BEAM-TRACKING STUDIES WITH RINGBEARER II.

UCID--19771

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DE83 008907

November 22, 1982

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ABSTRACT

This report presents results from the RINGBEARER II linearized monopole/dipole particle simulation for an intense relativistic electron beam propagating in a gas near three types of channels: 1) pre-existing conductivity, 2) density, and 3) density with pre-existing conductivity. Comparisons are made with earlier analytic results for the initial conditions for the pre-existing conductivity channel.

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Work performed jointly under the auspices of the U. S. Department of Energy by Lawrence Livermore National Laboratory under contract W-7405-ENG-4B and for the Department of Defense under Defense Advanced Research Projects Agency ARPA Order No. 4395 A#12, monitored by Naval Surface Weapons Center under document number N60921-83-WR-W0113.

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I. Introduction - Tracking Calculation Scenarios

This report was written as a companion report to Ref. (1) and will refer freely to Ref. (1) with regard to its analytic description of the electrostatic tracking. The results presented here, however, do not depend directly on Ref. (1) and may be viewed simply as results from the RINGBEARER II particle simulation.

RINGBEARER II is a monopole/dipole particle simulation code for an intense relativistic electron beam (Ref. 2). The code is fully linearized and all quantities related to the beam displacement are of first order. A primary reason for the development of RINGBEARER II was the study of the resistive hose instability (Ref. 3). It is currently being applied to the issue of beam tracking near a channel.

In Ref. (1), analytic estimates of the tracking force in the presence of a pre-existing conductivity channel were obtained in the purely electrostatic limit. These calculations were performed for a beam rigidly displaced from a conductivity channel. The present paper extends the earlier analysis in several directions.

First, with RINGBEARER II the calculation of the dipole force for a pre-existing conductivity channel is performed without the approximations used in Ref. (1). Hence both electrostatic and magnetic effects are included (within the approximation of "New Field Equations," (Ref. (4))). In addition the analysis is extended to include a beam which has responded to the dipole forces and has a curvature in its shape similar to that illustrated in Figure 1. Evolution of the beam profile and fields (both monopole and dipole) are calculated self-consistently and includes the magnetic coupling of body of the beam to the head of the beam.

We also consider the two cases of beam tracking in the presence of a density channel and in the presence of a density channel with finite pre-existing conductivity. The tracking of a beam into a density channel may occur via the creation of a conductivity channel by preferential avalanche ionization in the lower density regions of the channel. With the RINGBEARER II simulation, we are able to study this effect quantitatively.

II. Linearized Code Implementation, Limitations

In previous studies the tracking force has been computed numerically (Ref. 5) but the dynamic response of the beam to this force has not been computed. Utilizing RINGBEARER II we compute the dynamics of the beam response to the tracking force self-consistently. However, the linearization in RINGBEARER II places constraints on the propagation distance over which results are reliable. Furthermore, it requires special diagnostics for assessing tracking. The limitations implied by linearized dynamics are discussed in this section. Tracking diagnostics are discussed in Section III.

As explained in Ref. (1), at the beam front the portion of the beam within the channel will track into the channel while the portion outside the channel will be repelled from the channel. In the simulation, the beam usually has broad "wings" on the beam radial profile; hence some portion of the beam is always outside the channel in the detracking region. In RINGBEARER II all dipole quantities are of first order in the linearization. The magnitude of dipole quantities are therefore always infinitesimal when compared to the magnitude of monopole quantities which are of zero order in the linearization.

This ordering places limitations on the applicability of RINGBEARER II to the study of tracking. To illustrate these limitations consider the following scenario from Ref. (1) in which initially the inner core of the beam in the channel tracks while particles in the wings detrack. If this separation continued, two distinguishable beams would eventually result. As this separation increased the mutual interaction of these two beams would diminish. As computed in RINGBEARER II the initial response would be the same as above, although the response is only in the dipole quantities. Some of the beam would have a diminishing dipole displacement and some of the beam would have a growing dipole displacement. The limitation of RINGBEARER II is that there can never be two distinguishable beams, because the monopole orbits of the beam particles are zero order quantities and are by definition unaffected by the first order dipole displacements.

