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TOROIDAL EQUILIBRIUM WITH LOW FREQUENCY WAVE DRIVEN CURRENTS

by

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ABSTRACT

In the absence of an emf the parallel current, $j_{\parallel}$, in a steady state tokamak will consist of a neoclassical portion plus a wave-driven contribution. Using the drift kinetic equation, the quasilinear (wave-driven) current is computed for high phase speed waves in a torus, and this is combined with the neoclassical term to obtain the general expression for the flux surface average $\langle j_{\parallel} B \rangle$. For a given pressure profile this technique fully determines the MHD equilibrium, permitting the study of a new class of toroidal equilibria.

1.0 INTRODUCTION

In calculations of tokamak MHD equilibrium two functions of ψ , the poloidal flux, must be specified at the outset. In one conventional approach,⁽¹⁾ for example, the pressure, p , and diamagnetism, $F = RB_z$, are input to the problem, and the current density, $j_{\parallel}$, and safety factor, q , are determined by solving the Grad-Shafranov equation. Inasmuch as the input functions are frequently taken to be independent of each other an unlimited variety of equilibria results. Thus, by choosing different models for $F(\psi)$, various current profiles $j_{\parallel}(r)$ can satisfy equilibrium for a given $p(\psi)$. This degree of arbitrariness can, however, be removed if the equilibrium is sustained by steady-state rf (wave) current drive. In this case, as we will show, the current density consists of two distinct terms, $j_{\parallel} = j_{\parallel}^{NC} + j_{\parallel}^{QL}$. The neoclassical current, $j_{\parallel}^{NC}$, arises from the magnetic drifts of charged particles in the inhomogeneous magnetic field and is a known function of the plasma density and temperature profiles.⁽²⁾ The quasilinear current, $j_{\parallel}^{QL}$, due to velocity space diffusion with an rf current driver, can also be calculated from a knowledge of the density and temperature profiles if the spatial deposition of the driver power is computed.⁽³⁾ Consequently, for a

given density and temperature [and, hence, $p(\psi)$] profile a unique $\underline{j}$ exists with steady-state rf current drive, and it is possible to infer the actual $F(\psi)$ which is needed to solve for MHD equilibrium. The goal of this brief communication is the derivation of that equation which determines $F(\psi)$; actual numerical computation of two dimensional rf-sustained equilibria will be deferred to future reports. In our concluding remarks we will note that rf current drive may permit physicists to generate equilibria with favorable profile characteristics, and an application to a high beta reactor will be suggested.

2.0 TOROIDAL GEOMETRY AND MHD PRESSURE BALANCE

We consider an axisymmetric tokamak plasma with orthogonal right-handed coordinates ψ, χ, ζ where the toroidal angle ζ is in the direction of the plasma current density, $\hat{\zeta} \cdot \underline{j} > 0$. (The caret indicates a unit vector.) The poloidal flux, ψ , has a minimum on the magnetic axis, and the tokamak magnetic field can be expressed as $\underline{B} = F\nabla\zeta + \nabla\zeta \times \nabla\psi/2\pi$, where $\nabla\zeta = \hat{\zeta}/R$, $\nabla\psi = \hat{\psi}2\pi RB_p$, and $F = RB_t$. R is the distance from the symmetry axis and B_t and B_p are respectively the toroidal (ζ) and poloidal (χ) field components.

For a steady state equilibrium the plasma must satisfy the usual Maxwell-pressure balance equations:

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \times \underline{B} = \mu_0 \underline{j}$$

$$\underline{j} \times \underline{B} = \nabla p,$$

where p is the total plasma pressure and μ_0 is the free space permeability. It is easily shown⁽¹⁾ that the pressure and diamagnetism, F , are only functions of the poloidal flux, $p = p(\psi)$ and $F = F(\psi)$, and that the poloidal flux satisfies the Grad-Shafranov equation:

$$-4\pi^2 \left(\mu_0 R^2 \frac{dp}{d\psi} + F \frac{dF}{d\psi} \right) = R^2 \nabla \cdot \left(\frac{\nabla\psi}{R^2} \right). \quad (1)$$

The object of our study is to find the function $F(\psi)$ which is generated by noninductively driven tokamak currents, which, with the aid of Eq. (1), can be used to fully determine the two-dimensional equilibrium.

