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A MULTICELL SLUG FLOW HEAT TRANSFER
ANALYSIS OF FINITE LMFBR BUNDLES

By

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ABSTRACT

An analytical two-dimensional, multi-region, multi-cell technique has been developed for the thermal analysis of LMFBR rod bundles. Local temperature fields of various unit cells were obtained for 7, 19, and 37-rod bundles of different geometries and power distributions. The validity of the technique has been verified by its excellent agreement with the THTB calculational result.

By comparing the calculated fully-developed circumferential clad temperature distribution with those of the experimental measurements, an axial correction factor has been derived to account for the entrance effect for practical considerations. Moreover, the knowledge of the local temperature field of the rod bundle leads to the determination of the effective mixing lengths L_{ij} for adjacent subchannels of various geometries. It was shown that the implementation of the accurately determined L_{ij} into COBRA-IIIC calculations has fairly significant effects on intersubchannel mixing.

In addition, a scheme has been proposed to couple the 2-D distributed and lumped parameter calculation by COBRA-IIIC such that the entrance effect can be implanted into the distributed parameter analysis. The technique has demonstrated its applicability for a 7-rod bundle and the results of calculation were compared to those of three-dimensional analyses and experimental measurements.

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CHAPTER 1

INTRODUCTION

1.1 DESCRIPTION OF THE PROBLEM

The growth of nuclear power in the world's electric utility industry in the late 1960's has primarily relied on the light water reactors. These are reactors which consume fissionable fuel with low conversion ratios. The expected continued growth in electric power and consequent depletion of available fissionable fuels have prompted the effort of seeking ways to conserve uranium supplies. The fast breeder reactor is the most logical step in this direction among others.

A breeder reactor is one in which more fissionable fuel is produced than is consumed or one that has a conversion ratio greater than one. In order to reduce parasitic capture, the breeder reactor must be operated at the high neutron energy spectrum. Because of the absence of the moderator, fast reactor cores are much more compact than thermal reactors of the same power. Therefore they have much higher power densities and specific powers. Also the cores have more restricted coolant flow passages which may lead to severe heat removal problems.

Fig. 1.1 is a schematic diagram for the configuration of a pool type liquid metal fast breeder reactor. The cold primary sodium is discharged from each pump into a single pipe and flows down to a collector surrounding the subassembly support grid. Subassemblies are fed by nozzles in their lower part. The sodium leaving the subassemblies flows up through the upper part of the primary tank and enters the intermediate heat exchangers. At the outlet of these IHX, it comes back into the main tank and flows to the pump. A cross-sectional view of a typical LMFBR core design is shown in Fig. 1.2 the reactor core consists of approximately 200 hexagonal fuel assemblies arranged to form essentially a right-circular cylinder surrounded by a blanket and a reflector in both the radial and axial directions. A schematic diagram of a typical fuel assembly is illustrated in Fig. 1.3.

Approximately 200 and 60 rods are housed in the fuel and blanket assembly respectively. Continuous wire wrap are placed along the flow path to serve as supporting structure and to enhance mixing and heat transfer.

For turbulent flow of a non-metallic fluid, the major resistance of heat transfer is the thin viscous region adjacent to the clad surface. This explains why for these fluids local heat transfer rates can be described in terms of a heat transfer coefficient which is, to a very large extent, independent of the geometric configuration of the flow channel. The situation is entirely different for liquid

Fig.1.1.1 Schematic Diagram of a Pool Type Liquid Metal Fast Breeder Reactor

Fig.1.2 Cross-Sectional View of a Typical LMFBR Core

Fig.1.3 Schematic Diagram of a Typical Fuel Assembly

metals due to its high thermal conductivity, and consequently low Prandtl number, which suggests that conduction effect is no longer negligible and is also an important mode of heat transport when compared with turbulent transport characterized by eddy diffusivity. Furthermore the thickness of the thermal boundary layer in the liquid metal is substantially thicker than that of the hydrodynamic boundary layer. Therefore the heat transfer process in liquid metal is radially long-ranged and thus sensitive to the geometry of the coolant channel.

For neutronic considerations, rods in both fuel and blanket assemblies of present LMFBR designs are tightly packed and have fairly small pitch-to-diameter ratios; i.e., 1.24 for fuel and 1.08 for blanket assembly respectively. Close spacing of rods, accompanied by the high thermal conductivity of sodium, entails significant peripheral temperature variations in cladding and fuel, giving rise to structural problems such as rod bowing which may pose a severe safety problem. Even much higher variation in clad temperature can be expected for those rods near the assembly wall as the irregularity of the coolant channel geometry increases greatly. Naturally the clad temperature variation becomes even more severe if a reduction of coolant flow is experienced, i.e., forced convection effects become secondary and conduction effect dominates. Moreover, the fuel element must be designed under various neutronic,

thermal-hydraulic and material constraints to meet certain economical objectives. For instance, it has been estimated that a decrease of 5°C in the hot spot temperature is expected to result in a saving of $\$3 \times 10^6$ for the CRBR [1]. In order to predict quantitatively the magnitude of the circumferential variations of temperature and heat flux in the clad and coolant regions requires the simultaneous solutions of the momentum and energy equations for the velocity and temperature fields in all regions of the core as well as the blanket.

1.2 Review of Methods for Thermal Analysis of Rod Bundles

Because of the large number of the fuel and blanket assemblies, a complete thermal analysis for the entire core region is an enormous computational effort. In order to reduce the problem to a more manageable size, the research concerning the thermal analysis for LMFBR has been pursued in three major directions : 1. Porous Body Model Analysis, 2. Subchannel analysis, and 3. Distributed parameter analysis. These three kinds of analysis are designed to handle problems of multiassembly, multirod and detailed 2-D or 3-D temperature field, respectively. Application of these three different analyses depends on the designer's interest and focus.

1.2.1. Porous Body Model Analysis

In the porous body model analysis [2,3], the axial flow field of the assembly is characterized by an effective eddy diffusivity in the interior region and an average swirl flow in the peripheral region. By further neglecting the axial and transverse momentum coupling the problem is formulated and solved in terms of the energy equation only. Such a step is highly beneficial in terms of the simplicity which it returns to bundle analysis and the computation storage and time savings it yields. However, with both the axial and transverse momentum equations decoupled, the flow distribution must be supplied as input. This requires enormous amount of experimental data to cover normal and off-normal design cases [4].

1.2.2 Subchannel Models

Historically, subchannel analysis has been an important tool to establish the thermal performance of rod bundles for design purposes. Subchannel analysis usually divides the assembly into a number of subchannels, each of which is characterized by a bulk temperature, flow rate and pressure. By making appropriate assumptions concerning the flow and cross flow in these subchannels, the conservation equations of mass, momentum and energy are derived for each subchannel and the resulting set of equations obtained by appropriately coupling them is to be solved numerically.

The current digital computer codes like COBRA-IIIC [5] and THI-3D [6,7] used for LMFBR bundle analysis are outgrowths of the early COBRA [8] and THINC [9] developments. The conservation equations include a transverse momentum equation to account for the diversion cross flow and turbulent mixing. In these codes, wire wrap models are also introduced to account for the forced diversion cross flow effect.

Although both the subchannel analysis and the porous body model method have shown their capabilities for certain design purposes it is generally recognized that they have the following disadvantages:

- Detailed calculations [10,11,12] have shown that large uncertainties exist with respect to the transport properties for momentum and energy transfer which are used for the calculation of the respective quantity between subchannels. It was found that these properties cannot be described generally by simple correlations but are dependent upon various boundary conditions [13].
- The calculated clad temperature distributions are mostly not accurate enough for subsequent structural analysis because:
 1. the resolution in the azimuthal direction is not high enough.
 2. empirical correlations are used to establish the heat transfer coefficient in order to calculate an average clad temperature over an angular section. Mostly these correlations do not reflect the realistic geometry. As was

pointed out by Hsu [14], the use of the concept of the heat transfer coefficient under strongly varying geometries is highly questionable and in some cases even useless because it may generate locally infinite value for h .

It is evident that the only way to obtain the detailed 2-D clad and coolant temperature fields is to perform the distributed parameter analysis which solves the momentum and energy equations simultaneously without lumping the bundle into discrete subchannels. As a result, the outcomes of the distributed parameter analysis are pointwise values for local velocity, temperature and heat flux.

1.2.3 Distributed Parameter Methods

All previous research on distributed parameter analysis have employed a common procedure which divides the bundle into a number of unit cells. The cell selected is considered to be isolated from the other cells, i.e., there is no momentum or heat transfer across the boundaries of the cells. This procedure of analysis is generally designed as "single-cell analysis", which is only correct in the case of an infinite array of rods each with axisymmetric and identical mean power generation. However, if the bundle is enclosed by a wall which is always the case for LMFBRs, rod bowing occurs, or strong power gradients exist in the bundle, the single-cell analysis can no longer give satisfactory results.

Despite the needs for a multicell thermal analysis nothing is available in the open literature thus far. The only progress made so far is in the area of laminar flow. Mottaghian and Wolf [15,16] obtained solutions for round rod bundles using functions with special periodicity properties [17] which lead to a totally analytical solution for pin bundles where the rods are arranged on circles. Ullrich used an improved point-matching technique [18] to obtain the laminar flow solution in the symmetry section of a hexagonal 127 rod-bundle.

For two-dimensional slug flow and laminar flow heat transfer, solutions have been obtained for isolated unit cells in a hexagonal bundle. Sparrow, et.al. [19] in their work assumed a uniform clad outside temperature and performed an analysis for laminar flow. This kind of single region analysis becomes questionable if the irregularity of the flow area increases [20,21]. In order to predict quantitatively the magnitude of the circumferential variations of the temperature and the heat flux in the clad and coolant regions requires setting up the momentum and energy equations for the velocity and temperature fields in all regions of the cell and solving the equations simultaneously. This kind of multiregion analysis was performed by Axford [22] for the internal unit cell with coolant both in slug and laminar regimes. Wolf and Johannsen [23,24] obtained solutions resulting from a multiregion analysis for various

isolated unit cells in a hexagonal bundle for slug flow with special attention to the effect of rod displacement. In addition, Hsu [25,26] performed the multiregion analysis in the same geometries for both laminar and slug flow. All the aforementioned references treated the heat source density in the fuel as being constant over the cross section. This was extended by Wolf [27] in the case of slug flow to account for a tilted flux across the fuel in order to simulate the core/blanket flux gradients.

For fully developed turbulent flow situations, the review shows that a lot of research has been devoted to analyze two-dimensional fluid flow and heat transfer for the internal cell of bare rod bundles [28,29,30]. Due to the success of the hydrodynamics code VELASCO [31], Nijssing and Eifler [11] further extended the analysis to include the velocity fields in the edge and corner cells.

All aforementioned research has been only limited to bare rod bundles. No analysis can be found handling wire-wrapped bundles with the exception of the recently built FATHOM Code [32]. However, in this model, the velocity field is taken from the bare rod bundle calculation and the wire has no effect on momentum or energy transport mechanisms other than simply displacing fluid. Furthermore, the code assumes that the eddy diffusivities for momentum and energy are equal, which is not even true in the bare rod bundles [33,34].

For the three-dimensional solution, Meier [35] obtained the solution for the heat transfer problem in the various unit cells of a hexagonal bundle for slug flow. The method is a single region analysis with a constant heat flux imposed around the clad outside boundary. Furthermore, the analysis takes into account an arbitrarily axially varying heat flux distribution by using Duhamel's theorem [36]. The solution technique relied on the improved point-matching procedure [37] using an orthonormalization method for solving the overdetermined set of equations [38]. Surprisingly the results of the analysis agree quite well with the far more sophisticated turbulent analysis done by Nijssing and Eifler [12] in the thermal entrance region. This outcome is quite encouraging as it seems promising to start with a simple method in the first place. Recently, the 3-D problem was further extended by Wong [39] to a multicell slug flow analysis.

The review shows that there are relatively few developments in the area of the laminar mixed convection problem in bundle related geometries. Westinghouse developed a model [40] to simulate the buoyancy effects on the velocity and temperature distributions in the radial blanket assembly. Iqbal [41] considered the internal cell with boundary conditions of

the first and second kind around the circumference of the outside clad surface for a larger range of Ra-numbers. In Davis and Perona's work [42], the hexagonal geometry was approximated by an equivalent annulus. This approximation gives quite reasonable results for large P/D values which were studied by the authors. Similar analysis can also be found in Axford's work [43]. Kim [44] extended the analysis to include the clad region and simulated the power tilting effect by varying the heat flux at the clad inside surface.

In all these aforementioned distributed parameter analysis calculations, the computational effort involved is extremely high. In order to reduce these efforts, several attempts were made to overcome the disadvantages of the finite-difference method by using the curvilinear coordinate approach [45,46]. As was shown for internal cells [45], this method reduces the necessary nodes by a factor of 10. However, the generation of the orthogonal net may create some difficulties in the vicinity of the bundle wall and intersections of cells [46].

On the experimental side, the review indicates that there is relatively few data of temperature measurements. An experimental study was done by Dutton and Welty [47] for natural convection in an array of uniformly heated vertical cylinders.

In addition, clad temperature measurements were made by Subbotin, et.al. [48] for all types of cells in hexagonal

bundle for Pe numbers as low as 20. Recent 3-D temperature mappings in edge and corner fuel rods of a sodium cooled, bare 19-rod bundle obtained by Möller and Tschöke [49] show that the temperature variations are even higher than measured by the Russian authors.

The data in [49] seems to serve as a reliable base for comparison with any theoretical, 3D model which solves the heat transfer problem in vicinity of the bundle wall.

The foregoing section shows that there exists substantial gaps in:

- two-dimensional coupled multi-cell calculational capabilities for slug, laminar and turbulent flow situations as well as for mixed convection.
- effective computational tools in order to handle turbulent flow situations.
- An effective model to account for the wire-wrapped geometry.
- An analysis to fully integrate the subchannel analysis such as COBRA-IIIC and the distributed parameter analysis.
- The knowledge of effective transport parameters.

It is understood that these gaps cannot be closed at once. Therefore, the attached work schedule consists of small subsequent steps which can be solved in the near future and may serve as a basis for further decisions.

1.3 OBJECTIVES OF THIS THESIS

The objectives of the thesis can be summarized as :

1. Develop an efficient method to extend the present single cell analysis to multicell, multiregion analysis which includes effects of flow split and intraassembly power gradient.
2. Utilize the local temperature field resulted from the multicell calculation to determine effective transport parameters which can be implemented into current subchannel codes to upgrade their reliability with respect to the transport process.
3. Develop a scheme to combine the current subchannel codes, i.e., COBRA-IIIC and the distributed-parameter methods.

CHAPTER 2

SOLUTION OF TEMPERATURE FIELDS IN A UNIT CELL

It was pointed out earlier that the most common practice for thermal analysis of rod bundle is dividing the bundle into a number of unit cells which serves as the smallest basic unit for the analysis. For a multicell analysis, temperature fields in the various regions of all unit cells being considered in the analysis have to be derived and their solutions are to be solved either by a simultaneous direct solution technique or by a cell-to-cell iterative procedure. The solution technique will be presented and discussed in detail in chapter 4. In this chapter, the solution of temperature fields in fuel, clad and coolant for an individual unit cell is formulated for a finite hexagonal bundle. Different flow rates are assigned to different rods to simulate the flow split and intraassembly power tilting effects respectively.

2.1 GEOMETRIC AND THERMAL PARAMETERS OF A UNIT CELL

The geometry of an individual unit cell can be described by its number of sides, vertex angles (β_i), normal angles, (α_i), and normals to the sides (a_i). As shown in Fig. 2.1 the vertex angle is the angle between the line connecting the center and the vertex and the X-axis; the normal is the distance from the center to the side and

the normal angle is the angle between the normal and the X-axis. With these parameter the distance from the cell center to any point on any cell boundary is given by the relation:

$$c = a_i / \cos(\phi - \alpha_i) \quad (2.1)$$

$$\text{for } \beta_i \leq \phi \leq \beta_i + 1 \quad i = 1, 2, \dots, I$$

Thus the location of any point on the cell boundary can be precisely described by these geometric parameters.

A one-twelfth symmetry section of a 127-rod bundle is shown in Fig. 2.2. Geometric symmetry considerations enables one to subdivide this 30° section into a number of small unit cells. The cell boundaries are shown in Fig. 2.2 as dashed lines. It can be seen that there are seven types of unit cells based on their geometric configurations. However, closer examination indicates that some unit cells with the same geometric configurations are subject to different thermal boundary conditions because one of its boundary lines lies on the section symmetry line and consequently has an adiabatic boundary condition. Therefore, by taking the thermal boundary condition into consideration a total of ten unit cells can be identified in a one-twelfth symmetry section of a typical 127-rod bundle. The classification of these unit cells are summarized in table 2.1. They are

Fig.2.1 Geometric Parameters of a Unit Cell

Fig.2.2 Cross-sectional View of 1/12 of a 127 Rod Fuel Bundle

Table 2.1: Library of Unit Cells Considered in Analysis

type	unit cell	β_1	α_1	a_1	boundary ** conditions
1		$0, \frac{\pi}{6}$	0	$\frac{P}{2}$	2
2		$0, \beta_2, \frac{5}{6}\pi, \pi$ $\beta_2 = \frac{\pi}{6} + \tan^{-1} \frac{P}{2W}$	$\frac{\pi}{6}, \frac{2}{3}\pi, \pi$	$W, \frac{P}{2}, \frac{P}{2}$	0, 2, 2
3		$0, \frac{\pi}{3}, \beta_3, \pi$ $\beta_3 = \frac{\pi}{2} + \tan^{-1} \frac{2W}{P}$	$\frac{\pi}{6}, \frac{\pi}{2}, \pi,$	$\frac{P}{2}, \frac{P}{2}, W$	2, 2, 0
4		$0, \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \pi$	$0, \frac{\pi}{3}, \frac{2}{3}, \pi$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}$	2, 2, 0, 2
5					2, 2, 2, 2

Table 2.1: Library of Unit Cells Considered in Analysis (continued)

type	unit cell	β_1	α_1	a_1	boundary ** conditions
6		$0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$	$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}$	2, 2, 2
7		$0, \frac{\pi}{3}, \beta_3, \beta_4, \frac{5\pi}{3}, 2\pi$	$\frac{\pi}{6}, \frac{\pi}{2}, \pi$	$\frac{P}{2}, \frac{P}{2}, W, \frac{P}{2}, \frac{P}{2}$	2, 2, 0, 2, 2
8		$\beta_3 = \frac{\pi}{2} + \tan^{-1} \frac{2W}{P}$ $\beta_4 = 2\pi - \beta_3$			2, 2, 0, 0, 2
9		$0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$	$\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$	$\frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}, \frac{P}{2}$	2, 2, 2, 2, 2
10		$\frac{4}{3}, \frac{5}{3}, 2\pi$	$\frac{3}{2}\pi, \frac{11}{6}\pi$	$\frac{P}{2}$	2, 2, 2, 2, 0, 2

** Boundary Conditions: 0 - Adiabatic
1 - Dirichlet type
2 - Neumann type

characterized by their geometries as well as their thermal boundary conditions. Naturally the boundary conditions for each unit cell have to be prescribed during the course of the analysis.

2.2 GOVERNING DIFFERENTIAL EQUATIONS AND THEIR SOLUTIONS

By considering an arbitrary polygonal unit cell as shown in Fig. 2.1 three regions can be identified. These regions are defined for the nuclear fuel as $0 \leq r \leq a$, and $0 \leq \phi \leq \phi_m$; for the clad for $a \leq r \leq b$ and $0 \leq \phi \leq \phi_m$; and the coolant for $b \leq r \leq a_i / \cos(\phi - \alpha_i)$. The range of ϕ from 0 to ϕ_m is the angular section of the unit cell being considered. The governing differential equation for all these regions are the general energy equation [50].

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + q''' + T\beta \frac{DP}{Dt} + \mu \Phi^*, \quad (2.2)$$

where

T - temperature

k - thermal conductivity

q''' - power density

ρ - density

c_p - specific heat

β - compressibility

μ - viscosity

and ϕ^* is the viscous term given by:

$$\begin{aligned} \Phi^* = & 2 \left[\left(\frac{\partial V_x}{\partial x} \right)^2 + \left(\frac{\partial V_y}{\partial y} \right)^2 + \left(\frac{\partial V_z}{\partial z} \right)^2 \right] + \left[\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} \right)^2 + \left(\frac{\partial V_y}{\partial z} - \frac{\partial V_z}{\partial y} \right)^2 \right. \\ & \left. + \left(\frac{\partial V_z}{\partial x} + \frac{\partial V_x}{\partial z} \right)^2 \right] - \frac{2}{3} \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} \right)^2 \end{aligned} \quad (2.3)$$

The general energy equation is highly complex and its explicit solution requires enormous computational effort. However, Eq.(2.2) can be greatly simplified if appropriate assumptions are made:

1. only steady state condition is considered
2. liquid metal is incompressible
3. viscous effect is neglected
4. only thermally and hydrodynamically fully developed one phase flow is considered
5. the turbulent flow can be approximated by slug flow
6. neutron flux self-shielding effect is negligible
7. axial conduction effect is neglected
8. thermal conductivity and gap conductance are invariant with temperature and space
9. rod or duct bowing effects are neglected
10. no fuel cracking is assumed

Consequently, the temperature field in the fuel region reduces to the Poisson equation as:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \varphi^2} + \frac{q''}{k_f} = 0, \quad (2.4)$$

where k_f is the thermal conductivity of the fuel and V denotes the temperature field in the fuel region. In the clad region the energy equation reduces to the Laplace equation:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial t}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 t}{\partial \varphi^2} = 0, \quad (2.5)$$

where t denotes the temperature field in the clad region. Similarly the energy equation for the coolant region boils down to:

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} = \frac{\rho c_p u}{k_c} \left(\frac{\partial T}{\partial z} \right), \quad (2.6)$$

where

k_c - thermal conductivity of the coolant

u - Axial velocity of the coolant

$\frac{\partial T}{\partial z}$ - axial temperature gradient of the unit cell

and T denotes the temperature field in the coolant region. For a fully developed flow, the axial temperature gradient in the coolant region is constant and equal to the axial temperature gradient of the bundle which can be calculated by an energy balance relationship:

$$\dot{m}_B c_p \left(\frac{\partial T}{\partial z} \right)_B = (\rho \bar{u}_B A_T) c_p \left(\frac{\partial T}{\partial z} \right)_B = N \pi a^2 \bar{q}''' \quad (2.7)$$

where

$\dot{m}_B$ - bundle flow rate

$\bar{q}'''$ - averaged power density of the bundle

$\bar{u}_B$ - averaged bundle flow velocity

N - total number of rods in the bundle

and A_T - bundle flow area.

Upon rearranging Eq. (2.6) can be written as:

$$\left(\frac{\partial T}{\partial z} \right)_B = \frac{N \pi a^2}{\rho \bar{u}_B A_T c_p} \bar{q}''' = \left(\frac{\partial T}{\partial z} \right) \quad (2.8)$$

The axial temperature gradient as given by Eq. (2.8) can be substituted into Eq. (2.6) :

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} = \frac{N \pi a^2 \bar{q}'''}{k_c A_T} \left(\frac{u}{u_B} \right) \quad (2.9)$$

It has to be pointed out that the unit cell velocity ratio $u/\bar{u}_B$ is an input to the problem. A ratio of one implies a uniform velocity profile throughout the entire symmetry section. Naturally, $u/\bar{u}_B$ for a unit cell can also be inferred by the hydraulic characteristic of the unit cell and the bundle. This will be discussed further in chapter 4.

In the present study, Eqs. (2.4), (2.5), and (2.6) will be solved simultaneously and these coupled differential equations have to satisfy the following boundary conditions:

$$v(0, \phi) \text{ is finite.} \quad , \quad (2.10)$$

$$v(r, \phi) = v(r, \phi + 2\pi) \quad , \quad (2.11)$$

$$t(r, \phi) = t(r, \phi + 2\pi) \quad , \quad (2.12)$$

$$T(r, \phi) = T(r, \phi + 2\pi) \quad , \quad (2.13)$$

$$-k_f \frac{\partial}{\partial r} v(a, \phi) = h_s [v(a, \phi) - t(a, \phi)] \quad , \quad (2.14)$$

$$-k \frac{\partial}{\partial r} t(b, \phi) = h [t(b, \phi) - T(b, \phi)] \quad , \quad (2.15)$$

$$k_f \frac{\partial}{\partial r} v(a, \phi) = k \frac{\partial}{\partial r} t(a, \phi) \quad , \quad (2.16)$$

$$k \frac{\partial}{\partial r} t(b, \phi) = k_c \frac{\partial}{\partial r} T(b, \phi) \quad , \quad (2.17)$$

where h_s and h represent the thermal conductances at the fuel/clad and clad/coolant interfaces, respectively. Boundary conditions (2.10), (2.11), (2.12), and (2.13) require the solution of the temperature fields to be physically realistic and single-valued throughout the region being analyzed.

Boundary conditions (2.14) and (2.15) simulate the finite temperature drop at region interfaces by introducing the thermal conductances. Naturally, these conditions can be reduced to the more familiar forms of continuity of temperature if the interfacial conductances are sufficiently high. On the other hand, boundary conditions (2.16) and (2.17) are simply mathematical statements of continuity of heat fluxes at the fuel/clad and clad/coolant interfaces, respectively.

2.3 ANALYTICAL SOLUTION OF THE TEMPERATURE FIELDS

In this section, the solutions of the temperature fields in fuel, clad and coolant regions will be derived and obtained. It is felt that it is more logical to present the solution of the temperature field in the clad region first because of its relative mathematical simplicity. The solution of the Laplace equation for the temperature field in the clad region can be obtained by applying the technique of separation of variables [51]. Assuming the clad temperature field as a product of two functions:

$$t(r, \varphi) = R(r) \xi(\varphi) \quad , \quad (2.18)$$

with the proper choice of the separation constant yields:

$$\frac{d^2 \xi}{d\varphi^2} + \sigma^2 \varphi = 0 \quad , \quad (2.19)$$

and

$$r^2 \frac{d^2 R}{dr^2} + r \frac{dR}{dr} - \sigma^2 R = 0 \quad (2.20)$$

The general solution of Eq. (2.19) which satisfies boundary condition (2.12) is:

$$\xi(\varphi) = \begin{cases} C_0 + D_0 \varphi & \sigma = 0 \\ C_\sigma \cos \sigma \varphi + D_\sigma \sin \sigma \varphi & \sigma \neq 0 \end{cases} \quad (2.21)$$

and Eq. (2.20) is an equidimensional differential equation [52] whose general solution is given by:

$$R(r) = \begin{cases} C_0' + D_0' \ln r & \sigma = 0 \\ C_\sigma' r^\sigma + D_\sigma' r^{-\sigma} & \sigma \neq 0 \end{cases} \quad (2.22)$$

Thus the general solution of $t(r, \varphi)$ is a linear combination of the product of all these possible solutions:

$$t(r, \varphi) = C_0 + D_0 \ln r + D_0 (C_0' + D_0' \ln r) \varphi + \sum_{\sigma=1}^{\infty} \left[(C_\sigma r^\sigma + D_\sigma r^{-\sigma}) \cos \sigma \varphi + (C_\sigma' r^\sigma + D_\sigma' r^{-\sigma}) \sin \sigma \varphi \right] \quad (2.23)$$

In order that t be single-valued, the constants C_0' and D_0' have to be zero. Also for the same reason, the trigonometric functions must possess a common period 2π . This requirement

serves to determine the permissible values of the separation constant:

$$\sigma = n \quad ; \quad n = 1, 2, 3 \dots \quad (2.24)$$

Hence, the solution form of the temperature field in the clad region boils down to:

$$t(r, \varphi) = C_0 + D_0 \ln r + \sum_{n=1}^{\infty} \left[(C_n r^n + D_n r^{-n}) \cos n\varphi + (C'_n r^n + D'_n r^{-n}) \sin n\varphi \right] \quad (2.25)$$

where C_0 , D_0 , C_n , D_n , C'_n and D'_n are constants to be determined.

Eq.(2.25) is a complete Fourier series representation of the temperature field in the polar coordinate. However, it has to be pointed out that geometric symmetry of a unit cell, such as the $(0, \pi)$ symmetry of the corner cell, or a $(0, \pi/6)$ symmetry for the small internal cell, can give rise to further simplifications of the equation. The $(0, \pi)$ geometric symmetry implies that:

$$\frac{\partial}{\partial \varphi} t(r, 0) = \frac{\partial}{\partial \varphi} t(r, \pi) = 0 \quad (2.26)$$

Substituting Eq.(2.25) into (2.26) leads to:

$$\sum_{n=1}^{\infty} (C_n' r^n + D_n' r^{-n}) \cos n\varphi = 0, \quad \varphi = 0, \pi \quad (2.27)$$

The only possibility for $t(r, \varphi)$ to satisfy condition (2.27) is that C_n' and D_n' vanish. Thus Eq.(2.25) becomes

$$t(r, \varphi) = C_0 + D_0 \ln r + \sum_{n=1}^{\infty} (C_n r^n + D_n r^{-n}) \cos n\varphi \quad (2.28)$$

with the same argument the clad temperature field which satisfies the $(0, \pi/6)$ symmetry condition has the form:

$$t(r, \varphi) = C_0 + D_0 \ln r + \sum_{n=1}^{\infty} (C_n r^{6n} + D_n r^{-6n}) \cos 6n\varphi \quad (2.29)$$

It is evident that the separation of variable method is not directly applicable for the Poisson's equations for the temperature fields in the fuel and coolant regions because of its non-homogeneous nature. For this reason, an alternate method will be employed to obtain the solution. A new function will be used such that the Poisson's equation can be reduced to another Laplace equation [53].

Assuming the temperature field in the fuel region is a linear combination of two functions:

$$V(r, \varphi) = \Phi(r, \varphi) + \Omega(r) \quad (2.30)$$

Substituting Eq. (2.30) into Eq. (2.4), the following expression is obtained:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Omega}{\partial r^2} + \frac{1}{r} \frac{\partial \Omega}{\partial r} = -\frac{q'''}{k_f} \quad (2.31)$$

It can be seen that a Laplace equation can be obtained if Ω , when differentiated twice with respect to r , will give $-q'''/k_f$; that is:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \varphi^2} = 0 \quad (2.32)$$

if

$$\frac{d^2 \Omega}{dr^2} + \frac{1}{r} \frac{d\Omega}{dr} = -\frac{q'''}{k_f} \quad (2.33)$$

Since Ω is only a function of r , the solution of Eq.(2.33) can be readily obtained by direct integration. For mathematical convenience. Eq.(2.33) can be written as:

$$\frac{1}{r} \left(\frac{d}{dr} r \frac{d\Omega}{dr} \right) = - \frac{q'''}{k_f} \quad , \quad (2.34)$$

Integrating once with respect to r leads to:

$$\frac{d\Omega}{dr} = - \frac{q'''}{k_f} \frac{r}{2} + C_1 r^{-1} \quad , \quad (2.35)$$

In order to conserve the finiteness of the solution, the arbitrary constant C_1 has to vanish upon second integration, Ω becomes:

$$\Omega(r) = - \frac{q'''}{k_f} \frac{r^2}{4} + C_2 \quad , \quad (2.36)$$

where C_2 is another arbitrary constant. On the other hand, the solution of the Laplace equation(2.32) can be readily obtained by applying the technique of separation of variables as illustrated earlier. The function ϕ is given by the relation:

$$\Phi(r, \varphi) = a_0 + b_0 \ln r + \sum_{n=1}^{\infty} \left[(B_n r^n + A_n r^{-n}) \cos n\varphi + (B'_n r^n + A'_n r^{-n}) \sin n\varphi \right] \quad (2.37)$$

In order to satisfy boundary condition (2.10), the constants b_0 , as well as A_n and A'_n have to vanish; consequently Eq. (2.37) becomes:

$$\Phi(r, \varphi) = a_0 + \sum_{n=1}^{\infty} \left[B_n r^n \cos n\varphi + B'_n r^n \sin n\varphi \right], \quad (2.38)$$

where a_0 , B_n , and B'_n are constants to be determined.

Thus combining Eqs. (2.36) and (2.38) leads to the complete solution of the temperature field in the fuel region:

$$V(r, \varphi) = a_0 + c_2 - \frac{q'''_f}{k_f} \frac{r^2}{4} + \sum_{n=1}^{\infty} \left[B_n r^n \cos n\varphi + B'_n r^n \sin n\varphi \right], \quad (2.39)$$

or equivalently:

$$V(r, \varphi) = A + \frac{q'''_f a^2}{4k_f} \left[1 - \left(\frac{r}{a} \right)^2 \right] + \sum_{n=1}^{\infty} \left[B_n r^n \cos n\varphi + B'_n r^n \sin n\varphi \right] \quad (2.40)$$

where A is a new arbitrary constant. In case of geometric symmetry the coefficients of the sine series have to vanish and Eq.(2.40) reduces to the form:

$$v(r, \varphi) = A + \frac{q'' a^2}{4k_f} \left[1 - \left(\frac{r}{a} \right)^2 \right] + \sum_{n=1}^{\infty} B_n r^{\lambda_n} \cos \lambda_n \varphi, \quad (2.41)$$

where

$$\lambda = \begin{cases} 1 & \text{for } (0, \pi) \text{ symmetry} \\ 6 & \text{for } (0, \pi/6) \text{ symmetry.} \end{cases}$$

Applying the same procedure and arguments, the temperature field of the coolant regions is:

$$T(r, \varphi) = E_0 + F_0 \ln r + \frac{1}{4} S r^2 + \sum_{n=1}^{\infty} \left[(E_n r^n + F_n r^{-n}) \cos n\varphi + (E'_n r^n + F'_n r^{-n}) \sin n\varphi \right], \quad (2.42)$$

where E_0 , F_0 , E_n , F_n , E'_n and F'_n are constants to be determined. Also, the constant S is defined as:

$$S = \frac{\rho u c_p}{k_c} \left(\frac{\partial T}{\partial z} \right) = \frac{N \pi a^2 \bar{q}'''}{k_c A_T} \left(\frac{u}{u_B} \right) \quad (2.43)$$

For cases of geometric symmetry, Eq.(2.42) takes the form:

$$T(r, \varphi) = E_0 + F_0 \ln r + \frac{1}{4} S r^2 + \sum_{n=1}^{\infty} (E_n r^{\lambda_n} + F_n r^{-\lambda_n}) \cos \lambda_n \varphi, \quad (2.44)$$

where

$$\lambda = \begin{cases} 1 & \text{for } (0, \pi) \text{ symmetry} \\ 6 & \text{for } (0, \pi/6) \text{ symmetry} \end{cases}$$

Thus the derivation of the solutions of the temperature field in all regions in a unit cell has been completed. The determination of the constants of all temperature fields will be presented in the following section.

2.4 DETERMINATION OF THE INTEGRATION CONSTANTS

The determination of the constants appearing in Eqs.(2.29) (2.40) and (2.42) is accomplished by applying the boundary conditions imposed at the fuel/clad and clad/coolant interfaces. For generality, the constants associated with the temperature fields of all regions in a unit cell with no geometric symmetry will be expressed in terms of the Fourier coefficients of the fuel temperature field. Upon substituting Eqs.(2.28) and (2.40) into Eqs.(2.14) and (2.16) the following equations are obtained:

$$\begin{aligned}
& -k_f \left\{ \frac{q''' a}{2k_f} + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \cos n\varphi + B'_n a^{n-1} \sin n\varphi \right] \right\} = \\
& h_s \left\{ A - \sum_{n=1}^{\infty} \left[B_n a^n \cos n\varphi + B'_n a^n \sin n\varphi \right] - C_0 - D_0 \ln a \right. \\
& \left. - \sum_{n=1}^{\infty} \left[(C_n a^n + D_n a^{-n}) \cos n\varphi + (C'_n + D_n a^{-n}) \sin n\varphi \right] \right\}, \quad (2.45)
\end{aligned}$$

and

$$\begin{aligned}
& k_f \left\{ -\frac{q''' a}{2k_f} + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \cos n\varphi + B'_n a^{n-1} \sin n\varphi \right] \right\} = \\
& k \left\{ \frac{D_0}{a} + \sum_{n=1}^{\infty} n \left[(C_n a^{n-1} - D_n a^{-n-1}) \cos n\varphi + (C'_n a^{n-1} + D_n a^{-n-1}) \sin n\varphi \right] \right\} \quad (2.46)
\end{aligned}$$

The relationships between the unknown constants can be readily determined by taking advantage of the orthogonal nature of the Fourier series. Multiplying Eqs. (2.45) and (2.46) by $\cos m\phi d\phi$ and integrating with respect to ϕ from 0 to 2π , the following relations are obtained:

$$\begin{aligned}
& -k_f \left\{ -\frac{q'''a}{2k_f} \int_0^{2\pi} \cos m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
& \quad \left. \left. + B'_n a^{n-1} \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} = h_s \left\{ A \int_0^{2\pi} \cos m\varphi d\varphi + \right. \\
& \quad \left. \sum_{n=1}^{\infty} \left[B_n a^n \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi + B'_n a^n \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right. \\
& \quad \left. - (C_0 + D_0 \ln a) \int_0^{2\pi} \cos m\varphi d\varphi - \sum_{n=1}^{\infty} \left[(C_n a^n + D_n a^{-n}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
& \quad \left. \left. + (C'_n a^n + D'_n a^{-n}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} , \quad (2.47)
\end{aligned}$$

and

$$\begin{aligned}
& k_f \left\{ -\frac{q'''a}{2k_f} \int_0^{2\pi} \cos m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
& \quad \left. \left. + B'_n a^{n-1} \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} = k \left\{ \frac{D_0}{a} \int_0^{2\pi} \cos m\varphi d\varphi \right. \\
& \quad \left. + \sum_{n=1}^{\infty} n \left[(C_n a^{n-1} - D_n a^{-n-1}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
& \quad \left. \left. + (C'_n a^{n-1} - D'_n a^{-n-1}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} , \quad (2.48)
\end{aligned}$$

For the special case where $m = 0$, Eqs. (2.47) and (2.48) become:

$$\frac{q'''a}{2} = h_s (A - C_o - D_o \ln a) \quad , \quad (2.49)$$

and
$$-\frac{q'''a}{2} = k \frac{D_o}{a} \quad (2.50)$$

From Eq. (2.50) it can be easily seen that:

$$D_o = -\frac{q'''a^2}{2k} \quad (2.51)$$

Combining Eqs. (2.49) and (2.51) leads to:

$$C_o = A - \frac{q'''a^2}{2k_f} \left(\frac{1}{Bis} - \frac{k_f}{k} \ln a \right) \quad , \quad (2.52)$$

where Bis is the Biot number for the gap defined as:

$$Bis = \frac{h_s a}{k_f} \quad (2.53)$$

For the cases where $m \neq 0$, Eqs. (2.47) and (2.48) reduce to the forms:

$$-k_f n B_n a^{n-1} = h_s B_n a^n - h_s (C_n a^n + D_n a^{-n}) \quad , \quad (2.54)$$

and

$$k_f n B_n a^{n-1} = k n (C_n a^{n-1} - D_n a^{-n-1}) \quad , \quad (2.55)$$

where $n = 1, 2, 3, \dots$

Naturally, these equations cannot be obtained without utilizing the orthogonal properties of the Fourier integrals such as:

$$\int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi = \begin{cases} 0 & \text{if } m \neq 0 \\ \pi & \text{if } m = 0 \end{cases} \quad (2.56)$$

$$, \quad (2.57)$$

and

$$\int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi = 0$$

Solving the simultaneous equations (2.54) and (2.55) for C_n and D_n leads to:

$$C_n = \frac{B_n}{2} \left(1 + \frac{k_f}{k} + \frac{n}{Bis} \right) \quad , \quad (2.58)$$

and

$$D_n = \frac{B_n}{2} a^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{Bis} \right) \quad (2.59)$$

The next step is the determination of the Fourier coefficients of the sine series. The procedure is similar to the foregoing analysis. Multiplying Eqs. (2.45) and (2.46) by $\sin m\phi d\phi$ and integration with respect to ϕ from 0 to 2π , the following relations are obtained:

$$\begin{aligned} & -k_f \left\{ -\frac{q''' a}{2k_f} \int_0^{2\pi} \sin m\phi d\phi + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \int_0^{2\pi} \cos n\phi \sin m\phi d\phi \right. \right. \\ & \left. \left. + B'_n a^{n-1} \int_0^{2\pi} \sin n\phi \sin m\phi d\phi \right] \right\} = h_s \left\{ A \int_0^{2\pi} \sin m\phi d\phi + \right. \\ & \sum_{n=1}^{\infty} \left[B_n a^n \int_0^{2\pi} \cos n\phi \sin m\phi d\phi + B'_n a^n \int_0^{2\pi} \sin n\phi \sin m\phi d\phi \right] \\ & \left. - (C_0 + D_0 \ln a) \int_0^{2\pi} \sin m\phi d\phi - \sum_{n=1}^{\infty} \left[(C_n a^n + D_n a^{-n}) \int_0^{2\pi} \cos n\phi \sin m\phi d\phi \right. \right. \\ & \left. \left. + (C'_n a^n + D'_n a^{-n}) \int_0^{2\pi} \sin n\phi \sin m\phi d\phi \right] \right\} \quad (2.60) \end{aligned}$$

and

$$\begin{aligned}
 k_f \left\{ -\frac{q''_0 a}{2k_f} \int_0^{2\pi} \sin m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[B_n a^{n-1} \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi \right. \right. \\
 \left. \left. + B'_n a^{n-1} \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\} = k \left\{ \frac{D_0}{a} \int_0^{2\pi} \sin m\varphi d\varphi \right. \\
 \left. + \sum_{n=1}^{\infty} n \left[(C_n a^{n-1} - D_n a^{-n-1}) \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi + \right. \right. \\
 \left. \left. (C'_n a^{n-1} - D'_n a^{-n-1}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\} \quad (2.61)
 \end{aligned}$$

Notice that the Fourier sine integral has the following property:

$$\int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi = \begin{cases} 0 & m \neq n \\ \pi & m = n \end{cases} \quad (2.62)$$

consequently Eqs. (2.60) and (2.61) become:

$$-k_f n B'_n a^{n-1} = h_s B'_n a^n - h_s (C'_n a^n + D'_n a^{-n}) \quad (2.63)$$

and

$$k_f n B_n' a^{n-1} = k n (C_n' a^{n-1} - D_n' a^{-n-1}) \quad (2.64)$$

Solving the simultaneous equations (2.63) and (2.64) for C_n' and D_n' results in:

$$C_n' = \frac{B_n'}{2} \left(1 + \frac{k_f}{k} + \frac{n}{B_{is}} \right), \quad (2.65)$$

$$D_n' = \frac{B_n'}{2} a^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{B_{is}} \right) \quad (2.66)$$

The same procedure is extended to the interface between the clad and the coolant. Substituting Eqs. (2.29) and (2.42) into Eqs. (2.15) and (2.17), the following equations are obtained:

$$\begin{aligned} & -k \left\{ \frac{D_0}{b} + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \cos n\varphi + (C_n' b^{n-1} - D_n' b^{-n-1}) \sin n\varphi \right] \right\} \\ & = h_s \left\{ C_0 + D_0 \ln b + \sum_{n=1}^{\infty} \left[(C_n b^n + D_n b^{-n}) \cos n\varphi + (C_n' b^n + D_n' b^{-n}) \sin n\varphi \right] \right. \\ & \quad \left. - E_0 - F_0 \ln b - \frac{Sb^2}{4} - \sum_{n=1}^{\infty} \left[(E_n b^n + F_n b^{-n}) \cos n\varphi + \right. \right. \\ & \quad \left. \left. (E_n' b^n + F_n' b^{-n}) \sin n\varphi \right] \right\}, \quad (2.67) \end{aligned}$$

and

$$\begin{aligned}
 & k \left\{ \frac{D_0}{b} + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \cos n\varphi + (C'_n b^{n-1} - D'_n b^{-n-1}) \sin n\varphi \right] \right\} \\
 & = k_c \left\{ \frac{F_0}{b} + \frac{1}{2} S b + \sum_{n=1}^{\infty} n \left[(E_n b^{n-1} - F_n b^{-n-1}) \cos n\varphi \right. \right. \\
 & \quad \left. \left. + (E'_n b^{n-1} - F'_n b^{-n-1}) \sin n\varphi \right] \right\} \quad (2.68)
 \end{aligned}$$

Multiplying Eqs. (2.67) and (2.68) by $\cos m\varphi d\varphi$ and integrating from 0 to 2π , the following relations are obtained:

$$\begin{aligned}
 & -k \left\{ \frac{D_0}{b} \int_0^{2\pi} \cos m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
 & \quad \left. \left. + (C'_n b^{n-1} - D'_n b^{-n-1}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} = h \left\{ (C_0 + D_0 \ln b) \int_0^{2\pi} \cos m\varphi d\varphi \right. \\
 & \quad \left. + \sum_{n=1}^{\infty} \left[(C_n b^n + D_n b^{-n}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi + (C'_n b^n + D'_n b^{-n}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right. \\
 & \quad \left. - \left(E_0 + F_0 \ln b + \frac{S b^2}{4} \right) \int_0^{2\pi} \cos m\varphi d\varphi - \sum_{n=1}^{\infty} \left[(E_n b^n + F_n b^{-n}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
 & \quad \left. \left. + (E'_n b^n + F'_n b^{-n}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\}, \quad (2.69)
 \end{aligned}$$

and

$$\begin{aligned}
 & k \left\{ \frac{D_0}{b} \int_0^{2\pi} \cos m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi \right. \right. \\
 & \left. \left. + (C_n' b^{n-1} - D_n' b^{-n-1}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} = k_c \left\{ \left(\frac{F_0}{b} + \frac{1}{2} S b \right) \int_0^{2\pi} \cos m\varphi d\varphi + \right. \\
 & \left. \sum_{n=1}^{\infty} n \left[(E_n b^{n-1} - F_n b^{-n-1}) \int_0^{2\pi} \cos n\varphi \cos m\varphi d\varphi + (E_n' b^{n-1} + F_n' b^{-n-1}) \int_0^{2\pi} \sin n\varphi \cos m\varphi d\varphi \right] \right\} \quad (2.70)
 \end{aligned}$$

For the special case of $m = 0$, Eqs. (2.69) and (2.70) become:

$$-k \frac{D_0}{b} = h \left(C_0 + D_0 \ln b - E_0 - F_0 \ln b - \frac{S b^2}{4} \right), \quad (2.71)$$

and

$$k \frac{D_0}{b} = k_c \left(\frac{F_0}{b} + \frac{1}{2} S b \right), \quad (2.72)$$

respectively. Solving these two equations for E_0 and F_0 , the following relations are obtained:

$$F_0 = -\frac{q a^2}{2k_c} - \frac{S b^2}{2}, \quad (2.73)$$

and

$$E_0 = \frac{k}{h b} D_0 + C_0 + D_0 \ln b + \left(\frac{q a^2}{2k_c} + \frac{S b^2}{2} \right) \ln b + \frac{S b^2}{4}. \quad (2.74)$$

Substituting Eqs. (2.51) and (2.52) into Eq. (2.74) and rearranging, E_o becomes:

$$E_o = A + \frac{Sb^2}{2} \left(\ln b - \frac{1}{2} \right) + \frac{q'' a^2}{2k_c} \left[-\frac{k_c}{k_f} \frac{1}{B_{is}} + \frac{k_c}{k} \ln \frac{a}{b} - \frac{k_c}{k} \frac{1}{B_{i}} + \ln b \right] \quad (2.75)$$

where B_i is the Biot number of the clad/coolant interface defined as:

$$B_i = \frac{hb}{k} \quad (2.76)$$

For cases with $m > 0$, Eq. (2.69) and (2.70) become:

$$-\frac{k}{h} n (C_n b^{n-1} - D_n b^{-n-1}) = (C_n b^n + D_n b^{-n}) - (E_n b^n + F_n b^{-n}) \quad (2.77)$$

and

$$kn (C_n b^{n-1} - D_n b^{-n-1}) = k_c n (E_n b^{n-1} - F_n b^{-n-1}) \quad (2.78)$$

respectively. After rearranging, Eq. (2.78) can be rewritten as:

$$E_n = \frac{k}{k_c} (C_n - D_n b^{-2n}) + F_n b^{-2n} \quad (2.79)$$

Substituting Eq.(2.79) into Eq.(2.77) and performing the necessary manipulations, F_n is found to be:

$$F_n = \frac{C_n}{2} b^{2n} \left(1 - \frac{k}{k_c} + \frac{n}{\beta_i} \right) + \frac{D_n}{2} \left(1 + \frac{k}{k_c} - \frac{n}{\beta_i} \right) \quad (2.80)$$

upon substituting C_n and D_n given by Eqs.(2.58) and (2.59) respectively. Eqs(2.80) becomes:

$$F_n = \frac{B_n}{4} b^{2n} \left\{ \left(1 + \frac{k_f}{k} + \frac{n}{\beta_{is}} \right) \left(1 - \frac{k}{k_c} + \frac{n}{\beta_i} \right) + \left(\frac{a}{b} \right)^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{\beta_{is}} \right) \left(1 + \frac{k}{k_c} - \frac{n}{\beta_i} \right) \right\} \quad (2.80)$$

combining Eqs.(2.79) and (2.81), along with C_n and D_n are as given leads to:

$$E_n = \frac{B_n}{4} \left\{ \left(1 + \frac{k_f}{k} + \frac{n}{\beta_{is}} \right) \left(1 + \frac{k}{k_c} + \frac{n}{\beta_i} \right) + \left(\frac{a}{b} \right)^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{\beta_{is}} \right) \left(1 - \frac{k}{k_c} - \frac{n}{\beta_i} \right) \right\} \quad (2.82)$$

The determination of the coefficients of the Fourier sine series is similar to the foregoing procedure. Multiplying Eqs.(2.67) and (2.68) by $\sin m\phi d\phi$ and integration with respect to ϕ from 0 to 2π , the following relations are obtained:

$$\begin{aligned}
& -k \left\{ \frac{D_0}{b} \int_0^{2\pi} \sin m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi + \right. \right. \\
& \left. \left. (C'_n b^{n-1} - D'_n b^{-n-1}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\} = h_s \left\{ (C_0 + D_0 \ln b) \int_0^{2\pi} \sin m\varphi d\varphi \right. \\
& \left. + \sum_{n=1}^{\infty} \left[(C_n b^n + D_n b^{-n}) \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi + (C'_n b^n + D'_n b^{-n}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right. \\
& \left. - (E_0 + F_0 \ln b + \frac{Sb^2}{4}) \int_0^{2\pi} \sin m\varphi d\varphi - \sum_{n=1}^{\infty} \left[(E_n b^n + F_n b^{-n}) \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi \right. \right. \\
& \left. \left. + (E'_n b^n + F'_n b^{-n}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\}, \quad (2.83)
\end{aligned}$$

and

$$\begin{aligned}
& k \left\{ \frac{D_0}{b} \int_0^{2\pi} \sin m\varphi d\varphi + \sum_{n=1}^{\infty} n \left[(C_n b^{n-1} - D_n b^{-n-1}) \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi + \right. \right. \\
& \left. \left. (C'_n b^{n-1} - D'_n b^{-n-1}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\} = k_c \left\{ \left(\frac{F_0}{b} + \frac{1}{2} Sb \right) \int_0^{2\pi} \sin m\varphi d\varphi + \right. \\
& \left. \sum_{n=1}^{\infty} n \left[(E_n b^{n-1} - F_n b^{-n-1}) \int_0^{2\pi} \cos n\varphi \sin m\varphi d\varphi + (E'_n b^{n-1} - F'_n b^{-n-1}) \int_0^{2\pi} \sin n\varphi \sin m\varphi d\varphi \right] \right\} \quad (2.84)
\end{aligned}$$

utilizing the property of the Fourier integral and carrying out the necessary manipulations, the following equations are obtained:

$$-\frac{k}{h}n(c_n' b^{n-1} - D_n' b^{-n-1}) = (c_n' b^n + D_n' b^{-n}) - (E_n' b^n + F_n' b^{-n}) \quad (2.85)$$

and

$$kn(c_n' b^{n-1} - D_n' b^{-n-1}) = k_c n(E_n' b^{n-1} - F_n' b^{-n-1}) \quad (2.86)$$

Solving these two simultaneous equations, along with the definitions of C_n' and D_n' as given by Eqs. (2.65) and (2.66) respectively, the unknown coefficients E_n' and F_n' are found to be:

$$E_n' = \frac{B_n'}{4} \left\{ \left(1 + \frac{k_f}{k} + \frac{n}{B_{is}} \right) \left(1 + \frac{k}{k_c} + \frac{n}{B_{is}} \right) + \left(\frac{a}{b} \right)^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{B_{is}} \right) \left(1 - \frac{k}{k_c} - \frac{n}{B_{is}} \right) \right\} \quad (2.87)$$

and

$$F_n' = \frac{B_n'}{4} b^{2n} \left\{ \left(1 + \frac{k_f}{k} + \frac{n}{B_{is}} \right) \left(1 - \frac{k}{k_c} + \frac{n}{B_{is}} \right) + \left(\frac{a}{b} \right)^{2n} \left(1 - \frac{k_f}{k} + \frac{n}{B_{is}} \right) \left(1 + \frac{k}{k_c} - \frac{n}{B_{is}} \right) \right\} \quad (2.88)$$

The determination of the unknown constants in all temperature fields for unit cells with geometric symmetry is identical

to the previous procedure except that the coefficients of the Fourier sine series C_n', D_n', E_n' and F_n' have to vanish in order to satisfy the physical condition. However, the procedure is accomplished by multiplying $\cos \lambda n \phi d\phi$ and integrating from 0 to π/λ ($\lambda = 1$, for $0 - \pi$ symmetry and 6 for $0 - \pi/6$ symmetry) to the equations associated with the imposed boundary conditions at the interfaces. As a result, the constants C_0, D_0, E_0 and F_0 are given by the same expressions as denoted in the foregoing analysis while the coefficients of the Fourier series, C_n, D_n, E_n and F_n are given by:

$$C_n = \frac{B_n}{2} \left(1 + \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right), \quad (2.89)$$

$$D_n = \frac{B_n}{2} a^{2\lambda n} \left(1 - \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right), \quad (2.90)$$

$$E_n = \frac{B_n}{4} \left\{ \left(1 + \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) \left(1 + \frac{k}{k_c} + \frac{\lambda n}{B_{ic}} \right) + \left(\frac{a}{b} \right)^{2\lambda n} \left(1 - \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) \left(1 - \frac{k}{k_c} - \frac{\lambda n}{B_{ic}} \right) \right\}, \quad (2.91)$$

and

$$F_n = \frac{B_n}{4} b^{2\lambda n} \left\{ \left(1 + \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) \left(1 - \frac{k}{k_c} + \frac{\lambda n}{B_{ic}} \right) + \left(\frac{a}{b} \right)^{2\lambda n} \left(1 - \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) \left(1 + \frac{k}{k_c} - \frac{\lambda n}{B_{ic}} \right) \right\} \quad (2.92)$$

where the constant λ is defined as:

$$\lambda = \begin{cases} 1 & \text{for } 0 - \pi \text{ symmetry} \\ 6 & \text{for } 0 - \pi/6 \text{ symmetry} \end{cases}$$

Upon substituting all these constants into their respective equations, the solutions of temperature fields in all regions can be written in a more generalized form:

in the fuel:
$$V(r, \varphi) = A + \frac{q''' a^2}{2k_f} \left[1 - \left(\frac{r}{a} \right)^2 \right] +$$

$$\sum_{n=1}^{\infty} \left[B_n r^{\lambda n} \cos \lambda n \varphi + \delta_0 B_n' r^{\lambda n} \sin \lambda n \varphi \right] \quad (2.93)$$

in the clad:
$$t(r, \varphi) = A - \frac{q''' a^2}{2k_f} \left(\frac{1}{B_{is}} - \frac{k_f}{k} \ln a \right) - \frac{q''' a^2}{2k} \ln r$$

$$+ \sum_{n=1}^{\infty} \left\{ \left[\left(1 + \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) r^{\lambda n} + \left(1 - \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) a^{2\lambda n} r^{-\lambda n} \right] \frac{B_n}{2} \cos \lambda n \varphi \right. \\ \left. + \delta_0 \left[\left(1 + \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) r^{\lambda n} + \left(1 - \frac{k_f}{k} + \frac{\lambda n}{B_{is}} \right) a^{2\lambda n} r^{-\lambda n} \right] \frac{B_n'}{2} \sin \lambda n \varphi \right\} \quad (2.94)$$

in the coolant:

$$\begin{aligned}
 T(r, \varphi) = & A + \frac{Sb^2}{2} \left(\ln b - \frac{1}{2} \right) + \frac{Sr^2}{4} + \frac{q'' a^2}{2k_c} \left[-\frac{k_c}{k_f} \frac{1}{Bis} + \frac{k_c}{k} \ln \left(\frac{a}{b} \right) \right. \\
 & \left. - \frac{k_c}{k} \frac{1}{Bis} + \ln b \right] + \sum_{n=1}^{\infty} \left[B_n (f_n b^{2\lambda n} r^{-\lambda n} + g_n r^{\lambda n}) \cos \lambda n \varphi \right. \\
 & \left. + \delta_0 B'_n (f_n b^{2\lambda n} r^{-\lambda n} + g_n r^{\lambda n}) \sin \lambda n \varphi \right] \quad , \quad (2.95)
 \end{aligned}$$

where

$$f_n = \frac{1}{4} \left[\left(1 + \frac{k_f}{k} + \frac{\lambda n}{Bis} \right) \left(1 - \frac{k}{k_c} + \frac{\lambda n}{Bis} \right) + \left(\frac{a}{b} \right)^{2\lambda n} \left(1 - \frac{k_f}{k} + \frac{\lambda n}{Bis} \right) \left(1 + \frac{k}{k_c} - \frac{\lambda n}{Bis} \right) \right] ,$$

$$g_n = \frac{1}{4} \left[\left(1 + \frac{k_f}{k} + \frac{\lambda n}{Bis} \right) \left(1 + \frac{k}{k_c} + \frac{\lambda n}{Bis} \right) + \left(\frac{a}{b} \right)^{2\lambda n} \left(1 - \frac{k_f}{k} + \frac{\lambda n}{Bis} \right) \left(1 - \frac{k}{k_c} - \frac{\lambda n}{Bis} \right) \right] ,$$

and

$$\delta_0 = \begin{cases} 0 & \text{for unit cell with geometric symmetry} \\ 1 & \text{for unit cell without geometric symmetry} \end{cases}$$

Eqs. (2.93), (2.94) and (2.95) can be rearranged into dimensionless forms by moving A to the left hand side and dividing by $q'''a^2/2k_c$. This results in the following:

$$\frac{V(r, \varphi) - A}{q'''a^2/2k_c} = \Gamma = \frac{1}{2\kappa\kappa_w} \left[1 - \left(\frac{\rho}{t} \right)^2 \right] + \sum_{n=1}^{\infty} \left[\frac{Y_n}{n} \rho^{\lambda_n} \cos \lambda_n \varphi + \delta_0 \frac{Y_n'}{n} \rho^{\lambda_n} \sin \lambda_n \varphi \right] \quad (2.96)$$

$$\begin{aligned} \frac{T(r, \varphi) - A}{q'''a^2/2k_c} = \Psi = & -\frac{1}{\kappa\kappa_w} \left(\frac{1}{Bis} \right) + \frac{1}{\kappa} \ln \frac{t}{\rho} + \\ & \sum_{n=1}^{\infty} \left\{ \left[\left(1 + \kappa_w + \frac{\lambda_n}{Bis} \right) \rho^{\lambda_n} + \left(1 - \kappa_w + \frac{\lambda_n}{Bis} \right) \rho^{-\lambda_n} t^{2\lambda_n} \right] \frac{Y_n}{2n} \cos \lambda_n \varphi \right. \\ & \left. + \left[\left(1 + \kappa_w + \frac{\lambda_n}{Bis} \right) \rho^{\lambda_n} + \left(1 - \kappa_w + \frac{\lambda_n}{Bis} \right) \rho^{-\lambda_n} t^{2\lambda_n} \right] \delta_0 \frac{Y_n'}{2n} \sin \lambda_n \varphi \right\} \quad (2.97) \end{aligned}$$

and

$$\begin{aligned} \frac{T(r, \varphi) - A}{q'''a^2/2k_c} = \Theta = & \frac{1}{\kappa} \left(\ln t - \frac{1}{Bis} \right) - \frac{1}{\kappa\kappa_w} \frac{1}{Bis} - \left(1 + \frac{1}{I} \right) \ln \rho \\ & + \frac{1}{2I} (\rho^2 - 1) + \sum_{n=1}^{\infty} \left[\frac{Y_n}{\lambda_n} (f_n \rho^{-\lambda_n} + g_n \rho^{\lambda_n}) \cos \lambda_n \varphi \right. \\ & \left. + \delta_0 \frac{Y_n'}{\lambda_n} (f_n \rho^{-\lambda_n} + g_n \rho^{\lambda_n}) \sin \lambda_n \varphi \right] \quad (2.98) \end{aligned}$$

where

$$\rho = \frac{\gamma}{b} \quad , \quad (2.99)$$

$$t = \frac{a}{b} \quad , \quad (2.100)$$

$$\kappa_w = \frac{k_f}{k} \quad , \quad (2.101)$$

$$\kappa = \frac{k}{k_c} \quad , \quad (2.102)$$

$$I = \frac{q''' a^2 / 2 k_c}{S b^2 / 2} \quad , \quad (2.103)$$

$$Y_n = \frac{\lambda_n b^{\lambda_n} B_n}{q''' a^2 / 2 k_c} \quad , \quad (2.104)$$

and

$$Y_n' = \frac{\lambda_n b^{\lambda_n} B_n'}{q''' a^2 / 2 k_c} \quad . \quad (2.105)$$

As a result of the foregoing analysis, the constants appearing in the solutions of temperature fields in the clad and coolant regions have been expressed in terms of

an arbitrary constant A and two sets of unknown coefficients, namely Y_n and Y_n' . The determination of these unknowns requires additional boundary conditions. This will be illustrated in the following sections.

2.5 PHYSICAL MEANING OF THE CONSTANTS I AND A

In order to determine and interpret the physical meaning of the constant I as given by Eq.(2.43), it is necessary to recall the definition of S:

$$S = \frac{N\pi a^2 \bar{q}'''}{k_c A_T} \left(\frac{u}{\bar{u}_B} \right) \quad (2.106)$$

combining Eqs.(2.43) and (2.103) leads to:

$$I = \frac{\bar{q}'''^2 / 2k_c}{\frac{b^2}{2} \frac{N\pi a^2 \bar{q}'''}{k_c A_T}} \left(\frac{\bar{u}_B}{u} \right) = \left(\frac{A_T}{N\pi b^2} \right) \left(\frac{\bar{q}'''}{\bar{q}'''} \right) \left(\frac{\bar{u}_B}{u} \right) \quad (2.107)$$

It should be noticed that the first term of Eq.(2.107) represents the ratio of the total bundle flow area to the total rod cross-sectional area, while the second and third terms represent the power peaking factor and the reciprocal of the flow split factor for the unit cell being considered.

Naturally, the power peaking factor accounts for the intraassembly power tilting effect and has to be supplied as input to the analysis. On the other hand, total bundle flow area and rod area can be determined quite easily because they only depend on bundle geometry. In addition, the flow split factor accounts for the friction effect due to the different hydraulic characteristics in various unit cells has also to be furnished as input to the analysis and its relation with the cell hydraulic parameters will be discussed in later chapters.

The starting point for interpreting the physical meaning of the constant A is the general solution of the temperature field in the fuel region:

$$V(r, \varphi) = A + \frac{q'' a^2}{4k_c} \left[1 - \left(\frac{r}{a} \right)^2 \right] + \sum_{n=1}^{\infty} \left[B_n r^{\lambda_n} \cos \lambda_n \varphi + \delta_0 B_n' r^{\lambda_n} \sin \lambda_n \varphi \right] \quad (2.108)$$

Upon substituting a for r in the equation, the fuel surface temperature is given by:

$$V(a, \varphi) = A + \sum_{n=1}^{\infty} \left[B_n a^{\lambda_n} \cos \lambda_n \varphi + \delta_0 B_n' a^{\lambda_n} \sin \lambda_n \varphi \right] \quad (2.109)$$

Multiplying both sides by $d\phi$ and integrating with respect to ϕ from 0 to $\delta_{12}\pi/\lambda$ leads to:

$$\int_0^{\delta_{12}\pi/\lambda} V(a, \varphi) d\varphi = A \int_0^{\delta_{12}\pi/\lambda} d\varphi + \sum_{n=1}^{\infty} \left[B_n a^{\lambda n} \int_0^{\delta_{12}\pi/\lambda} \cos \lambda n \varphi d\varphi + \delta_0 B_n' a^{\lambda n} \int_0^{\delta_{12}\pi/\lambda} \sin \lambda n \varphi d\varphi \right], \quad (2.110)$$

where

$$\delta_{12} = \begin{cases} 1 & \text{with } 0 - \pi \text{ symmetry} \\ 2 & \text{without symmetry} \end{cases}$$

It can be easily verified that:

$$\int_0^{\delta_{12}\pi/\lambda} \cos \lambda n \varphi d\varphi = 0, \quad (2.111)$$

and

$$\int_0^{\delta_{12}\pi/\lambda} \sin \lambda n \varphi d\varphi = 0 \quad (2.112)$$

Thus Eq. (2.11) becomes:

$$\int_0^{\delta_{12}\pi/\lambda} V(a, \varphi) d\varphi = A \int_0^{\delta_{12}\pi/\lambda} d\varphi, \quad (2.113)$$

or equivalently:

$$A = \bar{V}(a, \varphi) \quad (2.114)$$

Hence it becomes clear that the arbitrary constant A , appearing in the solutions of the temperature fields of all regions is, in physical terms, the average fuel surface temperature. Therefore, there is no reason to expect that A can be found explicitly by using the boundary condition at the cell boundary without any further knowledge of the absolute temperature level of the system. The constant A will remain unspecified until a temperature normalization is chosen. The choice for the normalization will be discussed in Chapter 4.

