TECHNICAL MEMORANDUM

21
ANL-CT-81-29

①
MASTER

146
10-27-81
B7956
⑧

EXPERIMENT AND ANALYSIS OF INSTABILITY
OF TUBE ROWS SUBJECT TO LIQUID CROSSFLOW

by

S. S. Chen and J. A. Jendrzejczyk

BASE TECHNOLOGY

Prepared for
U. S. DEPARTMENT OF ENERGY
under Contract W-31-109-Eng-38

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

EXPERIMENT AND ANALYSIS OF INSTABILITY
OF TUBE ROWS SUBJECT TO LIQUID CROSSFLOW

by

S. S. Chen and J. A. Jendrzejczyk

Components Technology Division

DISCLAIMER

September 1981

TABLE OF CONTENTS

	<u>Page</u>
NOMENCLATURE	viii
ABSTRACT	ix
I. INTRODUCTION	1
II. TEST SECTION	2
III. TEST PROCEDURES	5
IV. EXPERIMENTAL DATA	6
V. ANALYTICAL AND EXPERIMENTAL RESULTS	24
VI. CONCLUSIONS	37
ACKNOWLEDGMENT	38
REFERENCES	39

LIST OF ILLUSTRATIONS

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	A row of five tubes	3
2	Two test cases	4
3	Tube displacements and frequency spectra of tube 3 in the drag direction at various temperatures in air and still water	7
4	Tube displacement as a function of flow velocity in Test a.1 at 23°C	12
5	Tube displacement as a function of flow velocity in Test a.1 at 8.9°C	13
6	Displacement components of tube 3 at 16.7°C (Tests b.1 and b.2)	14
7	Displacements of tube 2 in Test a.1 at 23°C	15
8	Displacement of tube 4 in Test a.1 at 16.7°C in the lift direction	16
9	Frequency spectra of tube displacements of tube 3 in the lift direction in Test a.1 at 16.7°C; sequence of tests is from bottom to top	18
10	Frequency spectra of tube displacements of tube 2 in Test a.1 at 8.9°C	19
11	Tube orbital paths of tube oscillations in Test a.1 at 23°C	20
12	Tube orbital paths of tubes at instability in Tests a.1, a.2, and a.3	21
13	Schematic of the tube row	25
14	Averaged modal damping ratio in still water	30
15	Averaged modal damping ratio in air	31
16	Calculated natural frequencies and mode shapes for Test b.1 in still water	33
17	Critical flow velocity for Case a	34
18	Critical flow velocity for Case b	35

LIST OF TABLES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1	Averaged natural frequency and damping for Test a.1	8
2	Averaged natural frequency and damping for Test a.2	9
3	Averaged natural frequency and damping for Test a.3	10
4	Averaged natural frequency and damping in the drag direction for Test b.1	11
5	Measured critical flow velocity and associated instability frequency	23

NOMENCLATURE

c_i, e_i	Damping coefficients of tube i in the x and y directions
D	Diameter of tubes
$E_i I_i$	Flexural rigidity of tube i
f	Averaged natural frequency (Hz) in air
l_e	Tube length
m_i	Mass per unit length of tube i
R	Radius of tubes
t	Time
U	Flow velocity
u_i	Displacement component in the lift direction
v_i	Displacement component in the drag direction
z	Axial coordinate
$\alpha_{ij}, \beta_{ij}, \sigma_{ij}, \tau_{ij}$	Fluid added mass coefficients
$\alpha'_{ij}, \beta'_{ij}, \sigma'_{ij}, \tau'_{ij}$	Flow-velocity-dependent damping coefficients
$\bar{\alpha}_{ij}, \bar{\beta}_{ij}, \bar{\sigma}_{ij}, \bar{\tau}_{ij}$	Fluid viscous damping coefficients
$\bar{\alpha}'_{ij}, \bar{\beta}'_{ij}, \bar{\sigma}'_{ij}, \bar{\tau}'_{ij}$	Fluid damping coefficients
$\alpha''_{ij}, \beta''_{ij}, \sigma''_{ij}, \tau''_{ij}$	Fluidelastic stiffness coefficients
λ_i	Eigenvalues of an added mass matrix
μ_{ij}, ν_{ij}	Parameters defined in Eqs. 10
δ_m	Mass-damping parameter ($= \frac{2\pi\zeta_m}{\rho D^2}$)
ζ	Averaged modal damping ratio in air
ζ_i, ζ'_i	Modal damping ratio in air of tube i
$\bar{\zeta}_i, \bar{\zeta}'_i$	Modal damping ratio in still fluid
ρ	Fluid density
ϕ_{im}	The m th orthonormal modal function of tube i
ω_i, ω'_i	Circular natural frequency of tube i in air
$\bar{\omega}_i, \bar{\omega}'_i$	Circular natural frequency of tube i in water
Ω_i	Coupled natural frequencies in water

ABSTRACT

A tube array subjected to crossflow may become unstable by either one or both of the two basic mechanisms: velocity mechanism and displacement mechanism. The significance of these two mechanisms depends on the mass-damping parameter. The velocity mechanism is dominant for tube arrays with a low mass-damping parameter, and the displacement mechanism is dominant for tube arrays with a high mass-damping parameter.

