LA-UR -91-939

CONF - 4

LA-UR--91-939

DE91 009960

APR 10 1991

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7400-ENG-84

TITLE ANYONS: A POSSIBLE MECHANISM FOR BINDING CHARGED PARTICLES

AUTHOR(S) D. Schmeltzer*
A. R. Bishop

SUBMITTED TO ANYONS: A POSSIBLE MECHANISM FOR BINDING CHARGED PARTICLES

*Dept. of Physics
City College of the City
University of New York
New York, NY 10031, USA

By submitting this manuscript, the author recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution or to allow others to do so for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

Los Alamos Los Alamos National Laboratory,
Los Alamos, New Mexico 87545

FORM NO. 836 Rev.
10-79 2020 1-01

DISTRIBUTION STATEMENT IS UNLIMITED

UNCLASSIFIED

Anyons: a possible mechanism for binding charged particles

D. Schmeltzer
Dept. Physics
City College of the City
University of New York
New York, NY 10031, USA

and

A. R. Bishop
Theoretical Division
Los Alamos National Laboratory
Los Alamos, NM 87545, USA

ABSTRACT

The hard core boson gas in $(2+1)$ -dimensions in the presence of a Chern-Simons term is considered, we find that the long wave description is given in terms of Coulomb (longitudinal) and transversal electrodynamics. The longitudinal part is described in terms of electric charges which carry vorticity. When binding of electrical charges occurs the transversal part is described by a photon field and corresponds to a superfluid. Using the same language we find that a spinless fermion is described by a massive photon. We use these concepts to understand the relation between the negative- U Hubbard model in the presence of a magnetic field, and vortices which carry electric charge induced by doping in the repulsive Hubbard model.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

I. Introduction

In this paper we consider the effect of the Chern-Simons term on a hard core boson system in $2+1$ dimensions. In addition we consider the effect of a Chern-Simons term on a repulsive Hubbard model and shows the similarity with a negative- U Hubbard model in an external magnetic field.

The major question of what is the microscopic mechanism which leads to the Chern-Simons term, or equivalently to the constraint that charged particle carry flux, remains unclear. In refs. 1, 2 we have suggested some answer to this central question. We have shown that frustration in the t - J model (next neighbor antiferromagnet couplings) leads to the constraint condition. Explicitly we have shown that holes leads to frustrations^(3,4) and therefore to the Chern-Simons term. Here we will not consider the question of a microscopic mechanism further here.

In Chapter II we begin our discussion with a hard core boson gas in the presence of a Chern-Simons term. Using the amplitude and phase representation for a spin- $\frac{1}{2}$ system we find the mapping of the hard core boson field. Integrating the short wave fluctuations we find a low energy action. This action is equivalent to a xy model in $2+1$ dimensions coupled to a C.S (Chern-Simons) term. Using the Villain transformation we construct an electrodynamic representation of the problem. The longitudinal part of the action is the Coulomb interactions with electric charges which carry vorticity, and neutral vortices. When binding occurs the transversal part is similar to a photon field with linear dispersion. In the language of bosons it corresponds to a superfluid.

In Chapter III we consider the relations between the repulsive Hubbard model with the C.S term and the attractive Hubbard model. Both models are described by a hard core boson hamiltonian. For the negative U case the hard core corresponds to a local pair with charge $2e$. When a magnetic field larger than H_{c1} is applied the flux penetrates forming a vortex lattice. For $T \rightarrow T_c$ and field $H \rightarrow H_{c2}$ the vortices form a normal or liquid

superfluid phase. For positive- U the boson operators describe spin excitations. Doping introduces frustration. For doping $\delta > \delta_c$, the vortices carry electric charge (this is the manifestation of the C.S. term in the Coulomb gauge). As in the negative- U case, a vortex lattice which carries electric charge is formed. As a result the charged vortex fluid can form the following phases: superfluid liquid with charge e , normal liquid, and a superconductor of charge $2e$.