2.6 BOUNDARY CONDITIONS USED TO DETERMINE THE UNKNOWN COEFFICIENTS

The general solutions of the temperature fields for the fuel, clad and coolant regions have been obtained in the foregoing sections. The unknown coefficients Y_n remain unknown until the boundary conditions in the coolant region at the cell boundary are specified and applied. There are two kinds of boundary conditions to be imposed, namely, Dirichlet and Neumann type boundary conditions. In practice these boundary conditions cannot be satisfied at all points of the boundaries because of the non-compatibility of the cell boundaries and the polar coordinate system. The present analysis employs an alternative which relaxes the rigorous mathematical formalism by allowing the coolant temperature field to satisfy the boundary condition only

at discrete collocation points.

At this point it should be recalled that the dimensionless temperature field in the coolant region is given by:

$$\begin{aligned} \frac{T(r, \varphi) - A}{q'' a^2 / 2k_c} = & \frac{1}{K} \left(\ln t - \frac{1}{Bi} \right) - \frac{1}{K K_w} \frac{1}{Bi_s} - \left(1 + \frac{1}{I} \right) \ln \rho + \\ & \frac{1}{2I} (\rho^2 - 1) + \sum_{n=1}^{\infty} \frac{Y_n}{\lambda_n} (f_n \rho^{-\lambda_n} + g_n \rho^{\lambda_n}) \cos \lambda_n \varphi \\ & + \sum_{n=1}^{\infty} \delta_n \frac{Y_n'}{\lambda_n} (f_n \rho^{-\lambda_n} + g_n \rho^{\lambda_n}) \sin \lambda_n \varphi \end{aligned} \quad (2.98)$$

The Dirichlet type boundary condition requires the temperature of the coolant to be equal to some prescribed values along the i th boundary of the cell; that is:

$$\left[T(r, \varphi) \right]_{r = a_i / \cos(\varphi - \varphi_i)} = T(c, \varphi) \quad (2.115)$$

where c is the distance from the origin of the unit cell to the i th cell boundary given by:

$$C = a_i / \cos(\varphi - d_i) \quad (2.116)$$

Applying the boundary condition as given by Eq.(2.115) leads to:

$$\begin{aligned} \frac{A}{q''' a^2 / 2k_c} + H(c) + \sum_{n=1}^{\infty} \frac{Y_n}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \cos \lambda_n \varphi \\ + \sum_{n=1}^{\infty} \delta_0 \frac{Y_n'}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \sin \lambda_n \varphi = \frac{T(c, \varphi)}{q''' a^2 / 2k_c} \end{aligned} \quad (2.117)$$

where

$$H(c) = \frac{1}{K} \left(\ln t - \frac{1}{\beta_i} \right) - \frac{1}{K K_w} \frac{1}{\beta_{is}} - \left(1 + \frac{1}{I} \right) \ln \left(\frac{c}{b} \right) + \frac{1}{\partial I} \left[\left(\frac{c}{b} \right)^2 - 1 \right] \quad (2.118)$$

During the course of a multicell analysis, it would be more convenient to normalize Eq.(2.117) to $\bar{q}''' a^2 / 2k_c$ instead of $q''' a^2 / 2k_c$ as $\bar{q}'''$ is the average power density of the section of the rod cluster being considered. Thus, Eq.(2.117), with the infinite series truncated at the N th term can be written in the form:

$$\frac{A}{\frac{q}{b} a^2/2k_c} + q^+ \left\{ \sum_{n=1}^N \frac{Y_n}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \cos \lambda_n \varphi + \right. \\ \left. \sum_{n=1}^N \delta_0 \frac{Y_n'}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \sin \lambda_n \varphi \right\} = \frac{T(c, \varphi)}{\frac{q}{b} a^2/2k_c} - q^+ H(c) \quad (2.119)$$

where q^+ is the dimensionless power peaking factor of the unit cell being considered defined as:

$$q^+ = \frac{q'''}{\frac{q}{b}} \quad (2.120)$$

Due to the non-compatibility of the cell boundary with the polar coordinate, Eq.(2.119) cannot be satisfied at all points along the cell boundary. In practice, Eq.(2.119) is only satisfied at $\phi = \phi_j$ with $J_0 \leq j \leq J$ where J_0 and J is the number of the first and last collocation point along the i th boundary of the cell. In order to distribute the boundary effect more evenly, the collocation points are equally spaced along the cell boundary.

The second kind of boundary condition being considered in the present analysis is the Neumann type boundary condition which requires the heat flux to be equal to some specified values along the i th boundary of the unit cell, that is;

$$\left[-k_c \nabla T \cdot \hat{n} \right]_{\rho = a_i / b \cos(\varphi - \alpha_i)} = q''(c, \varphi) \quad (2.121)$$

where ∇T is the temperature gradient of the coolant temperature field, $\hat{n}$ is the unit vector normal to the i th boundary of the cell and $q''(c, \phi)$ is the heat flux at the boundary which has to be specified. As shown in Fig. 2.3, the unit vector is given by:

$$\hat{n} = \cos \alpha_i \hat{i} + \sin \alpha_i \hat{j} \quad (2.122)$$

while the temperature gradient in the coolant region expressed in polar coordinate is:

$$\nabla T = \frac{\partial T}{\partial r} \hat{e}_r + \frac{1}{r} \frac{\partial T}{\partial \varphi} \hat{e}_\varphi \quad (2.123)$$

where $\hat{e}_r$ and $\hat{e}_\varphi$ are unit vectors in the radial and azimuthal direction, respectively. Their respective relations with the Cartesian unit vectors are:

$$\hat{e}_r = \cos \varphi \hat{i} + \sin \varphi \hat{j} \quad (2.124)$$

and

$$\hat{e}_\varphi = -\sin \varphi \hat{i} + \cos \varphi \hat{j} \quad (2.125)$$

Fig.2.3 Configuration of a Unit Vector Pointing Outward from a Unit Cell

Substituting Eq. (2.123) into Eq. (2.121) leads to:

$$-k_c \left\{ \frac{\partial T}{\partial r} (\hat{n} \cdot \hat{e}_r) + \frac{1}{r} \frac{\partial T}{\partial \varphi} (\hat{n} \cdot \hat{e}_\varphi) \right\} = q''(c, \varphi) \quad (2.126)$$

$\rho = a_i / b \cos(\varphi - \alpha_i)$

From Eqs. (2.122), (2.124) and (2.125), it is evident that:

$$\hat{n} \cdot \hat{e}_r = \cos \varphi \cos \alpha_i + \sin \varphi \sin \alpha_i = \cos(\varphi - \alpha_i) \quad , \quad (2.127)$$

$$\hat{n} \cdot \hat{e}_\varphi = -\sin \varphi \cos \alpha_i + \cos \varphi \sin \alpha_i = -\sin(\varphi - \alpha_i) \quad (2.128)$$

Therefore, Eq. (2.126) becomes:

$$-k_c \left\{ \cos(\varphi - \alpha_i) \frac{\partial}{\partial r} T(c, \varphi) - \sin(\varphi - \alpha_i) \frac{1}{r} \frac{\partial}{\partial \varphi} T(c, \varphi) \right\} = q''(c, \varphi) \quad (2.129)$$

Recalling the definition of the dimensionless temperature θ :

$$\theta = \frac{T(r, \varphi) - A}{q'' a^2 / 2 k_c} \quad , \quad (2.98)$$

it can be readily shown that:

$$\frac{\partial T}{\partial r} = \frac{q''' a^2}{2k_c} \frac{1}{b} \left(\frac{\partial \theta}{\partial \rho} \right) \quad (2.130)$$

and

$$\frac{1}{r} \frac{\partial T}{\partial \varphi} = \frac{q''' a^2}{2k_c} \frac{1}{b} \frac{1}{\rho} \left(\frac{\partial \theta}{\partial \varphi} \right) \quad (2.131)$$

Hence, Eq. (2.129) can be cast into the dimensionless form:

$$\cos(\varphi - \alpha_i) \frac{\partial}{\partial \rho} \theta(c, \varphi) - \sin(\varphi - \alpha_i) \frac{1}{\rho} \frac{\partial}{\partial \varphi} \theta(c, \varphi) = \frac{-1}{q^+} \frac{q''(c, \varphi)}{\frac{q''' a^2}{2b}} \quad (2.132)$$

where the partial derivatives of the dimensionless coolant temperature are given by the relations:

$$\begin{aligned} \frac{\partial \theta}{\partial \rho} = & -\left(1 + \frac{1}{I}\right) \frac{1}{\rho} + \frac{\rho}{I} + \sum_{n=1}^{\infty} Y_n \left(-f_n \rho^{-\lambda n - 1} + g_n \rho^{\lambda n - 1} \right) \cos \lambda n \varphi \\ & + \sum_{n=1}^{\infty} \delta_n Y_n' \left(-f_n \rho^{-\lambda n - 1} + g_n \rho^{\lambda n - 1} \right) \sin \lambda n \varphi \end{aligned} \quad (2.133)$$

and

$$\frac{\partial \theta}{\partial \varphi} = - \sum_{n=1}^{\infty} Y_n \left(f_n \rho^{-\lambda n} + g_n \rho^{\lambda n} \right) \sin \lambda n \varphi + \sum_{n=1}^{\infty} \delta_n Y_n' \left(f_n \rho^{-\lambda n} + g_n \rho^{\lambda n} \right) \cos \lambda n \varphi \quad (2.134)$$

Substituting Eqs. (2.133) and (2.134) into Eq. (2.132) and truncating the infinite series at the N th term results in:

$$\begin{aligned}
 & \cos(\varphi - \alpha_i) \left\{ -\left(1 + \frac{1}{I}\right) \frac{b}{c} + \frac{1}{I} \frac{c}{b} + \sum_{n=1}^N Y_n \left[-f_n \left(\frac{c}{b}\right)^{-\lambda n - 1} + g_n \left(\frac{c}{b}\right)^{\lambda n - 1} \right] \cos \lambda n \varphi \right. \\
 & \quad \left. + \sum_{n=1}^N \delta_0 Y_n' \left[-f_n \left(\frac{c}{b}\right)^{-\lambda n - 1} + g_n \left(\frac{c}{b}\right)^{\lambda n - 1} \right] \sin \lambda n \varphi \right\} - \sin(\varphi - \alpha_i) \frac{b}{c} \\
 & \left\{ -\sum_{n=1}^N Y_n \left[f_n \left(\frac{c}{b}\right)^{-\lambda n} + g_n \left(\frac{c}{b}\right)^{\lambda n} \right] \sin \lambda n \varphi + \sum_{n=1}^N \delta_0 Y_n' \left[f_n \left(\frac{c}{b}\right)^{-\lambda n} + g_n \left(\frac{c}{b}\right)^{\lambda n} \right] \cos \lambda n \varphi \right\} \\
 & = -\frac{1}{f^+} \frac{\frac{g''}{b}(c, \varphi)}{\frac{g''}{b} a^2 / 2b} \quad , \quad (2.135)
 \end{aligned}$$

or equivalently,

$$\begin{aligned}
 & \sum_{n=1}^N Y_n \left\{ g_n \left(\frac{c}{b}\right)^{\lambda n} \left[\sin \lambda n \varphi \sin(\varphi - \alpha_i) + \cos \lambda n \varphi \cos(\varphi - \alpha_i) \right] + \right. \\
 & \quad \left. f_n \left(\frac{c}{b}\right)^{-\lambda n} \left[-\cos \lambda n \varphi \cos(\varphi - \alpha_i) + \sin \lambda n \varphi \sin(\varphi - \alpha_i) \right] \right\} +
 \end{aligned}$$

$$\begin{aligned}
& \sum_{n=1}^N \delta_0 Y_n' \left\{ g_n \left(\frac{c}{b} \right)^{\lambda n} \left[\sin \lambda n \varphi \cos(\varphi - d_i) - \cos \lambda n \varphi \sin(\varphi - d_i) \right] + \right. \\
& \quad \left. f_n \left(\frac{c}{b} \right)^{-\lambda n} \left[-\sin \lambda n \varphi \cos(\varphi - d_i) - \cos \lambda n \varphi \sin(\varphi - d_i) \right] \right\} \\
& = \left[\left(1 + \frac{1}{I} \right) - \frac{1}{I} \left(\frac{c}{b} \right)^2 \right] \cos(\varphi - d_i) - \frac{1}{f} \left(\frac{c}{b} \right) \frac{f''(c, \varphi)}{f a^2 / 2b} \quad (2.136)
\end{aligned}$$

It has to be noticed that

$$\sin \lambda n \varphi \sin(\varphi - d_i) - \cos \lambda n \varphi \cos(\varphi - d_i) = -\cos[(\lambda n + 1)\varphi - d_i] \quad (2.137)$$

and

$$\sin \lambda n \varphi \sin(\varphi - d_i) + \cos \lambda n \varphi \cos(\varphi - d_i) = \cos[(\lambda n - 1)\varphi + d_i] \quad (2.138)$$

Thus Eq. (2.136) is reduced to a more compact form:

$$\begin{aligned}
& \sum_{n=1}^N Y_n \left\{ -f_n \left(\frac{c}{b} \right)^{-\lambda_n} \cos [(\lambda_{n+1})\varphi - \alpha_i] + g_n \left(\frac{c}{b} \right)^{\lambda_n} \cos [(\lambda_{n-1})\varphi + \alpha_i] \right\} + \\
& \sum_{n=1}^N \delta_o Y_n' \left\{ f_n \left(\frac{c}{b} \right)^{-\lambda_n} \sin [(\lambda_{n-1})\varphi + \alpha_i] - g_n \left(\frac{c}{b} \right)^{\lambda_n} \sin [(\lambda_{n+1})\varphi - \alpha_i] \right\} \\
& = \left[\left(1 + \frac{1}{I} \right) - \frac{1}{I} \left(\frac{c}{b} \right)^2 \right] \cos (\varphi - \alpha_i) - \frac{1}{g^+} \left(\frac{c}{b} \right) \frac{g''(c, \varphi)}{g^+ a^2 / 2b}
\end{aligned} \tag{2.139}$$

Naturally, Eq.(2.139) is only satisfied at a finite number of discrete collocation points along the i th boundary. Examples will be given in the following chapters to illustrate the determination of the solution coefficients by imposing appropriate boundary conditions at the cell boundaries.

CHAPTER 3

DETERMINATION OF THE UNKNOWN COEFFICIENTS FOR THE SINGLE-CELL ANALYSIS

In this chapter, the determination of the temperature fields of fuel, clad and coolant regions for different characteristic unit cell will be reduced to a problem of matrix inversion by imposing homogeneous Neuman type boundary condition at the imaginary cell boundaries. The solution technique for satisfying the boundary condition is illustrated and the numerical results for the azimuthal variations of clad temperature and heat flux are given. The results of the calculations will be compared to those available from open literature. In addition, parametric studies of pitch-to-diameter ratio and wall-to-diameter ratio are also presented. Furthermore, it should be pointed out that the result of this type of single-cell calculation will subsequently be used as the base case for two and multicell analyses which will be discussed in the following chapters.

3.1 BOUNDARY CONDITION FOR THE SINGLE-CELL ANALYSIS

In chapter 2, the general solutions for the nuclear fuel, the clad and the coolant temperature field have been obtained. All the temperature fields have been expressed

in terms of a single set of coefficients which cannot be determined until the boundary conditions at the cell boundaries are defined.

It was pointed out earlier in chapter 1 that the single cell analysis thermally and hydrodynamically decouples a unit cell from its neighbors. This is accomplished by requiring no heat transfer across the imaginary cell boundaries of any individual cell being considered, that is:

$$\left[-k_c \nabla T \cdot \hat{n} \right]_{r=a_i/\cos(\phi-\alpha_i)} = 0 \quad (3.1)$$

where a_i is the length of the normal to the i th cell boundary, α_i is the angle between the normal and the $\phi = 0$ axis the $\hat{n}$ is the unit vector pointing outward from the cell and is normal to the cell boundary as shown in Fig.2.3. Applying Eq.(2.139) with the heat flux $q''(c,\phi)$ set to zero, the following can be obtained:

$$\begin{aligned}
& \sum_{n=1}^N Y_n \left\{ -f_n \left(\frac{c}{b} \right)^{-\lambda n} \cos [(\lambda n + 1) \varphi - \alpha_i] + g_n \left(\frac{c}{b} \right)^{\lambda n} \cos [(\lambda n - 1) \varphi + \alpha_i] \right\} + \\
& \sum_{n=1}^N \delta_o Y_n' \left\{ f_n \left(\frac{c}{b} \right)^{-\lambda n} \sin [(\lambda n - 1) \varphi + \alpha_i] - g_n \left(\frac{c}{b} \right)^{\lambda n} \sin [(\lambda n + 1) \varphi - \alpha_i] \right\} \\
& = \left[\left(1 + \frac{1}{I} \right) - \frac{\left(\frac{c}{b} \right)^2}{I} \right] \cos (\varphi - \alpha_i) \quad (3.2)
\end{aligned}$$

In order to obtain a solution for the unknown coefficients, Eq.(3.2) is satisfied at angular positions ϕ_j ($j=1,2, \dots$) such that these points are equally spaced along the cell boundary. The uniform spacing of the collocation points provides a more even distribution of the geometric effect of the coolant channel. Following this procedure, the following finite system of linear simultaneous equations is obtained:

$$\sum_{n=1}^N Y_n G_{jn} + \sum_{n=1}^N \delta_o Y_n' G_{jn}' = P_j \quad (3.3)$$

where

$$P_j = \left[\left(1 + \frac{1}{I} \right) - \left(\frac{c}{b} \right)^2 \right] \cos(\varphi_j - \alpha_i) ,$$

$$G_{jn} = -f_n \left(\frac{c}{b} \right)^{-\lambda n} \cos [(\lambda n + 1) \varphi_j - \alpha_i] + g_n \left(\frac{c}{b} \right)^{\lambda n} \cos [(\lambda n - 1) \varphi_j + \alpha_i] ,$$

and

$$G'_{jn} = f_n \left(\frac{c}{b} \right)^{-\lambda n} \sin [(\lambda n - 1) \varphi_j + \alpha_i] - g_n \left(\frac{c}{b} \right)^{\lambda n} \sin [(\lambda n + 1) \varphi_j - \alpha_i]$$

Eq.(3.3) can be written in matrix notation as:

$$G_1 \underline{Y}_1 + G_2 \underline{Y}_2 = \underline{P} \quad . \quad (3.4)$$

or equivalently:

$$G \underline{Y} = \underline{P} \quad , \quad (3.5)$$

where

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$$G = \left[G_1 \mid G_2 \right]$$

and

$$\underline{Y} = \begin{bmatrix} \underline{Y_1} \\ \underline{Y_2} \end{bmatrix}$$

It is clear that if the total number of unknown coefficients is deliberately chosen to be equal to the total number of collocation points, a unique solution of $\underline{Y}$ can be obtained. This is the so-called point-matching technique which was used in a number of publications [54, 55]. This technique is quite simple but has its drawbacks. First, the boundary condition can only be satisfied at a limited number of arbitrarily chosen points, and the magnitude of the error at any other points along the boundaries may be quite large. Furthermore, there is no guarantee that the solution will represent the true solution as the number of points is increased because of round-off errors. Secondly, the approximate solution to a given boundary value problem usually changes appreciably with the choice and the distribution of the collocation points [56]. Depending on the type of boundary value problem considered, approximate solutions resulted from this kind of point-matching technique can differ as much as 100% for arbitrary choices of collocation points [57]. An improved

method is discussed by France and Ginsberg [58] which minimized the integral of the square of the error along the boundary. This technique was also used by Ullrich [18] for the multi-cell analysis for laminar flow. The results are quite satisfactory but the method involves a fairly large number of numerical integrations which makes the method less attractive for large bundle analysis.

It is evident that a reasonable compromise between the point-matching and the boundary integral least square method is the boundary least square method [59] which forces the solution to satisfy the boundary condition at J collocation points at the boundaries as defined by the physical problem. The number J should be greater than the number of unknown coefficients such that an overdetermined system of linear equation is obtained. However, it is impossible to find a unique set of unknown coefficients which exactly satisfies Eq.(3.5). In the present study, a least square procedure is used to obtain a consistent set of solution. The residual associated with the j th equation is given by:

$$r_j = \{GY\}_j - P_j, \quad (3.6)$$

and the sum of the square of the residual is given by:

$$\sum_{j=1}^J r_j^2 = \sum_{j=1}^J [\{G\underline{Y}\}_j - p_j]^2 \quad (3.7)$$

The least square method requires that the unknown coefficients are to be chosen such that the sum of the residuals is a minimum. Performing the necessary mathematical manipulations leads to the following relations [60]:

$$G^T G \underline{Y} = G^T \underline{P} \quad (3.8)$$

or

$$\underline{Y} = (G^T G)^{-1} G^T \underline{P} \quad (3.9)$$

Thus the overdetermined system of equations of N unknowns and J equations can be solved to yield, in the least square sense, the best adjusted values of the undetermined parameters simply by multiplying the overdetermined system of equations by the transpose of the coefficient matrix G^T .

Unfortunately the matrix $G^T G$ does not have any zero element and is not diagonally dominant. As a result an iterative solution procedure does not seem suitable for inverting the matrix [61]. Therefore, the determination of the solution vector in the present work is done by Gaussian elimination.

3.2 VALIDITY OF THE SOLUTION RESULTED FROM THE LEAST SQUARE METHOD

In order to illustrate the superiority of the least square method, temperature fields for an edge cell are obtained by applying the least square method with $N = 6, 12,$ and 36 respectively. The number of collocation points J for these cases is 36 such that the last case is really a point-matching method. In addition, the physical and thermal parameters used in the analysis are listed in table 3.1.

It was pointed out previously that the boundary condition imposed on this kind of single cell analysis is to require the dimensionless heat flux to vanish at the cell boundaries. Therefore, the dimensionless heat flux at the boundaries associated with the temperature field resulted from the least square procedure can be used as an indication of the error of the solution and a criterion for the validity of the solution.

Fig.3.1 illustrates the dimensionless heat flux distribution at the imaginary cell boundaries of the edge cell resulted from the least square procedure with $N = 36$. It can be seen from the figure that the dimensionless heat flux satisfies the boundary condition reasonably well in the region where $0 \leq \phi \leq 100^\circ$ but completely fails to give satisfactory

Table 3.1

Physical and Thermal Properties Used in the Analysis

Pitch-to-diameter ratio	$P/D = 1.24$
Dimensionless Wall Distance	$2W/D = 1.30$
Clad-to-coolant thermal conductivity ratio	$\kappa = \frac{k}{k_c} = 0.4$
Fuel-to-clad thermal conductivity ratio	$\kappa_W = \frac{k_f}{k} = 0.133$
Biot number at the fuel/clad interface	$B_{is} = \frac{h_{sa}}{k_f} = 10$
Biot number at the clad/coolant interface	$B_i = \frac{hb}{k} = 10^6$

Fig.3.1 Dimensionless Heat Flux Distribution at the Cell Boundary with $N=36$.

results near the corner region. The calculational result clearly illustrates the weakness of the method as large errors are concentrated locally in the vicinity of the relatively sharp corner. Therefore, it is clear that the point-matching method can only guarantee pointwise satisfaction of the boundary condition but has no control over the large fluctuation of the error between collocation points. As a result, the validity of the solution resulted from the point-matching method is highly questionable.

The dimensionless heat flux distributions at the cell boundaries for the cases with $N = 6, 12$ and 24 are given in Fig. 3.2. It is clear that the magnitude of fluctuation of the dimensionless heat flux at the boundaries of all these cases are far less than that of the point-matching method. Another important improvement is that the error of the solution associated with the least square method is distributed evenly over the entire boundary instead of being localized at the sharp corner. Naturally, the solution with only 6 terms requires much less computational effort, but the error is relatively large. Therefore it may not be acceptable for engineering purposes. The solution with 12 terms shows significant improvement but the solution with 24 terms is even better with a maximum error of 0.03 for the dimensionless heat flux.

Fig.3.2 Dimensionless Heat Flux Distribution at the Cell Boundary of the Edge Cell

A more general criterion for the evaluation of the validity of the solution resulted from the least square method can be devised by defining an "average error" along the boundary of a unit cell as:

$$\bar{\epsilon} = \frac{\sum_{i=1}^{\text{all sides}} \int_{S_i} \left| \frac{q''(c, \varphi)}{q''' a^2 / 2b} \right| d\left(\frac{s}{b}\right)}{(\mathcal{L}/b)} \quad (3.10)$$

where S is the length along the cell boundary and $\mathcal{L}$ is the total length of the cell boundary of the unit cell. The integrals are evaluated numerically by using the Gauss-Legendre ten-point method [62]. $\bar{\epsilon}$ given by Eq.(3.10) thus serves as a tool to quantitatively compare the validity of different solutions.

Fig.3.3 illustrates the average error plotted vs. the number of unknown coefficients used in the least square procedure for the three different unit cells. It can be seen that the average error of the internal cell is fairly insensitive to the number of unknown coefficients used and satisfactory result can be obtained over a very wide range of N . On the other hand, the error of the temperature field of the edge cell is extremely sensitive to the number of unknown coefficients. With less than 9 terms, the solution is not accurate enough to describe the temperature field of the cell.

Fig.3.3 "Average Error" vs. Number of Unknown Coefficients

The curve reaches its minimum at approximately $N = 24$, and $\bar{\epsilon}$ increases again as $N > 24$. This is because the number of terms is so close to the number of collocation points that the large fluctuation of error in the sharp corner region cannot be damped out sufficiently. For the corner cell, the average error also indicates a minimum at around $N = 24$. However, in this case, large fluctuation of the error when $N > 27$ is not experienced.

Finally, from the foregoing discussion it may be concluded that the least square method has successfully demonstrated its superiority over the point-matching method. Provided that the proper combination of number of collocation points and unknown coefficients is used, the local error in the heat flux at the cell boundary can be kept below 2-3% of the clad surface heat flux. Furthermore, it should be expected that the error of the clad temperature should be even considerably less because errors are damped out by radial and circumferential conduction in the coolant. The computational time for a single cell calculation with $J = 36$ and $N = 24$ is less than 10 seconds on an IBM 360/65 machine. Since the designer is mostly interested in the clad temperature in the first place, the least square method seems to be justified as it provides a fast and useful solution to the clad temperature of the rods placed in a highly complex geometry.

3.3 COMPARISON OF THE SLUG FLOW SOLUTION TO THE TURBULENT FLOW SOLUTION

It is desirable to compare the solution of the temperature field resulted from the present slug flow model to that resulted from a more sophisticated turbulent flow calculational model. Dwyer and Berry [63], in their work, obtained the fully developed temperature field of the coolant region of a 30° internal cell. This was accomplished by solving the energy equation, in conjunction with the Von Karman law-of-the-wall equation [64] to infer the velocity field and some empirical relations to determine the eddy diffusivities. In addition, the heat flux at the clad outside surface was assumed to be uniform. The comparison of the results of the two methods is given in Figs. 3.4 through 3.6 for different P/D ratios.

In these figures, the azimuthal clad temperature distributions resulted from Dwyer and Berry's calculation for two different values of Pe (100, and 1000), along with the azimuthal clad temperature distributed predicted by the slug flow model are presented. It has to be pointed out that the Peclet number is approximately to or slightly less than 1000 under normal operating conditions in an LMFBR. Therefore, the azimuthal clad temperature distribution should be somewhere in between the two curves for most normal and off-normal design cases.

It can be seen quite readily from Fig.3.5 that the clad temperature variation predicted by the slug flow model for $P/D = 1.05$ case agrees fairly well with the turbulent flow model calculation. On the other hand, the clad temperature variations predicted by the slug flow model as shown in Figs. 3.6 and 3.7 tend to be more and more conservative as the pitch-to-diameter ratio increases. It was pointed out by Levchenko, et al. [65,66] that the friction effect is relatively smaller in the unit cell of larger P/D ratios. This implies that the viscous sublayer is relatively small compared to the flow channel and the transport processes contributed by eddy diffusion in the unit cell of larger P/D ratios should be relatively more important. Therefore, the omission of the eddy diffusivities in the slug flow model leads to an underestimation of the heat transport in the coolant and consequently an overestimation of the clad temperature variation. Naturally, the degree of conservatism decreases as P/D ratio decreases and this is illustrated by the present calculated results. For the cases of very small P/D ratios, it would be highly questionable to use the slug flow model as the viscous effect becomes important and the slug flow model would fail to represent the physical system.

For present typical LMFBR designs, the P/D ratios for the fuel and blanket assemblies are approximately equal to 1.24

Fig.3.4 Comparison of Clad Temperature Distributions Resulted from Slug Flow and Turbulent Flow Model ($P/D=1.05$)

Fig.3.5 Comparison of Clad Temperature Distributions Resulted from Slug Flow and Turbulent Flow Model ($P/D=1.10$)

Fig.3.6 Comparison of Clad Temperature Distributions Resulted from Slug Flow and Turbulent Flow Model ($P/D=1.20$)

and 1.08, respectively. Therefore, it can be expected that the magnitude of the clad temperature variation from the present analysis should be fairly close to the turbulent flow solution for the rods in the blanket assembly and over conservative by a factor of 2 approximately for typical fuel assembly.

3.4 AZIMUTHAL CLAD TEMPERATURE DISTRIBUTIONS OF VARIOUS UNIT CELLS

Among all the informations provided by the local temperature analysis, the clad temperature fields are of particular interest from the designer's point of view. This is because the circumferential clad temperature variations may serve as input to the structural analysis for clad integrity studies.

The dimensionless clad temperature distributions of the internal cell are given in Fig.3.7 for 4 different P/D ratios. As shown by the figure, the azimuthal variation of the clad temperature decreases as P/D increases. It is because of the fact that larger coolant channel facilitates the transport process which damps out the disturbance effect due to the coolant channel irregularity. For $P/D \geq 1.40$, the clad temperature is essentially uniform and the local temperature analysis becomes unnecessary. On the other hand, the dimensionless clad temperature distributions of the corner cell for typical fuel and blanket assemblies are given in Figs.3.8 and 3.9. These quantities for the edge cell are also determined and presented in Figs.3.10 and 3.11. It can be seen from these

figures that the clad temperature distributions of the edge and corner cells are extremely sensitive to the wall distance. In addition, the magnitude of the clad temperature variations of the cells on the peripheral region of the bundle are much greater than that of the internal cell. With an unfavorable combination of P/D and $2W/D$ ratios, the maximum dimensionless clad temperature variation of the edge cell can be approximately as high as 2.0.

For a typical LMFBR under normal operating conditions, the numerator of the dimensionless group $q'' a^2 / 2k_c$ is approximately equal to 100°F . This implies a 200°F circumferential temperature variation for the rod on the periphery of the bundle. It must be recognized however that these values are for fully developed conditions. It is highly questionable whether this is the case at the core exit of recent LMFBR designs.

The results of this study thus highlights the fact that the rods subject to severe circumferential temperature variation are not those in the interior of the bundle but those on the edge of the bundle. Therefore, it becomes clear that the effort of developing highly sophisticated calculational models devoted solely to the solution of the coolant temperature in a small internal cell does not seem to be rewarding.

Fig.3.7 Clad Outside Temperature Distribution of the Small Internal Cell

Fig.3.8 Clad Outside Temperature Distributions of the Corner Cell with $P/D=1.24$

Fig.3.9 Clad Outside Temperature Distribution of the Corner Cell with $P/D=1.08$

Fig.3.10 Clad Outside Temperature Distribution of the Edge Cell with $P/D=1.24$

Fig.3.11 Clad Outside Temperature Distribution of the Edge Cell with $P/D=1.08$

3.5 AXIMUTHAL CLAD SURFACE HEAT FLUX DISTRIBUTION OF VARIOUS UNIT CELLS

In many references, for instance [12,58], local temperature analyses were always performed only for the coolant region with the assumption of an azimuthally uniform heat flux distribution at the clad outside surface. Evidently, this is not a very good assumption for LMFBR rods, especially for those on the periphery of the blanket assembly due to the geometric irregularity of those coolant channels. The determination of the local temperature as described in the preceeding section thus allows explicit calculation of the heat flux distributions, both at the clad inside and outside surfaces. It should be recalled that the heat flux in the clad region in the radial direction is given by:

$$q''(r, \varphi) = -k \frac{\partial}{\partial r} t(r, \varphi) \quad (3.11)$$

From the definition of the dimensionless clad temperature, it can be shown that:

$$\frac{\partial t}{\partial r} = \left(\frac{q''' a^2}{2k_c} \right) \frac{1}{b} \frac{\partial \Psi}{\partial \rho} \quad (3.12)$$

Substituting Eq. (3.12) into Eq. (3.11) leads to

$$q''(r, \varphi) = -\frac{k}{k_c} \left(\frac{q''' a^2}{2b} \right) \frac{\partial \Psi}{\partial \rho} \quad (3.13)$$

or in dimensionless form:

$$\frac{q''(r, \varphi)}{q'''a^2/2b} = q^*(\rho, \varphi) = -\kappa \left(\frac{r}{b}\right) \frac{\partial \Psi}{\partial \rho} \quad (3.14)$$

Physically, $q^*(\rho, \phi)$ represents the ratio of the local heat flux to the average heat flux of the surface. Performing the necessary manipulations, the dimensionless heat flux at the clad outside surface is given by the relation:

$$q^*(1, \varphi) = \frac{q''(b, \varphi)}{q'''a^2/2b} =$$

$$1 - \kappa \sum_{n=1}^N \frac{Y_n}{2} \left[\left(1 + \kappa_w + \frac{\lambda_n}{B_{is}}\right) - t^{2\lambda_n} \left(1 - \kappa_w + \frac{\lambda_n}{B_{is}}\right) \right] \cos \lambda_n \varphi$$

$$- \kappa \sum_{n=1}^N \delta_n \frac{Y_n}{2} \left[\left(1 + \kappa_w + \frac{\lambda_n}{B_{is}}\right) - t^{2\lambda_n} \left(1 - \kappa_w + \frac{\lambda_n}{B_{is}}\right) \right] \sin \lambda_n \varphi \quad (3.15)$$

and the dimensionless clad inside surface heat flux is given by:

$$q^*(t, \varphi) = \frac{q''(a, \varphi)}{q'' a^2 / 2b} =$$

$$1 - K K_w \sum_{n=1}^N Y_n t^{\lambda n} \cos \lambda n \varphi - K K_w \sum_{n=1}^N \delta_0 Y_n' t^{\lambda n} \sin \lambda n \varphi \quad (3.16)$$

Fig.3.12 illustrates the dimensionless clad outside heat flux distribution of the internal cell for different P/D ratios. On the other hand, the variation of the clad inside heat flux is so small that it cannot be shown in the same figure and the clad inside heat flux distributions are tabulated in table 3.2 for comparison. The decrease in the magnitude of azimuthal variation of heat flux is due to the heat conduction in the tangential direction in the clad region. Thus, the implication of this finding is that the maximum variation of heat flux occurs at the outside surface of the clad.

The heat flux distributions at the clad outside surface of the corner cell for the typical fuel and blanket assembly are given in Figs.3.13 and 3.14 respectively. Additionally, equivalent quantities of the edge cells are presented in Figs. 3.15 and 3.16. In general, the variation of the clad outside heat flux for both the edge and corner cells are much greater

than that of the internal cell. Furthermore, the magnitude of the clad surface heat flux variations for the edge and corner cell increase significantly as the coolant channel width decreases. For instance, the minimum and maximum clad outside heat flux may differ as much as 50% approximately for the blanket assembly while it is roughly 25% for the fuel assembly. Therefore, the clad outside surface heat flux distribution of the rods on the periphery of the assembly, especially blanket assembly, is highly non-uniform. Physically, this is because the heat flow adjusts its path by conducting in the azimuthal direction in the clad region such that more heat can be transferred to the relatively broader region of the channel.

It is quite obvious that a substantial error associated with the solution of the coolant temperature of the unit cells on the periphery of the assembly would be introduced if the uniform heat flux boundary condition were artificially imposed at the clad outside surface in a single-region analysis. This leads to the overestimation of heat transfer into the relatively narrow region of the coolant channel and vice versa for the relatively wider region of the channel. Naturally, this results in a clad temperature variation which is higher than what has to be expected in a realistic multiregion case. This excess conservatism can only be removed by performing the multiregion analysis.

Fig.3.12 Clad Outside Heat Flux Distribution of the Small Internal Cell

Table 3.2

Dimensionless Clad Inside Heat Flux Distribution for the
Internal Cell

ϕ	P/D = 1.05	P/D = 1.10	P/D = 1.15	P/D = 1.20
0°	0.980	0.990	0.996	0.998
6°	0.983	0.992	0.997	0.999
12°	0.994	0.997	0.999	1.000
18°	1.007	1.003	1.001	1.001
24°	1.016	1.008	1.003	1.001
30°	1.019	1.010	1.004	1.002

$$\frac{q''(b, \phi)}{q'' a^2 / 2b}$$

Fig.3.13 Clad Outside Heat Flux Distribution of the Corner Cell with $P/D = 1.24$

$$\frac{q''(b, \phi)}{q''' a^2 / 2b}$$

Fig.3.14 Clad Outside Heat Flux Distribution of the Corner Cell with $P/D = 1.08$

Fig.3.15 Clad Outside Heat Flux Distribution of the Edge Cell with $P/D = 1.24$

Fig.3.16 Clad Outside Heat Flux Distribution of the Edge Cell with $P/D=1.08$

3.6 DETERMINATION OF THE NUSSELT NUMBER

Another important quantity which can be determined quite readily from the local temperature field and the heat flux distribution is the dimensionless Nusselt number. The associated heat transfer coefficient can then be incorporated into subchannel codes which otherwise rely on experimental correlations which are usually based on "infinite array" geometry. The heat transfer coefficient is usually defined as:

$$\bar{h} = \frac{\bar{q}''(b)}{\bar{t}(b) - \bar{T}_M} \quad (3.17)$$

where $\bar{t}(b)$ is the average clad outside surface temperature, $\bar{T}_M$ is the average coolant temperature of the unit cell being considered and $\bar{q}''(b)$ is the average clad outside surface heat flux. In addition, a Nusselt number can then be defined based on the clad outside diameter ($2b$):

$$Nu = \frac{\bar{h}(2b)}{k_c} = \frac{2}{\frac{\bar{t}(b) - \bar{T}_M}{\bar{q}'' a^2 / 2k_c}} \quad (3.18)$$

or equivalently

$$Nu = \frac{2}{\left(\frac{\bar{T}(b) - A}{q'' a^2 / 2k_c} - \frac{\bar{T}_M - A}{q'' a^2 / 2k_c} \right)} \quad (3.19)$$

where the average dimensionless clad outside temperature is defined as:

$$\frac{\bar{T}(b) - A}{q'' a^2 / 2k_c} = \frac{\lambda}{\pi} \int_0^{\pi/\lambda} \frac{t(r, \varphi) - A}{q'' a^2 / 2k_c} \cdot d\varphi \quad (3.20)$$

Substituting Eq.(2.97) into Eq.(3.20) and carrying out the mathematical operations, the following relation is obtained:

$$\frac{\bar{T}(b) - A}{q'' a^2 / 2k_c} = -\frac{1}{\mathcal{K} \mathcal{K}_w} \frac{1}{Bis} + \frac{1}{\mathcal{K}} \ln\left(\frac{a}{b}\right) \quad (3.21)$$

The averaged coolant temperature is defined as:

$$\frac{\bar{T}_M - A}{q'' a^2 / 2k_c} = \frac{\int_0^{\pi/\lambda} d\varphi \int_b^{a/\cos(\varphi - d_i)} \frac{T(r, \varphi) - A}{q'' a^2 / 2k_c} dr}{\int_0^{\pi/\lambda} d\varphi \int_b^{a/\cos(\varphi - d_i)} r dr} \quad (3.22)$$

Dividing both the demoninator and numerator by b^2 , Eq. (3.22) becomes:

$$\frac{\bar{T}_M - A}{\frac{a^2}{2k_c}} = \frac{\int_0^{\pi/\lambda} d\varphi \int_1^{a_i/b \cos(\varphi - d_i)} \frac{T(\rho, \varphi) - A}{\frac{a^2}{2k_c}} \rho d\rho}{A_f} \quad (3.23)$$

where A_f is dimensionless flow area of the unit cell defined as:

$$A_f = \int_0^{\pi/\lambda} d\varphi \int_1^{a_i/b \cos(\varphi - d_i)} \rho d\rho = \int_0^{\pi/\lambda} \frac{1}{2} [\rho^2]_1^{a_i/b \cos(\varphi - d_i)} d\varphi \quad (3.24)$$

Upon substituting Eq. (2.98) into Eq. (3.23) and integrating leads to:

$$\begin{aligned}
\frac{\bar{T}_M - A}{\frac{1}{2} a^2 / k_c} &= \frac{1}{K} \left(\ln \frac{a}{b} - \frac{1}{B_i} \right) - \frac{1}{K K_w} \frac{1}{B_i s} - \frac{1}{2I} \\
&+ \frac{1}{2I A_f} \sum_{i=1}^{I_s} \int_{\beta_i}^{\beta_{i+1}} d\varphi \left[\frac{\rho^4}{4} \right]_1^{a_i/b \cos(\varphi - d_i)} \\
&- \frac{(I+1)}{I A_f} \sum_{i=1}^{I_s} \int_{\beta_i}^{\beta_{i+1}} d\varphi \left[\frac{\rho^2}{2} \left(\ln \rho - \frac{1}{2} \right) \right]_1^{a_i/b \cos(\varphi - d_i)} \\
&+ \frac{1}{A_f} \sum_{i=1}^{I_s} \int_{\beta_i}^{\beta_{i+1}} \cos \lambda n \varphi d\varphi \sum_{n=1}^N \frac{Y_n}{\lambda n} \left[\frac{g_n \rho^{\lambda n+2}}{\lambda n+2} + f_n \tau_n \right]_1^{a_i/b \cos(\varphi - d_i)} \\
&+ \frac{1}{A_f} \sum_{i=1}^{I_s} \int_{\beta_i}^{\beta_{i+1}} \sin \lambda n \varphi d\varphi \sum_{n=1}^N \delta_0 \frac{Y_n'}{\lambda n} \left[\frac{g_n \rho^{\lambda n+2}}{\lambda n+2} + f_n \tau_n \right]_1^{a_i/b \cos(\varphi - d_i)},
\end{aligned}
\tag{3.25}$$

and

$$\tau_n = \begin{cases} \ln \rho & \lambda n = 2 \\ \frac{\rho^{2-\lambda n}}{2-\lambda n} & \lambda n \neq 2 \end{cases},$$

and I_s is the total number of boundaries of the unit cell.

By defining the following quantities:

$$IA_i = \int_{\beta_i}^{\beta_{i+1}} \frac{d\varphi}{\cos^4(\varphi - d_i)} \quad , \quad (3.26)$$

$$IB_i = \int_{\beta_i}^{\beta_{i+1}} \frac{2 \ln \left[\frac{a_i/b}{\cos(\varphi - d_i)} \right] - 1}{\cos^2(\varphi - d_i)} d\varphi \quad , \quad (3.27)$$

$$IC_i = \int_{\beta_i}^{\beta_{i+1}} \cos \lambda n \varphi d\varphi \sum_{n=1}^N \frac{Y_n}{\lambda n} \left[\frac{g_n \rho^{\lambda n+2}}{\lambda n+2} + f_n \tau_n \right]_1^{a_i/b \cos(\varphi - d_i)} \quad , \quad (3.28)$$

and
$$IC'_i = \int_{\beta_i}^{\beta_{i+1}} \sin \lambda n \varphi d\varphi \sum_{n=1}^N \frac{Y'_n}{\lambda n} \left[\frac{g_n \rho^{\lambda n+2}}{\lambda n+2} + f_n \tau_n \right]_1^{a_i/b \cos(\varphi - d_i)} \quad , \quad (3.29)$$

the average coolant temperature of the unit cell can be written in the following form:

$$\begin{aligned}
\frac{\bar{T}_M - A}{f a^2 / 2 k_c} &= \frac{1}{K} \left(\ln \frac{a}{b} - \frac{1}{B_i} \right) - \frac{1}{K K_w} \frac{1}{B_{is}} - \frac{1}{2I} \\
&+ \frac{1}{8 I A_f} \left[\sum_{i=1}^{I_s} \left(\frac{a_i}{b} \right)^4 I A_i - \Delta \psi \right] \\
&- \frac{(I+1)}{4 I A_f} \left[\sum_{i=1}^{I_s} \left(\frac{a_i}{b} \right)^2 I B_i + \Delta \psi \right] \\
&+ \frac{1}{A_f} \sum_{i=1}^{I_s} I C_i + \frac{1}{A_f} \delta_o \sum_{i=1}^{I_s} I C'_i, \quad (3.30)
\end{aligned}$$

where

$$\Delta \psi = \int_0^{\pi/\lambda} d\psi$$

and the integrals A_f , IA_i , IB_i and IC_i are evaluated numerically by using the Gauss-Legendre ten-point method [62]. Once the averaged clad and coolant temperatures are determined, the calculation of the Nusselt numbers becomes possible.

Due to the fact that the Nusselt numbers depend on seven independent parameters, namely, κ , κ_w , B_i , B_{is} , P/D , a/b and $2W/D$ ratios, the presentation of a complete parametric study for all these parameters is virtually impossible. Therefore, the present analysis has been limited only to parametric studies for the pitch-to-diameter ratio and the dimensionless wall distance, with some typical values for the thermal parameters which were given earlier in Table 3.1. Fig.3.17 shows the Nusselt number of the internal cell as a function of the pitch-to-diameter ratio. The curve clearly indicates a maximum of the Nusselt number of approximately 26 in the vicinity of $P/D = 1.09$ or so. The decrease of the Nusselt number in the range of larger P/D ratios implies an increase of the the difference between the average clad surface and channel average coolant temperature due to the widening of the flow channel. On the other hand, the decrease in the P/D ratio leads to drastic increase in the clad surface temperature in the gap region and subsequently results in greater difference between the average surface and channel average coolant temperature. Naturally, this also implies a decrease in the Nusselt number. The result of this parameter study agrees very well with that of Trombetta [67].

In Figs.3.18 and 3.19, the Nusselt numbers of the corner and edge cells are plotted against the dimensionless wall distance ($2W/D$) for two different P/D ratios, representing typical fuel and blanket configurations. It can be seen

from Fig.3.18 that the $P/D = 1.08$ and $P/D = 1.24$ curves for the corner cell have maximas approximately at $2W/D = 1.06$ and 1.18 respectively. In Fig.3.19, it is indicated that the most optimal values for the dimensionless wall distance is approximately 1.03 and 1.12 for the blanket and fuel assembly, respectively. Another important fact which has to be pointed out is that the Nusselt numbers for the corner and edge cells are generally lower than that of the internal cell for the same P/D ratio. Their relative poor heat transfer characteristics are the result of their highly irregular coolant geometry. In order to check the validity of the present calculational procedure, a special case with $P/D=1.3$, $a/b=0.9$, $\kappa=0.3$, $\kappa_w=1.0$, $B_{is}=10^6$ and $B_i=10^6$ was run and the results of calculation for several $2W/D$ ratios are compared to those obtained by Wolf and Johannsen [23] in Table 3.3. It can be seen that the agreement is excellent.

The present analysis has indicated that the Nusselt number is generally a strong functions of the geometric configuration of the flow channel. Most of the heat transfer coefficient correlations rely upon results which are usually obtained by averaging the experimental measurements in an infinite array of rods. For example, Subbotin Correlation[68] predicts a Nusselt number of 10.6 for the case of an internal cell with the parameters $P/D = 1.24$ and $Pe = 1000$. For a fairly wide range of $2W/D$ ratios, this value is considerably higher than the Nusselt numbers of the corner and edge cells resulted

from the present analysis. Therefore, it becomes apparent that using the Nusselt number from the heat transfer correlation for the clad temperature of unit cells with highly irregular flow channel may lead to misleading results.

Another important finding from the present study is the non-optimized combination of P/D and $2W/D$ ratios in current LMFBR fuel and blanket assemblies. From the present analysis, it is indicated that an optimal design can be obtained by choosing the wall distance such that $2W/D$ is approximately equal to P/D . However, the width between the rod and bundle wall for current design of LMFBR fuel and blanket assemblies is approximately equal to the gap distance between two adjacent rods; that is, the $2W/D$ ratios are approximately 1.4 and 1.2 for the fuel and blanket assembly, respectively. This is intended to accomodate the wire wrap but it is easy to see that this non-optimized combination of P/D and $2W/D$ ratios can lead to significant increase in the azimuthal variation in the clad temperature and subsequently a decrease in Nusselt number. Thus, the present analysis indeed provides a quantitative tool to determine the most desirable combination of the geometric parameters of the flow channel of any unit cell in a rod bundle.

Fig.3.17 Nusselt Number of the Internal Cell vs. P/D Ratio

Fig.3.18 Nusselt Number of the Corner Cell vs. $2W/D$ ratio

Fig.3.19 Nusselt Number of the Edge Cell vs. 2W/D Ratio

Table 3.3

Comparison of Nusselt Numbers for Rod Bundles with $P/D = 1.30$,
 $\kappa = 0.3$, $\kappa_w = 1.0$, $a/b = 0.9$, $B_1 = 10^6$ and $B_{1s} = 10^6$.

2W/D	corner cell		edge cell	
	Nu/Nu _∞ from Wolf's calculation	Nu/Nu _∞ from present calculation	Nu/Nu _∞ from Wolf's calculation	Nu/Nu _∞ from present calculation
1.1	0.355	0.371	0.677	0.668
1.2	0.629	0.628	0.738	0.737
1.3	0.661	0.662	0.601	0.612
1.4	0.531	0.528	0.467	0.466
1.5	0.399	0.400	0.361	0.357
1.6	0.305	0.311	0.275	0.282

CHAPTER 4

ITERATIVE SOLUTION FOR MULTICELL ANALYSIS

In the foregoing chapter, solutions for the temperature fields in fuel, clad and coolant regions have been obtained for an isolated unit cell by imposing adiabatic boundary conditions at the imaginary cell boundaries. In this chapter, the single cell analysis is extended to a multicell analysis by employing a cell-to-cell iterative procedure. The multicell analysis predicts the 2-D temperature field without imposing the unrealistic adiabatic boundary condition at the common boundary between unit cells. In addition, the multicell analysis accounts for different power densities and flow velocities in different unit cells. The result of the multicell analysis enables the determination of the lumped transport parameters which can be used in subchannel code calculations.

4.1 SOLUTION PROCEDURE FOR COUPLING NEIGHBORING CELLS

There are essentially two possibilities to perform the multicell analysis; namely, a direct solution and an iterative method. For the direct solution method, the temperature fields for all unit cells in the rod cluster being considered are first formulated and a system of linear equations can be set up by matching the temperatures and heat fluxes at some collocation points at the common boundaries of adjacent cells, along with other appropriate boundary condition (such as

adiabatic boundary condition at the bundle wall). The solutions of the temperature fields of all unit cells can be obtained by solving the system of the linear equations. This procedure was actually used by Ullrich [18] for the laminar flow problem. The direct solution technique is a more natural approach because neither temperature nor heat flux has to be specified at the common boundary between adjacent cells. However, its disadvantage is the inversion of a large matrix if a large section of the rod bundle is to be analyzed.

The second method is an iterative procedure in which the temperature field for each individual unit cell is first obtained by imposing the adiabatic boundary condition at the imaginary cell boundaries as was done for the single cell analysis. The solutions resulted from the adiabatic calculations generally result in discontinuity of temperature along the common boundary between neighboring cells. The iterative method then relaxes the initial adiabatic boundary condition by assuming the temperature along the common boundary to be the average of the temperature of the two neighboring cells resulted from the initial calculation. The new boundary condition thus yields an updated temperature field for one of the unit cells. Following this updated temperature field is the calculation of the updated heat flux distribution along the common boundary which is used as boundary condition for its neighboring cell. This leads to a new temperature field for the neighboring cell. Starting the second loop of the iteration the temperature boundary

condition will be applied again and so forth until an acceptable solution is achieved. A flow chart of this method is summarized in Fig.4.1. This method has the advantage over the direct solution technique because it is a cell-to-cell iteration procedure which does not have to invert a large matrix even if a large section of bundle is analyzed. Moreover, it provides the flexibility for the designer to choose the number of iterations depending on the accuracy of solution he desires.

4.2 IDENTIFICATION OF UNIT CELLS IN A ROD BUNDLE

In order to perform a multicell analysis for a rod bundle effectively, it is essential to develop a scheme which can identify any individual unit cell as a function of the cell number. The scheme used in the present analysis consists of two parts; namely, a cell numbering system and an identification technique. This procedure has already been used by Ullrich for solving the laminar flow problem in rod bundle sections [18]. However, in case of the thermal problem, the scheme has to be extended to cope with the various thermal boundary conditions.

Throughout this analysis a double index (m,n) system is used for the numbering scheme of the unit cells. As shown in Fig. 4.2, the first index designates the number of rows away from the center rod of the bundle and the second index designates twice the number of pitches measured from the symmetry line of the section.

Fig. 4.1 Schematic Diagram of Solution Strategy for Multicell Analysis

Fig.4.2 Double Indices of the Unit Cells of a 127-Rod Bundle

This double index system provides a convenient way for identifying the neighboring cells for a unit cell. For illustration, a typical hexagonal unit cell along with the indices of its neighboring cells is shown in Fig.4.3. It should be noticed that the cells on the outermost row do not have the hexagonal or part of the hexagonal geometry. Table 4.1 indicates the scheme of identifying the indices of the neighboring cells for all ten possible types of unit cells in a hexagonal bundle.

In addition to the double index system a scheme which relates the double index and the cell type is needed. The scheme is shown in table 4.2. By inputting the (m,n) index, it can automatically identify the cell type as well as the complete information of its geometric parameters (such as numbers of boundaries normal distances... etc.).

Since the temperature fields in each individual unit cell are expressed in terms of the individual polar coordinate system (ρ, ϕ) , it is necessary to have a scheme which can transform the location of a point along a common boundary from one coordinate system to another. In other words, it is to transform a point (ρ, ϕ) along the cell boundary to (ρ', ϕ') in the coordinate system of the neighboring cell as shown in Fig.4.4. This is accomplished by first transforming from the cell coordinate system to the "Bundle" coordinate system. It can be seen from the figure that the transformation relations are given by:

$$\frac{x}{b} = \frac{x_0}{b} + \rho \cos(\phi - \xi) \quad (4.1)$$

and

$$\frac{y}{b} = \frac{y_0}{b} + \rho \sin(\phi - \xi) \quad (4.2)$$

where ξ is the angular displacement by rotating the X-axis of the bundle coordinate to the $\phi = 0$ axis of the cell coordinate system x and y are the abscissa and coordinate of the point expressed in the "bundle" cartesian coordinate system respectively, and x_0 and y_0 are the abscissa and coordinate of the origin of the cell coordinate with double index (m, n) . The relations between (x_0, y_0) and (m, n) are:

$$\frac{x_0}{b} = n \left(\frac{P}{D} \right) \quad (4.3)$$

and

$$\frac{y_0}{b} = 2m \frac{\sqrt{3}}{2} \left(\frac{P}{D} \right) \quad (4.4)$$

Once the abscissa and ordinate of the point are obtained, the location of this point can be easily transformed into the coordinate system of any unit cell with double index

Fig.4.3 Indices for Neighboring Cells for a Typical Hexagonal Cell

Table 4.1

Indices of the Neighboring Cells of Unit Cell (m, n)

type	# of boundary (i)	m' of the neighboring cell	n' of the neighboring cell
1	1	$m + 1$	$n + 1$
2	1	*	*
	2	m	$n - 2$
	3	$m - 1$	$n - 1$
3	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	*	*
4	1	$m + 1$	$n + 1$
	2	$m + 1$	$n - 1$
	3	*	*
	4	$m - 1$	$n - 1$
5	1	$m + 1$	$n + 1$
	2	$m + 1$	$n - 1$
	3	m	$n - 2$
	4	$m - 1$	$n - 1$
6	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	$m + 1$	$n + 1$
7	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	*	*
	4	*	*
	5	$m - 1$	$n - 1$
8	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	*	*
	4	m	$n - 2$
	5	$m - 1$	$n - 1$
9	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	$m + 1$	$n + 1$
	4	$m + 1$	$n - 1$
	5	*	*
	6	$m - 1$	$n - 1$
10	1	$m - 1$	$n + 1$
	2	m	$n + 2$
	3	$m + 1$	$n + 1$
	4	$m + 1$	$n - 1$
	5	m	$n - 2$
	6	$m - 1$	$n - 1$

* No neighboring cell

Table 4.2

Relations Between Index (m, n) and Cell Type

m	n	Type
0	0	1
1	1	4
$1 < m < M$	0	6
	1	10
	$1 < n < N$	9
M	n	5
	0	3
	$1 < l < N$	8
	n	7
	M	2

where

M - No. of the outermost row

N - Max. No. of cell of a row

Fig.4.4 Transformation of Coordinates between Unit Cells

(m', n') by the relations:

$$\rho' = \sqrt{\left(\frac{x}{b} - \frac{x_0'}{b}\right)^2 + \left(\frac{y}{b} - \frac{y_0'}{b}\right)^2} \quad (4.5)$$

and

$$\phi' = -\xi' + \tan^{-1} \frac{\left(\frac{y}{b} - \frac{y_0'}{b}\right)}{\left(\frac{x}{b} - \frac{x_0'}{b}\right)} \quad (4.6)$$

where x_0' and y_0' are the abscissa and ordinate of the cell with index (m', n') , respectively. ξ' is the angular displacement by rotating the X-axis of the "bundle" coordinate to the $\phi = 0$ axis of the (m', n') polar coordinate system. As a result of this scheme of transformations points along any cell boundary can be expressed in any chosen coordinate system of a unit cell. This greatly facilitates the iterative procedures during the multicell coupling calculation which will be discussed in the following section.

4.3 BOUNDARY CONDITIONS USED TO COUPLE NEIGHBORING UNIT CELLS

Consider two neighboring unit cells, ℓ and m , connected by a common boundary as shown in Fig.4.5. The coolant temperature fields of the ℓ th and m th unit cells resulting from the initial adiabatic calculation are given by:

Fig.4.5 Configuration of Two Neighboring Cells Connected by a Common Boundary

$$T_{\ell}^{(0)}(\rho, \phi) = A_{\ell} + q_{\ell}^+ \frac{\bar{q}''' a^2}{2k_c} \theta_{\ell}^{(0)}(\rho, \phi), \quad (4.7)$$

and

$$T_m^{(0)}(\rho, \phi) = A_m + q_m^+ \frac{\bar{q}''' a^2}{2k_c} \theta_m^{(0)}(\rho, \phi), \quad (4.8)$$

respectively. The associated dimensionless temperature fields $\theta_{\ell}^{(0)}$ and $\theta_m^{(0)}$ are obtained by the procedure described in chapter 3. The subscripts and superscripts represent the unit cell number and the number of iteration, respectively. Moreover, the radius and azimuthal angle for the two temperature fields are expressed in two different coordinate systems.

It is obvious that $T_{\ell}^{(0)}(\rho, \phi)$ and $T_m^{(0)}(\rho, \phi)$ resulted from the initial adiabatic calculation do not necessarily satisfy the condition of continuity of temperature along the common boundary. In order to reduce the difference between these two temperatures at the common boundary, the iterative procedure imposes a new boundary condition which requires the temperature along the common boundary to be equal to the average of the temperatures of the two cells resulted from the initial calculation; that is:

$$T_{\ell}^{(1)}(c, \phi) = \frac{1}{2} [T_{\ell}^{(0)}(c, \phi) + T_m^{(0)}(c', \phi')] \quad (4.9)$$

where

$$c = a_i / \cos(\phi - \alpha_i) \quad ,$$

and c' and ϕ' are to be determined by the procedure described in the preceding section. Substituting Eq.(4.9) into Eq. (2.119) for the ℓ th cell leads to:

$$\frac{A_{\ell}}{\bar{q}''' a^2 / 2k_c} + q_e^+ \left\{ \sum_{n=1}^N \frac{Y_{n,\ell}^{(1)}}{\lambda_n} \left[f_n\left(\frac{c}{b}\right)^{-\lambda_n} + g_n\left(\frac{c}{b}\right)^{\lambda_n} \right] \cos \lambda_n \phi + \sum_{n=1}^N \delta_0 \frac{Y_{n,m}^{(1)}}{\lambda_n} \right\}$$

$$\left[f_n\left(\frac{c}{b}\right)^{-\lambda_n} + g_n\left(\frac{c}{b}\right)^{\lambda_n} \right] \sin \lambda_n \phi = \frac{\frac{1}{2} \left[T_{\ell}^{(0)}(c, \phi) + T_m^{(0)}(c', \phi') \right]}{\bar{q}''' a^2 / k_c} - q_{\ell}^+ H_{\ell}(c)$$

(4.10)

Upon substituting Eqs.(4.7) and (4.8) into Eq.(4.10), the following equality is obtained:

$$\begin{aligned}
& \frac{1}{2} \left[\frac{A_\ell - A_m}{\bar{q}'' a^2 / 2k_c} \right]^{(1)} + q_\ell^+ \sum_{n=1}^N \frac{Y_{n,\ell}^{(1)}}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \cos \lambda_n \phi \\
& + q_\ell^+ \sum_{n=1}^N \frac{Y_{n,2}^{(1)}}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right] \sin \lambda_n \phi \\
& = \frac{1}{2} \left[q_\ell^+ \theta_\ell^{(0)}(c, \phi) + q_m^+ \theta_m^{(n)}(c', \phi') \right] - q_\ell^+ H_\ell(c) \quad (4.11)
\end{aligned}$$

Since the iterative method is a repeating process, Eq.(4.5) can be written in a more generalized form, namely:

$$\begin{aligned}
& Y_{0,\ell m}^{(v+1)} + q_\ell^+ \sum_{n=1}^N Y_{n,\ell}^{(v+1)} K_n \cos \lambda_n \phi + q_\ell^+ \sum_{n=1}^N \delta_0 Y_{n,\ell}^{(v+1)} K_n \sin \lambda_n \phi \\
& = \frac{1}{2} \left[q_\ell^+ \theta_\ell^{(v)}(c, \phi) + q_m^+ \theta_m^{(v)}(c', \phi') \right] - q_\ell^+ H_\ell(c) \\
& \quad (4.12)
\end{aligned}$$

where

$$Y_{0,l}^{(v+1)} = \frac{1}{2} \left[\frac{A_l - A_m}{\bar{q}''' a^2 / 2k_c} \right]^{(v)}$$

and

$$K_n = \frac{q_l^+}{\lambda_n} \left[f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right]$$

satisfying Eq. (4.12) at a finite number of discrete points along the common boundary between the two unit cells results in a system of linear equations:

$$Y_{0,l_m}^{(v+1)} + \sum_{n=1}^N \left[Y_{n,l}^{(v+1)} M_{jn} + \delta_0 Y_{n,l}'^{(v+1)} M_{jn}' \right] = P_{j,l}^{(v)}, \quad (4.13)$$

where

$$M_{jn} = K_n \cos \lambda_n \phi_j, \quad ,$$

$$M_{jn}' = K_n \sin \lambda_n \phi_j, \quad ,$$

and

$$P_{j,l}^{(v)} = \left\{ \frac{1}{2} \left[q_{l,\theta_l}^{+(v)}(c, \phi) + q_{m,\theta_m}^{+(v)}(c', \phi') \right] - q_{l,\theta_l}^{+H}(c) \right\}_{\phi=\phi_j}$$

It should be noticed that the coupling of the temperature fields has introduced an extra constant $Y_{o, \ell m}^{(v+1)}$ which physically represents the difference between the average fuel outside surface temperatures of the two unit cells.