We begin by noting⁽¹⁾ that in equilibrium the current density parallel to the magnetic field is

$$j_{\parallel} = \frac{-2\pi F}{B} \frac{dp}{d\psi} - \frac{2\pi B}{\mu_0} \frac{dF}{d\psi}, \quad (2)$$

where the total field is $B = |\underline{B}| = (B_t^2 + B_p^2)^{1/2}$.

In order to make use of Eq. (2) we first define⁽²⁾ the flux surface average as the volume average over an infinitesimal volume between two poloidal flux surfaces:

$$\begin{aligned} \langle \mathcal{F} \rangle &= \int_{\Delta V} d^3 \underline{r} \mathcal{F}(\underline{r}) / \int_{\Delta V} d^3 \underline{r} \\ &= \oint_{\psi} \frac{dr_{\chi}}{B_p} \mathcal{F} / \oint_{\psi} \frac{dr_{\chi}}{B_p}. \end{aligned} \quad (3)$$

Here we integrate over r_{χ} , the distance along a flux surface in the χ direction. For later convenience we define the local safety factor as the ratio of the changes in toroidal and poloidal flux⁽¹⁾:

$$\begin{aligned} q(\psi) &= \frac{d\Phi}{d\psi} = \oint_{\psi} \frac{RB_{\chi}}{R^2} \frac{dr_{\chi}}{2\pi B_p} \\ &= \frac{F(\psi)}{2\pi} \oint_{\psi} \frac{dr_{\chi}}{R^2 B_p}. \end{aligned} \quad (4)$$

Multiplying Eq. (2) by B and taking the flux surface average we obtain

$$\frac{1}{\mu_0} \frac{dF}{d\psi} + \left[\frac{1}{\langle B^2 \rangle} \frac{dp}{d\psi} \right] F + \frac{\langle j_{\parallel} B \rangle}{2\pi \langle B^2 \rangle} = 0. \quad (5)$$

The boundary condition is $F(\psi_1) = R_0 B_0$, the vacuum value (at the limiter flux surface, ψ_1); B_0 is the vacuum field at the major radius R_0 . This differential equation provides a straightforward prescription for F once $\langle j_{\parallel} B \rangle$ is known. Note, if Eq. (5) is substituted into Eq. (2) we find the following expression:

$$j_{\parallel} = 2\pi F \frac{dp}{d\psi} \left[\frac{B}{\langle B^2 \rangle} - \frac{1}{B} \right] + \frac{B}{\langle B^2 \rangle} \langle j_{\parallel} B \rangle.$$

The first term is known as the Pfirsch-Schluter current⁽²⁾ and vanishes in the high aspect ratio (cylindrical) limit for which $B^2 \rightarrow \langle B^2 \rangle$. The principal contribution to $j_{\parallel}$ in this limit is due to $\langle j_{\parallel} B \rangle$, which in general includes inductive (Spitzer) as well as noninductive currents. We will show that $\langle j_{\parallel} B \rangle$ can be computed for noninductive current drive in an axisymmetric torus, so Eq. (5) can be solved for $F(\psi)$.

