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ELECTRON CYCLOTRON RESONANCE HEATING OF TANDEM MIRRORS

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ABSTRACT

Heating electrons by microwave power near either the electron cyclotron frequency or its harmonic is analyzed in some detail. The purpose of this study is to determine whether electrons in current or future mirror devices could be heated by electromagnetic waves in this frequency range. Absorption lengths, accessibility conditions, and appropriate polarization are among the concerns studied. It is found that the Tandem Mirror Experiment (TMX) may efficiently be heated by the extraordinary mode at $\omega = \omega_{ce}$ either in the solenoid-to-plug transition region or near the center of the plug at densities $n \approx 10^{13} \text{ cm}^{-3}$.

INTRODUCTION

Heating of electrons in a single-cell mirror or a tandem mirror reactor (TMR) is of interest for the following reasons:

- By creating energetic electrons in a single-cell mirror (or in the end plugs of a tandem mirror), we may be able to alter the potential locally so that low-energy ions can be trapped and their lifetime thereby increased. This would improve stabilization of the drift-cyclotron loss-cone (DCLC) mode and would allow a reduction of cold gas flow.¹

- As shown in Fig. 1, heating the bulk of the electrons in the central cell of a TMR would allow a reduction of the beam energies injected into the plug to about 0.5 MeV (from above 1 MeV), would reduce the total cost, and would reduce the magnetic field in the plug² from about 17 T to 12 T. In addition, the electron energy confinement time in the central solenoid, as well as the overall Q, (defined as the ratio of fusion power output to the neutral-beam power injected to sustain the reaction) should be enhanced.

All of these factors would improve the potential performance of tandem mirror devices as reactors.

Fig. 1. Cost of a tandem mirror reactor vs fraction of heating direct to electrons.

We have examined the following frequency regimes for heating electrons in a tandem mirror device: (a) the electron cyclotron frequency (or its harmonic), (b) the lower-hybrid frequency, and (c) the fast magnetosonic wave or the Alfvén wave. For the present and the near future, the best frequency range appears to be that near the electron cyclotron frequency or its harmonic. From the long-range (reactor) point of view, heating with the compressional Alfvén wave also appears favorable. Lower-hybrid wave heating has the disadvantage of depositing the energy too far out in the tail of the distribution function ($v_{ph}/v_{te} \approx 3$ or $W/nT_e \approx 9$) where confinement is no longer provided by the end plugs (unless this tail is thermalized in times short compared with the transit time of the energetic electrons). Other electron heating methods such as electron-beam/plasma interaction or ion-cyclotron heating have not been examined in this study.

Heating bulk electrons is possible using the fast magnetosonic wave ($\omega > \omega_{ci}$) or, if a parallel wavelength of the order of $\omega/k_{||} \approx v_{te}$ is imposed by external loops,³ the compressional Alfvén wave ($\omega < \omega_{ci}$). Although this technique is favorable at high temperatures ($T_e > 1$ keV) and in long devices (where $L \approx 100$ m, as in the TMR), in present-day devices the absorption length is too long to be of interest.⁴

For these reasons, perhaps the most promising (although not necessarily the cheapest) technique for heating electrons in both present and future mirror devices (including tandem mirrors) appears to be near the electron cyclotron frequency or its harmonic. This conclusion is further supported by the consideration of a new generation of high-power microwave devices,

namely gyrotrons, that will soon be available in the 28- to 35-GHz frequency range at power levels of 200 kW per tube and pulse lengths of 10 ms to CW.⁵⁻⁷

Since cyclotron resonance is at $f(\text{GHz}) = 28 \nu B_T(\text{T})$ (where $\nu = 1$ for $f = f_c$ or $\nu = 2$ for $f = 2f_c$), we see that this frequency range is suitable for heating at the fundamental at $B \approx 1$ T, and at the harmonic at $B \approx 0.5$ T. Thus, these sources could be used to heat a tandem mirror such as the Tandem Mirror Experiment (TMX) either near the middle of the plug or near the rising part of the magnetic field between the solenoid and the plug. This question will be examined in some detail in this report.

The next question concerns the future development of gyrotrons. It appears that gyrotrons with frequencies $f \approx 100$ GHz and power levels of $P \approx 100$ kW will become a reality within the next 3 years.⁸ This frequency would be appropriate to heat plasmas either at $B \approx 3.5$ T at $f = f_{ce}$ or at 1.8 T at $f = 2f_{ce}$. These parameters may be of interest for the central cell of tandem mirror reactors. We note that although the efficiency of these tubes is now 30%, in principle, 70% efficiency could be obtained by advanced design techniques. However, such a technology remains to be proven, especially if power of the order of 50 to 100 MW is required to heat a reactor.

