conical caustic surface which forms an angle α_S with the center-line. After passing through the lens system and the magnet, the beam forms a focused image at point C of the area at point S. Only focusing action in the central plane will be treated.

A. Magnet Optics

The image at C is formed if the following relation is satisfied:

$$(R_E + L_2 \tan \gamma)(L_1 \cos \phi - R_E \sin \phi) + L_2 (R_E \cos \phi - L_1 \sin \phi) = 0, \quad (32)$$

where

$$\gamma = 0.8^\circ \text{ (Cf. Table IX) },$$

$$\phi = 90.8^\circ ,$$

and

$$L_2 = 168 \text{ cm} .$$

This relation results from the $m_{1,2}$ element of the magnet transfer matrix, which will be discussed in detail in Section V. The radius R_E is the distance from O' to B (Fig. 9) and is very nearly equal to R_T (Fig. 8). R_E is uniquely definable for simplified calculations based on equivalent pole-face edges if the sector shunting is symmetrical. In the nonsymmetrical case, it becomes

trajectory crossed the center-line at point C. Some voltage combinations are listed in Table XI. These are also plotted in Fig. 10 and labeled $z = 0$. Two more curves are calculated in a similar way and included for trajectories originating at $z = -1.0$ cm and at $z = 1.0$ cm.

For each combination of values for V_C and V_L , obtained from Table III, the associated angular magnification, M_{ang} , of the electrostatic optics, or the ratio of α_1 to α_S in Fig. 9, can be obtained by direct interpolation from Table I. These values are plotted in Fig. 11 and also listed in Table XI.

TABLE XI
MATCHING VOLTAGES FOR FOCUSING AND THE RESULTING PERFORMANCE
OF THE COMBINED OPTICS FOR AN $E = 50$ BEAM

V_C	V_L	M_{ang}	M_{total}	FM
2.48	0.00	0.081	3.09	0.66
2.50	5.00	0.081	3.07	0.66
2.63	10.00	0.085	3.00	0.68
2.97	15.00	0.094	2.79	0.73
3.00	15.35	0.095	2.76	0.74
3.77	20.00	0.114	2.53	0.81
4.00	20.86	0.119	2.48	0.82
5.00	23.31	0.139	2.35	0.87
6.00	24.75	0.158	2.31	0.88
6.22	25.00	0.167	2.27	0.90
8.00	26.43	0.190	2.25	0.91
10.00	27.40	0.215	2.20	0.93
12.00	28.03	0.238	2.18	0.94
15.00	28.66	0.265	2.17	0.94
20.00	29.27	0.302	2.19	0.93
30.00	29.81	0.353	2.27	0.90
50.00	29.98	0.410	2.52	0.81

Fig. 10. Dependence of the lens voltage, V_L , on the cone draw-out voltage, V_C , for focus of an $E = 50$ beam and for several positions of the object relative to the cone opening.

Fig. 11. Dependence of the angular magnification on the cone draw-out voltage for an $E = 50$ beam.

Similarly, interpolation from Table II yields the linear magnification, M_{lin} , for the same voltages, which when multiplied by M_x [Eq. (34)], yields the total magnification, M_{total} . The latter is plotted in Fig. 12 and also listed in Table XI.

The dispersion⁴ at point C is the unitless, fixed quantity

$$\begin{aligned}
 D_E &= \frac{p}{R_E} \frac{dx}{dp} \\
 &= \left(1 + \frac{L_2}{R_E} \tan \gamma \right) (1 - \cos \phi) + \frac{L_2}{R_E} \sin \phi \\
 &= 2.04 \quad , \quad (35)
 \end{aligned}$$

where p is the momentum of the ion. The expression results from the $m_{1,3}$ element of the magnet transfer matrix, Section V. We define the figure of merit as the ratio

Fig. 12. Dependence of the total magnification on the cone draw-out voltage for an $E = 50$ beam.

$$FM = \left| D_E / M_{\text{total}} \right| . \quad (36)$$

The associated values for FM are also listed in Table XI. The largest value occurs at around $V_C = 15$, where the total magnification is smallest.

V. LINEAR SHAPING OF THE MAGNETIC FIELD

We introduce another optical parameter by shaping the magnetic field in the gap. The field at the median plane varies linearly with radius about the value B_0 at the central ray, according to the field index n in the relation

$$B = B_0 \left(1 - n \frac{x}{R} \right) , \quad (37)$$

where x is the distance from the central ray and R is the radius. In the present configuration, $n = 0$. We shall evaluate R later according to the discussion leading to Eq. (33).

