A DAMAGE MODEL FOR ROCK FRAGMENTATION
AND
COMPARISON OF CALCULATIONS WITH BLASTING EXPERIMENTS IN GRANITE

by

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Abstract

Early attempts at estimation of stress wave damage due to blasting by use of finite element calculations met with limited success due to numerical instabilities that prevented calculations from being carried to late times. An improved damage model allows finite element calculations which remain stable at late times. Reasonable agreement between crater profiles calculated with this model using the PRONTO finite element program and excavated crater profiles from blasting experiments in granite demonstrate a successful application of this model. Detailed instructions for use of this new damage model with the PRONTO finite element programs are included.

Introduction

In recent years there has been an increasing interest in the development of numerical models for the prediction of rock fragmentation by blasting. These efforts have varied from semi-empirical [1,2] to formulations based on shock wave physics [3,4]. Numerical modeling of blasting has brought into focus the complexity of the blasting process. Not only are there numerous physical processes occurring, but the host media is geologic and therefore highly variable with response characteristics that are difficult to quantify and often size dependent. This complexity requires that numerical models be developed in a highly systematic fashion in close coordination with controlled field and laboratory experimentation. It is also important that postulated physical mechanisms undergo thorough numerical evaluation before they can be accepted as being important to the blasting process.

Since 1985, the Advanced Technology Division at Sandia National Laboratories has been involved in the development of numerical models for describing the blasting process. This work has developed two separate computational thrusts. The early time stress wave propagation is treated through the use of a dynamic wave computer program PRONTO [5,6] and a constitutive damage model that attempts to describe the fracturing that occurs as a result of the stress induced material deformation. A separate modeling effort [7] is underway to describe the motion of individual rock fragments as a result of gas flow loading.

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This paper describes in detail the model used to estimate stress wave induced fracturing. The damage model has evolved [8,9,4,6] over a number of years. As will become apparent, stress wave induced fragmentation is a very complex process and any computational algorithm will only approximately represent a fracturing event. Thus, it is important that any postulated model be compared in detail against controlled experimental data. Only by such comparisons can it be determined if the numerical embodiment of the physics represents a useful engineering approximation.

In the past two years, as part of our model development program, an experimental verification effort has been conducted. The experimental work is being performed jointly with the Atlas Powder Company. Beginning in 1988, two sequences of cratering experiments were conducted for the express purpose of generating data for comparison with model predictions. The experiments examined systematic variations in geometry, rock type, and explosive. Data collected on the tests included detonation velocity, high speed photography of ground motion, muck pile profiles, excavated crater profiles and particle size information.

In addition to detailing the numerical model this paper discusses comparisons of the predictions with experimental data. These comparisons have been used to guide our current approach and are being used to identify weaknesses which should be targeted in future research. These studies also indicate the complexity of the mechanisms involved and are serving as a basis for developing complementary numerical descriptions that emphasize alternate physical processes.

Damage Model

The damage model is intended to simulate the dynamic fracture of brittle rock. It is based on work started by Kipp and Grady [8] and continued by Taylor, Chen and Kuszmaw [9,4]. It is modified here to extend it to large crack densities as suggested by Englman and Jaeger [10]. The essential feature of this model is the treatment of the dynamic fracture process as a continuous accrual of damage in tension, where the damage mechanism is attributed to microcracking in the rock. The fundamental assumption of the model is that the rock is an isotropic material which is permeated by an array of randomly distributed and oriented microcracks which grow and interact with one another under tensile loading.

Englman and Jaeger [10] introduce a regularized damage parameter, F , which is related to Budiansky and O'Connell's [11] crack density, C_d , but takes into account the overlap between the damage volumes of different cracks. To this end they define F by

$$F = 1 - \exp(-\alpha C_d), \quad (1)$$

where $\alpha=16/9$. The effective Poisson's ratio, ν_* , and bulk modulus, K_* , of a cracked medium with undamaged Poisson's ratio ν and bulk modulus K are given by

$$\nu_* = \nu(1 - F) = \nu \exp(-\alpha C_d) \quad (2)$$

and

$$K_* = (1 - f_1(\nu_*)F)K, \quad (3)$$

where

$$f_1(\nu_*) = \frac{1 - \nu_*^2}{1 - 2\nu_*}. \quad (4)$$

In order to relate stress to strain we will generate a system of equations which can be solved for the effective elastic moduli of the cracked medium. It is convenient to introduce a damage parameter, D , defined by

$$D = f_1(\nu_*)F. \quad (5)$$

Note that $0 \leq F \leq D \leq 1$ and

$$K_* = (1 - D)K. \quad (6)$$

The time derivative of D is given by

$$\frac{dD}{dt} = f_1(\nu_*) \frac{dF}{dt} + \frac{df_1(\nu_*)}{dt} F. \quad (7)$$

In order to express equation 7 in terms of crack density we note that

$$\frac{dF}{dt} = \alpha \exp(-\alpha C_d) \frac{dC_d}{dt} = \frac{\alpha \nu_*}{\nu} \frac{dC_d}{dt} \quad (8)$$

follows from equations 1 and 2. We apply the chain rule twice to get

$$\frac{df_1(\nu_*)}{dt} = \frac{df_1}{d\nu_*} \frac{d\nu_*}{dC_d} \frac{dC_d}{dt}. \quad (9)$$

Differentiating equation 4 yields

$$\frac{df_1}{d\nu_*} = \frac{2(1 - \nu_* + \nu_*^2)}{(1 - 2\nu_*)^2} = f_2(\nu_*) \quad (10)$$

and from equation 2 we get

$$\frac{d\nu_*}{dC_d} = -\alpha \nu \exp(-\alpha C_d) = -\alpha \nu_*. \quad (11)$$

Combining equations 7 through 11 yields

$$\frac{dD}{dt} = \frac{\alpha \nu_a}{\nu} [f_1(\nu_0) - (\nu - \nu_0) f_2(\nu_0)] \frac{dC_d}{dt}, \quad (12)$$

where we have used equations 1 and 2 to simplify the form of equation 12.

The crack density can be related to an average flaw size, a , by

$$C_d = \gamma N a^3, \quad (13)$$

where N is the number of active cracks and γ is a proportionality ratio. Differentiating equation 13 yields

$$\frac{dC_d}{dt} = \gamma \frac{dN}{dt} a^3 + 3\gamma N \frac{da}{dt} a^2, \quad (14)$$

where the forms of N and a are yet to be determined.