3.0 NEOCLASSICAL-QUASILINEAR CURRENT DRIVE

The analysis is simplified by considering only current drive by low frequency ($\omega \ll \Omega$) waves resonant with the electrons, where ω is the wave's angular frequency and $\Omega = eB/m$ is the electron cyclotron frequency. This eliminates many noninductive drivers (neutral beams,⁽⁴⁾ minority ion heating,⁽⁵⁾ electron cyclotron heating,⁽⁶⁾) from consideration but includes the important options of the lower hybrid⁽⁷⁾ and magnetosonic (fast wave)^(8,9) current drivers. The appropriate focus of attention is thus the electron drift kinetic equation,⁽²⁾

$$C(f) = \frac{\partial f}{\partial t} + \left(v_{\parallel} \hat{b} + \underline{v}^d \right) \cdot \nabla f + eE_{\parallel} v_{\parallel} \frac{\partial f}{\partial w}, \quad (6)$$

where $f = f(t, \underline{r}, \mu, w)$. The velocity space variables are kinetic energy, $w = \frac{mv^2}{2}$, and magnetic moment, $\mu = \frac{mv_{\perp}^2}{2B}$, where v is the electron speed and $v_{\perp}$ is its velocity perpendicular to $\underline{B}$; e and m are the electron charge and

rest mass, and $\hat{b} = \underline{B}/B$. The velocity along the field line is

$$v_{\parallel} = \sigma \left| \sqrt{\frac{2}{m} (w - \mu B)} \right|, \quad (7)$$

where $\sigma = \pm 1$ denotes the sign of $\underline{v} \cdot \hat{b}$. The magnetic drift is⁽²⁾

$$\underline{v}^d = -v_{\parallel} \hat{b} \times \nabla (v_{\parallel}/\Omega),$$

and, for large aspect ratio ($\epsilon \ll 1$),

$$\underline{v}^d \cdot \nabla f \approx \frac{v_{\parallel} B_t}{B} \frac{\partial (v_{\parallel}/\Omega)}{\partial r_{\chi}} \hat{\psi} \cdot \nabla f. \quad (8)$$

We have neglected the $\underline{E} \times \underline{B}$ drift term which, in general, should be added to the electron orbit. This neglect is only justified when there are no wave-particle bounce resonances which could result in enhanced radial transport; we assume $\omega \gg v_e/Rq$, where v_e is the thermal speed ($v_e = \sqrt{T_e/m}$) and restrict our interest to the current drive problem.

In a true steady state reactor there would be no transformer-induced electric field, and we thus stipulate that the electric field, $E_{\parallel}$, in Eq. (6) is solely due to the wave. Thus, Eq. (6) shows that the wave is expected to determine the energy dependence of f , and, for electrostatic waves (with negligible transit-time magnetic pumping), the $v_{\parallel}$ dependence of f can be inferred from the relation in Eq. (7). For the situation outlined we follow standard quasilinear theory⁽¹⁰⁾ to obtain, by iterating Eq. (6),

$$\frac{\partial \bar{f}}{\partial t} + (v_{\parallel} \hat{b} + \underline{v}^d) \cdot \nabla \bar{f} = C(\bar{f}) + v_{\parallel} \frac{\partial}{\partial w} \mathcal{D} v_{\parallel} \frac{\partial \bar{f}}{\partial w}, \quad (9)$$

where the bar indicates an average over the waves' random phases. The quasilinear diffusion coefficient is

$$D = e^2 \int_0^\infty d\tau \sum_{\underline{k}} |E_{\underline{k}}|^2 e^{i(\underline{k} \cdot \underline{r} - \omega\tau)} \overline{U(\tau)} e^{-i\underline{k} \cdot \underline{r}}$$

where the $\overline{U(\tau)}$ operator propagates particle position along averaged orbits. Tokamak collisionless particle orbits can be complicated by the nonuniform field magnitude; this modulates $v_{\parallel}$ of circulating electrons and leads to the bounce orbits of trapped electrons. We achieve a great simplification by considering only wave drivers with high phase speeds along the field, $v_e \ll \omega/k_{\parallel}$, and with frequencies in excess of the bounce frequency, $\sqrt{e} v_e / Rq \ll \omega$. In this case quasilinear diffusion mostly affects electrons with $v_{\perp} \lesssim v_e \ll v_{\parallel}$ (see Fig. 1), and these resonant electrons can be approximated to have straight line orbits, $\overline{U(\tau)} e^{-i\underline{k} \cdot \underline{r}} \approx e^{-i(\underline{k} \cdot \underline{r} + k_{\parallel} v_{\parallel} \tau)}$. Thus