This report presents an experimental and analytical investigation of tube rows in liquid crossflow. The main objective is to verify a mathematical model and the transition between the two mechanisms at the intermediate values of mass-damping parameter. Tests of two tube rows are conducted to determine the critical flow velocity as a function of system damping. Experimental and analytical results are found to be in good agreement.

Based on the results of this study, the following conclusions can be drawn:

1. In general, instability of a tube array in liquid flow is controlled by fluid damping force and in gas flow by fluidelastic force.
2. There exists a finite jump in the critical flow velocity at a certain value of mass-damping parameter.
3. The effect of detuning on the critical flow velocity caused by the displacement mechanism is much larger than that by the velocity mechanism.
4. Different instability criteria have to be employed for the two instability mechanisms.

The mathematical model developed is verified by the experiments and has established the foundation for the development of a design guide to avoid detrimental dynamic instability.

I. INTRODUCTION

A general theory of stability of tube arrays subjected to crossflow was developed recently.¹ It is shown that dynamic instability may be caused by two basic mechanisms:

(1) **Velocity Mechanism:** The dominant fluid force is proportional to the velocity of tubes; this force is called fluid-damping force. Depending on the reduced flow velocity, the dominant fluid-damping force may act as an energy-dissipation mechanism or an excitation mechanism for tube oscillations. When it acts as an excitation mechanism, the system damping is reduced. Once the modal damping becomes negative, the tubes lose stability. This type of instability is called fluid-damping-controlled instability.

(2) **Displacement Mechanism:** The dominant fluid force is proportional to the displacement of tubes; this force is called fluidelastic force. The dominant force may affect the natural frequency as well as modal damping. As the flow velocity increases, the fluidelastic force may reduce the modal damping. When the modal damping becomes negative because of the fluidelastic force, the tubes become unstable by the displacement mechanism; this type of instability is called fluidelastic-stiffness-controlled instability.

In most cases, either the velocity mechanism or the displacement mechanism will be dominant. In general, instability is attributed to the velocity mechanism for small mass-damping parameter (δ_m) and to the displacement mechanism for large δ_m .

This report presents an experimental and analytical investigation of the instability phenomena of tube rows in liquid crossflow. Two series of tests were performed to establish the critical flow velocity as a function of the mass-damping parameter and to verify the existence of the discontinuity of the critical flow velocity at a certain value of δ_m . Experimental data and analytical results are found to be in good agreement.

II. TEST SECTION

The experiments are performed in the rectangular flow channel, 10.7 cm (4.2 in.) wide and 25.9 cm (10.2 in.) high, of a test chamber connected to a water loop with a maximum flow rate of $0.052 \text{ m}^3/\text{s}$ (700 gpm). The rectangular channel extends for more than 20 equivalent diameters upstream of the test chamber. The detail of the test chamber is the same as that used for earlier experiments for tube arrays in crossflow.²

The arrangement of the tube row is shown in Fig. 1. Tube outside diameter (D) is 1.59 cm (5/8 in.), and tube wall thickness is 0.079 cm (1/32 in.). The transverse pitch-to-diameter ratio is 1.35.

Each tube element is suspended as a simply supported beam on two O-rings mounted 91.4 cm (36 in.) apart, permitting lateral vibration with equal flexibility in all directions. The O-rings are seated in the compression plates. Two sets of tubes with different lengths of overhang as shown in Fig. 2 are tested: Case (a), tubes with an overhang of 35.6 cm; and Case (b), tubes with an overhang of 5.1 cm..

Three sequences of tests are performed for Case a: (1) all tubes are in tune; (2) a weight of 66.3 grams is attached to the free end of tubes 2 and 4; and (3) a weight of 66.3 grams is attached to tube 3 only. These tests will be referred to as Tests a.1, a.2, and a.3.

Two sequences of tests are performed for Case b: (1) all tubes are in tune; and (2) tubes 1 and 2, and tubes 4 and 5 are connected together using a rubber plug, which is pushed into the gap between the overhang ends of these tubes. These tests are referred to as Tests b.1 and b.2.

The instrumentation for the tubes consists of two accelerometers mounted at the middle of a tube with orientation to be sensitive in the drag and lift directions. The accelerometers are installed by means of a small mounting block attached to the inside of the tubes and the leads brought to the outside. The tube accelerometer outputs are amplified by charge amplifiers and coupled to double-integration circuits and an analog tape recorder. Displacement signals obtained from the double integrators are then analyzed using the FFT analyzer.

Fig. 1. A row of five tubes

Fig. 2. Two test cases

III. TEST PROCEDURES

Tests for each case are made in air, still fluid, and flowing fluid.