At this point, it should be noted that there are a wide variety of assumptions which can be made as to the form of N , and there is almost no agreement as to the proper form for a . Kipp and Grady [8] and Kuszmaul [4] assume that the number of cracks activated at a volumetric strain, ϵ , is described by a Weibull distribution. We modify the form of that distribution to account for dilatancy strain, ϵ_d , by writing

$$N = k(\epsilon - \epsilon_d)^m, \quad (15)$$

where k and m are material dependent constants and the volumetric strain, ϵ , is one third of the time integral of the trace of the deformation tensor, d :

$$\epsilon = 1/3 \int \text{tr}(d) dt. \quad (16)$$

Note that ϵ is positive in tension. The dilatancy strain, $\epsilon_d \geq 0$, is the volumetric strain at zero volumetric stress. Equation 15 implies

$$C_d = k(\epsilon - \epsilon_d)^m a^3 \quad (17)$$

and

$$\frac{dN}{dt} = mk(\epsilon - \epsilon_d)^{m-1} \frac{d\epsilon}{dt} \quad (18)$$

where the proportionality ratio γ has been absorbed in the material constant k . Introducing equations 15 and 18 into equation 14 yields

$$\frac{dC_d}{dt} = mk(\epsilon - \epsilon_d)^{m-1} a^2 \left(a \frac{d\epsilon}{dt} + \frac{3(\epsilon - \epsilon_d)}{m} \frac{da}{dt} \right). \quad (19)$$

Englman and Jaeger [10] suggest that a reasonable approach to determining the average flaw size, a , would be to assume that all flaws start at zero size and grow at some constant rate, perhaps related to the shear wave velocity of the material. Kipp and Grady [8] developed a three parameter model based on parameters m and k defined as above and a third parameter which was determined to be approximately one third of the shear wave velocity of the material. This may indicate that their model was similar to Englman and Jaeger's suggestion. In the following we will take a totally different approach aimed at modeling the observed dependence of fracture strength and fragment size on strain rate.

Based on energy considerations at high strain rates, Grady [12] derives the following expression for the nominal fragment radius, r , for dynamic fragmentation of a brittle material:

$$r = \frac{1}{2} \left(\frac{\sqrt{20}K_{IC}}{\rho c R} \right)^{2/3} \quad (20)$$

Here K_{IC} , ρ and c are the fracture toughness, density and sound speed of the undamaged material and R is the strain rate, which is assumed in the derivation to be both constant and large. This is an average fragment radius for the global response of a uniformly expanding sphere. We will assume that the local average flaw size, a , is proportional to the value of r appropriate to the local strain rate.

In order to apply equation 20 to the case where the strain rate is not constant, Taylor, Chen and Kuszmaul [9,4] replace the constant strain rate, R , in equation 20 with the maximum strain rate, R_{max} , which the material has experienced. The derivation of equation 20 assumes that the strain rate R is reached prior to the time of interest, is large and remains constant during the fragmentation process. In the finite element program R_{max} usually occurs when the material first comes off the peak of a shock wave and remains constant after that time. In this case R_{max} is both constant and large, as in the derivation of equation 20, even though $d\epsilon/dt$ is not. To assure that unreasonably small values of R_{max} are not used in equation 20, a material dependent minimum significant strain rate R_{min} is used until $d\epsilon/dt$ exceeds R_{min} . R_{max} is usually much greater than R_{min} before the material experiences any significant tension. Use of equation 20 in this manner results in

$$C_d = \frac{5k(\epsilon - \epsilon_d)^m}{2} \left(\frac{K_{IC}}{\rho c R_{max}} \right)^2 \quad (21)$$

and $da/dt=0$. Again the proportionality ratio has been absorbed in the material constant k . Equation 19 becomes

$$\frac{dC_d}{dt} = \frac{5km(\epsilon - \epsilon_d)^{m-1}}{2} \left(\frac{K_{IC}}{\rho c R_{max}} \right)^2 \frac{d\epsilon}{dt} \quad (22)$$

The bulk response of the material is given by $P = K_0(\epsilon - \epsilon_d)$, where the volumetric stress P , which is positive in tension, is one third of the trace of the stress tensor. We assure that the deviatoric response of the material is consistent with the bulk response by defining a degraded shear modulus, μ_0 , such that

$$S = 2\mu_0 e = \frac{3K_0(1 - 2\nu_0)}{(1 + \nu_0)} e, \quad (23)$$

where S is the deviatoric part of the stress tensor and e is the deviatoric part of the deformation tensor d . It follows that

$$\frac{dP}{dt} = 3K(1-D) \frac{d\epsilon}{dt} - 3K(\epsilon - \epsilon_d) \frac{dD}{dt} \quad (24)$$

and

$$\frac{dS}{dt} = 2\mu_0 \frac{de}{dt} - 2e \frac{d\mu_0}{dt}, \quad (25)$$

where

$$\frac{d\mu_0}{dt} = \frac{8\nu_0 K_0}{(1 + \nu_0)^2} \frac{dC_d}{dt} - \frac{3K(1 - 2\nu_0)}{2(1 + \nu_0)} \frac{dD}{dt}. \quad (26)$$

Equations 12, 22, 24, 25 and 26 constitute a coupled system of ordinary differential equations which can be integrated to define the response of the damaged material. In previous versions [9,4,5,6] the corresponding equations were solved in the PRONTO finite element program using a simple first order difference algorithm. However, when these equations are solved in PRONTO with a first order integration operator, the resulting value of D is inconsistent with the value of C_d when strain rates are very large. This is due to the exponential dependence of D on C_d in the current version. In order to avoid using a higher order accurate integration operator, equation 12 is replaced with equations 2, 4 and 5 and the resulting mixed system of algebraic equations and ordinary differential equations is solved using a simple centered integration operator.

The parameter m relates tensile fracture to strain. The cube root strain rate dependence determined by Grady [12] dictates that $m=6$. Unfortunately, no general definition of k has been found. For the special case of a material with Poisson's ratio zero, it is easy to show that

$$k_0 = \frac{9}{280} \left[\frac{\rho c}{K_{IC}} \right]^2 \left[\frac{18K}{7} \right]^6 \frac{R^2}{P_{max}^6}, \quad (27)$$

where $P_{\max}$ is the maximum volumetric tensile stress achieved in a test to failure at a constant strain rate R . Here we have made use of the fact that $m=6$ and have written k_0 to emphasize the $\nu=0$ assumption.

Since a rock with a Poisson's ratio of zero would be very strange, a solution of the constant strain rate expansion case without the $\nu=0$ assumption would be more realistic. While it may be possible to solve the constant strain rate expansion case without the $\nu=0$ assumption, we know of no such solution at this writing. However, the same method used to derive equation 27 can be used to solve for $k'(\nu)$, the value of k for a material of constant Poisson's ratio ν . This contradicts equation 2, unless $\nu=0$, but it should lend some insight into how k might be expected to depend on ν . The result of this procedure is

$$k'(\nu) = \frac{1 - 2\nu}{1 - \nu^2} k_0. \quad (28)$$

Note that $k'(1/2)=0$ and $k'(1/3)=3/8k_0$. This indicates that the $\nu=0$ assumption might result in values of k which are too large.