II. Hard core bosons in the presence of a Chern-Simons term

We consider a hard core boson gas in the presence of a C.S. term. This represents the formulation of the anyon problem in the bosonic language,^(1,2,4,7,8) with action

$$S = \int d\tau \left\{ \sum_{\mathbf{r}} \phi^\dagger(\mathbf{r}, \tau) [\partial_\tau - i A_0 - \mu] \phi(\mathbf{r}, \tau) - t \sum_{\mathbf{r}, \mathbf{r}+\boldsymbol{\mu}} \phi^\dagger(\mathbf{r}, \tau) e^{i A_\mu} \phi(\mathbf{r} + \boldsymbol{\mu}, \tau) + H.C. \right. \\ \left. - i\theta/2 \int d^2x \varepsilon_{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \right\}. \quad (1a)$$

We consider a charged hole system with a concentration, δ , of holes. $\phi^\dagger, \phi$ are hard core bosons variables, A_0, A_1, A_2 and θ are the gauge field and the C.S. parameter which introduces the constraint that the electric charge carries flux, $\theta \nabla \times A = \phi^\dagger \phi$. The hard core bosons are described in terms of spin-1/2 variables: $\phi^\dagger = \frac{1}{2} L^+ = \frac{1}{2} \sin \theta(\mathbf{r}, \tau) e^{i\varphi(\mathbf{r}, \tau)}$, $\phi = \frac{1}{2} L^- = \frac{1}{2} \sin \theta(\mathbf{r}, \tau) e^{-i\varphi(\mathbf{r}, \tau)}$, $\phi^\dagger \phi = \frac{1}{2} (1 + L_z)$. We introduce the phase φ and the density $\rho(\mathbf{r}, \tau) = \left[\frac{1}{2} \sin \theta(\mathbf{r}, \tau) \right]^2$ as our new variables, with

$$\int D\phi^\dagger D\phi e^{-S} = \int D\varphi D(\cos \theta) e^{-S} = \int D\varphi D\rho J(\rho) e^{-S}. \quad (1b)$$

Here $J(\rho)$ is the Jacobian

$$J(\rho) = \exp \left\{ -\frac{\Lambda^2 \Lambda_f}{2} \text{Ln} \left[\frac{1 + 4\rho(\mathbf{r}, \tau)}{4\rho(\mathbf{r}, \tau)} \right] \right\}. \quad (1c)$$

Λ, Λ_T are the spatial and temporal cutoffs. In the continuum limit we have for Eq. 1a,

$$\hat{S} = \int d\tau \int d^2x L \quad (2a)$$

$$L = L_\varphi + L_\rho + L_{C.S.} \quad (2b)$$

$$L_\varphi = \Lambda^2 \rho(x, \tau) [i \partial_\tau \varphi + i \Lambda_T A_0] - 2 \Lambda_T \Lambda^2 \rho(x, \tau) \left[\sum_{\mu=1,2} \cos(\Lambda^{-1} \partial_\mu \varphi - A_\mu) - 1 \right] \quad (2c)$$

$$L_\rho = \frac{1}{2} \Lambda_T (\nabla r)^2 - \frac{1}{2} M^2(x, \tau) [\rho(x, \tau) - r^2(x, \tau)] - \frac{\mu}{2} [1 + (1 - 4\rho(x, \tau))^{1/2} - 2\rho] \\ + \frac{\Lambda^2 \Lambda_T}{2} \text{Ln} \left[\frac{1 - 4\rho(x, \tau)}{4\rho(x, \tau)} \right] \quad (2d)$$

$$L_{C.S.} = \Lambda \Lambda_T \{ -i\theta A_0 [\partial_1 A_2 - \partial_2 A_1] - i\theta \left(\frac{\Lambda}{\Lambda_T} \right) \{ A_2 \partial_\tau A_1 - A_1 \partial_\tau A_2 \} \} . \quad (2e)$$

L_φ is the phase dependent part in the presence of the gauge fields A_0, A_1, A_2 and density $\rho(x, \tau)$. L_ρ represents the density Lagrangian: it includes the Jacobian $J(\rho)$ and the chemical potential μ . $\partial L_\rho / \partial \mu = 0 \rightarrow \langle \rho \rangle = \kappa(1 - \kappa)$. $M^2(x, \tau)$ is the Langrange multiplier which fixes the condition $\rho(x, \tau) = r^2(x, \tau)$. $L_{C.S.}$ is the Chern-Simons term in the Coulomb gauge.