Recent experimental results on the TM-3 tokamak showed that heating at $f = f_{ce}$ or $f = 2f_{ce}$ could be obtained with absorption efficiencies of the order of $\eta \approx 0.30$ (where $\eta = \Delta(nT_e)/P_{\text{source}}$).⁹ These results were obtained on TM-3 under conditions that were not necessarily optimum from the point of view of plasma heating. In principle, with careful design higher heating efficiencies should be obtainable (perhaps of the order of $\eta \approx 0.70$).

We wish to emphasize that in the present work we are investigating heating relatively high-density plasmas ($n \approx 10^{13} \text{ cm}^{-3}$); unlike in earlier low-density experiments ($n \sim 10^{12} \text{ cm}^{-3}$), we must carefully evaluate questions such as accessibility and wave absorption. In particular, although penetration is relatively easy at low densities and long wavelengths, and absorption may proceed via stochastic processes in a multimode-cavity configuration¹⁰⁻¹³ (which may lead to a loss-cone distribution for electrons^{14,15}), in the present regime the densities are high, the wavelengths are short, and therefore accessibility must be satisfied. The absorption mechanism relevant to this regime is similar to that used in recent tokamak work, namely, cyclotron absorption in one pass in an inhomog-

eneous magnetic field.¹⁶⁻¹⁹ This type of heating may lead to a Maxwellian electron distribution (due to collisional isotropization) as observed in recent tokamak experiments.⁹

THE PHYSICS OF WAVE ABSORPTION AND ACCESSIBILITY

We shall now present detailed calculations on wave absorption and accessibility. For this discussion, we assume that the polarization and/or the angle of incidence can be varied by using appropriate microwave horns ($N_z \approx 1$) and/or waveguide arrays ($N_z > 1$).

ORDINARY MODE

In the ordinary mode (O-mode), the wave is incident along the x-direction, normal to the external magnetic field $B_0 \hat{z}$, ($E \parallel B_0$), and accessibility exists only if $f_0 > f_{pe}$, where f_{pe} is the local electron plasma frequency.²⁰ Since for absorption we need $f_0 = f_{ce}$, the condition for absorption and accessibility is

$$f_0 = f_{ce} > f_{pe} \quad (1a)$$

This translates into the following density limit for penetration at a given magnetic field:

$$n_{13} < 0.97 \nu^2 B_T^2 \quad (1b)$$

where $\nu = 1$ or 2 for $f = f_{ce}$ or $f = 2 f_{ce}$, respectively, n_{13} is the plasma density in units of 10^{13} cm^{-3} , and B_T is the magnetic field in units of tesla. Since absorption is negligibly small at the second harmonic (unless $T_e \approx 10 \text{ keV}$), for all practical purposes we take $\nu = 1$ ($f = f_{ce}$). Thus, we see that for a magnetic field of 1 T , the density at the resonance layer cannot exceed the value of $n \approx 10^{13} \text{ cm}^{-3}$, or the wave is reflected.

The absorption coefficient can be defined as¹⁹

$$A = 1 - T, \quad T = \exp(-\Gamma_0), \quad (2)$$

where T is the wave energy transmission coefficient, and Γ_0 characterizes the absorption as the wave propagates through the cyclotron resonance layer in the inhomogeneous magnetic field. If we designate the scale length of the magnetic field gradient at the resonance layer by $L_B = |B(dB/dX)^{-1}|$, we have¹⁹

$$\Gamma_0 = 2\pi L_B \frac{\omega}{c} \frac{T_e}{m_e c^2} Q_0, \quad (3a)$$

where

$$Q_0 = \frac{1}{4} \frac{\alpha^2 (1 - \alpha^2)^{1/2}}{1 + N_z^2 (1 - \alpha^2)}.$$

Here, $\alpha^2 = \omega_{pe}^2 / \omega^2$, $N_z = ck_z / \omega_0$ is the component of the index of refraction along the z direction (i.e., along the external magnetic field), and T_e is the electron temperature. Numerically, we find

$$\Gamma_0 = 2.56 \times 10^{-6} L_B(\text{cm}) f(\text{GHz}) T_e(\text{eV}) Q_0, \quad (3b)$$

where L_B is measured in units of cm, f is measured in GHz, and T_e is measured in eV. In Fig. 2, we plot numerical values of Q_0 as a function of $\alpha = \omega_{pe} / \omega_0$ for the 0-mode for two values of N_z , namely, $N_z = 0$ and $N_z = 0.5$. We see that maximum absorption is attained at $\alpha \approx 0.85$, and that above $\alpha \approx 0.95$ and below $\alpha \approx 0.4$ the absorption decreases rapidly. We note that the absorption is only weakly dependent upon N_z (note that $N_z = 0$ corresponds to normal incidence). In Fig. 3, we plot absorption coefficients for two types of machines, that corresponding to the central cell of the TMR and that corresponding to the plug of TMX. These curves are obtained for a single pass and one transition through the resonance zone. Actually, in a given machine if $f > f_{pe}(0)$, the wave would pass through the two resonance zones in one pass and the total absorption coefficient would be

$$A_{\text{total}} = 1 - T^2 = 1 - (1 - A)^2 = A(2 - A), \quad (4)$$

where A is that given in Fig. 3 (i.e., the absorption of a wave passing through one resonance layer). Similarly, if $f = f_{ce}$ is attained near the

center, L_B is considerably larger than 54 cm, and again more efficient absorption is obtained.