If laboratory data for maximum tensile stress as a function of strain rate are not available, it is possible to compute a value for k by using an expression derived by Kipp, Grady and Chen [14]. For a number of rocks, they found that a reasonable approximation to maximum tensile stress, $P_{\max}$, as a function of strain rate, R , is given by

$$P_{\max} = \left(\frac{9\pi E K_{IC}^2 R}{16 c_s N_s^2} \right)^{1/3}, \quad (29)$$

where c_s and E are the shear wave velocity and Young's modulus of the undamaged material, and N_s is a shape factor ($N_s=1.12$ for penny shaped cracks). It is possible to eliminate the ratio $R^2/P_{\max}^6$ between equations 27 and 29 to get

$$k_0 = \frac{29.4 N_s^4}{\pi^2} \left(\frac{K}{K_{IC}} \right)^6 \left(\frac{c_s}{c} \right)^2. \quad (30)$$

For $\nu=0$, $(c_s/c)^2=1/2$ and in general

$$\left(\frac{c_s}{c} \right)^2 = \frac{1}{2(1 + \nu)}, \quad (31)$$

so that use of equation 29 implies that $k'(\nu)$ should be replaced with

$$k''(\nu) = \frac{1 - 2\nu}{(1 - \nu^2)(1 + \nu)} k_0. \quad (32)$$

Note that the difference between $k''(\nu)$ and k_0 is greater than the difference between $k'(\nu)$ and k_0 , with $k''(1/3)=3/4k'(1/3)=9/32k_0$.

Figure 1 shows the accumulation of damage with strain for three uniaxial strain calculations past fragmentation at constant strain rates of 10/s, 100/s and 1000/s using material properties derived for the granite in which our blasting experiments were carried out. The main difference between this version of the model and previous versions is evident at large values of damage where exponential growth of damage is replaced with an asymptotic approach to the maximum value $D=1$. In practice this allows the finite element calculation to remain stable in situations which would have resulted in numerical instability with previous versions. Figure 2 gives volumetric stress as a function of strain for these same three calculations. Note the increase in peak stress with strain rate.

Thus far we have only considered expansion in tension. Equation 22 implies that damage is healed when damaged material is recompressed. Since this is not our intent, the damage model is used only when the material is expanding in tension. Previous versions assumed that the material response was governed by the undamaged elastic constants in all other situations. This is equivalent to assuming that activated microcracks effect the material only when it is expanding in tension. This is illustrated in figure 3, which shows volumetric stress as a function of strain for a cyclic uniaxial strain calculation in which the material is expanded and recompressed at strain rates of 200/s, 400/s and 600/s. Note the large amount of dilatancy.

The opposite extreme would be to use the reduced bulk modulus, K_r , and Poisson's ratio, ν_r , for recompression in tension, returning to the undamaged moduli only in compression. This is equivalent to assuming that all cracks close exactly and heal until tension is again applied. This is illustrated in figure 4, which shows volumetric stress as a function of strain for a cyclic uniaxial strain calculation. Note the absence of dilatancy.

Since dilatancy is observed in lab tests, it seems reasonable to assume that reality is somewhere between these two extremes. A damage dependent amount of dilatancy can be built into the model by using only a fraction of the damage to compute a partially reduced bulk modulus, K_r , to be used during recompression in tension. A fraction, β , with $0 \leq \beta \leq 1$, is used to compute this partially reduced bulk modulus according to

$$K_r = (1 - \beta D) K. \quad (33)$$

The damaged Poisson's ratio is retained in tension and the undamaged elastic moduli are used in compression. Figure 5 gives volumetric stress as a function of strain for a cyclic uniaxial strain calculation with $\beta=9/10$. When $\beta=1$ we have the bulk response illustrated in figure 4. Figure 3 illustrates the bulk response for $\beta=0$. Since the reduced Poisson's ratio is always used in tension, the deviatoric response differs from previous versions even for $\beta=0$.

Figure 1. Damage as a function of strain for uniaxial strain calculations at strain rates of 10/s, 100/s and 1000/s.

Figure 2. Volumetric stress as a function of strain for uniaxial strain calculations at strain rates of 10/s, 100/s and 1000/s.

Figure 3. Stress as a function of strain for a cyclic calculation with undamaged properties used for recompression in tension ($\beta=0$).

Figure 4. Stress as a function of strain for a cyclic calculation with damaged properties used for recompression in tension ($\beta=1$).

Figure 5. Stress as a function of strain for a cyclic uniaxial strain calculation with K_r computed using $\beta=9/10$.

Figure 6. Inverse parabola yield function fit to triaxial test data for sandstone.

At high pressures rocks are known to exhibit behavior similar to plasticity in metals. In previous versions [8,9,4,6] this was accounted for by assuming the compressive behavior of the material to be elastic-perfectly plastic. In this version we extend this to account for the increase in yield stress with confining pressure which is observed in triaxial tests. This is accomplished by allowing yield stress to be a function of volumetric stress. In the most general case, the yield stress in compression is assumed to be an inverse parabola in the absolute value of volumetric stress. This is illustrated in figure 6 using triaxial data for a sandstone. Olsson [15] reports two triaxial tests at each of the confining pressures 50, 100 and 200 MPa. All of these tests exhibit clear indications of plastic behavior. If it is assumed that the sample is in a uniform stress state, even though we know that it is not, then these data can be plotted as a function of the absolute value of volumetric stress at failure, as in figure 6. The average stresses at the three confining pressures determine an inverse parabola which can be used as a yield surface for finite element calculations. If only two confining pressures are used, the function becomes a straight line, and if only one confining pressure is used, elastic-perfectly plastic behavior results. In tension the yield stress is assumed to be very large in order to avoid plasticity.

Damage Model Implementation in PRONTO

In the PRONTO [5,6] finite element programs the new damage model is used for materials with material name "DAMAGE". As with the old damage model, the following material cues must be specified:

YOUNGS MODULUS	Young's modulus, E , for the undamaged material.
POISSONS RATIO	Poisson's ratio, ν , for the undamaged material.
K	Material constant, k , introduced in equation 15.
FRACTURE TOUGHNESS	Fracture toughness, K_{IC} , for the undamaged material.

If the cue K is specified as zero or negative, then k is computed from equation 32 using $N_s = -1.12$. In this case values of k which would result in arithmetic faults are avoided by requiring that $K_{IC} > E/10^6$. Since m is always taken to be 6, there is no need for the old material cue M .

In addition, the following new material cues must be specified:

RELOAD	The fraction, β , $0 \leq \beta \leq 1$ of the damage to be used for recompression in tension.
CP1	Confining pressure for a triaxial test.
Y1	Yield stress for the triaxial test at confining pressure CP1.
CP2	Confining pressure for a triaxial test.
Y2	Yield stress for the triaxial test at confining pressure CP2

CP3	Confining pressure for a triaxial test.
Y3	Yield stress for the triaxial test at confining pressure CP3.
RMIN	The minimum value, R_{min} , to use for R_{max} in equation 21.

These are initially stored in PROP(1) through PROP(12) in the above order. In previous versions R_{min} was taken to be 10^{-6} for all materials. A reasonable way of choosing R_{min} is illustrated in the section on granite properties. No theory has yet been developed for choosing β . Previous versions always used $\beta=0$; values of $\beta \leq 1/2$ have given the best match between calculations and field data.

