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TITLE FERMION LIQUID THEORY FOR HIGH TEMPERATURE SUPERCONDUCTORS

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FERMI LIQUID THEORY FOR HIGH TEMPERATURE SUPERCONDUCTORS

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ABSTRACT

In this article the Fermi liquid theory of metals is discussed starting from Luttinger's theorem. The content of Luttinger's Theorem and its implications for microscopic theories of high temperature superconductors are discussed. A simple quasi-2d Fermi liquid theory is introduced and some of its properties are calculated. It is argued that a number of experiments on $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $x > 0.5$, strongly suggest the existence of a Fermi surface and thereby a Fermi liquid normal state.

1. INTRODUCTION

The discovery of the new high temperature superconductors ^{1,2} (HTS) has resulted in an intense theoretical effort to understand the origin of the pairing mechanism responsible for the high transition temperatures, T_c . A crucial issue is whether the normal state, where $T > T_c$, can be characterized as a Fermi liquid. This is important for understanding the superconducting transition in these materials. In the standard picture it is the Cooper pairing of the Fermi liquid quasiparticles that results in a superconducting transition. In some of the more exotic theories ^{3, 4, 5} of HTS one is far removed from the standard theory. For example, in the RVB theories ^{3, 4} the superconducting transition is understood as a Bose condensation of pre-formed Bosons in the normal state. These exotic theories of HTS are most imaginative however, it is not at all clear if they are applicable to the high temperature superconductors.

In addition to the high transition temperatures the unusual normal state properties of the HTS have also been sources of inspiration for theory. Some of the unusual normal state properties of $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ are, the apparent linear temperature dependence of the resistivity in the $a-b$ (Cu-O) plane and the semiconducting like feature in the c axis direction, ⁶ the linear temperature dependence of the Hall coefficient, ⁷ and the temperature independent thermopower, ⁷ to name a few. To date none of the theories of HTS has provided a consistent account of these properties. What I would like to address in this article is whether Fermi liquid theory can account for these unusual normal state properties. The type of Fermi liquid one can envisage for the HTS materials must include the strong anisotropy, i.e., quasi-2d character, and the strong electronic correlations. Very little is known about the properties

of such a Fermi liquid so it is not at all obvious if this is applicable. For Fermi liquid theory to be applicable we must have a Fermi surface. In the absence of the usual direct probes of the Fermi surface, e.g., the de Hass-van Alphen effect, we can not be sure that a Fermi surface exists and if it does, what the volume might be. As we will see there are a number of experiments that provide indirect evidence for a Fermi surface. Given this we will consider some of the consequences of a Fermi liquid theory for the high temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$.

II. LUTTINGER'S THEOREM AND FERMI LIQUID THEORY

Before we begin to study the Fermi liquid features of the high temperature superconductors we state what this means for a metal. To do this we outline below the exact statement of Luttinger's theorem.⁸ The theorem depends on certain analytic properties of the single particle propagator matrix, $\hat{G}(\omega_n)$, where ω_n is the Matsubara frequency. This is the exact propagator and it satisfies Dyson's equation,

$$\hat{G}^{-1}(\omega_n) = \hat{\omega}_n - \hat{\epsilon}^0 - \hat{\Sigma}(\omega_n) \quad (1)$$

with $\hat{\epsilon}^0$ the bare particle energy matrix and $\hat{\Sigma}(\omega_n)$ the self energy. For the band case $\hat{G}$, $\hat{\epsilon}$, and $\hat{\Sigma}$ are diagonal in the momentum $\vec{p}$ thus we can express the matrix elements as follows,

$$(\hat{\epsilon})_{\vec{p}\vec{p}'} = \epsilon_{\vec{p}\vec{v}}^0 \delta_{\vec{p}\vec{p}'} \delta_{\vec{v}\vec{v}'} = \left(\hat{\epsilon}_{\vec{p}}^0 \right)_{\vec{v}\vec{v}'} \delta_{\vec{p}\vec{p}'} \quad (2a)$$

and

$$(\hat{\Sigma})_{\vec{v}\vec{v}'} = \delta_{\vec{p}\vec{p}'} \hat{\Sigma}_{\vec{p},\vec{v}\vec{v}'}(\omega_n) \delta_{\vec{p}\vec{p}'} = (\hat{\Sigma}_{\vec{p}})_{\vec{v}\vec{v}'} \delta_{\vec{p}\vec{p}'} \quad (2b)$$