In order to construct a low energy action we integrate the density fluctuations $\tau(x, \tau)$, $\lambda(x, \tau) = \rho(x, \tau) - \rho_c(x, \tau) = \rho_c(x, \tau) + \tau(x, \tau)$, $M^2(x, \tau) = M_c^2 + \lambda(x, \tau)$.

We find for L_φ :

$$L_\varphi = \frac{1}{2} \Lambda^2 \Lambda_T^{-1} q_T [\partial_\tau + \Lambda_T A_0 + i(\rho_c/q_T) \Lambda_T^2]^2 \\ + 2\Lambda^2 \Lambda_T \sum_{\mu=1,2} q_\perp [\cos(\Lambda^{-1} \partial_\mu \varphi - A_\mu) - 1] . \quad (3a)$$

The coupling constant $g_r, g_\perp$ are a function of the density $\rho = \rho_c$: $g_\perp \approx \rho_c, g_r \approx (4\rho_c)^2$. Using the Villain representation,⁽⁶⁾ we perform duality transformations following the steps given in ref. 4. We find

$$\begin{aligned} \tilde{L} = & \frac{\Lambda^{-2}\Lambda_r}{2g_r} (\Lambda^{-1}\nabla_\perp \times a_\perp)^2 + \frac{\Lambda^{-2}\Lambda_r^{-1}}{2g_\perp} [(\partial_\tau a_\perp)^2 + (\Lambda^{-1}\nabla_\perp a_\tau)^2] \\ & - i\Lambda A_0[\Lambda^{-1}\nabla_\perp \times a_\perp + \theta\Lambda\nabla_\perp \times A_\perp] - i\Lambda^2\theta[(A_2\partial_\tau A_1 - A_1\partial_\tau A_2) + (A_2\partial_\tau a_2 - \\ & A_1\partial_\tau a_1)] + i a_\tau(\partial_1 A_2 - \partial_2 A_1) + i2\pi l_\perp a_\perp + i2\pi l_\tau a_\tau. \end{aligned} \quad (4)$$

$\vec{A} = (A_\perp, A_0)$ is the original gauge field and $\vec{a} = (a_\perp, a_\tau)$ are the gauge fields which describes neutral vortices and their currents,⁽⁴⁾ $n_0 = \partial_1 a_2 - \partial_2 a_1, n_\mu = \varepsilon_{\mu\nu\lambda} \partial_\nu a_\lambda$. $\vec{l} = l_\perp, l_\tau$ are integers fields which again can be expressed as $l_\mu = \varepsilon_{\mu\nu\lambda} \partial_\nu b_\lambda$, where $\vec{b}$ is a new gauge field.

In the Coulomb gauge we have

$$\delta \tilde{L} / \delta A_0 = \Lambda^{-1} \nabla_\perp \times a_\perp + \theta \Lambda \nabla_\perp \times A_\perp = 0. \quad (5a)$$

This allows the replacement $a_\perp = A_\perp$. In addition we have: $\theta(\nabla \times A) = \rho(r, \tau)$. This allows us to use the dual picture where $\theta(\nabla \times A)$ is replaced by the density ρ . In the Coulomb gauge we obtain from Eqs. 4 and 5a,

$$L = L_\perp + L_\parallel \quad (5b)$$

$$L_\perp = \frac{1}{2J_r} (\nabla_\perp A_\perp)^2 + \frac{1}{2J_\perp} (\partial_\tau A_\perp)^2 + i2\pi l_\perp A_\perp \quad (5c)$$

$$L_\parallel = \frac{1}{2J_\perp} (\nabla_\perp a_\tau)^2 + i2\pi a_\tau \left[\frac{1}{2\pi} (\nabla_\perp \times A_\perp) + l_\tau \right]. \quad (5d)$$

$J_r, J_\perp$ are the coupling constants given in term of g_r and $e_\perp$, $J_r^{-1} = \Lambda_r g_r^{-1}$, $J_\perp^{-1} = \Lambda^{-1} g_\perp^{-1}$. $L_\perp$ is the transversal part and $L_\parallel$ the longitudinal part with charge $Q(x, \tau)$

$$Q(x, \tau) = \frac{1}{2\pi} (\nabla_\perp \times A_\perp) + l_r = \frac{\rho(x, \tau)}{2\pi\theta} + l_r . \quad (5e)$$