In order to satisfy the continuity of heat fluxes, a Neumann type boundary conditions is imposed along the common boundary for the m th unit cell. Applying Eq.(2.139) for the m th cell leads to:

$$\begin{aligned} & \sum_{n=1}^N Y_{n,m}^{(v+1)} \left\{ -f_n \left(\frac{c}{b} \right)^{-\lambda n} \cos [(\lambda n + 1)\phi - \alpha_m] + g_n \left(\frac{c}{b} \right)^{\lambda n} \cos [(\lambda n - 1)\phi_m + \alpha_m] \right\} \\ & + \sum_{n=1}^N \delta_o Y'_{n,m}^{(v+1)} \left\{ f_n \left(\frac{c}{b} \right)^{-\lambda n} \sin [(\lambda n - 1)\phi + \alpha_m] - g_n \left(\frac{c}{b} \right)^{\lambda n} \sin [(\lambda n + 1)\phi - \alpha_m] \right\} \\ & = \left[\left(1 + \frac{1}{I_m} \right) - \frac{1}{I_m} \left(\frac{c}{b} \right)^2 \right] \cos(\phi - \alpha_m) - \frac{q''(c, \phi)}{\bar{q}''' a^2 / 2k_c} \left(\frac{1}{q_m^+} \frac{c}{b} \right) \end{aligned} \quad (4.14)$$

where α_m is the angular position of the normal to the common boundary from the center of the m th coordinate as shown in Fig.4.5. Naturally, the heat flux distribution in Eq.(4.14) is furnished by the updated temperature field of the ℓ th cell resulted from the $(v+1)$ th calculation; that is:

$$\begin{aligned}
 q''(c, \phi) &= -\hat{n} \cdot \left[-k_c \nabla T_\ell^{(v+1)}(c, \phi) \right] = \hat{n} \cdot k_c \nabla T_\ell^{(v+1)}(c', \phi') \\
 &= k_c \left\{ \cos(\phi' - \alpha_\ell) \frac{\partial}{\partial r} T_\ell^{(v+1)}(c', \phi') - \sin(\phi' - \alpha_\ell) \frac{1}{r} \frac{\partial}{\partial \phi} T_\ell^{(v+1)}(c', \phi') \right\} , \\
 &\quad (4.15)
 \end{aligned}$$

where the unit vector $-\hat{n}$ in Eq.(4.15) indicates that the direction of heat transfer is pointing into the ℓ th cell. Naturally, c' and ϕ' are also determined from the procedure described in sect.4.2. Eq.(4.15) can be equivalently expressed in dimensionless form as:

$$\begin{aligned}
 \frac{q''(c, \phi)}{\frac{\pi^2}{q a^2 / 2k_c}} &= q_\ell + \left\{ \cos(\phi' - \alpha_\ell) \frac{\partial}{\partial \rho} \theta_\ell^{(v+1)}(c', \phi') - \sin(\phi' - \alpha_\ell) \frac{1}{\rho} \frac{\partial}{\partial \phi} \theta_\ell^{(v+1)}(c', \phi') \right\} . \\
 &\quad (4.16)
 \end{aligned}$$

Again, Eq.(4.16) is satisfied at a finite number of collocation points along the common boundary. This leads to a system of simultaneous equations:

$$\sum_{n=1}^N \left[Y_{n,m}^{(v+1)} G_{jn} + \delta_{0,n} Y_{n,m}'^{(v+1)} G_{jn}' \right] = Q_{j,m}^{(v+1)} , \quad (4.17)$$

where

$$G_{jn} = -f_n \left(\frac{c}{b} \right)^{-\lambda_n} \cos [(\lambda_n + 1)\phi_j - \alpha_m] + g_n \left(\frac{c}{b} \right)^{\lambda_n} \cos [(\lambda_n - 1)\phi_j + \alpha_m] ,$$

$$G'_{jn} = f_n \left(\frac{c}{b} \right)^{-\lambda_n} \sin [(\lambda_n - 1)\phi_j + \alpha_m] - g_n \left(\frac{c}{b} \right)^{\lambda_n} \sin [(\lambda_n + 1)\phi_j - \alpha_m] ,$$

and

$$Q_{j,m}^{(v+1)} = \left\{ \left[\left(1 + \frac{1}{I_m} \right) - \frac{1}{I_m} \left(\frac{c}{b} \right)^2 \right] \cos(\phi - \alpha_m) - \frac{c/b}{q_m} \frac{q''(c, \phi)}{\bar{q} a^2 / 2k_c} \right\}_{\phi=\phi_j} .$$

Now it becomes clear that the temperature field of the m th unit cell is updated by the heat flux distribution at the common boundary generated from the updated temperature field of the neighboring cell. An example will be given to demonstrate the applicability of this coupling technique in the following section.

4.4 TEMPERATURE FIELDS OF THE 7-ROD BUNDLE

A small flow region near the corner which contains two unit cells as shown in Fig.4.6 is used in this section as an example to demonstrate the applicability of the aforementioned iterative method. By isolating the small 2 cell section from the rest of the bundle the problem becomes a symmetrical

section of a 7-rod bundle. As shown in Fig.4.7 the isolated section being analyzed consists of a small 30° internal cell and a corner cell which are designated as cell 1 and 2, respectively. In the present study, 36 and 8 collocation points are used for the corner and small internal cell, respectively. The distribution of these points are indicated in Fig.4.7.

By imposing the adiabatic boundary condition along the whole boundary of the two cells as described in the preceeding chapter, the dimensionless temperature fields of these two unit cells are obtained. The iterative procedure then proceeds to couple the two temperature fields by using the boundary conditions as indicated in Fig.4.7, in which T and q represents Dirichelet and Neumann type boundary condition respectively. For the corner cell, the boundary condition for the first cell boundary should remain adiabatic throughout the entire iteration procedure because the bundle wall is assumed to be adiabatic in this analysis.

Applying Eq.(4.17) at the collocation points along the first boundary of the corner cell, the following simultaneous equations are obtained:

$$\sum_{n=1}^N Y_{n,2}^{(v+1)} G_{jn} = Q_{j,2}^{(v+1)} = Q_{j,2}, \quad (4.18)$$

where $j = 1, 2, \dots, 16$. Due to the fact that the second

Fig.4.6 Configuration of Corner Section for Analysis

(a)

Fig.4.7 Symmetry Section of a 7-Rod Bundle

boundary of the corner lies on the symmetry line of the symmetry section of the 7-rod bundle, adiabatic boundary condition should be applied for this boundary. The following simultaneous equation can be obtained by applying Eq.(4.17) at the collocation points along the second boundary of the corner cell:

$$\sum_{n=1}^N y_{n,2}^{(\nu+1)} G_{jn} = Q_{j,2}^{(\nu+1)} = Q_{j,2} \quad (4.19)$$

where $j = 17, 18, \dots, 28$. For the third boundary of the corner cell, the temperature coupling technique is applied to link the two temperature fields resulted from one previous iteration. Upon applying Eq.(4.13) at the collocation points along the third boundary of the corner cell, the following simultaneous equations are obtained:

$$y_{0,21}^{(\nu+1)} + \sum_{n=1}^N y_{n,2}^{(\nu+1)} G_{jn} = P_{j,2}^{(\nu)} \quad (4.20)$$

where

$$y_{0,21}^{(\nu+1)} = \frac{1}{2} \left[\frac{A_2 - A_1}{\bar{q} a^2 / 2k_c} \right]^{(\nu+1)}$$

and $j = 29, 30, \dots, 36$.

Combining Eqs.(4.18), (4.19) and (4.20), the system of simultaneous equations can be arranged in matrix form as:

$$\underline{G_2} \underline{Y_2}^{(v+1)} = \underline{Q_2}^{(v+1)} \quad (4.21)$$

where

$$\underline{G_2} = \begin{bmatrix} 0 & G_{11} & G_{12} & \cdots & \cdots & \cdots & G_{1N} \\ \vdots & \vdots & \vdots & & & & \vdots \\ 0 & \vdots & \vdots & & & & \vdots \\ \hline 0 & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & & & & \vdots \\ 0 & \vdots & \vdots & & & & \vdots \\ \hline 1 & M_{291} & \cdots & \cdots & \cdots & \cdots & M_{29N} \\ \vdots & \vdots & \vdots & & & & \vdots \\ \vdots & \vdots & \vdots & & & & \vdots \\ 1 & M_{J1} & \cdots & \cdots & \cdots & \cdots & M_{JN} \end{bmatrix}$$

i=1

i=2

i=3

$$\underline{Y_2}^{(v+1)} = \begin{bmatrix} Y_{0,21}^{(v+1)} \\ Y_{1,2}^{(v+1)} \\ \vdots \\ Y_{N,2}^{(v+1)} \end{bmatrix}$$

and

$$\underline{Q}_2^{(v)} = \begin{bmatrix} Q_{1,2} \\ Q_{2,2} \\ \vdots \\ \vdots \\ \vdots \\ P_{29,2}^{(v)} \\ \vdots \\ \vdots \\ P_{36,2}^{(v)} \end{bmatrix} \quad \begin{matrix} i=1 \\ \\ \\ i=2 \\ \\ i=3 \end{matrix}$$

Letting $N = 23$ such that $(N+1)/J=2/3$ and applying the least square procedure as described in section 3.1, the solution vector is given by:

$$\underline{Y}_2^{(v+1)} = (G_2^T G_2)^{-1} G_2^T Q_2^{(v)} \quad (4.22)$$

Having determined the updated temperature field of the corner cell, the next step is to apply the Neumann type boundary condition for the internal cell along the connecting common boundary. Applying Eq.(4.17) at the collocation points along the only boundary of the small internal cell leads to:

$$\sum_{n=1}^N Y_{n,1}^{(v+1)} G_{jn} = Q_{j,1}^{(v+1)}, \quad (4.23)$$

where $j = 1, 2, \dots, 8$. Eq.(4.23) can be written in the equivalent matrix form:

$$G_1 \underline{Y}_1^{(v+1)} = \underline{Q}_1^{(v+1)},$$

where

$$G_1 = \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ G_{81} & G_{82} & \dots & G_{8N} \end{bmatrix},$$

$$\underline{Y}_1^{(v+1)} = \begin{bmatrix} Y_{1,1}^{(v+1)} \\ \vdots \\ Y_{N,1}^{(v+1)} \end{bmatrix}, \quad \text{and} \quad \underline{Q}_1^{(v+1)} = \begin{bmatrix} Q_{1,1}^{(v+1)} \\ \vdots \\ Q_{J,1}^{(v+1)} \end{bmatrix}.$$

In the present analysis, N is chosen to be 5 such that the ratio N/J is approximately equal to $2/3$. Applying the least square procedure as described in section 3.1, the updated solution vector for the small internal cell is given by:

$$\underline{Y}_1^{(v+1)} = (G_1^T G_1)^{-1} G_1^T Q_1^{(v+1)}$$

Thus the temperature field of the small internal cell is updated by the updated temperature field of the corner cell. In the next iteration the temperature coupling boundary condition will be applied again to further decrease the discontinuity in the temperature at the common boundary. The procedure is repeated until the change in the Nusselt number is less than 1%. The convergence of the solution will be discussed further in section 4.6.

4.5 NORMALIZATION OF TEMPERATURE FIELDS OF UNIT CELLS

The outcome of the aforementioned multicell coupling procedure is the solution coefficients for each unit cell. As a result, these coefficients yield the detailed 2-Dimensional dimensionless temperature fields. However, the dimensionless temperature for a unit cell as given in chapter 2 is the normalized temperature difference relative to the average fuel surface temperatures of the unit cell. Naturally, the average fuel surface temperatures for all the unit cells are not the same and it is meaningless to compare dimensionless tempera-

tures of different unit cells. Therefore it is desirable to rearrange the solution form of the temperature fields such that they are compared with respect to the same arbitrary constant.

In the present analysis, the bulk mean coolant temperature of the entire section ($\bar{T}_B$) is chosen to be the common arbitrary constant because $\bar{T}_B$ for a certain axial position can be easily obtained by a simple energy balance relation for the rod bundle. The bulk mean coolant temperature is defined as:

$$\bar{T}_B = \frac{1}{A_T} \sum_{\ell=1}^{\text{all cells}} \eta_{\ell} \bar{T}_{\ell} = \frac{1}{A_T} \sum_{\ell=1}^{\text{all cells}} \eta_{\ell} \left[A_{\ell} + \frac{\bar{q}_{\ell}'' a^2}{2k_c} \bar{\theta}_{\ell} \right] \quad (4.26)$$

where η_{ℓ} , is the flow area of the ℓ th cell and A_T is the total flow area of the section of the rod bundle being considered. In addition, $\bar{\theta}_{\ell}$ is the average dimensionless coolant temperature of the ℓ th unit cell. Dividing Eq.(4.26) by $\bar{q}'' a^2/2k_c$ leads to:

$$\frac{\bar{T}_B}{\bar{q}'' a^2/2k_c} = \sum_{\ell=1}^{\text{all cells}} \frac{A_{\ell}}{\bar{q}'' a^2/2k_c} A_{\ell}^{+} + \sum_{\ell=1}^{\text{all cells}} A_{\ell}^{+} \bar{q}_{\ell}^{+} \bar{\theta}_{\ell} \quad (4.27)$$

where A_ℓ^+ is the dimensionless cross-sectional flow area of the ℓ th unit cell given by:

$$A_\ell^+ = \frac{\eta_\ell}{A_T} \quad (4.28)$$

It should be pointed out that the calculation of the averaged dimensionless coolant temperature has already been discussed in chapter 3. Recalling the relationships between the absolute and dimensionless temperatures for the coolant and clad of the m th unit cell are:

$$\frac{T_m(\rho, \phi) - A_m}{q_m''' a^2 / 2k_c} = \Theta_m(\rho, \phi) \quad (4.29)$$

and

$$\frac{t_m(\rho, \phi) - A_m}{q_m''' a^2 / 2k_c} = \Psi_m(\rho, \phi) \quad (4.30)$$

respectively, where Ψ_m and Θ_m are the dimensionless clad and coolant temperature respectively. Multiplying Eqs.(4.29) and (4.30) by q_m^+ and leads after reaaranging to :

$$\frac{T_m(\rho, \phi)}{q_m''' a^2 / 2k_c} = \frac{A_m}{q_m''' a^2 / 2k_c} + q_m^+ \Theta_m(\rho, \phi) \quad (4.31)$$

and

$$\frac{t_m(\rho, \phi)}{\bar{q}'' a^2 / 2k_c} = \frac{A_m}{\bar{q}'' a^2 / 2k_c} + q_m^+ \bar{\psi}_m(\rho, \phi) \quad (4.32)$$

Subtracting Eq.(4.27) from Eqs.(4.31) and (4.32) yields the following relations:

$$\frac{T_m(\rho, \phi) - \bar{T}_B}{\bar{q}'' a^2 / 2k_c} = \frac{A_m - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ A_\ell}{\bar{q}'' a^2 / 2k_c} - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ q_\ell^+ \bar{\theta}_\ell + q_m^+ \theta_m(\rho, \phi), \quad (4.33)$$

$$\frac{t_m(\rho, \phi) - \bar{T}_B}{\bar{q}'' a^2 / 2k_c} = \frac{A_m - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ A_\ell}{\bar{q}'' a^2 / 2k_c} - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ q_\ell^+ \bar{\theta}_\ell + q_m^+ \theta_m(\rho, \phi). \quad (4.34)$$

Rewriting A_m as:

$$A_m = A_m \sum_{\ell=1}^{\text{all cells}} A_\ell^+ = \sum_{\ell=1}^{\text{all cells}} A_m A_\ell^+ \quad (4.35)$$

Eqs.(4.33) and (4.34) can be arranged into:

$$\frac{T_m(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} = \sum_{\ell=1}^{\text{all cells}} \frac{(A_m - A_\ell)}{\bar{q}''' a^2 / 2k_c} A_\ell^+ - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ q_\ell^+ \bar{\theta}_\ell + q_m^+ \theta_m(\rho, \phi), \quad (4.36)$$

and

$$\frac{t_m(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} = \sum_{\ell=1}^{\text{all cells}} \frac{(A_m - A_\ell)}{\bar{q}''' a^2 / 2k_c} A_\ell^+ - \sum_{\ell=1}^{\text{all cells}} A_\ell^+ q_\ell^+ \bar{\theta}_\ell + q_m^+ \psi_m(\rho, \phi). \quad (4.37)$$

Therefore, Eqs. (4.36) and (4.37) are general expressions which relate the temperature field of an individual unit cell to the bulk mean coolant temperature ($\bar{T}_B$) by carefully taking the differences between fuel surface temperatures of unit cells into consideration. Applying Eqs. (4.36) and (4.37) to the two-cell case and using the solutions resulted from the iteration yields:

$$\left[\frac{T_1(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} = A_2^+ \left[\frac{A_1 - A_2}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} - \sum_{\ell=1}^2 A_\ell^+ q_\ell^+ \bar{\theta}_\ell^{(v+1)} + q_1^+ \theta_1^{(v+1)}(\rho, \phi), \quad (4.38)$$

$$\left[\frac{T_2(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} = A_2^+ \left[\frac{A_2 - A_1}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} - \sum_{\ell=1}^2 A_\ell^+ q_\ell^+ \bar{\theta}_\ell^{(v+1)} + q_2^+ \bar{\theta}_2^{(v+1)}(\rho, \phi), \quad (4.39)$$

$$\left[\frac{t_1(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} = A_2^+ \left[\frac{A_1 - A_2}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} - \sum_{\ell=1}^2 A_\ell^+ q_\ell^+ \bar{\theta}_\ell^{(v+1)} + q_1^+ \bar{\psi}_1^{(v+1)}(\rho, \phi), \quad (4.40)$$

and

$$\left[\frac{t_2(\rho, \phi) - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} = A_1^+ \left[\frac{A_2 - A_1}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} - \sum_{\ell=1}^2 A_\ell^+ q_\ell^+ \bar{\theta}_\ell^{(v+1)} + q_2^+ \bar{\psi}_2^{(v+1)}(\rho, \phi), \quad (4.41)$$

It has to be noticed that

$$\left[\frac{A_2 - A_1}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)} = 2Y_{0,21}^{(v+1)}. \quad (4.42)$$

where $y_{0,21}^{(v+1)}$ is the constant resulted from the v th temperature coupling iteration. Numerical results for the normalized coolant and clad temperatures will be presented in the following section.

4.6 CONVERGENCE OF THE ITERATIVE METHOD

In order to demonstrate how the temperature fields of both unit cells converge during the iteration, the normalized dimensionless coolant temperature distributions as given by Eqs.(4.38) and (4.39) along the cell boundary for both unit cells are given in Figs.4.8 and 4.9. The flow regime is assumed to be thermally fully developed slug flow. Thus, the axial flow velocity is uniform throughout the entire symmetry section. From Figs.4.8 and 4.9, it can be readily seen that the dimensionless coolant temperature fields of both unit cells practically converge after two iterations; i.e, the maximum difference between the solutions resulted from the second and third iterations is only approximately 0.01, which means about 1.5°F for normal operation conditions.

On the other hand, the dimensionless heat flux distributions along the cell boundary as given by Eq.(3.12) for both cells are shown in Figs.4.10 and 4.11. Similar to the temperature distributions, the heat flux distributions along the cell boundary converge rapidly after approximately two iterations. However, this result should be anticipated because the

convergence of the temperature fields automatically implies the convergence of the heat flux distributions. In addition, it is shown in Fig.4.10 that the dimensionless heat flux along the first and second boundaries of the corner cell is so small (less than 1×10^{-3}) that it can hardly be shown in the figure. The small values for the heat fluxes along the bundle wall and the section symmetry thus seemingly satisfies the adiabatic boundary condition imposed by the problem.

The other boundary conditions which have to be satisfied for the present problem are the continuity of temperature and heat flux along the common boundary connecting the two cells. In order to illustrate the validity of the solution, the difference between the dimensionless coolant temperature along the common boundary given by the temperature field of the corner cell and that of the small internal cell after 3 iterations are presented in Fig.4.12. It can be seen quite readily that the maximum difference is approximately 1.11×10^{-3} , which implies a temperature discontinuity of about 0.17°F for typical operating conditions. In addition, the difference between the dimensionless heat flux normal to the common boundary given by the temperature field of the corner cell and that of the small internal cell is shown in Fig.4.13. It is indicated that the difference between the heat fluxes given by the temperature fields is extremely small (in the order of 10^{-5}) after 3 iterations. Therefore, the condition of continuity

$$\frac{T(c, \phi) - \bar{T}_B}{\bar{q}'' a^2 / 2k_c}$$

Fig.4.8 Coolant Temperature Distribution at the Cell Boundary of the Corner Cell

$$\frac{T(c, \phi) - \bar{T}_B}{\bar{q}'' a^2 / 2k_c}$$

Fig.4.9 Coolant Temperature Distribution at the Cell Boundary of the Internal Cell

Fig.4.10 Heat Flux Distribution at the Boundary of the Corner Cell

$$\frac{q''(c, \phi)}{q''' a^2 / 2b}$$

Fig.4.11 Heat Flux Distribution at the Boundary of the Small Internal Cell

Fig.4.12 Error of Temperature at the Common Boundary After 3 Iterations

$$\frac{\overset{\uparrow}{q''(c,\phi)} - [-q_2''(c,\phi)]}{\bar{q}''' a^2 / 2b} \times 10^5$$

Fig.4.13 Discontinuity of Heat Flux at the Common Boundary after 3 Iterations

of heat flux along the common boundary is satisfied to a very high degree of accuracy.

By virtue of these findings, it is clear that a quite satisfactory solution already exists after two or three iterations by using the aforementioned temperature coupling procedure. Moreover, it should be also pointed out that the iterative procedure can be terminated at any iterative step depending on the accuracy one desires. In the present study, the calculation procedure is terminated at the end of the fourth iteration because it is felt that more than 5 or 6 iterations are truly unnecessary in light of the aforementioned findings.

4.7 VERIFICATION OF THE ITERATIVE METHOD

In order to verify and test results of the calculation, the iterative method is applied to a two-cell calculation with a square array. The results are compared with the integral least square solution presented by France and Ginsberg [58], who successfully checked their solution method against the finite-difference solution of the THTB code [69].

The configuration of the two-cell section being considered is shown in Fig.4.14. The semi-infinite array is bounded by the bundle wall on the left side. The flow velocity is assumed to be constant throughout the entire flow channel. Furthermore, the boundary condition of aximuthally uniform heat flux distribution is also imposed at the clad outside surface.

Fig.4.15 shows the comparison of the clad temperature distributions as obtained by the three methods. It can be seen that the solution obtained from the present analysis is in excellent agreement with that of France and Ginsberg. In addition, it can be seen that the solutions of both the integral least square and the present method agree reasonably well with the finite-difference solution. The maximum error is approximately 0.05 which is roughly 5% of the maximum variation of the clad temperature. Furthermore, it is interesting to compare the computational efforts which are involved with the various methods. For instance, the finite-difference method used 720 nodes and required 2598 iterations which consumed approximately 45 minutes computation time on a CDC-3600 machine. The integral least square method of France and Ginsberg ran about 2 minutes on the same machine for 20 terms in the series expansion. The computation time for the present method with 36 collocation points and 24 terms in the Fourier Series expansion required less than 1 minute on an IBM 360/65 machine.

By virtue of these findings, it is obvious that the solution of the collocation least square method is at least just as good as other more sophisticated methods. However, its relatively shorter computation time definitely counts as in advantage over the other two methods.

Fig.4.14 A Two Cell Section of a Square Array Configuration

Fig.4.15 Comparison of Iterative Solution to Finite Difference Results

4.8 RESULTS OF THE CALCULATION FOR A 7-ROD BUNDLE

Graphs of the dimensionless clad temperature relative to $\bar{T}_B$ for both unit cells with $P/D = 1.24$ and $P/D = 1.08$ are given in Figs. 4.16 and 4.17, respectively. The former case simulates a typical 7-rod fuel assembly and the latter simulates that of a radial blanket assembly. For each pitch-to-diameter ratio, four cases with different $2W/D$ ratios are shown to demonstrate the effect of the dimensionless wall distance. Again, for these calculations the flow velocity is assumed to be constant throughout the symmetry section.

From these figures it is clear that the clad temperature distributions are extremely sensitive to the $2W/D$ ratio as the hot spot of the clad surface shifts from $\phi = 0$ of the corner cell to $\phi = 0$ of the internal cell when the wall distance increases. This is because an increase in the wall distance implies an increase in the heat sink in the region near the bundle wall and as a result strong undercooling occurs in this section of the cell an associated part of the rod.

Upon comparing Figs. 4.16 and 4.17, it can be seen that the magnitude of the dimensionless clad temperature variation of the blanket rods are generally higher than those of the fuel rods. However, this does not necessarily mean that the clad temperature variation of the blanket rods are higher than those of the fuel rods because the magnitude of the variation on the absolute temperature scale depends on the normalization group

Fig.4.16 Clad Outside Temperature Distributions of the Fuel Bundle for Different $2W/D$ Ratios

Fig.4.17 Clad Outside Temperature Distributions of the Blanket Bundle for Different $2W/D$ Ratios

$q'''a^2/2k_c$ and it is a well known fact that blanket rods usually have much lower power densities than fuel rods.

As pointed out by previous researches [23], it can also be seen that the magnitude of the azimuthal variation of the clad temperature of the small internal cell is much smaller than that of the corner cell. For example, for the blanket assembly, the maximum variation of the dimensionless clad temperature of the small internal unit cell is around 0.20 while the maximum variation of the dimensionless clad temperature of the corner cell can be as high as 4.00. Assuming the linear power of the blanket rod to be 2KW/ft, this yields a 180°F temperature variation around the circumference of the corner rod. Therefore the result of these calculation clearly indicate that the major problems for structural considerations are the cells next to the bundle wall.

Due to the enormous sensitivity of the clad temperature variation to the wall distance, the selection of the right wall distance which will minimize the maximum variation of clad temperature variation of a rod becomes very important for the assembly design. From Figs. 4.7 and 4.8, the most desirable dimensionless wall distance for the fuel assembly and blanket assembly are approximately $2W/D = 1.20$ and $2W/D = 1.05$ respectively, i.e., these optimum values turn out to be slightly less than the respectively P/D values.

It is known that the power gradient in the core of the LMFBR is relatively small but strong power gradient does exist in the neighborhood of the core/blanket interface. In the preceeding examples, the power densities in the two rods were assumed to be equal. In order to illustrate the effect of power tilting on the clad temperature behavior, an example with different cell-to-average power ratio (q^+) in different unit cells is given for a symmetry section of a 7-rod blanket bundle. Assuming that the pressure drop of all the unit cells are equal, it can be shown that the unit cell flow velocity is related to the hydraulic diameter by the following:

$$\frac{U_1}{U_2} = \left(\frac{D_1}{D_2} \right)^{5/7} \quad (4.43)$$

where U and D represents the flow velocity and the hydraulic diameter of the unit cell, respectively. It has to be pointed out the Eq.(4.43) is only a relatively simple approach and better flow factor can be implemented should a proper wire-wrapped model become available. In the present study, the cell-to-average velocity ratios U^+ resulted from Eq.(4.43) are 1.054 and 0.994 for the internal and corner cell, respectively.

The dimensionless clad outside surface temperature distributions of both unit cells of five cases with different degrees of power tilting are given in Fig.4.18. It can be seen that a maximum variation of the clad surface temperature results

with the rod power factor of the internal cell and the corner cell equal to 1.1 and 0.9 respectively. The reason for this is physically obvious. Even for the no-power tilting case (curve 3) the rod of the corner cell is already undercooled in the region near the bundle wall (i.e. $0 \leq \phi \leq 70^\circ$) due to the relatively large wall distance and subsequently wider coolant channel. Therefore, placing a rod with higher-than-average power in the internal cell and a rod with lower-than-average power in the corner cell can only make the variation worse. In this case, the geometric effect has been augmented by the high-inside-low-outside power tilting. On the other hand, the case with the power factor of 1.10 for the corner and 0.90 for the internal cell results in the smallest variation in clad temperature. This is because the geometric effect and the power tilting has been favorable combined such that they offset each other. From Fig.4.18 it can be seen that a 20% power tilting in the unfavorable direction can add an additional factor of approximately 1.50 to the dimensionless clad temperature variation in the corner rod. Depending on the power of the rod, this translates to about 50-150°F in terms of absolute temperature for typical operating conditions. The result of these calculations clearly indicates that the power tilting is an important factor in determining the azimuthal variation of clad temperature and its effect can by no means be estimated by using only the single-cell analysis. At this point, the advantage of the multicell analysis becomes apparent.

Fig.4.18 Clad Outside Temperature Distributions of the Blanket Bundle with Different Rod Power Densities

It has to be pointed out here that the clad surface temperature distributions for all calculations in this section do not take the entrance effect into consideration; that is, the azimuthal variations of the clad temperatures are those for the thermally fully-developed conditions. It is known that the coolant temperature distribution at the inlet of the core is essentially uniform and the azimuthal variations of both clad and coolant temperatures increase with increasing axial distance from the channel inlet until the entrance effect levels off and the fully-developed temperature distributions are achieved. From detailed 3-D calculations [39] as well as experiments it is known that thermally fully-developed conditions are not reached in LMFBR cores. Therefore the present 2-D analysis has to be considered to be over conservative and thus serving only as a upper bound estimation for the clad temperature variations.

CHAPTER 5

MULTICELL ANALYSIS FOR LARGER BUNDLES

The methodology of coupling neighboring unit cells for the two-cell case has been demonstrated in chapter 4. In the present chapter, the technique is extended one step further to extend the analysis for larger bundles. In addition, the calculational results of a 19-Rod bundle is compared to the experimentally measured data such that an axial correction factor can be determined to account for the thermal entrance effect.

5.1 BOUNDARY CONDITIONS USED TO COUPLE NEIGHBORING CELLS

A symmetry section of the 127-rod bundle as shown in Fig.5.1 is being considered in what follows. By isolating the 4-cell section from the remaining portion of the bundle, the section being analyzed is in effect a symmetry section of a 19-rod bundle. As shown in Fig.5.2, the symmetry section consists of a small 30° internal cell, a 4-sided internal cell, an edge cell and a corner cell. In order to minimize computation time, the number of collocation points used for each unit cell has to be reduced. The distribution of points at each boundary of the unit cells are also shown in Fig.5.2.

Similar to the 7-rod bundle case, the dimensionless temperature fields for all unit cells are initially obtained

Fig.5.1 Configuration of the Rod Cluster Used for the 19-Rod Bundle Analysis

Fig.5.2 Configuration of the Symmetry Section of a 19-Rod Bundle

as a result of imposing adiabatic boundary condition along all the cell boundaries. The iterative procedure then couples the temperature fields by using boundary conditions of continuity of temperature and heat flux. It has to be pointed out that the first boundary of the corner cell and the third boundary of the edge cell are actually formed by the bundle wall which is assumed to be adiabatic throughout the entire iteration procedure. Furthermore, the third boundary of the 4-sided internal cell has to be adiabatic because it lies on the geometric symmetry line. Satisfying Eq.(4.17) at the collocation points along the first boundary of the corner cell (with $q''=0$), the following simultaneous equations for the v th iteration are obtained:

$$\sum_{n=1}^N Y_{n,4}^{(v+1)} G_{jn,4} = Q_{j,4}^{(v)}, \quad (5.1)$$

where $j = 1, 2, \dots, 8$

On the other hand, the temperature coupling technique is applied to the second boundary such that the iterative procedure can be initiated. Applying the technique as illustrated in chapter 4, the following relations are obtained:

$$Y_{0,43}^{(v+1)} + \sum_{n=1}^N Y_{n,4}^{(v+1)} M_{jn,4} = P_{j,4}^{(v)}, \quad (5.2)$$

where

$$Y_{0,43}^{(v+1)} = \frac{1}{2} \left[\frac{A_4 - A_3}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)},$$

and $j = 9, 10, \dots, 14$

Similarly, the iterative relations for the third boundary are given by:

$$Y_{0,42}^{(v+1)} + \sum_{n=1}^N Y_{n,4}^{(v+1)} M_{jn,4} = P_{j,4}^{(v)}, \quad (5.3)$$

where

$$Y_{0,42}^{(v+1)} = \frac{1}{2} \left[\frac{A_4 - A_2}{\bar{q}''' a^2 / 2k_c} \right]^{(v+1)},$$

and $j = 15, 16, 17, 18$

Combining Eqs. (5.1), (5.3) and (5.6), the system of simultaneous equations can be written in a more compact form:

$$\underline{G}_4 Y_4^{(v+1)} = \underline{Q}_4^{(v)}, \quad (5.4)$$

where $\underline{G}_4$ is the coefficient matrix given by:

$$G_4 = \begin{bmatrix} 0 & G_{11} & G_{12} & \cdot & \cdot & \cdot & \cdot & \cdot & G_{1N} \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ 0 & \cdot & \cdot & & & & & & \cdot \\ \hline 1 & M_{91} & M_{92} & \cdot & \cdot & \cdot & \cdot & \cdot & M_{9N} \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ \hline \cdot & \cdot & \cdot & & & & & & \cdot \\ \cdot & \cdot & \cdot & & & & & & \cdot \\ 1 & M_{J1} & M_{J2} & \cdot & \cdot & \cdot & \cdot & \cdot & M_{JN} \end{bmatrix}$$

i=1
i=2
i=3

and the vectors $\underline{Q}_4^{(v)}$ and $\underline{Y}_4^{(v+1)}$ are given by:

$$\underline{Q}_4^{(v)} = \begin{bmatrix} Q_{1,4}^{(v)} \\ Q_{2,4}^{(v)} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hline P_{9,4}^{(v)} \\ \cdot \\ \cdot \\ \cdot \\ \hline \cdot \\ \cdot \\ \cdot \\ \hline P_{J,4}^{(v)} \end{bmatrix}$$

i=1
i=2
i=3

and

$$\underline{Y_4^{(v+1)}} \begin{bmatrix} Y_{0,43}^{(v+1)} \\ Y_{0,42}^{(v+1)} \\ Y_{1,4}^{(v+1)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ Y_{N,4}^{(v+1)} \end{bmatrix}$$

respectively. Since there is no information about the heat flux distribution at the first boundary of the edge cell, the temperature coupling technique has to be used to initiate the iteration procedure. For the second and third boundaries, the Neumann boundary condition can be used because the updated temperature field of the corner cell allows the determination of the heat flux distribution at the second boundary of the edge cell and the third boundary is assumed to be adiabatic at all time. Applying these boundary conditions, the following simultaneous equations are obtained for the v th iteration;

$$\text{for } i=1, \quad Y_{0,32}^{(v+1)} + \sum_{n=1}^N Y_{n,3}^{(v+1)} G_{jn,3} = P_{j,3}^{(v)} \quad (5.5)$$

where
$$Y_{0,32}^{(v+1)} = \frac{1}{2} \left[\frac{A_3 - A_2}{\frac{1}{q} a^2 / 2k_c} \right]^{(v+1)},$$

and $j = 1, 2, \dots, 8;$;

for $i=2,$
$$\sum_{n=1}^N Y_{n,3}^{(v+1)} G_{jn,3} = Q_{j,3}^{(v)}, \quad (5.6)$$

where $j = 9, 10, \dots, 14$;

and for $i=3,$
$$\sum_{n=1}^N Y_{n,3}^{(v+1)} G_{jn,3} = Q_{j,3}^{(v)}, \quad (5.7)$$

where $j = 15, 16, 17, 18$.

Naturally, the quantity $q''(\phi)$ contained in the $Q_{j,3}^{(v)}$ term of Eq.(5.6) for the second boundary has to be determined from the updated temperature field of the corner cell and for the third boundary, $q''(\phi)$ has to be set zero in order to satisfy the adiabatic boundary condition. By combining the system of simultaneous equations as given by Eqs. (5.5), (5.6) and (5.7), the iterative algorithm for the edge cell is given by:

$$G_3 Y_3^{(v+1)} = Q_3^{(v)}, \quad (5.8)$$

where

$$G_3 = \begin{bmatrix} 1 & G_{11} & G_{12} & & G_{1N} \\ \cdot & G_{21} & G_{22} & & G_{2N} \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ 1 & \cdot & & & \cdot \\ \hline 0 & \cdot & & & \cdot \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ \vdots & \vdots & & & \vdots \\ 0 & G_{J1} & G_{J2} & & G_{JN} \end{bmatrix} \begin{matrix} i=1 \\ \\ \\ \\ \\ i=2 \\ \\ i=3 \end{matrix},$$

$$l_3^{(v)} = \begin{bmatrix} p_{1,3}^{(v)} \\ p_{2,3}^{(v)} \\ \vdots \\ \vdots \\ q_{9,3}^{(v)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ q_{J,3}^{(v)} \end{bmatrix} \begin{matrix} i=1 \\ \\ \\ i=2 \\ \\ i=3 \end{matrix},$$

and

$$\underline{Y_3^{(v+1)}} = \begin{bmatrix} Y_0^{(v+1)} \\ Y_1^{(v+1)} \\ Y_{2,3}^{(v+1)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ Y_{N,3}^{(v+1)} \end{bmatrix}$$

For the 4-sided internal unit cell, the Neumann-type boundary condition is applied to the first, second and third boundaries. The heat flux distributions at the first and second boundaries are calculated by the updated temperature fields of the corner and edge cell respectively. Due to geometric symmetry the third boundary should remain adiabatic throughout the entire iteration procedure (a special case of the Neumann type boundary condition). For the fourth boundary, the technique of temperature fields coupling is used to couple the 4-sided internal and the small internal cell. Upon applying these appropriate boundary conditions the following simultaneous equations are obtained:

$$\text{for } i=1 : \sum_{n=1}^N Y_{n,2}^{(v+1)} G_{jn,2} = Q_{j,2}^{(v)} \quad , \quad (5.9)$$

where $j = 1, 2, 3, 4.$;

$$\text{for } i=2 : \sum_{n=1}^N Y_{n,2}^{(v+1)} G_{jn,2} = Q_{j,2}^{(v)} \quad , \quad (5.10)$$

where $j = 5, 6, \dots, 12$;

$$\text{for } i=3 : \sum_{n=1}^N Y_{n,2}^{(v+1)} G_{jn,2} = Q_{j,2}^{(v)} \quad , \quad (5.11)$$

where $j = 13, 14, \dots, 20$;

$$\text{and for } i=4 : Y_{0,21}^{(v+1)} + \sum_{n=1}^N Y_{n(2)}^{(v+1)} G_{jn,2} = P_{j,2}^{(v)} \quad , \quad (5.12)$$

$$\text{where } Y_{0,21}^{(v+1)} = \frac{1}{2} \left[\frac{A_2 - A_1}{\bar{q} a^2 / 2k_c} \right]^{(v+1)} \quad ,$$

and $j = 21, 22, 23, 24$.

Thus the iterative algorithm for the 4-sided internal cell is given by:

$$G_{\underline{2}} Y_{\underline{2}}^{(v+1)} = \underline{Q}_{\underline{2}}^{(v)} \quad , \quad (5.13)$$

where

$$G_2 = \begin{bmatrix} 0 & G_{11} & G_{12} & \cdots & G_{1N} \\ \cdot & G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdot & \cdot & & \cdot \\ 1 & \cdot & \cdot & & \cdot \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & G_{J1} & G_{J2} & & G_{JN} \end{bmatrix} \begin{matrix} i=1 \\ \\ \\ \\ \\ \\ \\ i=2 \\ \\ \\ i=3 \\ \\ \\ i=4 \end{matrix},$$

$$\underline{Q}_2^{(v)} = \begin{bmatrix} Q_{1,2}^{(v)} \\ Q_{2,2}^{(v)} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ P_{j,2}^{(v)} \\ \vdots \\ \vdots \\ P_{J,2}^{(v)} \end{bmatrix} \begin{matrix} i=1 \\ \\ \\ \\ \\ \\ i=2 \\ \\ \\ i=3 \\ \\ \\ i=4 \end{matrix},$$

and

$$\underline{Y}_2^{(v+1)} = \begin{bmatrix} Y_{0,21}^{(v+1)} \\ Y_{1,2}^{(v+1)} \\ \vdots \\ Y_{N,2}^{(v+1)} \end{bmatrix}$$

The final step of the procedure is to apply the Neumann type boundary condition to the only boundary of the small internal cell. Upon doing so, the following simultaneous equations are obtained:

$$\sum_{n=1}^N Y_{n,1}^{(v+1)} G_{jn,1} = Q_{j,1}^{(v)}, \quad (5.14)$$

where $j = 1, 2, \dots, 4$

Naturally, the $q''(\phi)$ contained in the $Q_{j,1}^{(v)}$ term has to be calculated from the updated temperature field of the 4-sided internal cell. The iterative algorithm of the small internal cell can be written as:

$$G Y_1^{(v+1)} = Q_1^{(v)}, \quad (5.15)$$

where

$$\underline{G}_1 = \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1N} \\ \vdots & \vdots & & \vdots \\ G_{J1} & G_{J2} & \dots & G_{JN} \end{bmatrix}$$

$$\underline{Q}_1^{(v)} = \begin{bmatrix} Q_{1,1}^{(v)} \\ \vdots \\ Q_{J,1}^{(v)} \end{bmatrix}$$

and

$$\underline{Y}_1^{(v+1)} = \begin{bmatrix} Y_{1,1}^{(v+1)} \\ \vdots \\ Y_{N,1}^{(v+1)} \end{bmatrix}$$

Thus, the calculation for updating the temperature fields of the four unit cells boils down to the general relation:

$$G_k \underline{Y}_k^{(v+1)} = \underline{Q}_k^{(v)}, \quad (5.16)$$

where k denotes the k th unit cell. Applying the least square technique as discussed in chapter 4, the updated temperature field solution vector $\underline{Y}_k^{(v+1)}$ is given by:

$$\underline{Y}_k^{(v+1)} = (G_k^T G_k)^{-1} G_k^T \underline{Q}_k^{(v)} \quad (5.17)$$

The updating calculation of the v th iteration has been completed. The temperature fields resulted from the calculation will be saved and used as the "old" temperature fields for the $v+1$ th iteration. This procedure is repeated until the solution converges. Despite its complexity, the entire procedure has been programmed (see Appendix C) and the calculation is quite automatic.

5.2 CIRCUMFERENTIAL CLAD TEMPERATURE DISTRIBUTIONS

Similar to chapter 4, the bulk mean coolant temperature of the four cell symmetry section ($\bar{T}_B$) is chosen as the common arbitrary constant such that the dimensionless temperature field of each unit cell can be compared to a common quantity. As shown in chapter 4, the difference between the clad temperature of the k th unit cell and the bulk coolant mean temperature is given by:

$$\frac{t_k(\rho, \varphi)}{\frac{q'' a^2}{2k_c}} = \sum_{l=1}^{\text{all cells}} A_l \frac{(A_l - A_k)}{\frac{q'' a^2}{2k_c}} - \sum_{l=1}^{\text{all cells}} A_l^+ q_l^+ \bar{\Theta}_l + q_k^+ \bar{\Psi}_k(\rho, \varphi) \quad (5.18)$$

Applying Eq. (5.18) to the special case of the 19-Rod bundle the normalized clad temperature for each individual unit cell becomes:

$$\begin{aligned} \frac{t_1 - \bar{T}_B}{\frac{q'' a^2}{2k_c}} = & A_2^+ \frac{(A_1 - A_2)}{\frac{q'' a^2}{2k_c}} + A_3^+ \left[\frac{(A_1 - A_2)}{\frac{q'' a^2}{2k_c}} + \frac{(A_2 - A_3)}{\frac{q'' a^2}{2k_c}} \right] \\ & + A_4^+ \left[\frac{(A_1 - A_2)}{\frac{q'' a^2}{2k_c}} + \frac{(A_2 - A_4)}{\frac{q'' a^2}{2k_c}} \right] - \sum_{l=1}^4 A_l^+ q_l^+ \bar{\Theta}_l + q_1^+ \bar{\Psi}_1, \end{aligned} \quad (5.19)$$

$$\begin{aligned} \frac{t_2 - \bar{T}_B}{\frac{q'' a^2}{2k_c}} = & A_1^+ \frac{(A_2 - A_1)}{\frac{q'' a^2}{2k_c}} + A_3^+ \frac{(A_2 - A_3)}{\frac{q'' a^2}{2k_c}} + A_4^+ \frac{(A_2 - A_4)}{\frac{q'' a^2}{2k_c}} \\ & - \sum_{l=1}^4 A_l^+ q_l^+ \bar{\Theta}_l + q_2^+ \bar{\Psi}_2, \end{aligned} \quad (5.20)$$

$$\frac{t_3 - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} = A_1^+ \frac{(A_3 - A_1)}{\bar{q}''' a^2 / 2k_c} + A_2^+ \frac{(A_3 - A_2)}{\bar{q}''' a^2 / 2k_c} + A_4^+ \frac{(A_3 - A_4)}{\bar{q}''' a^2 / 2k_c} - \sum_{\ell=1}^4 A_\ell^+ \bar{q}_\ell^+ \bar{\Theta}_\ell + \bar{q}_3^+ \bar{\Psi}_3 \quad (5.21)$$

$$\frac{t_4 - \bar{T}_B}{\bar{q}''' a^2 / 2k_c} = A_1^+ \left[\frac{(A_4 - A_2)}{\bar{q}''' a^2 / 2k_c} + \frac{(A_2 - A_1)}{\bar{q}''' a^2 / 2k_c} \right] + A_2^+ \frac{(A_4 - A_2)}{\bar{q}''' a^2 / 2k_c} + A_3^+ \frac{(A_4 - A_3)}{\bar{q}''' a^2 / 2k_c} - \sum_{\ell=1}^4 A_\ell^+ \bar{q}_\ell^+ \bar{\Theta}_\ell + \bar{q}_4^+ \bar{\Psi}_4 \quad (5.22)$$

Naturally, the temperature fields being considered here are those resulted from the solutions after v iterations. Also, Eqs. (5.19) through (5.22) can be written in the following equivalent forms:

$$\begin{aligned} \frac{t_1 - \bar{T}_B}{\frac{1}{2} a^2 / k_c} = & -A_2^+ [2Y_{0,21}] - A_3^+ [2Y_{0,21} + 2Y_{0,32}] \\ & - A_4^+ [2Y_{0,21} + 2Y_{0,42}] - \sum_{\ell=1}^4 A_\ell^+ f_\ell^+ \bar{\Theta}_\ell + f_1^+ \bar{\Psi}_1, \end{aligned} \quad (5.23)$$

$$\begin{aligned} \frac{t_2 - \bar{T}_B}{\frac{1}{2} a^2 / k_c} = & A_1^+ [2Y_{0,21}] - A_3^+ [2Y_{0,32}] - A_4^+ [2Y_{0,42}] \\ & - \sum_{\ell=1}^4 A_\ell^+ f_\ell^+ \bar{\Theta}_\ell + f_2^+ \bar{\Psi}_2, \end{aligned} \quad (5.24)$$

$$\begin{aligned} \frac{t_3 - \bar{T}_B}{\frac{1}{2} a^2 / k_c} = & A_1^+ [2Y_{0,32} + 2Y_{0,21}] + A_2^+ [2Y_{0,32}] - A_4^+ [2Y_{0,43}] \\ & - \sum_{\ell=1}^4 A_\ell^+ f_\ell^+ \bar{\Theta}_\ell + f_3^+ \bar{\Psi}_3, \end{aligned} \quad (5.25)$$

and

$$\frac{t_4 - \bar{T}_B}{\frac{\bar{q}''' a^2}{2k_c}} = A_1^+ [2Y_{0,42} + 2Y_{0,21}] + A_2^+ [2Y_{0,42}] + A_3^+ [2Y_{0,43}] - \sum_{\ell=1}^4 A_{\ell}^+ q_{\ell}^+ \bar{\theta}_{\ell} + q_4^+ \bar{\Psi}_4 \quad (5.26)$$

In order to demonstrate the applicability of the foregoing procedure, it is applied to a typical 19-rod fuel bundle with uniform power densities in the rods. The pitch-to-diameter ratio (P/D) and the dimensionless wall distance (2W/D) is fixed at 1.24 and 1.30, respectively. The other thermal parameters are the same as those used in the 7-rod bundle analysis. In addition, the flow split between unit cells obeys the 5/7 power relation as described previously. The results of the calculation is summarized in Table 5.1 and the circumferential clad temperature distributions for all four unit cells are plotted in Fig.5.3. It can be seen that the shape of the circumferential clad surface temperature distribution of the corner cell of the 19-rod bundle is fairly similar to the one obtained previously from the 7-rod bundle calculation. This implies that the effect of coupling the edge cell temperature field to the corner cell is not significant. The clad surfaces of both the edge and corner cells are undercooled in the region

Table 5.1

Summary of Calculation for the 19-Rod Fuel Bundle

cell k	$Y_{0,k2}$	$Y_{0,k2}$	$Y_{0,k3}$	$Y_{0,k4}$	A_k^+	q_k^+	$\bar{\theta}_k$	$A_k^+ q_k^+ \bar{\theta}_k$
1	-	0.283	-	-	0.045	1.0	-2.341	-0.105
2	-0.283	-	0.601	0.705	0.269	1.0	-2.341	-0.630
3	-	-0.601	-	0.151	0.338	1.0	-2.555	-0.864
4	-	-0.705	-0.151	-	0.348	1.0	-2.499	-0.870

Fig.5.3 Clad Outside Temperature Distributions of the 19-Rod Fuel Bundle

near the bundle wall and overheated on the side toward the center of the bundle. The calculation also reveals that the circumferential clad temperature variation of the corner cell is not the most severe case. As shown in Fig.5.3, the maximum variation of the clad surface temperature of the corner cell is approximately 1.5 while the counterpart of the edge cell is as high as 2.0, which corresponds to roughly 200°-300°F for typical operating conditions. Another important result revealed by the 19-rod bundle calculation is that the wall effect does not just stop at the first row from the peripheral of the bundle instead it propagates into the 4-sided internal unit cell. This is clearly illustrated by the difference between the circumferential clad surface temperature variations of the 4-sided internal and the small internal cell, i.e., maximum variation of approximately 1.0 for the former vs. 0.05 for the latter. The finding of this calculation further strengthens the argument that a single-cell analysis is not sufficient and does not necessarily give conservative results.

In addition to the 19-rod fuel bundle analysis, the procedure is applied to a typical 19-rod blanket assembly. The pitch-to-diameter ratio and the dimensionless wall distance are assumed to be 1.08 and 1.10 respectively. Three different cases of power peaking factors in the rod as shown in Table 5.2 are applied to simulate the strong power tilting effect

in the blanket assembly. These three cases represent a uniform case, a 20% power tilt with the high-powered rod in the bundle center and a 20% power tilt with the high-power rods next to the bundle wall respectively. The results of the calculation are summarized in Table 5.3 and the dimensionless clad surfaces temperature distributions are shown in Figs. 5.4, 5.5, 5.6 and 5.7 for the corner, edge, 4-sided internal and internal cell respectively. It can be seen from Fig. 5.4 that the circumferential clad temperature variation of the corner cell decreases as the power peaking moves from the inside to the peripheral of the bundle. Similar behavior of the edge cell is indicated by Fig. 5.5. This is due to the fact that the geometric effect is offset by the power distribution. That is, the rods with higher power peaking factors are placed in flow channels with relatively larger coolant flow areas. It can be seen from Figs. 5.4 and 5.5 that the difference between the maximum dimensionless clad temperature variations resulted from the favorable and unfavorable power tilting can be as high as 3.35 for the edge cell and 1.7 for the corner cell. Fig. 5.6 further indicates that the 4-sided internal cell shows even much greater dependence on intraassembly power tilting; i.e., a difference of 7.1 for the maximum dimensionless clad temperature variation between case 1 and case 3. The only unit cell that shows little dependence on power tilting is

Table 5.2

Power Peaking Factors for the 19-Rod Blanket Bundle

	case 1	case 2	case 3
q_1^+	1.12	1.0	0.88
q_2^+	1.02	1.0	0.98
q_3^+	0.93	1.0	1.07
q_4^+	0.93	1.0	1.07

Table 5.3

Results of Calculation for the 19-Rod Blanket Bundle

(a) case 1

cell k	$y_{0,k1}$	$y_{0,k2}$	$y_{0,k3}$	$y_{0,k4}$	A_k^+	q_k^+	$\bar{\theta}_k$	$A_k^+ q_k^+ \bar{\theta}_k$
1	-	4.574	-	-	0.042	1.12	-2.155	-0.101
2	-4.574	-	6.254	6.946	0.251	1.02	-2.130	-0.545
3	-	-6.254	-	1.457	0.350	0.93	-3.197	-1.041
4	-	-6.946	-1.457	-	0.357	0.93	-2.758	-0.916

Table 5.3 (cont.)

(b) case 2

cell k	$Y_{0,k1}$	$Y_{0,k2}$	$Y_{0,k3}$	$Y_{0,k4}$	A_k^+	q_k^+	$\bar{\theta}_k^+$	$A_k^+ q_k^+ \bar{\theta}_k^+$
1	-	2.141	-	-	0.042	1.0	-2.132	-0.0895
2	-2.141	-	4.161	4.557	0.251	1.0	-2.128	-0.534
3	-	-4.161	-	1.055	0.350	1.0	-2.953	-1.034
4	-	-4.557	-1.055	-	0.357	1.0	-2.647	-0.945

Table 5.3 (cont.)

(c) case 3

cell k	$Y_{0,k1}$	$Y_{0,k2}$	$Y_{0,k3}$	$Y_{0,k4}$	A_k^+	q_k^+	$\bar{\theta}_k$	$A_k^+ q_k^+ \bar{\theta}_k$
1	-	-0.288	-	-	0.042	0.88	-2.102	-0.0777
2	0.288	-	2.075	2.167	0.251	0.98	-2.125	-0.523
3	-	-2.075	-	0.656	0.350	1.07	-2.742	-1.027
4	-	-2.167	-0.656	-	0.357	1.07	-2.551	-0.974

Fig.5.4 Clad Outside Temperature Distributions of the Edge Cell of the 19-Rod Blanket Bundle

5.5 Clad Outside Temperature Distributions of the Corner Cell of the 19-Rod Blanket Bundle

Fig.5.6 Clad Outside Temperature Distributions of the 4-Sided Internal Cell of the 19-Rod Blandet Bundle

Fig.5.7 Clad Outside Temperature Distribution of the Small Internal Cell of the 19-Rod Blanket Bundle

the small internal cell. As shown by Fig.5.7, the change of bundle power tilting direction simply alters the value of the clad temperature but does not significantly change the shape of the temperature distributions. This analysis leads to the conclusion that the clad temperature variation of all unit cells, except the small internal cell, show enormous sensitivity to bundle power gradient for the thermally fully-developed conditions. Furthermore, due to the strong edge effect caused by the geometric irregularity of the unit cells next to the bundle wall, the procedure which approximates the temperature field of the internal cells of the second, or maybe even the third row from the bundle wall by single-cell analysis can lead to intolerable errors.

In order to illustrate the impact of the multicell analysis, the azimuthal clad temperature distribution resulted from multicell and single cell analyses for the corner, edge and 4-sided internal cells of the fuel bundle are compared in Figs.5.8,5.9, and 5.10, respectively. It is indicated by these figures that the magnitudes of the clad temperature variation for all these cells resulted from the multicell analysis are greater than those predicted by the single cell analysis. The difference between the clad temperature variation predicted by the single cell analysis and that by the multicell analysis is approximately 0.35 for both the corner and edge cells. For the 4-sided internal cell, the discrepancy is approximately 1.0 - an unacceptable underestimation

Fig.5.8 Comparison of Clad Temperature Distributions of the Corner Cell Resulted from Single and Multi Cell Analyses

Fig.5.9 Comparison of Clad Temperature Distributions of the Edge Cell Resulted from Single and Multi Cell Analyses

Fig.5.10 Comparison of Clad Temperature Distributions of the Internal Cell Resulted from Single and Multi Cell Analyses

by the single cell analysis. The difference in the clad temperature distributions is caused by the heat transfer at the cell boundaries resulted from the multicell analysis. As a consequence, the temperature of the coolant in the region near the boundaries where heat transfers in, is relatively higher than that resulted from the single cell analysis. Naturally, the opposite is true in the region near the boundaries where heat transfers out.

As a result of the findings of the present study, it becomes clear that the single cell analysis does not necessarily yield conservative results from clad temperature calculations. In order to obtain more realistic clad temperature distribution for various unit cells, a multicell analysis is absolutely necessary.

5.3 COMPARISON OF THE CALCULATIONAL RESULTS TO THE EXPERIMENTAL DATA

It has to be pointed out again that the calculations in the foregoing analysis are based on the assumption of hydrodynamically and thermally fully-developed conditions. This is a fairly good assumption if the axial plane being considered is sufficiently far from the inlet. However, the length of typical LMFBR cores is usually small and consequently the temperature field at the exit of the core may be still in the thermal entrance length. As a result, the circumferential variation of the clad temperature resulted from the

fully-developed 2-D calculation is too conservative. In this section, the analytically predicted temperature field of the clad of the corner cell will be compared with the experimental data taken by Möller and Tschöke [49]. By comparing the fully developed clad temperature distribution with the measurements at various axial positions, an axial correction factor can be obtained to account for the entrance effect in the entrance regime. The axial correction factor thus serves as a link between the fully-developed temperature field and the temperature field at any axial position in the entrance regime.

The experimental apparatus of Möller and Tschöke was a 19-rod electrically-heated test assembly cooled by liquid sodium. The geometric parameters of the test facility and the operating conditions are summarized in Table 5.4. The clad temperature at the four axial measurement planes was measured by the thermocouples embedded in the clad and the circumferential temperature variation around the clad was generated by continuously rotating the rod over 360° . Naturally, the temperature variation around the circumference of the rods increases with axial position and the maximum azimuthal temperature differences and gradients were measured at plane 4 ($\sim 97D_H$). This is illustrated by the clad temperature distributions of the corner rod in Fig.5.11. For comparison purpose, the fully-developed circumferential clad temperature distribution of the corner cell of a 19-rod assembly with the

Table 5.4

Geometric Parameters of the Test Facility

Number of rods	19
Rod Diameter	0.354"
Pitch-to-Diameter Ratio (P/D)	1.30
Dimensionless Wall Distance (2W/D)	1.38
Number of Grid Spacers	4
Number of Axial Measurement Planes	4
Positions of the measurement planes:	
$z_1 = 4.28"$	
$z_2 = 11.50"$	
$z_3 = 18.71"$	
$z_4 = 25.94"$

5.11 Measured Clad Temperature Distributions of the Corner Rod at Various Axial Positions

same geometric parameters and operating conditions is also given in Fig. 5.12. It is apparent that the shape of the developing clad temperature distributions look very much similar to the predicted fully-developed distribution except that their magnitude of variation are considerably smaller. Therefore, it is desirable to relate the magnitude of the clad temperature variation to that of the fully developed temperature field by a correction factor which is a function of the axial position. In the present study, the following relation is postulated:

$$t(r, \phi, z) = \Lambda + \frac{q''' a^2}{2k_c} F(z) \Psi_{FD}(r, \phi), \quad (5.27)$$

where Λ is a function of z only, $\Psi_{FD}(r, \phi)$ is the fully developed dimensionless clad temperature, $t(r, \phi, z)$ is the measured clad temperature at the axial position z , and $F(z)$ is the "axial correction factor" which relates the fully developed clad temperature and the measured clad temperature in the entrance region. Applying Eq. (5.27) at the hot spot ($\phi = \phi_{\max}$) and the cold spot ($\phi = \phi_{\min}$) of the clad results in the following relations:

$$t(\rho, \phi_{\max}, z) = \Lambda + \frac{q''' a^2}{2k_c} F(z) \Psi_{FD}(\rho, \phi_{\max}), \quad (5.28)$$

Fig.5.12 Clad Temperature Distribution of the Corner Rod at $\rho = 0.944$

$$t(\rho, \phi_{\min}, z) = \Lambda + \frac{q'' a^2}{2k_c} F(z) \Psi_{FD}(\rho, \phi_{\min}) \quad (5.29)$$

Subtracting Eq. (5.28) from Eq. (5.27) and letting $\rho = \rho^*$ leads to:

$$t(\rho^*, \phi_{\max}, z) - t(\rho^*, \phi_{\min}, z) = \frac{q'' a^2}{2k_c} [\Delta \Psi_{FD}]_{\max} F(z) \quad (5.30)$$

where ρ^* is the dimensionless radial distance at which the thermocouple is located ($\rho^* = 0.94$ in Möller and Tschöke's experiment) and $[\Delta \Psi_{FD}]_{\max}$ is the maximum clad temperature variation at $\rho = \rho^*$ resulted from the fully-developed analysis. Upon rearrangement, Eq. (5.30) can be written in the equivalent form:

$$F(z) = \frac{\frac{[\Delta t(\rho^*, z)]_{\max}}{q'' a^2 / 2k_c}}{[\Delta \Psi_{FD}]_{\max}} \quad (5.31)$$

where, $[t(\rho^*, z)]_{\max} = t(\rho^*, \phi_{\max}, z) - t(\rho^*, \phi_{\min}, z)$

Thus, the values of $F(z)$ at the axial planes can be determined readily by simply comparing the dimensionless clad

temperature variation with measurements. The values of $F(Z)$ at these positions are given in Table 5.5.

At this point, it is desirable to develop a functional relation for $F(Z)$ which is capable of prediction values of $F(Z)$ at all axial planes, as well as those measured values at the four measurement planes. Furthermore, it should be pointed out that physical constraint requires $F(Z)$ to be zero at $Z=0$ and approach unity as Z becomes sufficiently large. Therefore it is reasonable to approximate $F(Z)$ by an exponential relation as:

$$F(Z) = 1 - e^{-\gamma Z} \quad (5.32)$$

where γ is an unknown parameter to be determined. Physically γ governs the entrance effect and should be dependent on the Peclet number and the geometry. However, it should remain constant for a particular geometry as these parameters are fixed. By varying the value of γ , $F(Z)$ at all axial positions are readily obtained. The error associated with the calculated value of $F(Z)$ at the i th measured plane is just the difference between the measured and calculated values of $F(Z)$:

$$\Delta(Z_i) = F_{cal}(Z_i) - F_{exp}(Z_i) \quad ; \quad i=1,2,3,4 \quad (5.33)$$

And the sum of the square of the error at the four axial measurement planes is given by:

Table 5.5

F(Z) Values at the Four Axial Measurement Planes

	z (in)	$\frac{[\Delta t(r,z)]_{\max}}{\bar{q}''' a^2 / 2k_c}$	$F_{\exp}(Z_i)$
1	4.28	0.0518	0.0529
2	11.50	0.152	0.155
3	18.71	0.204	0.208
4	25.94	0.239	0.244

$$\Delta = \sum_{i=1}^4 [\Delta(Z_i)] = \sum_{i=1}^4 [F_{cal}(Z_i) - F_{exp}(Z_i)]^2 \quad (5.34)$$

Thus, the total error Δ becomes a criterion to measure how well the assumed function fits the experimental data. By systematically varying the value of γ , a curve of Δ as a function of γ can be obtained. This is shown in Fig. 5.13 and the plot clearly indicates a minimum at approximately $\gamma = 0.012 \text{ in}^{-1}$. With the determined optimal value of γ , the correction factors $F(Z)$ at various axial positions are calculated and listed in Table 5.6. Also, $F(Z)$ is plotted vs. Z in Fig. 5.14. It can be seen from the figure that $F(Z)$ fits the experimental data reasonably well. Moreover, it has to be pointed out that the result of the analysis has indicated that the clad temperature at the exit of most LMFBF cores is not even "half-developed"; i.e., $F(Z) < 0.5$. This finding implies that the clad temperature variation resulted from the fully-developed calculation is almost 100% overconservative for the present case. The determination of the empirical function $F(z)$ thus provides an estimate of the extent of the effect of the entrance effect as well as the degree of conservatism resulted from the 2-D analysis.

Fig.5.13 Total Error Resulted from Curve Fitting
vs. γ

Table 5.6

Values of $F(Z)$ at Various Axial Positions

$Z(\text{in})$	$F(Z)$
0	0
5	0.0582
10	0.113
15	0.165
20	0.213
25	0.295
30	0.302
35	0.343
40	0.381
45	0.417
50	0.451
55	0.483
60	0.513
65	0.542
70	0.568
75	0.593
80	0.617
85	0.639
90	0.660
95	0.680
100	0.699
105	0.716
110	0.733
115	0.748
120	0.763

Fig.5.14 Comparison of the Predicted and Measured Values of $F(z)$

5.4 APPLICATION OF THE MULTICELL TECHNIQUE TO A 37-ROD BUNDLE WITH CLINCH RIVER BREEDER REACTOR DESIGN PARAMETERS

In this section, the multicell analysis is extended to a 37-rod fuel bundle. The geometric and thermal parameters as given in the PSAR of the Clinch River Breeder Reactor (CRBR) was used for the analysis. In addition to the dimensionless circumferential clad temperature variation, the absolute temperature of the clad outside surface for various unit cells are also determined and presented.