Fig. 2. Q_0 vs α , for O-mode, $\omega = \omega_{ce}$: (a) $N_z = 0$, and (b) $N_z = 0.5$.

Fig. 3. Absorption coefficient A vs electron temperature T_e for O-mode, $\omega = \omega_{ce}$, and $\alpha = 0.80$:

- (a) TMR curve, $f = 70$ GHz, $N_z = 0$, $L = 50$ cm, $B = 2.5$ T, $n = 6 \times 10^{13}$ cm $^{-3}$;
- (b) TMX curve, $f = 35$ GHz, $L = 54$ cm, $B = 1.25$ T, $n = 1.5 \times 10^{13}$ cm $^{-3}$, $N_z = 0$; and
- (c) same as (b), but $N_z = 0.5$.

EXTRAORDINARY MODE

Accessibility

The extraordinary mode (EO-mode) may be efficiently absorbed at either $f = f_{ce}$ or $f = 2f_{ce}$. Accessibility in a uniform magnetic field and radially increasing density are available only at $f = 2f_{ce}$ ($\nu = 2$), and the plasma density must not be so high that the cut-off layer

$$\omega_{\pm} = \pm \frac{\omega_{ce}}{2} + \frac{1}{2} \left(\omega_{ce}^2 + 4\omega_{pe}^2 \right)^{1/2} \quad (5)$$

exceeds the incident frequency, i.e., $\omega_0 = 2\omega_{ce} > \omega_{+}$. This means that for accessibility

$$\omega_{pe}^2 / \omega_{ce}^2 < 2, \text{ or } n_{13} < 1.95 B_T^2. \quad (6)$$

We also note that in this case the upper-hybrid frequency $\omega_{uh} = \left(\omega_{pe}^2 + \omega_{ce}^2 \right)^{1/2}$ is in the range $\omega_{ce} < \omega_{uh} < \omega_{+}$.

If the magnetic field is decreasing radially toward the center (as when $\beta > 0$ or when a magnetic well exists), accessibility also exists as long as $\omega_{-} < \omega_{ce} = \omega_0$. This also gives $\omega_{pe}^2 / \omega_{ce}^2 < 2$ for accessibility, as in Eq. (6). This situation is shown schematically in Fig. 4, where we see that although in low- β tokamaks the power should be injected from the high magnetic field side (i.e., from inside the torus), in a mirror device in regions where a magnetic well exists propagation from the outside is ensured. However, we also note that in symmetric magnetic wells the upper-hybrid layer is not accessible (unlike in tokamaks, where $\omega_0 = \omega_{uh}$ is another potential absorption layer). We may also note that for a fixed frequency the highest density limit for the EO-mode is

$$n_{\max} (\text{cm}^{-3}) = 1.95 \times 10^{13} \frac{f(\text{GHz})}{28\nu}. \quad (7)$$

Thus, the highest accessible density for the EO-mode is at $\nu = 1$ (i.e., $f = f_{ce}$). However, we shall see that, at the limiting density, the absorption drops sharply so that one may have to contend with densities somewhat below that predicted by Eq. (7).

Absorption Lengths

We shall consider absorption lengths at the fundamental electron cyclotron frequency and at its harmonic.

Fig. 4. Sketched accessibility curves for (a) a magnetic field decreasing radially inward and (b) a magnetic field increasing radially inward, for EO-mode, $\omega_{pe}^2/\omega_{ce}^2 = 2$, $\omega_{+} = 2\omega_{ce}$, and $\omega_{-} = \omega_{ce}$. Shaded areas are regions of evanescence.

Absorption at the Fundamental Electron Cyclotron Frequency. The absorption at $f = f_{ce}$ is rather ineffective for normal incidence, and we have for the transmission coefficient in Eq. (2),

$$T_{EO-1} = 2\pi L_b \frac{T_e}{m_e c^2} Q_{EO-1}; \quad (8)$$

where for $N_z = 0$ we have¹⁹

$$Q_{EO-1} = \frac{1}{8} \frac{T_e}{m_e c^2} \alpha^2 (2 - \alpha^2)^{3/2}, \quad (9)$$

and for $N_z \neq 0$, $N_z (T_e/m_e c^2)^{1/2} \ll \alpha < 1$ we have^{16,18}

$$Q_{EO-1} = \frac{N_z^2 (1 + \alpha^2)^2 (2 - \alpha^2)^{3/2}}{4\alpha^2}. \quad (10)$$

