130/310/86
PPPL-2300

UC20-A

M.L.R.

2
25

PPPL-2300

PPPL--2300

DE86 007530

LOCALIZED STABILITY CRITERION FOR KINK MODES
IN SYSTEMS WITH SMALL SHEAR

By

R.J. Hastie and J.L. Johnson

FEBRUARY 1986

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,

UNDER CONTRACT DE-AC02-76-CH0-2023

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

LOCALIZED STABILITY CRITERION FOR KINK MODES
IN SYSTEMS WITH SMALL SHEAR

R. J. HASTIE

Culham Laboratory, Abingdon, Oxon

OX14 3DB, England

and

J. L. JOHNSON

Plasma Physics Laboratory, Princeton University

Princeton, New Jersey 08544 USA

ABSTRACT

A localized magnetohydrodynamic stability criterion for ideal kink instabilities is determined for systems where the safety factor has a local minimum on a rational surface with no pressure gradient. These modes are stable in the cylindrical limit, but toroidal effects can make them unstable. They could provide a partial explanation for the rapid current penetration observed in tokamaks.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

I. INTRODUCTION

The rapid current penetration observed in the start-up of plasma current in tokamak devices has never been well explained. Classical models would lead to a prediction of a large skin current that would persist nearly on a resistive time scale. This would provide a local minimum in the safety factor in this region so that magnetohydrodynamic kink instabilities could be set up. The energy available to drive ordinary kink and tearing modes is too diffuse in most models to provide a satisfactory instability driving mechanism. The double-tearing mode as a possible mechanism has been proposed,¹ but it is strongly localized and a more detailed study is needed. Here we investigate an ideal magnetohydrodynamic current-driven instability localized around a magnetic surface where the shear vanishes ($q' = 0$) near the plasma edge. We obtain a stability criterion for localized modes, analogous to the Mercier criterion for pressure-driven interchanges. This, or its resistive counterpart, could provide a mechanism for a rapid current redistribution.

In the next section we carry through the reduction of the potential energy for a general three-dimensional equilibrium and obtain a stability criterion for localized modes. We restrict ourselves to an axisymmetric configuration with a nearly circular cross section in Sec. III in order to evaluate it. Some speculations and interpretations are offered in Sec. IV.

II. GENERAL THREE-DIMENSIONAL CONFIGURATIONS

We consider an arbitrary three-dimensional magnetohydrodynamic equilibrium such that ^{2,3}

$$\begin{aligned} \underline{B} &= \frac{dx(V)}{dV} \nabla V \times [q(V) \nabla \theta - \nabla \phi] \\ &= \frac{dx(V)}{dV} \nabla V \times [\nabla u + (q - q_0) \nabla \theta] \end{aligned} \quad (1)$$

with $u \equiv q_0 \theta - \phi$. We work in a V, u, θ coordinate system with V the volume contained inside a magnetic surface, and θ and ϕ arbitrary variables increasing from 0 to 1 in the poloidal and toroidal directions, respectively, such that the magnetic field lines appear straight in the system. The poloidal flux inside a surface is $\chi(V)$; $q(V)$ is the safety factor, with q_0 its value on the rational surface $V = V_0$ of interest. The problem is simplified by introducing the coordinate u such that $\underline{B} \cdot \nabla u = 0$ on this rational surface. Then the Jacobian is $J = (\nabla V \times \nabla u \cdot \nabla \theta)^{-1}$. The basic hypothesis made in this work is that there is no shear at this surface so that, in its vicinity,

$$q(V) = q_0 + \frac{d^2 q(V)}{2dV^2} (V - V_0)^2 + \dots \quad (2)$$

With such little shear stabilization, the system would be unstable to localized interchanges unless $dp(V)/dV$ is locally zero. Since the pressure is small at early times in the discharge and also since one would expect these modes to flatten the distribution locally in any case, we assume that there is no local pressure gradient, so that all the current must flow along the field line;

$$\underline{J} = \sigma \underline{B} \quad ,$$

with $\sigma \equiv \underline{J} \cdot \underline{B}/B^2$ a function of V because $\nabla \cdot \underline{J} = 0$.