$$D \approx \pi e^2 \sum_{\underline{k}} |E_{\underline{k}}|^2 \delta(k_{\parallel} v_{\parallel} - \omega),$$

and $v_{\parallel}$ is given by Eq. (7) with the assumption $\mu B \ll w$. This derivation assumes that either a broad wave spectrum or collisions are sufficient to randomize orbits in order to validate particle diffusion in velocity space. We further posit that the wave amplitude is sufficiently small that neither particle-wave trapping nor resonance broadening occurs.

The limitation to high phase speeds eliminates the low frequency Alfvén wave^(8,11) from study. Although the high momentum content of this wave would yield very high current drive efficiency in a uniform field, we note that in a torus the presence of trapped electrons at a low phase speed resonance may lead to negligible current generation.⁽¹²⁾

Using the relation $\frac{\partial}{\partial w} = \frac{1}{mv} \cdot \frac{\partial}{\partial v_{\parallel}}$ and assuming a macroscopic steady state we can write the quasilinear equation finally as

$$v_{\parallel} \hat{b} \cdot \nabla \bar{f} - C(\bar{f}) = -\underline{v}^d \cdot \nabla \bar{f} - \frac{\partial}{\partial v_{\parallel}} \cdot \underline{r}, \quad (10)$$

where the velocity space flow is

Figure 1. Perturbed distribution of electrons due to quasilinear diffusion with high phase speed waves. [Reprinted from C. F. F. Karney and N. J. Fisch, *Phys. Fluids* 22, 1817 (1979), with permission.]

$$\underline{\Gamma} = - \underline{D} \cdot \frac{\partial \bar{f}}{\partial \underline{v}}, \quad (11)$$

with a diffusion tensor

$$\underline{D} = \frac{\pi e^2}{m^2} \sum_{\underline{k}} |E_{\underline{k}}|^2 \hat{b} \hat{b} \delta(k_{\parallel} v_{\parallel} - \omega).$$

This result is a generalization of the equation of Antonsen and Chu⁽³⁾ with the addition of the magnetic drift term on the right hand side. This additional term will be needed in order to fully determine the quantity $\langle j_{\parallel} B \rangle$ in the MHD problem. Note that in a steady state $\partial T_e / \partial t = 0$, and the quasilinear energy input (wave heating) is lost as radial electron thermal transport and energy transfer to the ions.

Equation (10) can be solved by linearizing $\bar{f}$ around a maxwellian: $\bar{f} = f_m + \tilde{f}$, where $f_m = n_e \left(\frac{m}{2\pi T_e} \right)^{3/2} e^{-\frac{mv^2}{2T_e}}$; $n_e(\psi)$ and $T_e(\psi)$, respectively the electron density and temperature, are constant on a flux surface. The linearization procedure is valid under the following circumstances: 1) toroidal effects are small assuming either a high collisionality or, in the banana-plateau regime, assuming a large aspect ratio; 2) quasilinear distortions of $\bar{f}$ are small or localized to untrapped electrons with $v_e \ll v_{\parallel}$. In practice these conditions can normally be satisfied. Thus, we obtain from Eq. (10)

$$v_{\parallel} \hat{b} \cdot \nabla \tilde{f} - C(\tilde{f}) = -\underline{v}^d \cdot \nabla f_m - \frac{\partial}{\partial \underline{v}} \cdot \underline{\Gamma}. \quad (12)$$

$C(\tilde{f})$ is the linearized collision operator, and we assume the ions are nearly maxwellian with $T_i \approx T_e$. A solution to this equation is readily found by setting $\tilde{f} = g + h$, where

$$v_{\parallel} \hat{b} \cdot \nabla g - C(g) = -\frac{\partial}{\partial \underline{v}} \cdot \underline{\Gamma} \quad (13)$$

and

$$v_{\parallel} \hat{b} \cdot \nabla h - C(h) = -\underline{v}^d \cdot \nabla f_m. \quad (14)$$

Our task has been thus greatly simplified by reducing the problem to two separate equations which have been previously studied. There are two distinct contributions to the electron current density, a neoclassical effect (h) and a quasilinear portion (g), and we will discuss these individually.