The configuration of the rod cluster being considered is shown in Fig.5.15. By isolating the corner section of the rod cluster being analyzed is effectively a symmetry section of a 37-rod bundle. As shown in Fig.5.16, the symmetry section consists of a small 30° internal cell, two 4-sided internal cells, a 3-sided internal cell, a 5-sided edge cell and a corner cell. To initiate the iteration procedure, dimensionless temperature fields for all cells are first obtained by imposing adiabatic boundary condition at all cell boundaries. Temperature fields of neighboring unit cells are coupled by using the procedure as described earlier. In addition, the first boundary of the corner cell and the third boundary of edge cell are always kept adiabatic because it is assumed that no heat transfer is allowed at the bundle wall. Also, the fourth boundary of the edge cell and third boundary of one of the 4-sided internal cell (No.2) have to remain

5.15 Configuration of the Rod Cluster Used for the 37-Rod Bundle Analysis

Fig.5.16 Configuration of the Symmetry Section of a 37-Rod Bundle

adiabatic throughout the iteration due to geometric symmetry. The thermal and geometric parameters of the CRBR core design are listed as follows:

$$P/D = 1.24, \quad 2W/D = 1.42, \quad D = 0.23"$$

$$a/b = 0.87$$

$$\text{core height} = 36"$$

$$Pe = 357$$

$$\text{average linear power } \bar{q}' = 6.6 \text{ Kw/ft}$$

$$\text{Axial power peaking factor} = 1.24$$

$$\text{Radial Power peaking Factor} = 1.20$$

The normalized circumferential clad temperature distributions of the six characteristic unit cells after 5 iterations are given in Fig.5.17. The results of the calculation indicate that the rod of the edge cell is subject to the most severe condition with a maximum variation of approximately 4.2 for the dimensionless clad temperature distribution. With the thermal and geometric parameters as given and assuming 1000°F for the bundle mean coolant temperature $\bar{T}_B$, the circumferential clad temperature distributions of various unit cells expressed in absolute temperature can be readily determined. The results of calculation are tabulated in Table 5.7. It is indicated that the circumferential clad outside temperature variations of the corner cell (No.6) and edge cell (No.5) are as high as

338°F and 403°F, respectively. The magnitudes of the clad temperature variation of the cells of the second row from the bundle wall decrease considerably but are still fairly significant; i.e., 338°F and 268°F for unit cells No.3 and 4, respectively. Even the internal unit cell two rows from the bundle wall (No.2) exhibits an approximately 171°F circumferential clad temperature variation. However, it must be pointed out that the magnitudes of all these clad temperature distributions are overestimated due to the fully developed nature of the 2-D analysis. The degree of conservatism of the present analysis can be reduced substantially by applying the "axial correction factor $F(z)$ " described in the preceding section. With the axial correction, the magnitude of the clad temperature variation for the worst rod is in vicinity of 140°F - still a fairly significant magnitude for a rod of only 0.23 diameter.

At this point, it is clear that the code has demonstrated its capability of coupling temperature fields of unit cells in large bundle. For economy reason, calculations for 61-rod and larger bundles have not been performed but the calculational process should be fairly straight-forward. In the next chapter, effective lumped transport parameters are determined from the local temperature field and their application on subchannel calculations will be presented.

Fig.5.17 Clad Outsided Temperature Distributions for the 37-Rod Fuel Bundle

Table 5.7

Clad Outside Temperature Distributions for Unit Cells of the 37-Rod Fuel Bundle

$\phi\lambda$	1	2	3	4	5	6
0	1493.69	1320.31	1376.39	1029.99	1109.61	681.20
12	1493.56	1320.31	1373.27	1030.32	1082.54	683.25
24	1493.43	1321.73	1363.38	1034.07	1028.34	688.82
36	1493.30	1317.95	1346.90	1045.79	961.87	696.82
48	1492.95	1343.96	1325.76	1065.63	867.87	706.94
60	1492.61	1361.16	1302.50	1089.36	775.65	720.44
72	1492.27	1376.76	1277.63	1112.55	723.85	739.86
84	1491.85	1391.27	1248.84	1133.44	707.14	767.88
96	1491.43	1407.85	1215.30	1156.04	706.79	805.91
108	1491.01	1427.25	1179.66	1185.25	722.90	852.37
120	1490.67	1446.45	1145.46	1217.81	774.83	901.64
132	1490.32	1461.92	1116.33	1246.56	867.87	945.85
144	1489.98	1473.24	1091.48	1268.52	963.10	979.44
156	1489.85	1482.10	1066.99	1284.13	1041.20	1001.78
168	1489.72	1448.79	1046.68	1294.03	1095.93	1014.72
180	1489.59	1491.48	1038.85	1297.56	1109.61	1019.08

CHAPTER 6

DETERMINATION OF EFFECTIVE CONDUCTION MIXING LENGTHS

Subchannel codes usually divide the assembly into a finite grid of "subchannels" each of which is characterized by a bulk temperature, flow rate and pressure. Energy transport between adjacent subchannels is governed by the mechanisms of diversion cross-flow, turbulence mixing and conduction. It should be noticed that the layout of the subchannels may not be necessarily the same as the unit cells used in the present analysis. For LMFBR design calculations, the energy transport by conduction constitutes a major contribution to the overall energy transport. This effect is even more pronounced for flows which are characterized by low-Re numbers. These flow situations especially arise in the case of a loss-of-flow accident. Despite this importance only scattered information is available for internal subchannels [12] but nothing is known for the subchannels located nearby the bundle wall regions. In this chapter, effective lumped parameters due to the conduction effect are determined by using the coolant temperature field resulted from the multicell analysis as given in the preceeding chapter.

6.1 DEFINITION OF EFFECTIVE CONDUCTION MIXING LENGTH

The linear heat transfer rate due to heat conduction between adjacent subchannels i and j as calculated in sub-

channel codes and needed for the energy equation is given by [5] :

$$Q_{ij} = k_c \frac{S_{ij} (\bar{T}_i - \bar{T}_j)}{l_{ij}} = k_c \frac{S_{ij}}{l_{ij}^*} (\bar{T}_i - \bar{T}_j) \frac{1}{L_{ij}} \quad (6.1)$$

where S_{ij} - the length of the common boundary
 l_{ij} - the effective conduction mixing length
 l_{ij}^* - the centroid-to-centroid distance of adjacent coolant channels

and L_{ij} is a dimensionless correction factor representing the ratio of the effective conduction mixing length to the centroid-to-centroid distance i.e.,

$$L_{ij} = \frac{l_{ij}}{l_{ij}^*} \quad (6.2)$$

In subchannel calculations, L_{ij} is usually set to be unity due to the lack of detailed knowledge of the local temperature field. It is quite obvious that the lumped parameter codes will not give reliable results if the correct effective mixing lengths are not known. However, by setting the L_{ij} to be unity implies that the effective conduction mixing length is equal to the centroid-to-centroid distance. This assumption is completely intuitive and lacks either analytical or experimental verification. With some rearrange-

ments Eq.(6.1) can be written as:

$$L_{ij} = k_c \frac{S_{ij}}{\ell_{ij}^*} \frac{(\bar{T}_i - \bar{T}_j)}{Q_{ij}} \quad (6.3)$$

In addition, the total heat transfer from subchannel i to j due to heat conduction can be expressed as:

$$Q_{ij} = \int_{S_{ij}} q_s'' ds \quad (6.4)$$

where q_s'' denotes the heat flux distribution along the common boundary between subchannel, i and j. Substituting Eq.(6.4) into Eq.(6.3) leads to:

$$L_{ij} = k_c \frac{S_{ij}}{\ell_{ij}^*} \frac{(\bar{T}_i - \bar{T}_j)}{\int_{S_{ij}} q_s'' ds} \quad (6.5)$$

Eq.(6.5) can be normalized to a dimensionless form by multiplying and dividing by the group $\bar{q}''' a^2 / 2k_c$ which results in:

$$\bar{L}_{ij} = k_c \frac{S_{ij}}{\ell_{ij}^*} \frac{\frac{(\bar{T}_i - \bar{T}_j)}{\bar{q}''' a^2 / 2k_c}}{\int_{S_{ij}} \frac{q_s''}{\bar{q}''' a^2 / 2k_c} ds} = \frac{S_{ij}}{\ell_{ij}^*} \frac{\frac{(\bar{T}_i - \bar{T}_j)}{\bar{q}''' a^2 / 2k_c}}{\int_{S_{ij}} \frac{q_s''}{\bar{q}''' a^2 / 2} ds} \quad (6.6)$$

Multiplying and dividing the integral in the numerator by the radius of the rod b leads to the form:

$$L_{ij} = \frac{s_{ij}}{l_{ij}} \frac{\frac{(\bar{T}_i - \bar{T}_j)}{\bar{q}''' a^2 / 2k_c}}{\int_{s_{ij}} \frac{q_s''}{\bar{q}''' a^2 / 2b} d\left(\frac{s}{b}\right)} \quad (6.7)$$

It is evident now that an accurate value of L_{ij} can be determined analytically by explicitly evaluating all quantities on the right hand side from the local coolant temperature field resulted from the multicell analysis. An example for a 7-rod bundle will be given in the following section.

6.2 CALCULATION OF L_{ij} IN SQUARE CELLS

Before the procedure of calculating L_{ij} in the hexagonal bundle array is given, the method is applied to a two cell section of a square cell array as shown in Fig.4.4. The purpose of this is to compare results of different calculational methods. This choice had to be made because this is the only case where sufficient information has been published in open literature. The clad temperature distribution of the two cells has already been compared to the result reported by France and Ginsberg [58] in chapter 4. The comparison of the effective mixing length between the two square cells serves as an additional

check for the validity of the present calculational procedure.

In the present analysis, only identical power densities are considered in the square array section. Slug flow is assumed within each cell but the variation in the velocity from cell to cell is given by the relation:

$$\frac{U_i}{U_j} = \left(\frac{D_i}{D_j} \right)^{5/7}, \quad (4.43)$$

where U_i and D_i are the cellular velocity and hydraulic diameter of cell i , respectively. It can be easily verified that the normalized average coolant temperature between the two cells is given by the relation:

$$\frac{\bar{T}_1 - \bar{T}_2}{\bar{q}''' a^2 / 2k_c} = 2Y_{0,21} + (\bar{\Theta}_1 - \bar{\Theta}_2) \quad (6.8)$$

In addition, it can be shown that the geometric parameter S_{12} / ℓ_{12}^* for the two cell section in square array geometry is given by the relation:

$$\frac{S_{12}}{\ell_{12}^*} = \frac{P/D}{\frac{3}{2} \left(\frac{P}{D} \right) + \frac{1}{2} \left(\frac{2W}{D} \right) + 2 \left[\frac{\frac{\pi}{8} \left(\frac{2W}{D} - \frac{P}{D} \right)}{\frac{P}{D} \left(\frac{P}{D} + \frac{2W}{D} \right) - \frac{\pi}{2}} \right]} \quad (6.9)$$

Naturally, the heat flux integral along the cell boundary in Eq.(6.7) has to be evaluated from the local temperature field of the 2-cell section. The results of calculation are presented in Fig.6.1 (curve 1) with the effective mixing length plotted vs. the quantity μ which is defined as:

$$\mu = \frac{\dot{m}_1}{\dot{m}_2} \quad , \quad (6.10)$$

where $\dot{m}_1$ and $\dot{m}_2$ are the mass flow rates for the cell next to the wall and the internal cell, respectively. Curve 2 in Fig.6.1 denotes the result given by France and Ginsberg [70] who used a boundary integral method for an eight cell configuration. For both cases, the boundary condition of uniform heat flux at the clad outside surface has been used. Naturally, the singular points as shown in Fig.6.1 imply that the net heat transfer between adjacent cells approaches zero. It can also be seen from the figure that L_{12} generally deviates quite substantially from one and exhibits strong dependence upon μ , which itself is a function of the geometry. However, the L_{12} values for a 2-cell configuration (curve 1) indicate fair agreement with those resulted from the 8-cell calculation except for cases where $\mu < 0.95$. This is because the effect on the flow due to the presence of the wall is more pronounced in the 2-cell case than in the 8-cell configuration. The agreement implies that most of the wall

Fig.6.1 Effective Mixing Lengths in Square Cell Configuration

effect has already been included by performing the 2-cell analysis. It is felt that the 8-cell analysis may be unnecessary due to the fact that the propagation of the wall effect diminishes remarkably beyond the second cell from the wall.

6.3 CALCULATION OF LUMPED QUANTITIES FROM THE LOCAL TEMPERATURE FIELD OF A 7-ROD BUNDLE

A cross-sectional view of a typical 7-rod bundle as shown in Fig.6.2 is used as an example to illustrate the applicability of the local temperature field for the evaluation of effective conduction mixing length. The dashed lines indicated in the figure denote the layout of the subchannels usually employed in most subchannel codes, such as COBRA-IIIC [5]. Obviously this kind of coolant-centered subchannel structure does not coincide with the unit cell structure which has been used so far in this analysis. As shown in Fig. 6.3, the symmetry section of the bundle consists of a 30° internal cell and a corner cell with their common cell boundary indicated as solid line. The local temperature fields of the coolant, the clad and the fuel regions are obtained by coupling the temperature fields of the two cells as described in the preceeding chapter.

From Fig.6.3, it can be seen that the internal subchannel (denoted as subchannel 1) consists of the entire internal unit cell and part of the corner cell where $2\pi/3 \leq \phi \leq \pi$. On the other hand, the corner subchannel (denoted as subchannel

Fig.6.2 Configuration of a 7-Pin Bundle

Fig.6.3 Symmetry Section of a 7-Pin Bundle

2) is actually the portion of the corner unit cell where $0 < \phi < 2\pi/3$. Upon multiplying and dividing the heat transfer integral of Eq.6.7 by the cell-to-average power factor q_2^+ , the normalized effective mixing length L_{12} can be written as:

$$L_{12} = \frac{S_{12}}{l_{12}^*} \frac{\frac{(\bar{T}_1 - \bar{T}_2)}{\bar{q}'' a^2 / 2k_c}}{q_2^+ \int_{S_{12}} \frac{q_s''}{\bar{q}_2'' a^2 / 2b} d\left(\frac{s}{b}\right)} \quad (6.11)$$

Recognizing that the subchannel boundary is the normal of the corner cell to its second boundary, it becomes evident that:

$$ds = dr \quad (6.12)$$

Furthermore, the heat flux distribution at the subchannel boundary S_{12} is given by the local temperature field of the corner cell as:

$$q_s'' = -k_c \hat{n} \cdot \nabla T_2(r, \alpha_2) = k_c \hat{\phi} \cdot \nabla T_2(r, \alpha_2) \quad (6.13)$$

or equivalently:

$$q_s'' = k_c \hat{\phi} \cdot \left[\frac{\partial T_2}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T_2}{\partial \phi} \hat{\phi} \right]_{\phi=\alpha_2} = k_c \frac{1}{r} \frac{\partial}{\partial \phi} T_2(r, \alpha_2) \quad (6.14)$$

Dividing Eq.6.14 by $q'''a^2/2b$ leads to:

$$\frac{q_s''}{q_2'''a^2/2b} = \frac{k_c \frac{1}{r} \frac{\partial}{\partial \phi} T_2(r, \alpha_2)}{q_2'''a^2/2b} = \frac{1}{\rho} \frac{\partial}{\partial \phi} \theta_2(\rho, \alpha_2) \quad (6.15)$$

where θ_2 is the dimensionless temperature of the coolant of the corner cell.

Substituting Eq.(6.15) into Eq.(6.11) and integrating from $\rho = 1$ to a_2/b , L_{12} becomes:

$$L_{12} = \frac{S_{12}}{l_{12}^*} \frac{\frac{(\bar{T}_1 - \bar{T}_2)}{\bar{q}'''a^2/2k_c}}{q_2^+ \int_1^{a_2/b} \left[\frac{1}{\rho} \frac{\partial}{\partial \phi} \theta_2(\rho, \alpha_2) \right] d\rho} \quad (6.16)$$

where the heat transfer integral is evaluated numerically by the Gauss-Legendre ten-point method [62].

The next lumped quantity to be calculated from the local temperature field is the normalized average coolant temperature difference between subchannels. Due to the different partitioning schemes used in the subchannel and the distributed parameter analyses, the determination of the subchannel temperature may involve lumping temperature fields of two or

three unit cells. From Fig.6.3, it is evident that the average temperature of subchannel 1 is given by the sum of the weighted average coolant temperature of the areas which constitute the subchannel; that is:

$$\bar{T}_1 = \frac{\langle A_1 \rangle_1 U_1 \langle T_1 \rangle_1 + \langle A_2 \rangle_1 U_2 \langle T_2 \rangle_1}{\langle A_1 \rangle_1 U_1 + \langle A_2 \rangle_1 U_2} \quad (6.17)$$

where $\langle A_i \rangle_j$ denotes the coolant area of unit cell i inside subchannel j , and $\langle T_i \rangle_j$ denotes the average coolant temperature of area $\langle A_i \rangle_j$. Recalling the relation between the coolant temperature and its dimensionless temperature as given in chapter 2, Eq.(6.17) can be written as:

$$\bar{T}_1 = \frac{\langle A_1 \rangle_1 U_1 \left[A_1 + \frac{q_1''' a^2}{2k_c} \langle \theta_1 \rangle_1 \right] + \langle A_2 \rangle_1 U_2 \left[A_2 + \frac{q_2''' a^2}{2k_c} \langle \theta_2 \rangle_1 \right]}{\langle A_1 \rangle_1 U_1 + \langle A_2 \rangle_1 U_2} \quad (6.18)$$

Similarly, the average coolant temperature of subchannel 2 is given by the relation:

$$\bar{T}_2 = A_2 + \frac{q_2''' a^2}{2k_c} \langle \theta_2 \rangle_2 \quad (6.19)$$

Therefore, the difference in the coolant temperature between the two subchannels can be obtained by combining Eqs.(6.18) and (6.19) as:

$$\begin{aligned} \bar{T}_1 - \bar{T}_2 = & \frac{\langle A_1 \rangle_1 U_1 A_1 + \langle A_2 \rangle_1 U_2 A_2}{\langle A_1 \rangle_1 U_1 + \langle A_2 \rangle_1 U_2} - A_2 \\ & + \frac{\langle A_1 \rangle_1 U_1 \frac{q_1''' a^2}{2k_c} \langle \theta_1 \rangle_1 + \langle A_2 \rangle_1 U_2 \frac{q_2''' a^2}{2k_c} \langle \theta_2 \rangle_1}{\langle A_1 \rangle_1 U_1 + \langle A_2 \rangle_1 U_2} - \frac{q_2''' a^2}{2k_c} \langle \theta_2 \rangle_2 \end{aligned} \quad (6.20)$$

Dividing Eq.6.20 by $\bar{q}''' a^2 / 2k_c$ and simplifying, the normalized average coolant temperature difference between subchannel 1 and 2 becomes:

$$\begin{aligned} \frac{\bar{T}_1 - \bar{T}_2}{\bar{q}''' a^2 / 2k_c} = & \frac{(A_1 - A_2)}{\bar{q}''' a^2 / 2k_c} \frac{1}{\left[1 + \frac{\langle A_2 \rangle_1}{\langle A_1 \rangle_1} \left(\frac{U_2^+}{U_1^+} \right) \right]} + \frac{q_1^+ \langle \theta_1 \rangle_1 + q_2^+ \langle \theta_2 \rangle_2}{1 + \frac{\langle A_2 \rangle_1}{\langle A_1 \rangle_1} \left(\frac{U_2^+}{U_1^+} \right)} \frac{\langle A_2 \rangle_1 U_2^+}{\langle A_1 \rangle_1 U_1^+} \\ & - q_2^+ \langle \theta_2 \rangle_2 \end{aligned} \quad (6.21)$$

Recalling that

$$\frac{A_1 - A_2}{\bar{q}''' a^2 / 2k_c} = -2 Y_{o,21} \quad (6.22)$$

where $Y_{0,21}$ is the constant resulting from the multicell coupling procedure. Thus, Eq.(6.22) can be written as:

$$\frac{\bar{T}_1 - \bar{T}_2}{\bar{q}''' a^2 / 2k_c} = \frac{-2Y_{0,21}}{\left[1 + \frac{\langle A_2 \rangle_1}{\langle A_1 \rangle_1} \left(\frac{U_2^+}{U_1^+} \right) \right]} + \frac{q_1^{+\langle \theta_1 \rangle_1} + q_2^{+\langle \theta_2 \rangle_1} \frac{\langle A_2 \rangle_1}{\langle A_1 \rangle_1} \left(\frac{U_2^+}{U_1^+} \right)}{1 + \frac{\langle A_2 \rangle_1}{\langle A_1 \rangle_1} \left(\frac{U_2^+}{U_1^+} \right)} - q^{+\langle \theta_2 \rangle_2} \quad (6.23)$$

Additionally, it can be shown that the geometric parameter S_{12}/ℓ_{12}^* is given by the relation:

$$\frac{S}{\ell_{12}^*} = \frac{\frac{1}{2} \left(\frac{P}{D} - 1 \right)}{\left[\frac{\frac{1}{16} \left(\frac{2W}{D} \right)^2 \left(\frac{P}{D} \right) + \frac{1}{24\sqrt{3}} \left(\frac{2W}{D} \right)^3 - \frac{1}{16}}{\frac{1}{4} \left(\frac{P}{D} \right) \left(\frac{2W}{D} \right) + \frac{1}{8\sqrt{3}} \left(\frac{2W}{D} \right)^2 - \frac{\pi}{12}} + \frac{\frac{1}{2} \left(\frac{P}{D} \right)^3 - \frac{\sqrt{3}\pi}{12} \left(\frac{P}{D} \right)}{\sqrt{3} \left(\frac{P}{D} \right)^2 - \frac{\pi}{2}} \right]} \quad (6.24)$$

Thus, all quantities in Eq.(6.16) are expressed in terms of the geometric parameters and quantities which can be obtained from the local temperature field resulted from the multicell analysis.

6.4 EFFECTIVE MIXING LENGTHS FOR THE 7-ROD BUNDLES

In this section, the effective mixing lengths for a 7-rod bundle with pitch-to-diameter ratios of 1.24 and 1.08 are determined as examples for typical fuel and blanket bundles, respectively. For the fuel bundle, equal power densities in the rods are assumed and for the blanket bundle, the power tilting effect is included in the analysis. In addition, the unit cell flow velocity is assumed to obey the 5/7 power relation as given by Eq.(4.43).

In order to provide an insight to the behavior of the effective mixing length, the normalized average intersubchannel coolant temperature difference and the integral of heat transport between subchannel 1 and 2 for the fuel bundle are plotted in Figs. 6.4 and 6.5, respectively. It can be seen readily that both the normalized coolant temperature difference and the integral of the heat transport between the subchannels increase monotonically with the dimensionless wall distance. Physically, this is because an increase in the wall distance implies an increase in the heat sink in the region near the bundle wall and as a result, a higher degree of non-uniform cooling results. Consequently, this leads to higher intersubchannel coolant temperature differences and thus heat transport. In addition, it has to be pointed out that the single-region analysis effectively eliminates circumferential heat conduction in the fuel and

Fig.6.4 Normalized Intersubchannel Temperature Difference vs. 2W/D Ratio (P/D=1.24)

Fig.6.5 Normalized Intersubchannel Heat Transport vs.
 $2W/D$ Ratios ($P/D=1.24$)

clad regions which would help to reduce the temperature gradient in the circumferential direction. As a result, the magnitude of both the normalized temperature difference and the heat transport between subchannels resulting from the single-region analysis are higher than that from the multi-region analysis. This effect is clearly illustrated in Figs.6.4 and 6.5.

It can also be seen from Figs.6.4 and 6.5 that the normalized average coolant temperature difference between the subchannels for both single and multiregion analyses are close to zero at approximately $2W/D \approx 1.16$. For smaller values of $2W/D$, $(\bar{T}_1 - \bar{T}_2) / \frac{\bar{q}'' a^2}{2k_c}$ becomes negative implying that the coolant temperature of subchannel 2 is higher than that of subchannel 1. On the other hand, the curve for the normalized heat transport integral intersects the abscissa at approximately $2W/D \approx 1.13$ for both the single and multiregion analyses. For values of $2W/D$ less than 1.13, the normalized heat transport integral becomes negative implying a reversal in the heat flow direction, i.e., heat is transported from subchannel 2 to 1.

Having determined the lumped quantities from the local temperature field, it is of prime interest to investigate the behavior of the effective mixing length L_{1j} . Fig.6.6 shows L_{1j} plotted as a function of $2W/D$ for the 7-rod fuel bundle.

It is obvious that the values of L_{ij} resulting from both the single and multiregion analyses deviate quite substantially from unity, the value which is usually employed in subchannel codes; for instance, COBRA-II [8]. Fig.6.6 also indicates that both curves have singularities in the vicinity of $2W/D \simeq 1.13$ as a result of zero net heat transport between subchannels 1 and 2. To the right of the singularities of both curves, there is a range in which L_{ij} has negative values. This is because of the fact that in this small range of $2W/D$, the average coolant temperature of subchannel 2 is higher than that of subchannel 1 but the net heat transport direction is from subchannel 1 to 2. At first sight, this is a surprising result but not a violation of basic heat transfer laws. The heat transfer at the subchannel boundary is a rather local phenomenon which is only governed by the temperature field near the gap region but not the subchannel average temperature difference. For larger values of $2W/D$, the L_{12} s approach their asymptotic values of 0.81 and 0.71 for multi and single region analysis, respectively. Over a fairly wide range of $2W/D$ ratios, the L_{12} values are basically independent of the wall distance and are only governed by the P/D ratio.

To the left of the singularities, asymptotic values of L_{12} are not seen in the range of $2W/D$ being considered here. However, it is apparent that values of L_{12} of both curves are generally greater than unity. The sudden change of L_{12} from

Fig.6.6 Effective Mixing Lengths for the 7-Rod Fuel Bundle

a value smaller than one to a value greater than one clearly indicates a geometric effect on the heat transport characteristic between subchannels; i.e., heat transport from subchannel 1 to 2 is not equal to the heat transport from subchannel 2 to 1 although the magnitudes of the coolant temperature differences are equal and differ only in sign.

Up to this point all calculations are based on the assumption of equal power generation in the rods. However, this is not necessarily the case especially for the blanket assembly where large power gradient may exist. In order to investigate the effect of interassembly power gradient three cases with different power peaking factors in the rods as indicated in table 5.2 are examined. Moreover, only multiregion analyses are performed for these calculations because it would be physically unrealistic to impose a uniform heat flux distribution at the clad outer surface in such a closely packed bundle. The normalized intersubchannel temperature difference and the heat transport are plotted vs. $2W/D$ in Figs. 6.7 and 6.8, respectively. As expected, case 1 results in the highest intersubchannel temperature difference and heat transport because of the unfavorable combination of the geometry and the power distribution. For case 3, the opposite is true. The normalized effective mixing lengths for these three cases are given in Fig. 6.9 as a function of $2W/D$ ratio. It can be seen that all three curves emerge to the same asymptotic value ($L_{12} = 0.64$). Therefore, it is clear that the power

Fig.6.7 Normalized Intersubchannel Temperature Difference
for Blanket Bundle with Power Tilting

Fig.6.8 Normalized Intersubchannel Heat Transport for Blanket Bundle with Power Tilting

Fig.6.9 Effective Mixing Lengths for the 7-Rod Blanket Bundle with Power Tilting

gradient in the bundle does not affect the asymptotic value of the effective mixing length between subchannels. In the following section, the analysis will be extended to the 19-rod fuel and blanket bundles and the effective mixing lengths of various unit cell pairs will be presented and examined at length.

6.5 EFFECTIVE MIXING LENGTHS OF LARGER BUNDLES

In this section, accurate effective mixing lengths for various subchannel pairs for a 19-rod bundle are determined by appropriately lumping the local temperature fields of the unit cells obtained from the multicell analysis as indicated in chapter 5. Fig.6.10 shows a schematic diagram of the symmetry section of a 19-rod bundle with dashed lines indicating the subchannel boundaries. The numbers used to identify these subchannels are also indicated in the figure. The procedure of determining the lumped quantities between the various subchannel pairs is essentially similar to the 7-rod bundle analysis except that it is more laborious. It can be shown that the lumped coolant temperature difference between the four different subchannel pairs are given by the following relations:

6.10 Subchannel Layout of the 19-Rod Bundle

$$\begin{aligned}
\frac{\bar{T}_1 - \bar{T}_2}{\bar{q}'' a^2 / 2k_c} &= -2Y_{0,21} \left[\frac{U_1^+ U_2^+ \langle A_1 \rangle_1 \langle A_2 \rangle_2}{\langle A \rangle_1 \langle A \rangle_2} + \frac{U_1^+ U_3^+ \langle A_1 \rangle_1 \langle A_3 \rangle_2}{\langle A \rangle_1 \langle A \rangle_2} \right] \\
&\quad - 2Y_{0,32} \left[\frac{U_1^+ U_3^+ \langle A_1 \rangle_1 \langle A_3 \rangle_2}{\langle A \rangle_1 \langle A \rangle_2} + \frac{U_2^+ U_3^+ \langle A_2 \rangle_1 \langle A_3 \rangle_2}{\langle A \rangle_1 \langle A \rangle_2} \right] \\
&\quad + \frac{q_1^+ U_1^+ \langle A_1 \rangle_1 \langle \theta_1 \rangle_1 + q_2^+ U_2^+ \langle A_2 \rangle_1 \langle \theta_2 \rangle_1}{\langle A \rangle_1} \\
&\quad - \frac{q_2^+ U_2^+ \langle A_2 \rangle_2 \langle \theta_2 \rangle_2 + q_3^+ U_3^+ \langle A_3 \rangle_2 \langle \theta_3 \rangle_2}{\langle A \rangle_2}
\end{aligned} \tag{6.25}$$

$$\begin{aligned}
\frac{\bar{T}_2 - \bar{T}_3}{\bar{q}''' a^2 / 2k_c} &= -2Y_{0,32} \left[\frac{U_2^+ U_3^+ \langle A_3 \rangle_3 \langle A_2 \rangle_2}{\langle A \rangle_2 \langle A \rangle_3} - \frac{U_2^+ U_3^+ \langle A_2 \rangle_3 \langle A_3 \rangle_2}{\langle A \rangle_2 \langle A \rangle_3} \right] \\
&\quad - 2Y_{0,42} \frac{U_2^+ U_4^+ \langle A_4 \rangle_3 \langle A_2 \rangle_2}{\langle A \rangle_2 \langle A \rangle_3} - 2Y_{0,43} \frac{U_3^+ U_4^+ \langle A_4 \rangle_3 \langle A_3 \rangle_2}{\langle A \rangle_2 \langle A \rangle_3} \\
&\quad + \frac{q_2^+ U_2^+ \langle A_2 \rangle_2 \langle \theta_2 \rangle_2 + q_3^+ U_3^+ \langle A_3 \rangle_2 \langle \theta_3 \rangle_2}{\langle A \rangle_2} \\
&\quad - \frac{q_2^+ U_2^+ \langle A_2 \rangle_3 \langle \theta_2 \rangle_3 + q_3^+ U_3^+ \langle A_3 \rangle_3 \langle \theta_3 \rangle_3 + q_4^+ U_4^+ \langle A_4 \rangle_3 \langle \theta_4 \rangle_3}{\langle A \rangle_3}
\end{aligned} \tag{6.26}$$

$$\begin{aligned}
\frac{\bar{T}_3 - \bar{T}_4}{\bar{q}''' a^2 / 2k_c} = & -2Y_{0,32} \frac{U_2^+ U_3^+ \langle A_2 \rangle_3 \langle A_3 \rangle_4}{\langle A \rangle_3 \langle A \rangle_4} - 2Y_{0,42} \frac{U_2^+ U_4^+ \langle A_2 \rangle_3 \langle A_4 \rangle_4}{\langle A \rangle_3 \langle A \rangle_4} \\
& - 2Y_{0,43} \frac{U_3^+ U_4^+ \langle A_4 \rangle_3 \langle A_3 \rangle_4}{\langle A \rangle_3 \langle A \rangle_4} - \frac{U_3^+ U_4^+ \langle A_3 \rangle_3 \langle A_4 \rangle_4}{\langle A \rangle_3 \langle A \rangle_4} \\
& + \frac{q_2^+ U_2^+ \langle A_2 \rangle_3 \langle \theta_2 \rangle_3 + q_3^+ U_3^+ \langle A_3 \rangle_3 \langle \theta_3 \rangle_3 + q_4^+ U_4^+ \langle A_4 \rangle_3 \langle \theta_4 \rangle_3}{\langle A \rangle_3} \\
& - \frac{q_3^+ U_3^+ \langle A_3 \rangle_4 \langle \theta_3 \rangle_4 + q_4^+ U_4^+ \langle A_4 \rangle_4 \langle \theta_4 \rangle_4}{\langle A \rangle_4} \quad (6.27)
\end{aligned}$$

where the subchannel $\langle A \rangle_1$, $\langle A \rangle_2$, $\langle A \rangle_3$ and $\langle A \rangle_4$ are defined as:

$$\langle A \rangle_1 = \langle A_1 \rangle_1 + \langle A_2 \rangle_1 ,$$

$$\langle A \rangle_2 = \langle A_2 \rangle_2 + \langle A_3 \rangle_2 ,$$

$$\langle A \rangle_3 = \langle A_2 \rangle_3 + \langle A_3 \rangle_3 + \langle A_4 \rangle_3 ,$$

and
$$\langle A \rangle_4 = \langle A_3 \rangle_4 + \langle A_4 \rangle_4$$

Naturally, the heat transport between the subchannel pairs are determined by integrating the appropriate temperature fields accordingly. In addition, the relations for the geometric parameters S_{ij} and ℓ_{ij}^* for the subchannel pairs are given in Table 6.1. The results of the calculations for the effective mixing lengths are shown in Fig.6.11 and 6.12 for the fuel and blanket bundles, respectively. From Fig.6.11, it can be seen that the singularities of L_{12} , L_{23} and L_{34} are approximately in the vicinity where $2W/D$ is equal to 1.16, 1.19 and 1.16, respectively. As pointed out in the foregoing section, the singularities are due to the fact that the heat transport between these subchannel pairs approaches zero. As $2W/D$ increases, all the three effective mixing lengths of the fuel bundle merge to an asymptotic value of 0.80. To the left of the singularities, establishment of asymptotic values of L_{12} , L_{23} , and L_{34} are not seen. On the other hand, it is indicated by Fig.6.12 that L_{12} and L_{23} of the blanket bundle have an almost constant value of 0.69 throughout the range of $2W/D$ being analyzed. Also, L_{34} has an asymptotic value of 0.70 as $2W/D$ is greater than 1.14 approximately. The values of L_{34} decreases as $2W/D$ decreases but a singularity is not seen in the range of $2W/D$ being analyzed.

Table 6.1

Geometric Parameters for L_{ij} Calculations

	General Form	Fuel Assembly	Blanket Assembly
l_{12}^*/D	$\frac{1}{\sqrt{3}} \left(\frac{P}{D} \right)$	0.716	0.624
l_{23}^*/D	$\frac{1}{\sqrt{3}} \left(\frac{P}{D} \right)$	0.716	0.624
l_{34}^*/D	$\frac{\frac{1}{2\sqrt{3}} \left(\frac{P}{D} \right) + \frac{\frac{1}{8} \left(\frac{2W}{D} \right)^2 \left(\frac{P}{D} \right) + \frac{1}{12\sqrt{3}} \left(\frac{2W}{D} \right)^3 - \frac{1}{16}}{\frac{1}{2} \left(\frac{P}{D} \right) \left(\frac{2W}{D} \right) + \frac{1}{4\sqrt{3}} \left(\frac{2W}{D} \right) - \frac{\pi}{12}}$	0.745	0.637
s_{12}/D	$\frac{1}{2} \left(\frac{P}{D} - 1 \right)$	0.12	0.04
s_{23}/D	$\frac{P}{D} - 1$	0.24	0.08
s_{34}/D	$\frac{P}{D} - 1$	0.24	0.08

Fig.6.11 Effective Mixing Lengths for the 19-Rod Fuel Bundle

Fig.6.12 Effective Mixing Lengths of the 19-Rod Blanket Bundle

From the results of the present calculation, it appears that the asymptotic value of the effective mixing lengths of various subchannel pairs are so close that they can be approximated by a single characteristic value for most practical purposes. For instance, values of 0.80 and 0.70 can be used to characterize the asymptotic effective mixing lengths in the fuel and blanket bundles, respectively. In the range of $2W/D$ ratios where heat flow reversal is experienced, special attention must be given to the wall dependence of the effective mixing length in order to obtain accurate results for heat transport between subchannels.

For general design calculations, it is desirable to derive some relations which give the values of the effective mixing lengths as a function of the wall distance for various bundles. This kind of correlating functions should include both the P/D and $2W/D$ ratios to cover a wide range of geometric configurations. However, this implies intensive parametric study for both the P/D and $2W/D$ ratios. For economy reason, such a parametric study is not to be performed but

instead correlations for the effective mixing lengths of the fuel and blanket bundles are derived separately. As a result, these correlations only contain the $2W/D$ ratio to account for the wall effect. In the present analysis, the following simple form is postulated from inspection of the curves for the L_{ij} of the 19-rod bundle:

$$L_{ij} = L_{ij\infty} + C_1^* \left[\frac{2W}{D} - \left(\frac{2W}{D} \right)_0 \right]^{-1} e^{-C_2^* \left| \frac{2W}{D} - \left(\frac{2W}{D} \right)_0 \right|} \quad (6.28)$$

where $(2W/D)_0$ - the value of $(2W/D)$ at which L_{ij} approaches infinity,

$L_{ij\infty}$ - the asymptotic value of L_{ij}

and C_1^* and C_2^* are the constants to be determined from the curve fitting procedure. The postulated form guarantees that L_{ij} goes to infinity as $(2W/D)$ approaches $(2W/D)_0$ and assumes its asymptotic values as $(2W/D)$ increases. Naturally, the asymptotic value of L_{ij} is a function of the P/D ratio. On the other hand, the values of L_{12} and L_{23} of the 19-rod blanket bundle are fairly constant throughout the range of $2W/D$ being considered. Therefore, it seems that a curve fit is unnecessary and constant L_{12} and L_{23} should be used

for the blanket bundle. It can also be seen that L_{34} of the blanket bundle does not have a singularity in the range of $2W/D$ being considered. However, it was found that the form of the correlating function given by Eq. (6.28) can still be used to fit the calculated results by setting $(2W/D)_0 \approx 0.97$. The results of curve fitting of the effective mixing lengths of all the subchannel pairs are summarized in Table 6.2.

In order to check the validity of these simple correlating functions, a comparison of the values of L_{ij} given by the multicell calculation and the correlating functions as given in Table 6.3. It can be seen that the simple correlating functions give very satisfactory results except in the vicinity of the singularities and in the range of very small $2W/D$ ratio. Certainly, better curve fitting results can be achieved if higher order polynomials or more sophisticated technique were used. But it is felt that these simple functions are extremely convenient to use and capable of giving reasonably good approximation to the effective mixing lengths throughout a wide range of $2W/D$ ratios of practical design interest. The accuracy of the calculated effective mixing lengths of the 19-rod bundles will be compared with the experimental results in the following section.

Table 6.2

Constants of the Correlation Functions for the Effective Mixing Lengths of the 19-Rod Bundles

		$(2W/D)_0$	Range	C_1^*	C_2^*
Fuel Bundle	L_{12}	1.16	$2W/D < 1.16$	0.012	24.45
			$2W/D > 1.16$	0.002	6.98
	L_{23}	1.19	$2W/D < 1.19$	0.0089	26.16
			$2W/D > 1.19$	0.0058	11.36
	L_{34}	1.16	$2W/D < 1.16$	0.0216	1.471
			$2W/D > 1.16$	0.0164	13.64
Blanket Bundle	L_{12}	0	$2W/D > 1$	0	0
	L_{23}	0	$2W/D > 1$	0	0
	L_{34}	0.97	$2W/D > 1$	0.00725	9.57

Table 6.3

Comparison of Calculated and Curve-Fitted Effective Mixing Lengths for the
19-Rod Bundle

	2W/D	L_{12}	L_{12}	L_{23}	L_{23}	L_{34}	L_{34}
		calculated	curve fit	calculated	curve fit	calculated	curve fit
Fuel Bundle	1.08	0.80	0.812	0.79	0.795	1.02	1.050
	1.10	0.85	0.845	0.80	0.799	1.12	1.14
	1.12	0.92	0.904	0.81	0.810	1.29	1.319
	1.14	1.15	1.15	0.84	0.838	1.85	1.859
	1.16	-	-	0.95	0.925	-	-
	1.18	0.67	0.704	1.46	1.475	0.25	0.186
	1.20	0.75	0.753	0.27	0.272	0.57	0.572
	1.22	0.77	0.769	0.56	0.653	0.70	0.689
	1.24	0.78	0.777	0.67	0.724	0.73	0.741
	1.28	0.785	0.784	0.74	0.767	0.78	0.783
	1.30	0.79	0.786	0.77	0.775	0.79	0.793
	1.32	0.791	0.787	0.78	0.780	0.80	0.798
	1.35	0.791	0.788	0.78	0.784	0.81	0.804
	1.40	0.791	0.789	0.79	0.789	0.81	0.807
Blanket Bundle	1.02	0.69	0.69	0.69	0.69	0.634	0.613
	1.03	0.69	0.69	0.69	0.69	0.645	0.645
	1.04	0.69	0.69	0.69	0.69	0.652	0.65
	1.06	0.69	0.69	0.69	0.69	0.666	0.666
	1.08	0.69	0.69	0.69	0.69	0.68	0.680
	1.10	0.69	0.69	0.69	0.69	0.69	0.690
	1.15	0.69	0.69	0.69	0.69	0.695	0.694
	1.20	0.69	0.69	0.69	0.69	0.70	0.699

6.6 COMPARISON OF THE CALCULATED EFFECTIVE MIXING LENGTHS WITH EXPERIMENTAL RESULTS

In addition to the comparison with the results of France and Ginsberg, the calculated results for the effective mixing length of a hexagonal bundle is compared with the experimental results obtained by Fukuda [71]. An empirical formula was obtained by this author by performing an electrostatic analog simulation for a single region analysis. The empirical relation for the mixing length of conduction was proposed as:

$$\frac{l_{1j}}{l_{1j}^*} = \frac{1}{\frac{1.38}{\sqrt{3}} \left(\frac{P-D}{D} \right)^{0.674} \left(\frac{P}{P-D} \right)} \quad (6.29)$$

Due to the fact that the empirical formula does not have the geometric parameter to account for the wall distance, it is presumed that the formula is only valid for the asymptotic values of the L_{1j} ; i.e., in the region where the wall distance is sufficiently large such that L_{1j} is fairly independent of $2W/D$. As a result, it is desirable to compare the asymptotic values of the effective mixing lengths determined from the multicell analysis with those predicted by the Fukuda's

formula. The results of comparison is summarized in Table 6.4. It can be seen that the difference between the calculated values from the multiregion analysis and those given by Fukuda's formula is approximately 20%. Part of the difference is due to the single region analysis nature of the simulation. Therefore, the agreement is generally better with the single region analysis results.

The satisfactory agreement between the results of the present analysis and that of France and Ginsberg for the square array geometry presented earlier and the comparison of the results of the present analysis with the electrostatic simulation in the hexagonal geometry should further strength the confidence in the analytical procedure. The impact of the accuracy of the effective mixing length for the subchannel calculations will be investigated in the following section.

6.7 IMPACT OF EFFECTIVE MIXING LENGTH ON SUBCHANNEL CALCULATIONS

In order to demonstrate the effect of introducing the analytically predicted mixing lengths on subchannel calculations, two cases with different flow conditions were performed with COBRA-IIIC for the 7-rod fuel bundle. For each case, the average channel exit temperatures at the axial position of 36" with and without proper correction factor for the

Table 6.4
Comparison of Calculated Effective Mixing Lengths with Experimental Results

		L_{12} internal to internal	L_{23} internal to internal	L_{34} internal to corner	L_{Fukuda}
P/D = 1.24	Multi-Region	0.791	0.79	0.810	0.636
	Single Region	-	-	0.710 [*]	
P/D = 1.08		0.69	0.69	0.70	0.51

* From the 7-Rod Calculation

mixing length is calculated. In COBRA-IIIC, a uniform heat flux distribution is always assumed at the clad outside surface in the azimuthal direction, therefore using a mixing length resulted from a multiregion analysis would not be consistent. As indicated in Fig. 6.6, the analytically predicted mixing length resulted from the single-region analysis is approximately 0.7. For the cases without proper correction, the centroid-to-centroid distance is used as the effective mixing length; i.e., $L_{12}=1.0$. For these calculations, diversion cross flow has been artificially suppressed. Other operating conditions are listed in Table 6.5 and the results of the calculation are summarized in Table 6.6.

The results of the calculations indicate that for both cases the use of the accurately predicted effective mixing length successfully results in reductions of the intersubchannel temperature difference as a consequence of an increase in the heat transport between the subchannels. As shown in Table 6.6 the intersubchannel temperature difference ($\bar{T}_1 - \bar{T}_2$) decreases from 70°F to 63.7°F for case 1 and from 94.29°F to 83.54°F for case 2. Therefore, the accurately predicted mixing length has indeed proven to remove excess conservatism due to the lack of knowledge of the local temperature field. However, it must be pointed out that calculations with analytically predicted values of the effective mixing lengths do not necessarily result in conservative results.

Table 6.5

Operating Conditions for COBRA-IIIC Calculations

system pressure	60 psia
inlet temperature	700°F
average heat flux	0.3×10^6 BTU/ft ² hr
uniform power in rods	
uniform heat flux distributions	
in both axial and azimuthal directions	
average mass flux $G =$	$\left\{ \begin{array}{l} 2 \times 10^6 \text{ lbm/ft}^2 \text{hr (case 1)} \\ 3 \times 10^6 \text{ lbm/ft}^2 \text{hr (case 2)} \end{array} \right.$

Table 6.6
 Subchannel Average Temperature at 36" from the Entrance of a
 of a 7-Rod LMFBR Bundle

case	mass flux (lbm/ft ² hr)	L ₁₂	$\bar{T}_1$	$\bar{T}_2$	$\bar{T}_1 - \bar{T}_2$
1	3x10 ⁶	1.0	963.47	893.47	70.0
		0.7	959.30	895.60	63.7
2	2x10 ⁶	1.0	1089.88	994.59	94.29
		0.7	1082.11	998.57	83.54

For example, using an effective mixing length greater than one for the subchannel calculations (which is the case for a fuel assembly with very small $2W/D$ ratio) would result in a decrease in intersubchannel heat transport for the same intersubchannel temperature gradient and lead to higher subchannel temperature difference than that predicted by the calculation without the correction.

Another important feature of the calculational results has to be addressed is that the effect of the correction in the mixing length is more pronounced as the mass flow rate decreases. As shown by Table 6.6, the intersubchannel temperature difference increases from 6.3°F to 10.75°F as the mass flux decreases from $3 \times 10^6 \text{ lbm/ft}^2 \text{ hr}$ to $2 \times 10^6 \text{ lbm/ft}^2 \text{ hr}$. Thus, it can be easily imagined that intolerable conservatism in heat transport and subsequent subchannel temperatures would result in a even lower flow rate case if the correct mixing length is not known.

It should be mentioned at this point that most ongoing efforts have been devoted to the diversion mixing area in order to obtain mixing coefficients which are of similar magnitude and it is suggested here that an equally significant gain in design margin is possible by properly accounting for the heat conduction effect.

CHAPTER 7

DISTRIBUTED-LUMPED PARAMETER COUPLING ANALYSIS

In this chapter, a scheme of coupling the distributed and lumped parameter is proposed. The analysis is applied to a 7-rod bundle and the results of the calculations are compared to 3-D calculation for checking the validity of the proposed approach.

7.1 MOTIVATION OF THE DISTRIBUTED-LUMPED COUPLING ANALYSIS

The multicell analysis has already demonstrated its applicability for the thermal analysis of rod bundles. The major advantage of the distributed parameter analysis is that it is capable of providing accurate local temperature fields for the fuel, clad and coolant. However, the distributed parameter analysis as given in the present work is only capable of accounting for the heat transport process by conduction effect. Naturally, the clad temperature distribution resulting from this analysis is too conservative. To obtain a more realistic solution, other transport mechanisms such as turbulence mixing and diversion cross-flow must be included in the analysis.

Another possible approach to improve the distributed parameter analysis is to couple the distributed parameter and subchannel analyses by properly using the results of the subchannel codes as boundary conditions for the distributed para-

meter analysis. Obviously, the most natural process would be to couple the present analysis with the COBRA-IIIC calculation with conduction mixing only but this is too conservative for practical purposes. Therefore, the most logical approach is to couple the present analysis with COBRA-IIIC calculations with all the momentum and heat transport mechanisms as normally used in design calculations. The results of the COBRA-IIIC calculations should provide a fairly good estimate for the average subchannel coolant temperatures. It is apparent that the intersubchannel coolant temperature difference from COBRA-IIIC calculation should be considerably less than that predicted by the 2-D fully-developed analysis because of the inclusion of the turbulence mixing and diversion crossflow. In the distributed-lumped coupling analysis, the subchannel temperatures obtained from the COBRA-IIIC calculations are imposed as boundary conditions for the coolant temperatures field in addition to the continuity of temperature and heat flux. Naturally, this will subsequently affect the fuel and clad temperature fields and a more realistic estimate for the clad temperature distribution can be obtained. The procedure of the analysis is summarized in Fig.7.1 and the details of the procedure will be illustrated in the following section.

7.2 METHOD OF THE ANALYSIS

The present coupled distributed-lumped parameters analysis

Fig.7.1 General Procedure of the Coupled Distributed-Lumped
Parameter Analysis

is applied to a 7-rod bundle as an example to demonstrate the applicability of the proposed method. As shown in Fig.6.2 the dashed line denotes the subchannel boundary used in the COBRA-IIIC calculation and the solid line denotes the cell boundary used in the multicell calculation of the distributed parameter analysis.

In the present analysis, the form of the coolant temperature of the coolant temperature field is assumed to obey the relation:

$$T(r, \phi, z) = \Delta + \frac{q'' a^2}{2k_c} \theta(r, \phi) \quad (7.1)$$

where $\theta(r, \phi)$ is the 2-D dimensionless temperature field of the coolant which has been defined in Eq.(2.98) and Δ is a function depending only on the axial position z . In addition, the form of the fuel and clad temperature fields are also assumed to be similar to Eq.(7.1) except that $\theta(r, \phi)$ is replaced by $\Gamma(r, \phi)$ and $\Psi(r, \phi)$, respectively.

Similar to the multicell analysis as described in the foregoing chapters, the distributed part of the analysis is initiated by imposing adiabatic boundary condition by applying Eq.(2.139) at the unit cell boundaries(solid lines). Substituting the coolant temperature as given by Eq.(7.1) into Eq.(2.139) and setting the heat flux term q'' equal to zero, the following relation is obtained:

$$-k_c \left[\cos(\phi - \alpha_1) \frac{\partial T}{\partial r}(c, \phi) - \sin(\phi - \alpha_1) \frac{1}{c} \frac{\partial}{\partial \phi} T(c, \phi) \right] = 0 \quad (7.3)$$

Dividing both sides by $q'' a^2 / k_c$, Eq.(7.3) reduces to the more familiar dimensionless form:

$$- \left[\cos(\phi - \alpha_1) \frac{\partial}{\partial \rho} \theta(c, \phi) - \sin(\phi - \alpha_1) \frac{b}{c} \frac{\partial}{\partial \phi} \theta(c, \phi) \right] = 0 \quad (7.4)$$

Which is identical to Eq.(3.2) derived for the fully developed temperature field (with $\delta_0=0$). Substituting the expression for $\theta(\rho, \phi)$ into Eq.(7.4) leads to the following relation:

$$\begin{aligned} \sum_{n=1}^N Y_n \left\{ -f_n \left(\frac{c}{b} \right)^{-\lambda_n} \cos[(\lambda_n + 1)\phi - \alpha_1] + g_n \left(\frac{c}{b} \right)^{\lambda_n} \cos[(\lambda_n - 1)\phi + \alpha_1] \right\} \\ = \left[\left(1 + \frac{1}{I} \right) - \frac{1}{I} \left(\frac{c}{b} \right)^2 \right] \cos(\phi - \alpha_1) \end{aligned} \quad (7.5)$$

It has to be pointed out that the Fourier sine series in the coolant temperature field has been omitted because of the geometric symmetry of the unit cells being analyzed. By satisfying Eq.(7.5) at the collocation points along the boundaries of the unit cells, an initial guess for the solution of the temperature fields is obtained.

The coupling of the two temperature fields is accomplished

by applying the condition of temperature continuity along the common boundary. The updating algorithm requires that the updated temperature is equal to the average of the temperatures given by the temperature fields of the corner and small internal cells from one previous calculation, that is:

$$T_2^{(v+1)}(\rho, \phi, z) = \frac{1}{2} \left[T_1^{(v)}(\rho', \phi', z') + T_2^{(v)}(\rho, \phi, z) \right] \quad (7.6)$$

Upon substituting Eq.(7.1) into Eq.(7.6) leads to:

$$\begin{aligned} \frac{\Delta_2}{\bar{q}''' a^2 / 2k_c} + q_2^+ \left[H_2(c) + \sum_{n=1}^N \frac{y_{n,2}^{(v+1)}}{\lambda_n} \left(f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right) \cos \lambda_n \phi \right] \\ = \frac{1}{2} \left(\frac{\Delta_2 + \Delta_1}{\bar{q}''' a^2 / 2k_c} \right) \frac{1}{2} \left[q_1^+ \theta_1^{(v)}(c', \phi') + q_2^+ \theta_2^{(v)}(c, \phi) \right] \quad (7.7) \end{aligned}$$

$$\text{where } H(c) = \frac{1}{\kappa} \left(\ln t - \frac{1}{B_1} \right) - \frac{1}{\kappa \kappa_w B_{1s}} - \left(1 + \frac{1}{I} \right) \ln \frac{c}{b} + \frac{1}{2I} \left[\left(\frac{c}{b} \right)^2 - 1 \right]$$

Dividing Eq.(7.7) by $\bar{q}''' a^2 / 2k_c$ leads to :

$$\begin{aligned}
& \frac{1}{2} \left[-\frac{\Delta_1}{\bar{q}''' a^2 / 2k_c} + \frac{\Delta_2}{\bar{q}''' a^2 / 2k_c} \right] + q_2^+ \sum_{n=1}^N \frac{Y_{n,2}^{(\nu+1)}}{\lambda_n} \left(f_n \left(\frac{c}{b} \right)^{-\lambda_n} + g_n \left(\frac{c}{b} \right)^{\lambda_n} \right) \cos \lambda_n \phi \\
& = -q_2^+ H_2(c) + \frac{1}{2} \left[q_1^+ \theta_1^{(\nu)}(c', \phi') + q_2^+ \theta_2^{(\nu)}(c, \phi) \right] \quad (7.8)
\end{aligned}$$

Satisfying Eq.(7.8) at a finite number of collocation points along the common boundary between the two unit cells, a system of linear equations is obtained.

In addition to the condition of continuity of temperature at the common boundary, the present analysis imposes two additional boundary conditions; that is, the temperature fields of the two unit cells, if properly lumped, should be equal to the two subchannel average temperatures given by COBRA-IIIC calculation at the particular axial position being considered.

It can be seen from Fig.7.1 that subchannel 2 is the region of the corner cell where $0 \leq \phi \leq \alpha_2$. As a result, the present analysis requires that:

$$\langle T_2 \rangle_2 = \bar{T}_2(z) \quad (7.9)$$

Where $\bar{T}_2(z)$ is the coolant temperature of subchannel 2 resulted from COBRA-IIIC calculation. Obviously, Eq.(7.9) can also be

expressed in the form:

$$\frac{\int_{\phi=0}^{\alpha_2} \int_b^{\hat{a}/\cos(\phi-\hat{\alpha})} T_2(r, \phi, z) r dr d\phi}{\int_{\phi=0}^{\alpha_2} \int_b^{\hat{a}/\cos(\phi-\hat{\alpha})} r dr d\phi} = \bar{T}_2(z) \quad (7.10)$$

where $\hat{a}=a_1$ and $\hat{\alpha}=\alpha_1$ for $\beta_1 \leq \phi \leq \beta_{i+1}$. Substituting Eq. (7.1) into Eq. (7.10) and dividing it by $\bar{q}''' a^2/2k_c$ leads to:

$$\frac{1}{\langle A_2 \rangle_2} \int_0^{\alpha_2} \int_1^{c/b} \left\{ \frac{\Delta_2}{\bar{q}''' a^2/2k_c} + \left[H_2(\rho) + q_2^+ \sum_{n=1}^N \frac{Y_{n,2}}{\lambda_n} (f_{n\rho}^{-\lambda_n} + q_{n\rho}^{\lambda_n}) \cos \lambda_n \phi \right] \right\} \rho d\rho d\phi = \frac{\bar{T}_2(z)}{\bar{q}''' a^2/2k_c} \quad (7.11)$$

where $\langle A_2 \rangle_2 = \int_0^{\alpha_2} \int_1^{c/b} \rho d\rho d\phi$,

and

$$c = a_i / \cos(\phi - \alpha_i) \quad .$$

Substituting the definition of $H_2(\rho)$ into Eq.(7.11), the local temperature field during the v th iteration should satisfy the following relation:

$$\begin{aligned} & \frac{\Delta_2}{\bar{q}'' a^2 / 2k_c} + \frac{1}{\kappa} \left(\ell_n t - \frac{1}{B_1} \right) - \frac{1}{\kappa \kappa_w} \frac{1}{B_{1s}} - \frac{1}{2I} \\ & + \frac{1}{2I \langle A_2 \rangle_2} \int_0^{\alpha_2} \int_1^{c/b} \rho^3 d\rho d\phi \\ & + \left(1 + \frac{1}{I} \right) \frac{1}{\langle A_2 \rangle_2} \int_0^{\alpha_2} \int_1^{c/b} \rho \ln \rho d\rho d\phi \\ & + \frac{1}{\langle A_2 \rangle_2} \int_0^{\alpha_2} \int_1^{c/b} q_2^+ \sum_{n=1}^{(v+1)} \frac{Y_{n,2}}{\lambda_n} (f_n \rho^{-\lambda_n+1} + g_n \rho^{\lambda_n+1}) \cdot \\ & \cos \lambda_n \phi d\rho d\phi = \frac{\bar{T}_2(z)}{\bar{q}'' a^2 / 2k_c} \quad . \end{aligned} \quad (7.12)$$

Interchanging the summation and the integration of the sixth term on the left hand side of Eq.(7.12) and defining the following quantities:

$$\langle \chi_2 \rangle_2 = \int_0^{\alpha_2} [\rho^4]_1^{c/b} d\phi, \quad (7.13)$$

$$\langle \gamma_2 \rangle_2 = \int_0^{\alpha_2} \left[2\rho^2 \left(\ell_n \rho - \frac{1}{2} \right) \right]_1^{c/b} d\phi, \quad (7.14)$$

$$\langle \Omega_{n,2} \rangle_2 = \frac{1}{\lambda_n} \int_0^{\alpha_2} \cos \lambda_n \phi \left[\frac{g_n^{\lambda_n+2}}{\lambda_n+2} + f_n \tau_n(\rho) \right]_1^{c/b} d\phi. \quad (7.15)$$

$$\text{where } \tau_n = \begin{cases} \ell_n \rho & \lambda_n = 2 \\ \frac{\rho^{2-\lambda_n}}{2-\lambda_n} & \lambda_n \neq 2 \end{cases},$$

the temperature field of the corner cell satisfies the follow-

ing relation:

$$\frac{\Delta_2}{\bar{q}'' a^2 / 2k_c} + \frac{1}{\kappa} \left(\ell_{nt} - \frac{1}{B_i} \right) - \frac{1}{\kappa \kappa_w} \frac{1}{B_{is}} - \frac{1}{2I} + \frac{1}{8I \langle A_2 \rangle_2} .$$

$$[\langle \chi_2 \rangle_2 - \langle \Delta \phi_2 \rangle_2] + \frac{(I+1)}{4I \langle A_2 \rangle_2} [\langle \gamma_2 \rangle_2 + \langle \Delta \phi_2 \rangle_1]$$

$$+ \frac{1}{\langle A_2 \rangle_2} \sum_{n=1}^N y_{n,2}^{(\nu+1)} \langle \Omega_{n,2} \rangle_1 = \frac{\bar{T}_2(z)}{\bar{q}'' a^2 / 2k_c} \quad (7.16)$$

or equivalently:

$$\frac{\Delta_2}{\bar{q}''' a^2 / 2k_c} + \frac{1}{\langle A_2 \rangle_2} \sum_{n=1}^N Y_{n,2}^{(\nu+1)} \langle \Omega_{n,2} \rangle_2 = \frac{\bar{T}_2(z)}{\bar{q}''' a^2 / 2k_c}$$

$$-\frac{1}{\kappa} \left(\ln t - \frac{1}{B_i} \right) + \frac{1}{\kappa \kappa_w} \frac{1}{B_{is}} + \frac{1}{2I}$$

$$-\frac{(I+1)}{4I \langle A_2 \rangle_2} [\langle \gamma_2 \rangle_2 + \langle \Delta \phi_2 \rangle_2] - \frac{1}{8I \langle A_2 \rangle_2} [\langle \chi_2 \rangle_2 - \langle \Delta \phi_2 \rangle_2] \quad (7.17)$$

$$\text{where } \langle \Delta \phi_2 \rangle_2 = \int_0^{\alpha_2} d\phi = \alpha_2$$

Another boundary condition that has to be satisfied is that the temperature fields of the two unit cells, upon appropriate lumping, would yield the coolant temperature of subchannel 1 predicted by COBRA-IIIC. From Fig.6.2, it can be seen that the average temperature of subchannel 1 can be given by the flow rate weighted sum of the coolant temperatures of the internal cell and the region of the corner cell where $\alpha_2 \leq \phi \leq \pi$, that is:

$$\frac{V_2^+ \langle A_2 \rangle_1}{V_2^+ \langle A_2 \rangle_1 + V_1^+ \langle A_1 \rangle_1} \langle T_2 \rangle_1 + \frac{V_1^+ \langle A_1 \rangle_1 \langle T_1 \rangle}{V_2^+ \langle A_2 \rangle_2 + V_1^+ \langle A_1 \rangle_1} = \bar{T}_1(z) \quad (7.18)$$

where V_1^+ and V_2^+ are dimensionless flow velocities normalized to the bundle average flow velocity for the small internal and corner unit cell, respectively. In addition, $\bar{T}_1(z)$ is the coolant temperature of subchannel 1 predicted by COBRA-IIIC at the axial position z . Due to the fact the present analysis is a cell-to-cell iterative method, it is obvious that the temperature fields of the unit cells cannot be updated simultaneously. Since the direction of the iteration is from the corner to the internal cell, it is natural to use the results from one previous iteration to update the temperature field of the corner cell. Therefore, the normalized Eq.(7.18) can be written as:

$$\frac{\langle T_2 \rangle_2^{(v+1)}}{\bar{q}'' a^2 / 2k_c} = \left[1 + \frac{V_1^+ \langle A_1 \rangle_1}{V_2^+ \langle A_2 \rangle_1} \right] \frac{\bar{T}_1(z)}{\bar{q}'' a^2 / 2k_c} - \frac{V_1^+ \langle A_1 \rangle_1}{V_2^+ \langle A_2 \rangle_1} \langle T_1 \rangle_1^{(v)} \quad (7.19)$$

Upon substituting the expression of the coolant temperature into Eq.(7.19), the following relation can be obtained:

$$\begin{aligned}
& \frac{1}{\langle A_2 \rangle_1} \int_{\alpha_2}^{\pi} \int_1^{c/b} \left\{ \frac{\Delta_2}{\bar{q}''' a^2 / 2k_c} + \left[H_2(\rho) + \sum_{n=1}^N \frac{Y_{n,2}^{(\nu+1)}}{\lambda_n} (f_n \rho^{-\lambda_n} + g_n \rho^{\lambda_n}) \right. \right. \\
& \left. \left. \cos \lambda_n \phi \right] \right\} \rho d\rho d\phi = \left[1 + \frac{v_1^+ \langle A_1 \rangle_1}{v_2^+ \langle A_2 \rangle_1} \right] \frac{\bar{T}_1(z)}{\bar{q}''' a^2 / 2k_c} \\
& - \frac{v_1^+ \langle A_1 \rangle_1}{v_2^+ \langle A_2 \rangle_1} \left[\frac{\Delta_1}{\bar{q}''' a^2 / 2k_c} + \langle \theta_1 \rangle_1^{(\nu)} \right] \quad (7.20)
\end{aligned}$$

Eq.(7.20) can be written in the equivalent form as:

$$\begin{aligned}
& \frac{v_1^+ \langle A_1 \rangle_1}{v_1^+ \langle A_2 \rangle_1} \cdot \frac{\Delta_1}{\bar{q}''' a^2 / 2k_c} + \frac{\Delta_2}{\bar{q}''' a^2 / 2k_c} + \frac{1}{\kappa} \left(\ln t - \frac{1}{B_i} \right) - \frac{1}{\kappa \kappa_w} \frac{1}{B_{is}} - \frac{1}{2I} \\
& + \frac{1}{8I \langle A_2 \rangle_1} [\langle \chi_2 \rangle_1 \langle \Delta \phi_2 \rangle_1] + \frac{(I+1)}{4I \langle A_2 \rangle_1} [\langle \gamma_2 \rangle_1 - \langle \Delta \phi_2 \rangle_1]
\end{aligned}$$

$$+ \frac{1}{\langle A_2 \rangle_1} \sum_{n=1}^N Y_{n,2}^{(v+1)} \langle \Omega_{n,2} \rangle_1 =$$

$$\left[1 + \frac{v_1^+ \langle A_1 \rangle_1}{v_1^+ \langle A_2 \rangle_1} \right] \frac{\bar{T}_1(z)}{\bar{q} a^2 / 2k_c} - \frac{v_1^+ \langle A_1 \rangle_1}{v_2^+ \langle A_2 \rangle_1} \langle \theta_1 \rangle_1^{(v)} \quad (7.21)$$

where

$$\langle \Delta \phi_2 \rangle_1 = \int_{\alpha_2}^{\pi} d\phi = \pi - \alpha_2$$

and $\langle \chi_2 \rangle_1$, $\langle \gamma_2 \rangle_1$ and $\langle \Omega_{n,2} \rangle_1$ are defined in a similar manner as those in Eqs. (7.13), (7.14) and (7.15) except that the limits of the integrals are from α_2 to π instead of 0 to α_2 . It has to be also pointed out that the physical configuration of the bundle requires the adiabatic boundary condition to be satisfied along the first and second boundaries of the corner cell throughout the entire iteration procedure.