The confining pressures and yield stresses are assumed to be specified such that $CP1 < CP2 < CP3$ and $0 < Y1 \leq Y2 \leq Y3$. When $Y1=Y2$, the compressive response is assumed to be elastic-perfectly plastic. When $Y1 < Y2=Y3$, the yield surface is a straight line. When $Y1 < Y2 < Y3$, the yield surface is the inverse of a parabola in the form

$$-P(Y) = aY^2 + bY + c \quad (35)$$

where Y is the yield stress at a compressive volumetric stress of P (recall that P is negative in compression). Only yield surfaces which result in a positive yield stress at a volumetric stress of zero are considered to be valid. This is the case when c is negative. A method of constructing a reasonable inverse parabola yield function when triaxial yield data are not available is illustrated in the section on granite properties.

The new damage model uses the following six internal state variables:

DAMAGE	Damage, D.
RMAX	Maximum strain rate, R_{max} .
EMAX	Maximum strain while not in tension, ϵ_d .
CRKDENS	Crack density, C_d .
EQPS	Equivalent plastic strain.
FRAGSIZE	Average fragment radius, $1/2(20K_{IC}^2/\rho^2c^2R_{max}^2)^{1/3}$

During execution the PROP array for the new damage model contains the following material constants:

PROP(1)	Shear modulus, μ , for the undamaged material.
PROP(2)	Poisson's ratio, ν , for the undamaged material.
PROP(3)	Yield stress at zero volumetric stress for linear and perfectly-plastic yield functions; otherwise 0.
PROP(4)	Slope of linear yield function; otherwise 0.
PROP(5)	Fraction of damage, β , for recompression in tension.
PROP(6)	Fragment radius coefficient, $1/2(20K_{IC}^2/\rho^2c^2)^{1/3}$.

PROP(7)	Coefficient a for inverse parabola yield function.
PROP(8)	Coefficient b for inverse parabola yield function.
PROP(9)	Coefficient c for inverse parabola yield function.
PROP(10)	1.0 for inverse parabolic yield function; otherwise 0.
PROP(11)	Not used
PROP(12)	Minimum value, R_{min} , to use for R_{max} in equation 21.
PROP(13)	Three times the bulk modulus of the undamaged material.
PROP(14)	Coefficient in equation 22, $15kK_{IC}^2/\rho^2c^2$.

Granite Properties

Olsson [15] reports properties for a granite in which blasting experiments were performed. Unfortunately, this granite exhibited a foliation which induced anisotropic material properties, violating the most basic assumption on which this model is constructed. In spite of this, we attempted to model this granite as an isotropic material using the damage model and average properties. Olsson's data give a 2680 kg/m³ density. Young's modulus measurements had a significant amount of directional dependence. After rejecting a suspicious sample measured at 30° to the foliation, the remaining 8 samples at 30° to the foliation and 8 samples at 90° to the foliation gave an average Young's modulus, E, of 62.2 GPa with standard deviation 16 GPa. The large standard deviation reflects not only the scatter in the data, which was large at both angles, but also the fact that these two data sets result in statistically distinct values for E when considered separately. In fact, Olsson [15] reports four different values for E. At 90° to the foliation three uniaxial tests give E=53.2 GPa while five triaxial tests result in E=77.2 GPa. At 30° to the foliation four uniaxial tests give E=43.7 GPa while four triaxial tests result in E=69.0 GPa. The use of E=62.2 is partially justified by two zero-stress bar velocity measurements on nonfoliated samples which gave an average Young's modulus of 62.8 GPa. Poisson's ratio, ν , was obtained for three samples measured at 90° to the foliation. The average of these three values was 0.29 with a standard deviation of 0.05. Measurements at 30° to the foliation did not yield meaningful values of Poisson's ratio, so the 90° value was used.

Kolsky bar tests at 90° to the foliation were used to determine the minimum strain rate, R_{min} , to use for R_{max} in equation 21. Figure 7 shows failure stress as a function of strain rate for three uniaxial tests and four Kolsky bar tests at strain rates between 100/s and 200/s. A cube root dependence of failure strength on strain rate drawn through the average of the four Kolsky bar tests results in the sloping line on the right in figure 7. This line drops below the average of the three uniaxial tests at a strain rate of 9.8/s. Assuming that the cube root dependence of failure stress on strain rate applies down to a strain rate of 20/s results in the piecewise linear fit shown in figure 7. The slope of the left hand segment is somewhat arbitrary, but seems reasonable.

Figure 7. Strain rate dependence of dynamic failure stress for granite.

Unfortunately, none of the triaxial tests exhibited indications of plasticity. We decided to treat the uniaxial and triaxial failure strengths as lower bounds on the plastic limit and construct an inverse parabola yield surface well above these data. Figure 8 shows failure strengths for 8 uniaxial tests, 9 triaxial tests at a confining pressure of 50 MPa and two triaxial tests at a confining pressure of 200 MPa (the two tests at 200 MPa were inadvertently omitted from reference 15). All of these data are plotted as a function of the absolute value of the stress which would result from assuming that the samples were in a uniform stress state at the time of failure. Note the large amount of scatter. The lower curve in figure 8 is the inverse parabola determined by the average failure stress at each confining pressure. The upper curve in figure 8 is the inverse parabola used as a yield surface in the damage model. This inverse parabola was generated by specifying CP1=-50MPa, Y1=150MPa, CP2=50MPa, Y2=750MPa, CP3=200MPa and Y3=1.05GPa. This choice is intended to give a yield stress of 150 MPa at a volumetric stress of zero and generate an inverse parabola which is approximately the same shape as the inverse parabola determined by the average failure stresses.

Chong [16] measured K_{IC} values for this granite at angles of 0° , 45° and 90° to the foliation. These measurements confirmed the anisotropic nature of the site. Ignoring the anisotropy and using divider geometry, an average value of 1.67 MPa/m with a standard deviation of 0.41 MPa/m was obtained for K_{IC} . In the total absence of any data on maximum tensile stress as a function of strain rate we specified zero for the material cue K and obtained the value $k=5.3 \times 10^{26}/m^3$ which resulted from using $N_s=1.12$ in equation 32.

Calculations

In an attempt to model blasting experiments in granite, axisymmetric finite element calculations were performed using the damage model and the PRONTO 2D [5] finite element program. The isotropic material properties discussed above were used without any adjustments being made to improve agreement between the calculations and the experiments. It was necessary to use one of the experiments as a basis for choosing the value of β (equation 33) to be used for recompression in tension. This will be discussed in the section which compares calculated and experimental results. These damage model parameters are summarized as follows:

YOUNGS MODULUS	E = 62.8 GPa
POISSONS RATIO	ν = 0.29
K	$k = 5.3 \times 10^{26}/m^3$
FRACTURE TOUGHNESS	$K_{IC} = 1.68$ MPa
RELOAD	$\beta = 0$
CP1	-50 Mpa
Y1	150 MPa

Figure 8. Granite failure stress data and inverse parabola yield function.