To emphasize consequences of the difference between a complete non-linear description and the linearized description of RINGBEARER II, consider the following simplistic example. Let the dividing line between the inside of a pre-existing conductivity channel and the outside of that channel be very definite. Consider now two neighboring particles in circular orbits, one just inside the channel and one just outside the channel. According to Ref. (1), one particle tracks and one detracks. Consider also that the displacement of the beam from the pre-existing conductivity channel is small but finite. In a non-linear description, the average displacement of the tracking particle would go to zero, but as the initial displacement was small the change in the particle's orbit would be almost imperceptible. On the other hand, the displacement of the detracking particle would grow and eventually change the particle's orbit as to separate from its neighbor which tracks.

In a linearized description, the same situation would be characterized as follows. The circular orbits of the particles just inside and just outside the channel would be described by the zero order monopole orbits of the particles. The initial displacement of the particles would be described by the first order dipole displacements of the particles. In the linearized treatment, only the dipole displacements of the particles are affected by the dipole tracking or detracking forces caused by the channel. Since the dipole displacements are of first order (i.e., infinitesimal when compared to the monopole orbits of zero order), the orbits of the two particles will never separate perceptibly due to the dipole displacements regardless of how large one of those displacements grows. The detracking particle will grow to a large first order amplitude and will produce a large dipole current at its monopole radial location. Thus in the linearized description, rather than leave the beam, the detracking particle would stay in the beam and could dominate the dipole forces in the simulation.

Despite these limitations, RINGBEARER II can be used to verify the existence and strength of the tracking region without resorting to the purely electrostatic approximation. Because magnetic forces and beam dynamics are included, we can demonstrate the coupling of the body of the beam to the beam front. By considering channels with large radii, we can minimize the non-linear nature of the problem because in these cases most of the beam particles will lie in the tracking region. Note that except in the context explained above, an infinitesimal beam displacement can be used to describe a real physical situation. An electron beam that is at some point coincident with the channel but is moving away from the channel would have a region in space in which the displacement of the beam from the channel can legitimately be considered small with respect to the beam radius.

III. Code Implementation for Tracking Studies

The analysis of Ref. (1) considered a centered beam and an offset channel so that the dipole quantities involved in the offset were limited to those of the channel. The general mode of operation of RINGBEARER II is to offset the beam from a centered channel and is used here. The alternate use of a centered beam and an offset channel has been checked in the case of the pre-existing conductivity channel and final results differ by no more than a few percent. Where analytic and code results are compared, the appropriate sign change due to the differing points of view has been inserted.

As we have emphasized, the mean displacement of the entire beam disk from a linearized code can be a misleading diagnostic, because this mean can be dominated by a few detracking particles in the wings. For analyzing the tracking results it was therefore useful to construct a special diagnostic. The displacement of the inner core is defined to be the mean displacement of all the particles whose total energy is less than a cutoff value. This diagnostic allows us to monitor the displacements of limited portions of the beam. Note that the diagnostic is not explicitly used in any part of the code calculation. The percentage of particles represented by the inner core is also monitored. In the case of pre-existing conductivity channel, that percentage is about 95% at the beam head. In the case of density channel that percentage is only about 50% at the beam head. As one moves back in X (distance from the beam head) and makes the transition into the magnetic regime, the inner core diagnostic encompasses more and more of the beam because the beam size is smaller. Our simulation results suggest that the beam body is guided by a part of the beam which is represented well by this inner core diagnostic.

IV. RINGBEARER II Results

The channel conditions to be considered here are displayed in Figure 2 along with the beam characteristics. In the RINGBEARER II calculations the beam was allowed to propagate 1 m in Z before the displacement of the beam from the channel was introduced. This procedure allows the beam, which is injected at a constant radius, to evolve to a steady state profile in X .