4.0 NEOCLASSICAL CURRENT

It is instructive to learn the functional form of the neoclassical current. With the aid of Eqs. (7) and (8) we rewrite Eq. (14) for $h(v_{\parallel}, v_{\perp})$,

$$\begin{aligned} v_{\parallel} \hat{b} \cdot \nabla h + \frac{1}{2} \hat{b} \cdot \nabla B \left(v_{\parallel} v_{\perp} \frac{\partial h}{\partial v_{\perp}} - v_{\perp}^2 \frac{\partial h}{\partial v_{\parallel}} \right) - C(h) \\ = -v_{\parallel} \frac{B_t}{B} \frac{\partial(v_{\parallel}/\Omega)}{\partial r_{\chi}} \hat{\psi} \cdot \nabla f_m \\ = \frac{F(\psi)}{RB_p} \hat{b} \cdot \nabla B \left(\frac{v_{\perp}^2}{2} + v_{\parallel}^2 \right) \frac{1}{\Omega} \hat{\psi} \cdot \nabla f_m. \end{aligned} \quad (15)$$

If we multiply this equation by e and integrate over all of velocity space we find the following expression for the neoclassical electron current, $j_{\parallel e}^{NC} \equiv e \int_{-\infty}^{\infty} dv_{\parallel} 2\pi \int_0^{\infty} dv_{\perp} v_{\perp} (v_{\parallel} h)$:

$$\hat{b} \cdot \nabla j_{\parallel e}^{NC} - \frac{j_{\parallel e}^{NC}}{B} \hat{b} \cdot \nabla B = 4\pi \frac{F}{B^2} \frac{dp_e}{d\psi} \hat{b} \cdot \nabla B,$$

where $p_e = n_e T_e$ is the electron pressure. An analogous equation results for the neoclassical ion current, and, if we add them, we obtain

$$\frac{\partial j_{\parallel}^{NC}}{\partial r_{\chi}} - \frac{j_{\parallel}^{NC}}{B} \frac{\partial B}{\partial r_{\chi}} = \frac{4\pi F}{B^2} \frac{dp}{d\psi} \frac{\partial B}{\partial r_{\chi}},$$

or,

$$j_{\parallel}^{NC} = \frac{-2\pi F}{B} \frac{dp}{d\psi} + BK(\psi),$$

where $p = p_e + p_i$ is the total pressure, $j_{\parallel}^{NC}$ is the total neoclassical current, and K is a constant of integration. Reference to Eq. (2) shows that neoclassical drifts are solely responsible for that part of $j_{\parallel}$ which is $\propto B^{-1}$.

The solution of Eq. (14) has been investigated in great detail, and a comprehensive review has been provided by Hinton and Hazeltine⁽²⁾. For our present purposes it suffices to merely repeat their result for the case without an emf, viz. $\langle j_{\parallel}^{NC} \rangle \equiv \langle B^2 \rangle H(\psi)$, where

$$H(\psi) \propto \frac{-B_0}{B_{p0}} \frac{n_e T_e}{\langle B^2 \rangle} \sqrt{\epsilon} \left\{ K_{13} A_{1e}' + K_{23} \frac{\partial}{\partial r_{\psi}} \ln T_e \right\}.$$