CP2	50 MPa
Y2	750 MPa
CP3	200 MPa
Y3	1.05 GPa
RMIN	$R_{min} = 20 \text{ MPa}$

Three depths of burial were simulated. Figure 9 shows part of the finite element configuration used for simulation of the three meter depth-of-burial blasting experiments. An axis of rotational symmetry was assumed along the center of the borehole. The 5 3/4" diameter borehole dictates that the center column of elements have a radial dimension of 0.073025 m. The radial dimension of the elements was gradually increased with radius, requiring 101 elements between the axis of symmetry and a nonreflecting boundary at a radius of 15 m. A vertical dimension of 1/16 of a meter was used for elements between the bottom of the borehole at a depth of 3.5 m and the free boundary at the surface. The vertical dimension of the elements below the borehole was gradually increased with depth. There are 101 elements between the surface and a nonreflecting boundary at a depth of 20 m, for a total of 10,201 elements. The explosive charge was modeled by the sixteen elements blacked out in figure 9 along the axis of symmetry between the 2.5 and 3.5 meter depths. The finite element configurations used for simulation of the two and four meter depth-of-burial blasting experiments are generated from the three meter depth-of-burial configuration by moving the depth of the explosive charge and the bottom boundary up by one meter in the two meter depth-of-burial case, resulting in 8,585 elements and down by one meter in the four meter depth-of-burial case, resulting in 11,817 elements.

Except for the sixteen elements representing the explosive charge, all elements are assumed to represent solid granite and are modeled using the above material properties in the damage model. This includes elements in the borehole above the charge. In the blast experiments, the borehole is filled with a loose packed stemming material which would be impossible to model in the finite element calculations. Assuming that the borehole above the charge is solid granite, rather than a loose packed stemming material, would invalidate gas flow and particle motion calculations, but should have little effect in the short time frame of these stress wave calculations.

The explosive charge was modeled using the JWL [17] equation-of-state subroutine in the PRONTO 2D [5] program. Atlas Powder APEX 1220 and ANFO charges were simulated at each depth-of-burial. The APEX 1220 was modeled as having 1250 kg/m³ density, 6065 m/s detonation velocity and $2.6 \times 10^9 \text{ J/m}^3$ energy density. The ANFO was modeled as having a density of 830 kg/m³, a detonation velocity of 3780 m/s and an energy density of $7.08 \times 10^9 \text{ J/m}^3$.

Figure 9. Finite element configuration for 3m depth-of-burial calculations.

All six calculations were carried out to a simulated time of 2 ms, which was well past the point at which computed damage had stopped growing. Each of these calculations required between 57 and 80 seconds on a Cray XMP/416.

Calculated Crater Profiles

Kuszmaul [13] applies three-dimensional percolation theory to derive a fragmentation criterion based on crack densities calculated with his fragmentation model [4]. The only modification of this theory necessary for use with the current damage model is that the fragmentation limit must be based on the regularized damage parameter rather than the crack density. This leads to a fragmentation limit of $F=0.693$. For a material with an initial Poisson's ratio of 0.29 this corresponds to a damage of $D=0.836$. In applying this interpretation it should be remembered that the theory assumes that cracks are randomly oriented. This probably is not a valid assumption for an anisotropic material. In addition, a radial orientation of activated cracks would be expected near the charge and horizontal orientation should dominate near the surface.

The dashed curve in figure 10 shows the fragmented region limit as predicted by the calculated $D=0.836$ contour for a 1 m APEX 1220 charge centered at a 3 m depth. The position of the explosive charge is indicated by the slant shaded region at the bottom of the borehole. This is compared to survey data for perpendicular profiles of the excavated crater from a blasting experiment. The difference between the two perpendicular profiles of the excavated crater is probably caused by the foliation at the site and is large enough to indicate that the foliation may have had a significant effect on the cratering process. The calculated edge of the fragmented region agrees with the excavated crater profiles as well as they agree with each other.

In this calculation a value of $\beta=0$ was used for recompression in tension. Equally good agreement was obtained with several values of $\beta \leq 1/2$. Values of $\beta \geq 3/4$ lead to damage near the borehole which was not observed in the excavated crater. We chose $\beta=0$ for the rest of the calculations based on this comparison. We may have been biased to choose $\beta=0$ rather than $\beta=1/2$ by the fact that all previous versions of this damage model had used undamaged elastic properties for recompression in tension.

Figure 11 compares the calculated fragmented region limit with perpendicular profiles of the excavated crater from an experiment using a 1 m APEX 1220 charge centered at a 2 m depth. The radius of the calculated fragmented region is somewhat larger than the excavated crater radius. The bottom of the excavated crater extends to the center of the charge, while the calculated fragmented region has a central dome. In fact the calculation predicted very little damage near the charge. This region experiences significant amounts of plastic strain while in

Figure 10. Calculated fragmented region limit and excavated crater profiles for a 1m APEX 1220 charge at a 3m depth.

Figure 11. Calculated fragmented region limit and excavated crater profiles for a 1m APEX 1220 charge at a 2m depth.

compression and this tends to limit tensile stresses. Since plasticity was not observed in any tests on this material, the way plasticity was treated is one of the more questionable aspects of the model. Improper treatment of plasticity could be the cause of the shallow depth of the calculated fragmented region. Break up of this region was predicted by calculations with large plastic limits and calculations with values of $\beta \geq 3/4$. However, this additional damage occurred before the stress wave reached the surface and these calculations predicted fragmentation well below the bottom of the charge. It may be that break up of the material near the charge is a result of gas flowing into cracks. Such gas flow related mechanisms cannot be predicted by any method that considers only shock wave effects.

Figure 12 compares the calculated fragmented region limit with perpendicular profiles of the excavated crater from an experiment using a 1 m APEX 1220 charge centered at a 4m depth. The radius of the calculated fragmented region is slightly larger than the excavated crater radius. The excavated crater extends to a depth of 1.3 m, while the calculated fragmented region is at most 1/2 that depth. The excavated crater was quite symmetric so it is difficult to argue that the disagreement is caused by anisotropy. This region experiences very little plastic strain and flow related fracture should not be effective this far from the charge. Notice the large fragmented region below the crater. This damage takes place before the stress wave has reached the surface. If the calculation is over predicting damage at this early time, then the excess energy removed from the stress wave could explain the shallow predicted crater. However, this argument would lead us to conclude that the agreement between calculated fragmented region and excavated crater depth shown in figure 10 was due to a fortuitous combination of too much damage at early time and too little damage from the surface release wave.

Figure 13 compares the calculated fragmented region limit with perpendicular profiles of the excavated crater from an experiment using a 1 m ANFO charge centered at a 3 m depth. The excavated crater was quite asymmetric and the predicted profile is within the scatter of the data. However, this experiment was in a wet region and may be inappropriate for comparison due to a measured detonation velocity of only 2/3 of that used in the calculation.

Figure 14 compares the calculated fragmented region limit with perpendicular profiles of the excavated crater from an experiment using a 1 m ANFO charge centered at a 2 m depth. Predicted radius and depth are both in excellent agreement with the excavated crater.

Figures 15 and 16 compare the limit of the calculated fragmented region with perpendicular profiles of excavated craters from two experiments using 1 m ANFO charges centered at a 4 m depth. Both excavated craters were small, but substantially deeper than predicted.

Figure 12. Calculated fragmented region limit and excavated crater profiles for a 1m APEX 1220 charge at a 4m depth.

Figure 13. Calculated fragmented region limit and excavated crater profiles for a 1m ANFO charge at a 3m depth.

Figure 14. Calculated fragmented region limit and excavated crater profiles for a 1m ANFO charge at a 2m depth.

