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Jim Wilson - Festschrift
Future Directions in Astrophysics

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About the Cover

Portrait of Jim Wilson, LLNL scientist and one of the world's leaders in astrophysics. In the 15 years since he began working in general relativity and numerical astrophysics, Wilson has used LLNL's high-speed computers to calculate the evolution in time of a wide variety of astrophysical phenomena. Jim has also contributed to other fields of astrophysics such as numerical general relativity and the numerical solution of problems in cosmology (the theory of the evolution of the universe). In late October 1982, on his 60th birthday, Wilson was honored by a three-day conference in his name at the University of Illinois, a tradition known in academia as a *festschrift*. The custom is to invite papers by experts who have worked with the honoree in his field. In Jim's case, there were 27 presentations in eight major fields of astrophysics. At Wilson's insistence, the *festschrift* focused not on past work, as is typical of such events, but on the future directions of astrophysics. The articles presented in this special issue of *Energy and Technology Review* were selected to present some of the recent work done by Wilson and his coworkers.

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About the Journal

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Energy and Technology Review

Lawrence Livermore National Laboratory

March 1983

Pioneer of Numerical Astrophysics

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Pioneer of Numerical Astrophysics

Jim Wilson's work in astrophysics has been characterized by his timely and significant contributions to problems of current interest; his work has been directed toward improving our understanding of some of the most puzzling astrophysical objects. He is recognized as one of the leaders in the world of astrophysics.

Scientists at LLNL have contributed to the fields of general relativity and astrophysics over a period of more than 15 years. In much of their work, they have made use of the Laboratory's high-speed computers to obtain numerical solutions to problems of interest.

Jim Wilson began working in general relativity and astrophysics in 1968. He was strongly influenced by the prior work done at LLNL by Stirling Colgate, Mike May, and Richard White. Colgate had investigated models of the explosion of supernovae. May and White did pioneering work in numerical general relativity, developing the first computer code to solve problems in general relativistic hydrodynamics. To obtain a better understanding of supernovae, Wilson began work on two projects. He used the methods of May and White to construct a similar computer code, to which he added the transport of neutrinos, which were thought to play a major role in supernova

explosions. He and his coworkers also started work on the collapse of rotating magnetic stars. Wilson's continuing work in numerical astrophysics and general relativity has made him a leader in these fields. He has used the techniques of numerical physics and mathematics to calculate on LLNL's large high-speed computers the evolution in time of a wide variety of astrophysical phenomena. He has investigated problems in gravitational collapse such as spherically symmetric stars and their explosion as supernovae, rotating and nonrotating supermassive stars (10^6 to 10^8 times the mass of the sun), neutron stars, black holes, and rotating magnetic stars. He has also studied the accretion of interstellar material onto normal stars, neutron stars, and black holes. These black holes can be characterized by their total mass, whether they are rotating or nonrotating, and whether they are electrically charged or not.

Jim has contributed to other fields of astrophysics including numerical general relativity and the numerical solution of problems in cosmology, the theory of the evolution of the universe. His work has centered on two aspects of cosmology: the development of a numerical model to represent the time interval during which general relativistic effects are important, and the study of the formation of galaxies.

This issue of the *Energy and Technology Review* includes some of the papers presented at an astrophysics meeting held to honor Jim Wilson on his 60th birthday (a *festschrift*). At the meeting, held in October 1982 at the University of Illinois at Urbana, 27 papers were presented by Jim's coworkers from the United States and Europe. The articles presented herein cover some of the recent work done by Wilson and his coworkers.

Larry Smarr describes a program in which he and John Hawley have been calculating the effects of material falling into the curved space-time surrounding a rotating black hole. The large curvature of space-time can cause very complicated flows of material. Both radiation and matter could be emitted from the region outside of but close to the black hole. They have collaborated with Jim Wilson in developing a two-dimensional, general-relativistic hydrodynamics code to simulate fluid flow in the gravitational field of a rotating black hole. Such calculations represent models that have been proposed for the energy sources of both quasars and jets from radiogalaxies, which are some of the most spectacular objects that have been observed in the universe, but still not well understood. In each of these cases, the black hole that powers the quasar or jet would have a mass of about 100 million times the mass of the sun. The black hole would be located in the center of a galaxy whose total mass is 1000 times greater than the black hole mass.

The article by Wilson reports on some recent calculations of a model of supernovae explosions. Jim's first work in astrophysics was the development of a code to calculate the explosions of supernovae; his work on this problem has continued to the present time. A supernova is a star that undergoes a violent explosion, during which it radiates energy at a rate equal to that of 100 billion suns. The radiative power of a single supernova is equal to that of an entire galaxy.

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The intensity of this radiation reaches a peak in about the first week after the explosion, then decays, with a half life of about 50 days. The Crab nebula and pulsar are the remnants of a supernova explosion. The nebula is the luminous matter ejected from the star during the explosion, and the pulsar is the remnant of the star itself.

Wilson postulates that a supernova is produced by the gravitational collapse of a star after it has burned all of its thermonuclear material. The end product of the burn is iron; thus, just before the star collapses, it has a central core composed of iron. During the collapse, the energy released by the gravitational field is used to decompose the iron into

lighter nuclei. Finally, the core consists of hot neutrinos, electrons, protons, neutrons, and helium nuclei. The pressure at the center of the star becomes sufficiently high to stop the collapse suddenly and to produce an outwardly directed shock wave. It is thought that this shock wave may be strong enough to eject the outer material of the star.

Numerical calculations made over the past 15 years have not successfully represented the supernova explosion. The complexity of the physics model in the codes has increased enormously, and our ideas of what drives the actual explosion have changed. Even so, as of early 1982, the numerical calculations still did not produce explosions. In his

article, Wilson describes his most recent calculations, which have resulted in a significant modification in our understanding of how some supernovae explode.

The article on magnetorotational collapse of an iron core by Eugene Symbalisty reports a study of the effects of rotation and magnetic fields on the gravitational collapse of the central iron core of old stars. In contrast to our sun, which has a small magnetic field and a small rotational velocity, most stars rotate much more rapidly and some stars have intense magnetic fields. There is little understanding of the evolution and gravitational collapse of such stars. The work Symbalisty describes is an investigation of the collapse of a star with a very high rotational rate and a very strong magnetic field. The objective of his calculations is to see whether, in this exceptional circumstance, neutron-rich matter could be ejected in an amount sufficient to account for the production of the neutron-rich isotopes much heavier than iron, called the r-process elements.

The work reported by Charles Evans is the development of a nonrotating, axisymmetric, general-relativistic hydrodynamics code. The effects of rotation are ignored in this code because their inclusion would greatly increase the code's complexity. Evans' approach in this code has been to develop numerical methods that allow the calculation of accurate solutions to nonspherical general-relativistic hydrodynamics problems when the relativistic effects are strong.

The code is a tool that can be used to solve a large array of interesting dynamic relativistic problems that have so far eluded solution. In one class of such problems, no matter is present. Two examples are the formation of a black hole by gravitational radiation and the interaction of gravitational waves with a black hole. In the class of problems that contain matter, the focus is on cases in which the distribution of matter is nonspherical. Examples are gravitational collapse, oscillation, and the formation of black holes. Here, the concern is to calculate the production of

gravitational radiation from such objects. This project is still in a developmental stage.

The article by Bob Barton describes a method he has used to determine how to organize and produce a large astrophysics code that can be executed on modern high-speed computers. Modern computers have a range of speeds for various types of numerical and logical operations. Fortran compilers with vector extensions supply a convenient tool to make use of these operations in a relatively easy way. Barton describes a code that is easy to understand, takes advantage of the fast vector operations of the computer, and is conservative in its use of the computer's memory capacity.

The article by Joan Centrella and Jim Wilson on numerical cosmology describes calculations that represent a recently proposed model for the formation of galaxies. The question of how and why galaxies form is one of the great unsolved problems of cosmology. The remarkable isotropy of the 3-K radiation (the cosmic background radiation field) implies that the material that coupled to the radiation field was in an extremely homogeneous spatial distribution at the time of decoupling. The difficulty facing scientists trying to understand galaxy formation is how this homogeneous matter evolves into the discrete structure of galaxies that exist at the present time. Centrella and Wilson discuss the model and then present numerical results of calculations that examine the evolution of perturbations on a cosmic scale and the subsequent coalescence of matter to form an individual galaxy.

In addition to the work presented in this issue of the *Energy and Technology Review*, scientists from LLNL gave three other papers at the *festschrift*. Tom Weaver presented results of calculations of the evolution of stars to the presupernova stage. Carl Lindl gave a presentation on numerical techniques used in the solution of the neutrino transport equation. The final discussion of the meeting, on the future of numerical astrophysics, was given by James Le Blanc. ■

Black H The Quas

Conventional schemes are inadequate to account for either the intrinsic brightness or the rapid variability of quasars. Computer simulations enable us to model these objects, revealing possible mechanisms to explain their tremendous energy output.

With the advent of radio telescopes in the early 1950s, astronomers began to detect many new classes of objects that conventional astronomical theories could not explain. Among the more intriguing are the quasars, extremely distant objects that radiate more energy than a dozen ordinary galaxies and yet must be relatively tiny (about the size of our solar system) because the intensity of their radiation can vary so rapidly (within hours).

Imagining what sort of power source could possibly be driving this enormous outpouring of radiation has been a continuing challenge. Part of the difficulty is observational—there are no nearby quasars and it is impossible to observe directly the central regions of any galaxy, including our own. Theoretical difficulties arise from the fact that evidently there are phenomena in progress that depend on conditions that are beyond our experience.

We can be sure, however, that the nuclear energy that heats our sun and all other common stars is totally inadequate to power a quasar. The energy radiated from some quasars over their estimated

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Black Hole Accretion: A Stellar Powerhouse

lifetime is equivalent to the total annihilation (conversion of mass into energy) of 10 million stars the size of our sun. We know that the nuclear processes in a star convert no more than 0.8% of its mass into energy, suggesting that a quasar powered by nuclear energy would involve the burning of more than a billion suns.

As it happens, there is a potentially far more efficient means of releasing energy, one that was originally suggested as the sun's power source, known as gravitational contraction. A gradual decrease in the sun's radius could provide enough energy to maintain the solar luminosity, but only for about 10 million years. For a body as small as the sun, the nuclear process proves to be more efficient; for a body with enough mass, gravity must eventually win out.

Our understanding of gravity as an energy source is hampered by the complexity of the mathematics that governs it. Astronomers using analytic techniques (mathematical manipulations that can be performed without a computer) now have a qualitative picture of the gravity power process, but they can only estimate such details as the actual net efficiency. Analytic techniques are restricted to the most simplified models of gravitational systems. Clearly then, to proceed further, we must use both computers and numerical calculations.

The most efficient gravity engine we can imagine is a black hole fueled by an external supply of matter provided by a companion star or surrounding interstellar gas. We can imagine the intense tidal forces near a supermassive black hole ripping apart entire stars, whose matter would then be drawn in by the black hole. In falling toward the black hole, this matter would acquire tremendous amounts of kinetic energy. The outstanding astrophysics problem in applying this concept to a quasar is to describe how this kinetic energy is transformed and radiated.

In the basic analytic scenario, matter orbits a black hole in a thin Keplerian disk similar to the rings of Saturn. Parts of such a ring near the center rotate much faster than those farther out; viscous shear forces between these differentially rotating layers cause a net outward transport of momentum, heating the gas as it gradually spirals inward. This heat is promptly radiated away at the surface of the disk.

Depending on the angular momentum of the black hole itself, this process could release between 5 and 40% of the rest-mass energy of the ring material, and the faster we supply material (up to a point), the higher the luminosity. The limit to this process is the Eddington luminosity, at which radiation pressure is enough to blow matter out of the

black hole's gravitational field and disrupt the disk. If we assume that quasars radiate at the Eddington limit, it would take a black hole a hundred million times as massive as the sun to power just one quasar.

At first, this analytic picture appears to be just what we need to explain the observed superluminous objects. However, it is highly oversimplified. For one thing, it is incapable of describing exactly how accreting matter will behave as it falls in toward the black hole. Velocities in this situation approach the speed of light, and the gravitational field becomes so intense that space and time are highly warped. The description of the physics can come only from a solution of the full general-relativistic equation of hydrodynamics in the gravitational field of the black hole, a solution that is simply beyond the reach of analytic dynamic calculations.

Fortunately, computers can produce numerical answers to even the most complicated set of equations without the need for an analytic solution. We have collaborated with Jim Wilson of LLNL to develop a two-dimensional, general-relativistic hydrodynamics code to simulate fluid flow in the gravitational field of a rotating black hole. We assume axial symmetry, an ideal-gas equation of state, and an unvarying gravitational field (i.e., we consider the mass of the material falling into the black hole to be negligible in comparison with that of the black hole itself).

Jim Wilson performed the first such numerical study of black-hole accretion in 1971.¹ He had made considerable progress in generalizing existing one-dimensional, general relativistic codes, using them to study stellar collapse and supernovae. Drawing on this experience, he designed a two-dimensional code to calculate axisymmetric hydrodynamic flows in the gravitational field of a black hole.

One new effect that he immediately discovered was that shock heating can drastically alter the structure of the flow of infalling cold material that has angular momentum with respect to the black hole. (Material without angular momentum falls directly in and vanishes. It is angular momentum that permits material

to orbit the black hole for a time). Pressure generated by a standing shock wave balloons the thin disk into a non-Keplerian orbiting torus (Fig. 1). Unfortunately, this pioneering numerical work went largely unnoticed.

Several years later, the toroidal disk was rediscovered and studied analytically. One of its most important qualitative features is a completely relativistic effect: accretion from such a torus can take place without viscosity.² (In a purely Newtonian gravitational field, viscosity is necessary to dissipate angular momentum and cause matter to spiral in.)

The reason for this inviscid (nonviscous) accretion is the fundamental difference between the gravitational potential around a black hole and the strict inverse-square law potential around a Newtonian object such as the Earth. To orbit the Earth, a body need have only enough angular momentum to stay above the atmosphere; satellites can orbit essentially forever only a few hundred miles above the surface.

Near a black hole, however, the gravitational field becomes so great that a body would need an orbital velocity greater than that of light. Relativistic effects thereupon would make the gravitational acceleration overcome centrifugal force, trapping the body in an inward spiral. Under these circumstances, the radius of the innermost stable circular orbit is about three times the radius to the black hole's event horizon (the surface at which the escape velocity becomes greater than the speed of light, so that not even light can get away).

Another way of describing this situation is to say that there is a pit in the center of the potential around a black hole,³ whereas the potential around a Newtonian object increases to infinity as the orbital radius decreases to zero. At the point where the gravitational acceleration begins to overcome centrifugal force, the potential "turns over" (stops increasing) and falls to zero (Fig. 2).

A circular orbit is a special case, of course, the orbit of minimum eccentricity. In a purely Newtonian system of

orbits around an infinitesimal point, there are infinitely many elongated orbits, and a body with any angular momentum at all (and less than escape velocity) will always follow a closed elliptical orbit, approaching the source of the gravitational field but never reaching it. In Fig. 2, these are represented by the horizontal line (between the colored dots denoting stable circular orbits) which intersects the curve of potential in two places (turning points).

The existence of an innermost stable circular orbit around a black hole implies an associated minimum angular momentum, which we will call l_{ms} (marginally stable, see Fig. 2). It also implies another limiting angular momentum, l_{mb} (material in eccentric orbits). This is the smallest angular momentum a bound particle (one with an ratio of energy to rest mass of 1 or less) can have. Any body with $l < l_{mb}$ that falls from rest at infinity must be captured by the hole. We find that l_{mb} is larger than l_{ms} . For a Schwarzschild (nonrotating) black hole, l_{mb} is 1.16 times l_{ms} .

Figure 3 presents a series of contour maps showing equipotential surfaces for material orbiting a black hole (derived from the analytical curves in Fig. 2) for various values of angular momentum.⁴ In much the same way that a geographical contour map shows the local slope of the land, these graphs show the local acceleration due to the combined gravitational and centrifugal forces. This acceleration can be countered by a pressure force, simply by filling the closed potentials with hot gas.