It is convenient to work with the energy principle in its standard form⁴

$$2\delta W = \int d\tau [Q^2 + \underline{J} \times \underline{\xi}^* \cdot \underline{Q} + \underline{\xi} \cdot \nabla p \nabla \cdot \underline{\xi}^* + \gamma p |\nabla \cdot \underline{\xi}|^2] , \quad (3)$$

with $\underline{Q} \equiv \nabla \times (\underline{\xi} \times \underline{B})$ the perturbed magnetic field associated with the displacement vector $\underline{\xi}$. Following Bussac et al.,⁵ we project the displacement vector into

$$\underline{\xi} = \frac{1}{\chi} [(\nabla u \times \nabla \theta) \xi_V + (\nabla \theta \times \nabla V) \xi_S] + B \xi_B \quad (4)$$

where ξ_V is the component along the surface normal, ξ_S is a part of the surface component in the $\nabla \theta \times \nabla V$ direction, and ξ_B is along $\underline{B}$. It is well known that ξ_B enters δW only through the compression term $\gamma p (\nabla \cdot \underline{\xi})^2$ and can be used to eliminate this term. Our assumption that ∇p is locally zero eliminates the third term. We make a localization assumption that

$$\frac{\partial}{\partial V} \gg \frac{\partial}{\partial u} \sim \frac{\partial}{\partial \theta} \sim 1$$

when operating on ξ_V or ξ_S , which we expand as $\xi_V = \xi_V^{(0)} + \xi_V^{(1)} + \xi_V^{(2)} + \dots$, etc.

In lowest order

$$2\delta W^{(-2)} = \int dV du d\theta \left| (\nabla V \times \nabla u) \frac{\partial \xi_V^{(0)}}{\partial V} \right|^2 , \quad (5)$$

which we minimize by setting $\xi_V^{(0)} = 0$. In the next significant order

$$\begin{aligned}
 2\delta W^{(0)} &= \int dV du d\theta \left[-(\nabla V \times \nabla u) \left(\frac{\partial \xi_V^{(1)}}{\partial V} + \frac{\partial \xi_S^{(0)}}{\partial u} \right) \right. \\
 &\quad \left. + (\nabla \theta \times \nabla V) \left(\frac{\partial \xi_S^{(0)}}{\partial \theta} \right) \right]^2 \\
 &= \int dV du d\theta \left[\left| \nabla V \times \nabla u \right|^2 \left| \frac{\partial \xi_V^{(1)}}{\partial V} + \frac{\partial \xi_S^{(0)}}{\partial u} + \frac{(\nabla V \times \nabla u) \cdot (\nabla V \times \nabla \theta)}{|\nabla V \times \nabla u|^2} \frac{\partial \xi_S^{(0)}}{\partial \theta} \right|^2 \right. \\
 &\quad \left. + \frac{|\nabla V \times (\nabla V \times \nabla u)|^2}{|\nabla V \times \nabla u|^4} \left| \frac{\partial \xi_S^{(0)}}{\partial \theta} \right|^2 \right] . \quad (6)
 \end{aligned}$$

This is minimized by setting

$$\begin{aligned}
 \xi_S^{(0)} &= \xi_S^{(0)}(V, u) , \\
 \frac{\partial \xi_V^{(1)}}{\partial V} + \frac{\partial \xi_S^{(0)}}{\partial u} &= 0 , \quad (7)
 \end{aligned}$$

which implies that $\xi_V^{(1)} = \xi_V^{(1)}(V, u)$. In the next order, $2\delta W^{(2)}$ is like $2\delta W^{(0)}$, but with $\xi_V^{(1)}$ replaced by $\xi_V^{(2)}$ and $\xi_S^{(0)}$ by $\xi_S^{(1)}$. Thus we must set

$$\begin{aligned}
 \xi_S^{(1)} &= \xi_S^{(1)}(V, u) , \\
 \frac{\partial \xi_V^{(2)}}{\partial V} + \frac{\partial \xi_S^{(1)}}{\partial u} &= 0 . \quad (8)
 \end{aligned}$$

