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FREE VIBRATION ANALYSIS OF A STEAM GENERATOR TUBE BUNDLE WITH AND WITHOUT LATERAL SUPPORT

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MASTER

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CONTENTS

Mathematical Abbreviations and Symbols	vii
Abstract	ix
Summary	xi
Introduction	1
Free Vibration Modeling and Analysis	3
Discussion	3
Modeling Philosophy	3
Analytical Model	6
Free Vibration Analysis--In Air	9
Free Vibration Analysis--Support Plates 10	
and 11 Constrained	9
Free Vibration Analysis--Support Plate 11 x-Translation	
and Support Plate 10 Constrained, and Both	
Support Plates Unconstrained	9
Free Vibration Analysis--Submerged	13
Definition of Hydrodynamic Mass and Viscous Damping	13
Added Mass	14
Analysis	18
Added Mass Coefficients for Uncoupled Tube Motion	20
Added Mass Coefficients for Coupled Tube Motion	22
Added Mass Coefficients for the Large Tube Bundle	
Interacting With the Steam Generator Shroud and	
Adjacent Tubes Not Supported by the Support Plates	26
Conclusions and Recommendations	35
References	37

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Appendix A - Symmetric Mode Shape Plots, Both Support Plates	
Constrained	39
Appendix B - Antisymmetric Mode Shape Plots, Both Support	
Plates Constrained	61
Appendix C - Symmetric Mode Shape Plots, Support Plate 11	
Free and Support Plate 10 Constrained	83
Appendix D - Antisymmetric Mode Shape Plots, Support Plate 11	
Free and Support Plate 10 Constrained	109
Appendix E - Symmetric Mode Shape Plots, Both Support	
Plates Unconstrained	131
Appendix F - Antisymmetric Mode Shape Plots, Both Support	
Plates Unconstrained	153

LIST OF ILLUSTRATIONS

1. Elevation view of Millstone Unit 2 steam generator	4
2. Top view of half of the bundle array showing tubes and support plates	5
3. Finite element grid of the symmetric half-model	7
4. Egg-crate tube support schematic	8
5. Tube in a compressible-fluid annulus	16
6. Added mass coefficients, c_m , for various rod/wall-diameter ratios and for $A = 0.001$	17
7. Theoretical and experimental values of added mass coefficients, c_m , for a seven-rod bundle	18
8. Basic array configuration assumed for added mass coefficient, c_m , determination	19
9. Configuration used to determine added mass coefficient, c_m , for coupled tube motion	27
10. Tube support plates and steam generator shroud	28
11. Idealization of support plate 10 and tubes as an isotropic body of length, L	29
12. Two body motions with fluid coupling	31
13. Real values of H as a function of R_2/R_1 for a cylinder vibrating in a viscous fluid annulus	33

LIST OF TABLES

1. Summary of constraint conditions and mode frequencies for support plates 10 and 11 in air and submerged in water (in Hz)	xiii
2. Symmetric frequencies for both support plates constrained--in air	10
3. Antisymmetric frequencies for both support plates constrained--in air	10
4. Symmetric frequencies for support plate 11 free and support plate 10 constrained--in air	11
5. Antisymmetric frequencies for support plate 11 free and support plate 10 constrained--in air	11
6. Symmetric frequencies for both support plates free--in air	12
7. Antisymmetric frequencies for both support plates free--in air	12
8. Symmetric frequencies for both support plates constrained--submerged	23
9. Antisymmetric frequencies for both support plates constrained--submerged	23
10. Symmetric frequencies for support plate 11 free and support plate 10 constrained--submerged	24
11. Antisymmetric frequencies for support plate 11 free and support plate 10 constrained--submerged	24
12. Symmetric frequencies for both support plates free--submerged	25
13. Antisymmetric frequencies for both support plates free--submerged	25

MATHEMATICAL ABBREVIATIONS AND SYMBOLS

a	$\omega d/2c$
b	$\omega D/2c$
c	velocity of sound in liquid
C_M, C_m	hydrodynamic mass coefficient
C_v	viscous damping coefficient
d	external or outer diameter of the tube
D	diameter of rigid duct to confine the fluid annulus
F	total force
f_c	frequency submerged
f_{e_i}	frequency in air for rod i
f_{u_i}	frequency of the uncoupled mode for rod i
G	gap
H	$H = 1 + \frac{4K_1(\alpha)}{\alpha K_0(\alpha)}$
J_i	Bessel function of the first kind of order i
K_0, K_1	modified Bessel functions of the second kind
L	length of the isotropic body
M	body mass
M_a, M_H	hydrodynamic mass (or added mass)
m_B	mass of the body being studied
m_F	added mass of fluid external to tube per unit length
m_{F_i}	mass of the fluid displaced by rod i
M_H, M_a	hydrodynamic mass (or added mass)
M_{R_i}	mass of rod i
r, R	radius
v	volume

Y_i	Bessel function of the second kind of order i
α	dimensionless parameter
ω	vibration frequency
ρ, ρ_F	mass density of the fluid
ρ_R	mass density of the rod

ABSTRACT

This analysis was initiated at the request of the U.S. Nuclear Regulatory Commission, Division of Operating Reactors. The purpose was to define the vibrational modes and frequency characteristics of a pressurized water reactor (PWR) steam generator tube bundle assembly with and without lateral support in a fluid environment. The idealized half-model was constructed using the SAP-IV finite element code. Free vibration analyses were performed for an in-air case and a submerged in-water case, each with different constraint conditions at steam generator tube bundle assembly support plates 10 and 11. These constraint conditions included having both support plates free, having both support plates fixed, and having support plate 11 free while support plate 10 was fixed. It was found that as the support plate constraints were removed, the frequency range for each case increased significantly.