For illustration purpose, the calculational procedure for the temperature field of the corner cell is as follows:

1. Satisfying Eq. (7.5) at the collocation points along the first and second boundaries.

2. Satisfying Eq.(7.8) at the collocation points along the common boundary between the two cells.
3. Satisfying Eqs.(7.17) and (7.22).

As a result, a system of simultaneous equations can be obtained and the unknown coefficients $Y_{n,2}^{(v+1)}$ of the updated solution vector can be determined by applying the least square technique described in the preceeding chapters.

On the other hand, the updating procedure for the internal cell is much more straight-forward because only one boundary is involved. Similar to the coupling analysis described in chapter 4, the internal cell uses the Neumann type boundary condition by substituting Eq.(7.1) into Eq. (2.139); that is:

$$\sum_{n=1}^N Y_{n,1}^{(v+1)} \left\{ -f_n \left(\frac{c}{b} \right)^{-\lambda_n} \cos [(\lambda_n + 1) \phi - \alpha_i] + g_n \left(\frac{c}{b} \right)^{\lambda_n} \cos [(\lambda_n - 1) \phi + \alpha_i] \right\}$$

$$= \left[\left(1 + \frac{1}{I} \right) - \frac{1}{I} \left(\frac{c}{b} \right)^2 \right] \cos (\phi - \alpha_i) - \frac{1}{q_1^+} \frac{c}{b} \frac{q''(c, \phi)}{q \frac{a^2}{2b}} \quad (7.22)$$

where the heat flux term $q''(c,\phi)$ is determined from the updated temperature field of the corner cell. Therefore, the updating scheme for the small internal cell is used to satisfy the condition of continuity of heat flux along the common boundary. A complete summary of the calculational procedure is given in Fig.7.2. The results of the calculation will be presented in the following sections.

7.3 RESULTS OF CALCULATION FOR A 7-ROD BUNDLE WITH CONDUCTION MIXING ONLY

Before the analysis is used for typical COBRA-IIIC design cases, it is first applied to a special case of a 7-rod bundle calculation with conduction mixing only. The purpose of this calculation is to compare the results of the coupled distributed-lumped parameter analysis with those obtained from a more sophisticated 3-D analysis performed by Wong [39] which is the only analytical results available for comparison.

The configuration of the geometry and subchannel layout of the 7-rod bundle being considered has already been shown in Fig.6.2. Other geometric and thermal parameters used in the COBRA-IIIC calculation are listed in Table 7.1 and coolant temperatures of the two subchannels at various axial positions are given in Table 7.2.

Fig.7.2 Calculational Scheme of the Coupled Distributed-Lumped Parameter Analysis

Table 7.1

Geometric and Thermal Parameters Used in COBRA-IIIC

Number of rods	= 7
Pitch-to-diameter ratio (P/D)	= 1.24
Dimensionless wall distance (2W/D)	= 1.30
Rod diameter (D)	= 0.25"
Core Height	= 36"
Inlet pressure	= 60 psia
Inlet temperature	= 700°F
Average mass velocity	= 3×10^6 lbm/ft ² hr
Average heat flux	= 0.3×10^6 BTU/ft ² hr
Turbulent mixing coefficient (β)	= 0
Normalized conduction mixing length	= 0.70
Uniform inlet temperature	
Uniform inlet mass Velocity	
No power tilting in the bundle	

Table 7. 2

Average Subchannel Temperatures at Various Axial Locations
Resulted from COBRA-IIIC Calculation

z (in)	T ₁ (°F)	T ₂ (°F)
0	700.00	700.00
2	715.12	710.37
4	730.12	720.80
6	745.00	731.30
8	759.82	741.85
10	774.55	752.46
12	789.18	763.17
14	803.73	773.93
16	818.22	784.74
18	832.61	795.59
20	846.94	806.51
22	861.21	817.49
24	875.40	828.51
26	889.54	839.56
28	903.61	850.70
30	917.61	861.86
32	931.57	837.07
34	945.46	884.31
36	959.30	895.60

In the present analysis, the coupled distributed-lumped parameter analysis is applied at the axial positions where $z=24"$ and $36"$. The results of calculations are shown in Figs. 7.3 and 7.4 with the clad outside temperature distributions of the corner cell plotted vs. the angular position ϕ for the axial positions of $z=24"$ and $36"$, respectively. For comparison purpose, the calculational results of the 3-D analysis of Wong are also sketched in the figures. It can be seen that the coupled distributed-lumped parameter analysis generally agrees reasonably well with the 3-D calculations except in the corner region of the cell (where $\phi < 30^\circ$). The maximum differences between the two analyses are approximately 20°F for both cases. This is due to the fact that the present layout of the subchannels cannot properly account for the local effect in the corner region. Considerable improvement can be achieved by subdividing the corner subchannel (No. 2) into two subchannels such that more detailed information of the coolant temperature in the corner region can be obtained.

In addition, a special case is performed at the entrance of the core where the coolant temperature is 700°F uniformly throughout the coolant region. Due to continuity of temperature, the clad surface temperature should also be constant in the azimuthal direction. Therefore, it can be expected that any circumferential variation in the clad temperature resulted from the coupled distributed-lumped parameter analysis is an indication of the error associated with the technique of

Fig.7.3 Clad Temperature Distribution of the Corner Rod at $z=24''$

Fig.7.4 Clad Temperature Distribution of the Corner Rod at $z=36''$

the analysis. The result of calculation is shown in Fig.7.5 with the clad outside temperature plotted vs. ϕ . It is indicated that the calculation fails to give uniform temperature distributions from both the internal and corner cells. As shown by Fig.7.5, the clad outside temperature variations of the small internal cell and the corner cell are approximately 2°F and 20°F , respectively. Furthermore, the clad temperature of both cells is generally higher than the expected value- 700°F . The reason for this is that the two subchannel temperatures are not sufficient to bring the 2-D fully-developed solution to a uniform temperature distribution at the inlet. One possible way to improve this is to introduce a correction term for the 2-D coolant temperature solution $\theta(\rho, \phi)$ such that for more uniform temperature can be obtained in the axial positions near the entrance. In the next section, the technique will be applied to 7-rod bundle design cases and the clad temperature distributions will be presented and investigated.

7.4 CLAD TEMPERATURE DISTRIBUTIONS FOR TYPICAL COBRA-IIIC Design CASES

After the procedure has been compared with the 3-D analysis, it is used in this section to predict the clad temperature distributions of typical 7-rod COBRA-IIIC design cases. The geometric parameters used are the same as those shown in

Fig.7.5 Clad Temperature Distributions at the Core Entrance

Table 7.1. The mixing coefficient has been set to be 0.01 and the conduction mixing length has been assumed to be equal to the subchannel centroid-to-centroid distance. Furthermore, the diversion crossflow is not suppressed in the present calculation. The inclusion of all these mixing mechanisms thus provide the most realistic subchannel temperatures which serve as input to the coupled distributed-lumped parameter analysis.

Three cases of different flow rates are performed and the subchannel temperatures at various axial positions resulted from COBRA-IIIC calculation given in Table 7.3. The coupled distributed-lumped parameter analysis is applied at the core exit ($z=36"$) where the clad temperature variation is the greatest. The results of calculation are presented in Figs. 7.6, 7.7 and 7.8 for $\bar{G}=1 \times 10^6$, 2×10^6 , and 3×10^6 lbm/ft²hr, respectively. It can be seen that the clad temperature variations of the small internal cell for all these cases are quite small (approximately 5°F for the worst case) and should not be of major concerns. On the other hand, the calculational results reveal that the clad temperature distribution of the corner rod for the $G=1 \times 10^6$ lbm/ft²hr case exhibits a maximum variation of 64°F. For higher mass fluxes, the corner rod clad temperature variation is reduced to 54°F and 37°F for the $G=2 \times 10^6$ and 3×10^6 lbm/ft²hr cases, respectively.

In addition, it would be interesting to compare the clad

Table 7.3

Subchannel Temperatures at Various Axial Locations for Different Flow Rates

z(in)	G = 1 X 10 ⁶ lbm/ft ² hr		G = 2 X 10 ⁶ lbm/ft ² hr		G = 3 X 10 ⁶ lbm/ft ² hr	
	$\bar{T}_1(^{\circ}\text{F})$	$\bar{T}_2(^{\circ}\text{F})$	$\bar{T}_1(^{\circ}\text{F})$	$\bar{T}_2(^{\circ}\text{F})$	$\bar{T}_1(^{\circ}\text{F})$	$\bar{T}_2(^{\circ}\text{F})$
0	700.00	700.00	700.00	700.00	700.00	700.00
2	747.76	731.53	723.98	715.72	716.02	710.47
4	791.80	764.72	746.27	732.11	730.98	721.35
6	833.62	799.14	767.55	749.00	745.24	732.55
8	874.10	834.51	788.11	766.37	759.05	744.00
10	913.71	870.57	808.17	784.03	772.53	755.68
12	952.75	907.16	827.91	801.94	785.76	767.54
14	991.45	944.12	847.41	820.08	798.79	779.53
16	1029.92	981.36	866.76	838.35	811.72	791.61
18	1068.24	1018.84	885.98	856.79	824.55	803.79
20	1106.44	1056.47	905.12	875.31	837.29	816.05
22	1144.62	1094.20	924.19	893.93	850.01	828.37
24	1182.72	1132.01	943.22	912.61	862.68	840.72
26	1220.79	1169.86	962.22	931.35	875.32	853.15
28	1258.84	1207.72	981.21	950.13	887.94	865.60
30	1296.87	1245.56	1000.22	968.94	900.56	878.06
32	1334.79	1283.41	1019.18	987.80	913.16	890.57
34	1372.76	1321.78	1038.15	1006.70	925.76	903.09
36	1410.58	1358.90	1057.14	1025.60	938.36	915.63

Fig. 7.6 Clad Temperature Distributions at the Core Exit for $G=1 \times 10^6 \text{ lbm/ft}^2 \text{ hr}$

Fig.7.7 Clad Temperature Distributions at the Core Exit for $G=2 \times 10^6$ lbm/ft² hr

Fig.7.8 Clad Temperature Distributions at the Core Exit for $G=3 \times 10^6 \text{ lbm/ft}^2 \text{ hr}$

temperature variation of the corner cell resulted from the present analysis which couples distributed parameter analysis and COBRA-IIIC design cases with that predicted by the fully-developed analysis given in chapter 4. From Fig.4.16 it can be seen that the maximum dimensionless circumferential clad temperature variation for the corner cell is approximately 1.3. By multiplying the normalized group $q''a^2/2k_c$ (83.3°F in the present case), a 108.29°F of clad temperature variation is obtained. Therefore, it becomes apparent that the coupled distributed-lumped parameter analysis has successfully reduced the magnitude of the clad temperature variation by almost a factor of 3 - a significant reduction of excess conservatism inherited from the 2-D analysis.

The coupled distributed-lumped parameter analysis has demonstrated its applicability for the simple 7-rod bundle. Estimate of clad temperature distributions of a larger bundle can be achieved by simply extending the present analysis. Such an extension is not performed in the present study. However, it is felt that an estimate of the clad temperature distribution of the corner cell of a 19-rod bundle can be obtained by using the coupled distributed-lumped parameter technique for the 7-rod bundle and the results of the COBRA-IIIC 19-rod bundle calculation.

In the present study, the 19-rod experimental apparatus of Möller and Tschöke is used to demonstrate this analysis such that the results of calculation can be compared with the measurements. The geometric configuration and the subchannel layout of the bundle is shown in Fig.7.9. The mixing parameters used in COBRA-IIIC calculations are the same as those used in the examples described earlier in this section. As shown in Fig.7.9, a 7-rod section has been isolated out from the 19-rod bundle. To order to obtain a reasonable estimate for the clad temperature of the corner rod, it is necessary to have some knowledge for the coolant temperature of the two shaded areas. Unfortunately, this is not known directly from the COBRA-IIIC calculation and approximations have to be made to obtain these values. For the present example, it has been assumed that the average temperature of the shaded area which consists of $\langle A_4 \rangle_3$ and half of $\langle A_2 \rangle_3$ is equal to $\bar{T}_3$, the coolant temperature of subchannel 3. This approximation should be reasonably valid as long as large power gradient does not exist in the bundle. On the other hand, it would be questionable to use $\bar{T}_4$ as the temperature of the shaded area $\langle A_4 \rangle_4$ because of the geometric irregularity of subchannel 4. One possible way to determine $\langle T_4 \rangle_4$ is to utilize the calculational results of the fully-developed temperature field of the 19-rod bundle obtained previously in chapter 5. In addition, the axial correction

— SUBCHANNEL BOUNDARY

Fig.7.9 Subchannel Layout of the 19-Rod Bundle

$F(z)$ has also been used to account for the entrance effect. It can be easily shown that the relation between $\langle T_4 \rangle_4$ and $\bar{T}_4$ is given by the following:

$$\frac{\langle T_4 \rangle_4 - \bar{T}_4}{\bar{q}'' a^2 / 2k_c} = \frac{\langle A_3 \rangle_4}{\langle A_3 \rangle_4 + \langle A_4 \rangle_4} \left\{ 2Y_{0,42} + F(z) \left[\langle \theta_4 \rangle_4 - \langle \theta_3 \rangle_4 \right] \right\} \quad (7.23)$$

Havign established the values of the coolant temperatures necessary for the analysis for the 7-rod section, the coupled distributed-lumped parameter technique is applied at the axial position of $z=25.94''$ and the result of calculation is shown in Fig.7.10 in which the clad temperature distribution of the corner cell is plotted vs. the angular position ϕ . For comparison, the clad temperature distribution of the corner cell measured by Möller and Tschöke is also presented. It can be seen that the clad temperature distribution (at $\rho=0.944$) predicted by the coupled matches the experimental data in the near-wall region fairly well but tends to over-estimate the temperature in the away-from-wall-region. In addition, the predicted clad temperature distribution has a maximum circumferential variation of approximately 50°F which is about 16°F higher than the measurement. The discrepancy between the calculated and the measured clad temperature distributions may be a result of the following reasons:

Fig.7.10 Clad Temperature Distribution of the Corner Rod for the Experimental 19-Rod Bundle

1. Uncertainty in the geometric and thermal parameters of the experimental apparatus.
2. The rounded corner of the experimental bundle (see Fig. 5.11) reduces the flow area in the region near the bundle wall and consequently drives the flow away from the near-wall region. The result of this is an increase in the temperature in the region away from the bundle wall as indicated by the experimental data.
3. Underestimation of the mixing parameters (conduction and turbulence) used in the subchannel calculation.

Among these three possibilities, it is felt that the mixing parameters probably has the greatest impact on the results of the calculation. It is because the input of the present coupled distributed-lumped parameter analysis requires the subchannel temperatures which strongly depend on the mixing parameters used in the subchannel calculation. For example, an increase in the turbulence mixing coefficient β would enhance intersubchannel mixing and lower the difference of subchannel temperature accordingly.

Therefore, it becomes apparent that the determination of a realistic clad temperature distribution relies on the validity of the subchannel calculation which cannot be achieved without proper knowledge of the various mixing parameters. Based on the present findings, it can be seen that the

Present analysis is only a first step effort to link the distributed and lumped parameter analyses and improvements in many areas can be made to extend and upgrade the analysis. This will be addressed in the next chapter which summarizes all major accomplishments in this thesis and makes recommendations for future studies.

CHAPTER 8

CONCLUSION AND RECOMMENDATIONS

8.1 CONCLUSION

The multicell analysis has already demonstrated its applicability for determination of clad temperature distributions for hexagonal rod bundles. Based on the available calculational results, the following conclusions can be made:

1). Circumferential Clad Temperature Variation

From the results of both the single and multi-cell calculation, it is found that both the edge and corner cells have severe problems with respect to the circumferential clad temperature variations. On the other hand, the small internal cell situated in the center of the bundle has a higher degree of symmetry with respect to the coolant channel geometry and does not impose a very serious problem. It has also been indicated that the wall effect does not only terminate at the first row next to the wall but propagates into the second and even third row of cells from the bundle wall. In addition, it has also been indicated that the clad temperature distributions are extremely sensitive to both the wall distance and the power densities in the rods. Parametric studies showed that there is always an optimal com-

ination of P/D and 2W/D ratios for both fuel and blanket assemblies. Another important finding revealed by the isolated and multicell analysis is that the single cell analysis does not necessarily yield the conservative clad temperature distribution. It was indicated in some cases that the coupling of the internal cell to corner cell actually increased the clad temperature variation of the corner cell. The implication of this finding is the necessity of the multicell analysis.

2). Axial Correction Factor $F(z)$

Upon comparing the clad temperature distribution predicted for the fully-developed condition with the experimental data of Möller and Tschöke, it has been shown that the fully-developed analysis is truly too conservative. An axial correction factor $F(z)$ has been derived to account for the entrance effect. It was found that the magnitude of the clad temperature variation at the core exit is only approximately 1/3 of that predicted by the fully-developed analysis.

3). Effective Mixing Lengths for Subchannel Calculations

The determination of the effective mixing lengths between subchannels is of prime importance for the accurate prediction of the bulk temperature differences between adjacent subchannels in subchannel calculations. From the results of the calculation,

it has been found that the effective mixing length is a strong function of the geometric parameters. Furthermore, it has also been shown that the effective mixing lengths are, in general, not equal to the geometric centroid-to-centroid distance between adjacent subchannels. In addition, COBRA-IIIC calculation results have indicated that the impact of the implementation of the correct effective mixing lengths on COBRA-IIIC calculations is fairly significant.

4). Coupled Distributed-Lumped parameter Analysis

The coupled distributed-lumped parameter analysis presented in the present study has successfully linked the results of the COBRA-IIIC with the distributed parameter analysis to give a more realistic estimate of the clad temperature distributions for normal operating conditions. At the present stage, the analysis is still a fairly crude approach and is only limited to a relatively simple 7-rod bundle geometry. In addition, the present analysis is not capable of giving satisfactory results in the entrance region where the temperature is relatively uniform. Despite these shortcomings, the analysis represents a first step toward the full integration of the subchannel lumped parameter and the distributed parameter analyses.

8.2 Recommendations

Recommendations for future work can be summarized as follows:

1. Introduction of a wire model to account for the effect caused by the presence of the wire-wrap in typical LMFBR rod bundles.
2. Extending the present multicell analysis to larger bundles such that the temperature field of the entire blanket and fuel bundles can be analyzed.
3. Extending the present analysis to a wider range of geometric and thermal parameters.
4. Developing a more generalized correlating function for the effective mixing lengths such that the effect of both the pitch-to-diameter ratio and the wall distance can be included in a single formula
5. Extending the present coupled distributed-lumped parameter analysis to 19, 37 or even larger bundles.
6. Introducing some form of axial correction to the solution form of the coupled distributed-lumped parameter analysis such that the analysis can yield more satisfactory results in the entrance region.
7. Fully integrating the distributed and lumped parameter analyses by including the coupled distributed-lumped analysis in subchannel codes as an option.

In addition, the multicell coupling technique can be further extended to problems of other flow regimes such as laminar and turbulent flows. Naturally, this implies that solutions for the momentum equations have to be obtained and additional information of the eddy diffusivities must be acquired for the turbulent flow.

APPENDIX A

LEAST SQUARE METHOD

The discretization of a boundary or initial value problem often results in the following matrix equation:

$$\underline{A}\underline{X} = \underline{Y} \quad (\text{A.1})$$

where $\underline{A}$ - the coefficient matrix,

$\underline{Y}$ - the given function vector at the boundary of the problem

and $\underline{X}$ - the unknown coefficient vector

Eq. (A.1) can also be in matrix form:

$$\begin{bmatrix} a_{11} & \cdot & \cdot & \cdot & \cdot & a_{1N} \\ a_{21} & \cdot & \cdot & \cdot & \cdot & a_{2N} \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ \cdot & & & & & \cdot \\ a_{M1} & \cdot & \cdot & \cdot & \cdot & a_{MN} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_N \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_M \end{bmatrix} \quad (\text{A.2})$$

In general, the dimensions of the vectors $\underline{X}$ and $\underline{Y}$ are not equal and for an over-determined linear system, the number of equations is greater than that of the unknown coefficient. As a result, a unique set of $\underline{X}$ cannot be determined. In order to obtain

a consistent set of $\underline{X}$, one introduces a residual to Eq.(A.1) such that it can be written as:

$$\underline{AX} = \underline{Y} + \underline{r} \quad (\text{A.3})$$

Thus, the i th component of the residual vector is given by the relation:

$$r_i = Y_i - \sum_j^N a_{ij} X_j \quad (\text{A.4})$$

and the sum of the square of the residuals of the all components is simply the following:

$$S = \sum_i^M r_i^2 = \sum_i^M (Y_i - \sum_j^N a_{ij} X_j)^2 \quad (\text{A.5})$$

The least square technique calls for the minimization of the S ; that is:

$$\frac{\partial S}{\partial X_j} = \sum_i^M 2(Y_i - \sum_j^N a_{ij} X_j) a_{ij} = 0 \quad (\text{A.6})$$

or equivalently:

$$\begin{aligned} \sum_j^M a_{ij} Y_i &= \sum_i^M a_{ij} \sum_j^N a_{ij} X_j \\ &= \sum_i^M a_{ij} \sum_k^N a_{ik} X_k \end{aligned} \quad (A.7)$$

Interchanging subscripts i and j of Eq.(A.7) leads to:

$$\sum_j^M a_{ji} Y_j = \sum_j^M a_{ji} \sum_k^N a_{jk} X_k \quad (A.8)$$

Eq.(A.8) can be written in the matrix notation as:

$$A^T \underline{Y} = A^T A \underline{X} \quad (A.9)$$

In the presently study, the unknown coefficient vector X is solved by using the Gauss Elimination Technique.

APPENDIX B
GAUSSIAN QUADRATURE
(GAUSS-LEGENDRE QUADRATURE)

In order to estimate the value of the integral $\int_a^b f(x) dx$, one can approximate the function $f(x)$ with an n th-degree interpolating polynomial $P_n(x)$ and integrate as follows:

$$\int_a^b f(x) dx = \int_a^b P_n(x) dx + \int_a^b R_n(x) dx \quad (B.1)$$

Here, $R_n(x)$ is the error term for the n th- degree interpolating polynomial. If the Lagrangian form of the interpolating polynomial is used, it becomes:

$$\begin{aligned} f(x) &= P_n(x) + R_n(x) \\ &= \sum_{i=1}^n L_i(x) f(x_i) + \prod_{i=0}^n (x-x_i) \frac{f^{(n+1)}(\xi)}{(n+1)!} \\ &\quad a < \xi < b \end{aligned} \quad (B.2)$$

where

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \left(\frac{x-x_j}{x_i-x_j} \right)$$

To simplify the problem, the interval of integration will be changed from $[a,b]$ to $[-1,1]$ by a suitable transformation of variables. Then the new variable z will be defined by

$$z = \frac{2x - (a+b)}{b-a} \quad (\text{B.3})$$

Similarly, the new function will be defined as

$$F(z) = f(x) = f\left[\frac{(b-a)z + (a+b)}{2}\right] \quad (\text{B.4})$$

As a result, Eq. (B.2) becomes

$$F(z) = \sum_{i=0}^n L_i(z) F(z_i) + \left[\sum_{i=0}^n (z-z_i) \right] \frac{F^{(n+1)}(\bar{\xi})}{(n+1)} \quad (\text{B.5})$$

where

$$L_i(z) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{(z-z_j)}{(z_i-z_j)} \quad \text{and} \quad -1 < \bar{\xi} < 1.$$

Here, z_i is simply the base-point value x_i transformed by Eq. (B.3). Now, if $f(x)$ is assumed to be a polynomial of degree $2n+1$, then the term $F^{(n+1)}(\bar{\xi})/(n+1)!$ must be a polynomial of degree n , since $\sum_{i=0}^n L_i(z) F(z_i)$ is a polynomial of degree n at most, and $\prod_{i=1}^n (z-z_i)$ is a polynomial of degree $n+1$. Let

$$\frac{F^{(n+1)}(\bar{\xi})}{(n+1)!} = q_n(z) \quad (\text{B.6})$$

where $q_n(z)$ is a polynomial of degree n . Then

$$F(z) = \sum_{i=1}^n L_i(z) F(z_i) + \left[\sum_{i=0}^n (z-z_i) \right] q_i(z) \quad (\text{B.7})$$

By integrating both sides of Eq. (B.7) between the integration limits -1 and 1 , it follows that

$$\begin{aligned} \int_{-1}^1 F(z) dz &= \int_{-1}^1 \sum_{i=0}^n L_i(z) F(z_i) dz \\ &+ \int_{-1}^1 \left[\prod_{i=0}^n (z-z_i) \right] q_n(z) dz \end{aligned} \quad (\text{B.8})$$

Since the $F(z_i)$ are fixed values, the integral can be simplified as

$$\begin{aligned} \int_{-1}^1 F(z) dz &= \sum_{i=0}^n F(z_i) \int_{-1}^1 L_i(z) dz \\ &= \sum_{i=0}^n W_i F(z_i) \end{aligned} \quad (\text{B.9})$$

where

$$W_i = \int_{-1}^1 L_i(z) dz = \int_{-1}^1 \prod_{\substack{j=0 \\ j \neq i}}^n \left[\frac{z-z_j}{z_i-z_j} \right] dz. \quad (\text{B.10})$$

One has to select z_i in such a way that the second integral on the right hand side of Eq. (B.8) , i.e.,

$$\int_{-1}^1 \prod_{i=0}^n (z-z_i) q_n(z) dz \quad (B.11)$$

vanishes in order to have a perfectly working procedure, i.e., Eq. (B.9) . The orthogonality property of the Legendre polynomials will be used to establish such values as z_i . By expanding the two polynomials $q_n(z)$ and $\prod_{i=0}^n (z-z_i)$ in terms of the Legendre polynomials, one obtains:

$$\begin{aligned} \prod_{i=0}^n (z-z_i) &= b_0 P_0(z) + b_1 P_1(z) + \dots + b_n P_n(z) + b_{n+1} P_{n+1}(z) \\ &= \sum_{i=0}^{n+1} b_i P_i(z) \end{aligned} \quad (B.12)$$

$$\text{and } q_n(z) = c_0 P_0(z) + c_1 P_1(z) + \dots + c_n P_n(z)$$

$$= \sum_{i=0}^n c_i P_i(z) \quad (B.13)$$

The product $q_n(z) \cdot \prod_{i=0}^n (z-z_i)$ becomes

$$\sum_{i=0}^n \sum_{j=0}^n b_i c_j P_i(z) P_j(z) + b_{n+1} \sum_{i=0}^n c_i P_i(z) P_{n+1}(z). \quad (B.14)$$

The integral of Eq. (B.11) will assume the form

$$\int_{-1}^1 \left[\sum_{i=0}^n \sum_{j=0}^n b_i c_j P_i(z) P_j(z) + b_{n+1} \sum_{i=0}^n c_i P_i(z) P_{n+1}(z) \right] dz. \quad (\text{B.15})$$

Because of the orthogonality properties of the Legendre polynomials, all terms of this integral that are of the form

$$b_i c_j \int_{-1}^1 P_i(z) P_j(z) dz \quad i \neq j \quad (\text{B.16})$$

will vanish. Thus the error term (Eq. B.11) for the quadrature formula of Eq. (B.9) may be written:

$$\begin{aligned} \int_{-1}^1 \left[\prod_{i=0}^n (z - z_i) \right] q_n(z) dz &= \int_{-1}^1 \sum_{i=0}^n b_i c_i [P_i(z)]^2 dz \\ &= \sum_{i=0}^n b_i c_i \int_{-1}^1 [P_i(z)]^2 dz. \quad (\text{B.17}) \end{aligned}$$

One way to make this expression vanish is to specify that the first $n+1$ of the b_i , $i=0, 1, \dots, n$ are zero. The coefficient b_{n+1} of $P_{n+1}(z)$ is still unspecified, but from Eq. (B.13) it must be given by :

$$\prod_{i=1}^n (z - z_i) = b_{n+1} P_{n+1}(z) \quad (\text{B.18})$$

The important feature of this equation is that the polynomial

$\prod_{i=0}^n (z-z_i)$ is already in factored form, that is, it has the $n+1$ roots z_i , $i=0,1,2,\dots,n$. Since $b_{n+1}P_{n+1}(z)$ is the same polynomial, the z must be the roots of $b_{n+1}P_{n+1}(z)$ as well, or equivalently, of $P_{n+1}(z)$. Thus the $n+1$ base points to be used in the integration formula of Eq.(B.9) are the $n+1$ roots of the appropriate $(n+1)$ th-degree Legendre polynomial. The relative weight assigned to each functional value $F(z_i)$ is given by Eq.(B.10). Values of the appropriate base points (roots) and weight factors are shown in Table (B.1).

Table B.1

Roots of the Legendre Polynomials $P_{n+1}(z)$ and the Weight Factors for the Gauss-Legendre Quadrature

Roots(z_i)	$\int_{-1}^1 F(z) dz = \sum_{i=0}^n w_i F(z_i)$	Weight Factors(w_i)
two-point formula $n=1$		
$\pm 0.57735 \ 02691 \ 89626$		1.00000 00000 00000
three-point formula $n=2$		
0.00000 00000 00000		0.88888 88888 88889
$\pm 0.77459 \ 66692 \ 41483$		0.55555 55555 55556
four-point formula $n=3$		
$\pm 0.33998 \ 10435 \ 84856$		0.65214 51548 62546
$\pm 0.86113 \ 63115 \ 94053$		0.34785 48451 37454
five-point formula $n=4$		
0.00000 00000 00000		0.56888 88888 88889
$\pm 0.53846 \ 93101 \ 05683$		0.47862 86704 99366
$\pm 0.90617 \ 98459 \ 38664$		0.23692 68850 56189
six-point formula $n=5$		
$\pm 0.23861 \ 91860 \ 83197$		0.46791 39345 72691
$\pm 0.66120 \ 93864 \ 66265$		0.36076 15730 48136
$\pm 0.93246 \ 95142 \ 03152$		0.17132 44923 79170
ten-point formula $n=9$		
$\pm 0.14887 \ 43389 \ 81631$		0.29552 42247 14753
$\pm 0.43339 \ 53941 \ 29247$		0.26926 67193 09996
$\pm 0.67940 \ 95682 \ 99024$		0.21908 63625 15982
$\pm 0.86506 \ 33666 \ 88985$		0.14945 13491 50581
$\pm 0.97390 \ 65285 \ 17172$		0.06667 13443 08688

APPDENDIX C LØCAL CODE

C.1 INTRODUCTION

The LØCAL Code was written by the author to calculate the temperature field of unit cells in a finite hexagonal rod bundle. The code carries out the numerical computations which has been discussed in Chapters 4 and 5 of this thesis.

In Section C.3, a listing of a typical 19-rod calculation is presented as specified in Section C.2. The output for the run is shown in Section C.4.

C.2 INPUT DESCRIPTION OF THE LOCAL CODE

<u>Card No.</u>	<u>Column</u>	<u>Symbol</u>	<u>Description</u>	<u>Format</u>
1	1-5	NI	Maximum No. of points	I5
1	5-10	NK	Maximum No. of terms	I5
1	11-15	MMA	Control option	I5
			1 - 7-rod bundle	
			2 - 19-rod bundle	
			3 - 37-rod bundle	
			4 - 61-rod bundle	
			5 - 127-rod bundle	
1	16-20	IT	Maximum No. of iterations	I5
1	21-25	NT	Maximum No. of angular intervals to be printed	I5
2	1-10	PD	Pitch-to-Diameter ratio	E10.4
2	11-20	T	Clad inside to outside diameter ratio	E10.4
2	21-30	VE	Dimensionless wall distance	E10.4
2	31-40	DAW	Fuel to clad thermal conductivity ratio	E10.4
2	41-50	DA	Clad to coolant thermal conductivity ratio	E10.4
2	51-60	BIS	Biot number at the gap	E10.4
2	61-70	BI	Biot number at the clad /coolant interface	E10.4
3-4	1-80	Z(I)	Roots of the Legendre Polynomials	E10.4
5-6	1-80	W(I)	Weights of the Gauss-Legendre quadrature	E10.4
7	1-80	Q(K)	Power peaking Factors	E10.4

LOCALCR YEUNG MR 07/07/78 09.20.59 7600 C
 PROGRAM LOCAL 76/76 OPT=1

FTN 4.6+439

07/07/78 09.21.00

PAGE 1

C.3 CODE LISTING

```

1      PROGRAM LOCAL (INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)
      *      *      *      *      *      *      *      *
      *
      *      LOCAL IS A COMPUTER CODE WRITTEN FOR MULTI-
      *      CELL ANALYSIS OF LMFBR ROD BUNDLES
      *
10     INPUT PARAMETERS
      *
      NI      - MAX NUMBER OF POINTS
      NK      - MAX NUMBER OF TERMS
15     MMAX   - CONTROL OPTION
      *          1 7-ROD BUNDLE ANALYSIS
      *          2 19-ROD BUNDLE ANALYSIS
      *          3 37-ROD BUNDLE ANALYSIS
      *          4 61-ROD BUNDLE ANALYSIS
20     *          5 127-ROD BUNDLE ANALYSIS
      *
      IT      - MAX NUMBER OF ITERATION
      NT      - NUMBER OF ANGULAR INTERVALS TO BE PRINTED
      PC      - PITCH TO DIAMETER RATIO
25     T      - CLAD INSIDE TO OUTSIDE DIAMETER RATIO
      VE      - DIMENSIONLESS WALL DISTANCE
      OAM     - FUEL TO CLAD CONDUCTIVITY RATIO
      DA      - CLAD TO COOLANT CONDUCTIVITY RATIO
      BIS     - BIOT NUMBER AT THE GAP
30     BI      - BIOT NUMBER AT THE CLAD/COOLANT INTERFACE
      Z(I)    - ROOTS OF THE LENGENDRE POLYNOMIALS
      W(I)    - WEIGHTS OF THE GAUSS-LEGENDRE QUADRATURE
      Q(K)    - POWER PEAKING FACTOR OF RODS
      *      *      *      *      *      *      *      *
35     *
      *      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      *      COMMON /GUN1/ M(10),Z(10)
      *      COMMON /GUN2/ DAM,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
      *      COMMON /GUN3/ Q(10),MT(10)
      *      DIMENSION A(5000)
      *      PI=3.1415926536
      *      EPS=1.00-12
40     READ (5,101) NI,NK,MMAX,IT,NT
      *      READ (5,100) PD,T,VE,OAM,DA,BIS,BI
      *      READ (5,100) (Z(I),I=1,10)
      *      READ (5,100) (W(I),I=1,10)
      *      GO TO (1,2,3,4,5),MMAX
50     1 KK=2
      *      GO TO 10
      *      2 KK=4
      *      GO TO 10
      *      3 KK=6
      *      GO TO 10
55     4 KK=9
      *      GO TO 10
  
```

```

5 KK=12
10 CONTINUE
60 READ (5,100) (Q(K),K=1,KK)
100 FORMAT (8E10.4)
101 FORMAT (16I5)
    MZ=(NK*(NK+1))/2
    PK=NK-1
65 ROD=1.0
    DO 7 K=1,MMAX
7 ROD=ROD+6.*K
    AF=(MMAX*PD*1.732/2.+VE/2.)*(MMAX*PD+VE/1.732)-ROD*PI/12.
    YT=AF/(ROD*PI/12.)
78 CH=AF/(ROD*PI/12.+(MMAX*PD+VE/1.732)/2.)
    JPHI=10
    JYJ=JPHI+NI
    JGJN=JYJ+NI
    JR=JGJN+NI*NK
75 JGTG=JR+NK
    JAUX=JGTG+MZ
    JDY=JAUX+MK
    JR1=JDY+KK*NK
    JVP=JR1+MK
80 JGP=JVP+NI
    JX=JGP+NI*NK
    JF=JX+NK
    JG=JF+NK
    JGF=JG+NK
85 JDG=JGF+KK*NK
    JB=JCG+KK*NK
    CALL COUPLE (MMAX,NI,NK,MZ,MK,KK,A(JPHI),A(JYJ),A(JGJN),A(JR),A(JG
    17G),A(JAUX),A(JDY),A(JR1),A(JVP),A(JGP),A(JX),A(JF),A(JG),A(JGF),A
    1(JCG),IT,NT)
90 END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS

46 LOCAL

VARIABLES	SN	TYPE	RELOCATION			
461 A		DOUBLE	ARRAY	437 AF	DOUBLE	
14 01		DOUBLE	BUN2	12 BIS	DOUBLE	BUN2
2 0A		DOUBLE	BUN2	0 DAW	DOUBLE	BUN2
22 0H		DOUBLE	BUN2	24 EPS	DOUBLE	BUN2
430 I		INTEGER		426 IT	INTEGER	
446 JAUX		INTEGER		460 JB	INTEGER	
456 JDF		INTEGER		457 JDG	INTEGER	
447 JDY		INTEGER		454 JF	INTEGER	
455 JG		INTEGER		443 JGJN	INTEGER	
452 JGP		INTEGER		445 JGTG	INTEGER	
441 JPHI		INTEGER		444 JR	INTEGER	
450 JR1		INTEGER		453 JX	INTEGER	
442 JYJ		INTEGER		451 JVP	INTEGER	

PROGRAM LOCAL

76/76 OPT=1

FTN 4.6+439

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PAGE 3

VARIABLES

SN TYPE

RELOCATION

432 K INTEGER
 434 MK INTEGER
 24 NT INTEGER
 423 NI INTEGER
 427 NT INTEGER
 16 PI DOUBLE
 435 ROD DOUBLE
 10 VE DOUBLE
 20 VT DOUBLE

ARRAY BUN3

BUN2

BUN2

BUN2

431 KK INTEGER
 425 HMAX INTEGER
 433 HZ INTEGER
 424 NK INTEGER
 4 PD DOUBLE
 0 Q DOUBLE
 6 T DOUBLE
 0 W DOUBLE
 24 Z DOUBLE

INTEGER
 INTEGER
 INTEGER
 INTEGER
 DOUBLE
 DOUBLE
 DOUBLE
 DOUBLE
 DOUBLE

ARRAY

ARRAY

ARRAY

BUN2

BUN3

BUN2

BUN1

BUN1

FILE NAMES

MODE

0 INPUT

20 OUTPUT

0 TAPES

FMT

20 TAPE6

EXTERNALS

TYPE

ARGS

COUPLE

23

STATEMENT LABELS

75 1
 103 4
 106 10

77 2

105 5

405 100

FMT

101 3

0 7

407 101

FMT

LOOPS LABEL

INDEX

FROM-TO

LENGTH

PROPERTIES

124 7

K

66 67

78

INSTACK

COMMON BLOCKS

LENGTH

BUN1

40

BUN2

22

BUN3

30

STATISTICS

PROGRAM LENGTH

240418

10273

BUFFER LENGTH

408

32

SCM LABELED COMMON LENGTH

1348

92

1 SUBROUTINE COUPLE (HMAX,NI,NK,MZ,MK,KK,PHI,VJ,GJN,R,GTG,AUX,DYN,R1
1,VP,GP,X,FN,GN,DFN,DGN,IT,NT)

C
C
C
C
C

5 SUBROUTINE COUPLE CALLS ALL SUBROUTINES AND COUPLES ALL
UNIT CELLS TOGETHER THROUGH VARIOUS BOUNDARY CONDITIONS

10 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /BUN1/ M(10),Z(10)
COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
COMMON /BUN3/ Q(10),HT(10)
DIMENSION PHI(NI),VJ(NI),GJN(NI,NK),R(NK),GTG(MZ),AUX(MK),DYN(KK,N
1K),R1(NK),X(NK),VP(NI),GP(NI,NK),FN(NK),GN(NK),DFN(KK,NK),DGN(KK,N
15 2K)
DIMENSION AI(8),AL(8),AB(8),AT(8),AL1(8),AB1(8),AP(8),MJ(10)

C
C
C

20 WRITE (6,100)
GO TO (1010,1020,1030,1040,1050),HMAX
1010 WRITE (6,101)
GO TO 1060
1020 WRITE (6,102)
GO TO 1060
25 1030 WRITE (6,103)
GO TO 1060
1040 WRITE (6,104)
GO TO 1060
30 1050 WRITE (6,105)
1060 WRITE (6,106) NI,NK,IT
WRITE (6,111) PD,T,VE
WRITE (6,112) DAW,DA
WRITE (6,113) BIS,BI
35 WRITE (6,114)
WRITE (6,273) Z
WRITE (6,115)
WRITE (6,273) M
100 FORMAT (1H1,' LISTING OF INPUT PARAMETERS ',/)
101 FORMAT (' 7-ROD ANALYSIS ',/)
102 FORMAT (' 19-ROD ANALYSIS ',/)
103 FORMAT (' 37-ROD ANALYSIS ',/)
104 FORMAT (' 37-ROD ANALYSIS ',/)
105 FORMAT (' 127-ROD ANALYSIS ',/)
40 106 FORMAT (' MAX NO OF POINTS= ',I5,5X,' MAX NO OF TERMS = ',I5,5X,'
1 NC CF ITERATION = ',I5,5X,/)
111 FORMAT (' P/D = ',F12.5,5X,' A/B = ',F12.5,5X,' 2W/D = ',F12.5,/)
112 FORMAT (' KF/K = ',F12.5,5X,' K/KC = ',F12.5,/)
113 FORMAT (' BIS = ',E12.5,5X,' BI = ',E12.5,/)
50 114 FORMAT (' ROOTS OF LEGENDRE POLYNOMIALS ',/)
115 FORMAT ('/, ' WEIGHTS OF GAUSS-LEGENDRE ORADRATURE ',/)

C
C
C
C
C

55 START OF ITERATION

DO 1000 KI=1,IT


```

      WRITE (6,121) KI
121  FORMAT (1H1,* ITERATION *,I5,/)
      MM=MMAX
2000  NN=MM
      C
      C      CELL IDENTIFICATION
      C
65    3000  CALL TYPE (MM,NN,MMAX,K,IK,CF,XC,YC,AX)
      CALL CHAN (K,IK,IS,AI,AL,AB,U)
      CALL POTN (IK,IS,MJ,NP,LF)
      NF=LF
      CALL NTB(MMAX,IK,JB)
70    IF (KI.GT.1) NF=LF-JB
      IF (IK.GE.7) NF=NF/2
      MT(K)=NF
      YNT=YT*Q(K)/U
      IF (KI-1) 7,7,8
75    7 DO 9 N=1,NK
      A=CF*N
      CFN(K,N)=(1.+A/BI-DA)*(1.+DAM+A/BIS)+(1.-DA-A/BI)*(1.-DAM+A/BIS)*
      1T*(2.*A)/4.
      9 DGN(K,N)=(1.+A/BI+DA)*(1.+DAM+A/BIS)+(1.-DA-A/BI)*(1.-DAM+A/BIS)*
      1T*(2.*A)/4.
80    8 CO 10 N=1,NK
      FN(N)=DFN(K,N)
      10 GN(N)=DGN(K,N)
      WRITE (6,122) K,IK,Q(K),U
85    122  FORMAT (1///,* CELL *,I4,5X,* TYPE *,I5,5X,* POWER = *,F10.4,5X,* F
      1LOW = *,F10.4,/)
      C
      C      DETERMINATION OF COLLOCATION POINTS
      C
90    CALL POINTS (IS,NP,MJ,AI,AL,AB,PHI)
      JJ=0
      IC=1
      DO 4800 I=1,IS
      JM=MJ(I)
95    C
      C      IDENTIFYING NEIGHBORING CELL AND BOUNDARY CONDITION
      C
      CALL NEIGH (MMAX,MM,NN,IK,I,M1,N1)
      IF (M1-MM) 300,320,310
100   300  IB=1
      GO TO 350
      310  IB=2
      GO TO 350
      320  IF (N1-NN) 330,330,340
105   330  IB=1
      GO TO 350
      340  IB=2
110   350  IF ((M1.EQ.-1).AND.(N1.EQ.-1)) IB=0
      IF (M1.EQ.1) IB=0
      IF (IB.EQ.0) GO TO 63
      CALL TYPE (M1,N1,MMAX,K1,IK1,CF1,X1,Y1,A1)
      CALL CHAN (K1,IK1,IS1,AI1,AL1,AB1,U1)
      Y1=YT*Q(K1)/U1
      NF1=MT(K1)

```

```

115      DO 22 N=1,NF
          R(N)=OYN(K,N)
          IF (IK-7) 22,210,210
210      R(NF+N)=OYN(K,NF+N)
          22 CONTINUE
120      DO 74 N=1,NF1
          R1(N)=OYN(K1,N)
          IF (IK1-7) 74,76,76
          76 R1(NF1+N)=OYN(K1,NF1+N)
          74 CONTINUE
125      C
          C      CONSTRUCTION OF THE COEFFICIENT MATRIX
          C
          63 DO 61 J=1,JH
              JP=J+JJ
              RM=A1(I)/DCOS(PHI(JP)-AL(I))
              IF (IB-1) 60,70,70
          70      XX=XC+RM*DCOS(PHI(JP)+AX)
              YY=YC+RM*DSIN(PHI(JP)+AX)
              RM1=DSORT((XX-X1)**2+(YY-Y1)**2)
              PH1=-1.*A1/DATAN((YY-Y1)/(XX-X1))
              IF ((XX-X1).LE.0.0) PH1=-1.*A1+PI+DATAN((YY-Y1)/(XX-X1))
              IF (IB-1) 73,73,75
          135      C
              C      COUPLING BY USING CONTINUITY OF TEMPERATURE
              C
          140      73 CALL TEMP (3,IK,CF,NF,R,YNT,RM,PHI(JP),TI)
              CALL TEMP (3,IK1,CF1,NF1,R1,YT1,RM1,PH1,T1)
              VJ(JP)=.5*(TT*Q(K)+T1*Q(K1))-Q(K)*((RM**2-1.)/(2.*YNT)-(1./YNT+1.))
              1*DOLOG(RM)+(DOLOG(T)-1./BI)/DA-(1./BIS)/(DA*DAW)
          145      IF (JE.EQ.0) GO TO 34
              CO 36 NB=1,J9
              IF (NB.EQ.1C) GO TO 31
              GJN(JP,N)=0.0
              GO TO 36
          150      31 GJN(JP,NB)=1.0
              36 CONTINUE
              34 CO 62 N=1,NF
              A=CF*N
          155      62 GJN(JP,N+JB)=(FN(N)*RH**(-1.*A)+GN(N)*RH**A)*DCOS(A*PHI(JP))*Q(K)/
              1A
              IF (IK.LT.7) GO TO 61
              DO 27 N=1,NF
              A=CF*N
          160      27 GJN(JP,NF+JB+N)=(FN(N)*RH**(-1.*A)+GN(N)*RH**A)*DSIN(A*PHI(JP))*Q
              1K1/A
              GO TO 61
          75 I1=1
          82 IF (PH1-AB1(I1+1)) 80,80,81
          81 I1=I1+1
          165      GO TO 82
          C
          C      COUPLING BY USING CONTINUITY OF HEAT FLUX
          C
          80 CALL FLUX (3,IK1,CF1,NF1,R1,YT1,AL1(I1),RM1,PH1,Q2N)
          Q2N=C2N*Q(K1)/Q(K)
          170      GO TO 77

```

```

60 Q2N=0.0
77 YJ(JP)=((1.+1./YNT)-(RH**2)/YNT)*DCOS(PHI(JP)-AL(I))+RH*Q2N
IF (KI-1) 52,52,54
175 52 DO 29 N=1,NF
29 GJN(JP,N)=GN(N)*RH**((CF*N)*DCOS((CF*N-1.)*PHI(JP)+AL(I))-FN(N)*RH*
1*(-1.*CF*N)*DCOS((CF*N+1.)*PHI(JP)-AL(I))
IF (IK.LT.7) GO TO 61
DO 28 N=1,NF
180 A=CF*N
28 GJN(JP,NF+N)=FN(N)*RH**((-1.*A)*DSIN((A-1.)*PHI(JP)+AL(I))-GN(N)*RH
1**A*DSIN((A+1.)*PHI(JP)-AL(I))
GO TO 61
54 CO 32 NB=1,JB
185 32 GJN(JP,NB)=0.0
DO 72 N=1,NF
72 GJN(JP,N+JB)=GN(N)*RH**((CF*N)*DCOS((CF*N-1.)*PHI(JP)+AL(I))-FN(N)*
1RH**((-1.*CF*N)*DCOS((CF*N+1.)*PHI(JP)-AL(I))
IF (IK.LT.7) GO TO 61
190 DO 19 N=1,NF
A=CF*N
19 GJN(JP,NF+JB+N)=FN(N)*RH**((-1.*A)*DSIN((A-1.)*PHI(JP)+AL(I))-GN(N)
1*RH**A*DSIN((A+1.)*PHI(JP)-AL(I))
195 61 CONTINUE
IF (IO.EQ.1) IC=IC+1
400 JJ=JJ+MJ(I)
C
C
C SOLVING THE UNKNOWN VECTOR
200 CALL SIZE (NI,NK,MK,NZ,NP,LF,GJN,YJ,GP,YP,GTG,AUX,EPS,R)
WRITE (6,401)
401 FORMAT (/, ' COEFFICIENTS FOR THE FOURIER SERIES ',/)
WRITE (6,273) (R(N),N=1,LF)
DO 21 N=1,NF
205 QYN(K,N)=R(N+JB)
X(N)=R(N+JB)
IF (IK-7) 21,23,23
23 DYN(K,NF+N)=R(JB+NF+N)
X(NF+N)=R(JB+NF+N)
210 21 CONTINUE
C
C
C DETERMINATION OF NUSSELT NO
215 CALL TAVG (1,IS,AD,IK,NF,FN,GN,X,AI,AL,CF,YNT,T3,FA)
OU=2./(DLOG(T)/DA-1.)/(DA*DA*8IS)-T3)
WRITE (6,51) T3,FA,OU
51 FORMAT (/, ' T MEAN = ',F12.5,5X, ' AREA = ',F12.5,5X, ' NU = ',F12.5)
C
C
C AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX
220 WRITE (6,650)
650 FORMAT (/, ' AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX ')
WRITE (6,651)
651 FORMAT (//,5X, ' DEGREES ',8X, ' C/B ',5X, ' TCLAD IN ',5X, ' OCLAD IN
1 ',5X, ' TCLAD OUT ',5X, ' OCLAD OUT ',5X, ' TIC ',5X, ' OIC ',/,)
I=1
DO 65 J=1,NT
225 PH=(J-1)*(AB(IS+1)-AB(1))/(INT-1)

```

```

230      IF (PH.GT.AB(I+1)) I=I+1
      RA=AI(I)/DCOS(PH-AL(I))
      FS=T
      CALL TEMP (2,IK,CF,NF,X,YNT,FS,PH,TV)
      CALL TEMP (2,IK,CF,NF,X,YNT,1.00,PH,TC2)
      CALL TEMP (3,IK,CF,NF,X,YNT,RA,PH,TK2)
235      CALL FLUX (1,IK,CF,NF,X,YNT,AL(I),FS,PH,Q1)
      CALL FLUX (2,IK,CF,NF,X,YNT,AL(I),1.00,PH,Q2)
      CALL FLUX (3,IK,CF,NF,X,YNT,AL(I),RA,PH,Q3)
      DG=PH*180./PI
65      WRITE (6,117) DG,RA,TV,Q1,TC2,Q2,TK2,Q3
240      WRITE (6,700)
700      FORMAT (/, ' CALCULATIONAL RESULTS FOR LUMPED PARAMETER ',/)
      DO 15 I=1,IS
C
C      CALCULATION OF HEAT TRANSFER AT THE GAP
245      SGAP=0.0
      DO 40 IZ=1,10
      PP=(2*IZ)*(AI(IZ)-1.)+(AI(IZ)+1.)/2.
      SUMQ=0.0
      DO 42 N=1,NF
      A=CF*N
      FG=FN(N)*PP*(-1.*A)+GN(N)*PP*A
      SUMQ=SUMQ+X(N)*FG*DSIN(A*AL(IZ))
      IF (IK-7) 42,44,44
255      44 SUMQ=SUMQ-X(NF+N)*FG*DCOS(A*AL(IZ))
      42 CONTINUE
      QGAP=SUMQ/PP
      40 SGAP=SGAP+QGAP*W(IZ)
      QIGP=SGAP*(AI(I)-1.)/2.
260      WRITE (6,701) I,QIGP
701      FORMAT (/, ' SIDE ',I5,' Q GAP = ',F10.4,/)
C
C      LUMPED TEMPERATURE CALCULATION FOR SUBCHANNEL GEOMETRY
265      AP(1)=AB(I)
      AP(2)=AL(I)
      IF (CABS(AP(1)-AP(2)).LE.1.00-3) GO TO 107
      CALL TAVG (1,I,AP,IK,NF,FN,GN,X,AI,AL,CF,YNT,T1,F1)
      WRITE (6,153) I,AP(1),AP(2),F1,T1
270      107 AP(1)=AL(I)
      AP(2)=AB(I+1)
      IF (DABS(AP(1)-AP(2)).LE.1.00-3) GO TO 15
      CALL TAVG (1,I,AP,IK,NF,FN,GN,X,AI,AL,CF,YNT,T2,F2)
      WRITE (6,153) I,AP(1),AP(2),F2,T2
275      153 FORMAT (2X, ' SIDE ',I2,2X, ' FROM ',F10.4,5X, ' TO ',F10.4,5X, ' AREA
      1 = ',F10.4,5X, ' TAVG = ',F10.4)
      15 CONTINUE
      14 NN=NN-2
      IF (NN) 142,3000,3000
280      142 NN=NN-1
      IF (NN) 1000,2000,2000
      1000 CONTINUE
      117 FORMAT (8I2X,F12.5)
      273 FORMAT (8I2X,E12.5)
285      RETURN

```

END

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 COUPLE

VARIABLES	SN	TYPE	RELOCATION
3614 A		DOUBLE	
4066 AB1		DOUBLE	ARRAY
4926 AI1		DOUBLE	ARRAY
4066 AL1		DOUBLE	ARRAY
0 AUX		DOUBLE	ARRAY F.P.
3635 A1		DOUBLE	
12 BIS		DOUBLE	BUN2
3627 CF1		DOUBLE	
0 CAM		DOUBLE	BUN2
3721 DG		DOUBLE	
22 OH		DOUBLE	BUN2
0 OYN		DOUBLE	ARRAY F.P.
3673 FA		DOUBLE	
0 FN		DOUBLE	ARRAY F.P.
3740 F1		DOUBLE	
0 GJN		DOUBLE	ARRAY F.P.
0 GP		DOUBLE	ARRAY F.P.
3620 I		INTEGER	
3617 IC		INTEGER	
3626 IK1		INTEGER	
3637 IS1		INTEGER	
3725 IZ		INTEGER	
3648 J		INTEGER	
3616 JJ		INTEGER	
3646 JP		INTEGER	
3565 KI		INTEGER	
3625 K1		INTEGER	
4126 MJ		INTEGER	ARRAY
3566 MM		INTEGER	
24 MT		INTEGER	ARRAY BUN3
3622 M1		INTEGER	
3665 NB		INTEGER	
3644 NF1		INTEGER	
0 NK		INTEGER	F.P.
3605 NP		INTEGER	
3623 N1		INTEGER	
3677 PH		DOUBLE	
3657 PH1		DOUBLE	
3726 PP		DOUBLE	
3734 QGAP		DOUBLE	
3713 C1		DOUBLE	
3667 Q2N		DOUBLE	
0 R		DOUBLE	ARRAY F.P.
3647 RH		DOUBLE	
0 R1		DOUBLE	ARRAY F.P.
4006 AB		DOUBLE	ARRAY
3746 AI		DOUBLE	ARRAY
3766 AL		DOUBLE	ARRAY
4106 AP		DOUBLE	ARRAY
3600 AX		DOUBLE	
14 BI		DOUBLE	BUN2
3572 CF		DOUBLE	
2 OA		DOUBLE	
0 OFN		DOUBLE	ARRAY F.P.
0 DGN		DOUBLE	ARRAY F.P.
3675 OU		DOUBLE	
24 EPS		DOUBLE	BUN2
3732 FG		DOUBLE	
3703 FS		DOUBLE	
3744 FZ		DOUBLE	
0 GN		DOUBLE	ARRAY F.P.
0 GTG		DOUBLE	ARRAY F.P.
3624 IB		INTEGER	
3571 IK		INTEGER	
3602 IS		INTEGER	
0 IT		INTEGER	F.P.
3666 I1		INTEGER	
3610 JB		INTEGER	
3621 JM		INTEGER	
3570 K		INTEGER	
0 KK		INTEGER	F.P.
3606 LF		INTEGER	
0 HK		INTEGER	F.P.
0 HMAX		INTEGER	F.P.
0 HZ		INTEGER	F.P.
3613 N		INTEGER	
3607 NF		INTEGER	
0 NI		INTEGER	F.P.
3567 NN		INTEGER	
0 HT		INTEGER	F.P.
4 PD		DOUBLE	BUN2
0 PHI		DOUBLE	F.P.
16 PI		DOUBLE	BUN2
0 O		DOUBLE	ARRAY BUN3
3736 OIGP		DOUBLE	
3715 OZ		DOUBLE	
3717 Q3		DOUBLE	
3701 RA		DOUBLE	
3655 RH1		DOUBLE	
3723 SGAP		DOUBLE	

6	T	DOUBLE	BUN2
3711	TK2	DOUBLE	
3705	TV	DOUBLE	
3742	T2	DOUBLE	
3603	U	DOUBLE	
10	VE	DOUBLE	BUN2
0	X	DOUBLE	F.P.
3651	XX	DOUBLE	ARRAY
3576	YC	DOUBLE	
3611	YNT	DOUBLE	
20	YT	DOUBLE	BUN2
3653	YY	DOUBLE	
24	Z	DOUBLE	ARRAY BUN1

MODE
FMT

DATAN	DOUBLE	1	LIBRARY
DLOG	DOUBLE	1	LIBRARY
DSORT	DOUBLE	1	LIBRARY
NEIGH		7	
PDTN		5	
SIZE		14	
TEMP		9	

```

TYPE      ARGS
DOUBLE    1  INTRIN

```

FNT

INACTIVE
INACTIVE

FNT

FMT

FMT

FMT

FMT

INACTIVE

FMT

FNT

SUBROUTINE COUPLE

76/76

OPT=1

FTN 4.E+439

07/07/78 09.21.008

PAGE 8

STATEMENT LABELS

0 4000

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
74	100C	* KI	57 202	25018	EXT REFS NOT INNER
145	9	* N	75 79	.2148	EXT REFS
371	10	N	81 83	68	INSTACK
413	4000	* I	93 196	14318	EXT REFS NOT INNER
500	22	N	115 119	118	INSTACK
524	74	N	120 124	118	INSTACK
537	61	* J	128 194	12768	EXT REFS NOT INNER
1075	36	NB	146 151	68	INSTACK
1105	62	* N	152 154	658	EXT REFS
1175	27	* N	157 159	648	EXT REFS
1370	29	* N	175 176	1078	EXT REFS
1502	28	* N	179 181	1018	EXT REFS
1612	32	NB	184 185	28	INSTACK
1616	72	* N	186 187	1108	EXT REFS
1731	19	* N	190 192	1018	EXT REFS
2110	21	N	204 210	158	OPT
2201	65	* J	227 239	1158	EXT REFS
2320	15	* I	242 277	2478	EXT REFS NOT INNER
2323	40	* IZ	247 258	1508	EXT REFS NOT INNER
2351	42	* N	250 256	1018	EXT REFS

COMMON	BLOCKS	LENGTH
BUN1		48
BUN2		22
BUN3		30

STATISTICS

PROGRAM LENGTH	43428	2274
SCN LABELED COMMON LENGTH	1348	92

```

1      SUBROUTINE SIZE (NI,NK,MK,MZ,NP,NC,GJN,VJ,G,Y,GTG,AUX,EPS,R)
      C
      C
      C      SUBROUTINE SIZE TAILORS THE SIZE OF THE COEFFICIENT
      C      MATRIX G AND THE VECTOR Y
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      DIMENSION GJN(NI,NK),VJ(NI),Y(NP),G(NP,NC),GTG(MZ),AUX(NK),R(NK)
      C      DO 10 J=1,NP
      C      V(J)=V(J)
      C      DO 10 N=1,NC
10      G(J,N)=GJN(J,N)
      C      CALL GTPRD (G,Y,R,NP,NC,1)
      C      CALL MATA (G,GTG,NP,NC,0)
      C      CALL OGELS (R,GTG,NC,1,EPS,IER,AUX)
      C      RETURN
      C      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 SIZE

VARIABLES	SN	TYPE	RELOCATION						
0 AUX		DOUBLE	ARRAY	F.P.	0 EPS	DOUBLE			F.P.
0 G		DOUBLE	APRAY	F.P.	0 GJN	DOUBLE	ARRAY		F.P.
0 GYG		DOUBLE	ARRAY	F.P.	130 IER	INTEGER			
126 J		INTEGER			0 HK	INTEGER			F.P.
0 MZ		INTEGER		F.P.	127 N	INTEGER			
0 NC		INTEGER		F.P.	0 NI	INTEGER			F.P.
0 NK		INTEGER		F.P.	0 NP	INTEGER			F.P.
0 R		DOUBLE	ARRAY	F.P.	0 Y	DOUBLE	ARRAY		F.P.
0 YJ		DOUBLE	ARRAY	F.P.					