For more general values of the density (i.e., for $\alpha \approx 1$) and N_z , the absorption is given by the complicated expressions

$$Q_{EO-1} = \frac{N_z^2}{\alpha^2 N_x} \frac{\left\{ \left[1 - (\alpha^2/2) - N_z^2 \right] \left[1 - \alpha^2 \right] - N_x^2 \right\}^2}{\left[\left(1 - \alpha^2 - N_x^2 \right)^2 + \left(1 - \alpha^2 \right) N_z^2 \right]},$$

where N_z is fixed by the angle of incidence, and N_x is the cold perpendicular index of refraction; namely,

$$N_x^2 = \frac{(1 - \alpha^2)^2 - v^2}{(1 - \alpha^2 - v^2)}, \quad (12)$$

where $\alpha^2 = \omega_{pe}^2/\omega_0^2$ and $v^2 = \omega_{ce}^2/\omega_0^2$. From Eq. (9) we note that the absorption of the EO-mode for normal incidence [i.e., Eq. (9)] at $f = f_{ce}$ is negligibly small for temperatures $T_e \lesssim 10$ keV (similar to the O-mode absorption at $f = 2f_{ce}$), and we shall ignore this case here. Furthermore, in an actual experiment we may expect $N_z \gtrsim 0.1$ due to the finite width of the microwave horn; hence, Eqs. (10) or (11) are the ones relevant to experimental situations.

In Fig. 5, we plot Eq. (10) for three values of N_z as a function of α . We see that for efficient absorption at high density (i.e., for $\alpha \approx 1$), substantial values of N_z are necessary to obtain efficient absorption. Thus, to increase the absorption efficiency it may be advantageous to tilt the microwave horn relative to the magnetic field. The Fig. 5 curve also shows the danger of surface absorption in tokamaks where cyclotron resonance exists everywhere across the plasma column at a fixed value of the major radius. We also wish to point out from Ref. 18, Fig. 6, that Eq. (11) predicts rapid loss of absorption for $\alpha^2 > 1.2$. Thus, for efficient absorption we must assume $\alpha^2 \sim 1$ (even though penetration exists for $\alpha^2 > 2$).

Fig. 5. $Q_{EO} n_z^2$ vs α for EO-mode, $\omega = \omega_{ce}$:

- (a) $N_z = 0.5$, $\theta \approx 60^\circ$;
- (b) $N_z = 0.3$, $\theta \approx 72^\circ$; and
- (c) $N_z = 0.1$, $\theta \approx 84^\circ$.

In Figs. 6 and 7 we show absorption curves for $\alpha = 0.4$ and 0.8 for three different values of N_z (i.e., for $N_z = 0.1, 0.3$, and 0.5) as a function of electron temperature. These curves can be used in conjunction with present-day gyrotrons, namely, either at $f = 28$ GHz, $B = 1.0$ T, $L_B = 62.5$ cm, or at $f = 35$ GHz, $B = 1.25$ T, $L_B = 50$ cm. The corresponding densities of interest are $n = 6.2 \times 10^{12} \text{ cm}^{-3}$ ($\alpha = 0.8$, $f = 28$ GHz) or $n = 1.0 \times 10^{13} \text{ cm}^{-3}$ ($\alpha = 0.8$, $f = 35$ GHz). For $\alpha = 0.4$, the relevant densities are correspondingly lower. Considering $N_z \approx 0.5$, $\alpha \approx 1$, we obtain similar curves but approximately 40% higher densities than obtained for the $\alpha = 0.8$

Fig. 6. Absorption coefficient A vs T_e for EO-mode, $\omega = \omega_{ce}$, $\alpha = 0.4$, $L_B = 50$ cm, $f = 35$ GHz:

- (a) $N_z = 0.5$ ($\theta = 60^\circ$);
- (b) $N_z = 0.3$ ($\theta = 72^\circ$); and
- (c) $N_z = 0.1$ ($\theta = 84^\circ$).

Fig. 7. Absorption coefficient A vs T_e for EO-mode, $\omega = \omega_{ce}$, $\alpha = 0.8$:

- (a) TMX, $f = 35$ GHz, $L_B = 200$ cm, $N_z = 0.3$;
- (b) $N_z = 0.5$ ($\theta = 60^\circ$);
- (c) $N_z = 0.3$ ($\theta = 72^\circ$); and
- (d) $N_z = 0.1$ ($\theta = 84^\circ$).

In (b) through (d), $L_B = 50$ cm, $f = 35$ GHz.

curves shown here. If we assume that the waves transit two resonance layers in one pass (as in a magnetic well of a mirror device), correspondingly higher absorption is obtained: namely; $A_{eff} = A(2 - A)$ [see Eq. (4)].