SUMMARY

An analysis of an idealized half-model of a pressurized water reactor (PWR) steam generator tube bundle assembly was performed to define the vibrational modes and frequency characteristics of a steam generator tube bundle with and without lateral support at tube support plates 10 and 11 in a fluid environment. The analysis was conducted for an in-air case and a submerged in-water case, each with different constraint conditions at support plates 10 and 11. These conditions included having

- Both support plates constrained or fixed against any motion.
- Only support plate 11 free to translate while support plate 10 was constrained.
- Both support plates 10 and 11 free to translate.

It was found that as the support plate constraints were removed, the frequency range for each case increased significantly. The results for each case are summarized in Table 1, which follows.

TABLE 1. Summary of constraint conditions and mode frequencies for support plates 10 and 11 in air and submerged in water. (in Hz)

Mode number	Both support plates 10 and 11 fixed	Support plate 11 and support plate 10 fixed	Both support plates 10 and 11 free
1	A ^a 7.081 (A 6.580) ^b	A 5.121 (A 4.582)	A 3.668 (A 3.296)
2	A 7.573 (A 7.019)	A 7.258 (A 6.783)	A 7.189 (A 6.694)
3	S ^a 7.619 (S 7.217)	S 7.554 (S 7.152)	S 7.517 (A 7.073)
4	S 7.922 (S 7.504)	A 7.802 (A 7.232)	A 7.609 (S 7.118)
5	A 8.111 (A 7.517)	S 7.863 (S 7.446)	S 7.840 (A 7.362)
6	S 8.241 (A 7.723)	S 8.180 (A 7.723)	A 7.921 (S 7.424)
7	A 8.330 (S 7.806)	A 8.330 (S 7.746)	S 8.157 (S 7.724)
8	S 8.581 (A 8.071)	A 8.393 (A 7.780)	A 8.382 (A 7.790)
9	A 8.708 (S 8.128)	S 8.516 (S 8.064)	S 8.491 (S 8.040)
10	A 8.939 (A 8.287)	S 8.873 (A 8.287)	A 8.656 (A 8.046)

^aA and S denote antisymmetric and symmetric mode frequencies, respectively.

^bNumbers in parentheses refer to submerged frequencies.

INTRODUCTION

This analysis was initiated at the request of the U.S. Nuclear Regulatory Commission, Division of Operating Reactors to define the vibrational modes and frequency characteristics of a PWR steam generator tube bundle assembly with and without lateral support in a fluid environment.

The dynamic response of nuclear reactor components is an area where analytical and experimental interest is growing. Some of these components are submerged, and dynamic analyses of the solid motions must take into account the presence of the liquids in order to provide meaningful results.

Current design practices consider the heat exchanger tubes to be simply-supported at the baffles without clearance.¹ Under this assumption, simple beam theory can be used to predict the natural frequencies of the tubing. Baffle spacing can then be chosen to keep the tube natural frequencies higher than the estimated excitation frequencies. If the tube/baffle clearances are small, such a design approach appears to be adequate to prevent gross tube vibration.

Generally, the spacing between the tubes in a steam generator is small. The vibration of a single element will interact with the surrounding elements because of fluid coupling. Therefore, any motion of an element in a steam generator will excite all the other elements, and the tubes will respond as a group rather than as a single element.

Few researchers have considered such a coupled* response of submerged structures. However, Chen^{2,3,4} has done extensive work in this area and has developed a consistent methodology. To account for coupling, Chen calculated the added mass coefficients for a group of circular cylinders based on potential flow theory. The equations of motion for rod bundles were then derived including hydrodynamic force, drag force, fluid pressure, gravity effect, axial tension, and damping. Based on these equations, a method of analysis was presented to study the free and forced vibration of rod bundles.

One consideration in the analysis of tube arrays is their inherent complexity. Because of this, it is often necessary to make simplifying assumptions about the array so that it can be modeled. The notion of added mass is one such simplification. Also when considering arrays, the absolute values of the added mass coefficients increase as the number of tubes increases. The implication is that the coupling between a tube and other tubes which are not immediately surrounding it can be neglected. For example, in a hexagonal arrangement, only the effects of the six tubes surrounding the central tube have to be considered if the motion of the central tube is of interest.

What trend the model is intended to demonstrate is another point to be considered. For this system, the trend is determined for natural frequencies as the lateral restraints on the tube support plates are removed. A model of a representative row of tubes was judged to be sufficient to show the desired behavior. It should be pointed out that if a more accurate representation of the tube bundle dynamic response or tube stresses is desired, then a more detailed and representative model should be used.

*A coupled mode refers to a natural mode in which all cylinders vibrate as a group with the same frequency and with a definite phase relationship among the cylinders. On the other hand, an uncoupled mode refers to a natural mode in which only a single cylinder is oscillating, while all the others are stationary.

FREE VIBRATION MODELING AND ANALYSIS

DISCUSSION

Due to the cracking phenomenon observed at the ligaments between tube holes and flow holes in the tube support plates of nuclear power plant steam generators, particular solutions have been implemented to lessen the problem. Plate expansion cannot occur uniformly since the support plates are welded to the tube bundle shroud and because of a solid metal rim at the outer periphery of the plates. Consequently, crack lines were found at tube-denting magnitudes less than would normally be expected to cause cracks in an unrestrained support plate.

In order to prevent further plate shifting from causing tube deformation, the restraints provided by the welded attachments of all tube support plates and by the continuous rim section of the upper (Level 11) tube support plates were removed. These modifications of the rim section and welded attachments from the tube support plates were considered sufficient to eliminate the development of successive cracking. The effects of the removal of restraints on the dynamic response of the tube bundle are evaluated in the subject analysis.

Modeling Philosophy

The steam generator analyzed is in the Millstone Unit 2 nuclear power plant (licensed to the Northeast Nuclear Energy Company) (Fig. 1). It was of interest to determine the dynamic response of the tube bundle when the lateral restraint of the support plates was removed. The number of tubes contained within the support plates is quite large, and the array pattern appears to be complex (Fig. 2). If one attempts to model such a system, the following questions have to be addressed:

- How can the actual physical system be represented analytically?
- What type of behavior is the model intended to show?

FIG. 1. Elevation view of Millstone Unit 2 steam generator.