In equilibrium, pressure contours in the gas coincide with the potential contours shown. The pressure maximum will be at the "lowest point" in the potential (i.e., the center of the closed contours), which coincides with the point where the Keplerian orbit of a test body of the same angular momentum would be. By comparing the various maps, we can visualize how a change in the angular momentum changes the equipotential surfaces and consequently the expected behavior of the gas.

In Fig. 3a, for example, the angular momentum is less than l_{ms} and, hence, there are no closed contours. All the equipotential surfaces end on the black hole. As a result, fluid falling toward the hole in this condition goes directly in without having to lose any angular momentum. This provides a new and quite plausible description for the final dynamic infall at the inner edge of the accretion torus as well as a description of the complete infall process for matter that arrives with insufficient angular momentum.

In general, however, fluid falling toward the black hole will have a large angular momentum and will orbit until some of its kinetic energy is dissipated by viscosity. That portion of the energy that is not immediately radiated away will instead go into heating the orbiting material. The resulting pressure will puff the gas away from the equatorial plane of its orbit to fill the

Fig. 1

Early computer simulation of conditions around a black hole. Infalling cold fluid (gas), arriving with some angular momentum, is heated by hydrodynamic shocks and forms a toroidal ring around the black hole's equator (red area). (Analytic calculations of the time pictured the material forming a thin equatorial disk, like Saturn's rings.) The black hole in this model rotates (about the z axis) with its maximum possible angular momentum. (From Ref. 1.)

Fig. 2

The effective potential for various angular momenta (indicated) as a function of radius along the equator for a Schwarzschild (nonrotating) black hole. For a nonrelativistic situation, the potential goes to infinity as we approach the center of mass. Around a black hole, however, relativistic effects take over, reducing the potential to zero and trapping infalling material in an ever descending spiral. (Material rotates about the z axis.) Stable circular orbits are located at the dots (gold). The path of an eccentric orbit is indicated by the horizontal line (blue).

equipotential surfaces pictured in Figs. 3b to 3d, forming a pressure-supported torus from initially cold gas. In Fig. 4, we show a computer-generated cross section of such a toroidal disk orbiting a black hole.⁵

At the same time, angular momentum transported outward in the torus by viscous drag decreases the angular momentum of the material at the inner edge until it goes below l_{ms} , at which point it falls inward without further

energy extraction. This tends to reduce the net efficiency of the energy release because it widens the region over which viscous drag ceases to heat the gas and make it radiate (which also coincides with the region of maximum gravitational attraction). On the plus side, however, the toroidal structure can support a much larger luminosity than the thin disk (up to 100 times as much, according to some investigators).⁶ Thus, although the torus may be

Fig. 3

Contour maps of constant total potential (due to combined gravitational and centrifugal accelerations) around a black hole, for various values of the infalling material's angular momentum l (material rotates about the z axis). (a) For l less than l_{ms} , potentials are open to the hole and fluid can flow directly in. (b) For l equal to l_{ms} , a stable circular orbit is possible, but material in eccentric orbits goes directly into the hole. (c) For l equal to l_{mb} , both circular and eccentric orbits are possible, and material can spread out into an accretion torus. (d) For l greater than l_{mb} , all bound orbits remain stable in the absence of viscous dissipation. (e) View of (d) in three dimensions. (From Ref. 4.)

less efficient than a thin disk, it may be considerably brighter, given a large enough accretion rate.

Clearly, these new analytic scenarios can have important observational consequences. They cannot, however, predict exactly what flows are actually possible and in what manner they might proceed. How does flow vary with angular momentum? Do structures resembling those described in the equilibrium analytic theory really develop in hydrodynamic situations? To approach these questions in the greater generality permitted by numerical (computer) analysis, we expanded on Wilson's original 1971 study, with his collaboration.

In this work, we used a time-explicit, fixed-grid Eulerian accretion code with monotonic transport. To model accretion-disk formation under various conditions, we introduce fluid on the outer boundary of the finite-difference grid with some value of angular momentum and allow it to flow into the grid. We are able to present several examples of calculated inviscid fluid flow close to the black hole.

We first consider the flow of a fluid with angular momentum less than l_{ms} (Fig. 3a), one that should fall directly into the black hole. The computer simulation shows this happening, of course, but it also shows much more. If we continuously resupply the outer boundary of the problem with fresh fluid, for example, the simulation rapidly establishes a steady-state nonradial flow that in many ways resembles the transonic solution for radial accretion of a hot ideal gas (the Bondi critical-point solution).

Furthermore, if we stop the inflowing gas at the outer boundary after this flow is established, we find that the fluid drains into the black hole in about the same time it would take a free-falling body with no angular momentum to do the same. Figure 5 consists of two graphs that show density contours and flow directions at two different times for this model. (For the particular value of angular momentum we used, the fluid at the indicated radius had time to complete 2.4 orbits around the

black hole between Figs. 5a and 5b.) Thus our calculations verify that, for low angular momentum, fluid spirals rapidly into the hole even without viscous drag.

Fig. 4

A full two-dimensional picture of a pressure-supported torus orbiting a black hole. The vacuum region at the hole axis is the "funnel," which may be important for collimating outgoing radiation. This illustration shows all four quadrants; subsequent model illustrations show only the upper right quadrant.

Fig. 5

Computer simulation showing density contours and flow directions for material falling into a black hole without enough angular momentum to find a stable orbit, shown at two different times. (a) An early stage of the process. (b) The same material after it has circled the black hole a few times.

By increasing the angular momentum slightly to a value between l_{ms} and l_{mb} (between Figs. 3b and 3c), we can obtain an entirely different type of behavior from our inflow model (Fig. 6). An angular momentum in this range permits a stable circular orbit and, up to a certain energy, stable eccentric orbits as well.

In the model, a portion of the fluid "bounces" as it hits a turning point between the radii for stable and unstable circular orbits. Infalling fluid begins to fill up the available volume and gradually forms a torus with a pressure maximum at the predicted radius. A compression wave moves outward and shuts off the inflow at the outer boundary, allowing the density in the torus to drop as fluid continues to drain into the black hole.

This drop in density permits material to flow in again, and the process

repeats itself over and over, though with diminishing amplitude. The torus, once formed, remains, and its maximum density fluctuates, but the rate of inflow into the hole remains relatively steady (Fig. 7).

It would be a mistake to attach too much significance to the inflow oscillations at the outer boundary, since these probably reflect conditions of the calculation (the calculational discontinuity at the outer boundary, for example) rather than a likely physical phenomenon. However, the relative constancy of the flow into the hole is suggestive. It is quite possible that the final inflow mechanism can support only a certain maximum flow for a given combination of angular momentum and fluid temperature, much like the flow in a nozzle. Since the rate at which material arrives from outside is not so limited, but may vary from nothing to very

Fig. 6

Computer simulation showing density contours and flow directions at several stages of the development of an accretion zone for material falling into a black hole with slightly more angular momentum than that needed for a marginally stable circular orbit. (a) The material first enters. (b) The material funnels inward, forming a pressure concentration. (c) An eddy develops and there is an outward-traveling compression wave. (d) The compression wave reaches the outer boundary of the simulation, temporarily blocking the inflow of more material and allowing the continual draining into the hole to reduce the pressure in the accretion torus.

large values, it is unlikely to match exactly the capacity of the "nozzle." Thus a smooth, steady-state flow may be an ideal rarely attained, and in general the flow may well be completely dynamic and time-dependent.

Finally, there is the especially simple case in which the angular momentum exceeds l_{mb} (Fig. 3d), and we can completely specify, analytically, a stationary axisymmetric toroidal accretion structure. Figure 8 shows the inner portion of such a torus. For this case, some viscosity would be necessary to redistribute the angular momentum and permit accretion.

In addition to being an interesting solution of the hydrodynamic equations, this case also serves the practical purpose of testing the code. To maintain such a stationary solution with a finite-difference code, all of the computed accelerations in both dimensions must repeatedly cancel. Such a test, combined with an ensemble of others, serves to calibrate the code and permit a proper evaluation of the results.

Conclusion

From this work, we can conclude that the dynamic flow of a hot fluid with angular momentum tends naturally to form a toroidal structure around a black hole and that, once the angular momentum has dropped below l_{mb} , the final accretion into the hole is probably precipitous rather than depending on the slow effects of viscous drag. In future work, we plan to incorporate magnetohydrodynamic equations into the model to study the important effects of magnetic fields on the infalling material. Radiation and viscosity are also of obvious importance, and work to include terms dealing with those phenomena is underway.

This work is but the beginning of attempts to understand gravity power, and many of our results are preliminary. The advent of supercomputers and advanced numerical hydrodynamic techniques has finally made possible the necessary studies. There is every reason to expect that our understanding of these gravity-driven energy sources will soon increase manifold. ■

Fig. 7

Mass accretion rate as a function of time for the model pictured in Fig. 6. The inflow at the outer boundary oscillates in response to dynamic changes within the accretion volume. The inflow to the hole remains constant; the "potential spout" allows only a certain flow rate, much like a nozzle.

Fig. 8

Density contours and flow directions when the infalling material has enough angular momentum to form a stable, pressure-supported torus that remains in equilibrium (disregarding dissipative forces such as viscosity). This case also provides a stringent test of the computer code, since such a stable solution requires that all the calculated accelerations in both dimensions, for every node in the calculational grid, repeatedly cancel one another.

Key Words: analytic calculation; black hole; computer simulation; Eulerian finite-difference calculation; magnetohydrodynamics; quasar; relativistic correction.

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Does Stellar Collapse Produce Supernovae or Black Holes?

New calculations indicate that the collapse of the iron core of a massive star leads to the explosion of the outer parts of the star. Although these results are encouraging, the energy of the explosion is five to ten times less than that observed in supernova explosions.

For more than 15 years, LLNL has been using computers to model the evolution of exploding stars. Three years ago, *Energy and Technology Review* reported some of our results.¹ At that time, we noted that because some of the underlying physics was poorly understood, our computer model could not be considered definitive. Among this physics was the equation of state of hot nuclear matter just above nuclear density. Subsequent work has led to an improved version of the equation of state, which is now regarded by workers in the field as well understood. Because models are sensitive to small

changes in input physics and methods of numerical calculation, improvements in the physics can produce qualitative changes in the evolutionary processes they describe. In our original article, we noted that changes in some of the physics can convert a stellar model that explodes into one that does not. It now appears that the latter model describes stellar evolution more accurately, a result that raises further intriguing issues.

A supernova of the type we are considering (Type II) is thought to represent the explosion of a massive star at the end of its nuclear-burning stage, the period in which lighter elements in

the star's interior undergo thermonuclear fusion to produce iron. (Because iron is the most tightly bound nucleus in nature, no further energy release by fusion is possible.) Stars with a mass of about 10 to 30 $M_{\odot}$ (where $M_{\odot}$ is the mass of the sun, equal to 2×10^{33} g) burn their inner 1.5 $M_{\odot}$ of material to iron in a quiet manner; that is, during nuclear burning, the star is always in approximate hydrostatic equilibrium. When all the available matter is burned, the remaining iron core then cools.

As the core cools, the pressures (produced by the electrons) that supported it against gravitational forces weaken. The outcome is sudden: gravitational collapse, a considerable fraction of the core falling inward at supersonic speeds. The collapse is halted by a sharp rise in pressure when the density of the central region exceeds the density of nuclear matter (2×10^{14} g/cm³), which is nearly incompressible. The abrupt termination of the collapse produces an elastic rebound of the inner core (known as core bounce) and a subsequent shock wave that propagates outward through the infalling matter. If the rebound is large enough and the shock wave is not dissipated by neutrino processes in the stellar envelope, it will continue to propagate outward, eventually blowing off the outer layers of the star. The explosive disruption that ensues can be identified with what we observe as a supernova event.

Stellar Collapse Revisited

The foregoing model of stellar evolution, which seemed plausible at the time of our earlier article, has now been modified by calculations based on a better understanding of the equation of state of hot nuclear matter. Our new calculations produce only a very small core bounce; as a result, the subsequent shock wave is not strong enough to disrupt the outer parts of the star immediately. Rather than rebounding, the inner part of the stellar core now comes to a smooth halt at the density of nuclear matter, forming a hot neutron star. (A neutron star consists of matter at or above nuclear density with most

of its baryons in the neutron state.) Because the outer portions of the star continue to fall in at supersonic speeds, an outward-moving shock wave forms at the point where the infalling matter impacts on the high-density core (see the box on p. 18).

In its early stages, much of the energy of the shock wave is dissipated. Because the temperatures behind the shock are so high, an appreciable fraction of the kinetic energy of the infalling matter is converted by the shock wave into internal energy and then is radiated in the form of neutrinos. An additional large amount of energy is absorbed by iron nuclei that dissociate into protons and neutrons and becomes unavailable for pressure forces. The net result is that matter continues to fall onto the hot neutron star, slowly accumulating around its core.

Figure 1 shows the evolution, around bounce time, of the 2- $M_{\odot}$ core of a 15- $M_{\odot}$ collapsing star during its transition to a hot neutron star. Material falling into the core is traveling inward much faster than the outward-moving shock wave. (In our 1980 calculation, the shock wave moved out with a speed equal to or greater than that of infalling matter.) As the shock wave moves out, the density of infalling matter it encounters decreases since the density of the precollapse configuration falls off rapidly with increasing radius. Further, the accreting matter is falling into a less deep gravitational potential, which lowers its post-shock temperature. Neutrino emission is very temperature sensitive. For the long neutrino mean free path in the region just behind the shock at late times, the rate of cooling is proportional to the sixth power

Fig. 1

Evolution of the inner $2.2\text{-}M_{\odot}$ core of a $15\text{-}M_{\odot}$ star near the time it collapses to form a hot neutron star. Each line represents the trajectory of an observer placed initially at a given radius (in kilometres) from the star's center and moving with the stellar material as the star evolves. The mass inside each line is given in multiples of one solar mass ($M_{\odot} = 2 \times 10^{33}$ g) contained within a sphere whose instantaneous radius coincides with the observer's frame of reference. The dark red line indicates the extent of the photosphere, which is the radius at which on the average a neutrino has its last interaction with matter. The dark green line indicates the shock wave produced by stellar collapse. The colored bands show the dominant material in a region: free neutrons and protons (blue), iron (red), oxygen and carbon (green), and silicon (yellow). For the time shown, no region has more than 50% helium. The inner region containing neutrons and protons is relatively cool (about 10 MeV) and too dense to tolerate individual nuclei. At 0.29 s, this region is composed of 69% neutrons and 31% protons. The outer region of neutrons and protons results from iron that has been decomposed by shock heating. As neutrinos are emitted by the inner core, it becomes enriched in neutrons, forming a neutron star.

of the temperature. This means that thermal energy losses behind the shock drop rapidly as it moves out.

While the shock wave continues to propagate outward, the hot neutron core begins to shrink under its own gravitational forces, radiating energetic neutrinos at a high rate (about 10^{53} erg/s). As the core continues to contract, however, its temperature again rises, generating pressures that serve to support it in the stronger gravitational field. As the temperature rises, the rate at which neutrinos are emitted from the core declines but their energy increases. This energy increase plays a crucial role in the model, as we will see in a moment, because the strength with which a neutrino interacts with matter is proportional to the square of the neutrino's energy.

A hot neutron star radiates its thermal energy in a few seconds. During this time, the rate of energy loss by neutrino emission from the hot core is almost constant. This energy becomes available to heat the outer portions of the star. Should the material encountered by the radiated neutrinos on their outward journey be too hot, however, this material will undergo net cooling. When the outward-moving shock wave has reached a point (about 300 km) where the temperature of the material behind it has cooled sufficiently by thermal emission, the high-energy neutrinos (which, as we have noted, interact strongly with matter) then begin to heat it. This additional heating may or may not accelerate the shock wave sufficiently to produce an explosion.

A Site for Nucleosynthesis?