Here $\vec{v}$ and $\vec{v}'$ represent both the spin and the band indices.

For each index $\vec{v}$ we can define a single particle distribution function $n_{\vec{p}\vec{v}}$ where,⁸

$$n_{\vec{p}\vec{v}} = \frac{1}{\beta} \sum_n \exp(\omega_n 0^+) (\hat{G}_{\vec{p}})_{\vec{v}\vec{v}} \quad (3a)$$

with

$$\hat{G}_{\vec{p}}^{-1} = \hat{\omega}_n - \hat{\epsilon}_{\vec{p}}^0 - \hat{\Sigma}_{\vec{p}}(\omega_n) \quad (3b)$$

and

$$\beta = 1/k_B T$$

From this we define the momentum distribution function $n_{\vec{p}}$,

$$n_{\vec{p}} = \sum_{\vec{v}} n_{\vec{p}\vec{v}} = \frac{1}{\beta} \sum_n \exp(\omega_n 0^+) \text{Tr}(\hat{G}_{\vec{p}}) \quad (4)$$

If we define the eigenvalues of the matrix $\hat{\epsilon}_{\vec{p}}^{-1} = \hat{\Sigma}_{\vec{p}}^{-1}(\omega_n)$ as $L_{\vec{p}\vec{p}}(\omega_n)$, where $\vec{p}$ represents the "true" band index, we can write Eq. (4) as follows,

$$n_{\vec{p}} = \frac{1}{\beta} \sum_n \sum_{\vec{p}} \exp(\omega_n \tau) \frac{1}{\omega_n - L_{\vec{p}\vec{p}}(\omega_n)} \quad (5a)$$

If we now let $T \rightarrow 0$ we can write the distribution function as,

$$n_{\vec{p}} = \frac{1}{2\pi i} \sum_{\vec{p}} \int_{-\infty}^{\mu} d\epsilon \left(\frac{1}{\epsilon - i\eta - L_{\vec{p}\vec{p}}(\epsilon - i\eta)} - \frac{1}{\epsilon + i\eta - L_{\vec{p}\vec{p}}(\epsilon + i\eta)} \right) \quad (5b)$$

where μ is the chemical potential.

The function $L_{\vec{p}\vec{p}}(\epsilon \pm i\eta)$ has the following analytic properties,⁸

$$L_{\vec{p}\vec{p}}(\epsilon \pm i\eta) = Q_{\vec{p}\vec{p}}(\epsilon) \mp i J_{\vec{p}\vec{p}}(\epsilon)$$

and

$$J_{\vec{p}\vec{p}}(\epsilon) > 0. \quad (6)$$

If we now assume that,

$$J_{\vec{p}\vec{p}}(\epsilon) = \begin{cases} C_{\vec{p}\vec{p}}^{(2)} (\epsilon - \mu)^2 \ln|\epsilon - \mu|, & 2d \\ C_{\vec{p}\vec{p}}^{(3)} (\epsilon - \mu)^2, & 3d \end{cases} \quad (7)$$

a number of rigorous consequences follow. To begin with there is a set of momentum values, $\vec{p}_F$, that satisfy the equation,

$$\mu - Q_{\vec{p}_F\vec{p}_F}(\mu) = 0 \quad (8)$$

where the momentum distribution function is discontinuous. This set of momentum values define the Fermi surface of the metal. In general only a few of the bands in a metal will satisfy Eq. (8) and these will be referred to as the "unfilled bands". It also follows that the total number of electrons, N , is given by,

$$N = \sum_{\vec{p}\vec{p}} \Theta(\mu - Q_{\vec{p}\vec{p}}(\mu)) = N_e + N_f, \quad (9)$$

where N_e is the number of electrons in the partially filled bands and N_f is the number of electrons in the full bands. The volume of the Fermi surface is determined by N_e and this is the same as for the non-interacting system.