EXTERNALS
CGELS
MATA

TYPE	ARGS		
	7	GTPRO	6
	5		

STATEMENT LABELS
0 10

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
22	10	J	10 13	228	NOT INNER
36	10	N	12 13	38	INSTACK

STATISTICS

PROGRAM LENGTH	1658	117
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```

1      SUBROUTINE TYPE (M,N,MHAX,K,IK,CF,XC,YC,AX)
    C
    C
    C          SUBROUTINE TYPE GIVES THE TYPE, CELL NUMBER AND THE
    C          LOCATION OF A UNIT CELL
    C
10     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        COMMON /DUM2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
        IF (M-1) 30,40,50
20     M1=M-1
        JSUM=0
        DO 15 J=1,M1
15     JSUM=JSUM+J/2+1
        K=2+N/2+JSUM
        GO TO 25
40     K=2
        GO TO 25
20     K=1
25     IF (M.EQ.0) GO TO 2
        IF (M.EQ.MHAX) GO TO 4
        IF (M.EQ.1) GO TO 3
        IF (N.EQ.0) GO TO 8
25     IF (N.EQ.1) GO TO 9
        IF (N.EQ.M) GO TO 10
        IK=9
        GO TO 20
8     IK=6
        GO TO 20
9     IK=10
        GO TO 20
10    IK=5
        GO TO 20
35     IF (N.EQ.0) GO TO 5
        IF (N.EQ.M) GO TO 7
        IF (N.EQ.1) GO TO 6
        IK=7
        GO TO 20
40     IF (N.EQ.2)
            GO TO 20
6     IK=8
        GO TO 20
5     IK=3
        GO TO 20
45     IF (N.EQ.3)
            GO TO 20
3     IK=4
        GO TO 20
2     IK=1
20    CF=1.0
        IF (IK.EQ.1) CF=6.0
        XC=N*PD
        YC=M*2.*PD*DSIN(PI/3.)
        AX=-1.*PI/2.
        IF (M.EQ.N) AX=PI/3.
        RETURN
        END

```

SUBROUTINE TYPE 76/76 OPT=1

FTN 4.6439

07/07/78 09.21.013

PAGE 2

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 TYPE

VARIABLES	SN	TYPE	RELOCATION				
0 AX		DOUBLE	F.P.	14 BI	DOUBLE	BUN2	F.P.
12 BIS		DOUBLE	BUN2	0 CF	DOUBLE		
2 DA		DOUBLE	BUN2	0 DAW	DOUBLE	BUN2	
22 DH		DOUBLE	BUN2	24 EPS	DOUBLE	BUN2	
0 IK		INTEGER	F.P.	150 J	INTEGER		
147 JSUM		INTEGER		0 K	INTEGER	F.P.	
0 H		INTEGER	F.P.	0 HMAX	INTEGER	F.P.	
146 M1		INTEGER		0 N	INTEGER	F.P.	
4 PD		DOUBLE	BUN2	16 PI	DOUBLE	BUN2	
6 T		DOUBLE	BUN2	10 VE	DOUBLE	BUN2	
0 XC		DOUBLE	F.P.	0 YC	DOUBLE	F.P.	
20 YT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS
DSIN	DOUBLE	1 LIBRARY

STATEMENT LABELS

67 2	65 3	50 4
63 5	61 6	57 7
42 0	44 9	46 10
0 15	71 20	30 25
26 30	24 40	0 50

INACTIVE

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
14	15	J	14 15	38	INSTACK

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS

PROGRAM LENGTH	1518	105
SCM LABELED COMMON LENGTH	268	22

```

1      SUBROUTINE CHAN (K,IK,IS,AI,AL,AG,UI)
      C
      C
      C
5      SUBROUTINE CHAN GIVES THE GEOMETRIC PARAMETERS OF A UNIT
      CELL
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
      COMMON /BUN3/ Q(10),MT(18)
      DIMENSION AI(10),AL(10),AG(10)
      GO TO (1,2,3,4,4,6,7,7,9,9),IK
1      IS=1
      AI(1)=PD
      AL(1)=0.
      AG(1)=0.0
15     AB(2)=PI/6.0
      D=(3.464*PD**2-PI)/PI
      GO TO 500
2      IS=3
      AI(1)=VE
      AI(2)=PD
      AI(3)=PD
      AL(1)=PI/6.
      AL(2)=PI*12./18.
      AL(3)=PI
25     AG(1)=0.
      AB(2)=AL(1)+DATAN(AI(2)/AI(1))
      AG(3)=5.*PI/6.
      AB(4)=PI
      AF=(PD+VE/1.732)*(1.732*PD/2.+VE/2.)-PD**2/3.464-PI/2.
30     D=AF/(PI/2.+(PD+VE/1.732)/2.)
      GO TO 500
3      IS=3
      AI(1)=PD
      AI(2)=PD
      AI(3)=VE
      AL(1)=PI/6.
      AL(2)=PI/2.
      AL(3)=PI
      AB(1)=0.0
40     AB(2)=PI/3.
      AG(3)=PI/2.+DATAN(VE/AI(2))
      AB(4)=PI
      D=(PD*(1.732*PD/2.+VE)-PI/2.)/(PI/2.+PD/2.)
      GO TO 500
45     4      IS=4
      DO 12 I=1,IS
      AI(I)=PD
12     AL(I)=(I-1)*PI/3.
      AG(1)=0.0
50     AB(2)=PI/6.
      AB(3)=PI/2.
      AB(4)=5.*PI/6.
      AG(5)=PI
      C=(3.464*PD**2-PI)/PI
      GO TO 500
55     6      IS=3
      AB(1)=0.0

```

```

      DO 13 I=1,IS
      AI(I)=PD
      AL(I)=(I-1)*PI/3.+PI/6.
13  AB(I+1)=PI*I/3.
      D=(3.464*PD**2-PI)/PI
      GO TO 500
      7  IS=5
      AI(1)=PD
      AI(2)=PD
      AI(3)=VE
      AI(4)=PD
      AI(5)=PD
      78  AL(1)=PI/6.
      AL(2)=PI/2.
      AL(3)=PI
      AL(4)=3.*PI/2.
      AL(5)=11.*PI/6.
      75  AB(1)=0.0
      AB(2)=PI/3.
      AB(3)=PI/2.+DATAN(VE/PD)
      AB(4)=2.*PI-AB(3)
      AB(5)=5.*PI/3.
      88  AB(6)=2.*PI
      D=(PD*(1.732*PD/2.+VE)-PI/2.)/(PI/2.+PD/2.)
      GO TO 500
      9  IS=6
      AB(1)=0.0
      85  DO 14 I=1,IS
      AI(I)=PD
      AL(I)=PI*(I-1)/3.+PI/6.
      14  AB(I+1)=PI*I/3.
      D=(3.464*PD**2-PI)/PI
      50  U=(D/DH)**(5./7.)
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY PCINTS
3 CHAN

VARIABLES	SM	TYPE	RELOCATION					
0 AB	DOUBLE	ARRAY	F.P.	1031	AF	DOUBLE		
0 AI	DOUBLE	ARRAY	F.P.	0	AL	DOUBLE	ARRAY	F.P.
14 BI	DOUBLE		BUN2	12	BIS	DOUBLE		BUN2
1027 D	DOUBLE			2	DA	DOUBLE		BUN2
0 DAW	DOUBLE		BUN2	22	DH	DOUBLE		BUN2
24 EPS	DOUBLE		BUN2	1033	I	INTEGER		
0 IK	INTEGER		F.P.	0	IS	INTEGER		F.P.
0 K	INTEGER	UNUSED	F.P.	24	MT	INTEGER	ARRAY	BUN3
4 PD	DOUBLE		BUN2	16	PI	DOUBLE		BUN2
0 Q	DOUBLE	ARRAY	BUN3	6	T	DOUBLE		BUN2
0 U	DOUBLE		F.P.	10	VE	DOUBLE		BUN2

SUBROUTINE CHAN

76/76 OPT=1

FTN 4.6+439

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VARIABLES SN TYPE
20 VT DOUBLERELOCATION
BUN2EXTERNALS TYPE ARGS
DATAN DOUBLE 1 LIBRARY

STATEMENT LABELS

24 1
344 4
701 9
0 1457 2
433 6
0 12
771 500231 3
525 7
0 13

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
362	12	I	46 48	108	INSTACK
462	13	I	58 61	248	OPT
722	14	I	85 88	308	OPT

COMMON BLOCKS LENGTH
BUN2 22
BUN3 30

STATISTICS

PROGRAM LENGTH	10348	540
SCN LABELED COMMON LENGTH	648	52

SUBROUTINE PDTH

76/76 OPT=1

FTN 4.6+439

07/07/78 09.21.005

PAGE 1

1 SUBROUTINE PDTH (IK,IS,MJ,NP,NF)

C
C
C
C

5

SUBROUTINE PDTH GIVES THE COLOCATION POINT DISTRIBUTION
OF A UNIT CELL

DIMENSION MJ(10)

GO TO (1,2,3,4,4,6,7,7,9,9),IK

10

1 MJ(1)=4
GO TO 1002 MJ(1)=8
MJ(2)=6
MJ(3)=4
GO TO 100

15

3 MJ(1)=8
MJ(2)=6
MJ(3)=4
GO TO 100

20

4 MJ(1)=4
MJ(2)=8
MJ(3)=8
MJ(4)=4
GO TO 100

25

6 MJ(1)=8
MJ(2)=8
MJ(3)=8
GO TO 100

30

7 MJ(1)=8
MJ(2)=6
MJ(3)=8
MJ(4)=6
MJ(5)=8
GO TO 100

35

9 MJ(1)=8
MJ(2)=8
MJ(3)=8
MJ(4)=8
MJ(5)=8
MJ(6)=8

40

100 NP=8
DO 16 I=1,IS16 NP=NP+MJ(I)
NF=NP*2/3

45

RETURN
END

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 PDTH

SUBROUTINE PDYN

76/76 OPT=1

FTN 4.6+439

07/07/78 09.21.033

PAGE 2

VARIABLES

74 I

0 IS

0 NF

SN TYPE

INTEGER

INTEGER

INTEGER

RELOCATION

F.P.

F.P.

0 IK

0 MJ

0 NP

INTEGER

INTEGER

INTEGER

ARRAY

F.P.

F.P.

F.P.

STATEMENT LABELS

24 1

36 4

52 9

26 2

42 6

0 16

32 3

45 7

56 100

LOOPS

LABEL

INDEX

FROM-TO

LENGTH

PROPERTIES

64 16

I

41 42

28

INSTACK

STATISTICS

PROGRAM LENGTH

758

61

```

1      SUBROUTINE NTB (MMAX,IK,JB)
      C
      C
      C      SUBROUTINE NTB GIVES THE NUMBER OF BOUNDARIES WHICH
5      C      WHICH ARE COUPLED TO NEIGHBORING CELLS BY THE CONTINUITY
      C      OF TEMPERATURE
      C
      C
10     GO TC (1,2,3,4,5,6,7,8,9,10),IK
      1 JB=0
      RETURN
      2 JB=2
      IF (MMAX.EQ.1) JB=1
      RETURN
15     3 JB=1
      RETURN
      4 JB=1
      RETURN
20     5 JB=2
      RETURN
      6 JB=1
      RETURN
      7 JB=3
      RETURN
25     8 JB=2
      RETURN
      9 JB=3
      RETURN
30     10 JB=2
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1) .

ENTRY POINTS
3 NTB

VARIABLES	SN	TYPE	RELOCATION				
0 IK		INTEGER	F.P.				
8 HMAX		INTEGER	F.P.	0	JB	INTEGER	F.P.

STATEMENT LABELS

24 1	26 2	33 3
35 4	37 5	41 6
43 7	45 8	47 9
51 10		

STATISTICS

PROGRAM LENGTH	538	43
----------------	-----	----


```

1      SUBROUTINE POINTS (ISD,NI,NJ,AI,AL,AB,PHI)
      C
      C
      C      SUBROUTINE POINTS GIVES THE POSITIONS OF THE COLLOCATION
      C      POINTS
      C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
      COMMON /BUN3/ Q(10),MT(10)
      DIMENSION AI(10),AL(10),AB(10),XS(10),YS(10),NJ(10)
      DIMENSION PHI(NI)
      ISC=ISD+1
      XS(1)=AI(1)/DCOS(AL(1))
      YS(1)=0.0
      DO 150 I=2,ISC
      XS(I)=(AI(I-1)/DCOS(AB(I)-AL(I-1)))*DCOS(AB(I))
      YS(I)=(AI(I-1)/DCOS(AB(I)-AL(I-1)))*DSIN(AB(I))
      K=1
      DO 120 I=1,ISD
      NIS=NJ(I)
      DO 120 J=1,NIS
      X=((NIS-J+.5)*XS(I)+(J-.5)*XS(I+1))/NIS
      Y=((NIS-J+.5)*YS(I)+(J-.5)*YS(I+1))/NIS
      IF (Y) 10,10,20
      10 IF (X.GE.0.0) PHI(K)=2.*PI+DATAN(Y/X)
      IF (X.LT.0.0) PHI(K)=PI+DATAN(Y/X)
      GO TO 120
      20 IF (X.GE.0.0) PHI(K)=DATAN(Y/X)
      IF (X.LT.0.0) PHI(K)=PI+DATAN(Y/X)
      120 K=K+1
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 POINTS

VARIABLES	SN	TYPE	RELOCATION					
0 AB		DOUBLE	ARRAY	F.P.	0 AI	DOUBLE	ARRAY	F.P.
0 AL		DOUBLE	ARRAY	F.P.	14 BI	DOUBLE		BUN2
12 BIS		DOUBLE		BUN2	2 DA	DOUBLE		BUN2
0 DAW		DOUBLE		BUN2	22 DH	DOUBLE		BUN2
24 EPS		DOUBLE		BUN2	276 I	INTEGER		
275 ISC		INTEGER			0 ISD	INTEGER		F.P.
301 J		INTEGER			277 K	INTEGER		
24 MT		INTEGER	ARRAY	BUN3	0 NI	INTEGER		F.P.
300 NIS		INTEGER			0 NJ	INTEGER	ARRAY	F.P.
4 PC		DOUBLE		BUN2	0 PHI	DOUBLE	ARRAY	F.P.
16 PI		DOUBLE		BUN2	0 Q	DOUBLE	ARRAY	BUN3
6 T		DOUBLE		BUN2	10 VE	DOUBLE		BUN2
302 X		DOUBLE			306 XS	DOUBLE	ARRAY	

SUBROUTINE POINTS

76/76 OPT=1

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VARIABLES	SN	TYPE	RELOCATION				
304 Y		DOUBLE		332	YS	DOUBLE	ARRAY
20 VT		DOUBLE	BUN2				
EXTERNALS	TYPE	ARGS					
CATAN	DOUBLE	1 LIBRARY		DCOS	DOUBLE	1 LIBRARY	
DSIN	DOUBLE	1 LIBRARY					

STATEMENT LABELS

0 10	INACTIVE	222 28	256 120
0 150			

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
34	150	* I	16 18	568	EXT REFS
113	120	* I	20 31	1528	EXT REFS NOT INNER
116	120	* J	22 31	1448	EXT REFS

COMMON	RLCKS	LENGTH
BUN2		22
BUN3		30

STATISTICS

PROGRAM LENGTH	4048	260
SCH LABELED COMMON LENGTH	648	52

```
1      SUBROUTINE NEIGH (MMAX,M,N,IK,IS,MH,NN)
      C
      C
      C      SUBROUTINE NEIGH IDENTIFIES NEIGHBORING CELLS
      C
      C
      C      GO TO (1,2,3,4,4,6,7,7,9,9),IK
      1  MH=M+1
      NN=N+1
      RETURN
      2  GO TO (11,12,13),IS
      11 MH=-1
      NN=-1
      RETURN
      12 MH=M
      NN=N-2
      IF (MMAX.EQ.1) MH=-1
      IF (MMAX.EQ.1) NN=-1
      RETURN
      13 MH=M-1
      NN=N-1
      RETURN
      3  GO TO (14,15,16),IS
      14 MH=M-1
      NN=N+1
      RETURN
      15 MH=M
      NN=N+2
      RETURN
      30 16 MH=-1
      NN=-1
      RETURN
      4  GO TO (17,18,19,20),IS
      17 MH=M+1
      NN=N+1
      RETURN
      35 18 MH=M+1
      NN=N-1
      RETURN
      40 19 MH=M
      NN=N-2
      IF (IK.EQ.4) MH=-1
      IF (IK.EQ.4) NN=-1
      RETURN
      45 20 MH=N-1
      NN=N-1
      RETURN
      6  GO TO (21,22,23),IS
      21 MH=M-1
      NN=N+1
      RETURN
      50 22 MH=M
      NN=N+2
      RETURN
      55 23 MH=M+1
      NN=N+1
      RETURN
```

```

      7 GO TO (24,25,26,27,28),IS
      24 MM=N-1
      60 NN=N+1
      RETURN
      25 MM=N
      NN=N+2
      RETURN
      65 26 MM=-1
      NN=-1
      RETURN
      27 MM=N
      NN=N-2
      70 IF (IK.EQ.8) MM=-1
      IF (IK.EQ.8) NN=-1
      RETURN
      28 MM=N-1
      NN=N-1
      75 RETURN
      9 GO TO (29,30,31,32,33,34),IS
      29 MM=N-1
      NN=N+1
      RETURN
      80 30 MM=N
      NN=N+2
      RETURN
      31 MM=N+1
      NN=N+1
      85 RETURN
      32 MM=N+1
      NN=N-1
      RETURN
      33 MM=N
      NN=N-2
      90 IF (IK.EQ.10) MM=-1
      IF (IK.EQ.10) NN=-1
      RETURN
      34 MM=N-1
      NN=N-1
      95 RETURN
      END

```

CARD NO. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

```

      11 I      AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.
      23 I      AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.
      48 I      AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

```

SYMBOLIC REFERENCE MAP (R=1)

SUBROUTINE NEIGH

76/76 OPT=1

FTN 4.6+439

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PAGE 3

ENTRY PCINTS
3 NEIGH

VARIABLES	SN	TYPE	RELOCATION	0	IS	INTEGER	F.P.
0 IK		INTEGER	F.P.	0	IS	INTEGER	F.P.
0 M		INTEGER	F.P.	0	MM	INTEGER	F.P.
0 NMAX		INTEGER	F.P.	0	N	INTEGER	F.P.
0 NN		INTEGER	F.P.				

STATEMENT LABELS

24 1	31 2	64 3
111 4	154 6	203 7
252 9	41 11	44 12
57 13	74 14	101 15
106 16	122 17	127 18
134 19	147 20	164 21
171 22	176 23	215 24
222 25	227 26	232 27
245 28	265 29	272 30
277 31	304 32	311 33
324 34		

STATISTICS

PROGRAM LENGTH

3318 217

```

1      SUBROUTINE GTPROD(A,B,R,N,M,L)
          C
          C
5         C
          C   SUBROUTINE GTPROD GIVES THE PRODUCT OF THE TRANSPOSE OF
          C   G AND Y
          C
          C
          C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
          DIMENSION A(1),B(1),R(1)
          IR=0
          IK=-N
          DO 10 K=1,L
              IJ=0
              IK=IK+N
              OO 10 J=1,M
                  IB=IK
                  IR=IR+1
                  R(IR)=0.
                  OO 10 I=1,M
                      IJ=IJ+1
                      IB=IB+1
                      10 R(IR)=R(IR)+A(IJ)*R(IB)
              RETURN
          END
```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY PCINTS
J GTPRD

VARIABLES	SN	TYPE	RELOCATION					
0 A		DOUBLE	ARRAY	F.P.	0	0	DOUBLE	ARRAY F.P.
71 I		INTEGER			70	10	INTEGER	
66 IJ		INTEGER			64	IK	INTEGER	
63 IR		INTEGER			67	J	INTEGER	
65 K		INTEGER			0	L	INTEGER	F.P.
0 M		INTEGER		F.P.	0	N	INTEGER	F.P.
0 R		DOUBLE	ARRAY	F.P.				

STATEMENT LABELS
0 10

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
22	10	* K	12 22	408	NOT INNER
26	10	* J	15 22	328	NOT INNER
42	10	I	19 22	128	INSTACK

STATISTICS

PROGRAM LENGTH	1108	72
----------------	------	----

```

1      SUBROUTINE NATA(A,R,N,M,MS)
      C
      C
      C      SUBROUTINE NATA GIVES THE PRODUCT OF THE TRANSPOSE
      C      OF G AND G
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      DIMENSION A(1),R(1)
      C      CO 70 K=1,M
      C      KX=(K*K-K)/2
      C      CO 70 J=1,M
      C      IF (J-K) 10,10,70
      C      10 IR=J*KX
      C      R(IR)=0.0
      C      CO 60 I=1,N
      C      IF(MS) 20,40,20
      C      20 CALL LOC(I,J,IA,N,M,MS)
      C      CALL LOC(I,K,IB,N,M,MS)
      C      20 IF(IA) 30,60,30
      C      30 IF(IB) 50,60,50
      C      40 IA=N*(J-1)+I
      C      IB=N*(K-1)+I
      C      50 R(IR)=R(IR)+A(IA)*A(IB)
      C      25 60 CONTINUE
      C      70 CONTINUE
      C      RETURN
      C      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 NATA

VARIABLES	SN	TYPE	RELOCATION					
0 A		DOUBLE	ARRAY	F.P.	121 I	INTEGER		
122 IA		INTEGER			123 IB	INTEGER		
120 IR		INTEGER			117 J	INTEGER		
115 K		INTEGER			116 KX	INTEGER		
0 N		INTEGER		F.P.	0 MS	INTEGER		F.P.
0 N		INTEGER		F.P.	0 R	DOUBLE	ARRAY	F.P.

EXTERNALS
LOCTYPE ARCS
6

STATEMENT LABELS

	INACTIVE		INACTIVE		INACTIVE
0 10		0 20		0 30	
46 40		53 50		66 60	
71 70					

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
17	70	* K	10 26	578	EXT REFS NOT INNER
22	70	* J	12 26	528	EXT REFS NOT INNER

SUBROUTINE DATA

76/76

OPT=1

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LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES	EXT REFS
30	60	• I	16 25	418		

STATISTICS

PROGRAM LENGTH

1589

184

SUBROUTINE LOC

76/76 OPT=1

FTN 4.64439

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PAGE 1

```

1      SUBROUTINE LOC (I,J,IR,N,M,MS)
      IX=I
      JX=J
      IF (MS-1) 10,20,30
5      10  IRX=N*(JX-1)+IX
      GOTO 36
      20  IF (IX-JX) 22,24,24
      22  IRX=IX+JX*(JX-JX)/2
      GOTO 36
10     24  IRX=JX+(IX*IX-IX)/2
      GOTO 36
      30  IRX=0
      IF (IX-JX) 36,32,36
      32  IRX=IX
15     36  IR=IRX
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 LOC

VARIABLES	SN	TYPE	RELOCATION			
0 I		INTEGER	F.P.	0 IR	INTEGER	F.P.
34 IRX		INTEGER		32 IX	INTEGER	
0 J		INTEGER	F.P.	33 JX	INTEGER	
0 N		INTEGER	UNUSED F.P.	0 MS	INTEGER	F.P.
0 N		INTEGER	F.P.			

STATEMENT LABELS

0 10	INACTIVE	15 20	0 22	INACTIVE	
22 24		25 30	0 32	INACTIVE	
30 36					

STATISTICS

PROGRAM LENGTH

358 29

```

SUBROUTINE DGELS(R,A,M,N,EPS,IER,AUX)
      C
      C SUBROUTINE DGELS SOLVES THE UNKNOWN COEFFICIENT VECTOR
      C BY GAUSSIAN ELIMINATION
      C
      DIMENSION A(1),R(1),AUX(1)
      DOUBLE PRECISION R,A,AUX,TB,TOL,PIVI,EPS
      IF(M)24,24,1
      IER=0
      PIV=0.00
      L=0
      DO 3 K=1,M
      L=L+K
      TB=0ABS(A(L))
      IF(TB-PIV)3,3,2
      PIV=TB
      I=L
      J=K
      CONTINUE
      TOL=EPS*PIV
      LST=0
      NM=N*M
      LEND=M-1
      DO 18 K=1,M
      IF(PIV)24,24,4
      IF(IER)7,5,7
      IF(PIV-TOL)6,6,7
      IER=K-1
      LT=J-K
      LST=LST+K
      PIVI=1.00/A(I)
      DO 9 L=K,NM,M
      LL=L+LT
      TR=PIVI*R(LL)
      R(LL)=R(LL)-TR
      R(LL)=TB
      IF(K-M)9,19,19
      LR=LST+(LT*(K-J-1))/2
      LL=LR
      L=LST
      DO 14 II=K,LEND
      LL=LL+1
      IF(L-LR)12,10,11
      A(LL)=A(LST)
      TB=A(L)
      GOTO 13
      LL=L+LT
      TB=A(LL)
      A(LL)=A(L)
      AUX(II)=TB
      A(L)=PIVI*TB
      A(LST)=LT
      PIV=0.00
      LST=LST

```

```

      LT=0
      DO 10 II=K,LEND
60      PIVI=-AUX(II)
      LL=LLST
      LT=LT+1
      DO 15 LLD=II,LEND
      LL=LL+LLD
65      L=LL+LT
      15 A(LL)=A(LL)+PIVI*A(LL)
      LLST=LLST+II
      LR=LLST+LT
      TB=DABS(A(LR))
70      IF (TB-PIVI) 17,17,16
      16 PIV=TB
      I=LR
      J=II+1
      17 DO 18 LR=K,NH,M
75      LL=LR+LT
      18 R(LL)=R(LL)+PIVI*R(LR)
      19 IF (LEND) 24,23,20
      20 II=M
      DO 22 I=2,M
80      LST=LST-II
      II=II-1
      L=A(LST)*.500
      DO 21 J=II,NH,M
      TB=R(J)
85      LL=J
      K=LST
      DO 21 LT=II,LEND
      LL=LL+1
      K=K+LT
90      21 TB=TB-A(K)*R(LL)
      R=J+L
      R(J)=R(K)
      22 R(K)=TB
      23 RETURN
95      24 IFR=-1
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 OGELS

VARIABLES	SN	TYPE	RELOCATION				
0 A		DOUBLE	ARRAY	F.P.	0	AUX	DOUBLE
0 EPS		DOUBLE		F.P.	407	I	INTEGER
0 IER		INTEGER		F.P.	417	II	INTEGER
410 J		INTEGER			406	K	INTEGER
405 L		INTEGER			413	LEND	INTEGER
415 LL		INTEGER			421	LLD	INTEGER
							ARRAY
							F.P.

SUBROUTINE DGELS

76/76 OPT=1

FTN 4.6+439

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VARIABLES	SH	TYPE	RELOCATION
420 LLST		INTEGER	
411 LST		INTEGER	
0 M		INTEGER	F.P.
412 NM		INTEGER	
403 PIVI		DOUBLE	
377 T0		DOUBLE	

416 LR	INTEGER	
414 LT	INTEGER	
0 N	INTEGER	F.P.
375 PIV	DOUBLE	
0 R	DOUBLE	ARRAY F.P.
401 TOL	DOUBLE	

INLINE FUNCTIONS	TYPE	ARGS
CABS	DOUBLE	1 INTRIN

STATEMENT LABELS

0 1	INACTIVE	0 2	INACTIVE	43 3	
0 4	INACTIVE	0 5	INACTIVE	0 6	INACTIVE
72 7		0 8		0 9	INACTIVE
0 10	INACTIVE	156 11		160 12	
165 13		0 14		0 15	
0 16	INACTIVE	251 17		0 18	
331 19		0 20	INACTIVE	0 21	
0 22		364 23		365 24	

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
10 3	K		14 21	148	OPT
60 10	* K		26 76	2218	EXITS NOT INNER
113 4	L		34 38	128	INSTACK
144 14	II		43 54	318	OPT
234 10	* II		59 76	728	NOT INNER
216 15	LLD		63 66	148	OPT
260 19	LR		74 76	138	OPT
335 22	* I		79 93	578	NOT INNER
316 22	* J		83 93	448	NOT INNER
332 21	LT		87 90	148	OPT

STATISTICS

PROGRAM LENGTH	4738	315
----------------	------	-----

```

SUBROUTINE TAVG (K1,K2,AB,IK,NF,FN,GN,X,AI,AL,CF,YNT,T3,FA)
C
C C C C C
      SLBROUTINE TAVG GIVES THE AVERAGE COOLANT TEMPERATURE
      OF A PARTICULAR REGION OF THE COOLANT CHANNEL

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /OUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,VT,DH,EPS
      DIMENSION X(NF),FN(NF),GN(NF)
      DIMENSION AB(10),AI(10),AL(10)
      DTAN(X)=DSIN(X)/DCOS(X)
      SUMA=0.0
      SUMB=0.0
      SUNC=0.0
      SUMY=0.0
      I=1
      DO 31 L=K1,K2
        CALL SIMPS (SIMPA,AB(I),AB(I+1),NF,FN,GN,X,AI(L),AL(L),CF,YNT,IK,1)
        SUMA=SUMA+(SIMPA*AI(L)**4)
        CALL SIMPS (SIMPB,AB(I),AB(I+1),NF,FN,GN,X,AI(L),AL(L),CF,YNT,IK,2)
        SUMB=SUMB+(SIMPB*AI(L)**2)
        CALL SIMPS (SIMPC,AB(I),AB(I+1),NF,FN,GN,X,AI(L),AL(L),CF,YNT,IK,3)
        SUNC=SUNC+SIMPC
        SUMY=SUMY+AI(L)*AI(L)*(DTAN(AB(I+1)-AL(L))-DTAN(AB(I)-AL(L)))
      31 Y=Y+1
      FA=(SUMY-(AB(I)-AB(1)))/2.
      T3=-1./(2.*YNT)+(DLOG(T)-1./BI)/DA-1./(BIS*DA*DAW)
      T3=T3+((SUMA-(AB(I)-AB(1)))/(8.*YNT)-(YNT+1.)*SUMB*(AB(I)-AB(1))
      /(4.*YNT)+SUNC)/FA
      RETURN
      END
```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 TAGS

VARIABLES	SN	TYPE	RELOCATION						
0	AB	DOUBLE	ARRAY	F.P.	0	AI	DOUBLE	ARRAY	F.P.
0	AL	DOUBLE	ARRAY	F.P.	14	BI	DOUBLE		BUN2
12	BIS	DOUBLE		BUN2	0	CF	DOUBLE		F.P.
2	DA	DOUBLE		BUN2	0	DAM	DOUBLE		BUN2
22	DM	DOUBLE		BUN2	24	EPS	DOUBLE		BUN2
0	FA	DOUBLE		F.P.	0	FN	DOUBLE	ARRAY	F.P.
0	GN	DOUBLE	ARRAY	F.P.	550	I	INTEGER		
0	IK	INTEGER		F.P.	0	KI	INTEGER		F.P.
0	K2	INTEGER		F.P.	551	L	INTEGER		
0	NF	INTEGER		F.P.	4	PD	DOUBLE		BUN2
16	PI	DOUBLE		BUN2	552	SIMPA	DOUBLE		

SUBROUTINE YAVG

76/76 OPT=1

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VARIABLES	SN	TYPE	RELOCATION
554 SIMPB		DOUBLE	
540 SUMA		DOUBLE	
544 SUMC		DOUBLE	
6 T		DOUBLE	BUN2
10 VE		DOUBLE	BUN2
0 VNT		DOUBLE	F.P.

556 SIMPC	DOUBLE	
542 SUMB	DOUBLE	
546 SUMY	DOUBLE	
0 T3	DOUBLE	F.P.
0 X	DOUBLE	F.P.
20 YT	DOUBLE	BUN2

EXTERNALS	TYPE	ARGS
OCOS	DOUBLE	1 LIBRARY
OSIN	DOUBLE	1 LIBRARY

DLOG	DOUBLE	1 LIBRARY
SINPS		13

INLINE FUNCTIONS	TYPE	ARGS
DTAN	DOUBLE	1 SF

STATEMENT LABELS
0 31

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
25	31	* L	18 29	2178	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS		
PROGRAM LENGTH	6608	432
SCN LABELED COMMON LENGTH	260	22

```

1      SUBROUTINE SIMPS (SIMP,XA,XB,NK,FN,GN,R,AI,AL,CF,YNT,IK,JA)
      C
      C
      C
      C      SUBROUTINE SIMPS PERFORMS NUMERICAL INTEGRATIONS
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      COMMON /BUN1/ M(10),Z(10)
      C      COMMON /DUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
      C      DIMENSION R(NK),FN(NK),GN(NK)
      C      SUMP=0.0
      C      DO 2 I=1,10
      C      PH=(Z(I)*(XB-XA)+(XA+XB))/2.
      C      GO TO (10,11,12,13),JA
      C      10 Y=(1./DCOS(PH-AL))**4
      C      GO TO 2
      C      11 Y=(2.*DLOG(AI/DCOS(PH-AL))-1.)/DCOS(PH-AL)**2)
      C      GO TO 2
      C      12 Y=0.0
      C      DO 21 N=1,NK
      C      A=CF*N
      C      G=GN(N)/(A+2.)*((AI/DCOS(PH-AL))**(A+2.))-1.)
      C      IF (A.EQ.2.0) GO TO 22
      C      F=FN(N)/(2.-A)*((AI/DCOS(PH-AL))**(2.-A))-1.)
      C      GO TO 25
      C      22 F=FN(N)*DLOG(AI/DCOS(PH-AL))
      C      25 Y=Y+(G+F)*R(N)/A*DCOS(A*PH)
      C      IF (IK=7) 21,23,23
      C      23 Y=Y+(G+F)*R(N)*DSIN(A*PH)/A
      C      21 CONTINUE
      C      GO TO 2
      C      13 RH=AI/DCOS(PH-AL)
      C      CALL FLUX (3,IK,CF,NK,R,YNT,AL,RH,PH,Q2)
      C      Y=C2*AI/DCOS(PH-AL)**2)
      C      2 SUMP=SUMP+M(I)*Y
      C      SIMP=SUMP*(XB-XA)/2.0
      C      RETURN
      C      ENC

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 SIMPS

VARIABLES	SN	TYPE	RELOCATION			
523 A		DOUBLE		0	AI	DOUBLE F.P.
0 AL		DOUBLE	F.P.	14	BI	DOUBLE BUN2
12 BIS		DOUBLE	BUN2	0	CF	DOUBLE F.P.
2 DA		DOUBLE	BUN2	0	DAW	DOUBLE BUN2
22 OH		DOUBLE	BUN2	24	EPS	DOUBLE BUN2
527 F		DOUBLE		0	FN	DOUBLE ARRAY F.P.
525 G		DOUBLE		0	GN	DOUBLE ARRAY F.P.
515 I		INTEGER		0	IK	INTEGER F.P.

SUBROUTINE SIMPS

76/76 OPT=1

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PAGE 2

VARIABLES	SN	TYPE	RELOCATION
0 JA		INTEGER	F.P.
0 NK		INTEGER	F.P.
516 PH		DOUBLE	
533 Q2		DOUBLE	
531 RH		DOUBLE	
513 SHMP		DOUBLE	
10 VE		DOUBLE	BUN2
0 XA		DOUBLE	F.P.
520 Y		DOUBLE	
20 YT		DOUBLE	BUN2

522 N	INTEGER				
4 PD	DOUBLE				BUN2
16 PI	DOUBLE				BUN2
0 R	DOUBLE	ARRAY			F.P.
0 SIMP	DOUBLE				F.P.
6 T	DOUBLE				BUN2
0 W	DOUBLE	ARRAY			BUN1
0 XB	DOUBLE				F.P.
0 YNT	DOUBLE				F.P.
24 Z	DOUBLE	ARRAY			BUN1

EXTERNALS	TYPE	ARGS
CCOS	DOUBLE	1 LIBRARY
DSIN	DOUBLE	1 LIBRARY

DLOG	DOUBLE	1 LIBRARY
FLUX		10

STATEMENT LABELS

434 2		56 10		73 11	
132 12		370 13		365 21	
247 22		0 23	INACTIVE	267 25	

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
21 2		• I	12 35	4278	EXT REFS NOT INNER
135 21		• N	20 30	2338	EXT REFS

COMMON BLOCKS	LENGTH
BUN1	40
BUN2	22

STATISTICS

PROGRAM LENGTH	6139	395
SCN LABELED COMMON LENGTH	768	62


```

1      SUBROUTINE TEMP (JA,IK,CF,NK,R,VNT,P,PH,TDMP)
      C
      C      SUBROUTINE TEMP GIVES THE DIMENSIONLESS FUEL, CLAD, AND
      C      COOLANT TEMPERATURES AT VARIOUS RADIAL AND ANGULAR
      C      POSITIONS
      C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DM,EPS
10     DIMENSION R(1)
      GO TO (2,3,4),JA
      2 SUMV=0.0
      DO 10 N=1,NK
      A=CF*N
      15     SUMV=SUMV+R(N)*P**A*DCOS(A*PH)/A
      IF (IK-7) 10,12,12
      12     SUMV=SUMV+R(NK+1)*P**A*DSIN(A*PH)/A
      10     CONTINUE
      TDMP=SUMV*(1.-(P/T)**2)/(2.*DA*DAW)
      RETURN
      3 SUMT=0.0
      DO 20 N=1,NK
      A=CF*N
      S1=(1.+DAW*A/BIS)*P**A
      25     S2=(1.-DAW*A/BIS)*T**(2.*A)/(P**A)
      SUMT=SUMT+(S1+S2)*R(N)*DCOS(A*PH)/(2.*A)
      IF (IK-7) 20,22,22
      22     SUMT=SUMT+R(NK+1)*(S1+S2)*DSIN(A*PH)/(2.*A)
      20     CONTINUE
      TDMP=SUMT*(DLOG(T/P))/DA*(1./BIS)/(DA*DAW)
      RETURN
      4 SUM=0.0
      DO 30 N=1,NK
      A=CF*N
      35     FN=((1.+A/BIS-DA)*(1.+DAW*A/BIS)+(1.-DA-A/BIS)*T**(2.
      1*A))/4.
      GN=((1.+A/BIS+DA)*(1.+DAW*A/BIS)+(1.-DA-A/BIS)*T**(2.
      1*A))/4.
      SUM=SUM+(FN*P**(-1.*A)+GN*P**A)*R(N)*DCOS(A*PH)/A
      40     IF (IK-7) 30,32,32
      32     SUM=SUM+(FN*P**(-1.*A)+GN*P**A)*R(NK+1)*DSIN(A*PH)/A
      30     CONTINUE
      TDMP=SUM*(P**2-1.1)/(2.*VNT)-(1./VNT+1.1)*DLOG(P)/((-1./BI*DLOG(T))/D
      1A-(1./BIS)/(DA*DAW))
      45     RETURN
      END

```

CARD NO. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

11 I

AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 TEMP

VARIABLES	SN	TYPE	RELOCATION				
1252 A		DOUBLE		14	BI	DOUBLE	BUN2
12 B1S		DOUBLE	BUN2	0	CF	DOUBLE	F.P.
2 DA		DOUBLE	BUN2	0	DAM	DOUBLE	BUN2
22 DM		DOUBLE	BUN2	24	EPS	DOUBLE	BUN2
1264 FN		DOUBLE		1266	GN	DOUBLE	
0 IK		INTEGER	F.P.	0	JA	INTEGER	F.P.
1251 N		INTEGER		0	NK	INTEGER	F.P.
0 P		DOUBLE	F.P.	4	PD	DOUBLE	BUN2
0 PH		DOUBLE	F.P.	16	PI	DOUBLE	BUN2
0 R		DOUBLE	ARRAY	1262	SUM	DOUBLE	
1254 SUMT		DOUBLE		1247	SUMV	DOUBLE	
1256 S1		DOUBLE		1260	S2	DOUBLE	
6 T		DOUBLE	BUN2	0	TOMP	DOUBLE	F.P.
10 VE		DOUBLE	BUN2	0	YNT	DOUBLE	F.P.
20 VT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS			
DCOS	DOUBLE	1 LIBRARY	DLOG	DOUBLE	1 LIBRARY
DSIN	DOUBLE	1 LIBRARY			

STATEMENT LABELS

26 2		206 3		475 4			
145 10		0 12	INACTIVE	420 20			
0 22	INACTIVE	1052 30		0 32	INACTIVE		

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
31	10	* N	13 10	1178	EXT REFS
211	20	* N	22 29	2128	EXT REFS
500	30	* N	33 42	3558	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS

PROGRAM LENGTH	13448	740
SCN LABELED COMMON LENGTH	268	22

```

1      SUBROUTINE FLUX (JA,IK,CF,NK,R,YNT,AL,P,PH,Q2)
      C
      C
      C
      C
      C
      SUBROUTINE FLUX GIVES RADIAL HEAT FLUX IN THE FUEL, CLAD
      AND COOLANT REGIONS AT VARIOUS RADIAL AND ANGULAR POSITIONS

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /SUN2/ DAW,DA,PD,Y,VE,BIS,BI,PI,YT,DH,EPS
      DIMENSION R(NK)
      GO TO (1,2,3),JA
      1 SUM1=0.0
      DO 10 N=1,NK
      A=CF*N
      SUM1=SUM1+R(N)*T**(CF*N)*DCOS(CF*N*PH)
      IF (IK-7) 10,12,12
      12 SUM1=SUM1+R(NK+N)*T**A*DSIN(A*PH)
      10 CONTINUE
      Q2=1.-DA*DAW*SUM1
      RETURN
      2 SUM2=0.0
      DO 20 N=1,NK
      A=CF*N
      S1=(1.-DAW+A/BIS)
      S2=(1.-DAW+A/BIS)*T**(2.*A)
      SUM2=SUM2+(S1-S2)*R(N)*DCOS(A*PH)/2.
      IF (IK-7) 20,22,22
      22 SUM2=SUM2+(S1-S2)*R(NK+N)*DSIN(A*PH)/2.
      20 CONTINUE
      Q2=1.-DA*SUM2
      RETURN
      3 SUMA=0.0
      SUMB=0.0
      DO 30 N=1,NK
      A=CF*N
      FN=((1.+A/BIS-DA)*(1.-DAW+A/BIS)+(1.-DA-A/BIS)*T**(2.
      1*A))/4.
      GN=((1.+A/BIS+DA)*(1.-DAW+A/BIS)+(1.-DA-A/BIS)*T**(2.
      1*A))/4.
      SUMA=SUMA+R(N)*(GN*P**A-1.-FN*P**(-1.-A))*DCOS(A*PH)
      SUMB=SUMB+R(N)*(FN*P**(-1.-A)+GN*P**A)*DSIN(A*PH)
      IF (IK-7) 30,33,33
      33 SUMA=SUMA+R(NK+N)*(GN*P**A-1.-FN*P**(-1.-A))*DSIN(A*PH)
      SUMB=SUMB+R(NK+N)*(FN*P**(-1.-A)+GN*P**A)*DCOS(A*PH)
      30 CONTINUE
      Q2=SUMB*DSIN(PH-AL)/P-(SUMA+P/YNT-(1.+1./YNT)/P)*DCOS(PH-AL)
      RETURN
      END

```

CARD NO. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

11 I

AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 FLUX

VARIABLES	SN	TYPE	RELOCATION				
1142 A		DOUBLE		0	AL	DOUBLE	F.P.
14 BI		DOUBLE	BUN2	12	BIS	DOUBLE	BUN2
0 CF		DOUBLE	F.P.	2	DA	DOUBLE	BUN2
0 CAN		DOUBLE	BUN2	22	DM	DOUBLE	BUN2
24 EPS		DOUBLE	BUN2	1156	FN	DOUBLE	
1160 GN		DOUBLE		0	IK	INTEGER	F.P.
0 JA		INTEGER	F.P.	1141	N	INTEGER	
0 NK		INTEGER	F.P.	0	P	DOUBLE	F.P.
4 PD		DOUBLE	BUN2	0	PH	DOUBLE	F.P.
16 PI		DOUBLE	BUN2	0	Q2	DOUBLE	F.P.
0 P		DOUBLE	ARRAY	1152	SUM4	DOUBLE	
1154 SUM3		DOUBLE		1137	SUM1	DOUBLE	
1144 SUM2		DOUBLE		1146	S1	DOUBLE	
1150 S2		DOUBLE		6	T	DOUBLE	BUN2
18 VE		DOUBLE	BUN2	0	YNT	DOUBLE	F.P.
20 VT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS		DSIN	DOUBLE	1 LIBRARY
DCOS	DOUBLE	1 LIBRARY				

STATEMENT LABELS

26 1	133 2	310 3
115 10	0 12	275 20
0 22	1006 30	0 33

INACTIVE INACTIVE INACTIVE

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
31	10	* N	13 18	678	EXT REFS
136	20	* N	22 29	1428	EXT REFS
314	30	* N	34 45	4758	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS		
PROGRAM LENGTH	12468	678
SCN LABELED COMMON LENGTH	268	22

SCOPE 2 LOAD MAP

PROGRAM WILL BE ENTERED AT LOCAL (302)

LOADER VERSION 1.0 07/07/78 09.21.54 PAGE 1

SCN LENGTH 47223 LCH LENGTH 0

BLOCK	ADDRESS	LENGTH
/BUN1/	100	50
/BUN2/	150	26
/BUN3/	176	36
LOCAL	234	24101
COUPLE	24335	4342
SIZE	30677	165
TYPE	31064	151
CHAN	31235	1034
POTH	32271	75
NTB	32366	53
POINTS	32441	404
NEIGH	33045	331
GTPRD	33376	110
HATA	33506	150
LOC	33656	35
CGELS	33713	473
TAVG	34406	660
SIMPS	35266	613
TEMP	36101	1344
FLUX	37445	1246
/STP.END/	40713	1
/FCL.C./	40714	23
/O.IO./	40737	140
QENTPV=	41077	1
COMIO=	41100	44
FECMSK=	41144	41
FLTIN=	41205	156
FLTOUT=	41363	315
FMTAP=	41708	372
FORSYS=	42272	561
FORUTL=	43053	16
GETFIT=	43071	43
INCOM=	43134	256
INFC=	43412	173
KOCFR=	43605	467
KRAKER=	44274	454
/HFAIDING/	44750	16
/LIN.CNT/	44766	1
OUTC=	44767	1054
OUTCOM=	46043	204
GOTOER=	46247	14
QATAN=	46263	17
QATCOM=	46302	106
DEULER=	46410	60
DEXP=	46470	34
QALOG=	46524	146
DSKOS=	46672	150
OSCRY=	47042	32
DTOD=	47074	13
DTCI=	47107	21
DTGX=	47130	10
SYSID=	47140	1

SCOPE 2 LOAD MAP

SYS=1ST 47141 62

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**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

LOCALOR

LISTING OF INPUT PARAMETERS

19-ROD ANALYSIS

MAX NO OF POINTS = 36 MAX NO OF TERMS = 24 NO OF ITERATION = 5

P/D = 1.24000 A/B = .07000 2W/D = 1.40000

KP/K = .13300 K/KC = .40000

BIS = .10000E+02 BI = .10000E+07

ROOTS OF LEGENDRE POLYNOMIALS

.14897E+00	-.14897E+00	.43340E+00	-.43340E+00	.67941E+00	-.67941E+00	.86506E+00	-.86506E+00
.97391E+00	-.97391E+00						

WEIGHTS OF GAUSS-LEGENDRE QUADRATURE

.29552E+00	.29552E+00	.26927E+00	.26927E+00	.21909E+00	.21909E+00	.14945E+00	.14945E+00
.66671E-01	.66671E-01						

C.4 OUTPUT FOR TYPICAL RUN

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALOR
 ITERATION 1

CELL 4 TYPE 2 POWER = 1.0000 FLOW = .9571

COEFFICIENTS FOR THE FOURIER SERIES

-.96755E+00 .30141E+00 .20202E+00 -.14237E+00 -.77935E-01 .50519E-01 -.29802E-01 .67071E-02
 -.11484E-02 .40235E-04 .20907E-03 -.21355E-03

T MEAN = -2.30284 AREA = 1.61874 NU = 26.67189

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.61658	-1.76690	.99823	-2.11636	1.00956	-2.28267	-.29586
12.00000	1.47205	-1.76286	.99762	-2.11168	1.00637	-2.30296	-.12453
24.00000	1.40771	-1.75258	.99601	-2.09974	.99832	-2.28236	-.04131
36.00000	1.40771	-1.74074	.99394	-2.08583	.98843	-2.25606	-.07235
48.00000	1.47205	-1.73310	.99202	-2.07631	.97945	-2.22245	-.23729
60.00000	1.61658	-1.73503	.99075	-2.07688	.97290	-2.15056	-.50546
72.00000	1.85315	-1.75066	.99051	-2.09177	.96881	-1.96490	-.67431
84.00000	1.53272	-1.78289	.99167	-2.12410	.96810	-2.22350	-.31737
96.00000	1.35735	-1.83276	.99455	-2.17555	.97417	-2.32682	.03438
108.00000	1.26770	-1.89740	.99917	-2.24372	.98993	-2.40030	.27433
120.00000	1.24000	-1.96826	1.00474	-2.31964	1.01381	-2.48322	.37452
132.00000	1.26770	-2.03209	1.00970	-2.39827	1.03700	-2.57885	.34382
144.00000	1.35735	-2.07630	1.01235	-2.43445	1.04590	-2.66565	.23603
156.00000	1.35735	-2.09673	1.01224	-2.45298	1.03469	-2.66634	.15994
168.00000	1.26770	-2.19362	1.01070	-2.45328	1.01477	-2.62196	.25239
180.00000	1.24000	-2.10000	1.00985	-2.45091	1.00515	-2.60399	.29370

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = -.0304

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.3040	TAVG =	-2.2523
SIDE 1 FROM	.5236	TO	1.2485	AREA =	.5056	TAVG =	-2.1943

SIDE 2 Q GAP = .0833

SIDE 2 FROM	1.2485	TO	2.0944	AREA =	.4450	TAVG =	-2.2575
SIDE 2 FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG =	-2.5299

SIDE 3 Q GAP = .0000

SIDE 3 FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG =	-2.5723
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CELL 3 TYPE 3 POWER = 1.0000 FLOW = 1.0214

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALOR

COEFFICIENTS FOR THE FOURIER SERIES

.11427E+01 -.14072E+00 -.20545E+00 .10740E+00 -.16071E-01 -.72364E-01 -.34052E-01 -.26519E-02
 .33229E-02 .17704E-02 -.24373E-03 -.62534E-03

T MEAN = -2.31897 AREA = 1.49680 NU = 21.95085

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.43183	-2.07976	1.01176	-2.43698	1.04143	-2.67456	.10900
12.00000	1.30381	-2.06973	1.01052	-2.42462	1.02858	-2.61394	.19122
24.00000	1.24683	-2.04677	1.00817	-2.39748	1.00573	-2.54955	.23223
36.00000	1.24683	-2.02338	1.00716	-2.37323	1.00145	-2.52123	.22136
48.00000	1.30381	-2.00103	1.00784	-2.35478	1.02437	-2.53843	.19536
60.00000	1.43183	-1.96578	1.00745	-2.32307	1.04662	-2.58309	.25775
72.00000	1.30381	-1.90448	1.00366	-2.25848	1.03184	-2.46170	.20622
84.00000	1.24683	-1.82456	.99532	-2.16946	.98571	-2.31573	.17739
96.00000	1.24683	-1.75149	.98804	-2.08820	.94512	-2.20813	.13606
108.00000	1.30381	-1.70880	.98444	-2.04236	.93061	-2.15096	-.12431
120.00000	1.43183	-1.70520	.99541	-2.04068	.94105	-2.12560	-.48123
132.00000	1.66858	-1.73453	.98985	-2.07508	.96631	-2.08102	-.80234
144.00000	1.73050	-1.78197	.99586	-2.12842	.99449	-2.15617	-.43927
156.00000	1.53249	-1.83091	1.00163	-2.19266	1.01933	-2.36747	-.09253
168.00000	1.43128	-1.86692	1.00574	-2.22238	1.03668	-2.44535	.09104
180.00000	1.40000	-1.88010	1.00722	-2.23696	1.04277	-2.46348	.14015

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = -.0266

SIDE	1	FROM	TO	AREA	TAVG
SIDE 1	FROM	0.0000	.5236	.1821	-2.5386
SIDE 1	FROM	.5236	1.0472	.1821	-2.4740

SIDE 2 Q GAP = -.0949

SIDE	2	FROM	TO	AREA	TAVG
SIDE 2	FROM	1.0472	1.5708	.1821	-2.3675
SIDE 2	FROM	1.5708	2.4167	.4450	-2.1552

SIDE 3 Q GAP = -.0000

SIDE	3	FROM	TO	AREA	TAVG
SIDE 3	FROM	2.4167	3.1416	.5056	-2.3107

CELL 2 TYPE 4 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

-.10334E-11 .40636E-11 -.68681E-12 -.13094E-11 .92605E-12 .78958E-01 -.17464E-11 .83956E-12
 -.59342E-12 .34143E-12 -.33730E-12 -.21373E-03 .23571E-14 .13915E-12 -.10669E-13 -.86832E-13

T MEAN = -2.33654 AREA = 1.09240 NU = 18.40191

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALOR

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/O	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-1.86790	.99791	-2.21230	.97967	-2.34943	.19148
12.00000	1.26770	-1.87379	.99895	-2.22005	.98973	-2.37021	.13490
24.00000	1.35735	-1.88560	1.00105	-2.23564	1.01029	-2.42105	.02046
36.00000	1.35735	-1.89151	1.00210	-2.24345	1.02050	-2.44473	.15457
48.00000	1.26770	-1.88559	1.00104	-2.23562	1.01012	-2.40409	.25539
60.00000	1.24000	-1.87380	.99895	-2.22008	.98983	-2.36474	.24490
72.00000	1.26770	-1.86790	.99791	-2.21229	.97960	-2.35345	.16612
84.00000	1.35735	-1.87379	.99895	-2.22005	.98970	-2.37415	-.12956
96.00000	1.35735	-1.84561	1.00105	-2.21566	1.01030	-2.42151	.14665
108.00000	1.26770	-1.89150	1.00209	-2.24342	1.02040	-2.42120	.33909
120.00000	1.24000	-1.88560	1.00105	-2.23563	1.01017	-2.39535	.35175
132.00000	1.26770	-1.87380	.99896	-2.22009	.98988	-2.37056	.14983
144.00000	1.35735	-1.86789	.99790	-2.21226	.97950	-2.35093	-.14147
156.00000	1.35735	-1.87380	.99895	-2.22007	.98980	-2.37461	-.07337
168.00000	1.26770	-1.84561	1.00105	-2.23566	1.01027	-2.40444	.30632
180.00000	1.24000	-1.89150	1.00209	-2.24341	1.02033	-2.41066	.40517

CALCULATIONAL RESULTS FOR LUMPEE PARAMETER

SIDE 1	G GAP =	0.0000					
SIDE 1	FROM	0.0000	TO	.5236	AREA =	.1821	TAVG = -2.3369
SIDE 2	G GAP =	-.0196					
SIDE 2	FROM	.5236	TO	1.0472	AREA =	.1821	TAVG = -2.3531
SIDE 2	FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG = -2.3204
SIDE 3	G GAP =	-.0196					
SIDE 3	FROM	1.5708	TO	2.0944	AREA =	.1821	TAVG = -2.3527
SIDE 3	FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG = -2.3200
SIDE 4	G GAP =	-.0000					
SIDE 4	FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG = -2.3361

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

.78958E-01 -.21373E-03

T MEAN = -2.34310 AREA = .18207 NU = 17.35456

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOCALDR
DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)		
0.00000	1.24000	-1.87058	.99818	-2.21466	.97709	-2.34795	.15647		
2.00000	1.24076	-1.87078	.99822	-2.21495	.97758	-2.34876	.15583		
4.00000	1.24303	-1.87136	.99834	-2.21579	.97904	-2.35118	.15395		
6.00000	1.24683	-1.87232	.99853	-2.21717	.98141	-2.35516	.15096		
8.00000	1.25219	-1.87359	.99878	-2.21901	.98459	-2.36066	.14708		
10.00000	1.25913	-1.87513	.99909	-2.22123	.98845	-2.36759	.14259		
12.00000	1.26770	-1.87687	.99944	-2.22375	.99281	-2.37583	.13781		
14.00000	1.27796	-1.87874	.99981	-2.22645	.99750	-2.38526	.13311		
16.00000	1.28997	-1.88065	1.00019	-2.22922	1.00231	-2.39569	.12884		
18.00000	1.30381	-1.88251	1.00056	-2.23192	1.00703	-2.40686	.12537		
20.00000	1.31958	-1.88426	1.00091	-2.23445	1.01146	-2.41853	.12301		
22.00000	1.33738	-1.88581	1.00122	-2.23670	1.01539	-2.43032	.12204		
24.00000	1.35735	-1.88709	1.00147	-2.23855	1.01865	-2.44175	.12262		
26.00000	1.37963	-1.88805	1.00167	-2.23994	1.02109	-2.45226	.12480		
28.00000	1.40439	-1.88864	1.00178	-2.24080	1.02260	-2.46108	.12841		
30.00000	1.43183	-1.88864	1.00182	-2.24109	1.02311	-2.46723	.13301		

CALCULATIONAL RESULTS FOR LUMPEC PARAMETER

SIDE 1 Q GAP = 0.0000

SIDE 1 FROM 0.3000 TO .5236 AREA = .1821 TAVG = -2.3431

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOCALDR
 ITERATION 2

CELL 4 TYPE 2 POWER = 1.0000 FLOW = .9571

COEFFICIENTS FOR THE FOURIER SERIES

-.10075E+00 -.24465E-01 -.63063E-01 -.35759E+00 -.26700E-01 .07207E-01 -.51340E-01 -.36507E-02
 -.13086E-01 .11929E-02 .22059E-02 -.73012E-03

T MEAN = -2.34707 AREA = 1.61074 NU = 16.77572

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.61650	-2.10170	1.01726	-2.46333	1.06101	-2.75079	.01065
12.00000	1.47205	-2.08560	1.01573	-2.44552	1.05107	-2.69574	-.00517
24.00000	1.40771	-2.04250	1.01191	-2.39854	1.03319	-2.60786	.00202
36.00000	1.43771	-1.98201	1.00714	-2.33481	1.01935	-2.52952	.00482
48.00000	1.47205	-1.91130	1.00184	-2.26156	1.01064	-2.45963	-.00501
60.00000	1.61650	-1.83482	.99559	-2.18046	.99285	-2.34181	.00260
72.00000	1.85315	-1.76230	.98887	-2.10172	.96075	-1.94118	-1.51071
84.00000	1.53272	-1.71134	.98389	-2.04516	.93270	-2.05575	-.79799
96.00000	1.35735	-1.69757	.98291	-2.03042	.92814	-2.13444	-.21315
108.00000	1.26770	-1.72400	.98630	-2.06041	.94562	-2.18999	.12034
120.00000	1.24000	-1.75056	.99285	-2.12358	.97727	-2.26772	.25356
132.00000	1.26770	-1.84842	1.00035	-2.19938	1.01629	-2.37875	.26731
144.00000	1.35735	-1.90530	1.00593	-2.26177	1.04269	-2.50000	.23799
156.00000	1.35735	-1.93733	1.00781	-2.29346	1.03876	-2.51927	.22965
168.00000	1.26770	-1.94728	1.00707	-2.29966	1.01682	-2.47148	.26862
180.00000	1.24000	-1.94027	1.00637	-2.29860	1.00515	-2.45241	.29651

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = -.1117

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.3040	TAVG =	-2.6036
SIDE 1 FROM	.5236	TO	1.2485	AREA =	.5056	TAVG =	-2.3500

SIDE 2 Q GAP = .0843

SIDE 2 FROM	1.2485	TO	2.0944	AREA =	.4450	TAVG =	-2.1386
SIDE 2 FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG =	-2.3467

SIDE 3 Q GAP = .0000

SIDE 3 FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG =	-2.4185
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CELL 3 TYPE 3 POWER = 1.0000 FLOW = 1.0214

**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

LOCALDR

COEFFICIENTS FOR THE FOURIER SERIES

-.27077E+01 .31632E+01 -.40024E+00 -.11072E+01 .15032E+00 .60004E-01 -.50107E-01 .22401E-01
 .34377E-01 .09347E-02 -.30726E-02 -.27950E-02

T MEAN = -3.10510 AREA = 1.49600 NU = 2.08915

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/R	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.43183	.66305	.90526	.34794	.90585	.21261	-.20411
12.00000	1.39381	.66177	.90180	.34935	.89487	.20320	.11617
24.00000	1.24683	.64909	.89158	.34840	.83817	.24152	.00965
36.00000	1.24683	.58707	.87826	.30792	.72436	.28379	-.44387
48.00000	1.30381	.48657	.87148	.14452	.62994	.25586	-1.45011
60.00000	1.43183	.04744	.87996	-.21449	.62208	.07229	-2.35135
72.00000	1.33381	-.50697	.90595	-.73194	.67054	-.70237	-.21371
84.00000	1.24683	-1.23810	.94936	-1.54021	.78147	-1.55481	-.25250
96.00000	1.24683	-2.03764	1.00694	-2.43675	.99729	-2.58128	-.24632
108.00000	1.30381	-2.94090	1.06625	-3.34226	1.24361	-3.71496	.05604
120.00000	1.43183	-3.66712	1.11129	-4.10223	1.39407	-4.82649	.50257
132.00000	1.66050	-4.18807	1.13409	-4.63065	1.41238	-5.74829	1.12522
144.00000	1.73050	-4.50185	1.13756	-4.93316	1.34134	-5.71412	-.03731
156.00000	1.53249	-4.65963	1.13068	-5.07260	1.24648	-5.47074	.01759
168.00000	1.41129	-4.72135	1.12256	-5.12170	1.17543	-5.33426	.00333
180.00000	1.40000	-4.73665	1.11918	-5.13195	1.14940	-5.35582	-.02723

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = .0061

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.1821	TAVG =	.2571
SIDE 1 FROM	.5236	TO	1.0472	AREA =	.1821	TAVG =	.1285

SIDE 2 Q GAP = .9775

SIDE 2 FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG =	-.8995
SIDE 2 FROM	1.5708	TO	2.4167	AREA =	.4450	TAVG =	-4.4180

SIDE 3 Q GAP = .0000

SIDE 3 FROM	2.4167	TO	3.1416	AREA =	.5056	TAVG =	-5.3561
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CELL 2 TYPE 4 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

-.10487E+01 -.11153E+01 -.62225E-01 .23036E+00 .29011E+00 .32625E+00 .14429E+00 -.22705E-01
 -.55071E-01 -.96370E-02 -.13530E-01 .28460E-02 -.21445E-03 .34006E-02 .70895E-03 .12225E-02

T MEAN = -2.34850 AREA = 1.09240 NU = 16.57733

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOCALDR

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/S	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-2.79307	1.02690	-3.13272	.91870	-3.20335	-.31203
12.00000	1.26770	-2.82736	1.03406	-3.17924	.98250	-3.29073	-.25149
24.00000	1.35735	-2.89042	1.05266	-3.27277	1.14432	-3.54646	-.28998
36.00000	1.35735	-2.88332	1.06398	-3.29146	1.28983	-3.85465	2.05650
48.00000	1.26770	-2.72579	1.05403	-3.12140	1.22414	-3.44660	.86059
60.00000	1.24000	-2.45101	1.02889	-2.81001	1.02938	-2.95102	.12006
72.00000	1.26770	-2.16381	1.00712	-2.50188	.92956	-2.57809	-.14711
84.00000	1.35735	-1.92141	.99611	-2.26151	.95718	-2.38856	-.00660
96.00000	1.35735	-1.70820	.98933	-2.05323	.99537	-2.26069	-.00276
108.00000	1.26770	-1.49834	.98010	-1.83748	.96916	-1.98954	.00649
120.00000	1.24000	-1.30023	.96966	-1.62914	.91813	-1.73507	-.01305
132.00000	1.26770	-1.13791	.96226	-1.46295	.90400	-1.56586	.01396
144.00000	1.35735	-1.01856	.95823	-1.34404	.91213	-1.46073	-.01765
156.00000	1.35735	-.93730	.95586	-1.26142	.90547	-1.35664	-.43535
168.00000	1.26770	-.89401	.95546	-1.21928	.91221	-1.32815	-.06461
180.00000	1.24000	-.88259	.95604	-1.21132	.93448	-1.35189	.32327

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1	Q GAP =	0.0000					
SIDE 1	FROM	0.0000	TO	.5236	AREA =	.1821	TAVG = -3.3497
SIDE 2	Q GAP =	-.3855					
SIDE 2	FROM	.5236	TO	1.0472	AREA =	.1821	TAVG = -3.3745
SIDE 2	FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG = -2.5037
SIDE 3	Q GAP =	-.2856					
SIDE 3	FROM	1.5708	TO	2.0944	AREA =	.1821	TAVG = -2.0326
SIDE 3	FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG = -1.5055
SIDE 4	Q GAP =	-.0000					
SIDE 4	FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG = -1.3250

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

.27224E+00 .14671E-01

T MEAN = -2.35704 AREA = .18207 NU = 15.48157

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOCALOR
DEGREES	C/O	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	TIC)	QIC)		
0.00000	1.24008	-1.84771	.99357	-2.18081	.91403	-2.25532	-.32630		
2.00000	1.24076	-1.84844	.99372	-2.18193	.91634	-2.25868	-.31429		
4.00000	1.24303	-1.85060	.99416	-2.18524	.92308	-2.26863	-.27880		
6.00000	1.24683	-1.85407	.99487	-2.19053	.93376	-2.28476	-.22156		
8.00000	1.25219	-1.85869	.99581	-2.19751	.94760	-2.30642	-.14544		
10.00000	1.25913	-1.86421	.99693	-2.20580	.96365	-2.33272	-.05453		
12.00000	1.26770	-1.87038	.99818	-2.21496	.98085	-2.36254	.04593		
14.00000	1.27756	-1.87690	.99949	-2.22453	.99819	-2.39455	.14953		
16.00000	1.28997	-1.88348	1.00080	-2.23406	1.01478	-2.42733	.24886		
18.00000	1.30381	-1.88984	1.00206	-2.24313	1.02988	-2.45933	.33575		
20.00000	1.31958	-1.89569	1.00321	-2.25138	1.04299	-2.48903	.40174		
22.00000	1.33738	-1.90082	1.00422	-2.25850	1.05378	-2.51508	.43379		
24.00000	1.35735	-1.90501	1.00504	-2.26427	1.06214	-2.53643	.44035		
26.00000	1.37963	-1.90812	1.00564	-2.26850	1.06804	-2.55260	.40297		
28.00000	1.40439	-1.91003	1.00601	-2.27109	1.07155	-2.56395	.32959		
30.00000	1.43183	-1.91067	1.00613	-2.27196	1.07271	-2.57220	.22777		