The propagation of a shock wave in a star is a complex process that may be affected by factors other than neutrino production. Among these are the rate at which density falls off with radius and the effects of nuclear burn and decomposition in the regions through which the shock passes. (The latter process can add or subtract energy in the shock wave.) Figure 2 shows the behavior of the shock front in a $10\text{-}M_{\odot}$ stellar model, in which density falls off more steeply with radius than in the $15\text{-}M_{\odot}$

Fig. 2

Evolution of the inner $1.5\text{-}M_{\odot}$ core of a $10\text{-}M_{\odot}$ star from the time just before collapse until an outward motion of matter is evident. Color keying is the same as in Fig. 1, with the addition of helium (orange). As in Fig. 1, the mass inside each line is given in multiples of one solar mass ($M_{\odot} = 2 \times 10^{33}$ g). At a radius of few hundred kilometres, iron and silicon falling through the shock front are decomposed into either helium or neutrons and protons. Later, as this matter moves out behind the shock front, it recombines to form first helium and then iron. This process releases a considerable amount of latent heat that helps drive the explosion. Farther out, silicon and oxygen are burned to iron as they pass through the shock front.

model (discussed above). Because of its lower density, this model is more likely to produce an explosion.

As thermal energy accumulates behind the shock wave, it slowly accelerates, moving into material ahead of it (Fig. 3). When the shock advances to a point beyond the region of neutrino heating (between about 300 and 1000 km), the production of thermal

energy for the explosion stops (Fig. 4). (Heating by the core neutrinos falls off as the inverse of the radius squared.) The total energy generated by such an explosion is rather small by supernova standards, 1.5×10^{50} erg vs 10^{51} erg estimated from observations of supernova explosions. The total amount of energy emitted in neutrinos from the cooling neutron star is about 3×10^{53} erg. The

Fig. 3

Instantaneous structure of the $10-M_{\odot}$ star shown in Fig. 2 at 0.29 s, when energy deposition in the region between 300 and 1000 km is at maximum. Red curves show the fractional percent composition of matter (right-hand scale) as a function of radius. Dark green and blue curves show the velocity (in 10^3 km/s) of inward- and outward-bound matter, respectively. Orange and light green curves show temperature and density, respectively. The gold band at the left indicates the upper extent of the photosphere. The sharp drop in the curves toward the right is the shock front.

efficiency of the explosion process—the ratio of the explosion energy to the total energy emitted by the forming neutron star—is thus very low, about 0.05%.

Although the $10\text{-}M_{\odot}$ model produces an explosion, it does not provide a plausible site for the nucleosynthesis of heavy elements.² The density of matter in the oxygen region (see Fig. 4) is so

low that the shock wave cannot heat it to its burn temperature, and most of the shock's energy resides in electromagnetic radiation. Nucleosynthesis requires explosions in more massive stars.

To study this process in more massive stars, we have carried the calculation of a $15\text{-}M_{\odot}$ star (whose behavior near bounce is shown in Fig. 1) to late times. In this case, the density of matter

Fig. 4

Instantaneous structure of the $10\text{-}M_{\odot}$ star shown in Figs. 2 and 3 at 0.51 s, when energy deposition has become negligible. Color keying is the same as in Fig. 3. Because heating of the shock wave is no longer strong enough to burn the silicon passing through its front, the radius of the silicon region is less than that of the shock wave. The density of matter falling into the shock wave is so low at this time that the bulk of the shock wave's thermal energy is emitted as electromagnetic radiation.

outside the initially collapsing iron core is much greater, offering more resistance to the outward-moving shock wave. As a result, the star reaches a configuration dominated by heating only a half-second after the hot neutron star is formed (Fig. 5). In this model, the material behind the shock wave begins to push it outward at a velocity that should lead to a weak

explosion only very late in the process. Very recently, the 15- $M_{\odot}$ and 25- $M_{\odot}$ stellar models have been run on our computer code to very late times—0.75 s after bounce—and strong outward shocks develop with escape energies of 3.5×10^{50} erg and 4.5×10^{50} erg, respectively. These shocks originate by heating in a manner similar to the 10- $M_{\odot}$ model, but the

Stellar Collapse

Evolutionary sequence of core collapse of a massive star. The lengths of the arrows indicate relative velocities. (a) At 0.025 s after collapse begins, matter is falling inward. (b) At 0.05 s, the core reaches the density of nuclear matter and comes to a smooth halt, forming a hot neutron star while infalling matter impacts on its surface. (c) At 0.10 s, a shock wave forms and propagates outward

through the infalling matter. (d) At 0.30 s, the core is undergoing further shrinkage by radiating energetic neutrin^os, which begin to heat and accelerate the matter behind the shock wave. (e) At 0.60 s after collapse begins, the shock wave has propagated through the outer regions of the star; if the shock wave is strong enough, it causes an explosion.

way the star evolves into a configuration in which heating can occur is much more complicated. As can be seen in Fig. 5, the outward progress of the shock wave is hesitant. Should an explosion occur under these conditions, it would provide a feasible site for nucleosynthesis.

Supernovae or Black Holes?

What have we learned from these new calculations? Although our current computer model does indeed produce a weak stellar explosion, the results should be treated with caution. Neutrino interactions that produce net heating are represented in our current model by a simplified version of the full equations. Such interactions have a complex dependence on temperature, density, and composition. Before our calculations can be considered valid, it will be necessary to incorporate more of these complexities into the model. The result could be an explosion that is stronger, or perhaps weaker, than currently predicted. Beware! Should a mechanism for turning collapsing stars into supernovae prove elusive, the universe of supernovae modelers may become populated with black holes.

Summary

The Laboratory has been attempting to model supernovae on computers for more than 15 years. We have pursued the hypothesis that a supernova is related to the collapse of a massive star after its fuel is depleted. Our new calculations of stellar collapse, using an improved equation of state of nuclear matter, produce explosions whose energy is slightly too low to explain most observed supernova. One stellar model, however, may provide a feasible site for nucleosynthesis. Further study of the evolutionary behavior of massive stars will require a more detailed treatment of neutrino interactions that contribute to heating the shock wave. ■

Key Words: black hole; neutrino heating; neutron star; stellar collapse; supernova.

Fig. 5

Evolution of the inner $2.2\text{-}M_{\odot}$ core of the $15\text{-}M_{\odot}$ stellar model shown in Fig. 1 from the time just before collapse to 0.9 s after collapse. Color keying is the same as in Fig. 1, with the addition of helium (orange). The motion of the shock wave appears more complicated than in the $10\text{-}M_{\odot}$ model. The shock begins to move out rapidly only after 0.8 s ; the matter behind it is still almost stationary. This matter becomes unbound by the end of the computation, when its internal energy is greater than its potential energy. The density of matter in the 1000-km region is much higher than in the $10\text{-}M_{\odot}$ model. As a result, the line separating the iron and silicon regions crosses mass-point trajectories because the infalling silicon has been heated sufficiently by compression to burn it to iron. Adiabatic heating of infalling oxygen by the convergence rise in density causes it to burn before it encounters the shock wave.

Notes and References

1. *Energy and Technology Review* (UCRL-52000-80-2), February 1980, p. 17.
2. *Energy and Technology Review* (UCRL-52000-80-2), February 1980, p. 1.

Magnetorotational Collapse of a Stellar Core

Stellar-collapse models with high rotation rates and very strong magnetic fields reveal a possible astrophysical site for the nucleosynthesis of elements heavier than iron.

According to current astrophysical theory, the core of a massive star undergoes rapid collapse during its final evolutionary stages, thereby triggering a sequence of catastrophic events that results either in a Type-II supernova explosion, leaving a remnant neutron star, or in a black hole. Such stars typically have a total mass between 10 and 25 $M_{\odot}$ (where $M_{\odot}$ is the mass of the sun, equal to 2×10^{33} g) and an inner core of about 2.5 $M_{\odot}$. Stellar collapse occurs at the end of the core's nuclear-burning stage, when thermonuclear fusion has consumed the lighter elements present. At this stage, the core is composed of iron.

Since the time it was first recognized that all elements heavier than boron are synthesized by stellar processes, astrophysicists have been searching for the site of the nucleosynthesis of neutron-rich elements much heavier than iron. Such elements are formed in a star when an iron nucleus undergoes a successive process of rapid

neutron capture and beta emission. Although the physics of this sequence of events, known as the r process, has been extensively studied and is now well understood, its actual stellar site remains a tantalizing puzzle.¹

The most sophisticated models of stellar evolution assume spherical symmetry and include a detailed equation of state for the density of nuclear matter, the effects of three varieties of neutrinos—electron, muon, and tau (ν_e , ν_{μ} , and ν_{τ}), thermonuclear burning, and relativistic corrections to the gravitational potential. Current one-dimensional models do not produce a supernova, at least not immediately after the core reaches the density of nuclear matter. (See the article beginning on p. 12.) Although the role of magnetic fields and rotation following stellar collapse is essentially a three-dimensional problem, by assuming symmetry about the rotational (z) axis we can begin to study it in two dimensions.

tional n Iron

LeBlanc and Wilson numerically modeled two-dimensional collapses in 1970 and discovered that, in some cases, mass was ejected from the core in the form of a jet spreading from the z axis.² For a time, such a jet was a candidate for the sought-for site of the r process. Subsequent developments, however, especially with regard to the equation of state of nuclear matter, the physics of neutrino transport, and the initial configuration of the core, have made these models obsolete. Here, we report on several two-dimensional models

of the collapse of an iron core that incorporate a better understanding of these features. We have artificially given the models rotational and magnetic energy. We find that a rapidly rotating core may provide an important site for the r process and that magnetic effects probably are unimportant.

Model Parameters

The initial configuration in our two-dimensional models consists of the stellar core, that is, the inner $2.5 M_{\odot}$ of the $15 M_{\odot}$ model developed by Weaver, Zimmerman, and Woosley.³ Computations begin with the core matter largely consisting of iron-56 nuclei and free neutrons and protons. Our equation of state is essentially the same as the one used by Bowers and Wilson⁴ but with two modifications: it excludes both

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Fig. 1

Mass points in our one-dimensional test plotted on a radius-vs-time diagram. Each black curve represents the trajectory of an observer placed initially at a given radius (in kilometres) from the star's center and moving with the stellar material as the star evolves. The mass inside each line is given in multiples of one solar mass ($M_{\odot} = 2 \times 10^{33}$ g) contained within a sphere whose instantaneous radius coincides with the observer's frame of reference. There is no explosion after the core bounce at 0.353 s. The gold curves show the results of Bowers and Wilson (Ref. 4) for solar masses of (from bottom to top) 0.57, 0.989, 1.35, 1.58, and 1.66. At the end of the computation, the core density at the origin was 3×10^{14} g/cm³, the temperature 13.1 MeV, the electron-neutrino energy 133 MeV, and the charge per baryon 0.2891.

thermonuclear burn and the presence of helium-4 nuclei.

Our models include only electron neutrinos (ν_e), tracing the evolution of their energy and number density and their emission and absorption by free nucleons and heavy nuclei. The number density n evolves under the flux-limited diffusion approximation, and energy flow is estimated from the number flow. Because in one-dimensional models the emission of ν_μ and ν_τ neutrinos occurs deep in the core and requires very high temperatures, their effects are subtle and are not included.

Our magnetohydrodynamics (MHD) computer code, written in spherical coordinates (r, θ, ψ), is a Newtonian, Eulerian, adaptive mesh in the radial (r) direction. Our simplifying assumptions (no nuclear burn, ν_e neutrinos only, and a Newtonian gravitational potential) are justified if the model reproduces previous one-dimensional results and is used only on physical problems whose matter is similar to

one-dimensional matter. Figure 1 shows plots of radius vs time of various mass points generated by our one-dimensional model of core collapse, compared to the one-dimensional results of Bowers and Wilson.⁴ The plots are quite similar. Neither model produces a supernova, at least within 30 ms after core bounce. The absence of a thermonuclear burn results in a shock wave that decomposes directly to free nucleons when hot enough, rather than going through a helium stage. Our simplifying neutrino assumption results in a core that is slightly less rich in electrons than the Bowers and Wilson core.⁴

Four additional parameters complete our MHD model: first, the initial angular velocity distribution $\Omega(r, \theta)$; second, the initial rotational energy E_{ROT} (which we normalize to the gravitational energy E_G using $\beta_{\text{ROT}} = |E_{\text{ROT}}/E_G|$); third, the initial magnetic field profile $H(r, \theta)$; and, finally, the initial magnetic field energy E_H (normalized to the gravitational energy defining the parameter $\beta_H = |E_H/E_G|$).

Observations of pulsars, believed to be rapidly rotating neutron stars with magnetic fields, enabled us to estimate the angular velocity and magnetic field strength of the core in its final state as a neutron star. Models of pulsar magnetospheres imply surface fields of 10^{12} to 10^{15} T. The shortest pulsar period observed so far (in the recently discovered pulsar PSR1927+21) is about 1.55 ms, corresponding to a uniform angular velocity of about 4000 rad/s. Using the $15-M_{\odot}$ model of Weaver *et al.*,² we derived $\Omega(r)$ by starting with a model in which the central density was 10^9 g/cm³ (assuming uniform Ω at this time) and then following it to a precollapse density of 6×10^9 g/cm³, which served as the initial central core density in our one-dimensional model. This density distribution provided the shape of $\Omega(r)$.

The scale β_{ROT} was determined by a coarse parameter search, assuming that $\beta_H = 0$ (no magnetic field). We found that models with β_{ROT} less than about 0.01 collapsed to nuclear density, producing a small bounce and no immediate explosion. Insofar as they produce

no supernovae—an important feature—our two-dimensional core models with low rotational velocities are similar to the one-dimensional model. Because the dramatic effects of rotation upon iron-core collapse appear at a β_{ROT} of about 4.5%, the initial value of β_{ROT} in our models is 4.5%.

We chose the simplest possible initial magnetic field $H(r, \theta)$ for the core, basically a polar field that smoothly becomes a dipole field at the outer edge of the computational grid. We again did a coarse parameter search to find interesting values of β_H , setting $\beta_{\text{ROT}} = 4.5\%$. A β_H of less than 0.05% gives an evolutionary sequence virtually indistinguishable from that in which β_H is zero, except for the presence of fields of 10^{17} T in the core. A β_H of 0.56% did produce a hydrodynamic sequence distinctly different from those of rotation-only models, with maximum field strengths of 5×10^{18} T. Thus, it is clear that only extremely strong magnetic fields will affect the collapse of iron cores.

Computational Results

We computed the evolutionary sequence of four two-dimensional stellar models incorporating the foregoing parameters, designated ROTA, ROTB, MRA, and MRB. The value of β_{ROT} was initially 4.5% in all four models; β_H was zero in ROTA and ROTB, 0.053% in MRA, and 0.56% in MRB. Electron neutrinos were excluded from ROTA but included in the other three models. Because of the computational expense (several hours per problem on the Cray 1 computer), we ran only these four models. Table 1 summarizes the initial and final energies of the rotational and magnetic fields.

Rotation-Only Models

ROTA model. ROTA collapsed to a core density of 3×10^{11} g/cm³ in 0.21 s, producing a shock wave that spread from a radius of 350 km. When the computation was ended 32 ms later, a slowly evolving, quasi-equilibrium core had been established. Figure 2 shows the highly nonspherical isodensity contours in one quadrant of the r - z plane

at the end of the computation. The shock wave in this figure is spreading from the quasi-equilibrium core both along the z axis and in angular extent from the z axis. This sequence, however, is an artifact of the omission of

Table 1 Initial and final rotational (β_{ROT}) and magnetic field (β_H) energies, and final core density (ρ_c), in four models of stellar collapse. Energies are normalized to the gravitational energy.