The theorem of Luttinger⁸ is exact, depending only on certain analytic properties of the single particle propagator. It is sometimes claimed that this is a theorem which depends on a perturbative relationship between the interacting and non-interacting system. This is clearly wrong. The only place a perturbative approach is necessary is in the actual calculation of the single particle propagator for the Hamiltonian of the system, this is the "If" part of the theorem, Eq. (7). This is the part of the theorem that microscopic theories must concentrate on. If it can be demonstrated that a particular microscopic theory yields a self-energy that satisfies Eq. (7) then Luttinger's theorem guarantees that we have a Fermi liquid whose volume is fixed by N_C . It has not yet been demonstrated in any of the models of HTS that claim the existence or non-existence of a Fermi liquid that Eq. (7) is or is not satisfied. Of course to unambiguously show that a given Hamiltonian does or does not have a Fermi liquid regime the exact propagator must be calculated. This is a task that is currently beyond most theoretical approaches, with the possible exception of Monte Carlo techniques, thus, microscopic models have not been helpful in deciding the issue of the existence or non-existence of a Fermi liquid regime in the HTS.

This issue is particularly complicated in the high temperature superconductors due to the strong electronic correlations as well as the quasi-2d nature of the CuO_2 planes. For example the compound $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, with $x < 0.4$ is an antiferromagnetic insulator,⁹ which results from the strong electronic correlations, this is clearly not a Fermi liquid. As oxygen is added the antiferromagnetism is suppressed and for $x > 0.4$ a metallic normal phase appears above T_c .⁹ The exact nature of this metallic phase is not yet certain. I will argue that the normal phase is a Fermi liquid. However, due to the strong correlations, the quasi-2d nature of the CuO_2 planes, as well as the possibility of several bands crossing the Fermi level, this Fermi liquid will most likely have a number of unusual properties. Until detailed calculations are carried out for this Fermi liquid we cannot decide if some unusual property, e.g., the temperature dependence of the Hall coefficient,⁷ is or is not a signature for a Fermi liquid.

III. A FERMIL LIQUID THEORY FOR $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $x > 0.5$.

For Fermi liquid theory to be applicable to $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $x > 0.5$, we must have a Fermi surface. The traditional experiments, e.g., the de Hass-van Alphen effect, are not available, however, there is some strong indirect evidence for a Fermi surface. Some of the most striking evidence for a Fermi surface, although this is somewhat indirect, is the photoemission experiment of List et al.¹⁰ This experiment shows clear evidence for a sharp Fermi edge in $\text{EuBa}_2\text{Cu}_3\text{O}_{6+x}$, for $x > 0.5$. This is certainly consistent with the existence of a Fermi surface. More recently List et al. have also seen evidence for band dispersion.¹¹ Another experiment that strongly suggests the existence of a Fermi surface is the position-annihilation experiment of Bansil et al.¹² on $\text{YBa}_2\text{Cu}_3\text{O}_7$. The structure of the Fermi surface suggested by these experiments^{11, 12} is quite similar. I will come back to this point later on.

Given the possibility of a Fermi surface a question we can ask is, what is the volume enclosed by this surface? For a simple metal with a spherical Fermi surface the number density, n_C , of conduction electrons can be obtained from the Hall coefficient. In general the Hall coefficient can not be used to determine n_C since it depends on details of the band structure etc. (see section IV). We can envisage a number of Fermi liquid scenarios for the high temperature superconductors with rather different Fermi surface volumes, two of these I will outline below.