Absorption at the Harmonic of the Electron Cyclotron Frequency. At the harmonic of the cyclotron frequency, $f = 2f_{ce}$, the absorption coefficient is given by¹⁹

$$\Gamma_{EO-2} = 2\pi L_B \frac{\omega}{c} \frac{T_e}{m_e c^2} Q_{EO-2} \quad (13a)$$

For $N_z \approx 0$ in Eq. (13a), we have

$$Q_{EO-2} = \alpha^2 \left[\frac{3 - 2\alpha^2}{3 - 4\alpha^2} \right]^2 \left[\frac{4(1 - \alpha^2)^2 - 1}{3 - 4\alpha^2} \right]^{1/2}, \quad (13b)$$

where $\alpha^2 = \omega_{pe}^2 / \omega^2$.

In Fig. 8, we give numerical plots of Q_{EO-2} as a function of α . We see that the peak absorption occurs at densities such that $\alpha \approx 0.65$ and that the absorption decreases rather rapidly away from this peak value. In Fig. 9, we show absorption curves for some typical α and L_B values as a function of electron temperature. These curves are mostly relevant to present-day devices, and are such as may be expected in TMX. We see that the absorption efficiencies increase rapidly with L_B above $T_e \approx 400$ eV. We should again emphasize that when the wave passes through two resonance zones the total absorption increases as predicted by Eq. (4). For example, if we take curve (c) in Fig. 9 (i.e., $L_B = 50$ cm, $\alpha = 0.60$), at $T_e = 400$ eV $A \approx 0.60$, and the total efficiency of absorption per pass is $A_{total} = A(2 - A) = 0.84$. At $T_e = 500$ eV, however, $A = 0.70$, so that $A_{total} \approx 0.91$.

Fig. 8. Q_{EO-2} vs α for EO-mode, $\omega = 2\omega_{ce}$.

Fig. 9. Absorption coefficient A vs T_e ,
for EO-mode, $\omega = 2\omega_{ce}$:

(a) TM₁₁ parameters, $\alpha = 0.60$, $l_B = 50$

cm, $B = 2.5$ T, $f = 140$ GHz;

(b) $\alpha = 0.60$, $l_B = 100$ cm;

(c) $\alpha = 0.60$, $l_B = 50$ cm;

(d) $\alpha = 0.60$, $l_B = 25$ cm;

(e) $\alpha = 0.40$, $l_B = 50$ cm;

(f) $\alpha = 0.20$, $l_B = 50$ cm.

In (b) through (f), $r = 38$ GHz.

Heating Rates

The heating rates have been given by Elridge et al.,¹⁸ and the results may be summarized as follows:

•Perpendicular heating:

$$\frac{d^2 W_{\perp}}{dt dV} = \sum_{\text{modes}} \frac{\omega_{pe}^2}{8} \frac{\omega_{ce}}{\omega^2} \frac{l_B}{\Delta} \frac{\exp(-X^2/2\Delta^2)}{(2\pi)^{1/2}} \left| E^- \right|^2. \quad (14a)$$

•Parallel heating:

$$\frac{d^2 W_{\parallel}}{dt dV} = \sum_{\text{modes}} \frac{\omega_{pe}^2}{8\omega} \frac{X}{\Delta} \frac{\exp(-X^2/2\Delta^2)}{(2\pi)^{1/2}} \left| E^- \right|^2, \quad (14b)$$

where $E^- = E_x - iE_y$, and

$$\Delta = N_z l_B \left(\frac{T_e}{m_e c^2} \right)^{1/2} \quad (15)$$

Here Eq. (15) gives the initial localization of heat deposition. For example, at $T_e = 1$ keV, $L_B = 100$ cm, $N_z = 0.5$, and $\Delta = 2.2$ cm; whereas for $T_e = 500$ eV, $L_B = 100$ cm, $N_z = 0.2$, and $\Delta = 0.63$ cm. Thus, we see that initially the heat deposition is strongly localized. On the other hand, we expect that this heat will spread rapidly due to heat conductivity. We may also conclude that at $f = f_{ce}$, these waves heat bulk electrons, whereas at $\omega = 2\omega_{ce}$ considerable tail heating may also occur.¹⁸

A useful way of looking at heating is to consider the relationship

$$P_{abs}(\text{electron}) = \frac{3}{2} \frac{\Delta T_e V}{\tau_E}, \quad (16)$$

which tells us the required heating power to achieve a given temperature T_e . In Eq. (16), τ_E is the energy confinement time, V is the volume, and ΔT_e is the change in electron energy. In the TMX central cell, for example if $V \approx 1.5 \times 10^3$ litre, $\tau_E \approx 3$ ms, and $n \approx 1 \times 10^{13} \text{ cm}^{-3}$ with $P_{abs} \approx 100$ kW of absorbed power, $\Delta T_e \approx 100$ eV; and with 0.5 MW of absorbed power, $\Delta T_e \approx 500$ eV. On the other hand, in the TMR central cell, $n \approx 8 \times 10^{14} \text{ s} \cdot \text{cm}^{-3}$, $n = 10^{14} \text{ cm}^{-3}$, and $T_e \approx 30$ keV; therefore, $P_{abs} \approx 40$ MW. Assuming 60% overall efficiency, we see that roughly 70 MW of source power will be necessary to achieve these electron temperatures (where we considered transmission losses and absorption efficiencies).