FIG. 2. Top view of half of the bundle array showing tubes and support plates.

Depending on the desired behavior, there are a wide variety of modeling techniques that can be used to represent the actual physical system analytically. For example, the entire tube assembly could be considered. Clearly, such a model would be unfeasible from both a numerical and an analytical standpoint. Another approach would be to limit the system such that its response could be predicted by a more simplified model. The motion to one direction could be limited by assuming that the first few modes will actually respond in such a manner. There may be actual physical restraints that limit the structural response.

After careful examination of the system at hand, it was decided that structural restraints allowed for in-plane motion only. This being the case, a simplified model of the structure was constructed. The intent of the model was now of major importance. At the outset, it was agreed that the model should be constructed to show the relative change in frequencies for different restraint conditions at the tube support plates 10 and 11 when the tube bundle is submerged in water. In order to show this trend a row of tubes was modeled (Fig. 2). Because the tube bundles represent a symmetric structure, the analytical system was modeled as a symmetric half-model with symmetric and antisymmetric boundary conditions applied along the plane of symmetry (Fig. 3).

Analytical Model

The analytical model was constructed using the SAP-IV finite element code. Three-dimensional beam elements were used to represent the tubes, and concentrated masses were input to represent the mass and inertia of the two support plates. The model has a total of 404 node points, 1040 degrees of freedom, and 377 beam elements.

Additional lateral support is supplied to the model at the egg-crate tube supports (Fig. 4). Following the methodology of Rodgers and Pick,¹ the tubes were assumed to be pinned at these locations. In order to restrict structural response to plane motion, only z-rotation and y-translation are permitted at the egg-crate locations and the support plates (see Fig. 3 for coordinate directions). All other node points have x-translation, y-translation, and z-rotation. The model extends only to the egg-crate-7

FIG. 3. Finite element grid of the symmetric half-model.

FIG. 4. Egg-crate tube support schematic.

location. This was done primarily to reduce the size of the problem, and because it was felt that the type of behavior desired could be captured by such a model. The model is pinned at egg-crate 7. The mass and stiffness information were obtained from the Northeast Nuclear Energy Company.⁵

FREE VIBRATION ANALYSIS--IN AIR

Free Vibration Analysis--Support Plates 10 and 11 Constrained

The first case analyzed had both support plates 10 and 11 fixed to the steam generator shroud. Analytically, this was accomplished by allowing only y-translation and z-rotation and no x-translation for the tubes at the support plate locations. The frequencies and mode shapes were computed, and the results are shown in the Appendices and in Tables 2 and 3.

Free Vibration Analysis--Support Plate 11 x-Translation and Support Plate 10 Constrained, and Both Support Plates Unconstrained

As described earlier, in order to prevent nonuniform plate expansion, the welded attachments of all tube support plates were removed. In the finite element model this was accomplished by allowing x-translation at the support plate locations using the master-slave node option within SAP-IV. For the case being analyzed, Node 209 is the master node and all other nodes at this location are the slave nodes (Fig. 3). From a displacement point of view, specified slave node displacements are adopted from the master node. Then, in order to simulate x-translation of the support plate, all slave nodes have the same x-displacement as the master node, while all other specified displacements remain independent of the master node. The results are shown in Tables 4 and 5. As can be seen, there is a slight reduction in the frequencies for both the symmetric and antisymmetric cases from the constrained case, i.e., welded attachments removed. It is immediately obvious that the antisymmetric modes are more sensitive to any changes in the support plate constraint conditions. The analysis for both support plates 10 and 11 unconstrained is identical to this analysis except that, in addition to Node 209, Node 157 is also a master node for plate 10 (see Tables 6 and 7).

TABLE 2. Symmetric frequencies for both support plates constrained--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4787E+02	.7619E-01	.1313E+00
2	.4978E+02	.7922E+01	.1262E+00
3	.5178E+02	.8241E+01	.1213E+00
4	.5392E+02	.8581E+01	.1165E+00
5	.5619E+02	.8944E+01	.1118E+00
6	.5863E+02	.9311E+01	.1072E+00
7	.6123E+02	.9745E+01	.1026E+00
8	.6401E+02	.1019E+02	.9815E-01
9	.6701E+02	.1066E+02	.9377E-01
10	.7023E+02	.1118E+02	.8947E-01

TABLE 3. Antisymmetric frequencies for both support plates constrained--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4449E+02	.7081E+01	.1412E+00
2	.4758E+02	.7573E+01	.1321E+00
3	.5096E+02	.8111E+01	.1233E+00
4	.5234E+02	.8330E+01	.1200E+00
5	.5471E+02	.8708E+01	.1148E+00
6	.5617E+02	.8939E+01	.1119E+00
7	.5889E+02	.9373E+01	.1067E+00
8	.6042E+02	.9617E+01	.1040E+00
9	.6357E+02	.1012E+02	.9884E-01
10	.6519E+02	.1037E+02	.9639E-01

TABLE 4. Symmetric frequencies for support plate 11 free and support plate 10 constrained--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4745E+02	.7553E+01	.1324E+00
2	.4941E+02	.7863E+01	.1272E+00
3	.5140E+02	.8180E+01	.1222E+00
4	.5351E+02	.8516E+01	.1174E+00
5	.5575E+02	.8873E+01	.1127E+00
6	.5814E+02	.9253E+01	.1081E+00
7	.6069E+02	.9659E+01	.1035E+00
8	.6342E+02	.1009E+02	.9908E-01
9	.6635E+02	.1056E+02	.9470E-01

TABLE 5. Antisymmetric frequencies for support plate 11 free and support plate 10 constrained--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.3218E+02	.5121E+01	.1953E+00
2	.4560E+02	.7258E+01	.1378E+00
3	.4902E+02	.7802E+01	.1282E+00
4	.5234E+02	.8330E+01	.1200E+00
5	.5274E+02	.8393E+01	.1191E+00
6	.5617E+02	.8939E+01	.1119E+00
7	.5684E+02	.9046E+01	.1105E+00
8	.6042E+02	.9617E+01	.1040E+00
9	.6141E+02	.9774E+01	.1023E+00
10	.6519E+02	.1037E+02	.9639E-01