In the third order,

$$\begin{aligned}
 2\delta W^{(3)} &= \int dV du d\theta \left[(q - q_0) \xi_S^{(0)} \frac{\partial \xi_V^{(1)}}{\partial u} + \xi_V^{(1)} \frac{\partial}{\partial V} (q - q_0) \xi_V^{(1)} \right] \\
 &= \int dV du d\theta \left[\frac{\partial}{\partial V} (q - q_0) \left| \xi_V^{(1)} \right|^2 \right] , \quad (9)
 \end{aligned}$$

where we have used Eq. (7). An integration by parts shows that this term is small and contributes to $\delta W^{(4)}$. The same vector manipulation that was used in getting Eq. (6) and substitution for ξ_S as in getting to Eq. (9) reduces $2\delta W^{(4)}$ to

$$\begin{aligned}
 2\delta W^{(4)} = & \int dV du d\theta \left\{ \oint |\nabla V \times \nabla u|^2 \left| \frac{\partial \xi_V^{(3)}}{\partial V} + \frac{\partial \xi_S^{(2)}}{\partial u} + \frac{d^2 q}{dV^2} (V - V_0)^2 \frac{\partial \xi_S^{(0)}}{\partial u} \right. \right. \\
 & - \frac{(\nabla V \times \nabla u) \cdot (\nabla \theta \times \nabla V)}{|\nabla V \times \nabla u|^2} \left(\frac{\partial \xi_S^{(0)}}{\partial \theta} + \frac{d^2 q}{2dV^2} \frac{d}{dV} (V - V_0)^2 \xi_V^{(1)} \right) \Big|^2 \\
 & + \frac{|\nabla V \times (\nabla V \times \nabla u)|^2}{\oint |\nabla V \times \nabla u|^4} \left| \frac{\partial \xi_S^{(2)}}{\partial \theta} + \frac{d^2 q}{dV^2} \frac{d}{dV} (V - V_0)^2 \xi_V^{(1)} \right|^2 - \frac{d\sigma}{dV} \frac{d^2 q}{2dV^2} (V - V_0)^2 |\xi_V^{(1)}|^2 \Big\} .
 \end{aligned} \quad (10)$$

The last term is what remained from $2\delta W^{(3)}$. The first term is eliminated by choosing $\xi_V^{(3)}$. The Euler-Lagrange equation obtained by varying $\xi_S^{(2)*}$ can be integrated to give

$$\frac{\partial \xi_S^{(2)}}{\partial \theta} = - \frac{d^2 q}{2dV^2} \left| \frac{\partial}{\partial V} (V - V_0)^2 \xi_V^{(1)} \right| \left[1 - \frac{\oint |\nabla V \times \nabla u|^2 / |\nabla V|^2}{\oint \oint (|\nabla V \times \nabla u|^2 / |\nabla V|^2) d\theta} \right] , \quad (11)$$

where the integration constant has been chosen so that $\xi_S^{(2)}$ is periodic in θ . Since $\oint d\theta = \oint d\ell / B$ on a magnetic field line $V = V_0$, $u = u_0$,

$$\begin{aligned}
 2\delta W^{(4)} = & \int dV du \left\{ \frac{(d^2 q / 2dV^2)^2}{\oint \oint (|\nabla V \times \nabla u|^2) / |\nabla V|^2 d\ell / B} \left| \frac{d}{dV} (V - V_0)^2 \xi_V^{(1)} \right|^2 \right. \\
 & - \left(\frac{d\sigma}{dV} \right) \left(\frac{d^2 q}{2dV^2} \right) (V - V_0)^2 \left(\oint \frac{d\ell}{B} \right) |\xi_V^{(1)}|^2 \Big\} .
 \end{aligned} \quad (12)$$