1. Air Environment

Testing in air is performed to determine the natural frequency and damping of each instrumented tube. The tube is excited by plucking it at the free end. The transient response of the tube is recorded. Then, the damping is obtained from the log decrement of the displacement trace, and the natural frequency is determined from the power spectrum of the displacement. Each instrumented tube is tested in two orthogonal directions.

2. Still Fluid

Uncoupled natural frequency and modal damping ratio are determined using the same procedure as in air. In this test, all the tubes surrounding the tubes being tested are held at the overhung end by hands to eliminate responses of those tubes.

3. Flowing Fluid

The flow velocity is increased at small intervals. At each flow velocity, the displacement signals in the lift and drag directions are recorded on a magnetic tape for several minutes for subsequent analysis. The test is terminated when tube impact occurs.

Tube damping depends on water temperature; this is attributed to O-rings, whose characteristics are a function of temperature.² Therefore, tube damping can be controlled by controlling water temperature. In each test sequence, the tube row is tested under several different temperatures: in-air test at room temperature, and in-fluid and in-flow tests at several loop temperatures with one of them being equal to room temperature.

IV. EXPERIMENTAL DATA

1. Natural Frequency and Damping

Typical tube responses in air and in still water at several different temperatures are shown in Fig. 3. Tube natural frequency can be determined from the frequency spectra and damping from the displacement-time traces. Natural frequency and damping are measured for each tube in the lift and drag directions and then averaged. The results are given in Tables 1 to 4.

Both natural frequency and damping in still water increase with decreasing temperature. The effect on natural frequency is not very significant. But damping increases significantly at lower temperatures.

2. Tube Responses in Flow

Figures 4 to 6 show the tube displacements of several tests as a function of flow velocity. The movements of the tubes are strongly dependent on flow velocity. At low flow velocities, the motion is small. As the flow velocity increases, tube displacement increases. When the flow velocity exceeds a certain value, rapid increase in vibration occurs; the threshold flow velocity is called the critical flow velocity. In the flow-velocity range tested, tube displacement in the lift direction is larger than in the drag direction.

Tube responses depend on damping, which is a function of water temperature. Figures 4 and 5 show significant differences in responses at two different temperatures. At 23°C, the tubes become unstable at 1.22 m/s, but at 8.9°C, up to about 4.5 m/s, the tubes are still stable.

Tube responses also depend on frequency variation among the constituent tubes. Figure 6 shows the displacements of tube 3 in Tests b.1 and b.2. When the tubes are in tune, the tubes become unstable at a lower flow velocity (Test b.1). When the tubes are out of tune, the critical flow velocity increases (Test b.2).

Tube responses exhibit nonlinear characteristics as illustrated in Figs. 7 and 8, in which tube displacements are given as a function of flow velocity. In these tests, the flow velocity is increased at several intervals until instability occurs. Then the flow velocity is gradually reduced. Tube displacements with decreasing flow rate are different from those of increasing flow rate. The tubes are still in unstable motion, even though the flow velocity is reduced to those lower than the critical value. With further decrease in flow velocity, the tubes eventually regain stability, but the flow velocity to regain stability is smaller than the critical value.

Fig. 3. Tube displacements and frequency spectra of tube 3 in the drag direction at various temperatures in air and still water

Table 1. Averaged natural frequency and damping for Test a.1

Medium		In Air	In Still Water								
Temperature (°C)		23.0	42.8	23.0	16.7	11.1	9.4	9.1	8.9	8.3	7.8
Averaged Natural Frequency (Hz)		31.0	24.8	25.4	26.2	26.1	26.5	26.6	26.6	26.9	26.9
Averaged Modal Damping Ratio (%)	Lift Direction	2.90	2.40	4.51	6.50	8.04	13.70	11.56	12.08	11.79	12.57
	Drag Direction	3.02	1.56	3.43	4.81	6.74	10.07	11.26	10.66	11.65	12.97

Table 2. Averaged natural frequency and damping for Test a.2

Medium		In Air	In Still Water		
Temperature (°C)		23.1	42.8	23.1	11.7
Averaged Natural Frequency (Hz)	Tube 1,3,5	30.8	24.5	25.3	25.7
	Tube 2,4	24.0	20.9	21.3	21.8
Averaged Modal Damping Ratio (%)	Lift Direction	Tube 1,3,5	2.68	1.75	3.45
		Tube 2,4	2.38	2.91	5.03
	Drag Direction	Tube 1,3,5	2.94	1.47	3.52
		Tube 2,4	2.32	1.21	2.77
					6.19

Table 3. Averaged natural frequency and damping for Test a.3

Medium		In Air	In Still Water			
Temperature (°C)		22.8	42.8	22.8	11.0	
Averaged Natural Frequency (Hz)	Tube 1,2,4,5	31.1	24.5	25.2	25.9	
	Tube 3	24.5	21.0	21.2	21.9	
Averaged Modal Damping Ratio (%)	Lift Direction	Tube 1,2,4,5	3.05	2.12	4.20	8.11
		Tube 3	2.31	2.69	4.29	6.97
	Drag Direction	Tube 1,2,4,5	3.02	1.63	3.35	7.31
		Tube 3	2.38	0.99	2.43	6.21