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = 0.0000

SIDE 1 FROM 0.0000 TO .5236 AREA = .1821 TAVG = -2.3570

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOCAL OR
 ITERATION 3

CELL 4 TYPE 2 POWER = 1.0000 FLOW = .9571

COEFFICIENTS FOR THE FOURIER SERIES

-.47449E+00 -.23424E+01 -.15388E+01 .66484E+00 .72926E-01 -.15850E+00 -.50304E-01 .38832E-01
 -.22883E-01 .33769E-02 .25366E-03 -.21370E-03

T MEAN = -2.78739 AREA = 1.61874 NU = 4.17073

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.61658	-3.06625	1.04756	-3.43788	1.08799	-3.73044	.00838
12.00000	1.47205	-3.05138	1.04682	-3.42154	1.07982	-3.67687	-.00400
24.00000	1.40771	-3.01132	1.04553	-3.37889	1.06545	-3.59429	.00172
36.00000	1.40771	-2.95390	1.04546	-3.32129	1.06415	-3.52939	.00310
48.00000	1.47205	-2.87692	1.04722	-3.25020	1.08507	-3.49377	-.00326
60.00000	1.61658	-2.77236	1.04917	-3.14999	1.12041	-3.48475	.00150
72.00000	1.85315	-2.61035	1.04796	-2.99305	1.15176	-3.48299	-.01964
84.00000	1.53272	-2.37024	1.03988	-2.75218	1.15635	-3.22257	.02802
96.00000	1.35735	-2.04481	1.02269	-2.41617	1.11273	-2.72760	-.02506
108.00000	1.26770	-1.65488	.99789	-2.00615	1.02049	-2.18989	-.02187
120.00000	1.24000	-1.24845	.97104	-1.57769	.91861	-1.68887	.07668
132.00000	1.26770	-.87818	.94827	-1.19268	.85647	-1.26846	.04426
144.00000	1.35735	-.57461	.93160	-.89251	.83699	-.93566	-.01902
156.00000	1.35735	-.34511	.91920	-.64764	.81926	-.70021	-.66411
168.00000	1.26770	-.19616	.91019	-.49145	.78540	-.54003	-.44413
180.00000	1.24000	-.14339	.90665	-.43498	.76608	-.47667	-.36040

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 G GAP = -.0962

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.3040	TAVG =	-3.5850
SIDE 1 FROM	.5236	TO	1.2485	AREA =	.5056	TAVG =	-3.4065

SIDE 2 C GAP = -.4467

SIDE 2 FROM	1.2485	TO	2.0944	AREA =	.4450	TAVG =	-2.8057
SIDE 2 FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG =	-1.1456

SIDE 3 G GAP = .0000

SIDE 3 FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG =	-.6224
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CELL 3 TYPE 3 POWER = 1.0000 FLOW = 1.0214

**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

**** LOCALDR

COEFFICIENTS FOR THE FOURIER SERIES

-.44845E+00 .16903E+01 .20297E-01 -.23520E+00 .21995E+00 .41257E-01 -.56239E-01 -.33526E-01
 .17727E-02 .31237E-02 -.13616E-02 -.11949E-02

T MEAN = -2.77645 AREA = 1.49680 NU = 3.64570

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/O	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.43183	-.27989	.92331	-.58564	.83791	-.61041	-.45105
12.00000	1.38381	-.31306	.92483	-.61946	.83469	-.68244	-.26353
24.00000	1.24683	-.41618	.93013	-.72535	.84465	-.80356	-.10162
36.00000	1.24683	-.58299	.93915	-.90005	.88223	-1.00424	-.08190
48.00000	1.30381	-.79476	.94913	-1.11956	.91831	-1.25746	-.21329
60.00000	1.43183	-1.03564	.95755	-1.36153	.91566	-1.47945	-.44130
72.00000	1.30381	-1.31285	.96654	-1.63567	.88607	-1.71004	-.03905
84.00000	1.24683	-1.64788	.98182	-1.97585	.89886	-2.05852	-.04142
96.00000	1.24683	-2.03820	1.00537	-2.38556	.98918	-2.52813	-.00582
108.00000	1.30381	-2.43857	1.03152	-2.80939	1.10054	-3.06056	.02369
120.00000	1.43183	-2.79015	1.05258	-3.17653	1.16927	-3.57111	.01018
132.00000	1.65858	-3.05656	1.06489	-3.44827	1.18689	-3.96522	-.04739
144.00000	1.73059	-3.23160	1.06911	-3.62118	1.16967	-4.05100	-.01478
156.00000	1.53249	-3.32977	1.06812	-3.71343	1.13585	-4.01973	.00731
168.00000	1.43128	-3.37446	1.06549	-3.75223	1.10390	-3.98829	.00186
180.00000	1.40000	-3.38655	1.06421	-3.76184	1.09058	-3.97385	-.01003

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = .1986

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.1821	TAVG =	-.7007
SIDE 1 FROM	.5236	TO	1.0472	AREA =	.1821	TAVG =	-1.1916

SIDE 2 Q GAP = .4466

SIDE 2 FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG =	-1.7611
SIDE 2 FROM	1.5708	TO	2.4167	AREA =	.4450	TAVG =	-3.3858

SIDE 3 Q GAP = -.0000

SIDE 3 FROM	2.4167	TO	3.1416	AREA =	.5056	TAVG =	-3.9210
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CELL 2 TYPE 4 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

-.54424E+00 -.10919E+01 -.66367E-01 -.62590E-01 -.24015E-01 .82184E-01 .12071E+00 -.59683E-02
 -.20983E-02 -.13556E-02 -.11194E-02 .39757E-03 .99632E-03 .23412E-03 .47247E-04 -.42009E-04

T MEAN = -2.34638 AREA = 1.09240 NU = 16.87330

**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

**** LOCAL DR

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-2.95201	1.05135	-3.32301	1.00242	-3.49793	.36292
12.00000	1.25770	-2.93660	1.05220	-3.31265	1.10750	-3.52288	.44551
24.00000	1.35735	-2.87614	1.05172	-3.25918	1.14506	-3.57242	.66425
36.00000	1.35735	-2.74055	1.04407	-3.12729	1.13353	-3.44247	.32428
48.00000	1.26770	-2.55912	1.03143	-2.92324	1.06025	-3.11152	.12679
60.00000	1.24000	-2.34678	1.01750	-2.69658	.98908	-2.82311	.07154
72.00000	1.26770	-2.14020	1.00877	-2.49558	.98508	-2.63164	.17304
84.00000	1.35735	-1.96325	1.00378	-2.31542	1.02050	-2.51972	.35586
96.00000	1.35735	-1.76870	.99697	-2.12075	1.02774	-2.35127	.00445
108.00000	1.26770	-1.55669	.98562	-1.89903	.98142	-2.05449	-.00220
120.00000	1.24000	-1.34947	.97346	-1.69001	.92355	-1.78785	.00050
132.00000	1.26770	-1.17621	.96493	-1.50233	.90710	-1.60389	.00035
144.00000	1.35735	-1.04446	.96015	-1.37195	.92174	-1.49546	.00011
156.00000	1.35735	-.94479	.95612	-1.27144	.92135	-1.40192	-.23144
168.00000	1.26770	-.87565	.95194	-1.19733	.89553	-1.30177	-.12604
180.00000	1.24000	-.84974	.94998	-1.16810	.87853	-1.25881	-.08908

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1	Q GAP =	0.0000					
SIDE 1	FROM	0.0000	TO	.5236	AREA =	.1821	TAVG = -3.4344
SIDE 2	Q GAP =	-.2337					
SIDE 2	FROM	.5236	TO	1.0472	AREA =	.1821	TAVG = -3.1429
SIDE 2	FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG = -2.5375
SIDE 3	Q GAP =	-.2186					
SIDE 3	FROM	1.5708	TO	2.0944	AREA =	.1821	TAVG = -2.1031
SIDE 3	FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG = -1.5423
SIDE 4	Q GAP =	.0000					
SIDE 4	FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG = -1.3182

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

.13163E+00 .93436E-03

T MEAN = -2.34730 AREA = .18207 NU = 16.74459

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOCALDR
DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)		
0.00000	1.24000	-1.86445	.99695	-2.20573	.96122	-2.32537	.05050		
2.00000	1.24076	-1.86478	.99702	-2.20622	.96209	-2.32666	.05160		
4.00000	1.24303	-1.86577	.99722	-2.20766	.96467	-2.33051	.05494		
6.00000	1.24683	-1.86738	.99754	-2.21000	.96884	-2.33685	.06052		
8.00000	1.25219	-1.86952	.99797	-2.21312	.97438	-2.34557	.06938		
10.00000	1.25913	-1.87211	.99849	-2.21688	.98103	-2.35649	.07953		
12.00000	1.26770	-1.87532	.99907	-2.22112	.98849	-2.36941	.09394		
14.00000	1.27796	-1.87914	.99969	-2.22564	.99640	-2.38406	.10551		
16.00000	1.28997	-1.88132	1.00033	-2.23025	1.00442	-2.40013	.12207		
18.00000	1.30381	-1.88443	1.00095	-2.23474	1.01220	-2.41724	.14036		
20.00000	1.31958	-1.88733	1.00152	-2.23892	1.01939	-2.43495	.16003		
22.00000	1.33738	-1.88989	1.00203	-2.24261	1.02571	-2.45273	.18069		
24.00000	1.35735	-1.89200	1.00245	-2.24565	1.03090	-2.46998	.20194		
26.00000	1.37963	-1.89358	1.00277	-2.24792	1.03476	-2.48598	.22344		
28.00000	1.40439	-1.89456	1.00296	-2.24932	1.03714	-2.49990	.24503		
30.00000	1.43183	-1.89489	1.00303	-2.24980	1.03794	-2.51072	.26689		

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = 0.0000

SIDE 1 FROM 0.0000 TO .5236 AREA = .1821 TAVG = -2.3473

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALDR

ITERATION 4

CELL 4 TYPE 2 POWER = 1.0000 FLOW = .9571

COEFFICIENTS FOR THE FOURIER SERIES

-.32225E+00 -.13440E+01 -.14825E+01 .58570E+00 .14386E+00 -.21191E+00 -.20073E-01 .30902E-01
 -.26824E-01 .A5488E-02 -.21710E-02 .33735E-03

T MEAN = -2.69608 AREA = 1.61874 NU = 4.27146

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	OCLAD IN	TCLAD OUT	OCLAD OUT	T(C)	Q(C)
0.00000	1.61658	-3.03532	1.04669	-3.40667	1.08722	-3.69863	.00529
12.00000	1.47205	-3.02041	1.04593	-3.39028	1.07904	-3.64564	-.00257
24.00000	1.40771	-2.98024	1.04458	-3.34748	1.06455	-3.56266	.00126
36.00000	1.40771	-2.92267	1.04441	-3.29969	1.06304	-3.49734	.00183
48.00000	1.47205	-2.84762	1.04602	-3.21844	1.08361	-3.46151	-.00220
60.00000	1.61658	-2.74125	1.04778	-3.11825	1.11821	-3.45096	.00116
72.00000	1.85315	-2.58011	1.04636	-2.96197	1.14848	-3.44855	-.00071
84.00000	1.53272	-2.34212	1.03811	-2.72310	1.15255	-3.18502	.00075
96.00000	1.35735	-2.02022	1.02079	-2.39076	1.11004	-2.70093	-.00186
108.00000	1.26770	-1.63471	.99558	-1.98434	1.01659	-2.16902	-.00690
120.00000	1.24000	-1.23515	.96808	-1.56135	.90411	-1.66514	-.00355
132.00000	1.26770	-.88091	.94557	-1.19058	.83053	-1.24398	-.00118
144.00000	1.35735	-.60894	.93161	-.91414	.82055	-.93816	-.27625
156.00000	1.35735	-.42135	.92396	-.72706	.83299	-.79460	-.46137
168.00000	1.26770	-.30781	.91922	-.61144	.82505	-.68848	-.25063
180.00000	1.24000	-.26864	.91733	-.57023	.81411	-.63990	-.18569

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = -.0966

SIDE	FROM	TO	AREA	TAVG
1	0.0000	.5236	.3040	-3.5536
1	.5236	1.2485	.5056	-3.3738

SIDE 2 Q GAP = -.4435

SIDE	FROM	TO	AREA	TAVG
2	1.2485	2.0944	.4450	-2.7764
2	2.0944	2.6180	.1821	-1.1509

SIDE 3 Q GAP = .0000

SIDE	FROM	TO	AREA	TAVG
3	2.6180	3.1416	.1821	-.7314

CELL 3 TYPE 3 POWER = 1.0000 FLOW = 1.0214

**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

**** LOCALCR

COEFFICIENTS FOR THE FOURIER SERIES

-.10092E+01 .17899E+01 .11113E+00 -.21241E+00 .19290E+00 .10124E-01 -.58924E-01 -.20488E-01
 .88141E-02 .31881E-02 -.21815E-02 -.13375E-02

T MEAN = -2.79432 AREA = 1.49680 NU = 3.53064

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.43183	-.14474	.91559	-.44671	.81917	-.44749	-.53611
12.00000	1.33391	-.18284	.91730	-.48492	.81747	-.53790	-.28894
24.00000	1.24683	-.29801	.92304	-.60276	.82653	-.67335	-.13193
36.00000	1.24683	-.44610	.93305	-.79809	.85873	-.88779	-.19111
48.00000	1.33381	-.73142	.94551	-1.05262	.90040	-1.17017	-.40051
60.00000	1.43183	-1.01469	.95786	-1.34175	.92226	-1.47249	-.58964
72.00000	1.33381	-1.32989	.97015	-1.65783	.91291	-1.76197	.17681
84.00000	1.24683	-1.68773	.98637	-2.02040	.92178	-2.11766	.04952
96.00000	1.24683	-2.04773	1.00921	-2.43736	.99806	-2.58225	-.00320
108.00000	1.33381	-2.49275	1.03461	-2.86471	1.10379	-3.11516	.09548
120.00000	1.43183	-2.84903	1.05533	-3.23561	1.17343	-3.63530	.02132
132.00000	1.66858	-3.11711	1.06746	-3.51003	1.19136	-4.03529	-.02381
144.00000	1.73050	-3.29348	1.07146	-3.69408	1.17316	-4.11754	-.01338
156.00000	1.53249	-3.39207	1.07027	-3.77656	1.13839	-4.09406	.00667
168.00000	1.43128	-3.43682	1.06750	-3.81530	1.10592	-4.05187	.09166
180.00000	1.40000	-3.44891	1.06617	-3.82487	1.09247	-4.03738	-.00925

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = .2090

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.1821	TAVG =	-.5716
SIDE 1 FROM	.5236	TO	1.0472	AREA =	.1821	TAVG =	-1.1189

SIDE 2 Q GAP = .4489

SIDE 2 FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG =	-1.7903
SIDE 2 FROM	1.5708	TO	2.4167	AREA =	.4450	TAVG =	-3.4470

SIDE 3 Q GAP = -.0000

SIDE 3 FROM	2.4167	TO	3.1416	AREA =	.5056	TAVG =	-3.9852
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CELL 2 TYPE 4 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

-.49411E+00 -.10187E+01 .34315E-01 .16936E-01 .17097E-01 .99695E-01 .12590E+00 -.81478E-02
 -.60157E-02 -.36280E-02 -.99749E-03 .89530E-03 .10765E-02 -.25468E-04 -.26143E-04 -.68285E-04

T MEAN = -2.14680 AREA = 1.09240 NU = 16.81397

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALDR

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	OCLAD IN	TCLAD OUT	OCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-2.80083	1.03926	-3.16230	1.03281	-3.30803	.18843
12.00000	1.26770	-2.79920	1.04166	-3.16620	1.06549	-3.34740	.24779
24.00000	1.35735	-2.77537	1.04527	-3.15293	1.12557	-3.44806	.46475
36.00000	1.35735	-2.69405	1.04329	-3.07377	1.14126	-3.40475	.51731
48.00000	1.26770	-2.54525	1.03340	-2.91338	1.08208	-3.12690	.27710
60.00000	1.24000	-2.36022	1.02092	-2.71376	1.00724	-2.85024	.12952
72.00000	1.26770	-2.17764	1.01241	-2.52785	.99755	-2.67015	.18940
84.00000	1.35735	-2.00128	1.00718	-2.35587	1.03109	-2.57196	.40954
96.00000	1.35735	-1.80997	.99971	-2.16365	1.03424	-2.39818	.60174
108.00000	1.26770	-1.59823	.98784	-1.94151	.98454	-2.09780	-.00135
120.00000	1.24000	-1.39032	.97519	-1.72146	.92523	-1.82955	.60095
132.00000	1.26770	-1.21632	.96634	-1.54236	.90813	-1.64437	-.00076
144.00000	1.35735	-1.09405	.96138	-1.41192	.92266	-1.53544	.00135
156.00000	1.35735	-.98408	.95725	-1.31110	.92239	-1.44173	-.23080
168.00000	1.26770	-.91474	.95301	-1.23677	.89653	-1.34158	-.12402
180.00000	1.24000	-.85971	.95102	-1.20758	.87918	-1.29786	-.09139

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1	C GAP =	0.0000					
SIDE 1	FROM	0.0000	TO	.5236	AREA =	.1821	TAVG = -3.2979
SIDE 2	O GAP =	-.2163					
SIDE 2	FROM	.5236	TO	1.0472	AREA =	.1821	TAVG = -3.1193
SIDE 2	FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG = -2.5760
SIDE 3	O GAP =	-.2199					
SIDE 3	FROM	1.5708	TO	2.0944	AREA =	.1821	TAVG = -2.1470
SIDE 3	FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG = -1.5827
SIDE 4	O GAP =	.0000					
SIDE 4	FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG = -1.3578

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

.13133E+00 .65411E-03
T MEAN = -2.34731 AREA = .18207 NU = 16.74261

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOCALGR
DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)		
0.00000	1.24088	-1.86449	.99696	-2.20580	.96143	-2.32574	.05350		
2.00000	1.24876	-1.86482	.99703	-2.20629	.96229	-2.32701	.05443		
4.00000	1.24303	-1.86581	.99723	-2.20773	.96484	-2.33080	.05727		
6.00000	1.24683	-1.86741	.99755	-2.21005	.96894	-2.33704	.06207		
8.00000	1.25219	-1.86954	.99797	-2.21315	.97442	-2.34562	.06892		
10.00000	1.25913	-1.87212	.99849	-2.21689	.98101	-2.35640	.07792		
12.00000	1.26770	-1.87502	.99907	-2.22111	.98841	-2.36918	.08915		
14.00000	1.27796	-1.87813	.99969	-2.22562	.99629	-2.38372	.10265		
16.00000	1.28997	-1.88131	1.00032	-2.23022	1.00429	-2.39972	.11840		
18.00000	1.30381	-1.88441	1.00094	-2.23470	1.01207	-2.41682	.13629		
20.00000	1.31958	-1.88730	1.00152	-2.23888	1.01928	-2.43459	.15616		
22.00000	1.33738	-1.88986	1.00203	-2.24257	1.02564	-2.45254	.17776		
24.00000	1.35735	-1.89198	1.00245	-2.24562	1.03087	-2.47005	.20081		
26.00000	1.37961	-1.89356	1.00276	-2.24790	1.03477	-2.48640	.22504		
28.00000	1.40439	-1.89453	1.00296	-2.24930	1.03717	-2.50070	.25022		
30.00000	1.43123	-1.89486	1.00302	-2.24977	1.03798	-2.51188	.27631		

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = 0.0000

SIDE 1 FROM 0.0000 TO .5236 AREA = .1821 TAVG = -2.3473

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOCAL DR
 ITERATION 5

CELL 4 TYPE 2 POWER = 1.0000 FLOW = .9571

COEFFICIENTS FOR THE FOURIER SERIES

-.33942E+00 -.13253E+01 -.14804E+01 .58282E+00 .14610E+00 -.21336E+00 -.19269E-01 .38618E-01
 -.26866E-01 .86662E-02 -.22360E-02 .35369E-03

T MEAN = -2.69565 AREA = 1.61874 NU = 4.27541

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.61658	-3.03413	1.04666	-3.40546	1.08719	-3.69741	.00519
12.00000	1.47205	-3.01922	1.04589	-3.38907	1.07902	-3.64443	-.00252
24.00000	1.40771	-2.97904	1.04454	-3.34627	1.06451	-3.56144	.00125
36.00000	1.40771	-2.92146	1.04437	-3.28847	1.06300	-3.49610	.00179
48.00000	1.47205	-2.84641	1.04597	-3.21721	1.08356	-3.46027	-.00217
60.00000	1.61658	-2.74804	1.04772	-3.11702	1.11813	-3.44968	.00115
72.00000	1.85315	-2.57892	1.04629	-2.96074	1.14835	-3.44734	-.00787
84.00000	1.53272	-2.34100	1.03803	-2.72194	1.15236	-3.18353	-.00038
96.00000	1.35735	-2.01925	1.02070	-2.38974	1.10983	-2.69971	-.01732
108.00000	1.26770	-1.63400	.99550	-1.99407	1.01634	-2.16815	-.00695
120.00000	1.24000	-1.23484	.96799	-1.56094	.90366	-1.66450	-.00490
132.00000	1.26770	-.88129	.94550	-1.19083	.82985	-1.24364	-.11235
144.00000	1.35735	-.61043	.93164	-.91557	.82015	-.93881	-.29152
156.00000	1.35735	-.42420	.92414	-.73003	.83346	-.79802	-.45539
168.00000	1.26770	-.31180	.91953	-.61571	.82634	-.69365	-.24435
180.00000	1.24000	-.27307	.91769	-.57499	.81569	-.64558	-.17989

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = -.0967

SIDE 1 FROM	0.0000	TO	.5236	AREA =	.3040	TAVG =	-3.5523
SIDE 1 FROM	.5236	TO	1.2485	AREA =	.5056	TAVG =	-3.3725

SIDE 2 Q GAP = -.4431

SIDE 2 FROM	1.2485	TO	2.8944	AREA =	.4450	TAVG =	-2.7752
SIDE 2 FROM	2.8944	TO	2.6180	AREA =	.1821	TAVG =	-1.1515

SIDE 3 Q GAP = .0000

SIDE 3 FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG =	-.7353
-------------	--------	----	--------	--------	-------	--------	--------

CELL 3 TYPE 3 POWER = 1.0000 FLOW = 1.0214

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOCALR

COEFFICIENTS FOR THE FOURIER SERIES

-.10445E+01 .17933E+01 .11402E+00 -.21187E+00 .19240E+00 .98229E-02 -.58565E-01 -.19799E-01
 .92678E-02 .33423E-02 -.21463E-02 -.13288E-02

T MEAN = -2.79492 AREA = 1.49680 NU = 3.52691

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/O	TCLAD IN	OCLAD IN	TCLAD OUT	OCLAD OUT	TIC1	OIC1
0.00000	1.43183	-.13996	.91528	-.44167	.81793	-.43971	-.54944
12.00000	1.30381	-.17835	.91703	-.44027	.81677	-.53281	-.29843
24.00000	1.24683	-.29417	.92284	-.59884	.82637	-.66960	-.12953
36.00000	1.24683	-.48297	.93288	-.79445	.85822	-.88424	-.13317
48.00000	1.30381	-.72924	.94539	-1.05032	.89976	-1.16708	-.40727
60.00000	1.43183	-1.01377	.95785	-1.34084	.92231	-1.47183	-.59423
72.00000	1.33381	-1.33018	.97023	-1.65825	.91357	-1.76315	.12266
84.00000	1.24683	-1.68887	.98650	-2.02168	.92241	-2.11932	.05203
96.00000	1.24683	-2.08935	1.00934	-2.43907	.99841	-2.58410	-.00270
108.00000	1.30381	-2.49466	1.03473	-2.86669	1.10403	-3.11725	.02561
120.00000	1.43183	-2.85011	1.05544	-3.23775	1.17366	-3.63772	.02219
132.00000	1.65858	-3.11928	1.06755	-3.51225	1.19153	-4.03770	-.02311
144.00000	1.73050	-3.29568	1.07155	-3.68631	1.17327	-4.11981	-.01325
156.00000	1.51249	-3.39428	1.07014	-3.77879	1.13847	-4.08632	.00661
168.00000	1.41128	-3.43903	1.06757	-3.81753	1.10599	-4.05412	.02164
180.00000	1.40800	-3.45111	1.06624	-3.82710	1.09254	-4.03563	-.02315

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 O GAP = .2095

SIDE 1 FROM 0.0000 TO .5236 AREA = .1821 TAVG = -.5667
 SIDE 1 FROM .5236 TO 1.0472 AREA = .1821 TAVG = -1.1165

SIDE 2 C GAP = .4492

SIDE 2 FROM 1.0472 TO 1.5708 AREA = .1821 TAVG = -1.7909
 SIDE 2 FROM 1.5708 TO 2.4167 AREA = .4450 TAVG = -3.4492

SIDE 3 O GAP = -.0000

SIDE 3 FROM 2.4167 TO 3.1416 AREA = .5056 TAVG = -3.9875

CELL 2 TYPE 4 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

-.51258E+00 -.10161E+01 .41699E-01 .18747E-01 .18138E-01 .10059E+00 .12658E+00 -.82206E-02
 -.63529E-02 -.37872E-02 -.93821E-03 .98912E-03 .10707E-02 -.52272E-04 -.34797E-04 -.59581E-04

T MEAN = -2.34686 AREA = 1.09240 NU = 16.80588

**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

LOCALDR

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-2.79590	1.03888	-3.15674	1.03121	-3.30103	.18253
12.00000	1.26770	-2.79472	1.04133	-3.16144	1.06419	-3.34173	.24163
24.00000	1.35735	-2.77202	1.04507	-3.14946	1.12504	-3.44410	.45993
36.00000	1.35735	-2.64209	1.04324	-3.07186	1.14160	-3.40350	.52400
48.00000	1.26770	-2.54445	1.03343	-2.91268	1.08270	-3.12085	.28221
60.00000	1.24000	-2.36020	1.02098	-2.71378	1.00739	-2.85025	.12905
72.00000	1.26770	-2.17829	1.01249	-2.52852	.99763	-2.67066	.15636
84.00000	1.35735	-2.03251	1.00730	-2.35722	1.03166	-2.57438	.41487
96.00000	1.35735	-1.81153	.99933	-2.16531	1.03471	-2.40022	.00118
108.00000	1.26770	-1.59985	.98794	-1.94318	.95475	-2.09955	-.05110
120.00000	1.24000	-1.39191	.97527	-1.72307	.92531	-1.83117	.00387
132.00000	1.26770	-1.21785	.96641	-1.54442	.90418	-1.64593	-.00076
144.00000	1.35735	-1.08553	.96143	-1.41342	.92273	-1.53697	.00129
156.00000	1.35735	-.98549	.95729	-1.31253	.92244	-1.44326	-.23338
168.00000	1.26770	-.91608	.95304	-1.23912	.89650	-1.34287	-.12466
180.00000	1.24000	-.89003	.95104	-1.20889	.87913	-1.29914	-.09153

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1	Q GAP =	0.0000					
SIDE 1	FROM	0.0000	TO	.5236	AREA =	.1821	TAVG = -3.2935
SIDE 2	Q GAP =	-.2159					
SIDE 2	FROM	.5236	TO	1.0472	AREA =	.1821	TAVG = -3.1183
SIDE 2	FROM	1.0472	TO	1.5708	AREA =	.1821	TAVG = -2.5771
SIDE 3	Q GAP =	-.2199					
SIDE 3	FROM	1.5708	TO	2.0944	AREA =	.1821	TAVG = -2.1488
SIDE 3	FROM	2.0944	TO	2.6180	AREA =	.1821	TAVG = -1.5843
SIDE 4	Q GAP =	.0000					
SIDE 4	FROM	2.6180	TO	3.1416	AREA =	.1821	TAVG = -1.3592

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0343

COEFFICIENTS FOR THE FOURIER SERIES

.13105E+00 .68420E-03

T MEAN = -2.34728 AREA = .18207 NU = 16.74613

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOCALR
DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)		
0.00000	1.24000	-1.86452	.99697	-2.20584	.96149	-2.32582	.05369		
2.00000	1.24076	-1.86485	.99704	-2.20633	.96235	-2.32709	.05464		
4.00000	1.24383	-1.86584	.99723	-2.20776	.96490	-2.33088	.05752		
6.00000	1.24683	-1.86743	.99755	-2.21008	.96900	-2.33711	.06237		
8.00000	1.25219	-1.86956	.99798	-2.21318	.97447	-2.34570	.06929		
10.00000	1.25913	-1.87213	.99849	-2.21692	.98105	-2.35648	.07835		
12.00000	1.26770	-1.87503	.99907	-2.22113	.98945	-2.36925	.08964		
14.00000	1.27796	-1.87814	.99969	-2.22563	.99631	-2.38377	.10317		
16.00000	1.28997	-1.88131	1.00032	-2.23022	1.00430	-2.39975	.11891		
18.00000	1.30381	-1.88440	1.00094	-2.23469	1.01205	-2.41681	.13673		
20.00000	1.31958	-1.88729	1.00152	-2.23886	1.01925	-2.43455	.15645		
22.00000	1.33738	-1.88984	1.00202	-2.24254	1.02559	-2.45244	.17732		
24.00000	1.35735	-1.89195	1.00244	-2.24559	1.03081	-2.46988	.20052		
26.00000	1.37963	-1.89353	1.00276	-2.24795	1.03469	-2.48615	.22429		
28.00000	1.40439	-1.89450	1.00295	-2.24926	1.03708	-2.50038	.24991		
30.00000	1.43183	-1.89483	1.00302	-2.24973	1.03789	-2.51149	.27434		

CALCULATIONAL RESULTS FOR LUMPED PARAMETER

SIDE 1 Q GAP = 0.0000

SIDE 1 FROM 0.0000 TO .5236 AREA = .1821 TAVG = -2.3473

***** LOCALR //// END OF LIST ////
 ***** LOCALR //// END OF LIST ////

APPENDIX D LØSUB CODE

D.1 INTRODUCTION

LØSUB Code is basically a modified version of LØCAL used to handle the coupled distributed-lumped parameter analysis for the 7-rod bundles. The code essentially carries out the computation as described in Chapter 7.

The listing of the input parameters and the FØRTRAN algorithm of the code are presented in Section D.2 and D.3, respectively. The corresponding output is given in Section D.4.

D.2 INPUT DESCRIPTION OF THE LØSUB CODE

Card No.	Column	Symbol	Description	Format
1	1-5	NI	Maximum No. of points	15
1	5-10	NK	Maximum No. of terms	15
1	11-15	MMA	Control option	15
			1 - 7-rod bundle	
			2 - 19-rod bundle	
			3 - 37-rod bundle	
			4 - 61-rod bundle	
			5 - 127-rod bundle	
1	16-20	IT	Maximum No. of iterations	15
1	21-25	NT	Maximum No. of angular intervals to be printed	15
2	1-10	PD	Pitch-to Diameter ratio	E10.4
2	11-20	T	Clad inside to outside diameter ratio	E10.4
2	21-30	VE	Dimensionless wall distance	E10.4
2	31-40	DAW	Fuel to clad thermal conductivity ratio	E10.4
2	41-50	DA	Clad to coolant thermal conductivity ratio	E10.4
2	51-60	BIS	Biot number at the gap	E10.4
2	61-70	BI	Biot number at the clad /coolant interface	E10.4
3-4	1-80	Z(I)	Roots of the Legendre Polynomials	E10.4
5-6	1-80	W(I)	Weights of the Gauss-Legendre quadrature	E10.4
7	1-80	TSUB(K)	Subchannel temperatures	E10.4
8	1-10	QAK	Normalization factor	E10.4
9	1-80	Q(K)	Power peaking factor	E10.4

407

```

IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /BUN1/ M(10),Z(10)
COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
COMMON /RUN3/ Q(10),MT(10),TSUB(10),QAK
DIMENSION A(5000)
PI=3.1415926536
EPS=1.00-12
READ (5,101) NI,NK,MMAX,IT,NT
READ (5,100) PD,T,VE,DAW,DA,BIS,BI
READ (5,100) IZ(I),I=1,10)
READ (5,100) IW(I),I=1,10)
READ (5,100) (TSUB(K),K=1,2)
READ (5,100) QAK
GO TO (1,2,3,4,5),MMAX
KK=2
GO TC 10
KK=4
GO TO 10

```

```

3 KK=6
GO TO 18
60 4 KK=9
GO TO 18
5 KK=12
18 CONTINUE
READ (5,100) (Q(K),K=1,KK)
65 100 FORMAT (8E10.4)
101 FORMAT (16I5)
MZ=(NK*(NK+1))/2
MK=K-1
ROD=1.0
70 DO 7 K=1,MMAK
7 ROD=ROD+6.*K
AF=(HMAX*PD*1.732/2.+VE/2.)*(HMAX*PD+VE/1.732)-ROD*PI/12.
YT=AF/(ROD*PI/12.)
DH=AF/(ROD*PI/12.+ (HMAX*PD+VE/1.732)/2.)
75 JPHI=10.
JYJ=JPHI+NI
JGJN=JYJ+NI
JR=JGJN+NI*NK
JGTG=JR+HK
80 JAUX=JGTG+MZ
JDY=JAUX+MK
JR1=JDY+KK*NK
JYP=JR1+HK
JGP=JYP+NI
85 JX=JGP+NI*NK
JF=JX+NK
JG=JF+NK
JDF=JG+NK
JDG=JDG+KK*NK
90 JB=JDG+KK*NK
WRITE (6,101) NI,NK,MMAK,IT,KK,JB
CALL COUPLE (MMAK,NI,NK,MZ,NK,KK,A(JPHI),A(JYJ),A(JGJN),A(JR),A(JG
1TG),A(JAUX),A(JDY),A(JR1),A(JYP),A(JGP),A(JX),A(JF),A(JG),A(JDF),A
1(JDG),IT,NT)
95 END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
46 LOCAL

VARIABLES	SN	TYPE	RELOCATION	ARRAY	473	AF	DOUBLE	BUN2
515 A		DOUBLE			12	BIS	DOUBLE	BUN2
14 BI		DOUBLE		BUN2	0	DAW	DOUBLE	BUN2
2 OA		DOUBLE		BUN2	24	EPS	DOUBLE	BUN2
22 OH		DOUBLE			462	IT	INTEGER	
464 I		INTEGER			514	JB	INTEGER	
502 JAUX		INTEGER			513	JDG	INTEGER	
512 JDF		INTEGER			510	JF	INTEGER	
503 JDY		INTEGER						

PROGRAM LOCAL

76/76 OPT=1

FTN 4.6+439

07/10/78 13.16.598

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VARIABLES SN TYPE RELOCATION

511 JG INTEGER
 506 JGP INTEGER
 475 JPHI INTEGER
 504 JRI INTEGER
 476 JVJ INTEGER
 465 K INTEGER
 478 MK INTEGER
 24 MT INTEGER
 457 NI INTEGER
 463 NY INTEGER
 16 PI DOUBLE
 62 QAK DOUBLE
 6 T DOUBLE
 10 VE DOUBLE
 28 YT DOUBLE

ARRAY BUN3

BUN2

BUN3

BUN2

BUN2

BUN2

477 JGJN INTEGER
 501 JGTG INTEGER
 500 JR INTEGER
 507 JX INTEGER
 505 JYP INTEGER
 466 KK INTEGER
 461 MHAX INTEGER
 467 HZ INTEGER
 460 NK INTEGER
 4 PD DOUBLE
 0 Q DOUBLE
 471 ROD DOUBLE
 36 TSUB DOUBLE
 8 W DOUBLE
 24 Z DOUBLE

ARRAY

BUN2

BUN3

ARRAY

BUN3

ARRAY

BUN1

ARRAY

BUN1

FILE NAMES

MODE

0 INPUT

28 OUTPUT

0 TAPE5

FMT

20 TAPE6

FMT

EXTERNALS

TYPE

AR65

COUPLE

23

STATEMENT LABELS

101 1
 107 4
 112 10

103 2
 111 5
 430 100 FMT

105 3
 0 7
 432 101 FMT

LOOPS

LABEL

INDEX

FROM-TO

LENGTH

PROPERTIES

130 7

K

70 71

78

INSTACK

COMMON BLOCKS

LENGTH

BUN1

48

BUN2

22

BUN3

52

STATISTICS

PROGRAM LENGTH

248758

10301

BUFFER LENGTH

408

32

SCH LABELED COMMON LENGTH

1628

114


```

1      SUBROUTINE COUPLE (HMAX,N1,NK,MZ,HK,KK,PHI,VJ,GJN,R,GTG,AUX,DYN,R,
    1  YP,GP,X,FN,GN,DFN,DGN,IT,NT)
C
C C C C C
5      SUBROUTINE COUPLE CALLS ALL SUBROUTINES AND COUPLES ALL
        UNIT CELLS TOGETHER THROUGH VARIOUS BOUNDARY CONDITIONS
C
C
10     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
        COMMON /BUN1/ M(10),Z(10)
        COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
        COMMON /BUN3/ Q(10),MT(10),TSUB(10),QAK
        DIMENSION PHI(N1),VJ(N1),GJN(N1,NK),R(NK),GTG(MZ),AUX(MK),DYN(KK,M
    1  K),R1(NK),X(NK),YP(N1),GP(N1,NK),FN(NK),GN(NK),DFN(KK,NK),DGN(KK,M
    2  ZK)
        DIMENSION AI(8),AL(8),AB(8),AI1(8),AL1(8),AB1(8),AP(8),MJ(10)
        DIMENSION AREA(10,10),TLUMP(10,10),THEAN(10),UC(10)
C
C C C
20     WRITE (6,100)
        GO TO (1010,1020,1030,1040,1050),HMAX
    1010 WRITE (6,101)
        GO TO 1060
    1020 WRITE (6,102)
        GO TO 1060
    1030 WRITE (6,103)
        GO TO 1060
    1040 WRITE (6,104)
        GO TO 1060
    1050 WRITE (6,105)
    1060 WRITE (6,106) NI,NK,IT
        WRITE (6,111) PD,T,VE
        WRITE (6,112) DAW,DA
    35     WRITE (6,113) BIS,BI
        WRITE (6,114)
        WRITE (6,273) Z
        WRITE (6,115)
        WRITE (6,273) M
    40     WRITE (6,116)
        WRITE (6,273)(TSUB(K),K=1,Z)
        WRITE (6,117) QAK
    180 FORMAT (1H1,* LISTING OF INPUT PARAMETERS *,/)
    181 FORMAT (* 7-ROD ANALYSIS *,/)
    45     182 FORMAT (* 19-ROD ANALYSIS *,/)
    183 FORMAT (* 37-ROD ANALYSIS *,/)
    184 FORMAT (* 37-ROD ANALYSIS *,/)
    185 FORMAT (* 127-RJD ANALYSIS *,/)
    186 FORMAT (* MAX NO OF POINTS= *,I5,5X,* MAX NO OF TERMS = *,I5,5X,*
    50     1 NO OF ITERATION = *,I5,5X,/)
    111 FORMAT (* P/D = *,F12.5,5X,* A/B = *,F12.5,5X,* 2W/D =*,F12.5,/)
    112 FORMAT (* KF/K = *,F12.5,5X,* K/KC = *,F12.5,/)
    113 FORMAT (* BIS = *,E12.5,5X,* BI = *,E12.5,/)
    114 FORMAT (* ROOTS OF LEGENDRE POLYNOMIALS *,/)
    115 FORMAT (/,* WEIGHTS OF GAUSS-LEGENDRE QUADRATURE *,/)
    117 FORMAT (/,* NORMALIZATION FACTOR = *,F10.4,* DEGREE F *,/)
    116 FORMAT (/,* SUBCHANNEL TEMPERATURES *,/)

```



```

115      340 IB=2
      350 IF ((M1.EQ.-1).AND.(M1.EQ.-1)) IB=0
      IF (KI.EQ.1) IB=0
      IF (IB.EQ.0) GO TO 63
      CALL TYPE (M1,N1,MMAX,K1,IK1,CF1,X1,Y1,A1)
120      CALL CHAN (K1,IK1,ISI,ATI,ALI,ABI,UI)
      VT1=VT*Q(K1)/UI
      NF1=NT(K1)
      DO 22 N=1,NF
      R(N)=DYN(K,N)
125      IF (IK-7) 22,210,210
      210 R(NF+N)=DYN(K,NF+N)
      22 CONTINUE
      DO 74 N=1,NF1
      R1(N)=DYN(K1,N)
130      IF (IK1-7) 74,76,76
      76 R1(NF1+N)=DYN(K1,NF1+N)
      74 CONTINUE

      C
      C      CONSTRUCTION OF THE COEFFICIENT MATRIX
      C
135      63 DO 61 J=1,JM
      JP=J+JJ
      RH=A1(I)/OCOS(PHI(JP)-AL(I))
      IF (IB-1) 60,78,78
140      78 XX=XC+RH*OCOS(PHI(JP)+AX)
      YY=YC+RH*DSIN(PHI(JP)+AX)
      RH1=DSORT((XX-X1)**2+(YY-Y1)**2)
      PHI=-1.*A1/DATAN((YY-Y1)/(XX-X1))
      IF ((XX-X1).LE.0.0) PHI=-1.*A1+PI+DATAN((YY-Y1)/(XX-X1))
145      IF (IB-1) 73,73,75

      C
      C      COUPLING BY USING CONTINUITY OF TEMPERATURE
      C
150      73 CALL TEMP (3,IK,CF,NF,R,YNT,RH,PHI(JP),TT)
      CALL TEMP (3,IK1,CF1,NF1,R1,YT1,RH1,PH1,T1)
      MH=(RH**2-1./((2.*YNT)-(1./YNT+1.))*DLOG(RH))/(DLOG(T)-1./BI)/DA-(1.
      1/BIS)/(DA*DAW)
      VJ(JP)=-1.*Q(K1)*MH+(TT*Q(K1)+T1*Q(K1))/2.
      IF (JB.EQ.0) GO TO 34
155      DO 36 NB=1,JB
      31 ZZ=1.0
      IF (NB.EQ.1) ZZ=-1.
      GJN(JP,NB)=1.0*ZZ/2./FZ
      36 CONTINUE
      34 DO 62 N=1,NF
      A=CF*N
160      62 GJN(JP,N+JB)=(FN(N)*RH**(-1.*A)+GN(N)*RH**A)*OCOS(A*PHI(JP))*Q(K)/
      1A
      IF (IK.LT.7) GO TO 61
      DO 27 N=1,NF
      A=CF*N
165      27 GJN(JP,NF+JB+N)=(FN(N)*RH**(-1.*A)+GN(N)*RH**A)*DSIN(A*PHI(JP))*Q(
      1K1/A
      GO TO 61
170      75 II=1
      62 IF (PH1-ABI(II+1)) 60,60,61

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```

      81 I1=I1+1
      GO TO 82
175  C
      C      COUPLING BY USING CONTINUITY OF HEAT FLUX
      C
      88 CALL FLUX (3,IK1,CF1,NF1,R1,VT1,AL1(I1),RH1,PH1,Q2N)
      Q2N=Q2N*Q(K1)/Q(K)
      GO TO 77
180  60 Q2N=0.0
      77 YJ(JP)=(1.+1./YNT)-(RH**2)/YNT)*DCOS(PHI(JP)-AL(I1))+RH*Q2N
      IF (KI-1) 52,52,54
      52 DO 29 N=1,NF
      29 GJN(JP,N)=GN(N)*RH**((CF*N)*DCOS((CF*N-1.)*PHI(JP)+AL(I1))-FN(N)*RH*
185  1*(-1.*CF*N)*DCOS((CF*N+1.)*PHI(JP)-AL(I1))
      IF (IK.LT.7) GO TO 61
      DO 28 N=1,NF
      A=CF*N
      28 GJN(JP,NF+N)=FN(N)*RH**(-1.*A)*DSIN((A-1.)*PHI(JP)+AL(I1))-GN(N)*RH
190  1**A*DSIN((A+1.)*PHI(JP)-AL(I1))
      GO TO 61
      54 DO 32 NB=1,JB
      32 GJN(JP,NB)=0.0
      DO 72 N=1,NF
      72 GJN(JP,N+JB)=GN(N)*RH**((CF*N)*DCOS((CF*N-1.)*PHI(JP)+AL(I1))-FN(N)*
195  1RH**(-1.*CF*N)*DCOS((CF*N+1.)*PHI(JP)-AL(I1))
      IF (IK.LT.7) GO TO 61
      DO 19 N=1,NF
      A=CF*N
      19 GJN(JP,NF+JB+N)=FN(N)*RH**(-1.*A)*DSIN((A-1.)*PHI(JP)+AL(I1))-GN(N)
200  1**RH**A*DSIN((A+1.)*PHI(JP)-AL(I1))
      61 CONTINUE
      IF (I8.EQ.1) IC=IC+1
      4888 JJ=JJ+MJ(I)
205  IF (KI.EQ.1) GO TO 5000
      C
      C      IMPLEMENTATION OF THE SUBCHANNEL TEMPERATURES
      C
      CALL SIMPS (SIMP1,AB(1),AL(2),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,1)
210  CALL SIMPS (SIMP2,AB(1),AL(2),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,2)
      CALL SIMPS (SIMP4,AB(1),AL(2),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,4)
      DX=AL(2)-AB(1)
      FA=(SIMP4-DX)/2.
      TH=-1./(2.*YNT)+(LOG(1)-1./8I)/DA-1./(8IS*DA*DAW)
      TH=TH+((SIMP1-DX)/(8.*YNT)-(YNT+1.)*(SIMP2-DX)/(4.*YNT))/FA
215  YJ(JJ+1)=TSUB(2)/(QAK*FZ)-TH
      GJN(JJ+1,1)=0.0
      GJN(JJ+1,2)=1.0/FZ
      DO 41 N=1,NF
      CALL LUMP (CK,AB(1),AL(2),N,FN,GN,AI,AL,AB,CF,YNT)
220  41 GJN(JJ+1,N+2)=CK/FA
      CALL SIMPS (SIMP1,AL(2),AB(4),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,1)
      CALL SIMPS (SIMP2,AL(2),AB(4),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,2)
      CALL SIMPS (SIMP4,AL(2),AB(4),NF,FN,GN,R,AI,AL,AB,CF,YNT,IK,4)
225  DX=AB(4)-AL(2)
      FA=(SIMP4-DX)/2.
      TH=-1./(2.*YNT)+(LOG(1)-1./8I)/DA-1./(8IS*DA*DAW)
      TH=TH+((SIMP1-DX)/(8.*YNT)-(YNT+1.)*(SIMP2-DX)/(4.*YNT))/FA

```

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230      YJ(JJ+2)=(1.+AREA(1,1)*UC(1)/UC(2)/AREA(2,1))*TSUB(1)/(QAK*FZ)
1      -TLUMP(1,1)*AREA(1,1)*UC(1)/(AREA(2,1)*UC(2))-TH
      GJN(JJ+2,1)=AREA(1,1)/AREA(2,1)/FZ*UC(1)/UC(2)
      GJN(JJ+2,2)=1.0/FZ
      DO 42 N=1,NF
      CALL LUMP ICK,AL(2),AB(4),N,FN,GN,AI,AL,AB,CF,YNT)
235      42 GJN(JJ+2,N+2)=CK/FA
5000     LJ=NP+JB
      IF (KI.EQ.1) LJ=NP
C
C      SOLVING THE UNKNOWN VECTOR
240     C
      CALL SIZE (NI,NK,HK,MZ,LJ,LF,GJN,YJ,GP,YP,GTG,AUX,EPS,R)
      WRITE (6,401)
401     FORMAT (/,* COEFFICIENTS FOR THE FOURIER SERIES *,/)
      WRITE (6,273)(R(N),N=1,LF)
245     DO 21 N=1,NF
      JL=JB
      IF (KI.EQ.1) JL=0
      DYN(K,N)=R(N+JL)
      X(N)=R(N+JL)
250     IF (IK=7) 21,23,23
      23 DYN(K,NF+N)=R(JL+NF+N)
      X(NF+N)=R(JL+NF+N)
      21 CONTINUE
      IF (KI.EQ.1) GO TO 600
      IF (K.EQ.1) GO TO 600
      XA1=R(1)
      XA2=R(2)
255     600 CONTINUE
      ISI=IS+1
260     C
C      DETERMINATION OF NUSSELY NO
C
      CALL TAVG (AB(1),AB(ISI),IK,NF,FN,GN,X,AI,AL,AB,CF,YNT,T3,FA)
      TMEAN(K)=T3
      OU=2./(DLOG(1)/DA-1.)/(DA*QAK*8ISI)-T3)
265     WRITE (6,51) T3,FA,OU
      51 FORMAT (/,* T MEAN = *,F12.5,5X,* AREA = *,F12.5,5X,* NU =*,F12.5)
C
C      AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX
270     C
      WRITE (6,650)
650     FORMAT (/,* AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX *)
      WRITE (6,651)
651     FORMAT (///,5X,* DEGREES *,8X,* C/B *,5X,* TCLAD IN *,5X,* QCLAD IN
1      *,5X,* TCLAD OUT *,5X,* QCLAD OUT *,5X,* TIC) *,5X,* Q(IC) *,/)
      I=1
      DO 65 J=1,NT
      PH=(J-1)*(A9(IS+1)-AB(1))/(NT-1)
      IF (PH.GT.AR(I+1)) I=I+1
      RA=AI(I)/DCOS(PH-AL(I))
      FS=T
      CALL TEMP (2,IK,CF,NF,X,YNT,FS,PH,TU)
      CALL TEMP (2,IK,CF,NF,X,YNT,1.00,PH,TC2)
      CALL TEMP (3,IK,CF,NF,X,YNT,RA,PH,TK2)
280     CALL FLUX (2,IK,CF,NF,X,YNT,AL(I),FS,PH,Q1)
285

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      CALL FLUX (2,IK,CF,NF,X,YNT,AL(1),1.00,PH,Q2)
      CALL FLUX (3,IK,CF,NF,X,YNT,AL(1),RA,PH,Q3)
      DG=PH*180./PI
      IF (KI.EQ.1) GO TO 700
290  XA0=XA2
      IF (K.EQ.1) XA0=XA1
      TV=(XA0+TV)*QAK
      TC2=(XA0+TC2)*QAK
      TK2=(XA0+TK2)*QAK
295  700 CONTINUE
      65 WRITE (6,127) DG,RA,TV,Q1,TC2,Q2,TK2,Q3
      GO TO (151,150),IK
      151 CALL TAVG (AB(1),AB(2),IK,NF,GN,X,AI,AL,AB,CF,YNT,T3,FA)
      AREA (K,1)=FA
      TLUMP (K,1)=T3
300  GO TO 14
      150 CALL TAVG (AB(1),AL(2),IK,NF,GN,X,AI,AL,AB,CF,YNT,T3,FA)
      AREA (K,2)=FA
      TLUMP (K,2)=T3
305  CALL TAVG (AL(2),AB(4),IK,NF,GN,X,AI,AL,AB,CF,YNT,T3,FA)
      AREA (K,1)=FA
      TLUMP (K,1)=T3
      14 NN=NN-2
      IF (NN) 142,3000,3000
310  142 MM=MM-1
      IF (MM) 1000,2000,2000
      1000 CONTINUE
      127 FORMAT (8I2X,F12.5)
      273 FORMAT (8I2X,E12.5)
315  RETURN
      END

```

CARD NO. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

297 I

AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 COUPLE

VARIABLES	SM	TYPE	RELOCATION				
4510 A		DOUBLE		4707 AB	DOUBLE	ARRAY	
4767 AB1		DOUBLE	ARRAY	4667 AI	DOUBLE	ARRAY	
4727 AI1		DOUBLE	ARRAY	4667 AL	DOUBLE	ARRAY	
4747 AL1		DOUBLE	ARRAY	5007 AP	DOUBLE	UNDEF	
5041 AREA		DOUBLE	ARRAY	0 AUX	DOUBLE	ARRAY	F.P.
4474 AX		DOUBLE		4531 A1	DOUBLE		
14 BI		DOUBLE		12 BIS	DOUBLE		BUN2
4466 CF		DOUBLE		4523 CF1	DOUBLE		

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VARIABLES	SN	TYPE	RELOCATION						
4605 CK		DOUBLE		2	DA	DOUBLE		BUN2	
0 OAW		DOUBLE	BUN2	0	DFN	DOUBLE	ARRAY		F.P.
4643 OG		DOUBLE		0	OGN	DOUBLE	ARRAY		F.P.
22 OH		DOUBLE	BUN2	4617	OU	DOUBLE			
4577 OX		DOUBLE		0	DYN	DOUBLE	ARRAY		F.P.
24 EPS		DOUBLE	BUN2	4601	FA	DOUBLE			
0 FN		DOUBLE	ARRAY F.P.	4625	FS	DOUBLE			
4468 FZ		DOUBLE		0	GJN	DOUBLE	ARRAY		F.P.
0 GN		DOUBLE	ARRAY F.P.	0	GP	DOUBLE	ARRAY		F.P.
0 GTG		DOUBLE	ARRAY F.P.	4561	HH	DOUBLE			
4514 I		INTEGER		4520	IB	INTEGER			
4513 IC		INTEGER		4465	IK	INTEGER			
4522 IK1		INTEGER		4476	IS	INTEGER			
4533 IS1		INTEGER		0	IT	INTEGER			F.P.
4566 I1		INTEGER		4541	J	INTEGER			
4504 JB		INTEGER		4512	JJ	INTEGER			
4610 JL		INTEGER		4515	JN	INTEGER			
4542 JP		INTEGER		4457	K	INTEGER			
4462 KI		INTEGER		0	KK	INTEGER			F.P.
4521 K1		INTEGER		4502	LF	INTEGER			
4607 LJ		INTEGER		5027	HJ	INTEGER	ARRAY		
0 MK		INTEGER	F.P.	4463	HM	INTEGER			
0 MHAX		INTEGER	F.P.	24	HT	INTEGER	ARRAY		BUN3
0 MZ		INTEGER	F.P.	4516	H1	INTEGER			
4507 N		INTEGER		4563	NB	INTEGER			
4503 NF		INTEGER		4540	NF1	INTEGER			
0 NI		INTEGER	F.P.	0	NK	INTEGER			F.P.
4464 NN		INTEGER		4501	NP	INTEGER			
0 NT		INTEGER	F.P.	4517	N1	INTEGER			
4		DOUBLE	BUN2	4621	PH	DOUBLE			
0 PHI		DOUBLE	ARRAY F.P.	4553	PH1	DOUBLE			
16 PI		DOUBLE	BUN2	0	Q	DOUBLE	ARRAY		BUN3
62 QAK		DOUBLE	BUN3	4635	Q1	DOUBLE			
4637 Q2		DOUBLE		4567	Q2N	DOUBLE			
4641 Q3		DOUBLE		0	R	DOUBLE	ARRAY		F.P.
4623 RA		DOUBLE		4543	RH	DOUBLE			
4551 RH1		DOUBLE		0	R1	DOUBLE	ARRAY		F.P.
4571 SIMP1		DOUBLE		4573	SIMP2	DOUBLE			
4575 SIMP4		DOUBLE		6	T	DOUBLE			BUN2
4631 TC2		DOUBLE		4603	TH	DOUBLE			
4633 TK2		DOUBLE		5351	TLUMP	DOUBLE	ARRAY		
5661 THEAN		DOUBLE	ARRAY	36	TSUB	DOUBLE	ARRAY		BUN3
4555 TT		DOUBLE		4627	TV	DOUBLE			
4557 T1		DOUBLE		4615	T3	DOUBLE			
4477 U		DOUBLE		5705	UC	DOUBLE	ARRAY		
4534 U1		DOUBLE		10	VE	DOUBLE			BUN2
0 W		DOUBLE	ARRAY BUN1	0	X	DOUBLE	ARRAY		F.P.
4645 XA0		DOUBLE		4611	XA1	DOUBLE			
4613 XA2		DOUBLE		4470	XC	DOUBLE			
4545 XX		DOUBLE		4525	X1	DOUBLE			
4472 YC		DOUBLE		0	YJ	DOUBLE	ARRAY		F.P.
4505 YNT		DOUBLE		0	YP	DOUBLE	ARRAY		F.P.
20 YT		DOUBLE	BUN2	4536	YT1	DOUBLE			
4547 YY		DOUBLE		4527	Y1	DOUBLE			
24 Z		DOUBLE	ARRAY BUN1	4564	ZZ	DOUBLE			

FILE NAMES
TAPE6MODE
FMT

EXTERNALS

TYPE

ARGS

CHAN 7
OCOS DOUBLE 1 LIBRARY
OSIN DOUBLE 1 LIBRARY
FLUX 10
NEIGH 7
POTN 5
SIMPS 14
TAVG 16
TYPE 9

DATAN DOUBLE 1 LIBRARY
DLOG DOUBLE 1 LIBRARY
DSQRT DOUBLE 1 LIBRARY
LUMP 11
NTB 3
POINTS 7
SIZE 14
TEMP 9

STATEMENT LABELS

0 7	INACTIVE	375 8	0 9
0 10		3310 14	0 19
2757 21		522 22	0 23 INACTIVE
0 27		0 20	0 29
0 31	INACTIVE	0 32	1140 34
0 36		0 41	0 42
4361 51	FMT	0 52	1640 54
1350 60		2066 61	0 62
552 63		0 65	0 72
0 73	INACTIVE	546 74	1316 75
0 76	INACTIVE	1352 77	0 78 INACTIVE
1327 80		0 81	1317 82
4172 100	FMT	4177 101	4203 102 FMT
4207 103	FMT	4213 104	4217 105 FMT
4223 106	FMT	4235 111	4244 112 FMT
4252 113	FMT	4257 114	4264 115 FMT
4301 116	FMT	4272 117	4312 121 FMT
4325 122	FMT	4435 127	0 142 INACTIVE
3262 150		3246 151	0 210 INACTIVE
4440 273	FMT	0 300	440 310
442 320		0 330	446 340
447 350		4341 401	3003 600
4373 650	FMT	4405 651	3233 700
0 1000		35 1010	40 1020
43 1030		46 1040	51 1050
53 1060		111 2000	113 3000
0 4000		2703 5000	

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
105	1000	* KI	64 312	32128	EXT REFS NOT INNER
161	9	* N	83 87	2148	EXT REFS
405	10	N	89 91	68	INSTACK
427	4000	* I	101 204	14518	EXT REFS NOT INNER
514	22	N	123 127	118	INSTACK
540	74	N	128 132	118	INSTACK
553	61	* J	136 202	13168	EXT REFS NOT INNER
1116	36	NB	155 159	218	OPT
1141	62	* N	160 162	658	EXT REFS
1231	27	* N	165 167	648	EXT REFS
1424	29	* N	183 184	1078	EXT REFS
1536	28	* N	187 189	1018	EXT REFS
1646	32	NB	192 193	28	INSTACK

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LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
1652	72	* N	194 195	1108	EXT REFS
1765	19	* N	198 200	1018	EXT REFS
2331	41	* N	219 221	218	EXT REFS
2662	42	* N	233 235	218	EXT REFS
2751	21	* N	245 253	218	OPT
3065	65	* J	277 296	1538	EXT REFS

COMMON BLOCKS	LENGTH
BUN1	48
BUN2	22
BUN3	52

STATISTICS

PROGRAM LENGTH	61728	3194
SCH. LABELED COMMON LENGTH	1628	114

```

1      SUBROUTINE SIZE (NI,NK,MZ,NP,NC,GJN,YJ,G,Y,GTG,AUX,EPS,R)
      C
      C
      C      SUBROUTINE SIZE TAILORS THE SIZE OF THE COEFFICIENT
      C      MATRIX G AND THE VECTOR Y
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      DIMENSION GJN(NI,NK),YJ(NI),Y(NP),GINP,NC),GTG(MZ),AUX(NK),R(NK)
      C      DO 10 J=1,NP
      C      Y(J)=YJ(J)
      C      DO 10 N=1,NC
      C      G(J,N)=GJN(J,N)
      C      CALL GTPRD (G,Y,R,NP,NC,1)
      C      CALL MATA (G,GTG,NP,NC,0)
      C      CALL OGELS (R,GTG,NC,1,EPS,IER,AUX)
      C      RETURN
      C      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 SIZE

VARIABLES	SN	TYPE	RELOCATION						
0 AUX		DOUBLE	ARRAY	F.P.	0 EPS	DOUBLE		F.P.	
0 G		DOUBLE	ARRAY	F.P.	0 GJN	DOUBLE	ARRAY	F.P.	
0 GTG		DOUBLE	ARRAY	F.P.	130 IER	INTEGER			
126 J		INTEGER			0 HK	INTEGER		F.P.	
0 MZ		INTEGER		F.P.	127 N	INTEGER			
0 NC		INTEGER		F.P.	0 NI	INTEGER		F.P.	
0 NK		INTEGER		F.P.	0 NP	INTEGER		F.P.	
0 R		DOUBLE	ARRAY	F.P.	0 Y	DOUBLE	ARRAY	F.P.	
0 YJ		DOUBLE	ARRAY	F.P.					

EXTERNALS	TYPE	ARGS		
OGELS		7	GTPRD	6
MATA		5		

STATEMENT LABELS
0 10

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
22	10	J	10 13	228	NOT INNER
36	10	N	12 13	38	INSTACK

STATISTICS	PROGRAM LENGTH	
	1658	117

SUBROUTINE TYPE

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1 SUBROUTINE TYPE (M,N,MHAX,K,IK,CF,XC,YC,AX)
C
C
C
C
C
C
5 SUBROUTINE TYPE GIVES THE TYPE, CELL NUMBER AND THE
LOCATION OF A UNIT CELL

18 IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /BUN2/ DAW,OA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
IF (M-1) 30,40,50
50 M1=M-1
JSUM=0
00 15 J=1,M1
15 JSUM=JSUM+J/2+1
K=2+N/2+JSUM
GO TO 25
40 K=2
GO TO 25
30 K=1
25 IF (M.EQ.0) GO TO 2
IF (M.EQ.MHAX) GO TO 4
IF (M.EQ.1) GO TO 3
IF (M.EQ.0) GO TO 8
25 IF (M.EQ.1) GO TO 9
IF (M.EQ.M) GO TO 10
IK=9
GO TO 20
8 IK=6
30 GO TO 20
9 IK=10
GO TO 20
10 IK=5
GO TO 20
35 4 IF (M.EQ.0) GO TO 5
IF (M.EQ.M) GO TO 7
IF (M.EQ.1) GO TO 6
IK=7
GO TO 20
40 7 IK=2
GO TO 20
6 IK=8
GO TO 20
5 IK=3
45 GO TO 20
3 IK=4
GO TO 20
2 IK=1
20 CF=1.0
50 IF (IK.EQ.1) CF=6.0
XC=N*PD
YC=N*2.*PD*DSIN(PI/3.)
AX=-1.*PI/2.
IF (M.EQ.N) AX=PI/3.
55 RETURN
END

SUBROUTINE TYPE

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SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 TYPE

VARIABLES	SN	TYPE	RELOCATION				
0 AX		DOUBLE	F.P.	14	BI	DOUBLE	BUN2
12 BIS		DOUBLE	BUN2	0	CF	DOUBLE	F.P.
2 OA		DOUBLE	BUN2	0	DAN	DOUBLE	BUN2
22 OH		DOUBLE	BUN2	24	EPS	DOUBLE	BUN2
0 IK		INTEGER	F.P.	150	J	INTEGER	
147 JSUM		INTEGER		0	K	INTEGER	F.P.
0 H		INTEGER	F.P.	0	MHAX	INTEGER	F.P.
146 H1		INTEGER		0	N	INTEGER	F.P.
4 PD		DOUBLE	BUN2	16	PI	DOUBLE	BUN2
6 T		DOUBLE	BUN2	10	VE	DOUBLE	BUN2
0 XC		DOUBLE	F.P.	0	YC	DOUBLE	F.P.
20 YT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS
OSIN	DOUBLE	1 LIBRARY

STATEMENT LABELS

67 2	65 3	50 4
63 5	61 6	57 7
42 8	44 9	46 10
0 15	71 20	30 25
26 30	24 40	0 50

INACTIVE

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
14	15	J	14 15	38	INSTACK

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS

PROGRAM LENGTH	1518	105
SCH LABELED COMMON LENGTH	268	22

```

1      SUBROUTINE CHAN (K,IK,IS,AI,AL,AB,U)
      C
      C      SUBROUTINE CHAN GIVES THE GEOMETRIC PARAMETERS OF A UNIT
      C      CELL
5
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN2/ OAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
      COMMON /BUN3/ Q(10),HT(10),TSUB(10),QAK
      DIMENSION AI(10),AL(10),AB(10)
      GO TO (1,2,3,4,4,6,7,7,9,9),IK
1      IS=1
      AI(1)=PD
      AL(1)=0.
      AB(1)=0.0
15     AB(2)=PI/6.0
      D=(3.464*PD**2-PI)/PI
      GO TO 500
2      IS=3
      AI(1)=VE
      AI(2)=PD
      AI(3)=PD
      AL(1)=PI/6.
      AL(2)=PI*12./18.
      AL(3)=PI
      AB(1)=0.
25     AB(2)=AL(1)*DATAN(AI(2)/AI(1))
      AB(3)=5.*PI/6.
      AB(4)=PI
      AF=(PD+VE/1.732)*(1.732*PD/2.+VE/2.)-PD**2/3.464-PI/2.
      C=AF/(PI/2.+(PD+VE/1.732)/2.)
30     GO TO 500
3      IS=3
      AI(1)=PD
      AI(2)=PD
      AI(3)=VE
35     AL(1)=PI/6.
      AL(2)=PI/2.
      AL(3)=PI
      AB(1)=0.0
40     AB(2)=PI/3.
      AB(3)=PI/2.*DATAN(VE/AI(2))
      AB(4)=PI
      D=(PD*(1.732*PD/2.+VE)-PI/2.)/(PI/2.+PD/2.)
      GO TO 500
45     IS=4
      DO 12 I=1,IS
      AI(I)=PD
12     AL(I)=(I-1)*PI/3.
      AB(1)=0.0
50     AB(2)=PI/6.
      AB(3)=PI/2.
      AB(4)=5.*PI/6.
      AB(5)=PI
      D=(3.464*PD**2-PI)/PI
55     GO TO 500
6      IS=3
      AB(1)=0.0

```

```

      DO 13 I=1,IS
      AI(I)=PD
60      AL(I)=(I-1)*PI/3.+PI/6.
      AB(I+1)=PI*I/3.
      D=(3.464*PD**2-PI)/PI
      GO TO 500
      7 IS=5
65      AI(1)=PD
      AI(2)=PD
      AI(3)=VE
      AI(4)=PD
      AI(5)=PD
      AL(1)=PI/6.
      AL(2)=PI/2.
      AL(3)=PI
      AL(4)=3.*PI/2.
      AL(5)=11.*PI/6.
70      AB(1)=0.0
      AB(2)=PI/3.
      AB(3)=PI/2.+DATAN(VE/PD)
      AB(4)=2.*PI-AB(3)
      AB(5)=5.*PI/3.
75      AB(6)=2.*PI
      D=(PI*(11.732*PD/2.+VE)-PI/2.)/(PI/2.+PD/2.)
      GO TO 500
      9 IS=6
      AB(1)=0.0
80      DO 14 I=1,IS
      AI(I)=PD
      AL(I)=PI*(I-1)/3.+PI/6.
      14 AB(I+1)=PI*I/3.
      D=(3.464*PD**2-PI)/PI
90      500 U=1.0
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 CHAN

VARIABLES	SN	TYPE	RELOCATION	1020	AF	DOUBLE	ARRAY	F.P.
0 AB		DOUBLE	ARRAY	0	AL	DOUBLE		F.P.
0 AI		DOUBLE	ARRAY	12	BIS	DOUBLE		BUN2
14 BI		DOUBLE		2	DA	DOUBLE		BUN2
1016 D		DOUBLE		22	DH	DOUBLE		BUN2
0 DAW		DOUBLE	BUN2	1022	I	INTEGER		
24 EPS		DOUBLE	BUN2	0	IS	INTEGER		F.P.
0 IK		INTEGER		24	MT	INTEGER	ARRAY	BUN3
0 K		INTEGER	*UNUSED	16	PI	DOUBLE		BUN2
4 PD		DOUBLE	BUN2	62	QAK	DOUBLE		BUN3
0 Q		DOUBLE	ARRAY	36	TSUB	DOUBLE	ARRAY	BUN3
6 T		DOUBLE	BUN2					

SUBROUTINE CHAN

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VARIABLES SN TYPE
 0 U DOUBLE
 20 Y DOUBLE

RELOCATION
 F.P.
 BUN2

10 VE

DOUBLE

BUN2

EXTERNALS TYPE
 DATAN DOUBLE

ARGS
 1 LIBRARY

STATEMENT LABELS

24 1
 344 4
 701 9
 0 14

57 2
 433 6
 0 12
 771 500

231 3
 525 7
 0 13

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
362	12	I	46 48	108	INSTACK
462	13	I	58 61	240	OPT
722	14	I	85 88	308	OPT

COMMON BLOCKS LENGTH
 BUN2 22
 BUN3 52

STATISTICS

PROGRAM LENGTH	10238	531
SCM LABELED COMMON LENGTH	1128	74

2

SUBROUTINE PDTH (IK,IS,MJ,NP,NF)

5

C
C
C
C
CSUBROUTINE PDTH GIVES THE COLOCATION POINT DISTRIBUTION
OF A UNIT CELL

10

DIMENSION MJ(10)
GO TO (1,2,3,4,4,6,7,7,9,9),IK

15

1 MJ(1)=12

GO TO 100

2 MJ(1)=6

MJ(2)=6

MJ(3)=4

GO TO 100

3 MJ(1)=8

MJ(2)=6

MJ(3)=4

GO TO 100

20

4 MJ(1)=4

MJ(2)=8

MJ(3)=8

MJ(4)=4

GO TO 100

25

6 MJ(1)=8

MJ(2)=8

MJ(3)=8

GO TO 100

30

7 MJ(1)=8

MJ(2)=6

MJ(3)=8

MJ(4)=6

MJ(5)=8

GO TO 100

35

9 MJ(1)=8

MJ(2)=8

MJ(3)=8

MJ(4)=8

MJ(5)=8

MJ(6)=8

40

100 NP=0

DO 16 I=1,IS

16 NP=NP+MJ(I)

NF=NP*2/3

RETURN

45

END

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 PDTH

SUBROUTINE PDTH

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2

VARIABLES

SN TYPE

RELOCATION

74	I	INTEGER
0	IS	INTEGER
0	NF	INTEGER

F.P.
F.P.