For the reasons explained in Section II a large channel of 3 cm in radius was chosen. Channels with radial profiles that were composed of step functions (hereafter referred to simply as "square" channels) were deemed desirable for two reasons. Firstly, this choice allows a direct comparison between the analytic solution for the net dipole forces as presented in Ref. (1) and the net forces computed with RINGBEARER II. The second reason derives from the noise limitations of finite particle number and the attendant over importance of a relatively few particles that have extreme orbits induced by the time varying, anharmonic nature of the beam's self-generated potential. The use of the square channels skirts this limitation. Inside the channel the net dipole forces are independent of r ; (see Section IV in Ref. (1)), therefore the variation in the monopole orbits due to the time varying potential do not induce exponential growth in the beam dipole displacements. Also the equation for the beam displacement in the channel, Equation (27) in Ref. (1) for $r < a_c$, is of the form of a harmonic oscillator and should exhibit no secular growth. That this claim is reasonable can be seen from the following considerations. In the absence of the channel, there would be no preferred axis, and a rigidly displaced beam would move straight ahead. All the forces acting in the dipole

(transverse) direction, though they are from anharmonic, time varying potentials, are in balance. The introduction of the square channel upsets that balance in a particular way. Whereas the square channel does not introduce any radial dependence inside the channel, it is reasonable that the net dipole force is constant within the channel and only depends on the beam displacement as is predicted from Equation (27) in Ref. (1).

Without the square channel the initial tracking forces are of the same form but weaker, and the limitations imposed by finite particle noise and secularly growing orbits make the analysis more difficult and may actually obscure a positive tracking result.

Case 1: Pre-Existing Conductivity Channel

The case with a pre-existing conductivity channel as detailed in Figure 2 will be compared with the predictions of the analytic model discussed in Ref. (1). These comparisons will be made at the point in Z at which the beam offset from the channel is first introduced ($Z = 1$ m) so the beam has not responded dynamically. Figure 3 shows four plots of the tracking force on an electron versus the radial coordinate r at X positions of 2.25, 6.00, 9.75 and 19.75 cm. At the earliest X position displayed the shape of the tracking force in r is as expected from the analytic model. The force is in the tracking direction inside the channel and is nearly constant. Near the channel boundary the forces change to the detracking direction. At the 6 cm position in X the tracking forces have strengthened as would be predicted by the analytic model, but the magnitude of the tracking forces on axis is beginning to lag significantly behind the maximum tracking. Both these trends are seen to continue at the 9.75 cm position. Because the conductivity is

highest on axis, the low conductivity Ansatz for which the electrostatic model was developed is violated first on axis. The depressed values of the tracking force on axis at later X positions are the manifestation of the transition into the magnetic regime. At $X = 19.75$ cm the forces have become detracking except for a small residual of electrostatic tracking around 2 cm in radius.

Figure 4 shows the tracking force as calculated by RINGBEARER II and by use of Equation (27) in Ref. (1). Qualitatively the two curves are similar. The magnitude, however, of the tracking portion of the RINGBEARER II curve is a factor of 4 to 5 times less than the prediction of the analytic model at this position in X . The analytic prediction is expected to be higher because the low conductivity approximation, $4\pi\sigma\tau \ll 1$, used in obtaining Equation (27) of Ref. (1) tends to over-estimate the on-axis tracking force and because the magnetic detracking forces are ignored. The electrostatic tracking force seen by RINGBEARER II without making the low conductivity approximation is understandably less. Moreover, as the pre-existing conductivity ($1 \cdot 10^9 \text{ s}^{-1}$) is not negligible, the detracking associated with the onset of the magnetic regime is significant.

Figure 5 shows the on-axis tracking force on an electron versus X as calculated by RINGBEARER II and with the analytic model (Equation (26) in Ref. (1)). At early values of X the code and the analytic result both increase with increasing X . Beyond X of about 6 cm the code result rolls over due to the emergence of the detracking forces in the magnetic regime. The analytic prediction continues to increase. The beam radius at the point in X where the curve from RINGBEARER II peaks is about .8 cm and is just

below the range predicted in Section VI of Ref. (1) (Relative Size of Beam Neck to Channel). Note that although the tracking force peaks at about 6 cm in X , optimum movement of the beam into the channel does not necessarily occur at that point. Recall from Figures 3 and 4 that the on-axis value of the tracking force dies out more quickly than the off-axis values. The degree to which a segment of beam tracks into the channel is a function of both the radial structure of the tracking forces and of the radial distribution of the beam. The segment at the beam front which exhibits the strongest tracking in the code results is at 7 cm in X . This location is one of the points for which results will be presented. The other point to be considered is at $X = 14.75$ cm. At that point the initial forces are slightly detracking as indicated by the dashed line in Fig. 5.