WHISTLER WAVE PROPAGATION

Another wave that may be useful for heating mirror devices is the right-hand circularly polarized wave, or whistler wave.²⁰ This wave propagates almost parallel to the external magnetic field. The cold-plasma dispersion relationship for parallel propagation is given by²⁰

$$N_{||}^2 = 1 - \frac{\omega_{pe}^2}{\omega(\omega - \omega_{ce})} \quad (17)$$

In a hot plasma near resonance, $|\omega - \omega_{ce}| < k_{||} v_{te}$, and the dispersion relationship is¹⁰

$$N_{||}^2 = 1 - \frac{2\omega_{pe}^2}{\omega_{ce}} \frac{(\omega - \omega_{ce})}{k_{||}^2 v_{te}^2} + \frac{i\omega_{pe}^2 \sqrt{\pi}}{\omega_{ce} k_{||} v_{te}} \exp - \frac{(\omega - \omega_{ce})^2}{k_{||}^2 v_{te}^2}, \quad (18)$$

where $v_{te} = (2T_e/m_e)^{1/2}$, and $N_{||}^2 = c^2 k_{||}^2 / \omega^2$.

If we assume that the wave is launched from the high magnetic field side from the mirror plug end such that $\omega \ll \omega_{ce}$, as the region $\omega \approx \omega_{ce}$ is approached strong cyclotron absorption sets in. Solving Eq. (18), we obtain

$$N_{||} \approx \left[\frac{1}{2} + \frac{1}{2} \left(1 - \frac{4c^2}{v_{te}^2} \frac{\omega_p^2}{\omega^2} \frac{(\omega - \omega_{ce})}{\omega_{ce}} \right)^{1/2} \right]^{1/2}; \quad (19)$$

and at $\omega \approx \omega_{ce}$,

$$\text{Im } k_{||} \approx \frac{1}{2} \left(\frac{\pi}{2} \right)^{1/2} \frac{\omega_{pe}^2}{v_{te} \omega N_{||}^2}. \quad (20)$$

Thus, we see that at $\omega \approx \omega_{ce}$, $N_{||} \approx 1$; and for $\omega \approx \omega_{pe}$, $\text{Im } k^{-1} \approx 3D_e$, which corresponds to very strong absorption. More likely, the wave will be damped out at positions $\omega/\omega_{ce} \approx 0.95$.

The advantage of heating with a whistler wave is the lack of a density limit. A disadvantage may be the difficulty of launching the wave (the microwave horn must be in the high magnetic field, possibly in contact with hot plasma). A second potential problem with whistler waves at high powers is their sensitivity to a number of nonlinear effects, such as parametric decay,²² trapping, and self-focusing²³ while they propagate through the mirror region. We note that some of these effects could lead to the undesirable production of a parallel, superthermal tail of the electron distribution function.²² Only a thorough experimental test could tell if these nonlinear effects would be strong enough in a given experiment to cause pump-depletion before the wave arrived at the linear absorption region.

HEATING OF TANDEM MIRRORS

In Fig. 10, we consider the antenna geometries by which single-cell and tandem mirrors may be heated. As discussed before, accessibility requires (a) that $\omega_{pe}^2/\omega_0^2 \lesssim 1$ for the EO-mode (our prime candidate in the present dis-

Fig. 10. Axial plot of the magnetic field and possible transmitting microwave horn locations for $\beta = 0$:

(a) through (c) for EO-mode, (d) for whistler mode.

cussion) and (b) that at $\omega_0 = \omega_{ce}$ the magnetic field decrease radially inward. In the high- β case, these conditions are achieved at any location when a magnetic well exists near the center of the plasma column. In the low- β case, accessibility exists near the positions (a), (b), and (c) in Fig. 10. This may be seen from plots of the magnetic field lines shown in Fig. 11, which are representative of the Lawrence Livermore Laboratory TMX device. In Fig. 12, we have plotted the radial variation of the vacuum magnetic field intensities in TMX at three different axial locations, namely at $z = 150$ cm, $z = 220$ cm, and $z = 320$ cm, corresponding to positions (a), (b), and (c), respectively, in Fig. 11. Figure 13 provides expanded plots for $z = 150$ cm and $z = 320$ cm. Since these are the vacuum fields, they give typical gradient scale lengths only for low- β plasmas. Nevertheless, we can use these as guide lines for our estimates, since for high- β plasmas the accessibility for the EO-mode is even better (although the absorption would be more localized to the center where the gradient scale lengths are large).

Using the Fig. 13 curves, in Fig. 14 we plot the radial gradient scale lengths of the magnetic field L_B . We see that in both cases $L_B \gtrsim 100$ cm for $R \lesssim 12$ cm, and at $R \approx 5$ cm $L_B \approx 200$ cm for $z = 320$ cm (the center of the mirror plugs) and $L_B \approx 10^3$ cm at $z = 150$ cm (the center solenoid). Both values of L_B predict strong absorption.