Figure 15. Calculated fragmented region limit and excavated crater profiles for a 1m ANFO charge at a 4m depth.

Figure 16. Calculated fragmented region limit and excavated crater profiles for a 1m ANFO charge at a 4m depth.

Conclusions

The current damage model is a significant improvement over previous versions mainly because it allows numerically stable calculations even when a large amount of damage is calculated. This allows calculations to be extended to simulation times large enough to be sure that no shock wave induced damage can occur at later times. These calculations can be carried out in as little as one percent of the computer time required for calculations with previous versions. The increased speed is important, because it allows efficient parameter variation to evaluate the effects of variation in and/or uncertainties about material properties and should allow very large three-dimensional finite element models of complicated geometry to be used at a reasonable cost.

Comparison of calculations with blast experiments in granite indicate that reasonable results can be obtained with the current damage model even using average material properties for an anisotropic site. However, significant discrepancies between the calculated fragmented regions and the excavated craters are evident. Comparison with experiments in a wide variety of isotropic rocks must be carried out before it will be known whether or not the current version of the model is a useful engineering tool. The current version is based on many assumptions which cannot be verified or denied on the basis of the experimental data currently available. More precisely controlled experiments are likely to indicate areas where the current version of the damage model can be improved.

In the derivation of the model we have tried to indicate where future efforts are likely to yield improvements. A more realistic interpretation of the parameter k , introduced in equation 15, and either a means of calculating it from standard lab tests or a new lab test for measuring it seems essential. Of course, the proper solution to this might be the replacement of equation 15 with another form for the number of cracks activated. The use of equation 20 in determining the characteristic flaw size has the advantage of introducing standard material parameters into the model, but requires the acceptance of a characteristic flaw size that depends on a previously determined maximum strain rate and does change with the application of tensile stress at lower rates. Since the calculated strain rate is the average over a finite element, the use of the maximum strain rate in this manner caused some previous versions of the damage model to be extremely sensitive to element size. However, this does not seem to be a problem with the current version of the damage model. Essentially the same results presented here were obtained with finite element configurations using less than half the number of elements used in these calculations. While plasticity is a well established theory for metals, the assumption that a rock can flow plasticity without accumulating damage is questionable. Perhaps the theory can be modified to include a link between plastic

strain and damage accumulation due to shear deformation in compression. Certainly a more realistic treatment of dilatancy is needed.

Beyond these details, it might be necessary to examine some of the more basic assumptions of the model. For an anisotropic material the assumption that microcracks are randomly distributed and oriented is probably not valid. Of course, introduction of anisotropy would require that all calculations be three-dimensional and that a new anisotropic finite element program be developed. It seems that the activation of damage should depend on the full stress tensor and not be limited to situations which result in volumetric tension. Budiansky and O'Connell's equivalent elastic moduli and Engleman and Jaeger's extension to large crack densities are only intended to be approximations. It may be possible to replace these with more accurate approximations. Given the complexities of this problem we intend to explore other computational approaches as well as evolutionary modifications to the current model.

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APPENDIX A

PRONTO CHANGES TO IMPLEMENT THE NEW DAMAGE MODEL

In order to use the material cue R_{min} , subroutine SVINT must be changed to set SV(2,I) to PROP(12) rather than to 1.0e-6 for material model 4 as follows:

```

C
C KIPP/GRADY/TAYLOR/CHEN/KUSZMAUL/ENGLMAN/JAEGER/THORNE DAMAGE MODEL
C   ELSE IF(MKIND.EQ.4) THEN
C       DO 40 I=1,NEL
C           SV(2,I) = PROP(12)
C   40   CONTINUE
C

```

In subroutine MATINT at steps 4 through 7 make the following changes to material model 4 variables:

```

C
C*****
C   *
C   * STEP 4: NUMSV is the array which defines the number
C   *         of internal state variables for the material.
C   *         Enter the appropriate number for your material
C   *
C*****
C
C   DATA NUMSV/0,6,2,6,0,0,4,7,6,19,16,19/
C
C*****
C   *
C   * STEP 5: NCUES is the array that defines the number of
C   *         material constants that will be read in for
C   *         each material. Enter the appropriate value in
C   *         the data statement for your new material.
C   *
C*****
C
C   DATA NCUES/2,5,6,12,1,7,7,17,6,14,12,14/
C
C*****
C   *
C   * STEP 6: NCONS is the array that defines the number of
C   *         material constants that are needed in each
C   *         constitutive routine. Note that NCONS must
C   *         be greater than or equal to NCUES. If NCONS
C   *         exceeds NCUES, this implies that you are
C   *         going to define extra constants in STEP 12
C   *         in addition to supplying lambda plus two mu.
C   *
C*****
C
C   DATA NCONS/4,11,9,14,1,7,9,20,10,18,14,16/
C

```

```

C*****
C      *
C      * STEP 7: CUES is the CHARACTER array that contains
C      * the names of each material property that will
C      * be read for each material type. Note that the
C      * word length must be less than 20 characters
C      * and the range of the first index must not
C      * exceed MCUES defined above in STEP 2. Enter
C      * a DATA statement with the names for your
C      * material constants to be read. Blanks are
C      * allowed in the cue names.
C      *
C      * IMPORTANT: We allow the user to abbreviate the cues
C      * to three characters; so each word in your cue
C      * name must be unique to three characters. This
C      * means that C1, C2, C3, etc. are three valid cue
C      * names but CON1, CON2, CON3, etc. are not!
C      *
C*****

```

```

C*****
C
C-----
C
C KIPP/GRADY/TAYLOR/CHEN/KUSZMAUL/ENGLMAN/JAEGER/THORNE DAMAGE MODEL
C DATA (CUES(I,4),I=1,12)/'YOUNGS MODULUS','POISSONS RATIO',
C * 'K','FRACTURE TOUGHNESS','RELOAD',
C * 'CP1','CP2','CP3','Y1','Y2','Y3','RMIN'/
C
C-----

```

In subroutine MATINT at step 9 define the new internal state variable names for material model 4 as follows:

```

C
C*****
C      *
C      * STEP 9: SVNAME is the character data which defines
C      * the names of your internal state variables.
C      * Enter appropriate names for your internal
C      * state variables. Please use the same spelling
C      * for conventional quantities (ie EQPS for
C      * equivalent plastic strain or ALPA11 for the
C      * 11 component of back stress) NOTE: only eight
C      * characters are allowed and no blanks are
C      * allowed.
C      *
C*****

```

```

C*****
C
C-----
C
C KIPP/GRADY/TAYLOR/CHEN/KUSZMAUL/ENGLMAN/JAEGER/THORNE DAMAGE MODEL
C DATA (SVNAME(I,4),I=1,6)/'DAMAGE','RMAX','EMAX','CRKDENS',
C * 'EQPS','FRAGSIZE'/
C
C-----