Model	Rotational energy, %		Magnetic field energy, %		Final core density, g/cm ³
	Initial	Final	Initial	Final	
ROTA	4.5	8.9	0	0	3×10^{11}
ROTB	4.5	16.9	0	0	2×10^{13}
MRA	4.5	16.9	0.053	0.15	2×10^{13}
MRB	4.5	11.5	0.56	3.0	1.4×10^{14}

Fig. 2

Isodensity contours in one quadrant of the z - r plane (z is the axis of rotation, r is the radial distance) of the ROTA model at the end of the computation. The contours are given in units of 10^9 g/cm³. The absence of neutrinos in this model halts the collapse at an origin density of 3.0×10^{11} g/cm³. A shock wave is beginning to develop on the z axis at about 400 km. The contours in the remaining three quadrants can be inferred from symmetry about the r and z axes.

neutrinos from the model. Neutrino losses become appreciable at core densities of about 10^{11} g/cm³, as dramatically demonstrated by the ROTB model.

ROTB model. ROTB collapsed smoothly to a core density of 2×10^{13} g/cm³ in 0.255 s. In this model, energy losses through emission of neutrino radiation allowed the core to collapse to one tenth the density of nuclear matter, at which point a quasi-rotational equilibrium core was again established with a radius of 50 km. A shock wave formed outside the core. During the next 20 ms, an elongated vortex circulation pattern developed and began to spread from the core surface (Fig. 3). The vortex defines a circulation region whose radial velocity at the outer end exceeds escape velocity. Such a region forms an off-axis jet ploughing through the infalling matter, thus providing an escape avenue for a relatively small amount of the core's mass ($4 \times 10^{-4} M_{\odot}$).

Fig. 3

Velocity vector profile at the end of the ROTB computation. The longest vector corresponds to a velocity of 3.06×10^4 km/s; the remaining vector magnitudes are scaled linearly. The inclusion of neutrinos in this model allowed the core to collapse to an origin density of 2.0×10^{13} g/cm³, establishing a rotating equilibrium core that is concealed within about 50 km. Isodensity contours (gold) are in units of 10^5 g/cm³. The blue rectangle encloses a region with a radial velocity greater than escape velocity, interpreted here as a jet, with a total mass of about $10^{-4} M_{\odot}$. The velocity profile clearly shows that the jet marks the edge of a large circulation pattern that grows in radius after the formation of the equilibrium core.

We stopped ROTB at 0.276 s for two reasons. At that time, the Courant hydrodynamic time step (determined by core quantities) was 1 μ s, and modeling further appreciable movement of the jet would have required several thousand more computational cycles. In addition, the interface of the jet with the infalling matter is essentially a supersonic shock phenomenon, which our code can only crudely approximate. More specifically, a Richtmyer-von Neumann artificial viscosity is implemented separately in the r and θ directions. Together with the assumption of no thermonuclear burn, this approximation for the artificial viscosity may amount to an oversimplification as the circulation zone expands into lower density regions.

Figure 4 shows the structure of the region behind the outwardly propagating shock wave. Keeping in mind our simplified treatment of the shock wave and the coarseness of the computational grid, some general features are apparent. The region is neutron rich, the fraction of neutrons being as high as 95% at about 400 km and falling to 60% at 1300 km. If the r process is to proceed, a large number of neutrons must be injected into the iron-rich regions. The jet is thus of considerable interest as a possible site for nucleosynthesis. The temperature contours follow the jet, decreasing from 1.1 MeV at 200 km to 0.33 MeV at 1400 km. The density contours are only mildly peaked within the jet, emphasizing that it composes only a small part of a larger circulation zone rather than being part of a full-blown explosion.

Rotational Models with a Magnetic Field

MRA model. MRA started with an initial field ratio of $\beta_H = 0.053\%$, corresponding to a core field of 10^{14} T. MRA evolved quite similarly to ROTB, collapsing to a quasi-rotational equilibrium core, which has a density of 2×10^{13} g/cm³, and producing a jet-like circulation zone growing along the z axis from the surface of the equilibrium core. Figure 5 shows the velocity vector

profile at the end of the computation. Maximum field strengths reached 10^{17} T. The final rotational field energy (see Table 1) and the total mass of the core ejected in the jet ($4 \times 10^{-4} M_{\odot}$) were the same in ROTB and MRA, emphasizing their dynamical equivalence.

MRB model. We started MRB with a β_H of 0.56% and the magnetic field more concentrated in the core, corresponding to a field strength of 10^{15} T. The collapse first proceeded to the quasi-rotational equilibrium stage of models ROTB and MRA, with a core density of 2×10^{13} g/cm³. At this point, MRB departed from those models in two respects: the core underwent further collapse to 1.3 times nuclear density, and a wholesale explosion started at the edge of the equilibrium core. Figure 6 shows the velocity vectors and isodensity contours at the end of the computation. The peaking of isodensity contours along the z axis demonstrates that the model produced a jet that is more explosive than the dynamic circulation pattern of models ROTB and MRA.

The core density in MRB reached 1.4×10^{14} g/cm³, a value about equal to nuclear density in our equation of state for the very low proton fraction of only 0.19. The core magnetic field strength was 5×10^{18} T. The angular velocity within the core was quite nonuniform, peaking along the rotation axis at 10^4 rad/s and rapidly decreasing to between 10^2 and 10^3 rad/s off axis. If the first 6.2 km of the core (whose average density was 10^{14} g/cm³) is given a uniform velocity distribution, preserving angular momentum, its average angular velocity is 1.23×10^3 rad/s and its average magnetic field is 2×10^{18} T. Under similar conditions, the quasi-equilibrium core within a radius of 60 km has an average density of 2.5×10^{12} g/cm³, an average angular velocity of 1.63×10^2 rad/s, and an average magnetic field of 5×10^{17} T. These angular velocities are compatible with the rotation rates of the fastest rotating pulsars. The magnetic fields, however, are about a thousand times more intense than the strongest observed in nature.

Fig. 4

Structure of the large circulation pattern shown in Fig. 3. Isodensity contours (gold) are given in units of 10^8 g/cm³, isotherm contours (blue) are in units of MeV, and the red isocontours show charge per baryon. Because densities in the circulation pattern are less than 6×10^8 g/cm³, in contrast to core densities of 2×10^{13} g/cm³, it is considered an "atmospheric" flow. The charge-per-baryon contours are aligned along an axis parallel to the jet in Fig. 3. Matter is extremely electron poor (and hence neutron rich) at the base of the circulation pattern, where the temperature is still near 1 MeV.

Fig. 5

Velocity vector profile at the end of the MRA computation. (Fig. 3 shows a similar circulation pattern for the ROTB model.) As in Fig. 3, isodensity contours (gold) are in units of 10^8 g/cm³, and the mass within the jet region (the blue rectangle) is $4 \times 10^{-4} M_{\odot}$. The initial magnetic field strength of 0.053% has no substantial effect on the model's evolution as compared with the ROTB model, which has no magnetic field. The maximum field strength reaches about 10^{17} T within the rotation equilibrium core. In this model, the largest velocity vector has a magnitude of 3.5×10^4 km/s.

Figure 7 shows the structure behind the shock wave and within the jet in the MRB model. The isodensity contours reveal an explosion of matter that starts from a density of $3 \times 10^9 \text{ g/cm}^3$ and becomes a polar jet by the time its density declines to $2 \times 10^8 \text{ g/cm}^3$. The isotherm contours closely follow the structure of the jet, ranging from 3 MeV at a radius of 10 km to 0.73 MeV at 950 km. The matter in the jet is neutron rich, with a neutron fraction of 85% well behind the shock, rapidly

falling to 55% at a radius of 600 km. The neutron-rich matter is concentrated along the rotation axis, as in the ROTB and MRA models.

R-Process Nucleosynthesis

Because of the neutron-rich material ejected in their jets, the models most likely to provide an r-process site for the synthesis of heavy elements are ROTB, MRA, and MRB. However, the extremely intense magnetic fields modeled in MRA and MRB have never been observed in nature and (on theoretical grounds) may be difficult to maintain if established. We are thus left with ROTB as a candidate r-process site. Because collapsing stars with the extremely high rotation rates and intense magnetic fields needed to form a neutron-rich jet may occur only rarely in nature, it is necessary that, when they do occur, they produce an abundance of r-process material (i.e., our model must produce a fair amount of material in this rare event). We did not interface our MHD code with a nucleosynthetic network. Consequently, to determine the appropriateness of the circulation pattern shown in Fig. 3 as a prospective site for the r process, we must examine it in more detail.

At the base of the circulation pattern generated in ROTB, the prevailing high temperatures and densities decompose matter into a gas consisting of free neutrons and protons that expands and cools toward the outer edge of the pattern. Other investigators have shown that in spherically symmetric supernova models, the nuclear seeds of the r process (such as iron-56 nuclei) form on a time scale of less than a microsecond. All r process begins under these conditions if the initial ratio of neutrons to protons is greater than about 3:2.⁵ The base of our circulation pattern is analogous to the base analyzed in these models, with two important exceptions. First, in our models, a larger fraction of matter infalling along the equator provides an additional source of neutrons. Second, in our models, only the matter in the circulation pattern may participate in the r process. These features provide the conditions for synthesis of

Fig. 6

Velocity vector profile at the end of the MRB computation. Any zone with a positive radial velocity is an expansion region. As before, isodensity contours (gold) are given in units of 10^8 g/cm^3 . Portions of the jet within the blue rectangles have radial velocities that exceed escape velocity, the largest vector having a magnitude of $4.4 \times 10^4 \text{ km/s}$. The profile indicates that the model produces more an explosion than an energetic circulation flow. The shock front on the z axis is at a radius of about 1200 km. Behind the shock front, the isodensity contours are strongly peaked along the z axis. The density at the origin is $1.4 \times 10^{14} \text{ g/cm}^3$, which (because of the low charge per baryon of about 0.19 at this point) is about 1.3 times the density of nuclear matter. The table gives the mass (in solar masses), the radial kinetic energy (KE), the gravitational kinetic energy (KE_g), the thermal energy of the matter (E_{TH}), the magnetic field energy (E_H), and the total energy (E_{TOT}), both in the jet and in the expanding regions. Energies are in units of 10^{51} J . The radius of the equilibrium core is at about 50 km; maximum field strengths within the core reach $5 \times 10^{18} \text{ T}$.

the iron-group elements, the seeds of the r process.

Studies have indicated that the r process occurs at densities of 10^8 to 10^9 g/cm³ and temperatures of a few billion degrees.⁶ At the end of our ROTB computation, the edge of the circulation pattern was 10 to 100 times as dense as this and two to three times as hot. Moreover, it seems likely that the material will remain hot enough to support nucleosynthetic reactions even after it has expanded into regimes of lower density. The time scale for this expansion is about 20 to 50 ms, a value compatible with estimates of the time scale needed to produce r-process nucleosynthesis.

Summary

We have studied several two-dimensional computational models of stellar collapse in which a 15- $M_{\odot}$ iron core was artificially given rotational and magnetic energy. In a one-dimensional test, our basic MHD model produced results that agree with those of Bowers and Wilson, demonstrating that our simplifying assumptions are quantitatively consistent with more sophisticated treatments under conditions in which spherical symmetry can be assumed. The differences observed between models MOTA and ROTB show that a significant amount of energy can escape the core in the form of electron neutrinos, resulting in a deeper collapse. Models with low rotation (β_{ROT} less than 1%) collapse to nuclear density. Such models differ from the one-dimensional model primarily in producing a nonspherical rebounding shock wave and circulatory flows in the core.

Our high-rotation model, ROTB, evolves to a quasi-rotational equilibrium core, generating an expanding circulation pattern near the z axis that may provide an escape route for about $10^{-4} M_{\odot}$ of matter from the core. This behavior is largely unchanged when the initial ratio of magnetic to

Fig. 7

Structure behind the shock front and within the polar jet at the end of the MRB computation. The contours are color-coded as in Fig. 4. The peaking of the isodensity contours along the z-axis again shows an explosion. The material at the base of the explosion, where temperatures are still relatively high (about 1.3 MeV), is electron poor, with a baryon charge of about 0.15.

gravitational field energies is less than 0.05%, even though, in this case, the maximum magnetic field may reach 10^{17} T. The extreme initial field ratio of 0.56% used in the MRB model was sufficient to disrupt the quasi-equilibrium state of the ROTB core, producing further collapse to nuclear density and powering an explosion in the form of a polar jet expanding from the z axis.

ROTB seems the most likely candidate for the site of r-process nucleosynthesis. The temperatures and densities within the circulation pattern generated by this model are significantly greater (10 to 100 times in the case of density and two to three times in the case of temperature) than those thought necessary to initiate r-process nucleosynthesis in iron-group elements. Under such conditions, even after the material has expanded into lower-density regimes it is likely to remain hot enough to support nucleosynthetic reactions.

Key Words: nucleosynthesis; r process; stellar collapse; supernova.

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Modeling Gravitation

The effort to detect gravitational waves, now being pursued in various laboratories, depends in part on knowing beforehand the strength and form of the signals to be expected. We are developing a computer code to simulate mathematically some of the violent stellar collisions and collapses expected to generate these gravity waves.

Until a few decades ago, astronomers probed the secrets of the universe with just one major tool, the telescope. Since the 1940s, however, an ever-increasing array of instruments has come into use, capable of making measurements of all segments of the electromagnetic spectrum from radio waves to gamma radiation. Still other instruments are used to measure cosmic rays (high-energy charged particles), and even astrophysical neutrinos probably have been detected. Progress in opening these "windows" has led to myriad startling discoveries over the past 20 years: quasars and

radiogalaxies, pulsars and neutron stars, possible black hole candidates, astrophysical jets, cosmic fireball radiation, x-ray sources, gamma-ray "bursters," and others.

Over the past 15 years, a number of research groups has been attempting to open yet another window on the universe by detecting gravitational radiation. Gravitational radiation is a theoretical consequence of general relativity; it arises whenever a mass is accelerated, although the energy radiated becomes significant only for very large masses moving at velocities close to that of light. The potential astrophysical sources of gravitational radiation include the collapse of a rotating stellar core, the

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Sources of Gravitational Radiation

rapid rotation of a newly formed neutron star, the collision and disruption of two neutron stars, the collision of two black holes, and the spiralling-in and coalescence of two neutron stars or black holes.

Pioneering efforts to detect gravitational radiation used a 1-tonne aluminum cylinder studded with strain gauges operating at room temperature.¹ Neither this nor subsequent similar detectors, some cooled to cryogenic temperatures, has detected gravity waves; the consensus based on theoretical signal-strength estimates is that greater detector sensitivity will be required. More sensitive detectors are now being built² and even more sensitive ones (including the radio Doppler tracking of spacecraft)³ are on the drawing boards. When gravitational radiation is detected, it will represent not only a triumph of laboratory physics but should soon thereafter provide a new and powerful tool with which to probe nature's most violent processes.

Theoretical astrophysicists are working to determine the strengths and characteristics of gravitational waves emitted from the various possible sources, and to estimate the probability of a source occurrence and the likelihood of its detection and measurement in the laboratory. Such calculations not

only aid the design of the gravitational-wave detectors but also, once these wave forms can be measured and differentiated, may yield the only evidence for effects that are otherwise hidden from study through any of the other windows.

During the collapse of a stellar core in a supernova explosion, about 85% of the mass of the star lies outside the collapsing core. These outer layers completely block electromagnetic radiation coming from the star's core. Neutrinos and gravity waves, on the other hand, pass easily through the outer parts of the star and should carry with them some information about the collapse. However, only if these emissions can be detected and studied will we be able to observe details of the dynamics of supernova core collapse.

Gravitational waves radiate from the accelerated motion of matter and are strongest when the motion is organized. The most powerful sources are those whose bulk (or fluid) velocities (v_{fluid}) are close to that of light or whose self-gravity is "strong," i.e., comparable to that of a neutron star or a black hole. (The gravitational binding energy of a mass element on or near such a body is comparable to the mass element's rest energy, which is equivalent to saying that

the escape velocity from such a body is comparable to c , the velocity of light.) The first regime occurs when v_{fluid} approaches c , the second when ϕ , the Newtonian gravitational potential, approaches c^2 .