Fig. 11. Contours of the vacuum magnetic field lines in TMX for $\theta = 0$. The associated variation in axial field intensity is as in Fig. 10.

Fig. 12. Radial variation B_T of the vacuum magnetic field intensities in TMX for the conditions of Figs. 10 and 11. The appropriate radial locations of the transmitting microwave horns for the EO-mode of propagation are also shown [(a), (b), and (c) in Fig. 10]. θ refers to the relative angle from the plane of the B-fan.

Fig. 13. Expanded plot of B_T vs radius R at two fixed axial locations in TMX, $z = 150$ cm and $z = 320$ cm [(a) and (c), respectively, in Fig. 10].

Fig. 14. Radial gradient scale length $L_B = B |\partial B / \partial x|^{-1}$ vs radius R at two axial locations in TMX.

In Fig. 15, we show the calculated absorption at different radial positions at the center of the TMX mirror plug at $z = 320$ cm. Here we assumed that the magnetic field gradients are as given in Figs. 12 or 13 and that the density and temperature profiles obey the relationships

$$n(r) = n_0 \exp \left[- (R/10^2) \right], \quad (21a)$$

$$T(r) = T(0) \exp \left[- (R/10^2) \right]. \quad (21b)$$

Fig. 15. Absorption coefficient A vs R for TMX parameters for EO-mode, $\omega = \omega_{ce}$, where $f = 28$ GHz at $R = 2$ cm, $T_c(0) = 400$ eV, $n_e(0) = 1 \times 10^{13} \text{ cm}^{-3}$, $z = 320$ cm, $n_e(R) = n_0 \exp [- (R/10)^2]$, and $T(R) = T(0) \exp [- (R/10)^2]$: $N_z = 0.3$.

- (a) one pass through one resonance layer; and
- (b) one pass through two resonance layers.

We further took $N_z = 0.3$ (so that the angle of incidence is 72°), and assumed that $T_e(0) = 400$ eV, $n_e(0) = 1 \times 10^{13} \text{ cm}^{-3}$, and $f_0 = f_{ce}$ locally. In particular, $f_0 = 28$ GHz at $R \approx 2$ cm at low values of β in TMX.

Thus, we see that absorption efficiencies in excess of 80% are obtained for $R < 4$ cm when we consider one pass through the two resonance layers. The localization of the absorption according to Eq. (15) is shown in Fig. 16 by the quantity Δ . We see that when $R > 2.5$ cm, $\Delta < 2R$; hence, we take this value of R for estimating the maximum absorption efficiency, which in the present case is in excess of 90%. We remark that accessibility exists in the whole region $z \approx 290$ to 350 cm, and thus one might consider $z \approx 300$ cm as a particularly suitable location for heating since here the density is considerably lower than at $z = 320$ cm.

Fig. 16. Width of the absorption layer Δ vs radius R . Associated parameters are as listed in Figs. 12 through 14.

Although these results were obtained for low β , at high β the magnetic field would be depressed near the center and the corresponding plasma density may also be decreased. Taking

$$B_{\text{actual}} \approx (1 - \beta)^{1/2} B_{\text{vacuum}}, \quad (22)$$

we see that for $\beta \approx 0.5$, $B_{\text{actual}} \approx 0.7 B_{\text{vacuum}}$; hence, the maximum density at accessibility would be reduced by the factor $1 \approx \omega_{pe}^2 / \omega_{ce}^2 \propto n_e / B^2 \propto n_e / \beta B_V^2$ or by the value of β . Thus, high- β values will proportionally reduce the maximum density to which accessibility exists.

Heating the TMX central solenoid at high plasma densities at $z < 150$ cm would be particularly difficult at finite β . For example, in TMX the central vacuum solenoid may have a maximum field of $B = 0.2$ T (see Fig. 12). This would limit the frequency to $f = f_{ce} = 5.8$ GHz, $f = 2f_{ce} = 11.6$ GHz; thus, the accessible density would be $n \approx 3 \times 10^{11} \text{ cm}^{-3}$ at $f_0 = f_{ce}$, and $n \approx 10^{12} \text{ cm}^{-3}$

at $f_0 = 2f_{ce}$, both rather low values. Finite β would further depress these density values, and we thus conclude that electron cyclotron resonance heating (ECRH) in the center of the TMX solenoid is restricted to uninterestingly low values of the density. Therefore, heating the central cell at interesting densities (i.e., at $n > 5 \times 10^{12} \text{ cm}^{-3}$) could only be performed in the rising portion of the magnetic field, namely at $z \approx 210$ to 220 cm in Figs. 10 and 12. This would require that the microwave horns be placed rather close to the axis, i.e., at $R < 15$ cm. Because here typically $B_{\text{vacuum}} (R = 0) \approx 1$ T, heating at $n \approx 10^{13} \text{ cm}^{-3}$ and $f = f_{ce} = 28$ GHz appears feasible. We note that at $z \approx 220$ cm there are strong magnetic field gradients in the z -direction (larger than in the x -direction); hence, care must be exercised in injecting the microwave beam at the correct angle so as to absorb the microwave energy in the interior of the plasma (rather than at the plasma edge). We believe that detailed ray tracing would be of help to determine the correct angle of the microwave horns when one considers heating plasma.