The integration of a_r in Eq. 5d gives rise to the Coulomb interaction.

$$L_c \simeq J_\perp Q(x, \tau) [-\nabla^2]^{-1} Q(x, \tau) , \quad (5f)$$

with the neutrality condition

$$\sum_x Q(x, \tau) = 0 . \quad (5g)$$

When the charges in Eq. 5f are bound, $L_\perp$ describes a superfluid phase with a linear dispersion $\omega \sim K$. This correspond to a superconductor. (The electric charge of the superconductor is determined by θ ; for $\theta = \frac{1}{4}$ the charge is $2e$.)

When $\theta = \frac{1}{2\pi}$ the situation is very different. Then we have a fermionic system with $Q(x, \tau) = \rho(x, \tau) + l_r(x, \tau)$. In this case the constraint can be satisfied locally: $Q(x, \tau) \simeq 0$. We use $l_\mu = \frac{1}{2\pi} \varepsilon_{\mu\nu\lambda} \partial_\nu b_\lambda$ and find

$$\nabla \times A = - \nabla \times b , \quad (6a)$$

Therefore we find

$$L_\perp^{Fermion} = \frac{1}{2J_r} (\nabla \times A_\perp)^2 + \frac{1}{2J_\perp} (\partial_r A_\perp)^2 + i[A_1 \partial_r A_2 - A_2 \partial_r A_1] . \quad (6b)$$

In order see the similarity to a fermion system we add a weak external field $A_{\perp t}$. We replace $A \rightarrow A + A_{\perp t}$, integrate A and find to leading order $\delta L_{\perp} \sim \chi (\nabla_{\perp} \times A_{\perp t})^2$. This is the diamagnetic response of a free fermion in the presence of a weak external magnetic field.

For $\theta \neq \frac{1}{2\pi}$ only $\sum Q(x, \tau) = 0$ but not $Q(x, \tau) = 0$. When $\theta = 1/\pi$ we have $Q(x, \tau) = \frac{1}{2} \rho(x, \tau) + l_{\tau}(x, \tau)$. This is the *semion* case where two electric charges with $\pm 1/2$ and $\pm 1/2$ vorticity bind. In ref. 4 we have seen that for $\theta \neq \frac{1}{2\pi}$ integration of the neutral vortices l_{τ} leads to a corrections of the action of the form: $\rho(x)V(x, x')\rho(x')$ and the form of $V(x, x')$ is such that in the long wave limit it is a function of B^2 and not A^2 ($B = \nabla \times A$). Therefore to second order in A

$$L_{\perp}^{\theta \neq 1/2\pi} \simeq \frac{1}{2J_{\tau}} (\nabla_{\perp} \times A_{\perp})^2 + \frac{1}{2J_{\perp}} (\partial_{\tau} A_{\perp})^2. \quad (6c)$$

This means a superfluid excitation with linear dispersion and therefore superconductivity.

III. Duality relations in the repulsive and attractive 2d Hubbard models

Here the effect of the C.S. term on the repulsive Hubbard model is considered in analogy with the effect of a magnetic field on the negative-U Hubbard model. Our purpose is to investigate the possibility of dual flux lattice models for the +ve and -ve U Hubbard models.

In the $U \rightarrow 0$ limit we have exactly (in the half filled band case) the following two dimensional hardcore boson model (for generality, we also include intersite terms - this is the extended Hubbard model)

$$H = -J \sum_{r, r+\mu} b^{\dagger}(r) b(r+\mu) + J_{\perp} \sum_{r, r+\mu} n(r) n(r+\mu) - \mu \sum_r n(r) \quad (7a)$$

where $J_z = Jn$, $n \leq J = \frac{U^2}{4J}$, $b^\dagger(x)b(x) = n(x)$, $n(x) = 0, 1$. Here $b(x), b^\dagger(x)$ are hard core bosons which for the -ve U case correspond to local pairs of charge $2e$.⁽⁹⁾ In the -ve U case we have for a square lattice: $b(x) = e^{i\pi x} \tilde{b}(x)$, $b^\dagger(x) = e^{-i\pi x} \tilde{b}^\dagger(x)$, $x = (n_1 n_1, n_2 n_2)$. For $U > 0$, $b(x), b^\dagger(x)$ represent spin-1/2 spin excitations and Eq. (7a) represents the anisotropic spin-1/2 Heisenberg model.