0	IK
0	HJ
0	NP

INTEGER

F.P.

INTEGER

ARRAY

F.P.

INTEGER

F.P.

STATEMENT LABELS

24	1
36	4
52	9

26	2
42	6
0	16

32	3
45	7
56	100

LOOPS	LABEL
64	16

INDEX
I

FROM-TO
41 42

LENGTH
28

PROPERTIES
INSTACK

STATISTICS

PROGRAM LENGTH

758

61

SUBROUTINE NTB

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PAGE 1

1

SUBROUTINE NTB (HMAX,IK,JB)

5

SUBROUTINE NTB GIVES THE NUMBER OF BOUNDARIES WHICH
WHICH ARE COUPLED TO NEIGHBORING CELLS BY THE CONTINUITY
OF TEMPERATURE

10

GO TO (1,2,3,4,5,6,7,8,9,10),IK

1 JB=0
RETURN2 JB=2
RETURN3 JB=1
RETURN

15

4 JB=1
RETURN5 JB=2
RETURN

20

6 JB=1
RETURN7 JB=3
RETURN

25

8 JB=2
RETURN9 JB=3
RETURN10 JB=2
RETURN

30

END

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 NTB

VARIABLES	SN	TYPE	RELOCATION			
0 IK		INTEGER	F.P.	0	JB	INTEGER
0 HMAX		INTEGER	UNUSED			F.P.

STATEMENT LABELS

26 1	26 2	30 3
32 4	34 5	36 6
40 7	42 8	44 9
46 10		

STATISTICS

PROGRAM LENGTH	500	60
----------------	-----	----

```

1      SUBROUTINE POINTS (ISD,NJ,AI,AL,AB,PHI)
C
C
C
C
5      SUBROUTINE POINTS GIVES THE POSITIONS OF THE COLLOCATION
POINTS
C
C
C
C
10     IMPLICIT DOUBLE PRECISION (A-H,O-Z)
COMMON /BUN2/ DAN,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
COMMON /BUN3/ Q(10),NT(10),TSUB(10),QAK
DIMENSION AI(10),AL(10),AB(10),XS(10),YS(10),NJ(10)
DIMENSION PHI(NI)
ISC=ISD+1
XS(1)=AI(1)/DCOS(AL(1))
15     YS(1)=0.0
DO 150 I=2,ISC
XS(I)=(AI(I-1)/DCOS(AB(I))-AL(I-1))*DCOS(AB(I))
150   YS(I)=(AI(I-1)/DCOS(AB(I))-AL(I-1))*DSIN(AB(I))
K=1
DO 120 I=1,ISD
NIS=NJ(I)
DO 120 J=1,NIS
X=((NIS-J*.5)*XS(I)+(J-.5)*XS(I+1))/NIS
Y=((NIS-J*.5)*YS(I)+(J-.5)*YS(I+1))/NIS
25     IF (Y) 10,10,20
10    IF (X.GE.0.0) PHI(K)=2.*PI+DATAN(Y/X)
IF (X.LT.0.0) PHI(K)=PI+DATAN(Y/X)
GO TO 120
20    IF (X.GE.0.0) PHI(K)=DATAN(Y/X)
IF (X.LT.0.0) PHI(K)=PI+DATAN(Y/X)
30    K=K+1
RETURN
END
```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 POINTS

VARIABLES	SN	TYPE	RELOCATION					
0 AB		DOUBLE	ARRAY	F.P.	0	AI	DOUBLE	F.P.
0 AL		DOUBLE	ARRAY	F.P.	14	BI	DOUBLE	BUN2
12 BIS		DOUBLE		BUN2	2	OA	DOUBLE	BUN2
8 DAW		DOUBLE		BUN2	22	OH	DOUBLE	BUN2
24 EPS		DOUBLE		BUN2	276	I	INTEGER	
275 ISC		INTEGER			0	ISD	INTEGER	F.P.
301 J		INTEGER			277	K	INTEGER	
24 MT		INTEGER	ARRAY	BUN3	0	NI	INTEGER	F.P.
300 NIS		INTEGER			0	NJ	INTEGER	F.P.
4 PD		DOUBLE		BUN2	0	PHI	DOUBLE	F.P.
16 PI		DOUBLE		BUN2	0	Q	DOUBLE	BUN3
62 QAK		DOUBLE		BUN3	6	T	DOUBLE	BUN2
36 TSUB		DOUBLE	ARRAY	BUN3	10	VE	DOUBLE	BUN2

SUBROUTINE POINTS

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VARIABLES	SN	TYPE	RELOCATION
302 X		DOUBLE	
304 Y		DOUBLE	
20 YT		DOUBLE	BUN2

306 XS	DOUBLE	ARRAY
332 YS	DOUBLE	ARRAY

EXTERNALS	TYPE	ARGS
DATAN	DOUBLE	1 LIBRARY
DSIN	DOUBLE	1 LIBRARY

DCOS	DOUBLE	1 LIBRARY
------	--------	-----------

STATEMENT LABELS

0	10
0	150

INACTIVE

222 20

256 120

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
34	150	• I	16 18	568	EXT REFS
113	120	• I	20 31	1528	EXT REFS NOT INNER
116	120	• J	22 31	1448	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22
BUN3	52

STATISTICS

PROGRAM LENGTH	4048	260
SCM LABELED COMMON LENGTH	1128	74

```
1      SUBROUTINE NEIGH (MMAX,M,N,IK,IS,MN,NN)
      C
      C
      C      SUBROUTINE NEIGH IDENTIFIES NEIGHBORING CELLS
      C
      C
      C      GO TO (1,2,3,4,4,6,7,7,9,9),IK
      1  MN=M+1
      NN=N+1
      RETURN
      2  GO TO (11,12,13),IS
      11 MN=-1
      NN=-1
      RETURN
      12 MN=N
      NN=N-2
      IF (MMAX.EQ.1) MN=-1
      IF (MMAX.EQ.1) NN=-1
      RETURN
      13 MN=M-1
      NN=N-1
      RETURN
      3  GO TO (14,15,16),IS
      14 MN=M-1
      NN=N+1
      RETURN
      15 MN=M
      NN=N+2
      RETURN
      16 MN=-1
      NN=-1
      RETURN
      4  GO TO (17,18,19,20),IS
      17 MN=M+1
      NN=N+1
      RETURN
      18 MN=M+1
      NN=N-1
      RETURN
      19 MN=M
      NN=N-2
      IF (IK.EQ.4) MN=-1
      IF (IK.EQ.4) NN=-1
      RETURN
      20 MN=M-1
      NN=N-1
      RETURN
      6  GO TO (21,22,23),IS
      21 MN=M-1
      NN=N+1
      RETURN
      22 MN=M
      NN=N+2
      RETURN
      23 MN=M+1
      NN=N+1
      RETURN
```

```

60      7 GO TO (24,25,26,27,28),IS
      24 MM=M-1
        NN=N+1
        RETURN
      25 MM=M
        NN=N+2
        RETURN
      65      26 MM=-1
        NN=-1
        RETURN
      27 MM=M
        NN=N-2
      78      IF (IK.EQ.8) MM=-1
        IF (IK.EQ.8) NN=-1
        RETURN
      28 MM=M-1
        NN=N-1
      75      RETURN
      9 GO TO (29,30,31,32,33,34),IS
      29 MM=M-1
        NN=N+1
        RETURN
      80      30 MM=M
        NN=N+2
        RETURN
      31 MM=M+1
        NN=N+1
      85      RETURN
      32 MM=M+1
        NN=N-1
        RETURN
      33 MM=M
        NN=N-2
      90      IF (IK.EQ.10) MM=-1
        IF (IK.EQ.10) NN=-1
        RETURN
      34 MM=M-1
        NN=N-1
      95      RETURN
        END

```

CARD NO. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

11	I	AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.
23	I	AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.
48	I	AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

SUBROUTINE NEIGH

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3

ENTRY POINTS
3 NEIGH

VARIABLES	SN	TYPE
0 IK		INTEGER
0 M		INTEGER
0 MHAX		INTEGER
0 NN		INTEGER

RELOCATION
F.P.
F.P.
F.P.
F.P.

0	IS
0	MM
0	N

INTEGER
INTEGER
INTEGER

F.P.
F.P.
F.P.

STATEMENT LABELS

24	1
111	4
252	9
57	13
106	16
134	19
171	22
222	25
245	28
277	31
324	34

31	2
154	6
41	11
74	14
122	17
147	20
176	23
227	26
265	29
304	32

64	3
203	7
44	12
101	15
127	18
164	21
215	24
232	27
272	30
311	33

STATISTICS
PROGRAM LENGTH

3318 217

```

1      SUBROUTINE GTPRD(A,B,R,N,M,L)
      C
      C
      C      SUBROUTINE GTPRD GIVES THE PRODUCT OF THE TRANSPOSE OF
5      G AND Y
      C
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      DIMENSION A(1),B(1),R(1)
10     IR=0
      IK=-N
      DO 10 K=1,L
      IJ=0
      IK=IK+N
      DO 11 J=1,M
15     ID=IK
      IR=IR+1
      R(IR)=0.
      DO 10 I=1,N
      IJ=IJ+1
      IB=IB+1
20     R(IR)=R(IR)+A(IJ)*B(IB)
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 GTPRD

VARIABLES	SN	TYPE	RELOCATION					
0 A		DOUBLE	ARRAY	F.P.	0 B	DOUBLE	ARRAY	F.P.
71 I		INTEGER			70 IB	INTEGER		
66 IJ		INTEGER			64 IK	INTEGER		
63 IR		INTEGER			67 J	INTEGER		
65 K		INTEGER			0 L	INTEGER		F.P.
0 M		INTEGER		F.P.	0 N	INTEGER		F.P.
8 R		DOUBLE	ARRAY	F.P.				

STATEMENT LABELS
8 10

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
22	10	K	12 22	408	NOT INNER
26	10	J	15 22	328	NOT INNER
42	10	I	19 22	128	INSTACK

STATISTICS
PROGRAM LENGTH 1108 72


```

1      SUBROUTINE MATA(A,R,N,M,MS)
      C
      C
      C      SUBROUTINE MATA GIVES THE PRODUCT OF THE TRANSPOSE
      C      OF G AND G
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      DIMENSION A(1),R(1)
10     DO 70 K=1,M
      KX=(K*K-K)/2
      DO 70 J=1,M
      IF (J-K) 10,10,70
15     IR=J+KX
      R(IR)=0.0
      DO 60 I=1,N
      IF (MS) 20,40,20
20     CALL LOC(I,J,IA,N,M,MS)
      CALL LOC(I,K,IB,N,M,MS)
25     IF (IA) 30,60,30
      IF (IB) 50,60,50
      IA=N*(J-1)+I
      IB=N*(K-1)+I
      R(IR)=R(IR)+A(IA)*A(IB)
      60 CONTINUE
      70 CONTINUE
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 MATA

VARIABLES	SN	TYPE	RELOCATION					
0 A		DOUBLE	ARRAY	F.P.	121 I	INTEGER		
122 IA		INTEGER			123 IB	INTEGER		
120 IR		INTEGER			117 J	INTEGER		
115 K		INTEGER			116 KX	INTEGER		
0 M		INTEGER		F.P.	0 MS	INTEGER	F.P.	
0 N		INTEGER		F.P.	0 R	DOUBLE	ARRAY	F.P.

EXTERNALS
LOCTYPE
6

STATEMENT LABELS

	INACTIVE		INACTIVE		INACTIVE
0 10		0 20		0 30	
46 40		53 50		66 60	
71 70					

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
17	70	* K	10 26	578	EXT REFS NOT INNER
22	70	* J	12 26	528	EXT REFS NOT INNER

SUBROUTINE MAYA

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LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES	EXT REFS
30	60	* I	86 25	418		

STATISTICS		
PROGRAM LENGTH	1508	104

SUBROUTINE LOC

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PAGE 1

```

1      SUBROUTINE LOC (I,J,IR,N,M,MS)
      IX=I
      JX=J
      IF (MS=1) 10,20,30
5      10  IRX=N*(JX-1)+IX
      GOTO 36
      20  IF (IX-JX) 22,24,24
      22  IRX=IX+(JX-JX-JX)/2
      GOTO 36
10     24  IRX=JX+(IX-IX-IX)/2
      GOTO 36
      30  IRX=0
      IF (IX-JX) 36,32,36
      32  IRX=IX
15     36  IR=IRX
      RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 LOC

VARIABLES	SN	TYPE	RELOCATION	0	IR	INTEGER	F.P.
0 I		INTEGER	F.P.	0	IR	INTEGER	F.P.
36 IRX		INTEGER		32	IX	INTEGER	
0 J		INTEGER	F.P.	33	JX	INTEGER	
0 M		INTEGER	*UNUSED	0	MS	INTEGER	F.P.
0 N		INTEGER	F.P.				

STATEMENT LABELS

0	10	INACTIVE	15	20	0	22	INACTIVE
22	24		25	30	0	32	INACTIVE
30	36						

STATISTICS

PROGRAM LENGTH

358 29

```

1      SUBROUTINE OGELS(R,A,M,N,EPS,IER,AUX)
      C
      C      SUBROUTINE OGELS SOLVES THE UNKNOWN COEFFICIENT VECTOR
      C      BY GAUSSIAN ELIMINATION
      C
10     DIMENSION A(1),R(1),AUX(1)
      DOUBLE PRECISION R,A,AUX,PIV,TB,TOL,PIVI,EPS
      IF(M)24,24,1
      IER=0
      PIV=0.00
      L=0
      DO 3 K=1,M
15     L=L+K
      TB=DABS(A(L))
      IF(TB-PIV)3,3,2
      2   PIV=TB
      I=L
      J=K
      3   CONTINUE
      TOL=EPS*PIV
      LST=0
      NM=N*M
      LEND=M-1
      DO 10 K=1,M
      IF(PIV)24,24,4
      4   IF(IER)7,5,7
      5   IF(PIV-TOL)6,6,7
      6   IER=K-1
      7   LT=J-K
      LST=LST+K
      PIVI=1.00/A(I)
      DO 8 L=K,NM,M
35     LL=L+LT
      TB=PIVI*A(LL)
      R(LL)=R(LL)
      R(L)=TB
      IF(K-M)9,19,19
      9   LR=LST+(LT*(K+J-1))/2
      LL=LR
      L=LST
      DO 14 II=K,LEND
      L=L+II
45     LL=LL+1
      IF(L-LR)12,10,11
      10  A(LL)=A(LST)
      TB=A(L)
      GOTO 13
      11  LL=L+LT
      12  TB=A(LL)
      A(LL)=A(L)
      13  AUX(II)=TB
      14  A(L)=PIVI*TB
55     A(LST)=LT
      PIV=0.00
      LST=LST

```

```

        LY=0
        DO 18 II=K,LEND
60      PIVI=-AUX(II)
        LL=LLST
        LT=LT+1
        DO 15 LLD=II,LEND
65      LL=LL+LLD
        L=LL+LT
        15  A(LL)=A(LL)+PIVI*A(LL)
        LLST=LLST+II
        LR=LLST+LT
        TB=DABS(A(LR))
70      IF (TB-PIVI) 17,17,16
        16  PIV=TB
        I=LR
        J=II+1
        17  DO 18 LR=K,NM,M
75      LL=LR+LT
        18  R(LL)=R(LL)+PIVI*R(LR)
        19  IF (LEND) 24,23,20
        20  II=M
        DO 22 I=2,M
80      LST=LST-II
        II=II-1
        L=A(LST)+.500
        DO 22 J=II,NM,M
        TB=R(J)
85      LL=J
        K=LST
        DO 21 LT=II,LEND
        LL=LL+1
        K=K+LT
90      21  TB=TB-A(K)*R(LL)
        K=J+L
        R(J)=R(K)
        22  R(K)=TB
        23  RETURN
95      24  IER=-1
        RETURN
        END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 DGELS

VARIABLES	SN	TYPE	RELOCATION					
0 A		DOUBLE	ARRAY	F.P.	0 AUX	DOUBLE	ARRAY	F.P.
0 EPS		DOUBLE		F.P.	407 I	INTEGER		
0 IER		INTEGER		F.P.	417 II	INTEGER		
410 J		INTEGER			406 K	INTEGER		
405 L		INTEGER			413 LEND	INTEGER		
415 LL		INTEGER			421 LLD	INTEGER		

SUBROUTINE OGELS

76/76 OPT=1

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VARIABLES	SN	TYPE	RELOCATION
420 LLST		INTEGER	
411 LST		INTEGER	
0 M		INTEGER	F.P.
412 NM		INTEGER	
403 PIVI		DOUBLE	
377 TB		DOUBLE	

416 LR		INTEGER			
414 LT		INTEGER			
0 N		INTEGER		F.P.	
375 PIV		DOUBLE			
0 R		DOUBLE	ARRAY	F.P.	
401 TOL		DOUBLE			

INLINE FUNCTIONS	TYPE	ARGS
OABS	DOUBLE	1 INTRIN

STATEMENT LABELS

0 1	INACTIVE	0 2	INACTIVE	43 3	
0 4	INACTIVE	0 5	INACTIVE	0 6	INACTIVE
72 7		0 0		0 9	INACTIVE
0 10	INACTIVE	156 11		160 12	
165 13		0 14		0 15	
0 16	INACTIVE	251 17		0 18	
301 19		0 20	INACTIVE	0 21	
0 22		364 23		365 24	

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
30 3		K	14 21	148	OPT
60 18		* K	26 76	2218	EXITS NOT INNER
113 0		L	34 38	128	INSTACK
144 14		II	43 54	318	OPT
204 18		* II	59 76	728	NOT INNER
216 15		LLD	63 66	148	OPT
260 18		LR	74 76	138	OPT
305 22		* I	79 93	578	NOT INNER
316 22		* J	83 93	448	NOT INNER
332 21		LT	87 90	148	OPT

STATISTICS

PROGRAM LENGTH		
4738	315	

```

1      SUBROUTINE TAVG (XA,XB,XF,FN,GH,X,AI,AL,AB,CF,YNT,T3,FA)
        IMPLICIT DOUBLE PRECISION (A-H,O-Z)

5          C
            C
            C
            C
            C
            C
            C
            C

10         COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
           DIMENSION X(NF),FN(NF),GH(NF)
           DIMENSION AB(10),AI(10),AL(10)
           QYAN(X)=DSIN(X)/DCOS(X)
           D=XB-XA

15         CALL SIMPS (SIMPA,XA,XB,NF,FN,GH,X,AI,AL,AB,CF,YNT,IK,1)
           CALL SIMPS (SIMPB,XA,XB,NF,FN,GH,X,AI,AL,AB,CF,YNT,IK,2)
           CALL SIMPS (SIMPC,XA,XB,NF,FN,GH,X,AI,AL,AB,CF,YNT,IK,3)
           CALL SIMPS (SIMPF,XA,XB,NF,FN,GH,X,AI,AL,AB,CF,YNT,IK,4)
           FA=(SIMPF-DI)/2.

20         T3=-1./12.*YNT)+(LOG(T)-1./3I)/(DA-1./(BIS*DA*DAW))
           T3=T3+((SIMPA-DI)/(8.*YNT)-(YNT+1.)*(SIMPB-DI)/(4.*YNT)+SIMPC)/FA
           RETURN
           END
```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
J TANG

VARIABLES	SN	TYPE	RELOCATION						
0	AB	DOUBLE	ARRAY	F.P.	0	AI	DOUBLE	ARRAY	F.P.
0	AL	DOUBLE	ARRAY	F.P.	14	BI	DOUBLE		BUN2
12	BIS	DOUBLE		BUN2	0	CF	DOUBLE		F.P.
406	D	DOUBLE			2	DA	DOUBLE		BUN2
0	DAM	DOUBLE		BUN2	22	DH	DOUBLE		BUN2
24	EPS	DOUBLE		BUN2	0	FA	DOUBLE		F.P.
0	FN	DOUBLE	ARRAY	F.P.	0	GN	DOUBLE	ARRAY	F.P.
0	IK	INTEGER		F.P.	0	NF	INTEGER		F.P.
4	PD	DOUBLE		BUN2	0	PI	DOUBLE		BUN2
410	SIMPA	DOUBLE			412	SIMPB	DOUBLE		
414	SIMPC	DOUBLE			416	SIMPF	DOUBLE		
6	T	DOUBLE		BUN2	0	T3	DOUBLE		F.P.
10	VE	DOUBLE		BUN2	0	X	DOUBLE	ARRAY	F.P.
0	XA	DOUBLE		F.P.	0	XB	DOUBLE		F.P.
0	YNT	DOUBLE		F.P.	20	YT	DOUBLE		BUN2

EXTERNALS	TYPE	ARGS
OLOG	DOUBLE	1 LIBRARY

INLINE FUNCTIONS	TYPE	ARGS	SF
DTAN	DOUBLE	1	SF

SUBROUTINE TAVG

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PAGE 2

COMMON BLOCKS LENGTH
BUN2 22

STATISTICS

PROGRAM LENGTH	4208	272
SCH LABELED COMMON LENGTH	268	22


```

1      SUBROUTINE SIMPS (SIMP,XA,XB,NK,GN,R,AA,AF,AB,CF,YNT,IK,JA)
      C
      C
      C      SUBROUTINE SIMPS PERFORMS NUMERICAL INTEGRATIONS
      C
      C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      C      COMMON /BUN1/ M(10),Z(10)
      C      COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
      C      DIMENSION R(1),FN(1),GN(1)
      C      DIMENSION AA(10),AF(10),AB(10)
      C      SUMP=0.0
      C      DO 2 I=1,10
      C      PH=(Z(I))*(XB-XA)+(XA+XB))/2.
      C      K=1
      C      50 IF (PH-AB(K+1)) 30,30,40
      C      K=K+1
      C      GO TO 50
      C      30 AI=AA(K)
      C      AL=AF(K)
      C      GO TO (10,11,12,13),JA
      C      10 Y=AI**4/DCOS(PH-AL)**4
      C      GO TO 2
      C      11 V=(2.*DLOG(AI/DCOS(PH-AL))-1.)/(DCOS(PH-AL)**2)*AI**2
      C      GO TO 2
      C      25 Y=0.0
      C      DO 21 N=1,NK
      C      A=CF*N
      C      G=GN(N)/(A+2.)*((AI/DCOS(PH-AL))**(A+2.))-1.)
      C      IF (A.EQ.2.0) GO TO 22
      C      F=FN(N)/(2.-A)*((AI/DCOS(PH-AL))**(2.-A))-1.)
      C      GO TO 25
      C      22 F=FN(N)*DLOG(AI/DCOS(PH-AL))
      C      25 Y=Y+(G+F)*R(N)/A*DCOS(A*PH)
      C      IF (IK-7) 21,23,23
      C      23 Y=Y+(G+F)*R(NK*N)*DSIN(A*PH)/A
      C      21 CONTINUE
      C      GO TO 2
      C      13 Y=AI**2/DCOS(PH-AL)**2
      C      2 SUMP=SUMP+H(I)*Y
      C      SIMP=SUMP*(XB-XA)/2.0
      C      RETURN
      C      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 SIMPS

VARIABLES	SN	TYPE	RELOCATION				
516 A		DOUBLE		0	AA	DOUBLE	F.P.
0 AD		DOUBLE	ARRAY	0	AF	DOUBLE	F.P.
507 AI		DOUBLE		511	AL	DOUBLE	

SUBROUTINE SIMPS

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VARIABLES	SN	TYPE	RELOCATION						
14 BI		DOUBLE	BUN2	12	BIS	DOUBLE		BUN2	
8 CF		DOUBLE	F.P.	2	DA	DOUBLE		BUN2	
0 DAM		DOUBLE	BUN2	22	DM	DOUBLE		BUN2	
24 EPS		DOUBLE	BUN2	522	F	DOUBLE			
0 FN		DOUBLE	ARRAY	520	G	DOUBLE			
8 GN		DOUBLE	ARRAY	503	I	INTEGER			
0 IK		INTEGER	F.P.	0	JA	INTEGER		F.P.	
506 K		INTEGER		515	N	INTEGER			
0 NK		INTEGER	F.P.	4	PD	DOUBLE		BUN2	
504 PH		DOUBLE		16	PI	DOUBLE		BUN2	
0 R		DOUBLE	ARRAY	0	SIMP	DOUBLE		F.P.	
501 SUMP		DOUBLE		6	T	DOUBLE		BUN2	
10 VE		DOUBLE	BUN2	0	M	DOUBLE	ARRAY	BUN1	
8 XA		DOUBLE	F.P.	0	X8	DOUBLE		F.P.	
513 Y		DOUBLE		0	YNT	DOUBLE	*UNUSED	F.P.	
20 YT		DOUBLE	BUN2	24	Z	DOUBLE	ARRAY	BUN1	

EXTERNALS	TYPE	ARGS			
DCOS	DOUBLE	1 LIBRARY	DLOG	DOUBLE	1 LIBRARY
DSIN	DOUBLE	1 LIBRARY			

STATEMENT LABELS

436 2	74 18	115 11
161 12	417 13	414 21
276 22	0 23	316 25
55 38	0 40	45 50
	INACTIVE	
	INACTIVE	

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
21	2	* I	13 40	4318	EXT REFS NOT INNER
164	21	* N	27 37	2338	EXT REFS

COMMON	BLOCKS	LENGTH
BUN1		40
BUN2		22

STATISTICS		
PROGRAM LENGTH	5638	371
SCM LABELED COMMON LENGTH	768	62

```

1      SUBROUTINE LUMP (SIMP,XA,XB,N,FN,GN,AA,AF,AB,CF,YNT)
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN1/ W(10),Z(10)
      COMMON /BUN2/ DAW,DA,PD,Y,VE,BIS,BI,PI,YT,DH,EPS
5      DIMENSION FN(1),GN(1)
      DIMENSION AA(10),AF(10),AB(10)
      SUMP=0.0
      DO 2 I=1,10
      PH=(Z(I)*(XB-XA)+(XA+XB))/2.
10     K=1
      50 IF (PH-AB(K+1)) 30,30,40
      40 K=K+1
      GO TO 50
      30 AI=AA(K)
      AL=AF(K)
15     A=CF*N
      G=GN(I)/(A+2.)*((AI/DCOS(PH-AL))**(A+2.))-1.
      IF (A.EQ.2.0) GO TO 22
      F=FN(I)/(2.-A)*((AI/DCOS(PH-AL))**(2.-A))-1.
20     GO TO 25
      22 F=FN(I)*DLOG(AI/DCOS(PH-AL))
      25 Y=(G+F)/A*DCOS(A*PH)
      2 SUMP=SUMP+W(I)*Y
      SIMP=SUMP*(XB-XA)/2.0
25     RETURN
      END

```

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 LUMP

VARIABLES	SN	TYPE	RELOCATION					
306 A		DOUBLE		0	AA	DOUBLE	ARRAY	F.P.
0 AB		DOUBLE	ARRAY	0	AF	DOUBLE	ARRAY	F.P.
302 AI		DOUBLE		304	AL	DOUBLE		
14 BI		DOUBLE	BUN2	12	BIS	DOUBLE		BUN2
0 CF		DOUBLE	F.P.	2	DA	DOUBLE		BUN2
0 DAW		DOUBLE	BUN2	22	DH	DOUBLE		BUN2
24 EPS		DOUBLE	BUN2	312	F	DOUBLE		
0 FN		DOUBLE	ARRAY	310	G	DOUBLE		
0 GN		DOUBLE	ARRAY	276	I	INTEGER		
301 K		INTEGER		0	N	INTEGER		F.P.
4 PD		DOUBLE	BUN2	277	PH	DOUBLE		
16 PI		DOUBLE	BUN2	0	SIMP	DOUBLE		F.P.
274 SUMP		DOUBLE		6	T	DOUBLE		BUN2
10 VE		DOUBLE	BUN2	0	W	DOUBLE	ARRAY	BUN1
0 XA		DOUBLE	F.P.	0	XB	DOUBLE		F.P.
314 Y		DOUBLE		0	YNT	DOUBLE	*UNUSED	F.P.
20 YT		DOUBLE	BUN2	24	Z	DOUBLE	ARRAY	BUN1

SUBROUTINE LUMP

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PAGE 2

EXTERNALS
DCOSTYPE
DOUBLEARGS
1 LIBRARY

DLOG

DOUBLE

1 LIBRARY

STATEMENT LABELS

0 2
55 30176 22
0 40

INACTIVE

214 25
45 50LOOPS
21 2LABEL
2INDEX
+ 1FROM-TO
0 23LENGTH
2308

PROPERTIES

EXT REFS

COMMON BLOCKS

LENGTH

BUN1 48
BUN2 22

STATISTICS

PROGRAM LENGTH

3508

232

SCH LABELED COMMON LENGTH

768

62

```

1      SUBROUTINE TEMP (JA,IK,CF,NK,R,YNT,P,PH,TOMP)
      C
      C
      C
      C
      SUBROUTINE TEMP GIVES THE DIMENSIONLESS FUEL, CLAD, AND
      COOLANT TEMPERATURES AT VARIOUS RADIAL AND ANGULAR
      POSITIONS
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUN2/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,DH,EPS
10     DIMENSION R(1)
      GO TO (2,3,4),JA
      2 SUMV=0.0
      DO 10 N=1,NK
      A=CF*N
      15 SUMV=SUMV+R(N)*P**A*DCOS(A*PH)/A
      IF (IK-7) 10,12,12
      12 SUMV=SUMV+R(NK+N)*P**A*DSIN(A*PH)/A
      10 CONTINUE
      TOMP=SUMV*(1.-(P/T)**2)/(2.*DA*DAW)
      28 RETURN
      3 SUMT=0.0
      DO 20 N=1,NK
      A=CF*N
      S1=(1.+DAW+A/BIS)*P**A
      25 S2=(1.-DAW+A/BIS)*T**2/(P**A)
      SUMT=SUMT+(S1+S2)*R(N)*DCOS(A*PH)/(2.*A)
      IF (IK-7) 20,22,22
      22 SUMT=SUMT+R(NK+N)*(S1+S2)*DSIN(A*PH)/(2.*A)
      20 CONTINUE
      30 TOMP=SUMT*(DLOG(T/P))/(DA-(1./BIS)/(DA*DAW))
      RETURN
      4 SUM=0.0
      DO 30 N=1,NK
      A=CF*N
      35 FN=((1.+A/BIS-DA)*(1.+DAW+A/BIS)+(1.+DA-A/BIS)*(1.-DAW+A/BIS)*T**2.
      1*A))/4.
      GN=((1.+A/BIS+DA)*(1.+DAW+A/BIS)+(1.-DA-A/BIS)*(1.-DAW+A/BIS)*T**2.
      1*A))/4.
      SUM=SUM+(FN*P**(-1.*A)+GN*P**A)*(R(N)*DCOS(A*PH))/A
      40 IF (IK-7) 30,32,32
      32 SUM=SUM+(FN*P**(-1.*A)+GN*P**A)*R(NK+N)*DSIN(A*PH)/A
      30 CONTINUE
      45 TOMP=SUM*(P**2-1.)/(2.*YNT)-(1./YNT+1.)*DLOG(P)/(-1./BI+DLOG(T))/D
      1A-(1./BIS)/(DA*DAW)
      RETURN
      END

```

CARD NR. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

11 I

AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 TEMP

VARIABLES	SN	TYPE	RELOCATION				
1252 A		DOUBLE		14	BI	DOUBLE	BUN2
12 B1S		DOUBLE	BUN2	0	CF	DOUBLE	F.P.
2 DA		DOUBLE	BUN2	0	DAM	DOUBLE	BUN2
22 OH		DOUBLE	BUN2	24	EPS	DOUBLE	BUN2
1264 FN		DOUBLE		1266	GN	DOUBLE	
0 IK		INTEGER	F.P.	0	JA	INTEGER	F.P.
1251 N		INTEGER		0	NK	INTEGER	F.P.
8 P		DOUBLE	F.P.	4	PD	DOUBLE	BUN2
0 PH		DOUBLE	F.P.	16	PI	DOUBLE	BUN2
0 R		DOUBLE	ARRAY	1262	SUM	DOUBLE	
1254 SUMT		DOUBLE		1247	SUMV	DOUBLE	
1256 S1		DOUBLE		1260	S2	DOUBLE	
6 T		DOUBLE	BUN2	0	TOMP	DOUBLE	F.P.
10 VE		DOUBLE	BUN2	0	YNT	DOUBLE	F.P.
20 YT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS			
DCOS	DOUBLE	1 LIBRARY	DLOG	DOUBLE	1 LIBRARY
DSIN	DOUBLE	1 LIBRARY			

STATEMENT LABELS

26 2		206 3		475 4			
145 10		0 12	INACTIVE	420 20			
0 22	INACTIVE	1052 30		0 32	INACTIVE		

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
31	10	* N	13 18	1178	EXT REFS
211	20	* N	22 29	2120	EXT REFS
500	30	* N	33 42	3558	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS			
PROGRAM LENGTH	13448	740	
SCM LABELED COMMON LENGTH	260	22	

```

1      SUBROUTINE FLUX (JA,IK,CF,NK,R,YNT,AL,P,PH,Q2)
      C
      C
      C
      C
      C
      SUBROUTINE FLUX GIVES RADIAL HEAT FLUX IN THE FUEL, CLAD
      AND COOLANT REGIONS AT VARIOUS RADIAL AND ANGULAR POSITIONS

      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /BUNZ/ DAW,DA,PD,T,VE,BIS,BI,PI,YT,OH,EPS
      DIMENSION R(1)
      GO TO (1,2,3),JA
1      SUM1=0.0
      DO 10 N=1,NK
      A=CF*N
      SUM1=SUM1+R(N)*T** (CF*N)*DCOS(CF*N*PH)
      IF (IK-7) 10,12,12
12     SUM1=SUM1+R(NK+NI)*T**A*DSIN(A*PH)
10     CONTINUE
      Q2=1.-DA*DAW*SUM1
      RETURN
2      SUM2=0.0
      DO 20 N=1,NK
      A=CF*N
      S1=(1.+DAW+A/BIS)
      S2=(1.-DAW+A/BIS)*T** (2.*A)
      SUM2=SUM2+(S1-S2)*R(N)*DCOS(A*PH)/2.
      IF (IK-7) 20,22,22
22     SUM2=SUM2+(S1-S2)*R(NK+NI)*DSIN(A*PH)/2.
20     CONTINUE
      Q2=1.-DA*SUM2
      RETURN
3      SUMA=0.0
      SUMB=0.0
      DO 30 N=1,NK
      A=CF*N
      FN=((1.+A/BIS-DA)*(1.+DAW+A/BIS)+(1.-DA-A/BIS)*(1.-DAW+A/BIS)*T** (2.
1*A))/4.
      GN=((1.+A/BIS+DA)*(1.+DAW+A/BIS)+(1.-DA-A/BIS)*(1.-DAW+A/BIS)*T** (2.
1*A))/4.
      SUMA=SUMA+R(N)*(GN*P** (A-1.)-FN*P** (-1.-A))*DCOS(A*PH)
      SUMB=SUMB+R(N)*(FN*P** (-1.-A)+GN*P**A)*DSIN(A*PH)
      IF (IK-7) 30,33,33
33     SUMA=SUMA+R(NK+NI)*(GN*P** (A-1.)-FN*P** (-1.-A))*DSIN(A*PH)
      SUMB=SUMB+R(NK+NI)*(FN*P** (-1.-A)+GN*P**A)*DCOS(A*PH)
45     CONTINUE
      Q2=SLHD*DSIN(PH-AL)/P-(SUMA+P/YNT-(1.+1./YNT)/P)*DCOS(PH-AL)
      RETURN
      END

```

CARD NR. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

11 I

AN IF STATEMENT MAY BE MORE EFFICIENT THAN A 2 OR 3 BRANCH COMPUTED GO TO STATEMENT.

SYMBOLIC REFERENCE MAP (R=1)

ENTRY POINTS
3 FLUX

VARIABLES	SN	TYPE	RELOCATION				
1142 A		DOUBLE		0	AL	DOUBLE	F.P.
14 BI		DOUBLE	BUN2	12	BIS	DOUBLE	BUN2
0 CF		DOUBLE	F.P.	2	OA	DOUBLE	BUN2
0 DAM		DOUBLE	BUN2	22	OH	DOUBLE	BUN2
24 EPS		DOUBLE	BUN2	1156	FN	DOUBLE	
1160 GN		DOUBLE		0	IK	INTEGER	F.P.
0 JA		INTEGER	F.P.	1141	N	INTEGER	
0 NK		INTEGER	F.P.	0	P	DOUBLE	F.P.
4 PD		DOUBLE	BUN2	0	PH	DOUBLE	F.P.
16 PI		DOUBLE	BUN2	0	Q2	DOUBLE	F.P.
0 R		DOUBLE	ARRAY F.P.	1152	SUMA	DOUBLE	
1154 SUMB		DOUBLE		1137	SUM1	DOUBLE	
1144 SUM2		DOUBLE		1146	S1	DOUBLE	
1150 S2		DOUBLE		6	T	DOUBLE	BUN2
10 VE		DOUBLE	BUN2	0	VNT	DOUBLE	F.P.
20 YT		DOUBLE	BUN2				

EXTERNALS	TYPE	ARGS	OSIN	DOUBLE	1 LIBRARY
DCOS	DOUBLE	1 LIBRARY			

STATEMENT LABELS

26	1		133	2		310	3
115	10		0	12	INACTIVE	275	20
0	22	INACTIVE	1006	30		0	33
							INACTIVE

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
31	10	* N	13 18	678	EXT REFS
136	20	* N	22 29	1428	EXT REFS
314	30	* N	34 45	4758	EXT REFS

COMMON BLOCKS	LENGTH
BUN2	22

STATISTICS

PROGRAM LENGTH	12468	678
SCM LABELED COMMON LENGTH	268	22

SCOPE 2 LOAD MAP

PROGRAM WILL BE ENTERED AT LOCAL (330)

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SCN LENGTH 51171 LCM LENGTH 0

BLOCK	ADDRESS	LENGTH
/BUN1/	100	50
/BUN2/	150	26
/BUN3/	176	64
LOCAL	262	24135
COUPLE	24417	6172
SIZE	32611	165
TYPE	32776	151
CHAN	33147	1023
POTN	34172	75
NTB	34267	50
POINTS	34337	404
NEIGH	34743	331
GTPRD	35274	110
HATA	35404	150
LOC	35554	35
OGELS	35611	473
TAVG	36304	420
SIMPS	36724	563
LUMP	37507	350
TEMP	41057	1344
FLUX	41423	1246
/STP.END/	42671	1
/FCL.C./	42672	23
/CS.IO./	42715	140
QBNTY=	43055	1
CONIO=	43056	44
FECMSK=	43122	41
FLTIN=	43163	156
FLTOUT=	43341	315
FHTAP=	43656	372
FORSYS=	44250	561
FORUTL=	45031	16
GETFIT=	45047	43
INCOM=	45112	256
INPC=	45370	173
KODER=	45563	467
KRAKER=	46252	454
/HEADING/	46726	16
/LIN.CNT/	46744	1
OUTC=	46745	1054
OUTCOM=	51021	204
GOTCER=	50225	14
DATAN=	51241	17
DATCOM=	50260	106
DEULER=	50366	60
DEXP=	50446	34
DLNLOG=	51502	146
OSNCOS=	50650	150
OSORT=	51020	32
DTOD=	51052	13
OTOI=	51065	21
SYSID=	51106	1

SCOPE 2 LOAD MAP

SYS-1ST 51107 62

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**** NOT CERTIFIED

NOT CERTIFIED

NOT CERTIFIED

LOSUBVI

LISTING OF INPUT PARAMETERS

7-ROD ANALYSIS

MAX NO OF POINTS = 36 MAX NO OF TERMS = 24 NO OF ITERATION = 5

P/D = 1.24000 A/B = .87000 2H/D = 1.30000

KF/K = .13300 K/KC = .40000

BIS = .10000E+02 BI = .10000E+07

ROOTS OF LEGENDRE POLYNOMIALS

.14897E+00	-.14887E+00	.43340E+00	-.43340E+00	.67941E+00	-.67941E+00	.86506E+00	-.86506E+00
.97391E+00	-.97391E+00						

WEIGHTS OF GAUSS-LEGENDRE QUADRATURE

.29552E+00	.29552E+00	.26927E+00	.26927E+00	.21909E+00	.21909E+00	.14945E+00	.14945E+00
.66671E-01	.66671E-01						

SUBCHANNEL TEMPERATURES

.95930E+03 .89560E+03

NORMALIZATION FACTOR = 83.0000 DEGREE F

D.4 OUTPUT FOR TYPICAL RUN

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOSUBVI
 ITERATION 1

CELL 2 TYPE 2 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

-.50518E+00 .34221E+00 .17984E+00 -.11476E+00 -.97859E-01 .33204E-01 -.29546E-01 .29899E-02
 -.59903E-03 .23064E-03

T MEAN = -2.47094 AREA = 1.47422 NU = 6.22770

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/D	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.50111	-2.19297	1.01792	-2.54567	1.01792	-2.75525	-.01245
12.00000	1.36690	-2.19111	1.00545	-2.54169	1.00545	-2.71743	-.02809
24.00000	1.30716	-2.19251	.98702	-2.54010	.98702	-2.68342	-.02158
36.00000	1.30716	-2.20756	.99455	-2.55701	.99455	-2.70403	-.02523
48.00000	1.36690	-2.23205	1.03301	-2.58908	1.03301	-2.78345	-.03240
60.00000	1.50111	-2.24363	1.07846	-2.60947	1.07846	-2.89521	-.01327
72.00000	1.74932	-2.21470	1.10584	-2.58564	1.10584	-2.96159	-.03945
84.00000	1.51272	-2.12756	1.10094	-2.49691	1.10094	-2.85675	-.00509
96.00000	1.35735	-1.98384	1.05689	-2.34378	1.05689	-2.58464	-.03545
108.00000	1.26770	-1.80834	.98547	-2.15364	.98547	-2.30300	-.03279
120.00000	1.24000	-1.64073	.92708	-1.97393	.92708	-2.07984	-.01608
132.00000	1.26770	-1.51206	.91935	-1.84258	.91935	-1.94719	-.03605
144.00000	1.35735	-1.42461	.94826	-1.75936	.94826	-1.90382	-.02447
156.00000	1.35735	-1.36075	.95932	-1.69695	.95932	-1.86619	-.01152
168.00000	1.26770	-1.31281	.93295	-1.64419	.93295	-1.76910	-.01623
180.00000	1.24000	-1.29348	.91294	-1.62132	.91294	-1.72595	-.01428

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.00296E-01 .26106E-03 .25318E-04 -.37752E-05 -.30699E-05 -.19993E-05 -.48236E-06 -.39958E-07

T MEAN = -2.34404 AREA = .18207 NU = 17.21415

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/D	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	-1.87041	.97647	-2.21438	.97647	-2.34886	.17727
2.00000	1.24076	-1.87061	.97699	-2.21468	.97699	-2.34931	.16446
4.00000	1.24303	-1.87121	.97854	-2.21556	.97854	-2.35127	.14446
6.00000	1.24683	-1.87218	.98103	-2.21697	.98103	-2.35565	.14639
8.00000	1.25219	-1.87349	.98436	-2.21886	.98436	-2.36223	.16703

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOSUBVI
10.00000	1.25913	-1.87506	.98838	-2.22115	.98838	-2.36978	.17254		
12.00000	1.26770	-1.87684	.99288	-2.22372	.99288	-2.37775	.14907		
14.00000	1.27796	-1.87874	.99768	-2.22648	.99768	-2.38707	.13263		
16.00000	1.29997	-1.88068	1.00256	-2.22928	1.00256	-2.39847	.14879		
18.00000	1.30361	-1.88258	1.00731	-2.23203	1.00731	-2.41051	.16863		
20.00000	1.31958	-1.88435	1.01174	-2.23459	1.01174	-2.42144	.13921		
22.00000	1.33738	-1.88591	1.01566	-2.23685	1.01566	-2.43257	.10936		
24.00000	1.35735	-1.88721	1.01889	-2.23872	1.01889	-2.44543	.15747		
26.00000	1.37963	-1.88818	1.02130	-2.24012	1.02130	-2.45565	.14867		
28.00000	1.40439	-1.88877	1.02279	-2.24098	1.02279	-2.46333	.03680		
30.00000	1.43183	-1.88898	1.02330	-2.24127	1.02330	-2.47990	.45885		

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOSUBVI

ITERATION 2

CELL 2 TYPE 2 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.13951E+02 .13417E+02 -.47565E+00 .34286E+00 .17755E+00 -.11026E+00 -.96746E-01 .29029E-01
 -.26695E-01 .10233E-02

T MEAN = -2.46755 AREA = 1.47422 NU = 8.34405

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.50111	933.99813	1.01470	904.79174	1.01470	887.64270	-.01542
12.00000	1.36690	934.09315	1.00210	905.06379	1.00210	890.72171	-.03895
24.00000	1.30716	933.79577	.98396	905.80769	.98396	893.29911	-.03161
36.00000	1.30716	932.25743	.99256	903.29665	.99256	891.18323	-.03164
48.00000	1.36690	929.87261	1.03200	900.26584	1.03200	884.14565	-.03727
60.00000	1.50111	928.53815	1.07771	898.19356	1.07771	874.47481	-.01820
72.00000	1.74932	930.54974	1.10476	899.78460	1.10476	868.69440	-.03976
84.00000	1.53272	937.35292	1.09943	906.72233	1.09943	877.12362	-.00623
96.00000	1.35735	948.77944	1.05560	918.92297	1.05560	899.10132	-.03494
108.00000	1.26770	962.77721	.98589	934.10374	.98589	921.72773	-.02867
120.00000	1.24000	976.13396	.93058	948.41502	.93058	939.45529	-.00315
132.00000	1.26770	986.40878	.92427	958.88941	.92427	949.87157	-.01214
144.00000	1.35735	993.46162	.95046	965.62952	.95046	953.50288	-.00730
156.00000	1.35735	998.62956	.95905	970.71084	.95905	957.01494	-.00856
168.00000	1.26770	1002.44422	.93529	974.88832	.93529	964.43106	-.00519
180.00000	1.24000	1003.95843	.91795	976.65902	.91795	967.67391	.01302

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.96123E-01 .60914E-03 .38528E-04 -.34090E-05 -.26846E-05 .17580E-05 -.42397E-06 -.35130E-07

T MEAN = -2.34530 AREA = .18207 NU = 17.02850

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	1002.85965	.97170	974.37931	.97170	963.56712	.14279
2.00000	1.24076	1002.83934	.97233	974.34966	.97233	963.51268	.13213
4.00000	1.24303	1002.77929	.97421	974.26205	.97421	963.30630	.11630
6.00000	1.24683	1002.68221	.97725	974.12048	.97725	962.88332	.12080
8.00000	1.25219	1002.55242	.98130	973.93140	.98130	962.26211	.14265

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOSUBVI
10.00000	1.25913	1002.39569	.98616	973.70343	.98616	961.53648	.15192		
12.00000	1.26770	1002.21896	.99159	973.44683	.99159	960.75200	.13622		
14.00000	1.27796	1002.03004	.99735	973.17300	.99735	959.84510	.12526		
16.00000	1.28997	1001.83720	1.00319	972.89396	1.00319	958.76874	.14664		
18.00000	1.30381	1001.64886	1.00886	972.62185	1.00886	957.63206	.16954		
20.00000	1.31958	1001.47322	1.01412	972.36839	1.01412	956.56913	.14832		
22.00000	1.33738	1001.31787	1.01875	972.14446	1.01875	955.48893	.12774		
24.00000	1.35735	1001.18952	1.02257	971.95960	1.02257	954.28656	.17697		
26.00000	1.37963	1001.09369	1.02541	971.82165	1.02541	953.28587	.17614		
28.00000	1.40439	1001.03450	1.02717	971.73646	1.02717	952.48804	.08531		
30.00000	1.43183	1001.01448	1.02776	971.70766	1.02776	951.07534	.46797		

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOSUBVI

ITERATION 3

CELL 2 TYPE 2 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.13967E+02 .13414E+02 -.47175E+00 .34123E+00 .17813E+00 -.10992E+00 -.97043E-01 .29128E-01
 -.26753E-01 .10332E-02

T MEAN = -2.46709 AREA = 1.47422 NU = 8.36006

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.50111	934.03303	1.01442	904.83264	1.01442	887.70926	-.01611
12.00000	1.36690	934.12361	1.00183	905.10018	1.00183	890.77562	-.03960
24.00000	1.30716	933.81303	.98370	905.03050	.98370	893.33503	-.03225
36.00000	1.30716	932.25358	.99235	903.29753	.99235	891.19391	-.03230
48.00000	1.36690	929.84168	1.03184	900.23876	1.03184	884.12511	-.03791
60.00000	1.53111	928.47619	1.07756	898.13515	1.07756	874.42225	-.01870
72.00000	1.74932	930.45272	1.10457	899.63167	1.10457	868.62790	-.04009
84.00000	1.53272	937.21388	1.09917	906.58820	1.09917	877.02582	-.00658
96.00000	1.35735	948.58705	1.05530	918.73571	1.05530	898.94635	-.03522
108.00000	1.26770	962.51714	.98565	933.84750	.98565	921.48773	-.02906
120.00000	1.24000	975.73309	-.93052	948.07451	.93052	939.12093	-.00372
132.00000	1.26770	985.98177	.92450	958.45743	.92450	949.42971	-.01259
144.00000	1.35735	992.95646	.95098	965.11434	.95098	952.94413	-.00676
156.00000	1.35735	998.06687	.95969	970.13579	.95969	956.38816	-.03533
168.00000	1.26770	1001.84929	.93587	974.28146	.93587	963.79849	-.01355
180.00000	1.24000	1003.35363	.91848	976.04296	.91848	967.04021	.01410

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.95395E-01 .60151E-03 .38796E-04 -.34701E-05 -.27319E-05 -.17889E-05 -.43140E-06 -.35745E-07

T MEAN = -2.34525 AREA = .18207 NU = 17.03711

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	1004.12359	.97191	975.64008	.97191	964.81211	.14444
2.00000	1.24076	1004.10343	.97254	975.61065	.97254	964.75871	.13356
4.00000	1.24303	1004.04384	.97441	975.52371	.97441	964.55463	.11738
6.00000	1.24683	1003.94749	.97743	975.38322	.97743	964.13397	.12183
8.00000	1.25219	1003.81869	.98144	975.19559	.98144	963.51522	.14389

****	NOT	CERTIFIED	****	****	NOT	CERTIFIED	****	****	NOT	CERTIFIED	****	LOSUBVI
10.00000	1.25913	1003.66315	.98626	974.96935	.98626	962.79357	.15311					
12.00000	1.26778	1003.48777	.99165	974.71469	.99165	962.01503	.13687					
14.00000	1.27796	1003.30027	.99737	974.44294	.99737	961.11454	.12544					
16.00000	1.29997	1003.10889	1.00317	974.16602	1.00317	960.04374	.14686					
18.00000	1.30381	1002.92198	1.00879	973.89596	1.00879	958.91316	.16982					
20.00000	1.31958	1002.74767	1.01401	973.64442	1.01401	957.85860	.14785					
22.00000	1.33738	1002.59349	1.01861	973.42218	1.01861	956.78673	.12650					
24.00000	1.35735	1002.46610	1.02240	973.23870	1.02240	955.58997	.17617					
26.00000	1.37963	1002.37100	1.02522	973.10178	1.02522	954.59685	.17486					
28.00000	1.40439	1002.31225	1.02696	973.01723	1.02696	953.80732	.08296					
30.00000	1.43183	1002.29238	1.02755	972.98865	1.02755	952.38773	.47075					

**** NOT CERTIFIED **** **** NOT CERTIFIED **** **** NOT CERTIFIED **** LOSUBVI
ITERATION 4

CELL 2 TYPE 2 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.13969E+02 .13414E+02 -.47102E+00 .34091E+00 .17827E+00 -.10989E+00 -.97056E-01 .29104E-01
-.26743E-01 .10262E-02

T MEAN = -2.46700 AREA = 1.47422 NU = 8.36315

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.50111	934.04253	1.01438	904.84321	1.01438	887.72347	-.01617
12.00000	1.36690	934.13235	1.00178	905.11000	1.00178	890.78866	-.03973
24.00000	1.30716	933.81940	.98365	905.03791	.98365	893.34515	-.03239
36.00000	1.30716	932.25603	.99232	903.30085	.99232	891.19902	-.03242
48.00000	1.36690	929.83913	1.03181	900.23687	1.03181	884.12401	-.03400
60.00000	1.50111	928.46803	1.07754	898.12762	1.07754	874.41548	-.01877
72.00000	1.74932	930.43834	1.10453	899.67805	1.10453	868.61887	-.04016
84.00000	1.53272	937.19196	1.09912	906.56725	1.09912	877.01205	-.00667
96.00000	1.35735	948.55531	1.05525	918.70499	1.05525	898.91527	-.03532
108.00000	1.26770	962.47276	.98560	933.80380	.98560	921.44690	-.02911
120.00000	1.24000	975.73381	.93052	948.01518	.93052	939.06205	-.02375
132.00000	1.26770	985.90698	.92454	958.38192	.92454	949.35223	-.01263
144.00000	1.35735	992.86736	.95105	965.02380	.95105	952.84920	-.02583
156.00000	1.35735	997.96592	.95979	970.03286	.95979	956.27926	-.03487
168.00000	1.26770	1001.74023	.93600	974.16984	.93600	963.68663	-.02308
180.00000	1.24000	1003.24163	.91863	975.92813	.91863	966.91903	.01458

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.95381E-01 .60849E-03 .38840E-04 -.34822E-05 -.27416E-05 -.17952E-05 -.43293E-06 -.35872E-07

T MEAN = -2.34524 AREA = .18287 NU = 17.03725

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	1004.32256	.97192	975.83897	.97192	965.01052	.14453
2.00000	1.24076	1004.30240	.97255	975.80956	.97255	964.95724	.13361
4.00000	1.24303	1004.24282	.97441	975.72263	.97441	964.75337	.11736
6.00000	1.24683	1004.14649	.97743	975.58216	.97743	964.33275	.12182
8.00000	1.25219	1004.01770	.98144	975.39456	.98144	963.71390	.14394

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LCSUBVI
10.00000	1.25913	1003.86218	.98626	975.16836	.98626	962.99230	.15318		
12.00000	1.26770	1003.68683	.99165	974.91374	.99165	962.21407	.13687		
14.00000	1.27796	1003.49936	.99737	974.64203	.99737	961.31384	.12537		
16.00000	1.29997	1003.30801	1.00316	974.36515	1.00316	960.24301	.14566		
18.00000	1.30381	1003.12113	1.00879	974.03513	1.00879	959.11240	.16988		
20.00000	1.31958	1002.94603	1.01401	973.84363	1.01401	958.05819	.14781		
22.00000	1.33738	1002.79268	1.01861	973.62141	1.01861	956.98660	.12638		
24.00000	1.35735	1002.66531	1.02240	973.41795	1.02240	955.78954	.17620		
26.00000	1.37963	1002.57022	1.02522	973.30105	1.02522	954.79653	.17487		
28.00000	1.40439	1002.51148	1.02696	973.21652	1.02696	954.00734	.08262		
30.00000	1.43183	1002.49161	1.02755	973.18793	1.02755	952.58508	.47174		

**** NOT CERTIFIED **** NOT CERTIFIED **** NOT CERTIFIED **** LOSUBVI
 ITERATION 5

CELL 2 TYPE 2 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.13969E+02 .13414E+02 -.47000E+00 .34004E+00 .17829E+00 -.10988E+00 -.97058E-01 .29099E-01
 -.26742E-01 .10243E-02

T MEAN = -2.46698 AREA = 1.47422 NU = 8.36373

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.50111	934.04435	1.01437	904.84523	1.01437	887.72617	-.01618
12.00000	1.36690	934.13402	1.00177	905.11188	1.00177	890.79115	-.03975
24.00000	1.33716	933.82062	.98365	905.03933	.98365	893.34709	-.03242
36.00000	1.30716	932.25652	.99231	903.30150	.99231	891.20001	-.03244
48.00000	1.36690	929.83967	1.03181	900.23654	1.03181	884.12383	-.03802
60.00000	1.53111	928.46652	1.07753	898.12622	1.07753	874.41422	-.01878
72.00000	1.74932	930.43564	1.10452	899.67550	1.10452	868.61719	-.04017
84.00000	1.53272	937.18784	1.09911	906.56331	1.09911	877.00947	-.00669
96.00000	1.35735	948.54935	1.05523	918.69922	1.05523	898.91056	-.03534
108.00000	1.26770	962.46440	.98560	933.79556	.98560	921.43922	-.02912
120.00000	1.24000	975.72264	.93052	948.00399	.93052	939.05093	-.00376
132.00000	1.26770	985.89288	.92455	958.36756	.92455	949.33761	-.01263
144.00000	1.35735	992.85055	.95106	965.00673	.95106	952.81131	-.00684
156.00000	1.35735	997.94686	.95981	970.01343	.95981	956.25873	-.03478
168.00000	1.26770	1001.71962	.93603	974.14875	.93603	963.65835	-.00299
180.00000	1.24000	1003.22046	.91866	975.90642	.91866	966.89610	.01467

CELL 1 TYPE 1 POWER = 1.0000 FLOW = 1.0000

COEFFICIENTS FOR THE FOURIER SERIES

.95380E-01 .60030E-03 .38849E-04 -.34845E-05 -.27435E-05 -.17964E-05 -.43322E-06 -.35896E-07

T MEAN = -2.34524 AREA = .18207 NU = 17.03726

AZIMUTHAL DISTRIBUTION OF TEMPERATURE AND HEAT FLUX

DEGREES	C/B	TCLAD IN	QCLAD IN	TCLAD OUT	QCLAD OUT	T(C)	Q(C)
0.00000	1.24000	1004.36093	.97192	975.87734	.97192	965.04881	.14454
2.00000	1.24076	1004.34077	.97255	975.84792	.97255	964.99555	.13361
4.00000	1.24303	1004.28119	.97441	975.76100	.97441	964.79172	.11735
6.00000	1.24683	1004.18486	.97743	975.62053	.97743	964.37111	.12181
8.00000	1.25219	1004.05600	.98144	975.43293	.98144	963.75223	.14395

****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	****	NOT CERTIFIED	****	LOSUBVI
10.00000	1.25913	1003.90056	.98626	975.20673	.98626	963.03064	.15319		
12.00000	1.26770	1003.72521	.99165	974.95212	.99165	962.25246	.13687		
14.00000	1.27796	1003.53775	.99737	974.68042	.99737	961.35227	.12536		
16.00000	1.28997	1003.34640	1.00316	974.40354	1.00316	960.28143	.11486		
18.00000	1.30381	1003.15952	1.00879	974.13353	1.00879	959.15080	.10490		
20.00000	1.31958	1002.98523	1.01401	973.88203	1.01401	958.09664	.10481		
22.00000	1.33738	1002.83108	1.01861	973.65981	1.01861	957.02510	.10436		
24.00000	1.35735	1002.70371	1.02240	973.47636	1.02240	955.82797	.10421		
26.00000	1.37963	1002.60862	1.02522	973.33946	1.02522	954.83498	.10417		
28.00000	1.40439	1002.54988	1.02696	973.25493	1.02696	954.04584	.10425		
30.00000	1.43183	1002.53002	1.02755	973.22634	1.02755	952.62367	.10419		

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