TABLE 6. Symmetric frequencies for both support plates free--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4723E+02	.5717E+01	.1330E+00
2	.4926E+02	.7840E+01	.1276E+00
3	.5125E+02	.8157E+01	.1226E+00
4	.5335E+02	.8491E+01	.1178E+00
5	.5557E+02	.8845E+01	.1131E+00
6	.5794E+02	.9222E+01	.1084E+00
7	.6047E+02	.9624E+01	.1039E+00
8	.6318E+02	.1005E+02	.9945E-01
9	.6608E+02	.1052E+02	.9509E-01
10	.6919E+02	.1101E+02	.9080E-01

TABLE 7. Antisymmetric frequencies for both support plates free--in air.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.2305E+02	.3668E+01	.2726E+00
2	.4517E+02	.7189E+01	.1391E+00
3	.4781E+02	.7609E+01	.1314E+00
4	.4977E+02	.7921E+01	.1262E+00
5	.5267E+02	.8382E+01	.1193E+00
6	.5439E+02	.8656E+01	.1155E+00
7	.5673E+02	.9029E+01	.1108E+00
8	.5868E+02	.9340E+01	.1071E+00
9	.6126E+02	.9749E+01	.1026E+00
10	.6338E+02	.1009E+02	.9913E-01

FREE VIBRATION ANALYSIS--SUBMERGED

Definition of Hydrodynamic Mass and Viscous Damping

A structural component encounters no resistance when moving at a constant velocity in an infinite ideal fluid, assuming irrotational flow about the body. This phenomenon is commonly referred to as D'Alembert's paradox. A body moving at a variable velocity, even in a condition of potential flow, experiences a resistance. The body behaves as though an added mass of fluid were rigidly attached to and moving with it. When the body is subjected to an excitation, not only must the mass of the body be accelerated, but also the added fluid mass. The total force, F , required to accelerate the body is given by Chen⁶ as

$$F = (m + m_a) \frac{\partial^2 u}{\partial t^2} + c_v \frac{\partial u}{\partial t} \quad , \quad (1)$$

where

m = body mass

m_a = hydrodynamic mass (or added mass)

c_v = viscous damping coefficient.

The sum of the hydrodynamic mass and the body mass is called the virtual mass or the apparent mass. The viscous damping coefficient depends on fluid viscosity, fluid density, vibration frequency, and other factors. If the fluid is inviscid, c_v is zero. Note that two force components are associated with the fluid motion:

- $m_a \frac{\partial^2 u}{\partial t^2}$, in phase with the acceleration, arises because the fluid moves as the body oscillates.
- $c_v \frac{\partial u}{\partial t}$, opposing the movement of the body, is attributed to fluid viscosity.

The hydrodynamic mass is proportional to the fluid density, ρ , and the body volume, V , and is given by

$$m_a = \rho V c_m, \quad (2)$$

where

c_m = hydrodynamic mass coefficient.

Equation (1) corresponds to a body moving in a particular direction. For a body having three degrees of freedom both in translation and rotation, a complete description of the hydrodynamic mass requires a six-by-six matrix. The hydrodynamic masses for a body with N degrees of freedom are represented by $[m_{ij}]$ ($i, j = 1, 2, 3, \dots, N$). It can be shown that $[m_{ij}]$ is symmetric; thus, for a body with N degrees of freedom, the number of terms needed to completely describe the hydrodynamic mass is $N(N + 1)/2$.

In a structural component consisting of multiple bodies, the situation is more complicated. For example, consider a structure containing k bodies, in which each body possesses N' degrees of freedom. The hydrodynamic masses again can be represented by a matrix $[m_{ij}]$ ($i, j = 1, 2, 3, \dots, N$; $N = N' \times k$). As in the case of a single body, $[m_{ij}]$ is symmetric.

Since $[m_{ij}]$ is symmetric, one can find the N eigenvalues of the hydrodynamic mass matrix $[m_{ij}]$. These eigenvalues are called effective hydrodynamic masses. In many practical applications, the effective hydrodynamic masses for multiple bodies play the same role as the hydrodynamic mass for a single body.⁴

Added Mass

When a tube array is immersed in a shell-side fluid medium, the added mass of a tube is no longer the mass of fluid displaced by the tube, but it is

affected by the adjacent tubes and the shell-side wall or shroud that surrounds the tube bundle. The added mass can be expressed, in general, by

$$m_F = c_m \rho \frac{\pi d^2}{4} \quad , \quad (3)$$

where

m_F = added mass of fluid external to the tube per unit length

c_m = added mass coefficient

ρ = mass density of the fluid

d = external diameter of the tube.

If a single tube is immersed in an infinite fluid medium, $c_m = 1$. In other words, the added mass of the tube is the mass of fluid displaced by the tube.

The added mass of the tube in a compressible-fluid annulus (Fig. 5) has been reported by Chen and Wambsganss,⁷ and their expression of c_m is

$$c_m = \frac{[bJ_0(b) - J_1(b)]Y_1(a) - [bY_0(b) - Y_1(b)]J_1(a)}{[bY_0(b) - Y_1(b)][aJ_0(a) - J_1(a)] - [bJ_0(b) - J_1(b)][aY_0(a) - Y_1(a)]} \quad (4)$$

where

J_i, Y_i = Bessel functions of the first and second kinds, of order i

$a = \omega d/2c$

$b = \omega D/2c$

and

ω = vibration frequency

c = velocity of sound in fluid

d = outer diameter of tube

D = diameter of rigid duct to confine the fluid annulus.

FIG. 5. Tube in a compressible-fluid annulus.