The first is the model proposed by Emery.¹³ In this model the copper d band is split by the correlations with the lower Hubbard band full and the upper one empty. When holes

are added they go into the oxygen p-band, eventually suppressing the long range antiferromagnetic order. The Fermi liquid that forms has a Fermi surface that encloses a volume δn_c where $\delta < 1$ is the fraction of additional holes per unit cell and $n_c \approx 10^{22} \text{ cm}^{-3}$ is the number density of copper holes. Another Fermi liquid we can have is one in which the copper d-state and oxygen p-state are strongly hybridized. The addition of holes here eventually suppresses the long range antiferromagnetic order. However, the Fermi surface in this case will enclose a volume of $n_c + \delta n_c$ holes.

Of these two Fermi liquids I believe that the second version, with some modifications, is appropriate to the HTS, in particular to $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $x > 0.5$. This choice is supported by the photoemission ¹¹ as well as the positron annihilation ¹² experiments. From these experiments, in particular the photoemission ¹¹, it is clear that there is a strong copper d-state character at the Fermi level as well as oxygen p-state. It is also clear from these experiments that more than one band crosses the Fermi surface as suggested by bandstructure calculations. ^{12,14} This need for more than one band at the Fermi surface is also evident in the Knight shift ¹⁵ and NMR ¹⁶ experiments on $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$.

Before starting the next section some additional comments about the Fermi liquids we have discussed are needed. It has been argued by Kim et al. ¹⁷ that the first Fermi liquid we discussed above does not satisfy Luttinger's theorem. This claim is without justification since Emery ¹³ has not calculated the exact propagator in this model. The claim by Kim et al. ¹⁷ is probably based on the assumption that the non-interacting reference system begins with the density n_c , i.e., one hole per CuO_2 plane. Adding enough holes, δn_c , to suppress the antiferromagnetism and then turning on the interactions would yield a Fermi liquid state with $n_c + \delta n_c$ holes, if of course the self energy has the property given by Eq. (7). However, Luttinger's theorem does not tell us how to count the number of electrons in partially filled bands. The Fermi liquid scenario proposed by Emery ¹³ could also satisfy Luttinger's theorem if we make the right choice for the non-interacting reference system. The non-interacting system would be δn_c holes, enough to suppress the antiferromagnetism, in the oxygen p-band with the copper d-band split and the lower Hubbard band full. If we assume that the Hubbard bands remain rigid as we turn on the interactions between the holes in the oxygen p-band, this includes as well the interactions with the copper band, and the self energy satisfies Eq. (7) this would satisfy Luttinger's theorem, i.e., this would be a Fermi liquid. Of course experiment does not support this choice for the Fermi liquid state of $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$. However, it is possible that the model of Emery ¹³ does have a Fermi liquid of the type he proposed but this has not been demonstrated.

Another point should be made in regard to the dimensionality of the Fermi liquid. Luttinger's theorem is valid for a 2d system. It is straightforward to go through the analysis of Luttinger ⁸ for the 2d system using the 2d form for the imaginary part of the self energy, see Eq. (7). Thus, at the level of Luttinger's theorem there is no difference between a 2d or 3d Fermi liquid. However, the presence of the $\ln|\epsilon - \mu|$ in the 2d quasiparticle lifetime suggest that this is a marginal dimension for a Fermi liquid. It is more difficult in 2d to show that Eq. (7) is satisfied for a given Hamiltonian since all of the diagrams that appear in a perturbative calculation of $J_{pp}^{(2)}(\epsilon)$ are potentially important.

IV. A FERMIL LIQUID MODEL FOR $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$

In the previous section we argued that some experiments ^{10, 11, 12} on $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$, $x > 0.5$, suggest the existence of a Fermi surface that encloses a volume of the order

$n_c \approx 10^{22} \text{ cm}^{-3}$. It is also evident from several experiments 10, 11, 12, 15, 16 that more than one band crosses the Fermi surface. For now we will consider only a single band Fermi liquid to stress more the role of correlations and the quasi-2d nature of the quasiparticle excitations. We will come back and discuss the need for a multiple band Fermi liquid when considering the Hall effect and the Knight shift.