The sundry terms in this expression are defined elsewhere⁽²⁾, and we note the quantity in braces is a function of the electron and ion pressure and temperature gradients as well as the plasma collisionality. $H(\psi)$ may be characterized by the following properties⁽²⁾: 1) its derivation assumes $\epsilon \ll 1$, but neoclassical currents are theoretically significant provided the poloidal beta is of order $\epsilon^{-1/2}$; 2) in the collisional regime (e.g., at the cold edge of a reactor plasma) the value of $H(\psi)$ is vanishingly small; 3) in the plateau regime (e.g., near the magnetic axis of a reactor) the derivation assumes circular flux surfaces; and 4) in the banana regime (the bulk of a reactor plasma) this "bootstrap" current density may be substantial, and a generalization of this expression is available for application to noncircular flux surfaces. In principle $H(\psi)$ is well suited to our needs as a source term in Eq. (5).

5.0 QUASILINEAR CURRENT

The functional form of the quasilinear current is obtained by expressing Eq. (13) as an equation for $g(v_{\parallel}, v_{\perp})$. The result is similar to Eq. (15) except the right-hand side is now $-\frac{\partial}{\partial v_{\parallel}} \cdot \Gamma$. Since quasilinear diffusion conserves particles, when we integrate this equation we obtain for the quasilinear current density, $j_{\parallel}^{QL} = e \int d^3 \underline{v} v_{\parallel} g$, the following result⁽³⁾:

$$j_{\parallel}^{QL} = B G(\psi).$$

$G(\psi)$ is an integration constant, and we observe that quasilinear current drive contributes only to the term $\propto B$ in the general expression for $j_{\parallel}$ in Eq. (2).

For high phase speed waves the solution to Eq. (13) is obtained in the manner outlined by Antonsen and Chu,⁽³⁾ i.e., $\langle j_{\parallel}^{QL} \rangle = \langle B^2 \rangle G(\psi)$, where

$$G(\psi) = \frac{-e}{2\pi q(\psi)} \oint_{\psi} \frac{dr}{B_p} \int d^3 \underline{v} \underline{r} \cdot \frac{\partial}{\partial \underline{v}} \left[e^{w/T_e} f_s \right], \quad (16)$$

and with $f_s(-v_{\parallel})$ the solution to the toroidal Spitzer-Härm problem without radial gradients. That is,

$$-v_{\parallel} \hat{b} \cdot \nabla f_s - \mathcal{C}(f_s) = -v_{\parallel} \hat{b} \cdot \nabla \zeta e^{-w/T_e}.$$

As noted in Ref. 3, for high phase speed waves toroidal effects are small so the integrand of Eq. (16) may be well approximated with an alternative expression involving the Fisch-Boozer function.⁽¹³⁾ Expressing quantities in SI units, except temperatures in units of keV, this becomes

$$G(\psi) = A \frac{T_e}{n_e} \langle B^{-1} \rangle \int d^3 \underline{v} (\tilde{j}/\tilde{p}) \underline{r} \cdot \frac{\partial w}{\partial \underline{v}}, \quad (17)$$

where $A = 19.19 \times 10^{18}/\ln \Lambda$ and $\ln \Lambda$ is the Coulomb logarithm. In the limit of weak damping we can approximate $\underline{r}$ by $\underline{r}_m = -\underline{D} \cdot \frac{\partial \underline{r}_m}{\partial \underline{v}}$; this replacement will facilitate numerical (ray tracing) calculations of toroidal current drive.