**Table 4. Averaged natural frequency and damping
in the drag direction for Test b.1**

Medium	In Air	In Still Water			
Temperature (°C)	19.3	43.3	23.3	16.7	11.1
Averaged Natural Frequency (Hz)	36.9	26.9	27.3	27.5	28.4
Averaged Modal Damping Ratio (%)	4.20	1.64	3.57	5.76	8.22

Fig. 4. Tube displacement as a function of flow velocity in Test a.1 at 23°C

Fig. 5. Tube displacement as a function of flow velocity in Test a.1 at 8.9°C

Fig. 6. Displacement components of tube 3 at 16.7°C (Tests b.1 and b.2)

Fig. 7. Displacements of tube 2 in Test a.1 at 23°C

Fig. 8. Displacement of tube 4 in Test a.1 at 16.7°C in the lift direction

Additional tests have also been made to understand the significance of this nonlinear phenomenon using Test a.1. First, the flow velocity is increased to a little above the critical value. At this flow velocity, the tubes are excited to large oscillations (instability). Then, the flow velocity is decreased to a little less than the critical value; the tubes are still performing large oscillations. As soon as the tubes are held by hand at the overhung portion, the tubes regain stability. The tubes remain stable after the hand is removed. Therefore, the nonlinear characteristics possess little practical significance and the critical flow velocity should be established using the case of increasing flow rate.

Typical frequency spectra of tube displacements are given in Figs. 9 and 10. Tube response frequencies are contained within the frequency band of coupled modes. In the subcritical flow velocity ranges, tube response frequencies increase with flow velocity; this can be seen clearly from Fig. 10. Once the tubes become unstable, the response frequency is associated with a particular mode.

Figure 9 shows that there is a shift in frequency between 1.36 and 1.40 m/s. At 1.36 m/s, the tubes are stable, but at 1.40 m/s, the tubes become unstable. When the flow velocity is gradually decreased, the response frequency also gradually decreases and eventually returns to the value before instability occurs. Since at instability, the tubes respond at a particular well-defined frequency, the critical flow velocity can also be established by examining the frequency spectra in conjunction with the tube displacements.

Figure 11 shows the orbital paths of Test a.1 at 23°C for several flow velocities. In this test, the flow velocity is increased from 1.19 to 1.23 and 1.25 m/s, and then decreased to 1.19 m/s. The arrows on the figure show the relative positions of the tubes at a particular instant of time. The predominant mode is the out-of-phase mode in the lift direction. Note that in (d) for 1.19 m/s, tube displacements are larger than those for 1.23 m/s in (b); this is associated with the nonlinear behavior for increasing and decreasing flow velocity.

Figure 12 shows the instability modes of Tests a.1, a.2, and a.3. In Test a.1, all tubes are in tune; the amplitudes of the tubes are about the same except tube 5. In Test a.2, tubes 2 and 4 have a lower natural frequency; the displacements of these tubes are much larger than the

Fig. 9. Frequency spectra of tube displacements of tube 3 in the lift direction in Test a.1 at 16.7°C; sequence of tests is from bottom to top

Fig. 10. Frequency spectra of tube displacements of tube 2 in Test a.1 at 8.9°C

(a) 1.19 m/s

(b) 1.23 m/s

(c) 1.25 m/s

(d) 1.19 m/s

Fig. 11. Tube orbital paths of tube oscillations in Test a.1 at 23°C

Fig. 12. Tube orbital paths of tubes at instability in Tests a.1, a.2, and a.3

others. Apparently, tubes 2 and 4 first become unstable. In Test a.3, tube 3 has a lower natural frequency, but the displacement of tube 3 is smaller than the others. In this case, tubes 1 and 2, and 4 and 5, are in tune. The instability is apparently associated with those tubes.

3. The Critical Flow Velocity

One of the main objectives is to establish the critical flow velocity. It is defined as the flow velocity at which the tube displacements increase sharply. In all tests, the critical flow velocities are fairly well defined. The critical flow velocity is also characterized by a single tube-response frequency. The critical flow velocity and the oscillation frequency at instability for all tests are given in Table 5.