A. The Transfer Matrix

The end properties of an ion trajectory will be described by the three vector components

$\frac{x}{R}$ = fractional coordinate in the median plane,
 σ = slope in the median plane, and
 $\delta = \Delta p/p$ = fractional momentum deviation.

They will be called simply position, slope, and momentum, respectively. The momentum of an ion traveling along the central ray is $p = mv$. That of an ion with the same energy but differing slightly in mass is $p + \Delta p$, and

$$\delta = \frac{1}{2} \Delta m/m \quad . \quad (38)$$

An ion leaves the virtual object and travels a distance L_1 in free-flight to the magnet edge (Fig. 9) according to the transfer matrix

$$M_1 = \begin{pmatrix} 1 & L_1/R & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad . \quad (39)$$

Then the ion travels through the magnet designed for normal entry according to the matrix (Ref. 4, p. 213)

$$M_{\text{magnet}} = \begin{pmatrix} \cos \epsilon \phi & \epsilon^{-1} \sin \epsilon \phi & \epsilon^{-2} (1 - \cos \epsilon \phi) \\ -\epsilon \sin \epsilon \phi & \cos \epsilon \phi & \epsilon^{-1} \sin \epsilon \phi \\ 0 & 0 & 1 \end{pmatrix} \quad , \quad (40)$$

where ϕ is the sector angle, and

$$\epsilon = \sqrt{1 - n} \quad . \quad (41)$$

The exit boundary of the magnet forms an angle β with the central ray. The angle is positive if the outward normal from the magnet boundary points outside the beam with respect to the center of curvature. The effect is described by the matrix

$$M_{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ \tan\beta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (42)$$

Finally, the ion travels a distance L_2 in free-flight to the collector according to the matrix

$$M_2 = \begin{pmatrix} 1 & L_2/R & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (43)$$

The complete transfer matrix is the product

$$M_T = M_2 \cdot M_{\beta} \cdot M_{\text{magnet}} \cdot M_1 \quad (44)$$

whose elements are⁵

$$m_{1,1} = \left(1 + \frac{L_2}{R} \tan\beta\right) \cos\epsilon\phi - \frac{L_2}{R} \epsilon \cdot \sin\epsilon\phi, \quad (45)$$

$$m_{1,2} = \left(1 + \frac{L_2}{R} \tan\beta\right) \left(\frac{L_1}{R} \cos\epsilon\phi + \frac{\sin\epsilon\phi}{\epsilon}\right) + \frac{L_2}{R} \left(\cos\epsilon\phi - \frac{L_1}{R} \epsilon \cdot \sin\epsilon\phi\right), \quad (46)$$

$$m_{1,3} = \left(1 + \frac{L_2}{R} \tan\beta\right) \frac{(1 - \cos\epsilon\phi)}{\epsilon^2} + \frac{L_2}{R} \frac{\sin\epsilon\phi}{\epsilon}, \quad (47)$$

$$m_{2,1} = \cos\epsilon\phi \cdot \tan\beta - \epsilon \cdot \sin\epsilon\phi, \quad (48)$$

$$m_{2,2} = \left(\frac{L_1}{R} \cos\epsilon\phi + \frac{\sin\epsilon\phi}{\epsilon}\right) \tan\beta + \left(\cos\epsilon\phi - \frac{L_1}{R} \epsilon \cdot \sin\epsilon\phi\right), \quad (49)$$

$$m_{2,3} = \frac{(1 - \cos\epsilon\phi)}{\epsilon^2} \tan\beta + \frac{\sin\epsilon\phi}{\epsilon}, \quad (50)$$

$$m_{3,1} = m_{3,2} = 0, \text{ and } m_{3,3} = 1. \quad (51)$$

B. Varying n

The position component of the trajectory vector must be independent of α at the source for focusing to occur, or

$$m_{1,2} = 0 \quad . \quad (52)$$

If n can now be varied, thereby varying ε [Eq. (41)], L_1 is no longer the fixed quantity established by Eq. (32). The latter becomes a function of n determined by the focusing condition above. The relationship is

$$L_1 = R \cdot \frac{\frac{\sin \varepsilon \phi}{\varepsilon} \left(1 + \frac{L_2}{R} \tan \beta \right) + \frac{L_2}{R} \cos \varepsilon \phi}{\frac{L_2}{R} \varepsilon \cdot \sin \varepsilon \phi - \cos \varepsilon \phi \left(1 + \frac{L_2}{R} \tan \beta \right)} \quad , \quad (53)$$

and ε is defined by Eq. (41).