A stability criterion for localized modes can easily be obtained by varying Eq. (12) with respect to $y^* \equiv (V - V_0)^2 \xi_V^{(1)*}$,

$$\frac{d^2 y}{dV^2} - \frac{(d\sigma/dV)}{(d^2 q/dV^2)} \left(\frac{d}{dV} \int \frac{|\nabla V \times \nabla u|^2}{|\nabla V|^2} \frac{d\ell}{B} \right) \left(\int \frac{d\ell}{B} \right) \frac{V}{(V-V_0)^2} = 0 \quad (13)$$

If we assume that $y = (V-V_0)^s (1+\dots)$, we obtain an initial equation such that s is complex if

$$D_K = 1 - \frac{8(d\sigma/dV)}{(d^2 q/dV^2)} \left(\int \frac{|\nabla V \times \nabla u|^2}{|\nabla V|^2} \frac{d\ell}{B} \right) \left(\int \frac{d\ell}{B} \right) < 0. \quad (14)$$

As in similar calculations,³ it is easy to argue that an unstable test function can be constructed to demonstrate instability if this criterion is satisfied, and that otherwise $2\delta W^{(4)} > 0$ for localized modes with $\partial \xi / \partial V \gg \partial \xi / \partial \theta$. It follows from Eq. (1) that $|\nabla V \times \nabla u|^2 / |\nabla V|^2$ is closely associated with $(B/RB_{\text{poloidal}})^2$. Also, $d\sigma/dV$ and $d^2 q/dV^2$ are related by the condition that $dp/dV = 0$. These relationships are difficult to establish in the general case. Since these modes are of primary interest in tokamaks, we restrict consideration to axisymmetric systems with nearly circular cross sections. Then all field lines on a magnetic surface are identical and the line integrals can be changed to surface integrals, so that

$$D_K = 1 - \frac{2(d\sigma/dV)}{\pi^2(d^2 q/dV^2)} \left(\frac{d}{dV} \int \frac{|\nabla V \times \nabla u|^2}{|\nabla V|^2} d\tau \right) < 0 \quad (15)$$

The evaluation of Eq. (15) is done in the next section.

III. AXISYMMETRIC SYSTEMS

Since our interest centers on tokamak operation and a formalism for axisymmetric systems has been established,⁶ we assume a low- β tokamak ordering for the evaluation and interpretation of the stability criterion of Eq. (15).

In this section we write our magnetic field as

$$\vec{B} = RB_0 [f(r)\nabla\phi \times \nabla r + g(r)\nabla\phi] \quad , \quad (16)$$

where surfaces of constant r are magnetic surfaces. The relationship of the (r, θ, ϕ) coordinate system to the cylindrical (X, ϕ, Z) system is defined by the equations

$$X = R - r \cos \theta - \Delta(r) + \sum_{m \geq 2} S_m(r) \cos(m-1)\theta + P(r) \cos \theta + \dots, \quad (17)$$

$$Z = r \sin \theta + \sum_{m \geq 2} S_m(r) \sin(m-1)\theta - P(r) \sin \theta + \dots,$$

with X the distance from the cylindrical axis, Z the distance along it, and ϕ the azimuthal angle; R is the radius of the magnetic axis, $\Delta(r)$ is the Shafranov shift, $S_m(r)$ measures the noncircularity of the magnetic surfaces with the $m = 2$ term introducing ellipticity, $m = 3$ triangularity, etc., and $P(r)$ is a surface relabeling parameter which we choose to be

$$P = \frac{r^3}{8R^2} + \frac{r\Delta}{2R} - \sum_M \frac{(m-1)}{2r} S_m^2 \quad (18)$$

so that

$$q = rg/Rf. \quad (19)$$

Here we have assumed an ordering

$$\frac{r}{R} \sim \frac{\Delta}{r} \sim \frac{S_m}{r} \sim \frac{f}{g} \sim \left(\frac{P}{r}\right)^{1/2} \sim \beta^{1/2} < 1 \quad .$$