The beam displacement and beam radius at the 7 cm and 14.75 cm positions in X are shown as functions of Z in Figure 6. The displacement of the inner core of the beam is used as the beam displacement. The measure of the beam radius is the radius, $R_{1/2}$, which contains half the simulation particles. The total propagation distance in Z is 5 meters. For longer distances in Z , the results are suspect due to the limitations in RINGBEARER II. The beam head ($X = 7$ cm) is seen to track into the channel. The beam body ($X = 14.75$ cm) does not appear to move initially although an examination of the numbers show a slight detracking at that point. Recall from Figure 5 that the sign of the on-axis translational force was in the detracking direction by the 14.75 cm point in X . As Z increases the beam body moves in toward the channel in an oscillatory fashion. Initially the segment at $X = 14.75$ cm moves slightly away from the channel axis. During this time the earlier (in X) beam segments

have tracked and moved toward the channel axis. The new potential minimum or equilibrium point set up by the beam front is now nearer the channel axis and the 14.75 cm segment now moves toward and eventually overshoots the channel axis. This tracking is due to the magnetic coupling of this segment to the beam front. Note that the equilibrium position seen by the beam tail is not simply the position of the beam head. That equilibrium position depends on the history of all of the preceding beam which itself is overshooting and oscillating. The motion of the beam body between 4 and 5 meters in Z seems somewhat extreme and brings into question the validity of the results there. It is difficult at that point to distinguish between the complicated interaction of the beam body with all the preceding beam and a breakdown of the simulation.

Figures 7, 8, 9 and 10 are plots of beam displacement and beam radius versus X at fixed Z values. Again the displacement of the inner core of the beam is used to represent the entire beam displacement. For the points between 7 cm and 14.75 cm in X the core diagnostic represents at least 95% of the total beam. At even earlier points in X, less and less of the beam is represented by the core diagnostics as the beam radius expands. Note however that there are no marked inconsistencies in the results at the early X values. The uncertainty is in how well the results shown represent the whole beam.

Figure 7a illustrates the initial response of the beam. From the plot of on-axis tracking force shown in Figure 5, one expects that the tracking motion of the beam toward the channel would increase from $X = 0$ cm to a point between 6 and 7 cm in X. Beyond 7 cm the tracking motion would lessen until at late X, the initial response of the beam would be to detrack. This is the response seen in Figure 7a. The detracking of the beam is barely discernible at the

last X point shown. This detracking is more evident in code runs that involve longer beams. In Figure 7b the beam head continues to track and pulls the rest of beam inward with it. In this frame the body of the beam is still lagging behind the beam in front of it. In Figures 8 through 10 the beam head continues to track in toward the channel centered at zero. In these same frames the last segment of beam continues to move toward an equilibrium point provided by the prior segments of beam but overshoots. A careful examination of Figures 8 through 10 shows segments of the beam other than the last segment oscillating in a similar fashion about the equilibrium position established by preceding beam. The validity of the results at $Z = 478$ cm shown in Figure 10b is questionable. On one hand, strong gradients in X are sometimes an indication of the breakdown of the simulation. On the other hand, the segments of beam are oscillating at different frequencies, because of the shift in mean betatron wavelength with X . Therefore, sharp changes with X may be the manifestation of a mere phase difference in the oscillations.

Case 2: Density Channel

The detailed analysis of the case for tracking near a density channel is somewhat more complicated than that of the pre-existing conductivity channel. The gross result is that less of the beam tracks and that the portion that does track exhibits weaker tracking than in the pre-existing conductivity channel case.

Many of the results of the density channel case can be understood by comparison to the conductivity channel case. This comparison is not straightforward, however, because the monopole behavior of the beam differs going into a density channel in contrast to going into a conductivity channel. At the same point in X near the beam head the radii of the beams for the two cases

are quite different. The comparisons are therefore not made at the same X position but rather at two X positions where the beams have similar radii.