Useful approximations to general relativity exist in the limit of v_{fluid} very much less than c (the slow motion approximation) and in the limit of ϕ very much less than c^2 (the weak field limit). These perturbative techniques, which expand the Einstein equations of general relativity in powers of v/c or ϕ/c^2 and reduce intractable nonlinear equations to more simplified linear equations, have been used extensively to calculate gravitational radiation ever since Einstein's original calculations (1918). However, it is for precisely those systems that produce gravitational radiation most efficiently, those with large fluid velocities or strong self-gravity, that these approximations break down. If we are to understand the most violent astrophysical events, we must solve the full Einstein equations.

Problems arise in that, except in highly symmetric cases, these equations of relativistic hydrodynamics are too complex to be solved analytically. Instead, we must use finite-difference techniques to model the system on the computer as it develops. Large-scale finite-difference calculations have been applied to aircraft design, weather forecasting, weapons design, and so forth, since the advent of high-speed digital computers, but their use in relativity calculations is fairly recent.

During the mid-1970s, Larry Smarr (now of the University of Illinois) teamed with Jim Wilson to work on numerical relativity. Smarr and Ken Eppley were completing their pioneering calculation of the collision of two black holes, which Smarr began as a graduate student at the University of Texas. Wilson brought to numerical relativity a wealth of experience in physical simulations including hydrodynamics, supernovae, magnetohydrodynamics, and relativistic hydrodynamics in stationary general relativistic fields. This experience was a key factor in helping to develop stable

numerical-relativity algorithms such as the two-dimensional axisymmetric matter code described here. Later researchers have continued and refined this code.⁴

Choice of Model

Finite-difference techniques are so powerful that, under ordinary circumstances, they allow simulation of very complex systems. The equations of relativistic hydrodynamics are so numerous and complex, however, that calculation over even a modest-sized grid requires vast amounts of computer memory and computer time. Furthermore, the finite-difference algorithms are still something of an art form and can be applied only with great caution to new and complex problems. For these reasons, it is still necessary to seek nontrivial systems for study that are symmetric enough to allow some simplification.

The problems we have chosen to solve involve only nonrotating, axially-symmetric systems, enabling us to use a computational grid that has only two spatial dimensions (in addition to time). This restriction rules out most of the astrophysically interesting problems, leaving only a set of rather improbable situations. For this reason, we avoid treating the microphysics of the matter, and use simplified equations of state.

This simplification also reduces from two to one the number of polarization states in which gravitational radiation can be emitted in our simulations. On the other hand, for these special cases, we now can solve the full equations of general-relativistic hydrodynamics (to the accuracy of the finite-difference approximation) for problems involving strong self-gravity and speeds approaching that of light.

The development of this code represents an important step in advancing the techniques of numerical relativity calculations. This is because, in numerical relativity, the general-relativistic parts of the calculation have become much more difficult than the hydrodynamic parts. Our code, therefore, is a test bed in which we can refine the relativity calculations before proceeding to more ambitious codes that allow

rotation or that simulate all three space dimensions.

Model problems that can be simulated with this code include the gross features of the collapse of a prolate or oblate stellar core or neutron star into a black hole, the collapse and subsequent deep hydrodynamic rebound of such an object without forming a black hole, and the head-on collision of two neutron stars. Each of these improbable processes is expected to be a copious source of gravitational radiation, and calculation of their characteristics, in addition to its purely theoretical interest, may yield some hints on what can be expected from more realistic events such as the glancing collision of two neutron stars, the orbital spiralling and coalescence of two black holes or neutron stars, and the collapse of a rotating stellar core or neutron star. The full confirmation of such predictions will await either observational verification or the development of more sophisticated codes.

Numerical Relativity

Let us sketch briefly the technique of solving a problem in numerical relativity, indicating how our choice of symmetry simplifies the problem and how we have introduced further simplifications in the algorithm. What makes numerical relativity so difficult is that, unlike Newtonian gravitation with its single gravitational potential, numerical

relativity has ten separate fields that must be evaluated to determine the whole gravitational field. Adopting symmetries allows some of these fields to vanish, simplifying the calculations.

Another difference is that the equations of relativity, unlike those of Newtonian gravity, exhibit wave solutions analogous to those of electromagnetic theory. We expect that, just as wave solutions in electromagnetic theory describe the properties of electromagnetic radiation, the similar wave solutions in general relativity imply the existence and describe the properties of gravitational radiation. Except that both are composed of waves, however, the two types of radiation will be distinctly different.

The difference in their physics shows up most clearly in terms of polarity. Electromagnetics deals with positive and negative charges, north- and south-seeking magnetic poles, attraction and repulsion. In gravitation, there is no negative mass or gravitational repulsion.

Because of these physical differences, the simplest and most prevalent electromagnetic radiator is the dipole; there are no gravitational dipoles and the simplest gravitational radiator is the quadrupole. Even if we compare an axisymmetric electromagnetic quadrupole and an axisymmetric gravitational quadrupole, the radiation-emission patterns are different (Fig. 1).

Fig. 1

Axisymmetric quadrupole radiation patterns for (a) electromagnetic radiation and (b) gravitational radiation. The gold area indicates the quadrant covered in the numerical simulation shown in Fig. 3.

There are, on the other hand, certain aspects of the general-relativity field that are determined instantaneously, just like the gravitational potential of Newtonian gravity, from a knowledge of the rest of the gravitational field. Hence, the general-relativity description of gravity has features in common with both Newtonian gravity and electromagnetic theory.

Of the ten gravity fields noted above, only six are determined by the field equations of general relativity. Before we can begin a numerical calculation of some astrophysical event, we must find appropriate initial values of these six fields and their rates of change (first-time derivatives). We cannot assign these values arbitrarily, however, because the field equations also provide four constraints that relate these six fields to their first-time derivatives and to various hydrodynamic quantities. Only those field choices that satisfy the constraints can serve as proper starting values, and if they do so initially, then the rest of the field equations guarantee (at least theoretically) that they will continue to do so throughout the subsequent evolution of the problem.

These constraints have a parallel in electromagnetic theory, in which the initial value of the electric field is

constrained by the charge density. This means that one of the electric field's three components can be determined everywhere by a knowledge of the other two and the charge density. Charge, in other words, is a conserved quantity, and distant parts of the electric field must reflect the total charge. The four constraints of general relativity can be similarly related to the conservation of total energy and to the three components of total linear momentum.

The practical mathematical techniques that form the basis for constructing a numerical algorithm for solving these constraints were developed a decade ago.⁵ These techniques are a way of "bootstrapping" from initial values that may be chosen almost arbitrarily to new initial values that properly satisfy the four constraints, and, in so doing, identify the parts of the initial data that we are truly free to specify.

The process of specifying these fields at the initial instant also defines the "initial instant of time." The concept of time is very general in relativity; space-time is viewed as a collection of events (times and places), but how we give these events spatial and time coordinates or how we "slice up" space-time is somewhat arbitrary. In Fig. 2a,

Fig. 2

Schematic space-time maps illustrating various features of numerical relativity calculations. (a) How the choice of a coordinate system can influence the time sequence of events. (b) The choice of initial conditions defines time zero. (c) The roles of the scalar (lapse) function α and the vector (shift) function β illustrate the distortion of time and space. Although all points on the upper slice are at the same computa-

tional time ($t = \Delta t$), the scale α defines how fast physical time flows as a function of position, thus allowing portions of the slices to be closer or farther apart. Also, although every point on the second slice maps to a corresponding point on the first slice, the relative distances between points can vary as determined by the vector β .

for example, events A and B occur at the same time in one set of coordinates (gold) but at different times in another (blue).

The equations of general relativity apply regardless of which coordinate system we choose; however, when we begin to specify the various initial conditions, we begin to restrict our choices. Solving the initial-value constraints determines the first slice through space-time at $t = 0$ (Fig. 2b). In addition, the initial values of the various rates of change indicate the direction in which the gravitational field and the hydrodynamics will evolve in the next instant of time.

But if the choice of coordinates is arbitrary, what determines the "next" instant of time? Furthermore, what determines where the spatial coordinates will be at this future time? The answers to these questions lie in the specification of the other four of the ten fields alluded to earlier.

These four (known as gauge fields) consist of one scalar field and three vector fields, and they determine the shapes of our space-time slices and the way in which we can map locations on one slice onto the next. The scalar part (the lapse function α) determines the time interval between slices (which may vary as a function of position in the slice). The vector part (the shift vector β) determines how the spatial coordinates, once they are specified on the initial slice, propagate on to the next slice (Fig. 2c).

The choice of conditions to be placed on these gauge variables is very important, since many choices lead to singularities that terminate the calculation before it can yield results. Wilson and Smarr chose to use the so-called "maximal slicing" condition on the lapse function, which effectively stops the advance of time inside a black hole while allowing it to proceed steadily outside. This freezes the simulation inside the black hole, saving the computer from a futile attempt to simulate that region's unimaginable conditions and enabling it to complete its simulation in the observable region outside.

The choice of conditions to be placed on these gauge variables is very important, since many choices lead to singularities that terminate the calculation before it can yield results.

The choice for the shift vector is perhaps less fundamental, but it is still important in a practical sense. Wilson chose a shift vector that explicitly removes from the calculation several components of the gravitational field. This incidentally makes the calculational results easier to understand, but its main impact is in vastly simplifying the equations to be solved numerically. Without this simplification, the programming of the calculation would have been so difficult that it probably would never have been attempted.

Yet another simplification had to do with the structure of the relativity calculation. The choice of zero rotation and axisymmetry reduced the number of constraints to three, the shift-vector components to two, and the hydrodynamic momentum components to two. In addition, and more important, all state variables became functions of only two spatial coordinates, the radius and the polar angle, since symmetry made it possible to ignore azimuthal angles. This reduction in dimensionality had by far the greatest effect in making our calculation possible on existing computers.

Another of Wilson's contributions greatly improved the stability of the computation. In an analytic calculation, as mentioned above, the operation of the various equations will assure that initial solutions that satisfy all the constraints will remain valid throughout the subsequent evolution. In a numerical simulation, this is no longer necessarily true; the time-evolved quantities can drift away from the "true" solution and increasingly violate the constraints. Wilson arranged that only the true dynamic variables and the hydrodynamic quantities were evolved in the calculations and that the other fields were derived by explicitly solving the constraint equations on each successive time slice. Although this procedure slows the calculation somewhat, increasing the cost in computer time, it greatly diminishes the drift in the evolution and stabilizes the code.

Although relativistic hydrodynamics is considerably more complicated than the usual nonrelativistic version, the

two systems do have many features in common. In both, the material sources, rest energy density, internal energy density, and momentum density must all be accounted for as they evolve with time, and the pressure and stresses must be calculated at each time step according to an equation of state. Wilson was able to formulate the relativistic hydrodynamics code in a form that mimics Newtonian hydrodynamics. This enabled us to exploit the large body of knowledge accumulated from simulating nonrelativistic hydrodynamic systems.

Test and Conclusions

A brief example will serve to indicate the accuracy of the code and illustrate the improvements that are being made. Since the purpose of this test was to measure the accuracy of the simulation, we chose a relativistic problem whose overall result is known qualitatively although it could not be calculated analytically.

In Newtonian physics, gravity is directly related to matter; if there is no matter, there is no mass and hence no gravity. In general relativity, this relation no longer holds; there are spacetimes that contain only gravitational waves and no material sources. (There is still mass, however, since gravitational waves carry energy which is equivalent to mass.) Our code is able to simulate such spacetimes, which under our symmetry restrictions are called Brill waves.⁶

Apart from the scientific interest in Brill wave solutions, these models provide excellent development examples because they let us test the relativistic parts of the code without involving the hydrodynamic complications. (Hydrodynamics is matter in motion; without matter there is no hydrodynamics.) To be more specific, this problem tests the wave parts of the gravitational field, some of the constrained parts of the field, and the radiation-flux-measuring indicators of the code.

Figure 3 is a series of projected plots of a dynamic variable (namely, the ratio of two of the metric components that carry the gravitational wave) as a

Fig. 3

Computer simulation of a quadrupolar Brill (gravity) wave radiating away from an initial disturbance. Only one quadrant is shown; the other three may be inferred by symmetry. (a) The initial disturbance. (b) The wave has split into an outgoing and an ingoing part. (c) The ingoing wave has passed through the center and is now outgoing. (d-g) The wave progresses. (h) The last of the wave is passing beyond the borders of the simulation grid.

function of radius and polar angle. We used fairly coarse zoning in this simulation for a total of 360 zones. Since the other three quadrants may be inferred from symmetry, this calculation covers only one quadrant.

To begin the simulation, we introduce a quadrupole Brill wave of a given amplitude at time zero (Fig. 3a, compare with Fig. 1b). In Fig. 3b, the original wave has split into ingoing and outgoing parts. In Fig. 3c, the ingoing wave has passed through the center and is now moving out. Figures 3d to 3g continue the evolution. In Fig. 3h,

Fig. 4

Measures of radiation energy used to test the accuracy of the numerical simulation of gravitational-wave propagation in matter-free space. (a) Total flux as a function of radius, integrated over all angles, for cycle 150 of the calculation. The main pulses have not yet reached the outer edge of the grid, and small negative fluxes near the center indicate ingoing waves. (b) Mass-energy as a function of time. The blue curve indicates the mass-energy of the system in the region spanned by the grid. The gold curve indicates the mass-energy that has propagated off the grid. The essentially complete transfer of energy from one to the other as the wave moves the edge of the grid is a measure of the accuracy of the calculation.

the last of the wave is passing beyond the limits of the calculational grid.

The total radiation flux (for all angles combined) is calculated by a variety of means and is time-integrated as the wave goes over the edge of the grid. Figure 4a is a plot of one such total flux indicator at cycle 150 in the calculation. Note that the main pulses have not yet reached the edge of the grid. There are also small negative flux values near the origin, indicating an ingoing wave there.

As indicated earlier, one of the constraint equations can be related to the conservation of total mass-energy. The component of the gravitational field that this constraint determines can be used to indicate the total mass-energy contained in the region spanned by the grid. This mass is essentially constant until the gravitational wave begins to pass off the edge of the grid, and then decreases toward zero as the wave carries off mass-energy.

At the same time, an independent indicator measures the total mass-energy leaving the grid. This indicator starts at zero and rises to a maximum value as the wave moves past the edge. Figure 4b shows the histories of these two independent indicators during this test calculation.

One result of this test is that, despite the relatively coarse grid on which it was calculated, the final integrated energy flux agrees with the initial configuration of mass-energy to within 0.7%. Because this excellent agreement was achieved in a simulation that involved several independent calculations, we believe this is a good indicator of the accuracy of the parts of the code that involve the gravity-wave generation and propagation, the energy constraint, and the radiation-flux measures.

We have recently made several significant improvements in our two-dimensional numerical relativity code, implementing more efficiently the ideas discussed above. One such improvement is in the numerical solution of the nonlinear energy-constraint equation which, in previous simulations of black hole formation, would become unstable

and stop the calculation before the gravity waves generated in the black hole's formation could propagate to the edge of the grid and be measured. Our modification allows the integration of the constraint equation (and therefore the entire calculation) to continue much further along the event's history.

We are also improving the parts of the code that solve the momentum-constraint equations and those that solve for the shift-vector components. Improved solution of the momentum-constraint equations will be particularly important for simulations that involve material sources.

With these improvements, we expect soon to be able to undertake a complete survey of all the axisymmetric nonrotating models discussed above. In addition, with the experience gained on this project, we should soon be able to construct a new code that will allow us to simulate far more probable astrophysical phenomena, namely, the axisymmetric rotating systems that represent potentially observable sources of gravitational radiation.

Key Words: astrophysics; black hole; computer modeling; general relativity; gravitational

collapse; gravity wave; neutron star; numerical simulation.

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An Arbitrary-Mesh Computer Program With Applications to Astrophysics

We have developed a versatile computer program for finite-difference mesh calculations with unique features that enable us to calculate a variety of challenging problems in astrophysics.