The only difference between this one and that of Ref. 6 is that the noncircularity terms are ordered larger so that they contribute to the stability criterion. This change in ordering introduces a new set of equations for S_m ,

$$S_m'' + \left[\frac{2(rf^{(1)})'}{rf^{(1)}} - \frac{1}{r} \right] S_m' - \frac{(m^2-1)}{r^2} S_m = 0 \quad , \quad (20)$$

and adds the term $-\epsilon^2 \sum (m-1) S_m^2 / r^2$ into Eq. (30) and

$$\sum_m \left[\frac{m}{r} S_m'^2 + \frac{2m(m-1)}{r^2} S_m' S_m + \frac{m(m-1)^2}{r^3} S_m^2 + \frac{(rf^{(1)})'}{2rf^{(1)}} \left(-3S_m'^2 + \frac{(m-1)^2}{r^2} S_m^2 \right) \right]$$

into Eq. (39) of that work. The Jacobian is

$$\begin{aligned} J = rX \left[1 + \Delta' \cos \theta - \sum_m \left(S_m' - \frac{(m-1)S_m}{r} \right) \cos m\theta - \frac{r^2}{2R^2} \right. \\ \left. - \frac{r\Delta'}{2R} - \frac{\Delta}{R} + \sum_m \frac{\Delta' S_m}{r} \cos (m-1)\theta \dots \right] \quad , \quad (21) \end{aligned}$$

with primes denoting derivatives with respect to r , so that the volume inside a magnetic surface is

$$V(r) = 2\pi^2 Rr^2 \left(1 - \frac{r^2}{4R^2} - \frac{2\Delta}{R} + \dots \right) \quad . \quad (22)$$

We assume that q_0' , p_0' , and p_0'' vanish at the rational surface. Then

$$\sigma \equiv \underline{J} \cdot \underline{B} / B^2 = -g'/f \quad (23)$$

by direct calculation [using Eq. (15) of Ref. 6]. We can add Eqs. (36) and (39) of Ref. 6 to get an expression for g' to as high an order as we need:

$$\begin{aligned}
 & gg' \left(1 + \frac{r^2}{2R^2} + \frac{3r\Delta'}{R} + \frac{2\Delta}{R} \right) + ff' \left[1 + \frac{5r^2}{4R^2} + \frac{3r\Delta'}{R} + \frac{3\Delta}{R} - \frac{3\Delta'^2}{2} \right. \\
 & \quad \left. - \sum_m \left(\frac{3S_m'^2}{2} - \frac{(m^2-1)S_m}{2r^2} \right) \right] + \frac{f^2}{r} \left[1 + \frac{11r^2}{4R^2} + \frac{5r\Delta'}{R} + \frac{3\Delta}{R} - \frac{\Delta'^2}{2} \right. \\
 & \quad \left. - \sum_m \left(\frac{S_m'^2}{2} - \frac{2(m^2-1)S_m'S_m}{r} + \frac{(m^2-1)S_m^2}{2r^2} \right) \right] \quad (24)
 \end{aligned}$$

We take the derivative of this with respect to r to get an expression relating f'' and g'' :