Table 5. Measured critical flow velocity and associated instability frequency

Tests	Water Temperature, °C	Critical Flow Velocity, m/s	Oscillation Frequency (Hz) At Instability
a.1	42.8	1.07	24.3
	23.0	1.19	24.3
	16.7	1.28	24.3
	11.1	1.51	25.5
	9.4	1.83	26.8
	9.1	2.00	28.0
	8.9	5.18	Not measured
a.2	42.8	1.07	24.0
	23.0	1.23	24.4
	11.1	1.69	26.9
a.3	42.8	1.07	21.4
	23.0	1.23	22.0
	11.1	1.95	28.7
b.1	43.3	1.13	25.4
	23.3	1.30	25.7
	16.7	1.43	28.0
	11.1	1.65	29.7
b.2	43.3	1.30	28.0
	23.3	1.46	27.8
	16.7	1.61	28.3
	11.1	1.81	29.4

V. ANALYTICAL AND EXPERIMENTAL RESULTS

Equations of Motion

A schematic of the tube row is shown in Fig. 13. The axes of the tubes and flow are parallel to the z axis and y axis, respectively. The subscript i is used to denote a variable associated with tube i . The equations of motion in the x and y directions are¹

$$\begin{aligned}
 E_i I_i \frac{\partial^4 u_i}{\partial z^4} + c_i \frac{\partial u_i}{\partial t} + m_i \frac{\partial^2 u_i}{\partial t^2} + \rho \pi R^2 \sum_{j=1}^5 \left(\alpha_{ij} \frac{\partial^2 u_j}{\partial t^2} + \sigma_{ij} \frac{\partial^2 v_j}{\partial t^2} \right) \\
 + \sum_{j=1}^5 \left[(\bar{\alpha}_{ij} + \rho R U \alpha'_{ij}) \frac{\partial u_j}{\partial t} + (\bar{\sigma}_{ij} + \rho R U \sigma'_{ij}) \frac{\partial v_j}{\partial t} \right] \\
 - \rho U^2 \sum_{j=1}^5 (\alpha''_{ij} u_j + \sigma''_{ij} v_j) = 0
 \end{aligned} \quad (1)$$

and

$$\begin{aligned}
 E_i I_i \frac{\partial^4 v_i}{\partial z^4} + e_i \frac{\partial v_i}{\partial t} + m_i \frac{\partial^2 v_i}{\partial t^2} + \rho \pi R^2 \sum_{j=1}^5 \left(\tau_{ij} \frac{\partial^2 u_j}{\partial t^2} + \beta_{ij} \frac{\partial^2 v_j}{\partial t^2} \right) \\
 + \sum_{j=1}^5 \left[(\bar{\tau}_{ij} + \rho R U \tau'_{ij}) \frac{\partial u_j}{\partial t} + (\bar{\beta}_{ij} + \rho R U \beta'_{ij}) \frac{\partial v_j}{\partial t} \right] \\
 - \rho U^2 \sum_{j=1}^5 (\tau''_{ij} u_j + \beta''_{ij} v_j) = 0,
 \end{aligned} \quad (2)$$

where $E_i I_i$ is flexural rigidity; m_i is tube mass per unit length; c_i and e_i are damping coefficients in the x and y directions; ρ is fluid density; R is tube radius; U is flow velocity; t is time; u_i and v_i are tube displacements in the x and y directions; α_{ij} , β_{ij} , σ_{ij} , and τ_{ij} are added mass coefficients; $\bar{\alpha}_{ij}$, $\bar{\beta}_{ij}$, $\bar{\sigma}_{ij}$, and $\bar{\tau}_{ij}$ are viscous damping coefficients; α'_{ij} , β'_{ij} , σ'_{ij} , and τ'_{ij} are flow-velocity-dependent damping coefficients; and α''_{ij} , β''_{ij} , σ''_{ij} , and τ''_{ij} are fluidelastic stiffness coefficients.

Note that the tubes are in fluid from $z = 0$ to l_d , subjected to flow from $z = l_b$ to l_c , and in air from $z = l_d$ to l_e . Equations 1 and 2 can be applied to different portions of the tubes:

Fig. 13. Schematic of the tube row

- (a) $z = \ell_b$ to ℓ_c , use Eqs. 1 and 2.
 (b) $z = 0$ to ℓ_b and ℓ_d to ℓ_c , set $U = 0$ in Eqs. 1 and 2.
 (c) $z = \ell_d$ to ℓ_e , set $U = \rho = \bar{\alpha}_{ij} = \bar{\sigma}_{ij} = \bar{\tau}_{ij} = \bar{\beta}_{ij} = 0$.
 Let

$$u_i = \sum_{m=1}^{\infty} a_{im} \phi_{im}$$

and

$$v_i = \sum_{m=1}^{\infty} b_{im} \phi_{im} ,$$

(3)

where ϕ_{im} is the orthonormal modal function of tube i in vacuo; i.e.,

$$\frac{1}{\ell_e} \int_0^{\ell_e} \phi_{im} \phi_{in} dz = \delta_{mn} . \quad (4)$$