Chen and Wambsganss explained the added mass of a tube in a compressible fluid annulus as follows: The added mass coefficient, c_m , depends on the frequency, sound velocity, and diameter of tube and duct, as can be seen from Eq. (4). Results of computations for various diameter ratios (Fig. 6) show that the added mass coefficient, c_m , is insensitive to the parameter, $a = \omega d / 2c$. In general, a is small; therefore, the fluid may be assumed to be incompressible. Figure 6 shows that the boundary effect is important when the tube is relatively close to the duct wall, i.e., when $D/d < 3$.

FIG. 6. Added mass coefficients, c_m , for various rod/wall-diameter ratios and for $A = 0.001$.

If a tube is immersed in an incompressible fluid annulus, according to Chin and Wambsganss,⁸ Eq. (4) reduces to

$$c_m = \frac{(D/d)^2 + 1}{(D/d)^2 - 1} \quad (5)$$

The effect of fluid viscosity on the added mass and damping of a rod vibrating in a fluid annulus were also evaluated in a theoretical/experimental study.⁶

The added mass of a cylinder confined by hexagonal arrays of six similar cylinders has been studied experimentally.⁶ A method of analysis was developed at Argonne National Laboratory (ANL) for calculating the added mass tensor of a group of circular cylinders vibrating in a liquid. This method can be applied to a group of circular cylinders arranged in any tube pattern, and the cylinders may have different diameters. Typical results for a seven-tube hexagonal array are shown in Fig. 7.

FIG. 7. Theoretical and experimental values of added mass coefficients, c_m , for a seven-rod bundle.

Analysis

Because of the complexity of tube arrays (Fig. 8), a precise analysis is difficult. However, it is possible to simplify this situation by reducing the size of the problem and performing a detailed analysis of the reduced system. With submerged structures, the fluid coupling has to be accounted for, and, with the reduced system, this is a much easier task to accomplish. Figure 8 shows the basic hexagonal configuration used to formulate this phase of the analysis. Chen² has shown that to determine the dynamic response of a particular tube in an array only the effects of the six tubes surrounding the tube of interest need be considered, and all others can be neglected. This assumption was used to

- Determine the added mass coefficients for the tube row.
- Calculate the modes and frequencies for the system being studied.

FIG. 8. Basic array configuration assumed for added mass coefficient, c_m , determination.

Before beginning any detailed analysis, however, the question of fluid coupling in the problem must be addressed. It is known that for submerged structures, e.g., steam generator tube bundles, fluid coupling among structural components exists to some degree. Chen⁴ has shown that the degree of coupling is a function of the added mass coefficient, c_m , and that for a group of equally spaced identical tubes, it is dependent upon gap only.

When the results of the in-air part of the analysis are examined and the definition of coupled and uncoupled mode shapes is recalled, it can be seen that, for some cases, only individual tubes are excited for all modes considered. For this type of behavior it was assumed that the array exhibited uncoupled motion. For the remaining cases there appears to be a mix of uncoupled and coupled motion for the various modes, i.e., the first mode might show the array responding as a group, while other modes exhibit individual tube motion.

This is an important distinction, because the value of the added mass coefficient, c_m , will depend upon whether the modes are coupled or uncoupled. The basic assumptions are as follows:

- If the tubes respond individually, then the resulting modes are uncoupled.
- If they behave as a group, then the modes are coupled.

For the case where support plates 10 and 11 are fixed to the shroud, the tubes responded individually. (The mode shape lots for the various cases are shown in the Appendices.) For the cases where the tube support plate restraints were released, the mode shapes were mixed. Since these latter cases yield both coupled and uncoupled modes, the analysis is slightly more complicated than a case which has only one mode. With this in mind, calculation of the added mass coefficients is now undertaken.

Added Mass Coefficients for Uncoupled Tube Motion. For the case of individual tube response, or uncoupled response, the derivation of the added mass coefficient, c_m , is direct. Figure 7 shows the added mass coefficient, c_m , for different values of gap, g , to radius, R , ratio, G/R .⁴ This curve assumes that only tube 1, the center tube, moves while all the others remain rigid. As can be seen from the curve, c_m increases asymptotically as $G/R \rightarrow 0$ and likewise, $c_m \rightarrow 1$ as G/R becomes larger.

Referring to the mode shape plots in the Appendices for the first case, i.e., both plates constrained, it can be seen that the horizontal tubes are responding sequentially for the first ten modes. While the spacing is symmetric, the gap is different from the vertical tube gap, i.e., $G = .25$ in. for the vertical tubes when $G/R = .67$, and $G = .98$ in. for the horizontal tubes when $G/R = 2.6$. From Fig. 7 when $G/R = .67$, then $c_m = 1.38$; when $G/R = 2.6$, it is assumed that $c_m = 1.0$, which implies the surrounding tubes do not influence the motion of the central tube. To be absolutely correct, each mode should be examined and the proper added mass coefficient applied depending on which portion of the tube, vertical or horizontal, is responding. Using the derived values of c_m in the program, the system modes and frequencies can be a symmetric case of both support plates constrained by taking into account the added mass of the fluid. For computation the mass density used by SAP-IV is given by

$$\rho = \rho_R + c_m \rho_F \quad , \quad (6)$$

where

ρ_R = mass density of the rod

ρ_F = mass density of the fluid (water).

The accuracy of the method can easily be checked with a simple hand calculation.

From the in-air analysis the modes (as shown in the Appendices) appear to be uncoupled for the first symmetric mode with both plates constrained. The submerged frequency can be calculated by the following formula:⁴

$$f_{u_i} = \left(\frac{m_{R_i}}{m_{R_i} + c_m m_{F_i}} \right)^{1/2} f_{e_i} \quad , \quad (7)$$

where

f_{u_i} = frequency of the uncoupled mode for rod i

m_{R_i} = mass of rod i

c_m = added mass coefficient

m_{F_i} = mass of the fluid displaced by rod i

f_{e_i} = frequency in air for rod i.