Figures 11, 12 and 13 show the tracking forces and monopole conductivities as a function of r for the density channel case and the pre-existing conductivity channel case. The $X = 5.5$ cm point was taken for the density channel case and the $X = 4.5$ cm point was taken for the conductivity channel case in Figure 11. The beam radii are approximately equal at those points. The same plots are shown in Figures 12 and 13 at 0.75 cm earlier and 1 cm later in X , respectively. At the observable X positions (the code does not generate output for every X position) the on-axis tracking force is maximum at 5.5 cm point in X for the density channel case.

As can be seen in Figure 11c, the conductivity generated in the channel has reached a value of about $2 \cdot 10^9 \text{ sec}^{-1}$. This value is comparable to the corresponding pre-existing conductivity in case 1 shown in Figure 11d, although its profile is more narrow. Note that at a radius of about 0.5 cm the conductivity is above the limiting conductivity of $3 \cdot 10^9 \text{ s}^{-1}$ discussed in conjunction with the analytic model in Ref. (1). The tracking force on an electron for the density channel case shown in Figure 11a has a value on axis of about $1.8 \cdot 10^{-9}$ dyne. This is comparable to the magnitude of the corresponding force for the pre-existing conductivity channel case shown in Figure 11b. The force for the density channel case falls off away from axis due to the higher conductivity noted in Figure 11c.

At the earlier X position of Figure 12 the tracking force in the density channel case is down by an order of magnitude whereas the forces in the pre-existing conductivity channel case have dropped by only about 20%. Figure 12c shows that sufficient conductivity has yet to form in the density channel and

significant tracking forces are therefore delayed. At the later X position of Figure 13, the conductivity on axis in the density channel is well above the $3 \cdot 10^9 \text{ s}^{-1}$ limit as is evidenced by the detracking force on axis in Figure 13a. At the comparable point in the pre-existing conductivity channel case the tracking force is still increasing.

The peak of the positive tracking force occurs at a point in X where the beam radius is about 1.8 cm as compared to about 0.8 cm in the pre-existing conductivity channel case. This larger radius plus the fact that the tracking region is more narrow in r places much of the beam outside the tracking region. The tracking region is also shorter in X due to the requirements for generating conductivity preferentially in the channel before tracking can occur. The conductivity generation in the channel must come from avalanche ionization in the channel. Avalanching requires some significant electric field, which must be present before the axial electric field peaks heralding the transition into the magnetic regime. In the cases considered here, the radial electric field supports the avalanche ionization, but it is of insufficient magnitude until just prior to the entrance into the magnetic regime.

Figures 14 and 15 show beam displacement and beam radius as a function of Z for the 5.5, 6.0, 7.0 and 14.75 cm points in X . Figure 14a shows that at least half the beam at $X = 5.5 \text{ m}$ is undergoing weak tracking. The beam core in that case represents about 52% of the total beam. If fewer of the outer particles were included in the core, the beam displacement would move inward more quickly. Conversely if more of the outer particles are included the displacement would move inward more slowly. Figure 14c shows that that segment of the beam is neither well pinched nor is it coming apart ballistically. From

the profile of the tracking force shown in Figure 11a, the beam core is expected initially to move in the tracking direction. As the beam expands at later Z , all the forces weaken. The $X = 5.5$ cm segment is accelerated initially in the tracking direction and at later Z continues to coast in that direction. The $X = 6.0$ cm segment shown in Figure 14b does not move for about a meter in Z . After that short period of indecision it also moves in the tracking direction and eventually moves in farther. The $X = 7.0$ cm segment shown in Figure 15a initially moves in the detracking direction but turns around; by $Z = 5$ m it has moved inward slightly farther than the $X = 6.0$ cm segment. The progression from $X = 5.0$ cm to $X = 7.0$ cm shows the change from a segment which initially tracks to one which initially detracks but then couples to the tracking segments of the beam. Behind the $X = 7.0$ cm point the initial detracking increases as the conductivity increases. By the $X = 14.75$ cm point shown in Figure 15b the majority of the beam ahead of it is still detracking so that it continues to detrack for the entire propagation distance in Z .