$$\begin{aligned}
 & ff'' \left[1 + \frac{5r^2}{4R^2} + \frac{3r\Delta'}{R} + \frac{3\Delta}{R} - \frac{3\Delta'^2}{2} - \sum_m \left(\frac{3S_m'^2}{2} - \frac{(m^2-1)S_m^2}{2r^2} \right) \right] \\
 & \quad + gg'' \left(1 + \frac{r^2}{2R^2} + \frac{3r\Delta'}{R} + \frac{2\Delta}{R} \right) + \frac{g'^2}{f^2} \left(\frac{6r^2}{R^2} + f^2 \right) \\
 & \quad + \frac{12ff'g'}{f} \left(\frac{r^2}{R^2} - \frac{r\Delta'}{R} \right) + \frac{20g'r}{R^2} + f'^2 \left\{ 1 + \frac{29r^2}{4R^2} - \frac{9r\Delta'}{R} + \frac{3\Delta}{R} + \frac{9\Delta'^2}{2} \right. \\
 & \quad \left. + \sum_m \left(\frac{9S_m'^2}{2} + \frac{(m^2-1)S_m^2}{2r^2} \right) \right\} + \frac{2ff'}{r} \left[1 + \frac{27r^2}{2R^2} - \frac{4r\Delta'}{R} + \frac{3\Delta}{R} + 2\Delta'^2 \right. \\
 & \quad \left. + \sum_m \left(2S_m'^2 - \frac{(m^2-1)S_m'S_m}{r} - \frac{(m^2-1)S_m^2}{r^2} \right) \right] \\
 & \quad - \frac{f^2}{r^2} \left\{ 1 - \frac{71r^2}{4R^2} + \frac{5r\Delta'}{R} + \frac{3\Delta}{R} - \frac{3\Delta'^2}{2} - \sum_m \left(\frac{(4m^2-1)S_m'^2}{2} - \frac{8(m^2-1)S_m'S_m}{r} \right. \right. \\
 & \quad \left. \left. + \frac{(4m^2-1)(m^2-1)S_m^2}{2r^2} \right) \right\} = 0 \quad (25)
 \end{aligned}$$

The condition that $q' = 0$, the first derivative of Eq. (19), determines

$$ff' = \frac{f^2}{r} + \frac{fg'}{g} \quad (26)$$

We express g'' in terms of σ' by taking the derivative of Eq. (23).

Substituting all this into the second derivative of Eq. (19) then gives

$$\begin{aligned} \sigma' = & -\frac{Rf^2q''}{rg^2} \left[1 + \frac{3r^2}{4R^2} + \frac{4}{R} - \frac{3\Delta'^2}{2} - \sum_m \left(\frac{3S_m'^2}{2} - \frac{(m^2-1)S_m^2}{2r^2} \right) - f^2 \right] \\ & + \frac{f}{r^2} \left\{ \frac{8r^2}{R^2} - \frac{6r\Delta'}{R} + 12\Delta'^2 \right. \\ & \left. + \sum_m \left[2(m^2+5)S_m'^2 - \frac{12(m^2-1)S_m'S_m}{r} + \frac{2(m^2-1)^2S_m^2}{r^2} \right] - 4f^2 \right\} \quad (27) \end{aligned}$$

Similarly,

$$\begin{aligned} \left(\int \frac{|\nabla V \times \nabla U|^2}{|\nabla V|^2} d\tau \right)' = & \frac{4\pi^2 rg^2}{Rf^2} \left[1 - \frac{5r^2}{4R^2} - \frac{3r\Delta'}{R} - \frac{3\Delta}{R} + \frac{3\Delta'^2}{2} \right. \\ & \left. + \sum_m \left(\frac{3S_m'^2}{2} - \frac{(m^2-1)S_m^2}{2r^2} \right) + f^2 \right] \quad (28) \end{aligned}$$

Thus the stability criterion of Eq. (15) reduces to

$$\begin{aligned} D_K = & 9 \left(1 - \frac{4r^2}{9R^2} - \frac{24r\Delta'}{9R} - \frac{16\Delta}{9R} \right) - \frac{64 r^2}{R^2 (r^2 q''/q)} \left\{ 1 - \frac{3R\Delta'}{4r} + \frac{3R^2 \Delta'^2}{2r^2} \right. \\ & + \frac{R^2}{4r^2} \sum_m \left[(m^2+5)S_m'^2 - \frac{6(m^2-1)S_m'S_m}{r} \right. \\ & \left. \left. + \frac{(m^2-1)^2 S_m^2}{r^2} \right] - \frac{1}{2q^2} \right\} < 0 \quad (29) \end{aligned}$$