The magnification produced by the magnet is the effect on the position component of the trajectory vector, or

$$M_x = m_{1,1} \quad . \quad (54)$$

We assume that if one makes the effort to engineer a control for n , he also shunts the exit of the magnet for symmetry. The configuration parameters then assume the values listed in Table XII. The resulting values for L_1 and M_x are computed from Eqs. (53) and (45) and listed versus n in Table XIII.

TABLE XII
PARAMETERS FOR THE
SYMMETRICALLY SHUNTED MAGNET

R	165 cm
ϕ	90°
β	0°
L_2	172 cm

TABLE XIII
DEPENDENCE OF L_1 AND M_x ON n

n	L_1 (cm)	M_x
-0.5	43.4	-1.54
-0.4	57.9	-1.47
-0.3	75.2	-1.38
-0.2	96.4	-1.28
-0.1315	114.0	-1.20
-0.1	123.2	-1.17
0.0	158.3	-1.04
0.1	206.8	-0.91
0.2	278.8	-0.75
0.3	397.4	-0.59
0.4	631.7	-0.41
0.5	1318.8	-0.22
0.603	$\pm\infty$	0.0

C. Focusing with the Field Index

The magnet can now be made to accept a range of values for the virtual object distance presented to it by the electrostatic lens system by simply adjusting n , and one can obtain matching sets of values for n , V_C , and V_L by interpolating from Tables III and XIII for any common value of L_1 . It is possible to control focusing with the field index entirely and leave the lens voltage, V_L , turned off.

Given a value for V_C , one obtains in turn the virtual object distance that is generated (Table III) and then the required value for n by inverse interpolation from Table XIII. The latter is listed in Table XIV. The source object is magnified by M_{lin} (Table II), due to electrostatic action, and again by the magnet [Eq. (54)]. The total magnification, $M_{total} = M_{lin} \cdot M_x$, is also

TABLE XIV
PERFORMANCE OF THE COMBINED OPTICS
WHEN $V_L = 0$ AND n IS VARIED

V_C	n	M_{total}	D_E	FM
2	0.082	-4.26	2.10	0.49
3	-0.030	-2.39	2.02	0.85
4	-0.065	-1.84	2.00	1.08
5	-0.082	-1.59	1.98	1.25
6	-0.093	-1.45	1.97	1.36
8	-0.105	-1.28	1.97	1.54
10	-0.112	-1.20	1.96	1.63
12	-0.117	-1.15	1.96	1.70
15	-0.121	-1.11	1.95	1.76
20	-0.125	-1.09	1.95	1.79
30	-0.130	-1.11	1.95	1.76
50	-0.131	-1.20	1.95	1.62

listed in Table XIV. Finally, the dispersion D_E [Eq. (35)] is expressed in this more general case by Eq. (47). This and the figure of merit, Eq. (36), are also listed in Table XIV.

The figure of merit is plotted in Fig. 13 versus V_C together with the case for $n = 0$ from Table XI. The cases of focusing with n for several other values of V_L are also included for the curious reader. Because the dispersion is relatively insensitive to focusing parameters, FM is roughly inversely proportional to M_{total} .

VI. FIELD MODIFICATIONS BY ELECTRIC CURRENTS IN THE GAP

The analysis of the effects from field shaping have been considered to first order only. It can be argued that when an ion optical system produces imaging of first order quality, but where some resolution is lost due to second order effects, the latter can be cleared by introducing a matching second order perturbation. Such a field has been produced by Camplan and Meunier⁶. It is described by

$$B = B_0 \left[1 - \alpha \left(\frac{x}{R} \right) + \beta \left(\frac{x}{R} \right)^2 \right] , \quad (55)$$

Fig. 13. Dependence of the figure of merit on several focusing parameters for an $E = 50$ beam.

where B_0 is the field value at the central ray. We can set $\alpha = n$, by comparing Eq. (55) with Eq. (37), and thereby also call α the field index. The quantity β is the second order perturbation or curvature coefficient (not to be confused with the exit angle β).

In this and the following sections, we shall discuss practical ways of sloping and curving the field in the magnet gap by following the methods of Ref. 6.

A. Effect at a Distance from Two Conductors

Two conductors A and A' are located opposite each other and against the pole faces, Fig. 14, and at a distance x_c from the central ray. Each carries a current, I, directed out of the paper. Together, their contribution to the magnetic field in the median plane is⁷

$$b = \frac{\mu_0 I}{2h} \tanh\left[\frac{\pi(x-x_c)}{2h} \right], \quad (56)$$

where μ_0 is $4\pi \times 10^{-7}$ Henry/meter in the MKSA system of units.