Computer programs play a key role in much of the Laboratory's research. Often a program developed for a general class of problems—for example, finite-difference calculations—finds applications in other studies. Finite-difference calculations require a spatial mesh. In a Lagrangian treatment, mesh lines move with the material; in an Eulerian treatment, in contrast, the mesh lines remain fixed in space as the material moves through them. We have developed a large, multimaterial, two-dimensional computer program with an arbitrary Lagrangian-Eulerian (ALE) mesh.¹ The program is distinctive in enabling us to use, alternatively, an embedded Lagrangian mesh, a fixed Eulerian mesh, or a partially embedded, partially remapped mesh. Lagrangian mesh distortion is removed by remapping. Under the guidance of Jim Wilson, we have used our general-purpose program to model a variety of astrophysical problems. The process of developing such a large computer code highlights several important issues in program design.

Large programs (containing hundreds of thousands of Fortran statements) designed for many users have special requirements that may not be important for small, single-user programs. Large programs have long lives and pass through many hands; therefore, they must be understandable to programmers so that they can be adapted to evolving user requirements and machine resources. In addition, they can use considerable amounts of computer time and thus require careful attention to computational efficiency. We must often tailor the algorithms to the characteristics of the computer on which they are to be run. The nature of the calculational mesh is very important in determining what types of problem can be solved; it is also a major factor in determining the efficiency of the calculation.

The overall efficiency in carrying a problem from conception to completion depends on the algorithms used, the computer speed, and the time the users must spend interacting with the program. To maximize efficiency, programs must be written to make optimal use of the available computer resources. It is equally important, however, to provide both powerful and convenient ancillary service programs and understandable documentation.

We describe, here, some of the considerations in developing a large finite-difference computer program, the manner in which our program treats discontinuous velocities and how it moves mass, energy, and momentum from zone to zone as the mesh remaps, and finally how these features have proved useful in modeling some specific problems in astrophysics.

Goals in Developing Our Large Program

Developing a large program involves many design decisions that strike an appropriate balance between desirable but competing characteristics such as ease of modification, computer-resource efficiency, user efficiency, and accuracy.

Modularity

A modifiable program is modular, well-documented, and understandable. A large program evolves during its lifetime as new demands are made on it. Therefore, it must be easily modified as new computers and peripheral equipment are acquired. Because large programs require large investments of manpower, computer time, etc., a functionally modular, well-understood program must permit changes with minimum effort, thus retaining most of the prior investment.

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Modularity is achieved on several levels in our program. On the highest level, a controller, or master program, coordinates the operation of several independent programs by passing messages and data files between itself and the individual programs. These programs include utility routines for listing, editing, storing, and retrieving files. In addition, several programs perform independent functions. One generates the initial conditions and mesh configuration and can also refine, coarsen, or repair an existing mesh. Another program numerically and graphically edits the problem at any stage of evolution. For multidimensional problems,

graphics are very important because the amount of data can be overwhelming if only numerical edits are provided.

The physics-cycle program advances the physical variables in time. The differential equations relate time derivatives to space derivatives. When these are converted to finite-difference equations, a time interval is introduced. The process of advancing the problem by one time interval is called a computational cycle. Beginning with the initial conditions, the solution is advanced in time cycle by cycle.

The next level of modularity comprises the subroutines that make up the programs. Where possible, we use routines from already existing specialized libraries—graphical, equation-of-state, and mathematical functions, and input/output drivers. Unfortunately, most of the program must be coded specifically for this application. Although algorithms may be adapted from other programs, differences in the data bases require us to recast those algorithms.

When many people are involved in the development of a large program, clear definitions of variables and explanations of algorithms are vital. A powerful programming language greatly aids comprehension. We are fortunate to have the LRLtran language, an extension of Fortran that can concisely express complex one-dimensional arrays (vectors). Vector operations are crucial to obtaining dense data packing and maximum execution speed on pipeline computers such as the Cray 1. Arithmetic operations on vector arrays are much faster than the same operations done one element at a time.

Lagrangian vs Eulerian Meshes

Because some astrophysical objects are symmetric about an axis, only two coordinates (dimensions) are needed to define their geometry: the distance along the axis of symmetry and the radial distance out from the axis of symmetry. A two-dimensional model calculation requires less computer time and data storage than a three-dimensional model calculation.

Fig. 1

Applied to nonturbulent problems, a Lagrangian mesh (gold) often provides better accuracy than an Eulerian mesh (blue) with fewer zones to describe a particular shape.

A Lagrangian mesh (Fig. 1a) is usually more efficient than a fixed rectangular Eulerian mesh (Fig. 1b). In a Lagrangian mesh, zones are embedded in the material; thus, they can be put, and will remain, where they are needed most (Fig. 1a). A spherical Eulerian mesh, on the other hand, can be quite efficient for astrophysical problems. Our program combines the best features of Eulerian and Lagrangian meshes by using either mesh or a combination of both as appropriate. We can tailor the mesh to the problem by following the initial material contours instead of extending the mesh into uninteresting regions. This reduces the amount of data that must be stored. Later, we will see how this tailoring can improve accuracy. Unlike Eulerian meshes employed in other programs, the Eulerian portion of our mesh need not be rectangular or even orthogonal.

We have found that a Lagrangian mesh can calculate initial motion with maximum accuracy since little distortion results. Turbulence and shear motions usually appear and persist only after important transient phases have passed, causing mesh distortion and making it necessary to remap.

For a given number of zones, a Lagrangian mesh can give a more accurate representation than an Eulerian mesh so long as grid lines remain regular (usually quasi-orthogonal). As the mesh becomes badly distorted, however, difference formulations become inaccurate, and the mesh must be repaired. Because the Courant condition for hydrodynamic stability is improved when the mesh improves, our program can often run with larger time steps than a Lagrangian mesh program. This condition requires that the time step be less than the transit time of sound across a zone. Rezoning to remove distortion increases the shortest distance across a zone, thus increasing the time step. Were all other factors constant, a larger time step would reduce the calculation time. This is not necessarily the case, though, because the remapping calculations increase the time needed to calculate a cycle. We will see later that remapping to remove

distortion offers another benefit: it often improves the accuracy of the calculation.

Automatic Mesh Correction

One of our main goals in designing the program was to greatly reduce the labor needed for the user to repair Lagrangian meshes. Since a Lagrangian mesh is tied to material motion, kinking or necking off of the material will distort the mesh enough to cause large errors in the difference equations and can require a small time step. One of the main advantages of an ALE mesh program over Lagrangian mesh programs is its ability to detect and reduce mesh distortion automatically.

Two tests are made to detect unacceptable distortion. One test (Fig. 2) checks whether the angles at each grid-line intersection (node) are within specified limits. If any angles are outside the limits, the node is remapped unless it has

Fig. 2

A Lagrangian mesh tends to distort when used to model turbulent material. Our program can detect and reduce mesh distortion automatically. One test for mesh distortion compares the angles (A, B, C, D) formed by the intersection of mesh lines with a user-specified value. (a) If any angles are too large or too small, as are A and C, the program will remap to more closely approximate a configuration (b) in which all four angles are closer to 90 deg.

been exempted by the user. The other test (Fig. 3) compares the area of the smallest of the four triangles surrounding a node with the sum of all four areas. If the ratio is below a threshold value and the node has not been exempted, the node will be remapped. Only the angle test is made at the boundaries of the problem.

The user can remap nodes in many ways. Nodes can be returned either to specific coordinates or to their initial position (which produces an Eulerian mesh that is not necessarily orthogonal). Nodes can be interpolated between other nodes. Most of the remapped nodes are repositioned according to a weighted average of the coordinates of their eight neighbors; this tends to produce orthogonal mesh

lines. This is called an "equipotential" method.^{2,3} The mesh can be remapped in either polar or Cartesian coordinates, and mesh lines can be uniformly or geometrically spaced. Remapping can be prevented or reduced if it would make the smallest adjacent triangle smaller.

If the initial remapping leaves the mesh distorted, equipotential remapping can be repeated several times. If distortion persists, the program maximizes the area of the smallest triangle around the distorted node. However, remapping can cause diffusion of physical quantities. To reduce diffusion, the coordinate shift due to remapping can be limited to a multiple of the shift in position of the fastest neighbor node. If unconstrained, diffusion can obscure real gradients.

Distortion near nodes along the outer boundary must be removed by special means, since these nodes do not have the eight neighbors required by the equipotential method. Remapping a curved outer boundary brings material into or removes material from the problem; thus, it should be allowed only if the outer boundary is not critical to the problem. To remap boundary nodes, the program initially tries to space them more uniformly. If that fails to reduce interior-angle distortion, the program reduces the curvature of the boundary instead.

Departure from a Lagrangian mesh in a problem with several materials can result in "mixed" zones containing several components. Treatment of mixed zones usually requires additional approximations. In any case, as the mesh is moved through the materials, some diffusion is introduced. When our program is initially run Lagrangian, with only occasional remapping as zones distort, the amount of advection and mixing is kept small. This minimizes typical Eulerian-mesh errors and reduces distortion that could cause errors in a Lagrangian-mesh calculation.

A problem is usually run so that the mesh starts completely Lagrangian; that is, it is embedded in the material. Only after significant distortion is detected does remapping begin. Remapping is

Fig. 3

(a) A Lagrangian mesh can distort the four triangles (A, B, C, D) around a node so that their areas become quite different. Our program tests for this condition by automatically comparing the areas of the triangles and remapping the node if the area of the smallest triangle is less than a specified percentage, say 5%, of the total area of all four. (b) After remapping, the areas of the triangles are much more similar.

confined to the distorted region and typically involves only a small displacement relative to the material in each cycle. Any node can be prevented, either temporarily or permanently, from remapping.

Documentation and Service Programs

Expert knowledge is needed to make the many decisions required to set up a two-dimensional problem. A good user's manual is an essential aid in making such decisions and in informing the user about the many options available in the program. A powerful generator is necessary to permit the user to readily exercise these options. For example, the user must decide how many zones are needed to define each material adequately. Will the region be allowed to remap? Which details will be included and which ignored? Is material strength important? Where can tangential velocity components be different (that is, where can sliding be allowed to occur)? In addition, the user must supply key node coordinates in either Cartesian or polar form. The user can instruct the program to interpolate lines between nodes and to interpolate regions between the lines. The program can smooth lines and can shift them to generate other lines. The user must specify initial zone properties and node velocities. Boundary conditions and velocity constraints must be specified. Material properties must be either specified or obtained from data bases. Boundary conditions must be established when the problem is generated; the code applies these conditions during each calculational cycle. Boundaries may be fixed, reflection, or free. The pressure and temperature outside free boundaries may be specified as tabular functions of time and of the mesh-line indices. The velocities of the boundary nodes may be specified as functions of time or may be constrained to move only in certain directions. In addition to service programs for generating, we also provide programs for editing, plotting, and altering. These programs check extensively to guard against input errors that could invalidate results.

One of the main advantages of an ALE mesh program over Lagrangian mesh programs is its ability to detect and reduce mesh distortion automatically.

Typical Modeling Methods in Astrophysical Problems

Astrophysical problems often involve complex hydrodynamic flows that require special treatment. Two such treatments that we use in our code are velocity discontinuities and advection.

Velocity Discontinuities

When solids and fluids slide past each other, we need to describe discontinuities in tangential velocities at the interface (Fig. 4). If we can ignore the thin, relatively sluggish boundary layer and allow a velocity discontinuity at the interface, we need not use the small zones necessary to discriminate the boundary layer.

Our program allows discontinuities in tangential velocities at certain nodes called slide nodes (Fig. 5). Some pure-Lagrangian-mesh programs allow nodes to slide at an interface so that the mesh becomes discontinuous, in effect producing two separate meshes adjacent to each other. Our program, however, maintains a continuous mesh at the sliding interface by treating each slide node there as if it were actually two nodes that can temporarily move somewhat independently. We call one node the master component and the other the slave component. Each half of the master-slave pair moves at its own velocity and separates from the other when their velocities differ. To preserve the continuity of the mesh, however, each separated slave component is

remapped to its master position before the end of the calculational cycle. This ensures that the mesh lines will be continuous, which is advantageous for all of the other calculations that must be made.

Mesh continuity eliminates some disadvantages of discontinuous meshes (such as those produced by Lagrangian codes that allow sliding at interfaces): additional approximations caused by averaging and interpolation are avoided, coarse zone patterns will not be imprinted on fine zones, and narrow zones will not pass over wide zones, which could cause the wide zones to rotate unrealistically about their centers. Mesh continuity also greatly simplifies advection and heat flow calculations.

However, mesh continuity sacrifices the zoning flexibility of a discontinuous mesh, which, among other advantages, allows a transition to a coarser mesh in which zoning detail is a less stringent requirement. To overcome this, we can average the physical properties of several adjacent zones and velocities of adjacent nodes where the mesh resolution is not needed (such as near the center of a polar mesh). This averaging stabilizes the calculation, so a larger time step can be used. Mesh continuity at slides also means that on at least one side of the slide the mesh must remap. This may degrade accuracy because of the approximations introduced by remapping. Whether this error is less or greater than the errors that would be introduced by mesh discontinuity depends on the problem.

Advection

Advection is the process by which the velocity field moves mass, momentum, and energy from one computational zone to another. The Lagrangian part of our cycle is almost identical to the calculations done by other two-dimensional Lagrangian programs.^{4,5} However, since the mesh can be remapped in a very general way (not just back to an orthogonal grid), the rather straightforward advection calculations of most Eulerian programs are not applicable. In particular, some two-dimensional Eulerian programs use

Fig. 4

A portion of the flow of a low-density (slave) material past a high-density (master) material. Note the different tangential velocity components at the interface.

boundary-centered, single-component (x or y) velocities. Because our program uses velocity vectors at mesh nodes, the motion of advection boundaries is not usually one-dimensional as it is in a rectangular mesh.

The program calculates the volume swept from one zone into another each time a boundary shifts. The energy and mass density in that volume are determined in one of two ways. If a material boundary lies in or between the adjacent zones or if there is a large density change (as when a shock wave occurs), the density in the zone that loses the volume is used. Otherwise, the densities in the zones near the boundary are interpolated to the center of the advected volume. This interpolation is crucial to obtaining acceptable accuracy in an Eulerian or a remapped mesh because if the donor zone's density is always used, gradients are not preserved. The method of interpolating densities invokes monotonicity conditions to ensure that the new densities are neither less than the preadvection minimum density nor larger than the maximum density near the advection boundary.

Mixed-zone calculations are the most complex in the program. Advection that involves or produces mixed zones requires that the transferred volume be apportioned among the constituent materials in the new zones. Our program infers the information it needs on the positions of mixed-zone components from volume fractions of neighboring zones. To further simplify these calculations, they are done in directional sweeps. Thus, only two zones must be considered at each remapped boundary. Details of the calculation are quite complex but are based on the following constraints: first, a zone cannot donate more of a material than it contains; second, component mass and energy must be conserved; third, total volume must be conserved; and, finally, the sum of the component volumes must equal the total zone volume.⁶

Each node has a portion of the mass of the surrounding zones associated with it. Consequently, as mass is transferred from zone to zone by advection, there is a momentum transfer. The

velocity associated with the mass transfer between nodes is usually interpolated from the node velocities. In some exceptional cases, only one node velocity is used. This process is analogous to the advected volume density determination described above. Again, velocity interpolation is critical. Without it, shocks tend to diffuse and dissipate. Once the momentum transfer has been determined, a node's new velocity is simply its new momentum divided by its new mass. Slave velocities are calculated in the same way as boundary node velocities, since only the slave side remaps.

Astrophysical Applications

We have taken advantage of the great versatility of the ALE mesh to

Fig. 5

Our program can model discontinuities in tangential velocity components at material boundaries and still maintain a continuous mesh by means of special slide nodes. (a) Each slide node has a master (m) and slave (s) component, which can temporarily move somewhat independently according to the velocity of the material with which each is associated. (b) The displacement between master and slave components represents the difference in tangential velocity between the two materials. (c) Before the end of each computational cycle, the slave component remaps back to its master position so that the mesh remains continuous.

model a variety of specific problems in astrophysics.⁷ The astrophysical routines that were added to our general-purpose program were formulated by Jim Wilson.