In Fig. 17 we show the mod-B curves in the solenoid-plug transition region at $z \approx 212$ to 222 cm and $\theta = 90^\circ$. The half-width of the plasma in this region is typically 4 cm. Thus, we see that good accessibility exists in this whole region for the EO-mode. Since the radial magnetic field gradients in Fig. 17 are comparable to that in Fig. 14 (i.e., the center of the

Fig. 17. Mod-B surfaces at $\theta = 90^\circ, \Delta B = 0.072$ T, for the following values of B:

(a) 0.6257 T	(h) 0.1128 T
(b) 0.6975 T	(i) 0.1199 T
(c) 0.7692 T	(j) 0.1271 T
(d) 0.8409 T	(k) 0.1343 T
(e) 0.9126 T	(l) 0.1415 T
(f) 0.9843 T	(m) 0.1486 T
(g) 0.1056 T	(n) 0.1557 T
	(p) 0.1629 T

plug), we expect comparable absorption at the same frequency and temperature. We also note that even the 0-mode could be used to heat the plasma, although not with as great efficiency as the EO-mode. In future devices where $T_e \sim 1$ keV, the 0-mode will likely be also completely absorbed.

In Fig. 18, we show a general plot of frequencies and critical accessible densities as a function of magnetic field. For a given magnetic field one can directly determine the necessary frequency at either $f_0 = f_{ce}$ or $f_0 = 2f_{ce}$, and the associated critical (maximum accessible) density. This magnetic field should be taken as the net magnetic field, corrected for β . Thus, we see that in order to allow operations at higher densities, it would be advantageous to work at the harmonic of the cyclotron frequency. In TMX, for example, by working at $B = 10$ T, $f_0 = 2f_{ce} = 56$ GHz, a density of $n \approx 2 \times 10^{13}$ is accessible in the plug; at $B = 0.65$ T, $f_0 = 2f_{ce} = 36$ GHz, a density of $n \approx 1 \times 10^{13}$ is accessible toward the solenoid. In a device such as TMR, by working at $B = 2.25$ T, $f_0 = 2f_{ce} = 125$ GHz, a density of $n \approx 1 \times 10^{14} \text{ cm}^{-3}$ is accessible, which would correspond to typical values in the central solenoid. It is believed that such power sources may become available in the next 2 to 3 years. Taking a vacuum field of $B \approx 3.2$ T, $\beta \approx 0.5$, we obtain a local field of $B \approx 2.2$ T, which may be a typical value half way in radially, even if $\beta \rightarrow 1$ near the center. The absorption at high temperatures ($T_e > 1$ keV) is excellent at the second harmonic, even if the gradient scale lengths shorten due to the presence of the magnetic well.

Fig. 18. Critical frequencies and cut-off densities n_c vs magnetic field.

CONCLUSIONS

We have examined in some detail the predictions of the theory of electron cyclotron resonance heating of mirror devices. We found that heating with the extraordinary mode either near the fundamental or at the harmonic of the electron cyclotron frequency offered the best possibility for heating electrons in relatively low-density devices ($\omega_{pe} \lesssim \omega_{ce}$). In present-day tandem devices, either the mirror plug or the rising portion of the mirror field between the center cell and the plug offers the best place for heating electrons at $\omega_0 = \omega_{ce} \approx \omega_{pe}$. Care must be exercised in selecting the proper angle of microwave injection.

In future devices where the magnetic field will be considerably higher, heating the center cell is feasible both at $\omega_0 = \omega_{ce}$ and $\omega_0 = 2\omega_{ce}$. Once high-frequency sources become available, heating at the harmonic offers the best possibility for heating the higher-density plasmas. In particular, at a central magnetic field of $B \gtrsim 3.0$ T, $f \gtrsim 100$ GHz, densities approaching $n \approx 10^{14}$ cm⁻³ may be heated in devices with $\beta \approx 0.5$. In most cases considered, the absorption efficiencies were found to be very high ($\eta \approx 0.80$). The heating efficiency will further increase at higher electron temperatures.

Finally, heating with whistler waves offers interesting possibilities even in high-density devices ($\omega_{pe} \gg \omega_{ce}$). However, the waves must travel through the mirror to the central cell, and nonlinear effects may play an important role in determining the absorption mechanisms and the resulting distribution functions at the high-power levels considered here. Only experimental tests could determine how serious such effects may be.

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