Using the data for the finite element model,

$$f_{u_1} = .82 f_{e_1} = .82 (7.62 \text{ Hz})$$

or

$$f_{u_1} = 6.25 \text{ Hz} .$$

Comparing this to the finite-element-generated results shown in Table 8, there is good agreement. The procedure is the same for the remaining modes, but care must be taken to observe which portion of the tube responds and then to choose the appropriate value of c_m . The results for the submerged antisymmetric modes for the first case (both support plates fixed) are presented in Table 9. The results for both the submerged modes are presented in Tables 10 and 11 for the second case and in Tables 12 and 13 for the third case.

Added Mass Coefficients for Coupled Tube Motion. Consider the results of the in-air analysis for the second and third antisymmetric cases, i.e., support plate 10 fixed and support plate 11 free, and both support plates free, respectively. It is immediately obvious that the first mode is the primary structural mode for each case. In this mode, all tubes in the bundle vibrate as a group. This type of group behavior leads to a different formulation of the added mass coefficient.

TABLE 8. Symmetric frequencies for both support plates constrained--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4535E+02	.7217E+01	.1386E+00
2	.4715E+02	.7504E+01	.1333E+00
3	.4905E+02	.7806E+01	.1281E+00
4	.5107E+02	.8128E+01	.1230E+00
5	.5324E+02	.8473E+01	.1180E+00
6	.5554E+02	.8839E+01	.1131E+00
7	.6094E+02	.9700E+01	.1031E+00
8	.6652E+02	.1059E+02	.9446E-01
9	.7003E+02	.1115E+02	.8972E-01
10	.7332E+02	.1167E+02	.8579E-01

TABLE 9. Antisymmetric frequencies for both support plates constrained--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4134E+02	.6580E+01	.1520E+00
2	.4410E+02	.7019E+01	.1425E+00
3	.4723E+02	.7517E+01	.1330E+00
4	.4852E+02	.7723E+01	.1295E+00
5	.5071E+02	.8071E+01	.1239E+00
6	.5207E+02	.8287E+01	.1207E+00
7	.5459E+02	.8688E+01	.1151E+00
8	.5602E+02	.8916E+01	.1122E+00
9	.5892E+02	.9378E+01	.1066E+00
10	.6044E+02	.9619E+01	.1040E+00

TABLE 10. Symmetric frequencies for support plate
11 free and support plate 10 constrained--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4494E+02	.7152E+01	.1398E+00
2	.4679E+02	.7446E+01	.1343E+00
3	.4867E+02	.7746E+01	.1291E+00
4	.5067E+02	.8064E+01	.1240E+00
5	.5279E+02	.8402E+01	.1190E+00
6	.5508E+02	.8766E+01	.1141E+00
7	.5791E+02	.9217E+01	.1085E+00
8	.6242E+02	.9935E+01	.1007E+00
9	.6599E+02	.1050E+02	.9522E-01
10	.7001E+02	.1114E+02	.8975E-01

TABLE 11. Antisymmetric frequencies for support plate
11 free and support plate 10 constrained--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.2879E+02	.4582E+01	.2182E+00
2	.4234E+02	.6738E+01	.1484E+00
3	.4544E+02	.7232E+01	.1383E+00
4	.4852E+02	.7723E+01	.1295E+00
5	.4888E+02	.7780E+01	.1285E+00
6	.5207E+02	.8287E+01	.1207E+00
7	.5268E+02	.8385E+01	.1193E+00
8	.5602E+02	.8916E+01	.1122E+00
9	.5692E+02	.9059E+01	.1104E+00
10	.6044E+02	.9619E+01	.1040E+00

TABLE 12. Symmetric frequencies for both support plates free--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.4473E+02	.7118E+01	.1405E+00
2	.4665E+02	.7424E+01	.1347E+00
3	.4853E+02	.7724E+01	.1295E+00
4	.5052E+02	.8040E+01	.1244E+00
5	.5263E+02	.8376E+01	.1194E+00
6	.5488E+02	.8734E+01	.1145E+00
7	.5742E+02	.9139E+01	.1094E+00
8	.6088E+02	.9689E+01	.1032E+00
9	.6541E+02	.1041E+02	.9607E-01
10	.6895E+02	.1097E+02	.9112E-01

TABLE 13. Antisymmetric frequencies for both support plates free--submerged.

Mode number	Circular frequency, rad/s	Frequency, cps (Hz)	Period, s
1	.2071E+02	.3296E+01	.3034E+00
2	.4206E+02	.6694E+01	.1494E+00
3	.4444E+02	.7073E+01	.1414E+00
4	.4626E+02	.7362E+01	.1358E+00
5	.4894E+02	.7790E+01	.1284E+00
6	.5055E+02	.8046E+01	.1243E+00
7	.5272E+02	.8390E+01	.1192E+00
8	.5455E+02	.8681E+01	.1152E+00
9	.5693E+02	.9060E+01	.1104E+00
10	.5892E+02	.9378E+01	.1066E+00

For this type of behavior, as an approximation, the fluid can be considered to be "locked" as an annulus around each tube in the bundle, i.e., the fluid moves with the bundle. This is shown schematically in Fig. 9. The added mass coefficient, given by Eq. (5), for $D = 1.0$ in. and $d = .75$ in. is

$$c_m = \frac{(D/d)^2 + 1}{(D/d)^2 - 1}$$

or

$$c_m = 3.6 \quad .$$

As before, the density used in the program is a combination of the tube density and fluid density, or

$$\rho = \rho_R + c_m \rho_{wf} \quad .$$

Using this value for density, the effects of the fluid can now be determined. These results are presented in the Appendices. Up to now, the effects of the fluid on group motion and individual tube motion have been accounted for. However, another case exists which is as important. This case is the fluid/structure interaction between the tubes supported by support plates 10 and 11 moving as a group and the steam generator shroud.