Fig. 14. The two conductors A and A' produce the magnetic field contribution b in the median plane at x.

We define for convenience the quantity

$$\theta = \frac{\pi(x-x_c)}{2h} . \quad (57)$$

The first derivative of b taken in the direction normal to the central ray and in the median plane is

$$\frac{db}{dx} = \frac{\mu_0 \pi}{4h^2} I \operatorname{sech}^2 \theta , \quad (58)$$

and the second derivative is

$$\frac{d^2b}{dx^2} = - \frac{\mu_0 \pi^2}{4h^3} I \operatorname{sech}^2 \theta \tanh \theta . \quad (59)$$

In principle, one can produce a magnetic field with a desired index and second order perturbation by finding an appropriate distribution of electric currents along the pole faces.

The current distribution must not change the value of the field along the central ray, otherwise the position of the image will shift. This condition can be satisfied by deploying each return wire outside the gap on the same side of the central ray as its field-shaping conductor.

B. Constant Current Density Distribution

A constant current distribution can be expressed in terms of the differential current

$$dI = K dx_c . \quad (60)$$

The distribution extends symmetrically over both pole faces from $x_c = -L$ to $x_c = L$. The contribution to the field B_0 at $x = 0$ will only be a first derivate, obtained by substituting Eq. (60) for I in Eq. (58) and integrating over the extent of the distribution:

$$\begin{aligned} \frac{dB}{dx} &= \frac{\mu_0 \pi}{4h^2} K \int_{x_c=-L}^{x_c=L} \operatorname{sech}^2 \theta dx_c \\ &= \frac{\mu_0 K}{2h} (\tanh \theta_1 - \tanh \theta_2) , \end{aligned} \quad (61)$$

where

$$\theta_1 = \frac{\pi}{2h}(x + L) \quad , \quad (62)$$

and

$$\theta_2 = \frac{\pi}{2h}(x - L) \quad . \quad (63)$$

Integrating Eqs. (56) and (59) over a symmetrical distribution leads to zero contributions at $x = 0$.

Equation (61) is plotted in Fig. 15 versus x for the half-gap width $h = 5$ cm (see Fig. 4) and for several values of L .

The constant K of the distribution can be related to the field index α by equating the first derivative of the field at $x = 0$ from Eq. (55) to the value from Eq. (61) and noting that $\tanh(\pi L/2h) \cong 1.0$:

$$\alpha = - \frac{\mu_0 R}{B_0 h} K \quad . \quad (64)$$

C. Linear Current Density Distribution

A linear current density distribution can be expressed in terms of the differential current

$$dI = k x_c dx_c \quad . \quad (65)$$

As before, we let the distribution extend over both pole faces from $x_c = -L$ to $x_c = L$. This contribution to the field at $x = 0$ will be a second derivative obtained by substituting Eq. (65) into Eq. (59) and integrating over the extent of the distribution:

$$\begin{aligned} \frac{d B}{dx^2} &= - \frac{\mu_0 \pi}{4h^3} k \int_{x_c = -L}^{x_c = L} x_c \operatorname{sech}^2 \theta \tanh \theta dx_c \\ &= \frac{\mu_0 k}{2h} \left[\frac{\pi L}{2h} (\tanh^2 \theta_1 + \tanh^2 \theta_2 - 2) \right. \\ &\quad \left. + \tanh \theta_1 - \tanh \theta_2 \right] \end{aligned} \quad (66)$$

where θ_1 and θ_2 are defined by Eqs. (62) and (63).

Equation (66) is plotted in Fig. 16 versus x for the half-gap width $h = 5$ cm and for several values of L .

Fig. 15. Dependence of $\frac{dB}{dx}$ on the distance from the central ray for several values of L.

Fig. 16. Dependence of $\frac{d^2B}{dx^2}$ on the distance from the central ray for several values of L.

The constant k of the current distribution can be related to the curvature coefficient β by equating the second derivative at $x = 0$ from Eq. (55) to the value from Eq. (66) and assuming the limiting value for $\tanh(\pi L/2h)$:

$$\beta = \frac{\mu_0 R^2}{2hB_0} k \quad (67)$$

VII. POLE-FACE CORRECTION COILS

A. General Considerations

In order to focus entirely with the field index, we can expect to require a value of

$$\alpha = -0.13 \quad (68)$$

from the discussion in Section V-C (see Table XIII).