```

In subroutine MATINT at step 12 substitute the following FORTRAN
for the old material model 4 initiation:

```

-----
C
C KIPP/GRADY/TAYLOR/CHEN/KUSZMAUL/ENGLMAN/JAEGER/THORNE DAMAGE MODEL
  ELSE IF( MKIND .EQ. 4 ) THEN
C Check Poisson's ratio:
  IF( PROP(2) .LE. 0.0 .OR. PROP(2) .GE. 0.5 ) THEN
    WRITE( KOUT, 3040 )
    IERR = IERR + 1
    PROP(2) = 0.0
  END IF
C RELOAD:
  PROP(5) = MIN( PROP(5), 1.0 )
  PROP(5) = MAX( PROP(5), 0.0 )
C Bulk modulus:
  PROP(13) = PROP(1) / ( 1.0 - 2.0 * PROP(2) )
C CDCON:
  IF( PROP(3) .LE. 0.0 ) THEN
    PROP(3) = MAX( PROP(4), 1.0E-6 * PROP(13) )
    FUNC4 = (1.0 - 2.0 * PROP(2)) / ((1.0 + PROP(2)) * (1.0 - PROP(2)**2))
    PROP(3) = 0.0032 * FUNC4 * ( PROP(13) / PROP(3) )**6
    WRITE(KOUT,3061) PROP(3)
  END IF
C
  DENOM = PROP(1) * RHO0
  IF( DENOM .NE. 0.0 ) THEN
    CON = PROP(4) * PROP(4) / DENOM
  ELSE
    CON = 0.0
  END IF
C
  PROP(14) = 15.0 * PROP(3) * CON
C
C Lambda:
  XLAM = PROP(1) * PROP(2) / ((1.0 + PROP(2)) * (1.0 - 2.0 * PROP(2)))
C Mu:
  PROP(1) = PROP(1) / ( 2.0 * ( 1.0 + PROP(2) ) )
  TWOMU = 2.0 * PROP(1)
C Lambda plus two mu:
  DATMOD = XLAM + TWOMU
C
C Yield Function Pressures:
  P1 = PROP(6) + PROP(9)/3.0
  P2 = PROP(7) + PROP(10)/3.0
  P3 = PROP(8) + PROP(11)/3.0
C CACON:
  PROP(6) = 0.5 * ABS( 20.0 * CON ) ** (1.0/3.0)
C

```


APPENDIX B

DAMAGE MODEL SUBROUTINE

SUBROUTINE MAT4(NBLK,NINSV,DT,PROP,SIG,D,SV,STRAIN)

```

C
C*****
C
C KIPP/GRADY/TAYLOR/CHEN/KUSZMAUL/ENGLMAN/JAEGER/THORNE DAMAGE MODEL
C
C   A damage model based on micro-crack growth and interaction,
C   with Engleman and Jaeger's extention of Budianski and O'Connell's
C   bulk and shear moduli based on crack density.
C
C   Reloading of damaged material uses a fraction of the damage
C   to reduce the bulk modulus. Loading past dilatancy density
C   or into compression returns to elastic properties.
C
C   Plasticity is used in compression with yield radius a
C   function of volumetric stress, P:
C
C   The linear option,  $Y(P) = YIELD + DYDP \cdot ABS(P)$ , is based
C   on a line through the maximum stress difference for two
C   triaxial tests showing plastic behavior. The zero pressure
C   intercept of this line is YIELD and it's slope is DYDP.
C   DYDP=0 for perfectly plastic behavior.
C
C   The parabolic option,
C    $Y(P) = ( \sqrt{B^2 + 4 \cdot A \cdot (ABS(P) + C)} - B ) / 2 \cdot A$ ,
C   is based on an inverse parabola through the maximum stress
C   difference for three triaxial tests showing plastic behavior;
C    $-P(Y) = A \cdot Y^2 + B \cdot Y + C$ .
C
C   Yield radius is multiplied by 1e12 in tension to avoid plasticity
C
C FORMAL PARAMETERS:
C   NBLK   INTEGER   Number of Elements in this Block
C   NINSV  INTEGER   Number of Internal State Variables
C   DT     REAL      Time Increment
C   PROP   REAL      Material Properties
C   SIG    REAL      Stresses
C   D      REAL      Strain Rates
C   STRAIN REAL      Strains
C   SV     REAL      Internal State Variables:
C                       (1,*) - damage
C                       (2,*) - maximum volumetric strain rate
C                       (3,*) - maximum volumetric strain in compression
C                       (4,*) - crack density
C                       (5,*) - equivalent plastic strain
C                       (6,*) - fragment size rate factor
C
C CALLED BY: UPDSTR
C
C*****
C

```



```

PARAMETER (THIRD=1.0/3.0, TWOTHD=2.0/3.0, S9THS=16.0/9.0 )
C
PARAMETER (NSPC=2,NELNS=4,NESNS=2,NSYMM=4,NASYM=1,NONSYM=5,NHGM=1,
*          NEBLK=512,MFIELD=22)
C      NSPC      INTEGER      Number of Spatial Coordinate Components
C      NELNS      INTEGER      Number of Element Local Nodes
C      NESNS      INTEGER      Number of Element Side Nodes
C      NSYMM      INTEGER      Number of Symmetric Tensor Components
C      NASYM      INTEGER      Number of Asymmetric Tensor Components
C      NONSYM      INTEGER      Number of Non-Symmetric Tensor Components
C      NHGM      INTEGER      Number of Hourglass Modes
C      NEBLK      INTEGER      Number of Elements per Vector Block
C      MFIELD      INTEGER      Number of fields that the free field
C                                reader will allow on each line
C
DIMENSION PROP(*),SIG(NSYMM,NBLK),D(NSYMM,NBLK),SV(NINSV,NBLK),
*          STRAIN(NSYMM,NBLK)
C
F1(X) = ( 1.0 - X * X ) / ( 1.0 - 2.0 * X )
F3(X) = ( 1.0 - 2.0 * X ) / ( 1.0 + X )
C
C Material Properties
C1      = SQRT(TWOTHD)
TWOMU   = PROP(1) * 2.0
XNU     = PROP(2)
YIELD   = PROP(3)
DYDP    = PROP(4)
RELOAD  = PROP(5)
CACON   = PROP(6)
COEFA   = PROP(7)
COEFB   = PROP(8)
COEFC   = PROP(9)
PARAB   = PROP(10)
BLKX3   = PROP(13)
CDCON   = PROP(14) * DT
DT2MU   = DT * TWOMU
BLK3DT  = DT * BLKX3
HDT     = 0.5 * DT
XNUI    = 1.0 / XNU
TERM    = 1.0 / TWOMU
STBLK   = 3.0 * S9THS * BLKX3
C
DO 10 I = 1,NBLK
C
C Volumetric stress and strain at beginning of time step
PF = THIRD * ( SIG(1,I) + SIG(2,I) + SIG(3,I) )
EP = THIRD * ( STRAIN(1,I) + STRAIN(2,I) + STRAIN(3,I) )
C Volumetric and Deviatoric Strain Rates at center of time step
DAVG = THIRD * ( D(1,I) + D(2,I) + D(3,I) )
DDEV1 = D(1,I) - DAVG
DDEV2 = D(2,I) - DAVG
DDEV3 = D(3,I) - DAVG
C Elastic Volumetric Stress Increment
PEINC = BLK3DT * DAVG