The excitation spectra for a quasi-2d Fermi liquid is

$$\epsilon_{\vec{p}} - \mu = v_F (p_{\parallel} - k_F) + 2t_{\perp} \cos p_{\perp} c \quad (10)$$

Here $p_{\parallel}$ is the component of the momentum in the a-b plane and $p_{\perp}$ is the component along the c-axis and $c = 11.7 \text{ \AA}$ is the unit cell length in this direction. Here v_F is the Fermi velocity in the a-b plane and it is given by $v_F = k_F/m^*$ with m^* the effective mass in the a-b plane. From k_F , c and m^* we have the density $n = k_F^2/\pi c$, the density of states at the Fermi level $N(0) = k_F/(\pi v_F c)$, and the Fermi energy $T_F = k_F v_F/2$. The quasi-2d nature of this spectrum requires the hopping energy between nearest neighbor planes, $t_{\perp}$, to satisfy the condition $t_{\perp} \ll T_F$.

With this form for the quasiparticle excitation spectrum what are the characteristic signatures of this Fermi liquid? To begin with the low temperature specific heat $C_v = \gamma T$ with $\gamma \propto N(0)$. The linear specific heat has not been clearly seen in the high T_C materials, thus, we have no independent determination of $N(0)$. We can get an estimate of this from other experiments. The quasiparticle susceptibility, in the absence of spin orbit interactions is isotropic, and is given to order T^2 by,

$$\chi(T) = \chi(0) (1 - \beta (\frac{T}{T_F})^2) \quad (11)$$

with $\chi(0) = \mu^2 N(0)/(1 + F_0^a)$ and β is a dimensionless quantity of the order of $(1 + F_0^a)^{-2}$. The value of $\chi(0)$ is estimated to be $3 \times 10^{-4} \text{ emu/mole}$ after taking into account core diamagnetism etc. ¹⁸ The weak temperature dependence of $\chi(T)$ ^{15, 18} suggests that β is of the order of one or that F_0^a is close to zero. This gives a large effective mass enhancement $m^*/m \approx 10$. If we set $m^*/m = 10$ then $F_0^a \approx -0.3$ and $T_F = 1600 \text{ }^\circ\text{K}$, here we have used $n = n_c = 10^{22} \text{ cm}^{-3}$. This choice of the effective mass is consistent with the measured value of the London penetration depth ¹⁹ and reflectivity data ²⁰ on the $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ material.

With such a large effective mass we might expect a large contribution to transport properties coming from electron-electron scattering. For example the resistivity of a quasi-2d metal would behave in the following way: For $T \ll t_{\perp}$ we have a three dimensional metal, thus

$$\rho_{\parallel}(T) = \rho_{\parallel}^0 + A_{\parallel} T^2 \quad (12a)$$

and

$$\rho_{\perp}(T) = \rho_{\perp}^0 + B_{\perp} T^2 \quad (12b)$$

where $\rho_{\perp}$ is the resistivity in (perpendicular to) the a-b plane. For $T \gg T_F$ we have that ²¹

$$\rho_{\parallel}(T) = \rho_{\parallel}^{\circ} + A_2 T^2 \ln \frac{T_F}{T} \quad (13a)$$

and

$$\rho_{\perp}(T) = \rho_{\perp}^{\circ} + B_2 T^2 \ln \frac{T_F}{T} \quad (13b)$$