We now estimate a probable value for β by computing a value necessary to correct for the classical second-order magnet aberration. We assume a symmetrical 90° system for simplicity, which is nearly the case in Fig. 9, so that we can set $L_1 = R = L_2$. A ray leaving the source with an angle α with respect to the central ray is overbent by a magnet with a uniform field, and it intersects the collector at a distance equal to $-R\alpha^2$ from the central ray. The angle through which it is overbent is approximately $\Delta\phi = \alpha^2$. Inside the magnet, this ray travels at a distance between $R\alpha$ and $x = 230\alpha$ (see Table X) or about

$$x = 200\alpha \quad (69)$$

from the central ray. In order to undo the overbending, the field there must be weakened by

$$\begin{aligned} \Delta B &= B_0 \beta \left(\frac{x}{R}\right)^2 \\ &= B_0 \beta \left(\frac{200\alpha}{R}\right)^2, \end{aligned} \quad (70)$$

according to Eq. (55). By reasoning as for Eq. (25), the azimuthal correction becomes

$$\alpha^2 = -\frac{1}{B_0 R} \Delta B \frac{\pi}{2} R$$

$$= - \frac{\pi \Delta B}{2B_0} \quad (71)$$

Solving Eqs. (70) and (71) for β ,

$$\beta = -0.4 \quad (72)$$

One might expect routine separations of ^{175}Lu at 50 kV, and in the extreme one might envision separations of ^{254}Es at 60 kV. The values for B_0 for these cases are, respectively, 0.26 and 0.34 Tesla. We adopt half-width values of 17.5 cm and 5 cm for the coils and gap, respectively, and compute the expected current densities for the two cases from Eqs. (64) and (67). The results are listed in Table XV.

B. An α -Coil Design

We shall call the conductor pattern for field-index control the " α -coil." The number of conductors selected is 30 per pole face, because this number was employed successfully by Neyret and Parain (Ref. 7). When distributed uniformly across the 35-cm pole-face width, this will allow one-cm-wide conducting bands on a printed circuit board spaced 1.17 cm apart with about 1/16-inch separations in between. These bands will be connected in series and carry the same current in order to produce a constant linear current density along the pole face, as required by Eq. (60). The total length of the bands in series is 156 m for both coils, presenting a resistance of about 4.1Ω if the heaviest available printed circuit board (2 oz/ft²) is used. If one uses an equal length of #12 hookup copper wire and allows an additional 20% for looping, this would present an additional 1.0Ω of resistance.⁸ A power supply must deliver 9.5 A at 48 V for a ^{175}Lu separation, or 12.4 A at 63 V for a ^{254}Es separation.

TABLE XV
CURRENT DENSITIES FOR TWO SEPARATION CASES

Separation	Field (Tesla)	Volts (kV)	K (A/m) ($\alpha=-0.13$)	k (A/m ²) ($\beta=-0.4$)	kL (A/m) (L=0.175)
^{175}Lu	0.26	50	815	-3040	-530
^{254}Es	0.34	60	1070	-4000	-700

C. A β -Coil Design

We shall call the conductor pattern for the curvature coefficient the " β -coil." The distribution of linear current density for the β -coil is described by Eq. (65). The i -th circuit board band from the central ray has a theoretical width w_i , which includes the conductor and some spacing between it and the neighboring conductor. It is bounded by an outer radius r_i at $x = x_i$ and an inner radius $r_{i-1} = r_i - w_i$ at x_{i-1} . The current density distribution prescribed by Eq. (65) over this bandwidth amounts to a total current of

$$I = k \int_{x_{i-1}}^{x_i} x_c dx_c$$

$$= \frac{1}{2} k w_i (x_i + x_{i-1}) ; \quad (73)$$

or, the width of the region carrying a current I times its average distance from the central ray is a constant from band to band if all bands carry the same current. One can show from this that

$$x_i = \pm \sqrt{i} w_1 . \quad (74)$$

The centroid of the prescribed current distribution within the width w_i is displaced by a distance

$$g_i = \frac{1}{6} w_i [(4i-3)\sqrt{i} + (4i-1)\sqrt{i-1}]^{-1} \quad (75)$$

from the center of the band, and one should therefore space the edges of the conductor for this band equally about the centroid.

Because the α -coil used 30 conductors, a comparable number seems suitable for the β -coil. Starting with a value of $w_1 = 5$ cm, 24 bands result, spanning a pole-face distance of 34.6 cm, and the last band is $w_{12} = 0.74$ cm wide. A correction for the centroid, Eq. (75), is advisable for the first band, because the shift involved is 17% of w_1 . The correction can be omitted for the remaining bands because it is trivial. A power supply for this coil, using #12 return wires, should deliver 3.9 A at 16 V for a ^{175}Lu separation or 5.1 A at 20 V for a ^{254}Es separation.

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