The first term in Eq. (29) is normally of order ϵ^2 larger than the rest, implying stability. However, if $(r^2 q''/q)$ is small, the other terms can be important. The criterion then reduces to

$$1 - \frac{64r^2}{9R^2(r^2 q''/q)} \left\{ 1 - \frac{3R\Delta'}{4r} + \frac{3R^2 \Delta'^2}{2r^2} + \frac{R^2}{4r^2} \sum_m ((m^2 + 5) S_m'^2 - \frac{6(m^2 - 1) S_m' S_m}{r} + \frac{(m^2 - 1) S_m^2}{r^2} - \frac{1}{2q^2}) \right\} < 0 \quad (30)$$

In the limit of small r/R and S_m that is usually treated, only the last term in the bracket survives so that the system would be stable. However, the effects of toroidicity or noncircularity can be important. The criterion can be violated at early stages in a circular discharge when q is large and Δ' is small if

$$(r^2 q''/q) < 64r^2/9R^2 \quad (31)$$

IV. DISCUSSION

The ideal localized kink instability, occurring at a minimum of $q(r)$ when a skin current is present, could provide an explanation for the current penetration that is observed early in a tokamak discharge. What is surprising is that the effect must rely on toroidal or on shaping effects. The mode would be stable in a cylindrical discharge of circular cross section.

When the stability criterion for ideal modes that was derived in the previous section is satisfied, there still remains the possibility of unstable resistive modes. These are of two types: localized resistive interchange modes and tearing modes. The resistive interchange mode is driven by the same

current gradient which is responsible for the ideal mode, but lacks the field-line-bending stabilizing effect provided by the shear. By analogy with pressure-driven interchanges,⁷ where $D_I > 0$ is replaced by $D_R > 0$ as the stability criterion for resistive modes, we conjecture that the appropriate stability criterion for resistive current-driven interchanges at the $q' = 0$ surface to be unstable is

$$\frac{2(d\sigma/dV)}{\pi(d^2q/dV^2)} \left(\frac{d}{dV} \int \frac{|\nabla V \times \nabla u|^2}{|\nabla V|^2} d\tau \right) > 0, \quad (32)$$

i.e., the effect of resistivity is simply to remove the q'^2 field-line bending term. Again, making use of the equilibrium relationship between σ' and q'' [Eq. (27)], we see that this criterion,

$$r^2 q''/q < 8r^2/R^2$$

for instability, is only slightly more stringent than the corresponding ideal criterion.

The quantity $\Delta' \equiv (\psi'/\psi)_{r_0+} - (\psi'/\psi)_{r_0-}$ must be calculated to study tearing mode stability. Here ψ is the perturbed magnetic field normal to the surface. A local calculation is possible only when the poloidal mode number m is large. In this case field-line bending stabilizes the tearing modes. For low- m modes the eigenfunctions are radially extended, and require a solution of the tearing mode equation¹ from $r = 0$ to $r = a$.

The results obtained here lead to a surprisingly different conclusion from what was obtained in Ref. 1. There it was shown that in a cylinder an unstable mode can always be found for the "double tearing mode" if two surfaces exist such that $m - nq = 0$. The instability obtained there

corresponded to an interchange-like mode localized between the two layers. The model studied in this work corresponds to the limit where these two surfaces coalesce, which causes this mode to vanish. We see that as the region where Furth and his colleagues found the instability disappears, the mode is eliminated. Thus we are led to conclude that this localization of the kink mode is stronger than might have been anticipated.

However, it is clear that a system with a skin current should be more unstable than one with a monotonic decreasing $J(r)$ since inspection of the sign of the kink-driving term in δW shows that this is destabilizing in the region where $j' > 0$ inside the singular surface, as well as in the usual region outside the singular surface where $j' < 0$.

ACKNOWLEDGMENTS

We are indebted to Dr. A.H. Boozer who suggested this problem and showed much interest in its development. One of us (RJH) enjoyed the hospitality of the Princeton Plasma Physics Laboratory on a Libby-Cockroff exchange while this work was done. The other (JLJ) was supported by the U.S. DoE Contract No. DE-AC02-76-CHO-3073 with Princeton University, while on loan from the Westinghouse Research and Development Center.