Added Mass Coefficients for the Large Tube Bundle Interacting With the Steam Generator Shroud and Adjacent Tubes Not Supported by the Support Plates. For the group motion of the tube bank shown in the first modes of cases two and three, the interaction of the entire tube bank supported by the support plates, those tubes adjacent to but not supported by the support plates, and the steam generator shroud must be considered. In order to analyze this case, the portion of the tube bank under consideration must be idealized and established methods applied to obtain c_m . The tube bank can be considered as an idealization (Fig. 10). It is known that $G \approx 4$ in. and $g = .25$ in.⁵

FIG. 9. Configuration used to determine added mass coefficient, c_m , for coupled tube motion.

FIG. 10. Tube support plates and steam generator shroud.

Consider the lower support plate⁶ shown in Fig. 10. It was determined that $AB = 121.5$ in. and $CD = 28.6$ in.,⁵ from which the area (A), of the lower plate, is easily found, or

$$(A) = 2411.8 \text{ in.}^2$$

Likewise, the area surrounding support plate 10 bounded by the shroud and remaining tubes is

$$A' = 3008.9 \text{ in.}^2$$

It remains now to determine the value of c_m for the two-body problem with fluid coupling (Fig. 11).

FIG. 11. Idealization of support plate 10 and tubes as an isotropic body of length, L .

Fritz⁹ has analyzed a similar problem with two concentric cylinders separated by a fluid annulus (Fig.12). The inner cylinder of radius, a , is surrounded by an outer concentric cylindrical container of inner radius, b . The length of the annulus is L , where L is much greater than b . For this configuration Fritz found that the hydrodynamic mass, M_H , is given by

$$M_H = M_1 \frac{b^2 + a^2}{b^2 - a^2} + M_1 \frac{A' + A}{A' - A} + M_1 C_M, \quad (8)$$

where

M_1 = mass of the fluid displaced by the inner cylinder.⁹

Using the results obtained by Fritz, it is possible to achieve an approximation for c_m for the configuration shown in Fig. 11. This system must be transformed to the one analyzed by Fritz. It should be noted, however, that this technique is only an approximation and the results serve only to give a first order of magnitude estimate as to the value of c_m for the gross structural behavior being studied.

The inner cylinder (Fig. 12) has an area given by

$$A_i = \pi a^2.$$

Let $A_i = \textcircled{A}$, from which can be found an equivalent value of a for the configuration being studied, or

$$a = 27.7 \text{ in.}$$

Similarly, the inner area of the outer cylinder is given by

$$A_o = \pi b^2.$$

Likewise let $A_o = A'$, and it is found that

$$b = 30.95 \text{ in.}$$

FIG. 12. Two body motions with fluid coupling.⁹

Finally, the value of M_1 can be determined. The fluid mass displaced by the inner cylinder is

$$M_1 = \pi a^2 L \rho_{F200} \quad , \quad (9)$$

where

ρ_{F200} = fluid density at an assumed temperature of 200°F

L = length of the isotropic body.

Assuming $L = 46.67$ in., then from Eq. (9) it is found that

$$M_1 = 3914.7 \text{ lbm} \quad .$$

The hydrodynamic mass is then determined from Eq. (8) to be

$$M_H = (3914.7)(9.05) = 35427.7 \text{ lbm} \quad ,$$

where

$$c_m = 9.05 \quad .$$

To check these results, with $r = 27.7$ in. and $\omega = 23.05 \text{ s}^{-1}$, it can be found from Fig. 13 that

$$S = 3.59 \times 10^7 \quad .$$

For $R_2/R_1 = 1.12$, it can also be found from Fig. 13 that $R_e(H)$, the real part of the complex quantity H ,¹⁰ agrees with the value obtained above:

$$R_e(H) \equiv c_m = 9 \quad ,$$

FIG. 13. Real values of H as a function of R_2/R_1 for a cylinder vibrating in a viscous fluid annulus.

where

$$H = 1 + \frac{4K_1(\alpha)}{\alpha K_0(\alpha)}$$

and

K_0, K_1 = modified Bessel functions of the second kind

α = dimensionless parameter.

A hand calculation can be performed to determine the expected submerged frequency for this configuration, i.e., the antisymmetric case with both support plates unconstrained. Recall from Eq. (7) that the submerged frequency is given by

$$f_c = f_e \left(\frac{m_B}{m_B + c_m m_F} \right)^{1/2}$$

where

m_B = mass of the body being studied.

From previous analysis, it was found that when the tube array moves as a group, the fluid between the tubes is essentially "locked" in between the tubes and moves with them. The value of the added mass coefficient for this case was $c_m \approx 3.6$, and the density was assumed to be given by Eq. (6), i.e.,

$$\rho = \rho_R + c_m \rho_F$$

Using these results, the mass, m_B , is calculated from Eq. (9) to be

$$m_B = \rho \pi a^2 L = 42,500 \text{ lbm}$$

With these parameters and knowing that $f_e = 3.67 \text{ Hz}$, it can be found from Eq. (7)

$$f_c = 2.71 \text{ Hz.}$$

CONCLUSIONS AND RECOMMENDATIONS

A method of analysis was presented to compute the modes and frequencies of a PWR steam generator tube bundle array for submerged and in-air conditions. This free vibration analysis is based on a finite element half-model of the structure with various tube support plate constraint conditions. In this analysis, the modes and frequencies of the structure were found for the in-air and submerged cases.

Based on the results of the analysis, the frequency ranges for the various support plate conditions were found to be:

- Both plates constrained (in air): 7.081 to 11.180 Hz
- Both plates constrained (submerged): 6.580 to 11.670 Hz
- Support plate 11 unconstrained (in air): 5.121 to 10.560 Hz
- Support plate 11 unconstrained (submerged): 4.582 to 11.14 Hz
- Both support plates unconstrained (in air): 3.668 to 11.01 Hz
- Both support plates unconstrained (submerged): 3.296 to 10.97 Hz.

From these results, it can be concluded that as the support plate constraints were removed, the frequency range for each case increased significantly. With the frequencies being in such close proximity within the range, it would seem necessary to model the system to more accurately determine the modal resolution.