It is our goal here to construct the dual flux lattice for the Hamiltonian given in Eq. (7a). For the -ve U model we can apply an orbital magnetic field $B = (\vec{\nabla} \times \vec{a})$ in the z direction perpendicular to the 2-dimensional plane. The $2e$ charged pair becomes superconducting for $T < T_c$,⁽⁹⁾ the Kosterlitz-Thouless transition temperature. The neutral vortices are bound and only above T_c are they free, forming a plasma phase. In the presence of an external magnetic field above H_{c1} ($B > H_{c1}$ and $T < T_c$) the magnetic field penetrates, forming a vortex lattice. Using a dual representation^(11,12) of the Ginzburg-Landau and Pitaevskii equation (which form the continuum representation of the hard core boson Hamiltonian given in Eq. (7a)), one finds a 2d boson at $T=0$ with the boson world lines $\tau_i(\tau) = (x_i(\tau), y_i(\tau))$. The new bosons represent the vortices created by the magnetic field. This new boson system formed by vortex lines is either a superfluid or a normal liquid.

For the -ve U case, the *anisotropic* spin-1/2 Heisenberg model will yield similar results as for the -ve U model. T_c now corresponds to the spin wave Kosterlitz-Thouless transition. The analog of H_{c1} is the critical concentration of charged holes of density ρ_c , $\rho_c = 1 - \delta_c$. This transition is realized when we dope the anisotropic Heisenberg model. Above this critical concentration $\delta > \delta_c$ (with $\delta = (1 - \rho)$), the presence of charged holes induces a rotational magnetic field $B(x, \tau) = \frac{1}{g} \rho(x, \tau)$ (θ represents the statistical parameter⁽¹³⁾). For this case we introduce a gauge field which couples to the hard core boson $b^\dagger(x) \partial_\mu x \rightarrow \rho(x) b^\dagger(x) \exp i\theta \mu x (b(x))$ such that $\nabla \times \vec{a} = \frac{1}{g} (1 - \rho)$. By this construction we have, for $T < T_c$ and $\delta > \delta_c$, the same situation that we have for the -ve U model. This means that for $T < T_c$

duality transformations, we often obtain a boson representation for the charged vortices.^{1,5} The basic difference between +ve and -ve U situations is that in the latter case the vortices carry electric charge and represent the charged holes introduced by doping. Here again we can have charged holes in a superfluid or normal phase liquid state. In addition to the superfluid phase of charge e it is also possible to obtain an attractive interaction between the charged holes.⁽⁴⁾ (This attraction is obtained after integrating out the neutral vortices: see ref. 4).

Therefore as a function of J , δ and $\eta < 1$ we can have, for the charge holes, the following phases: superfluid with charge e , superconducting with charge $2e$, normal liquid with charge e , and in addition, for $\delta < \delta_c$, a solid phase representing the spin wave phase.

Although most of our conclusion for the +ve U case can be derived by analogy with the Ginzburg-Landau-Pitaevskii equation, we prefer to use the duality method. Thus we write Eq. (7a) in the following form:

$$H = -J \sum_{x, x+\mu} b^+(x) \exp[iu_\mu(x)] b(x+\mu) + J_2 \sum_x n(x)n(x+\mu) - \mu \sum_x n(x) + \frac{B^2}{8\pi} - \frac{\vec{H} \cdot \vec{B}}{4\pi}, \quad (7b)$$

where

$$H = \begin{cases} H_c \rho(x) & , \quad U < 0 \\ \frac{1}{\eta} \rho(x) & , \quad U > 0 \end{cases} \quad (7c)$$

Equation (7b), with the convention given in Eq. (7c), represents the duality mapping for the two cases. Performing the variation with respect $B(x, \tau) = \vec{\nabla} \times \vec{a}$ we obtain $B(x, \tau) = \frac{1}{\eta} \rho(x, \tau)$, for $U > 0$, and $B(x, \tau) = H_c$, for $U < 0$. Here $\rho(x, \tau) = e^+e^-$ is the large density of holes introduced by doping.