The residual resistivities are related if we assume that the impurity scattering is isotropic, thus

$$\rho_{\parallel}^{\circ} = \frac{m^*}{ne^2\tau_0} \quad (14a)$$

and

$$\rho_{\perp}^{\circ} = \frac{1}{2} \left(\frac{v_F}{c t_{\perp}} \right)^2 \rho_{\parallel}^{\circ} \quad (14b)$$

where τ_0 is the impurity scattering lifetime. If we further assume that the electron-electron scattering is also isotropic then,

$$B_2 = \frac{1}{2} \left(\frac{v_F}{c t_{\perp}} \right)^2 A_2 \quad (14c)$$

From the temperature dependence of $\rho_{\parallel}(T)$ for a quasi-2d metal it does not look as if this could be applicable to the HTS materials. In single crystal samples of YBa₂Cu₃O_{6.9} the resistivity in the a-b plane can be fit approximately with a straight line. ⁷ In Fig. 1 we plot a straight line (solid curve) which is a fit to the resistivity data of Tozer et al. ⁷ This fit to the data can be given as $\rho_{\parallel}(T) = \rho_{\parallel}^{\circ} + CT$ with $\rho_{\parallel}^{\circ} = 70 \mu\Omega - \text{cm}$, and $C = 1.3 \mu\Omega - \text{cm}/^{\circ}\text{K}$. Also shown in Fig. 1 is a plot of Eq. (13a) (dashed curve) with $\rho_{\parallel}^{\circ} = 122 \mu\Omega - \text{cm}$, $A = 30.3 \times 10^{-4} \mu\Omega - \text{cm}/^{\circ}\text{K}^2$ and $T_F = 10^3 \text{K}$. Note that this T_F is comparable to the value we obtained from $\chi(0)$. We also get a good fit to the data if we use $T_F = 1600 \text{K}$, $\rho_{\parallel}^{\circ} = 135 \mu\Omega - \text{cm}$, and $A = 21.8 \times 10^{-4} \mu\Omega - \text{cm}/^{\circ}\text{K}^2$. The main point to emphasize by this data fitting is that the cutoff in the logarithm is of the order of T_F . If F_0^{a} was closer to -1 we would expect the low temperature Fermi liquid behavior to persist over a much smaller temperature range. The fact that this logarithm appears to persist over such a broad temperature is again suggestive that the quasiparticle interaction F_0^{a} is small. Another point we note is that the size of the coefficient A_2 is comparable to the coefficient, A_1 , of the T^2 term in the strongly correlated A-15 compounds. ²¹ For example in Nb₃Sn we have that $A_1 = 70 \times 10^{-4} \mu\Omega - \text{cm}/^{\circ}\text{K}^2$.

We can estimate the size of A_2 if we assume that the resistivity lifetime, $\tau_p(T)$, can be approximated by the quasiparticle lifetime, $\tau_{qp}(T)$. This at best only gives an order of magnitude estimate for simple metals ²² and so we should not expect this to be any better for the HTS materials. If we assume that only s-wave scattering contributes we have, ¹⁹

$$\tau_{qp}^{-1}(T) \approx \frac{\pi}{2} \frac{\bar{W}}{T_F} T^2 \ln \frac{T_F}{T} \quad (15a)$$

and

$$\rho_{||}(T) - \rho_{||}^0 \approx \frac{\pi^2 c}{4e^2} \bar{W} \frac{T^2}{T_F^2} \ln \frac{T_F}{T} \quad (15b)$$

For s-wave scattering we have that $\bar{W} = \bar{W}^{\uparrow\downarrow} = |A_0^s - A_0^a|^2$ where for a charged system $A_0^s = 1$. If we only have s-wave scattering then the triplet scattering amplitude must vanish, thus $A_0^s + A_0^a = 0$ or $F_0^a = -0.5$. Note that this value of F_0^a is rather close to our earlier estimate. If we now plug these values of the Landau parameters into Eq. (15b) with $T_F = 10^3$ K we get that $A_2 \approx 40 \times 10^{-4} \mu\Omega \cdot \text{cm}/\text{K}^2$ and for $T_F = 1.6 \times 10^3$ K we get $A_2 \approx 16 \times 10^{-4} \mu\Omega \cdot \text{cm}/\text{K}^2$. Clearly, with these small values of T_F electron-electron scattering can contribute a large fraction to the resistivity.