It is noted again that the system as modeled was only intended to show the effect of removing the lateral support plate restraints in air and in water. The model should be refined if an accurate representation of stresses is desired, or if transient response to a particular forcing function is sought. It should also be noted that the ideas used to determine the added mass coefficients and, hence, the submerged frequencies are not adequate to handle complex mode shapes, such as symmetric modes 33 and 34 for both plates unconstrained. A more detailed theory is needed to address this problem.

As a result of these conclusions it is recommended that:

- A more refined finite element mesh be generated which is an accurate representation of the actual structure.
- A hydrodynamic mass code be implemented to determine added mass coefficients for more complex array patterns.
- Alternate methods of modeling the fluid/solid interaction problem be investigated.¹¹
- A test on a simple array be performed so that the validity of the analytical model can be verified.

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APPENDIX A

SYMMETRIC MODE SHAPE PLOTS, BOTH SUPPORT PLATES CONSTRAINED

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE *

DISP. S.F. = 7.00000E+00

•• CASE 1S - PLATES 10 & 11 FIXED ••
MODE * 2

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 3

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE *

4

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 5

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****

MODE *

6

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****

MODE *

7

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 8

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****

MODE *

9

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 10

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****

MODE *

30

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****

MODE * 31

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 32

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 33

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 34

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 35

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 36

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 37

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 38

DISP. S.F. = 7.00000E+00

**** CASE 1S - PLATES 10 & 11 FIXED ****
MODE * 39

DISP. S.F. = 7.00000E+00

APPENDIX B

ANTISYMMETRIC MODE SHAPE PLOTS, BOTH SUPPORT PLATES CONSTRAINED

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 1

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * **2**

DISP. S.F. = 7.00000E+00

** CASE 1 - BOTH PLATES 10 6 11 FIXED **
MODE * 3

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 4

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE # 5

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 6

DISP. S.F. = 7.00000E+00

•• CASE 1 - BOTH PLATES 10 6 11 FIXED ••
MODE * 7

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 8

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 9

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 6 11 FIXED ****
MODE * 10

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 11

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 12

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 13

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 14

DISP. S.F. = 7.00000E+00

•• CASE 1 - BOTH PLATES 10 & 11 FIXED ••
MODE • 15

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 16

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE • 17

DISP. S.F. = 7.00000E+00

**** CASE 1 - BOTH PLATES 10 & 11 FIXED ****
MODE * 18

DISP. S.F. = 7.00000E+00

•• CASE 1 - BOTH PLATES 10 & 11 FIXED ••
MODE * 19

DISP. S.F. $\approx 7.00000E+00$

•• CASE 1 - BOTH PLATES 10 & 11 FIXED ••
MODE • 20

DISP. S.F. = 7.00000E+00

APPENDIX C

SYMMETRIC MODE SHAPE PLOTS, SUPPORT PLATE 11
FREE AND SUPPORT PLATE 10 CONSTRAINED

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 1

DISP. S.F. = 7.00000E+00

•• CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ••
MODE • 2

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 3

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE *

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 5

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****

MODE *

6

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 7

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 8

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 9

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 10

DISP. S.F. $\approx 7.00000E+00$

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****

MODE *

27

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 28

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 29

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 30

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE • 31

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 32

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE • 33

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE # 34

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE • 35

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 36

DISP. S.F. = 7.00000E+00

•• CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ••
MODE * 37

DISP. S.F. = 7.00000E+00

**** CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 38

DISP. S.F. = 7.00000E+00

•• CASE 2S - PLATE 10 FIXED & PLATE 11 FREE ••
MODE * 39

DISP. S.F. = 7.00000E+00

APPENDIX D

ANTISYMMETRIC MODE SHAPE PLOTS, SUPPORT PLATE 11 FREE
AND SUPPORT PLATE 10 CONSTRAINED

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 1

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE • 2

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE *

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE * 4

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE * 5

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE *

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 7

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE *

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE * 9

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE * 10

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 30

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 31

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 32

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE *

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE •

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE *

CASE 1 *

08.33.37.

DISP. S.F. = 7.00000E+00

** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE **
MODE * 36

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 37

CASE 1**

08.33.37.

DISP. S.F. $\approx 7.00000E+00$

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE * 38

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

**** CASE 2 - PLATE 10 FIXED & PLATE 11 FREE ****
MODE •

CASE 1**

08.33.37.

DISP. S.F. = 7.00000E+00

APPENDIX E

SYMMETRIC MODE SHAPE PLOTS, BOTH SUPPORT
PLATES UNCONSTRAINED

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 1

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 3

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE *

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 5

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 6

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 7

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE • 8

DISP. S.F. = 7.00000E+00

•• CASE 3S - PLATES 10 & 11 FREE ••

MODE •

9

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 10

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 27

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 30

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****

MODE *

31

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 32

DISP. S.F. = 7.00000E+00

** CASE 3S - PLATES 10 & 11 FREE **
MODE * 33

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 34

DISP. S.F. = 7.00000E+00

**** CASE 35 - PLATES 10 & 11 FREE ****
MODE * 35

DISP. S.F. $\approx 7.00000E+00$

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 36

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 37

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 38

DISP. S.F. = 7.00000E+00

**** CASE 3S - PLATES 10 & 11 FREE ****
MODE * 39

DISP. S.F. = 7.00000E+00

APPENDIX F

ANTISYMMETRIC MODE SHAPE PLOTS, BOTH
SUPPORT PLATES UNCONSTRAINED

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 1

DISP. S.F. = 7.00000E+00

•• CASE 3 - PLATES 10 & 11 RELEASED ••
MODE •

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****

MODE *

4

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 5

DISP. S.F. $\approx 7.00000E+00$

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 6

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****

MODE * 7

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 8

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 9

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****

MODE *

10

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE # 30

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 31

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 32

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 33

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 34

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 35

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE * 36

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & .1 RELEASED ****
MODE * 37

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****
MODE # 38

DISP. S.F. = 7.00000E+00

**** CASE 3 - PLATES 10 & 11 RELEASED ****

MODE *

39

DISP. S.F. = 7.00000E+00