Several astrophysical problems run on our program required mixed-zone advection. One computation of core collapse in a star during its transition to a supernova involved turbulent mixing (Fig. 6).⁸ The instability was driven by gravitational acceleration of high-density layers overlying lower density layers. This calculation used a spherical Eulerian mesh to greatly simplify the gravitational acceleration calculation.

Another calculation, which employed both slide and mixed-zone advection, modeled the radiation produced by the collision of an asteroid with a neutron star (the remnant core of a supernova explosion).⁹ The initial phase of this problem followed an iron asteroid as it fell toward the star. The

mesh was initially spherical and Lagrangian. Shortly before the collision, small amounts of internal remapping occurred. This calculation included the iron's strength; as expected, it proved to have a negligible effect. Figure 7 shows the distortion of the asteroid that is caused by the strong gravitational field of the star. The result of the free-fall calculation was used to set up the initial conditions for the collision calculation.

In a follow-on problem, a slide along the surface of the neutron star decoupled the velocity of the asteroid from that of the stationary neutron star. The slide automatically provided a curved, rigid boundary at the interface. The asteroid exploded on impact and then flowed over the surface of the star (Fig. 8).

We also used a Lagrangian mesh to validate an Eulerian difference scheme used in an Eulerian mesh program with

Fig. 6

Computed Rayleigh-Taylor overturn during the core collapse of a star entering the supernova stage, showing motions of the fluid regions obtained with a 90-deg model for times of (a) 1.5 ms, (b) 2.5 ms, (c) 3.0 ms, and (d) 3.5 ms. Only the inner portion of the collapse is shown. The central region (at a radius less than 6 km) was excluded from the calculation. Boundaries separate the stellar matter according to its initial density distribution.

which we calculated the angular momentum of a rotating protostar undergoing collapse.¹⁰ Figure 9 shows a closeup view of the inner mesh. Because the Lagrangian-mesh solution had no diffusive effects due to advection, it could be relied on until mesh distortion was large.

Summary

We have successfully developed a large computer code that is quite simple to modify and that uses computer resources efficiently. With its arbitrary Lagrangian-Eulerian mesh, the code allows astrophysicists to model a variety of problems with accuracy and relative ease of use. The code employs a number of physics models such as advection and velocity discontinuities that are particularly useful for a number of astrophysical applications. □

Key Words: ALE; arbitrary-Lagrangian-Eulerian mesh; astrophysical modeling; Eulerian mesh; Lagrangian mesh.

Notes and References

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Fig. 7

Initial gravitational distortion of a spherical asteroid, computed with our program, just before it collides with a neutron star. Note that the scales of the r and z axes differ by a factor of 100. The asteroid is moving to the left toward the star.

Fig. 8

Result of the collision of an asteroid with a neutron star, computed with our program. The asteroid material is flowing around the surface of the neutron star.

Fig. 9

Collapse of a rotating protostar computed with our program. A portion of the Lagrangian grid is shown at the termination of the calculation, indicating the flattening induced by the collapse. At this time, the central region is compressed by a factor of 10^4 over its initial density. Arrows show the velocities of the zone corners. The initial radius of the protostar cloud was 7×10^{11} km.

Collisi and G

Calculations indicate that superclusters of galaxies and galaxies themselves may arise quite naturally in an expanding (Big Bang) universe in which most of the mass is provided by neutrinos with nonzero rest mass.

One hypothesis of the formation of galaxies and clusters of galaxies has been predicated for some years on the idea that the gravitational field responsible for the coalescence of matter into galaxies and clusters of galaxies may arise from unseen (i.e., dark or nonluminous) matter. The total mass of this invisible matter may be several times greater than that of the visible matter, and these two constituents interact only through their gravitational fields.

We have examined this hypothesis quantitatively by numerical experiments.¹ Our three-dimensional simulations of the gravitational clumping of the dark or invisible matter are highly suggestive of the observed distributions of galaxies in superclusters of galaxies. Our one-dimensional calculations, performed to model the gravitational interaction of visible matter with the

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onless Matter Galaxy Formation

invisible matter, are less encouraging. The visible matter tends to be so overheated as it falls into the gravitational potentials of the unseen matter that subsequent condensation of the visible matter into compact galaxies with stars appears a little difficult, but not impossible. Because these one-dimensional calculations may artificially enhance the heating, three-dimensional calculations with the full interactions of visible and invisible matter may be required to establish the feasibility of the dark-matter hypothesis of galaxy formation.

Galaxy Formation: Theories and Problems

Explaining the origin of galaxies and of their large associations (clusters and superclusters of galaxies) has been an intractable problem in astrophysics for a number of years.² A major difficulty arises from evidence that the visible matter in the universe was too homogeneous at red shift $z = 1000$ for such clusters to have formed by now ($z = 0$ now; see the box on p. 52). This evidence is provided by the high isotropy of the 3-K cosmic microwave background we observe today (deviations from isotropy are less than one part in a thousand), which reflects a similar isotropy in the temperature (and, we assume, in the density) of visible matter at the time when the radiation uncoupled from the visible matter (at about $z = 1000$).

We know that density perturbations in cold matter grow in proportion to the change in scale $(1 + z)$ of a normal expanding universe. Hence, unless the matter was isothermal but not isobaric (as has been assumed in a number of recent calculations), the largest possible perturbations in visible matter at $z = 1000$ could only now have become strong enough (i.e., the order of one) to have collapsed and formed galaxies. However, quasars, which may be early forms of galaxies, are observed at red shifts as far back as $z = 4$; by that time, perturbations in

visible matter could have grown only by a factor of 200, which is not enough to lead to the nonlinear collapse required to form galaxies.

Another aspect of the galaxy-formation problem is the question of the "missing mass."³ When the virial theorem (which relates the kinetic and gravitational energy of gravitationally bound systems) is applied to clusters of galaxies, which have the relaxed appearance of bound systems, the kinetic energy is often found to be too high for these systems to be bound. The inference is that these clusters contain dark, invisible matter in addition to the luminous, visible matter.

It has been suggested that objects made of ordinary baryonic matter but with low surface brightness (e.g., objects like the planet Jupiter, or black dwarf stars) could account for the missing mass. However, calculations of light-element synthesis in the Big Bang require a density of baryons in the universe about one-tenth as great as the total mass density inferred from dynamical considerations (such as the virial theorem). This strongly suggests that the dark matter is not baryonic, or, in other words, that it consists of neutral leptons of nonzero rest mass.

During the past few years, our picture of the three-dimensional distribution of galaxies in space (especially our knowledge of the distances to these galaxies) has been improved by extensive observations with both optical and radio telescopes. Astronomers have found that galaxies lie in very large, highly anisotropic distributions known as superclusters.^{4,6} A recently found supercluster spans more than 100 deg in the sky. Given the number of galaxies in them, supercluster masses must range from 10^{15} to $10^{17} M_{\odot}$ (where $M_{\odot}$ is the mass of the sun, equal to 2×10^{33} g). These superclusters can be filamentary or flattened "pancakes" and form cellular structures surrounding vast regions of space, known as holes or voids, in which no galaxies are observed (Fig. 1).^{7,8} These superclusters and voids have been

Fig. 1

Our solar system (black dot) is at the center of this projection, which shows the three regions of the sky in which galaxy red-shift surveys have been completed. The galaxies in these regions are clustered into pancakes, filaments, and voids: these structures are called superclusters, even though they do not represent true gravitational clustering. Superclusters have dimensions of tens to hundreds of

millions of light years, and contain 10^{15} to $10^{17} M_{\odot}$. Typical galaxies, by contrast, are 0.03 million light years across and contain 10^{11} to $10^{12} M_{\odot}$. Clusters of galaxies, which are true gravitational associations, are a few million light years across and contain hundreds or thousands of galaxies.

measured to be 30 to 300 million light years across. In comparison, ordinary clusters of galaxies are 5 million light years across and galaxies themselves are about 0.03 million light years across.

Galaxies, which are observed to be moving at speeds of hundreds of kilometres per second, could have crossed only a small fraction of the 30 to 300 million light years during the age of the universe (10 to 20 billion years). Thus, the observed cellular structure could not have formed purely from gravitational clustering of smaller objects. Rather, superclusters and voids must represent primordial structures or imprints of the initial Big Bang. They therefore provide important clues about the evolution of the universe. Such primordial structures would be contained in both the baryonic and nonbaryonic components of the universe from the beginning.

Are Neutrinos the "Missing Mass"?

The search for the nature of the invisible or dark matter takes us from the scale of the universe to the smallest scale known—that of elementary particles. A few years ago, some experimenters measured the rest mass of the neutrino emitted during the beta decay of tritium to be about 30 eV.^{9,10} While this experiment is very difficult and the results must be confirmed, a neutrino with nonzero rest mass provides a tempting source for the missing matter because, as discussed below, the preferred scale of gravitational clumping of these neutrinos is the same—tens of millions of light years—as the scale of the galaxy superclusters. Such particles could also carry the perturbations that induce gravitational collapse.¹¹⁻¹³

Neutrinos would have decoupled from ordinary matter and its associated electromagnetic radiation well before $z = 1000$. If massive electron neutrinos do not exist, other candidates for the invisible matter—such as other flavors of neutrinos, or perhaps photinos, or even gravitinos—are available in the particle physicists' bestiary. We assume the existence of some sort of weakly

interacting ("collisionless") particles, which, for convenience, we will hereafter call neutrinos, with rest mass of roughly 10 to 100 eV. We also assume that these particles have arbitrary abundance. With the above discussion as our starting material, we proceed to investigate the role of these neutrinos in the development of the present structure of the universe.

Our approach to the problem has been guided by the work of Zel'dovich and his colleagues, who proposed the "pancake" theory of galaxy formation.¹⁴ The essence of this idea as it applies to our work is as follows for the case of neutrinos of nonzero rest mass. The neutrinos would decouple from ordinary matter at a period during the expansion of the universe when the temperature is about 1 MeV (10^{10} K). (The interaction of neutrinos with matter varies inversely with the square of the temperature.) If, before decoupling, the neutrinos had a "white noise" density perturbation spectrum (i.e., one in which the amplitudes of all perturbation wave numbers have equal probability), then, after decoupling, all perturbations of wavelength smaller than the horizon distance will rapidly damp out by phase mixing.^{15,16} (The horizon distance is the distance over which light signals have been able to communicate up to that time.) In a density wave of relativistic neutrinos, all neutrinos have the same speed, but individual particles are moving in different directions; they will soon fall out of step and damp the wave.

As long as the neutrino energies are relativistic, the neutrino pressure is too high for appreciable gravitational growth of density perturbations to occur.¹⁷ The neutrinos become nonrelativistic when their temperature falls to a value equal to their rest mass; for 10- to 100-eV neutrinos, this occurs at a scale factor of about 60 000 (see figure in the box on p. 52). After the neutrinos become nonrelativistic, the neutrino temperature falls rapidly and large-scale perturbations can grow readily.

As long as ordinary matter and radiation are strongly coupled, the matter pressure is kept high and the growing

neutrino perturbation can impress only very small perturbations on the matter density. Thus, when radiation and matter decouple at about $z = 1000$, perturbations in the neutrino density can

be much larger than those in the matter density.

The perturbation length scale not damped out by velocity dispersion is given roughly by $\lambda = 10^6(30/m_\nu)$ light

Big Bang Cosmology

The evolution of the average features of a standard Big Bang cosmological model forms the background for our calculations on the formation of superclusters of galaxies and galaxies themselves.

At the top of the figure is the time, in seconds, since the start of the Big Bang. At the bottom is the scale factor, which is the ratio of the present value of any typical dimension to its value at the earlier time. The quantity z , the so-called red shift, is a measure of the amount by which any wavelength was shorter then than it is now. The scale factor is equal to $z + 1$, as indicated at the bottom of the figure.

(In astronomical observations, objects far away from us have very high red shifts. By definition, the red shift z is zero now; larger values of z correspond to times in the distant past. Therefore, distance is roughly proportional to red shift. The light we see from these distant objects actually left them a long time ago. Thus, we are seeing these objects as they were then, that is, we are actually looking back in time when we view high- z objects.)

As shown in this figure, electron-positron pairs recombine at about 100 s after the Big Bang. At about the same time, neutrinos cease to interact, except gravitationally, with all other matter (nuclei and electrons). At 10^9 s, neutrinos with a rest mass of 30 eV cool enough to become nonrelativistic; after this time, the neutrino temperature T_ν drops rapidly and the neutrino density ρ_ν becomes the dominant density in the universe. At 10^{13} s, protons and electrons combine to form hydrogen atoms (and a few helium atoms), and thereafter radiation is decoupled from matter, leaving the present-day 3-K background radiation. The atoms cool rapidly, as the neutrinos did earlier. The earliest galaxies (quasars) form at about 2×10^{16} s after the Big Bang. Horizon length is the distance over which a light signal could have travelled since the beginning of time. The associated horizon mass is the mass of all matter that could have been traversed by a causal signal (gravitational waves, neutrinos, sound waves) up to that time.

years, where m_ν is the neutrino rest mass in electron volts. The corresponding mass scale is of the order of $10^{16}(30/m_\nu)^2 M_\odot$. For 30-eV neutrinos, these length and mass scales are 100 million light years and 10^{16} solar masses, just the scales of superclusters. This remarkable agreement makes massive neutrinos an attractive candidate as the suppliers of the "missing mass" and has led to wide interest in these models.

Investigating the Distribution of Matter in the Universe

When a small perturbation grows to an amplitude of the order of unity, its growth rate increases. Out of all possible directions of collapse, one direction eventually dominates and a pancake-like configuration will evolve.¹⁴ The pancakes are the walls between the voids in superclusters. In our investigation, we considered the evolution of the universe over its last thousand-fold linear expansion, during which time it consists of neutrinos, ordinary matter made up of partially ionized hydrogen and helium atoms, and a uniform gas of photons that interact weakly with the electrons.

We followed two lines of numerical analysis to elucidate the above picture. First, we made one-dimensional calculations of the collapse of simple plane-wave neutrino density perturbations to form pancakes and followed the motion of ordinary matter as it fell into the resulting gravitational potential wells. We looked for evidence that the resulting pancakes of ordinary matter could fragment into objects of galactic mass. The behavior of both the neutrinos and ordinary matter, with all relevant details such as heat flow and the radiative cooling of matter, can be calculated accurately in one dimension.

Then, we made three-dimensional calculations of the gravitational growth of density perturbations in a pure neutrino gas. The three-dimensional calculations show the morphology of the resulting cellular structure and give some insight into the applicability of

the one-dimensional pancake model to the real three-dimensional universe.

One-Dimensional Calculations

The one-dimensional model for the study of collapse to form pancakes has several ingredients.^{18,19} The overall expansion of the universe is superimposed on the dynamics of our system. We calculate only a plane slice of the universe, with its boundaries comoving with the overall expansion. The thickness of the slice is taken to be the wavelength of the largest perturbation in neutrino density at the given time. This wavelength is close to the horizon size at the time the neutrinos became nonrelativistic. In the direction perpendicular to the slice, the universe is assumed to be periodic. The initial perturbation is taken in the form of a small-amplitude sinusoidal density wave; the peak of the distribution is taken at the origin of our coordinates so that, by symmetry, only one half of the pancake must be calculated.

For numerical convenience, we represent the neutrinos by about a thousand point particles whose masses are scaled so as to give the proper total neutrino mass. We are concerned only with times long after the decoupling of ordinary matter (electrons and nuclei) from electromagnetic radiation. The residual electromagnetic radiation is assumed to be uniformly distributed in space.

The matter is treated as a gas by the hydrodynamic equations. The matter interacts with the neutrinos and with itself through the gravitational field. The matter and radiation interact by emission processes (such as bremsstrahlung and recombination) and also by Compton energy exchanges by elastic electron-radiation collisions. After collapse, the matter may be hot enough that heat conduction by electrons and/or ions is important. (This leads to an ambiguity in the results, because a very small chaotic magnetic field could prevent heat conduction by electrons. Whether a magnetic field exists in the universe at such early times is unknown.)