The dimensionless quantity $\tilde{j}/\tilde{p}$ is generally a function of $v_{\parallel}$ and $v_{\perp}$, but, in the linear regime, for $v_e \ll v_{\parallel}$ and $\underline{r}_m$ parallel to $\hat{b}$ it is well represented⁽⁸⁾ for singly charged ions as $\tilde{j}/\tilde{p} = 2 + 1.4 u^2$, where $u = v_{\parallel}/v_e$ is the location of quasilinear diffusion. Further simplification may be achieved under two additional conditions. First, with reasonable care it may be possible, as well as desirable, to utilize narrow wave spectra for current drive. In this case $\underline{r}_m$ is localized in u and $\tilde{j}/\tilde{p}$ may be removed from the velocity

integral. Second, for many cases of interest it is possible to remove $\tilde{j}/\tilde{p}$ and B^{-1} from the flux surface integral. For example, if single pass wave absorption is accomplished with waves localized within a narrow spatial region then $\tilde{j}/\tilde{p}B$ can be evaluated at the poloidal position, χ_r , where the ray trajectory intersects the flux surface. Alternatively, multipass absorption may obtain, but if $\epsilon \ll 1$ then $k_{\parallel}$ and B may be sufficiently constant on the flux surface that $\tilde{j}/\tilde{p}B$ is independent of χ . Under these rather mild constraints our expression reduces to

$$G(\psi) = A \frac{T_e}{n_e} \left[\frac{\tilde{j}/\tilde{p}}{B} \right]_{\chi_r} \bar{p}^{QL}, \quad (18)$$

where $\bar{p}^{QL}(\psi) = \langle \int d^3\underline{v} \frac{\underline{r}_m}{r_m} \cdot \frac{\partial \underline{w}}{\partial \underline{v}} \rangle$ is the flux surface averaged wave heating power density.

The derivation of Eq. (16) is quite general with regard to flux surface shape. Thus, $G(\psi)$ may be included in Eq. (5) for the calculation of $F(\psi)$ for many interesting equilibria (ellipses, dees, beans, etc.). An accurate calculation of Eq. (17) or (18) would of course presuppose a knowledge of the two dimensional equilibrium; for example $\bar{p}^{QL}$ might be found by lower hybrid or fast wave ray tracing computation in noncircular flux coordinates. The solutions of the equilibrium equation, Eq. (1), and the wave damping calculation could in principle be iterated until the equilibrium $\psi(\underline{r})$ converges.

6.0 CONCLUSIONS

The diamagnetism function F which is an input to the MHD equilibrium (Grad-Shafranov) calculation is a solution to a first order ordinary differential equation with a driving term involving the flux surface averaged parallel current density, $\langle j_{\parallel} B \rangle / \langle B^2 \rangle$. For purely steady-state current drive with no emf there is no Spitzer current, the only contributions to $j_{\parallel}$ arising from neoclassical drifts and noninductive current generation. We have derived the general expression for $\langle j_{\parallel} B \rangle$ based on the drift kinetic equation, which is appropriate for low frequency wave driven currents. The expression for the

quasilinear portion of $\langle j_{\parallel} B \rangle$ assumes high phase speed waves are utilized, and our results should be useful for studies of MHD equilibrium with drivers such as lower hybrid and fast (magnetosonic) waves.

In particular, our results can be applied to tokamaks with noncircular boundaries. We note that highly shaped plasmas may have superior stability at high beta provided the toroidal current density maintains a specific, somewhat hollow, profile.⁽¹⁴⁾ A study of high beta bean-shaped equilibria for PBX⁽¹⁵⁾ showed the difficulty of obtaining the desired current density inductively. In contrast, previous calculations have suggested that single pass wave absorption can be arranged to provide wave-driven currents at specific minor radial locations. Thus, in a reactor, the fast wave, which is accessible to the high density interior, could provide current near the magnetic axis,⁽¹⁶⁾ while the lower hybrid wave might be employed to generate current near the low density plasma surface.⁽¹⁷⁾

In principle, noninductive current drive permits tokamak operation with tailored current density, controlled externally by parameters such as the wave frequency and mode spectrum. At this point it becomes necessary to couple the numerical solutions of the Grad-Shafranov equation to a two-dimensional ray tracing/wave damping calculation with arbitrary flux surfaces in order to make $\psi(r)$ self-consistent. This will permit a more accurate assessment of the requirements and benefits of wave driven currents in tokamaks.

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