Using Eqs. 1 to 4 yields

$$\begin{aligned} & m_i \omega_{in}^2 a_{in} + c_i \dot{a}_{in} + m_i \ddot{a}_{in} + \rho \pi R^2 \sum_{j=1}^5 \sum_{m=1}^{\infty} (a_{ij} \ddot{a}_{jm} + \sigma_{ij} \ddot{b}_{jm}) u_{ijnm} \\ & + \sum_{j=1}^k \sum_{m=1}^{\infty} [(\bar{\alpha}_{ij} u_{ijnm} + \rho R U \alpha'_{ij} v_{ijnm}) \dot{a}_{jm} \\ & + (\bar{\sigma}_{ij} u_{ijnm} + \rho R U \sigma'_{ij} v_{ijnm}) \dot{b}_{jm}] \\ & - \rho U^2 \sum_{j=1}^5 \sum_{m=1}^{\infty} (\alpha''_{ij} a_{jm} + \sigma''_{ij} b_{jm}) v_{ijnm} = 0 \end{aligned} \quad (5)$$

and

$$\begin{aligned} & m_i \omega_{in}^2 b_{in} + e_i \dot{b}_{in} + m_i \ddot{b}_{in} + \rho \pi R^2 \sum_{j=1}^5 \sum_{m=1}^{\infty} (\tau_{ij} \ddot{a}_{jm} + \beta_{ij} \ddot{b}_{jm}) v_{ijnm} \\ & + \sum_{j=1}^k \sum_{m=1}^{\infty} [(\bar{\tau}_{ij} v_{ijnm} + \rho R U \tau'_{ij} u_{ijnm}) \dot{a}_{jm} \\ & + (\bar{\beta}_{ij} v_{ijnm} + \rho R U \beta'_{ij} u_{ijnm}) \dot{b}_{jm}] \\ & - \rho U \sum_{j=1}^5 \sum_{m=1}^{\infty} (\tau''_{ij} a_{jm} + \beta''_{ij} b_{jm}) u_{ijnm} = 0 , \end{aligned} \quad (6)$$

where ω_{in} is the n th natural frequency of cylinder i in vacuo, and

$$\mu_{injm} = \frac{1}{\ell_e} \int_0^{\ell_d} \phi_{in} \phi_{jm} dz \quad , \quad (7)$$

$$v_{injm} = \frac{1}{\ell_e} \int_{\ell_b}^{\ell_c} \phi_{in} \phi_{jm} dz \quad .$$

Note that Eqs. 5 and 6 are coupled for different values of n and m . In experiments, it is found that the dominant tube responses are associated with $n = m = 1$. As an approximation, all terms not associated with $m = n = 1$ will be dropped. Eliminating the index m and n from Eqs. 5 and 6 yields

$$\begin{aligned} m_1 \omega_1^2 a_1 + c_1 \dot{a}_1 + m_1 \ddot{a}_1 + \rho \pi R^2 \sum_{j=1}^5 (\alpha_{1j} \ddot{a}_j + \sigma_{1j} \ddot{b}_j) \mu_{1j} \\ + \sum_{j=1}^k [(\bar{\sigma}_{1j} \mu_{1j} + \rho R U \alpha'_{1j} v_{1j}) \dot{a}_j + (\bar{\sigma}_{1j} \mu_{1j} + \rho R U \sigma'_{1j} v_{1j}) \dot{b}_j] \\ - \rho U^2 \sum_{j=1}^5 (\alpha''_{1j} a_j + \sigma''_{1j} b_j) v_{1j} = 0 \end{aligned} \quad (8)$$

and

$$\begin{aligned} m_1 \omega_1^2 b_1 + e_1 \dot{b}_1 + m_1 \ddot{b}_1 + \rho \pi R^2 \sum_{j=1}^5 (\bar{\tau}_{1j} \ddot{a}_j + \bar{\beta}_{1j} \ddot{b}_j) \mu_{1j} \\ + \sum_{j=1}^k [(\bar{\tau}_{1j} \mu_{1j} + \rho R U \tau'_{1j} v_{1j}) \dot{a}_j + (\bar{\beta}_{1j} \mu_{1j} + \rho R U \beta'_{1j} v_{1j}) \dot{b}_j] \\ - \rho U^2 \sum_{j=1}^5 (\tau''_{1j} a_j + \beta''_{1j} b_j) v_{1j} = 0 \quad , \end{aligned} \quad (9)$$

where

$$\mu_{1j} = \frac{1}{\ell_e} \int_0^{\ell_d} \phi_1 \phi_j dz \quad (10)$$

and

$$v_{1j} = \frac{1}{\ell_e} \int_{\ell_b}^{\ell_c} \phi_1 \phi_j dz \quad .$$

Equations 8 and 9 can be used to calculate the natural frequencies and mode shapes in still water, and critical flow velocity and instability modes in flow.

Natural Frequencies and Damping Characteristics of Uncoupled Modes

In experiments, natural frequencies and modal damping ratios in the lift and drag directions were measured at a particular temperature T for both in air and in still water. In addition, natural frequencies and modal damping ratio were also measured at a series of other temperatures T' in still water. From this data, the damping coefficients c_i and e_i can be calculated for different temperature T' .