In the c-axis direction the resistivity, in most of the early experiments on single crystals, ⁷ showed a semiconducting like up turn just above T_C . This is clearly inconsistent with the prediction of Eq. (13b). More recently this upturn has been questioned by Iye et al. ²³ In a number of their samples ²³ $\rho_{\perp}(T)$ is roughly parallel to $\rho_{||}(T)$ as a function of T . This is what we expect for a quasi-2d metal if Eqs. (14b) and (14c) are satisfied. As noted earlier for this quasi-2d band structure to be valid we must have $t_{\perp} \ll T_F$. We can estimate $t_{\perp}$ using Eq. (14b) and the experimental value of $\rho_{\perp}^0/\rho_{||}^0$ which is of the order of 10^2 . ^{7, 23} From this we have that $t_{\perp} \approx 20$ K ($T_F = 10^3$ K) or $t_{\perp} \approx 30$ K ($T_F = 1.6 \times 10^3$ K). Since T_C is larger than $t_{\perp}$ we are well into the regime where the resistivity is characterized by Eqs. (13a) and (13b). Given the likelihood of a Fermi surface and the potentially important electron-electron scattering contribution to the resistivity I believe the metallic value of $\rho_{\perp}(T)$ seen by Iye et al. ²³ is the correct behavior.

The one band Fermi liquid model described above leads to two rather general conclusions: The first is that the band structure is quasi-2d and the other is that these are **strongly correlated metals**. These features will remain regardless of the details that might be necessary for a more realistic Fermi liquid. A one band Fermi liquid is almost certainly incomplete. The Knight shift ¹⁵ on YBa₂Cu₃O_{6.9} shows clear evidence for at least two different bands at the Fermi level. The Hall effect also strongly suggests a more complicated band structure than a single band Fermi liquid.

If we consider a simple tight binding band structure with only nearest neighbor hopping then the Hall coefficient vanishes when the band is half full. Clearly, this is not the way to go about counting the number of conduction electrons! Including more realistic band structure effects, ^{17, 24} e.g., next nearest neighbor hopping, additional bands, etc. changes this in a rather non-trivial way. For example in the ab-plane it is found ^{12, 24} that R_{ab} is positive (hole-like) while in the c-axis direction R_c is negative (electron-like). Including a number of the details from the band structure is important for understanding some of the features of the Hall coefficient. However, the linear temperature dependence observed in the Hall coefficient is certainly beyond our current understanding of this effect. Most likely the

strong electronic correlations and the multi-band character of the Fermi surface are responsible for this effect. These must be included in the calculation of the Hall coefficient before any conclusions regarding the implications of the temperature dependence of this coefficient can be made.

V. CONCLUSION

In this article I have discussed the notion of Fermi liquid theory for layered metals and the application to HTS. One of the early Fermi liquid treatments of $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ was that proposed by Bedell and Pines.²⁵ This assumed an isotropic 3d Fermi liquid. This is clearly inconsistent with the resistivity anisotropy of the HTS. However, the strong electronic correlations leading to large effective masses was already evident from this analysis.²⁵ This is a feature that will survive as more realistic Fermi liquid descriptions emerge. It is also clear that the quasi-2d character of the quasiparticles of this Fermi liquid will be important. Many of the properties of a strongly correlated, quasi-2d, multi-band, Fermi liquid are not known and much research is still needed to elucidate these properties.

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Fig. 1 A plot of the resistivity in the a-b plane $\rho_{||}(T)$. The solid curve is a straight line fit to the data of reference 7 for a single crystal of $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$. The dashed curve is a plot of Eq. (13a) with $T_F = 10^3 \text{K}$, $A_2 = 30.3 \times 10^{-4} \mu\Omega \cdot \text{cm}/^\circ\text{K}^2$ and $\rho_{||}^0 = 122 \mu\Omega \cdot \text{cm}$.