Using the familiar Coulomb gas formalism we can derive the dual action for Eq. (7b).

We find

$$\begin{aligned}
H^{(dual)} = & -J' \sum_{x, x+\mu} \cos[\Delta_\mu \varphi(x) - \Delta_\mu \phi(x) - \chi_\mu(x)] \\
& + J_z \sum_{x, x+\mu} (\vec{\nabla} \times \vec{\chi})_x (\vec{\nabla} \times \vec{\chi})_{x+\mu} - \mu \sum_x (\vec{\nabla} \times \vec{\chi})_x + \sum_x \pi(x)^2 \\
& + \frac{1}{2} \sum_{x, x'} [\dot{N}(x) - \frac{1}{2\pi} B(x)] G(x, x') [\dot{N}(x') - \frac{1}{2\pi} B(x')] \\
& + \sum_x \left[\frac{B^2(x)}{8\pi} - \frac{H \cdot B(x)}{4\pi} \right]. \tag{8a}
\end{aligned}$$

Here $G(x, x') \sim -\ln|x - x'|$ is the 2d Coulomb potential with total vorticity $Q(x) = \dot{N}(x) - \frac{1}{2\pi} B(x)$. $N(x)$ represents the neutral vortices with the field operator $\exp[i\varphi(x)]$ such that $[\varphi(x), \dot{N}(x)] = i$. Similarly $\vec{\nabla} \times \vec{a} = B = \frac{1}{g} \rho(x)$ represents the charged vortex to which we attribute the field $\exp[i\phi(x)]$ with the commutation relations $[\phi(x), \rho(x')] = i$. $\chi_\mu(x)$ is the gauge field generated by the boson field $\chi(x) \sim b^\dagger(x)b(x + \mu)$, $\pi(x)$ is the momentum conjugate to $\chi_\mu(x)$, and $J' \sim \frac{1}{M_{core}}$, where M_{core} is the effective core mass. In order to find the effective action in terms of the charged vortex field $\rho(x)$, $\phi(x)$ and gauge field $\chi_\mu(x) \equiv \exp[iA_\mu(x)]$, we integrate out the neutral vortices $N(x)$, $\varphi(x)$. Following the same method as in ref. 4 we find

$$\begin{aligned}
S_{eff} = & \int d\tau \int d^d x \{ \psi^\dagger(x, \tau) (\partial_\tau - i A_\tau) \psi(x, \tau) \\
& + \frac{1}{2M_{core}} \psi^\dagger(x, \tau) (i \nabla^2 - A_\mu^2) \psi(x, \tau) \\
& - \mu \psi^\dagger(x, \tau) \psi(x, \tau) + \int d^d x' \psi^\dagger(x, \tau) \psi(x, \tau) V(x, x') \\
& \psi^\dagger(x', \tau) \psi(x', \tau) + \frac{1}{8\pi^2 J} (\nabla \times A)_x^2 \}. \tag{8b}
\end{aligned}$$

Here $\psi = \sqrt{\rho(x)}e^{i\phi(x)}$ represents the boson field which describes the charged hole vortices, $A_\mu(x, \tau)$ is the gauge field $\chi_\mu(x, \tau) \stackrel{\text{def}}{=} \exp[iA_\mu(x, \tau)]$, and $V(x, x')$ is the effective potential between the charged holes $\rho(x)$.