REFERENCES

- ¹H. P. Furth, P. H. Rutherford, and H. Selberg, Phys. Fluids 16, 1054 (1973).
- ²M. D. Kruskal and R. M. Kulsrud, Phys. Fluids 1, 251 (1958).
- ³J. M. Greene and J. L. Johnson, Plasma Phys. 10, 729 (1968).
- ⁴I. B. Bernstein, E. A. Frieman, M. D. Kruskal, and R. M. Kulsrud, Proc. Roy. Soc. (London) 244A, 17 (1958)
- ⁵M. N. Bussac, R. Pellat, D. Edery, and J. L. Soule, Phys. Rev. Lett. 35, 1638 (1975).
- ⁶J. M. Greene, J. L. Johnson, and K. E. Weimer, Phys. Fluids 14, 671 (1971).
- ⁷A. H. Glasser, J. M. Greene, and J. L. Johnson, Phys. Fluids 18, 875 (1975).

EXTERNAL DISTRIBUTION IN ADDITION TO UC-20

Plasma Res Lab, Austr Nat'l Univ, AUSTRALIA
 Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Prof. F. Cap, Inst Theo Phys, AUSTRIA
 Prof. Frank Verhaest, Inst theoretische, BELGIUM
 Dr. O. Palumbo, Dg XII Fusion Prog, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
 Dr. C.R. James, Univ of Alberta, CANADA
 Prof. J. Telchmann, Univ of Montreal, CANADA
 Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
 Prof. S.R. Sreenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Hannes Barnard, Univ British Columbia, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Chalk River, Nucl Lab, CANADA
 Zhengyu Li, SW Inst Physics, CHINA
 Library, Tsing Hua University, CHINA
 Librarian, Institute of Physics, CHINA
 Inst Plasma Phys, Academia Sinica, CHINA
 Dr. Peter Lukac, Komenského Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Prof. Schatzman, Observatoire de Nice, FRANCE
 J. Radet, CEN-BPG, FRANCE
 AM Dupas Library, AM Dupas Library, FRANCE
 Dr. Tom Muai, Academy Bibliographic, HONG KONG
 Preprint Library, Cent Res Inst Phys, HUNGARY
 Dr. R.K. Chhajlani, Vikram Univ, INDIA
 Dr. B. Dasgupta, Saha Inst, INDIA
 Dr. P. Kaw, Physical Research Lab, INDIA
 Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
 Prof. S. Oepman, Tel Aviv University, ISRAEL
 Prof. G. Rostagni, Univ Di Padova, ITALY
 Librarian, Int'l Ctr Theo Phys, ITALY
 Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
 Biblioteca, del CNR EURATOM, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Direc. Dept. Ig. Tokamak Dev. JAERI, JAPAN
 Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Prof. Sigeru Mori, JAERI, JAPAN
 Prof. S. Tanaka, Kyoto University, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Ichiro Kawakami, Nihon Univ, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Dr. D.I. Choi, Adv. Inst Sci & Tech, KOREA
 Tech Info Division, KAERI, KOREA
 Bibliothek, Fom-Inst Voor Plasma, NETHERLANDS
 Prof. R.S. Liley, University of Waikato, NEW ZEALAND
 Prof. J.A.C. Cabral, Inst Superior Tecn, PORTUGAL
 Dr. Octavian Petrus, ALI CUZA University, ROMANIA
 Prof. M.A. Hellberg, University of Natal, SO AFRICA
 Dr. Johan de Villiers, Plasma Physics, Nacoor, SO AFRICA
 Fusion Div. Library, JEN, SPAIN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
 Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glushikh, Inst Electro-Physical, USSR
 Institute Gen. Physics, USSR
 Prof. T.J.M. Boyd, Univ College N Wales, WALES
 Dr. K. Schindler, Ruhr Universitat, W. GERMANY
 Nuclear Res Estab, Julich Ltd, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Bibliothek, Inst Plasmeforschung, W. GERMANY
 Prof. R.K. Janev, Inst Phys, YUGOSLAVIA