As compared to the pre-existing conductivity channel, the density channel has two distinct disadvantages. The first is the need to generate the conductivity in the channel as has been discussed. The second is a radial profile of the beam which is not well suited to beam tracking. With a pre-existing conductivity channel charge neutralization of the beam occurs sooner in X and the beam is more easily pinched. The peak of the tracking forces in case 1 is in a region in X where a stable beam head has formed. In contrast the density channel which allows for the formation of a conductivity channel, also promotes stronger return current. The beam is not as well pinched, and the point in X where the maximum tracking forces occur is in a region of transition between a pinched beam and an unpinched beam.

Figures 16 and 17 show beam displacement and beam radius as a function of X for different values of Z . The dashed line on the beam displacement plots represent the initial displacement of the beam. The short region of tracking can be seen in these plots (the portions below the dashed lines). Although not obvious in these figures, the region of tracking is broader in X for later Z values. This can be seen from the plot of beam displacement versus Z at $X = 7.0$ cm in Figure 15a. Because the beam is detracking at earlier Z and tracking at later Z , the region where the beam is tracking is wider in X at later Z . Although the results do show tracking, albeit weaker than that in the pre-existing conductivity channel case, the more prominent feature of the plots in Figures 16 and 17 is the strong detracking at the tail of the beam. Further work is required to determine the extent to which the beam head can move the beam into the channel against the effects of the plasma return currents in the channel.

Case 3: Conductivity and Density Channel

As one would expect, the results for the case of a pre-existing conductivity channel in a density channel has similarities to those in the other two cases. There is a region at the front of the beam which tracks, as in the pre-existing conductivity case and a fast transition into a region of strong detracking as in the density channel case. Figures 18 and 19 compare the tracking forces for the pre-existing conductivity and density channel with those for the pre-existing conductivity channel. Figures 20 and 21 show beam displacement and beam radius versus Z for fixed X values and Figures 22 and 23 show beam displacement and beam radius versus X at fixed Z values.

IV. Summary

This report has compared the analytic results for electrostatic tracking force calculations developed in Ref. (1) with the more general forces calculations done in the particle simulation, RINGBEARER II and has found good qualitative agreement. The analysis of beam tracking was then extended to consider the response of an electron beam to the presence of three types of channels: pre-existing conductivity channel, density channel and a density channel with finite pre-existing conductivity.

Of the three cases considered, case 1 for the pre-existing conductivity channel case showed the strongest tracking of the beam head into the channel. Also in case 1 the coupling of the beam body to the beam head was quite evident, and the beam body moved in toward the channel. In cases 2 and 3 for the density channel and the pre-existing conductivity plus density channel, respectively, tracking at the beam head was observed to a lesser degree. The latter cases also exhibited magnetic coupling of the beam just behind the beam head to the beam head but also showed strong detracking of the beam body.

The results quoted here are from a linearized monopole/dipole particle simulation and do not include the non-linear effects that are certainly necessary for a full description of beam tracking.

The authors wish to acknowledge the guidance of W. A. Barletta in defining the scope of this study and in the preparation of this report.