Consider the i th tube oscillating in the lift direction. The uncoupled equation of motion in air is (see Eq. 8)

$$\ddot{a}_i + 2\zeta_i \omega_i \dot{a}_i + \omega_i^2 a_i = 0 \quad , \quad (11)$$

$$\zeta_i = \frac{c_i}{2m_i \omega_i} \quad ,$$

where ζ_i and ω_i are the modal damping ratio and natural frequency of tube i in air at the temperature T . At the same temperature T , the uncoupled equation of motion in the lift direction in still water is

$$\ddot{\bar{a}}_i + 2\bar{\zeta}_i \bar{\omega}_i \dot{\bar{a}}_i + \bar{\omega}_i^2 \bar{a}_i = 0 \quad ,$$

$$\bar{m}_i = m_i + \sigma_{ii} \mu_{ii} \quad ,$$

$$\bar{\omega}_i = \left(\frac{m_i}{\bar{m}_i} \right)^{1/2} \omega_i \quad , \quad (12)$$

$$\zeta_i = \frac{c_i + \sigma_{ii} \mu_{ii}}{2\bar{m}_i \bar{\omega}_i} \quad ,$$

where $\bar{\omega}_i$ and $\bar{\zeta}_i$ are the natural frequency and modal damping ratio in water; ζ_i , $\bar{\zeta}_i$, ω_i and $\bar{\omega}_i$ were measured; therefore, c_i and $\sigma_{ii} \mu_{ii}$ can be calculated.

At a different temperature T' , natural frequency $\bar{\omega}_i'$ and modal damping ratio $\bar{\zeta}_i'$ in water are also measured. The natural frequency ω_i' and modal damping ratio ζ_i' in air at T' can be calculated based on this data. Since the variation of natural frequency with temperature is small, it is reasonable to use the following relationship:

$$\frac{\omega_i'}{\bar{\omega}_i'} = \frac{\omega_i}{\bar{\omega}_i} \quad . \quad (13)$$

Using Eqs. 12 and 13, one can calculate the modal damping ratio and damping coefficient in air at a different temperature T' :

$$\zeta'_1 = \left(\frac{\omega_1}{\bar{\omega}_1} \right) \bar{\zeta}'_1 - \frac{\bar{\omega}'_1}{\omega_1} \left[\frac{\omega_1}{\bar{\omega}_1} \frac{\bar{\zeta}_1}{\bar{\zeta}'_1} - 1 \right] \zeta_1 , \quad (14)$$

$$c'_1 = 2m_1 \omega'_1 ,$$

where c'_1 is the damping coefficient in the lift direction at T' . Note that Equations 14 are also valid for tube oscillations in the drag direction.

The average modal damping ratio $\bar{\zeta}_1$ measured in still water is given in Fig. 14, which corresponds to those given in Tables 1 to 4. In general, the damping in the lift direction is smaller than that in the drag direction and there is no significant difference in damping values obtained from different tests.

Figure 15 shows the modal damping ratio in air calculated from Eq. 14. In still water, the modal damping values are different in the drag and lift directions. However, in air, their values in the two directions calculated from Eq. 14 are about the same. In Fig. 15, the averaged values of damping in the two directions are presented. For calculations of the critical flow velocity, the solid line given in Fig. 15 is used.

Natural Frequencies and Mode Shapes of Coupled Modes in Still Water

For a tube row, $\sigma_{ij} = \tau_{ij} = 0$.³ Neglecting the damping terms and flow-velocity-dependent terms in Eqs. 8 and 9 gives

$$m_1 \omega_1^2 a_1 + m_1 \ddot{a}_1 + \rho \pi R^2 \sum_{j=1}^5 \alpha_{ij} \mu_{ij} \ddot{a}_j = 0 \quad (15)$$

and

$$m_1 \omega_1^2 b_1 + m_1 \ddot{b}_1 + \rho \pi R^2 \sum_{j=1}^5 \beta_{ij} \mu_{ij} \ddot{b}_j = 0 . \quad (16)$$

Equations 15 and 16 show that the motions in the drag and lift directions are uncoupled. Natural frequencies and natural modes in still water can be calculated easily from these two equations.

Consider the case that all tubes are identical: $m_1 = m$ and $\omega_1 = \omega$. The coupled natural frequencies and its natural modes can be obtained as follows:⁴

$$\Omega_1 = \frac{\omega}{\lambda_1^{1/2} \left(1 + \frac{1}{m} \right)} , \quad i = 1 \text{ to } 5 , \quad (17)$$

Fig. 14. Averaged modal damping ratio in still water

Fig. 15. Averaged modal damping ratio in air

where λ_i 's are the eigenvalues of the added mass matrix α_{ij} or β_{ij} . The mode shapes are the same as the eigenvalues of the matrices α_{ij} and β_{ij} .

Figure 16 shows the analytical results of the natural frequencies and mode shapes of five tubes corresponding to Test b.1 in still water. There are 10 modes: 5 out-of-plane modes and 5 in-plane modes. Detailed comparisons of theoretical and experimental results for tube rows oscillating in still water is available⁵ and further study is not presented here.