```

```

C Update Strains to center of time step
  STRAIN(1,I) = STRAIN(1,I) + HDT * D(1,I)
  STRAIN(2,I) = STRAIN(2,I) + HDT * D(2,I)
  STRAIN(3,I) = STRAIN(3,I) + HDT * D(3,I)
  STRAIN(4,I) = STRAIN(4,I) + HDT * D(4,I)
C Volumetric and Deviatoric Strains at center of time step
  EAVG = THIRD * ( STRAIN(1,I) + STRAIN(2,I) + STRAIN(3,I) )
  EDEV1 = STRAIN(1,I) - EAVG
  EDEV2 = STRAIN(2,I) - EAVG
  EDEV3 = STRAIN(3,I) - EAVG
  EDEV4 = STRAIN(4,I) - EAVG
C Update Strains to end of time step
  STRAIN(1,I) = STRAIN(1,I) + HDT * D(1,I)
  STRAIN(2,I) = STRAIN(2,I) + HDT * D(2,I)
  STRAIN(3,I) = STRAIN(3,I) + HDT * D(3,I)
  STRAIN(4,I) = STRAIN(4,I) + HDT * D(4,I)
C----- Begin Damage Model Calculations -----
C Maximum strain rate
  SV(2,I) = MAX( SV(2,I), DAVG )
  EINV = 1.0 / ( SV(2,I) * SV(2,I) )
C Update Multiplier for Average Fragment Size
  SV(6,I) = CACON * EINV**THIRD
C RFAC = 1: When expanding at center of time step
  RFAC = 0.5 + SIGN( 0.5, DAVG )
C PFAC = 1: When in tension at beginning of time step
  PFAC = 0.5 + SIGN( 0.5, PF )
C Maximum strain in compression to allow for dilatancy
  SV(3,I) = MAX( SV(3,I), ( 1.0 - PFAC ) * EP )
  EEFECD = EAVG - SV(3,I)
C EFAC = 1: When expanded beyond dilatancy strain
  EFAC = 0.5 + SIGN( 0.5, EEFECD )
C Damage can occur only when all three conditions are in effect
  TFAC = EFAC * RFAC * PFAC
C
C Crack Density Increment
  CDINC = TFAC * CDCON * EINV * DAVG * EEFECD**5
C Update Crack Density to end of time step
  SV(4,I) = SV(4,I) + CDINC
C Effective Poisson's ratio at center of time step
C Use elastic Poisson's ratio above dilatancy density and in compression
  XNUB = XNU * EXP( S9THS*( 0.5*CDINC - SV(4,I) )*EFAC*PFAC )
  FUNC3 = F3( XNUB )
C Effective Poisson's ratio at end of time step
  XNUE = XNU * EXP( -S9THS * SV(4,I) )
  FUNC1 = F1( XNUE )
C Damage Increment
  DINC = TFAC * ( FUNC1 * ( 1.0 - XNUE * XNUI ) - SV(1,I) )
C Update Damage to end of time step
  SV(1,I) = SV(1,I) + DINC
C Effective damage at center of time step
  DFAC = 1.0 - ( SV(1,I) - 0.5 * DINC )
C Use elastic properties above dilatancy density and in compression
C Use a fraction of the damage to reduce elastic properties when reloading.
  DFAK = 1.0 - ( SV(1,I) - 0.5 * DINC ) * RELOAD * EFAC * PFAC

```

```

C Volumetric Stress Increment
  PINC = ( DFAC * PEINC - BLKX3 * EEFEC * DINC ) * TFAC
  *
  * + DFAK * PEINC * ( 1.0 - TFAC )
C Increment of deviatoric stress
  DMUDT = STBLK * DFAC * CDINC * XNUB / ( 1.0 + XNUB )**2
  *
  * - BLKX3 * FUNC3 * DINC
  S1INC = TFAC * ( DMUDT * EDEV1 + BLK3DT * DFAC * FUNC3 * DDEV1 )
  *
  * + ( 1.0 - TFAC ) * BLK3DT * DFAK * FUNC3 * DDEV1
  S2INC = TFAC * ( DMUDT * EDEV2 + BLK3DT * DFAC * FUNC3 * DDEV2 )
  *
  * + ( 1.0 - TFAC ) * BLK3DT * DFAK * FUNC3 * DDEV2
  S3INC = TFAC * ( DMUDT * EDEV3 + BLK3DT * DFAC * FUNC3 * DDEV3 )
  *
  * + ( 1.0 - TFAC ) * BLK3DT * DFAK * FUNC3 * DDEV3
  S4INC = TFAC * ( DMUDT * EDEV4 + BLK3DT * DFAC * FUNC3 * D(4,I) )
  *
  * + ( 1.0 - TFAC ) * BLK3DT * DFAK * FUNC3 * D(4,I)
C Total stress at end of time step
  SIG(1,I) = SIG(1,I) + S1INC + PINC
  SIG(2,I) = SIG(2,I) + S2INC + PINC
  SIG(3,I) = SIG(3,I) + S3INC + PINC
  SIG(4,I) = SIG(4,I) + S4INC
C
C Radius of yield surface (Parabolic, Linear or Constant)
  PF = THIRD * ( SIG(1,I) + SIG(2,I) + SIG(3,I) )
  RADIUS = C1 * ( YIELD + DYDP * ABS(PF)
  *
  * + PARAB * ( SQRT( COEFB**2 + 4.0 * COEFA
  *
  * * ( ABS(PF) - COEFC ) ) - COEFB )
  *
  * / ( COEFA + COEFA ) )
C Elastic when in tension
  PFAC = 0.5 + SIGN( 0.5, PF )
  RADIUS = RADIUS * ( 1.0 + PFAC * 1.0E12 )
C Deviatoric Trial Stress
  TRIAL1 = SIG(1,I) - PF
  TRIAL2 = SIG(2,I) - PF
  TRIAL3 = SIG(3,I) - PF
C Magnitude of the Trial Stress
  DSMAG = SQRT( TRIAL1**2 + TRIAL2**2 + TRIAL3**2 + 2.0 * SIG(4,I)**2 )
C Determine the Factor for Plasticity (Zero for Elastic).
  DIFF = DSMAG - RADIUS
  EFAC = 0.5 + SIGN( 0.5, DIFF )
C Determine the addition factor for when DSMAG is Zero.
C Since there is no Plasticity, EFAC will be Zero and ADDFAC One.
  ADDFAC = 1.0 - EFAC
  DSMAG = DSMAG + ADDFAC
C Plastic relaxation factor
  XLAM = EFAC * TERM * DIFF
C Update Stresses for Plasticity
  RFAC = 1.0 - XLAM * TWOMU / DSMAG
  SIG(1,I) = RFAC * TRIAL1 + PF
  SIG(2,I) = RFAC * TRIAL2 + PF
  SIG(3,I) = RFAC * TRIAL3 + PF
  SIG(4,I) = RFAC * SIG(4,I)
C Update equivalent plastic strain.
  SV(5,I) = SV(5,I) + C1 * XLAM
C
10 CONTINUE
C
RETURN
END

```

END

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