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Figure Captions

- Figure (1) Relative positions of beam to channel.
- Figure (2) Beam parameters and channel parameters for the three cases considered.
- Figure (3) Profiles of tracking force for pre-existing conductivity case at $X = 2.25, 6.00, 9.75$ and 19.75 cm.
- Figure (4) Comparison of tracking forces versus radius from electrostatic model of Ref. (1) and from RINGBEARER II for the pre-existing conductivity case.
- Figure (5) Comparison of on-axis tracking forces versus X (distance from beam head) from electrostatic model of Ref. (1) and from RINGBEARER II for the pre-existing conductivity case.
- Figure (6) Beam displacement and beam radius versus Z (propagation distance) at $X = 7$ cm and 14.75 cm for the pre-existing conductivity case.
- Figure (7) Beam displacement and beam radius versus X at $Z = 158$ cm and 222 cm for the pre-existing conductivity case.
- Figure (8) Same as Figure (7) for $Z = 286$ cm and 318 cm.
- Figure (9) Same as Figure (7) for $Z = 382$ cm and 414 cm.
- Figure (10) Same as Figure (7) for $Z = 446$ cm and 478 cm.
- Figure (11) Comparison of initial tracking forces and monopole conductivities at $Z = 1$ m between the density channel case at $X = 5.5$ cm and the pre-existing conductivity case at $X = 4.5$ cm.
- Figure (12) Same as Figure (11) for the density channel case at $X = 4.75$ cm and the pre-existing conductivity case at $X = 3.75$ cm.
- Figure (13) Same as Figure (11) for the density channel case at $X = 6.5$ cm and the pre-existing conductivity case at $X = 5.5$ cm.
- Figure (14) Beam displacement and beam radius versus Z at $X = 5.5$ cm and 6.0 cm for the density channel case.
- Figure (15) Same as Figure (14) at $X = 7$ cm and 14.75 cm.

- Figure (16) Beam displacement and beam radius versus X at $Z = 158$ cm and 318 cm for the density channel case.
- Figure (17) Same as Figure (16) at $Z = 478$ cm.
- Figure (18) Comparison of initial tracking forces at $Z = 1$ m between the density channel with pre-existing conductivity case and the pre-existing conductivity case at $X = 2.25$ cm and 4.75 cm.
- Figure (19) Same as Figure (18) at $X = 6.0$ cm and 9.75 cm.
- Figure (20) Beam displacement and beam radius versus Z at $X = 4.75$ cm and 7.00 cm for the density channel with pre-existing conductivity case.
- Figure (21) Same as Figure (20) at $X = 9.75$ cm and 14.75 cm.
- Figure (22) Beam displacement and beam radius versus X at $Z = 158$ cm and $Z = 318$ cm for the density channel with pre-existing conductivity case.
- Figure (23) Same as Figure (22) at $Z = 478$ cm.

Figure 1. Beam - channel geometry

BEAM

50 Mev beam
 No energy loss/no scattering
 0.5 cm radius
 15 cm rise length
 760 torr ambient pressure
 1.6×10^9 cm/sec mean transverse velocity

CHANNELS

Conductivity

Density

Conductivity and density

Figure 2. Cases to be considered

Figure 3. Initial tracking forces versus radius at $z = 1$ m – Pre-existing conductivity channel

Figure 4. Tracking force versus radius at $x = 6$ cm for pre-existing conductivity channel

Figure 5. On axis tracking force versus x -Pre-existing conductivity channel

Figure 6. Beam displacement and radius versus z at fixed x for a pre-existing conductivity channel

Figure 7. Beam displacement and radius versus x at fixed z for a pre-existing conductivity channel

Figure 8. Beam displacement and radius versus x at fixed z for a pre-existing conductivity channel

Figure 9. Beam displacement and radius versus x at fixed z for a pre-existing conductivity channel

Figure 10. Beam displacement and radius versus x at fixed z for a pre-existing conductivity channel.

Figure 11. Initial tracking forces and monopole conductivities versus radius at $z = 1$ m

Figure 12. Initial tracking forces and monopole conductivities versus radius at $z = 1$ m

Figure 13. Initial tracking forces and monopole conductivities versus radius at $z = 1$ m

Figure 14. Beam displacement and radius versus z at fixed x for a density channel

Figure 15. Beam displacement and radius versus z at fixed x -Density channel

Figure 16. Beam displacement and radius versus x at fixed z for a density channel

Figure 17. Beam displacement and radius versus x at fixed z for a density channel

Figure 18. Initial tracking forces and monopole conductivities versus radius at $z = 1$ m

Figure 19. Initial tracking forces and monopole conductivities versus radius at $z = 1$ m

Figure 20. Beam displacement and radius versus z at fixed x for pre-existing conductivity and density channel

Figure 21. Beam displacement and radius versus z at fixed x for pre-existing conductivity and density channel

Figure 22. Beam displacement and radius versus x at fixed z for pre-existing conductivity and density channel

Figure 23. Beam displacement and radius versus x at fixed z for pre-existing conductivity and density channels