Another important characteristic of matter is its degree of ionization, which we estimated both by assuming complete equilibrium (and applying the Saha equation) and by assuming that the degree of ionization is determined by the balance of recombination by emission of radiation and ionization by electron-atom collisions. The latter is the most appropriate model for ionization in the pancake gas, but, in the mass range of interest, it turns out to matter little which ionization model is used. The dynamics of the neutrinos and the matter gas are treated in the Newtonian approximation.

The behavior of the collapse is as follows (see Figs. 2 and 3). The neutrino perturbations grow larger and larger, and the neutrinos move toward the central symmetry plane. Initially, the matter is rather cold (with a temperature of only a few tens of kelvins) and the corresponding matter pressure is so low that the matter almost keeps up with the infalling neutrinos. The trajectories of the neutrinos eventually form a caustic, i.e., neutrinos pass through each other at the central plane. At the moment of caustic formation, the neutrino density increases to infinity (in the idealized case of a cold, smooth neutrino distribution); then, the particles begin passing through the central plane and the density falls. The matter dragged by the neutrinos becomes increasingly dense, but it is so strongly self-interacting that it cannot pass through itself.

Fig. 2

Density and temperature profiles during the collapse of a typical supercluster formed by $4 \times 10^{15} M_{\odot}$ of neutrinos and $4 \times 10^{14} M_{\odot}$ of ordinary matter, for a neutrino mass of about 60 eV. Each graph shows the neutrino and matter densities ρ_{ν} and ρ_m , in g/cm^3 , and the matter temperature T_m , in millions of kelvins, as a function of distance from the symmetry plane. (a) Conditions at the time the neutrinos start passing through themselves, 0.478 billion years after the Big Bang. (b) 0.499 billion years, (c) 0.533 billion years, and (d) 0.582 billion years after the Big Bang. The wavelength of the initial neutrino density perturbation was about 15 million light years, 100 times the full width of the graph.

After the neutrinos pass through the origin, a shock wave forms in the matter that is trying to follow. The shock wave moves out from the origin and becomes increasingly strong as it encounters matter that has higher kinetic energy (because it has had farther to fall into a deeper gravitational potential). The matter near the origin is usually fairly cool (a few thousand kelvins), but matter falling in late (which has fallen farther and into a deeper gravitational potential) may reach tens of millions of kelvins as a result of being stopped by the shock wave. The cool, dense central region tries to cool

Fig. 3

Density and matter temperature profiles for the calculation of Fig. 2 at 6.5 billion years after the Big Bang, a time not long ago. The peak neutrino density is $2 \times 10^{-27} \text{ g}/\text{cm}^3$, and the peak matter density is $1.3 \times 10^{-27} \text{ g}/\text{cm}^3$. The peak temperature, decreased by the overall cosmic expansion, is still high enough to produce a soft x-ray background of the order of magnitude of the observed x-ray background.

further by emitting radiation, but enough heat flows in by conduction from the outer regions (if they are too hot) that a cool inner region cannot persist.

The principal goal of these calculations is to determine whether the matter will be cool enough that it can fragment into objects of galactic mass after it has collapsed into a pancake in the field of the neutrinos. A uniform cloud of gas is gravitationally unstable if its mass is at least equal to the Jeans mass, for which the gravitational potential energy and the thermal energy of the cloud are approximately equal. For the pancakes to fragment into galaxies, we require that the Jeans mass be rather less than $10^{11} M_{\odot}$.

The Jeans mass depends only on the temperature and density of the cloud and is proportional to $T^{3/2}/\rho^{1/2}$. We calculate the Jeans mass from the temperature and density of our central zone. Although this is not the best possible estimate of the mass at which our matter cloud will fragment, it is easily calculated and is useful for comparing the results of calculations with different initial conditions and parameters. The Jeans mass is the minimum mass that can fragment; the true fragmentation mass is probably as much as an order

of magnitude greater. (The Jeans mass definition assumes a spherical gas distribution; in the pancake, the matter associated with a Jeans mass is very flat.)

A number of crucial quantities needed in these calculations are not well known. The rest mass of the neutrino is uncertain by a factor of three or so. The present density of the universe is uncertain to within an order of magnitude. The expansion rate of the universe is uncertain by at least a factor of two. Because of these observational uncertainties, we carried out our calculations for a range of parameters, rather than for unique values. We studied the following parameters:

- The wavelength of the perturbation λ and the related mass $M_{\text{TOT}} = \rho \lambda^3$, where ρ is the neutrino density.
- The ratio W of mass in neutrinos to mass in baryons; W must be about ten to account for the missing mass.
- The time t_c at which neutrinos begin passing through themselves. This time is related to the perturbation amplitude—greater amplitude leads to earlier crossing. The associated red shift is called z_c .
- The ratio of thermal conductivity to its usual no-magnetic-field value.

To narrow the range of parameters, we took the age of the universe to be 18 billion years²⁰ and assumed a spatially flat (marginally closed) universe. Together, these assumptions imply a Hubble constant of 40 km/s · Mpc (where Mpc stands for megaparsecs).

Figure 4 summarizes the results of calculations performed with no heat conduction. The Jeans mass is comfortably small compared to galactic masses (i.e., less than the $10^{11} M_{\odot}$ value noted above), so that fragmentation is possible. The fraction of cold matter does become rather small for the high-mass range, $10^{16} M_{\odot}$. The no-conduction models look very encouraging, but they may be unrealistic.

Next, we ran a sequence of calculations with various masses and with heat conduction. In the high-mass cases, the matter falling in late heats the originally cold inner region enough that its Jeans mass is large compared to

Fig. 4

The Jeans mass (gold) of the central cooled matter vs the mass of baryons in the supercluster for no heat conduction, a crossing red shift $z_c = 5$, and a ratio of mass in neutrinos to mass in baryons $W = 10$. The figure also shows the fraction of baryonic matter that is cool (blue) for two different times, 7 and 18 billion years.

galactic masses. Figure 5, which summarizes these calculations, shows that the time available for the cold gas to collapse becomes shorter for high-mass systems. The free-fall time of the cold central gas is about 5×10^{15} s, so that the cold matter could collapse in these times. However, because the matter contained in a galactic mass is in a flat pancake of large radius, across which the sound travel time is about 10^{17} s, galaxy formation is questionable under these circumstances.

We also took a baryon mass of $4 \times 10^{15} M_{\odot}$ and varied the heat conduction coefficient to see how sensitive our results are to heat conduction. Figure 6 shows a high degree of sensitivity to the assumed heat conduction. A strong, tilted, straight magnetic field could produce conductivity reductions such as these.

For a baryon mass of $1.3 \times 10^{15} M_{\odot}$, we varied the crossing or caustic time to see if the results were sensitive to the initial amplitude. For crossing times from 5×10^{15} s to 8×10^{16} s, the ratio of time of heating to time of crossing is constant to 5% at a value of 1.7; the Jeans mass is also fairly insensitive to crossing times. This means that the results are insensitive to the initial amplitude of the perturbation, which is good because the initial amplitude of perturbations is hard to justify except that the magnitude selected works.

All the above calculations were done with a ratio of mass in neutrinos to mass in baryons of $W = 10$, because we wanted a large fraction of neutrino mass to explain the "missing mass." We also ran a series of calculations with the ratio mass in neutrinos to mass in baryons of 3:1 with heat conduction. The ratio in times of heating to crossing times (i.e., the relative time available for galaxy formation) is about the same as for baryon cluster masses ten times as small. That is, the heating problem goes away rapidly as the fraction of mass in neutrinos is reduced. To produce superclusters of $10^{16} M_{\odot}$, therefore, it would appear that we must either have heat conduction suppressed or only have the mass in neutrinos a little greater than the mass in baryons.

The Three-Dimensional Model

We also made three-dimensional numerical calculations to model the gravitational dynamics of heavy (i.e., 30-eV) neutrinos.²¹⁻²³ By going to three dimensions, we are able to study the evolution of a system of interconnected superclusters and voids. In this work, we address only the problem of the distribution of the heavy neutrinos, which form the potential wells within which galaxies will form. We did not

Fig. 5

Times t_h for heating central matter enough that it cannot fragment into galaxies, as a function of baryon mass. The figure also shows the ratio of time that matter is cool to crossing time t_c . The crossing time is characteristic of times for large changes, so this ratio must be of order unity for fragmentation to be likely. For scale we include the neutrino mass associated with the baryonic mass. As in Fig. 4, the ratio of mass in neutrinos to mass in baryons is $W = 10$.

Fig. 6

Heating times t_h for a supercluster of baryonic mass $1.3 \times 10^{15} M_{\odot}$ and neutrino mass $1.3 \times 10^{16} M_{\odot}$ as a function of the ratio of the heat conduction C to its no-field value C_0 . The crossing time is t_c .

calculate the details of the matter collapse, such as shock heating and x-ray cooling, that we examined in great detail with our one-dimensional code. However, in three dimensions, we are not constrained to assume a pancake shape for a supercluster. Rather, we can put in a spectrum of density perturbation waves moving in a variety of directions, let the system evolve, and study the shapes of the resulting structures. In this way, we can learn about the generic shapes of the superclusters and voids that constitute the large-scale structure of the universe.

The Numerical Method

The numerical method we use in our three-dimensional work is known as the cloud-in-cell technique. We use a three-dimensional grid, typically of $32 \times 32 \times 32$ cells, in a Cartesian (rectangular) coordinate system. At the beginning of the calculation, we place a number of overlapping clouds in each cell. Each cloud is a cube, the same size as a cell, with a mass scaled to give the desired total neutrino mass.

When the system is perturbed, the clouds migrate from cell to cell under the force of their own gravity. As dense clusters form, there will be more clouds per cell in the dense regions where the superclusters lie than in the empty (void) regions. To adequately cover all regions of space with the clouds through all phases of evolution of the system, we usually take 27 clouds per cell at the start of the calculation. With our $32 \times 32 \times 32$ grid, this gives nearly 900 000 clouds.

As in the one-dimensional calculations, we use co-moving coordinates, so that our grid expands along with the overall expansion of the universe. This means that we calculate the motion of the particles relative to the overall expansion. It is convenient to use these "peculiar velocities" because they are responsible for the clumping of galaxies that astronomers observe.

Schematically, the code works as follows. On each time step, the cloud velocities carry them to their new positions. We then calculate the new density in each grid cell; since the clouds

are extended distributions rather than point particles, we include only the portion of a cloud within a cell when computing the density within that cell. (The cell size is thus the limiting resolution of our code.) Knowing the density, we then use Fourier transform techniques to solve Poisson's equation for the gravitational potential. This potential gives us the new gravitational acceleration, with which we update the particle's velocity for its motion in the next step.

Because we run our code on Cray computers, we wanted to take advantage of the fast vector operations available on these machines. Most of our computing time is taken up in updating the density array as just described—finding the new positions of the particles and "scattering" these positions into the density array. Since we do not know beforehand where each particle will go on each time step, we cannot vectorize this part of the code by conventional means. Instead, we used the STACKLIB software routines,²⁴ which permit such scattering operations to be done while exploiting the full power of the Cray hardware instruction set. STACKLIB helped us to increase the speed of our code by a factor of three.

The Physical Problem

Our three-dimensional calculations start at a red shift of about $z = 1000$, just after the era of matter and radiation decoupling. At this time, the three major components of the cosmos—invisible matter (in the form of heavy neutrinos), ordinary matter (mainly partially ionized hydrogen), and electromagnetic radiation—are all decoupled from one another. The mass in the cosmos is dominated by the heavy neutrinos; the hydrogen gas that will later form the luminous galaxies pours into the potential wells formed by the neutrinos, as discussed above. In these three-dimensional calculations, we follow only the gravitational dynamics of the neutrinos in order to map out the large-scale potential wells in the universe.

The neutrino clouds are distributed in a spectrum of density enhancements

or perturbations, with a short-wavelength cutoff λ_s due to earlier phase mixing, as discussed above. We let this shortest wavelength fit across our grid four times, thus resolving it with eight cells on our $32 \times 32 \times 32$ grid. It appears from the results that the cutoff is responsible for the appearance of a cellular structure in these models. Certainly, λ_s gives the characteristic size of supercluster clumps and voids: for a neutrino mass of about 30 eV, λ_s is about 100 million light years, a typical supercluster or void dimension.

We assume small initial amplitudes for the neutrino density perturbations, typically 0.003 of the background density at $z = 1000$. This amplitude is consistent with the measurements of the isotropy of the 3-K cosmic background radiation. Since the neutrinos are not strongly coupled to the photons at $z = 1000$, when the photons decouple, a neutrino density perturbation this big will not distort the radiation enough that we could measure it. The overdense regions of neutrinos continue to collapse as our calculation proceeds. In these local areas, the self-gravity of the neutrinos is winning the battle against

the expansion of the universe, allowing the region to grow more and more dense. Simultaneously, the underdense regions are losing the battle and are being evacuated as neutrinos flow from these "voids" into the collapsing, denser regions.

Results

The neutrino collapse is highly anisotropic, proceeding at different rates in the three directions. If one axis collapses first, a pancake results. If two axes collapse faster than the third (which may even be expanding), a filament is formed.

Figure 7a shows neutrino isodensity surfaces (within which the density is at least twice the average value) at a scale factor of five ($z = 4$). The first galaxies would be expected to form at about this time. As expected, the neutrinos are distributed in a cellular structure of interconnecting pancakes and filaments, on scales of about 100 million light years.

In Fig. 7b, the shape of the voids becomes clearer. Within the isodensity surfaces in this figure, densities are at most half the average value. The voids

Fig. 7

(a) Isodensity contours of dense regions (within which neutrinos have densities at least twice the average value) at $z = 4$. Galaxies would begin forming at about this time. The neutrinos are distributed in a cellular structure of interconnecting pancakes and filaments on scales of millions of light years. (b) Isodensity contours of underdense regions (within which neutrinos have densities at most half the average density), also at $z = 4$. The voids are approximately spherical, giving the overall structure a Swiss-cheese appearance.

Fig. 8

isodensity contours at the end of the simulation (intended to represent the present time). (a) The green contours (ρ at least $2\rho_{avg}$) clearly show the cellular structure. (b) The blue contours (ρ at least $4\rho_{avg}$) show that the filaments become denser toward their centers. (c) The red

contours (ρ at least $10\rho_{avg}$) show that the greatest densities are found at the intersections of the filaments. (d) Green, blue, and red contours have been superimposed.

are roughly spherical, so that the overall neutrino distribution is somewhat like a Swiss cheese.

Conditions at the present time ($z = 0$) are given in Fig. 8. This figure shows that the densest regions are at the intersection of structures. The green isodensity surfaces (density at least twice the average value) clearly show the cellular structure. The surfaces shown in blue (density at least four times average) show that, as might be expected, the centers of filaments are denser than the outer regions. Finally, the surfaces in red (density at least ten times average) show that the densest regions of all are at the intersections of the filaments. This last feature arises not only because the intersection of two structures is denser than either structure, but also because the neutrinos are flowing down the filaments into the dense clusters at the intersections. Such three-dimensional flow prevents the pancakes from staying too flat and could be an important modification to the one-dimensional cooling calculation described above. The matter in pancakes will flow adiabatically into these deeper potentials, where it will adiabatically compress to higher density, thus favoring Jeans fragmentation.

The Future

The calculations presented here represent important first steps toward modeling the large-scale structure of the universe. We have a detailed one-dimensional model that includes microphysics and a larger-scale, three-dimensional model that gives us the macrophysics of the neutrinos. The next step is to combine these approaches to produce a three-dimensional simulation with hydrodynamics plus neutrinos. At that stage, we will calculate the radiation losses and heat flows on a fully three-dimensional slice of the universe. This will allow us to resolve the question of whether galaxies and clusters of galaxies can form in this model in a manner similar to the real universe.

Key Words: Big Bang; computer modeling; cosmology; galaxy formation; neutrino collapse; neutrinos; numerical simulation; STACKLIBE; supercluster; universe—evolution, structure.

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