As a function of temperature and hole concentration, $V(x, x')$ can be attractive or repulsive. For short distances, we assume a repulsive potential $V(x, x') \simeq V_0\delta(x, x')$. We substitute $\psi = \sqrt{\rho(x)}e^{i\phi(x)} \simeq \sqrt{\rho_0}[1 + \frac{\pi(x)}{2\rho_0}]e^{i\phi(x)}$ and find S_{eff} as a function of $\pi(x)$ and $\phi(x)$:

$$\begin{aligned} S_{eff} &\approx \sum_{K, \omega_n} \left[\frac{1}{2} \eta^*(K, \omega_n) G^{-1}(K, \omega_n) \eta(K, \omega_n) \right. \\ &- \frac{\rho_0}{2M_{corr}} \sqrt{T} K^2 \cdot [\tilde{A}(K) \phi^*(K) - \tilde{A}(-K) \phi(K)] \\ &- \frac{\rho_0}{2M_{corr}} T \tilde{A}(K, \omega_n) \cdot \tilde{A}(-K, -\omega_n) \} + \\ &+ \sum_{i, J} \sum_{K, \omega_n} A_i(K, \omega_n) \frac{\delta_{iJ} - \frac{K_i K_J}{K^2}}{[\omega_n^2 + J^2 K^2]} A_J(-K, -\omega_n) , \end{aligned} \quad (83)$$

where $\eta^* = (\phi, \pi)$ and G^{-1} is given by:

$$G^{-1}(K, \omega_n) = \begin{bmatrix} \frac{\rho_0 K^2}{2M_{corr}} & 0 \\ 0 & V_0 + \frac{-\omega_n}{16\rho_0 M_{corr}} \end{bmatrix} . \quad (84)$$

We diagonalize the free particle boson $G^{-1}(K, \omega_n)$ and find the well known Bogliubov spectrum of a weakly interacting superfluid:

$$E(K) = \sqrt{\left(\frac{K^2}{2M_{corr}}\right)^2 + \left(\frac{\rho_0 V_0}{2M_{corr}}\right) K^2} . \quad (85)$$

The photon field can be integrated in Eq. (84). (A Gaussian integration with respect to A_i). Due to the nonzero value of ρ_0 the photon field is massive. The result of this is

that it will produce *two* different frequencies for the bosons. The phase excitation will be acoustic, $\omega \sim K \sim K$ and the density excitation $K \rightarrow 0$ will be $E_\pi(K) \sim \omega_n$, where ω_n is the plasma ($K \rightarrow 0$) frequency: $\omega_n^2 \sim 4\pi e^2 \rho_0^2/M_{core}$. In order to study relaxation of these bosons in the normal phase, we have to take into account higher order terms in the expansion of $\psi(x, \tau) = \sqrt{\rho_0 + \pi(x, \tau)}e^{i\phi(x)}$. These terms couple to the gauge field A and lead to relaxation of the boson field. This channel of relaxation is controlled by the plasma frequency ω_n , and becomes important for $T/\omega_n \gtrsim 1$. (ω_n is typical frequency of the photon field used by the boson to relax). In the region $T > \omega_n$ we expect that the boson relaxation will give rise to linear resistivity.⁽¹³⁾

To conclude, we have shown that the charged vortices can be described by an interacting bose gas coupled to a gauge field. It is this interplay of interaction and coupling which gives rise to superfluid and normal fluid behaviors.

REFERENCES

1. D. Schmeltzer, J. Mod. Phys. Vol. 3, No. 12 (1989) 2009-2018.
2. D. Schmeltzer, Physica A**108** (1990) 375-381.
3. D. Schmeltzer and A. R. Bishop, Phys. Rev. B **41**, 9603 (1980).
4. D. Schmeltzer and A. R. Bishop, Phys.Rev. B **10159** (1990).
5. A. R. Bishop et al. Europhys. Letters (1990).
6. J. Villain, J. Phys. (Paris) **36**, 581 (1988).
7. M. P. A. Fisher and D. H. Lee, Phys. Rev. B **30**, 2756 (1989).
8. D. H. Lee and M. P. A. Fisher, Phys. Rev. Lett. **63**, 903 (1989).
9. R. Menas, J. Ramnager and S. Robuskiewicz, Rev. Mod. Phys. **62**, 111 (1990).
10. D. Schmeltzer and A. R. Bishop, Phys. Rev. B**41**, 9603 (1990).
11. D. R. Nelson, Phys. Rev. Lett. **60**, 1973 (1988).

12. M. P. A. Fisher and D. H. Lee, Phys. Rev. **B39**, 2756 (1989).
13. N. Nagosa and P. A. Lee, Phys. Rev. Lett. **64**, 2450 (1990).