The Critical Flow Velocity

The lowest critical flow velocity at which large tube oscillations occur can be calculated based on Eqs. 8 and 9. The procedures to obtain the critical flow velocity are the same as those given in Ref. 1. Note, in Eqs. 8 and 9, the added mass terms and fluid viscous damping terms are multiplied by μ_{ij} , and the terms associated with fluidelastic force and flow-velocity-dependent damping force are multiplied by ν_{ij} . This is due to the fact that the tube row is not completely submerged in flow. The values of μ_{ij} and ν_{ij} can be calculated from Eqs. 10. For tubes without added weight at the free end, $\mu_{ij} = 0.705$ and $\nu_{ij} = 0.361$ for Case a and $\mu_{ij} = 1.0$ and $\nu_{ij} = 0.522$ for Case b.

Calculations of the "exact" critical flow velocity based on a row of five tubes is straightforward.¹ Computations are made for a row of five identical tubes corresponding to Tests a.1 and b.1 for different values of damping ratio; the results are presented by the solid lines given in Figs. 17 and 18. The instability modes are also given.

Dashed lines in Figs. 17 and 18 present the theoretical critical flow velocity based on constrained modes. For smaller values of mass-damping parameter, the dashed line corresponds to a single elastic tube in a rigid tube row, but for large values of mass-damping parameter, the dashed line corresponds to a group of three elastic tubes in a tube row with the middle tube moving in the drag direction and the two wing tubes moving in the lift direction.¹

The experimental values of the critical reduced flow velocity (U/fD) and mass-damping parameter ($2\pi m\zeta/\rho D^2$) are evaluated based on the experimental data of natural frequency (f) given in Tables 1 to 4, critical flow velocity given in Table 5, and damping values given in Fig. 15. The experimental results are also presented in Figs. 17 and 18.

OUT-OF-PLANE MODES

IN- PLANE MODES

Fig. 16. Calculated natural frequencies and mode shapes for Test b.1 in still water

Fig. 17. Critical flow velocity for Case a

Fig. 18. Critical flow velocity for Case b

Comparison of Analytical and Experimental Results

The experimental and analytical results for the critical flow velocity given in Figs. 17 and 18 agree well. Note that the exact solution in Fig. 17 is corresponding to Test a.1 and the solid line in Fig. 18 is corresponding to Test b.1. Both of them agree fairly well, although the analytical critical flow velocity is a little higher.

At a certain value of $\delta_m (= 2\pi m \zeta / \rho D^2)$, there is a jump of the critical flow velocity. The jump is attributed to the transition from one dominant instability mechanism to another dominant instability mechanism. This jump is demonstrated in Test a.1. With a small increase in damping by decreasing water temperature less than one degree, the critical flow velocity increases several times. The analytical result for δ_m at which jump occurs agrees with the experimental data reasonably well.

Experimental instability modes given in Figs. 11 and 12 show that the most critical mode is the out-of-phase mode in the lift direction, in which the adjacent tubes are moving out of phase to each other. This is in agreement with the analytical instability modes given in Figs. 17 and 18. For tube rows with low δ_m , this mode is the critical instability mode.

The critical flow velocities for Test a.1 are lower than those for Tests a.2 and a.3, and the critical flow velocities for Test b.1 are lower than those for Test b.2. This demonstrates the effect of detuning, which tends to stabilize the tube row. Analytical results also show that detuning is beneficial in the stability of a tube array.¹

Calculations also have been made to predict the coupled natural frequencies of the tube row as a function of flow velocity. The general characteristics are similar to those given in Fig. 10; i.e., the coupled natural frequencies increase with flow velocity.

VI. CONCLUSIONS

Based on the experimental and analytical results of this study, the following conclusions can be made.

1. There are two basic instability mechanisms: fluid-damping-controlled instability and fluidelastic-stiffness-controlled instability. The former is dominant for small mass-damping parameter, and the latter is dominant for large mass-damping parameter.

2. There is a finite jump in the critical flow velocity at a certain value of mass-damping parameter associated with the transition from one instability mechanism to another. The jump phenomenon is demonstrated in experiment by varying tube damping.

3. At low mass-damping parameter, fluid-damping force is dominant and, for a tube row, the dominant instability mode is associated with the out-of-phase mode in the lift direction.

4. Detuning of the tubes tends to increase the critical flow velocity. However, for fluid-damping-controlled instability, its effect is not very significant.

5. In practical situations, fluid-damping-controlled instability will be dominant for liquid flow, and fluidelastic-stiffness-controlled instability for gas flow. The transition region will most likely appear in two-phase flows.

The experimental data have verified the validity of the mathematical model;¹ thus, the model has established the foundation for the development of a design guide to avoid detrimental instability.

ACKNOWLEDGMENT

This work was performed under the sponsorship of the Office of Reactor Research and Technology, U. S. Department of Energy.

REFERENCES

1. Chen, S. S., "Mathematical Model, Instability Mechanisms, and Stability Criteria of a Group of Circular Cylinders Subjected to Crossflow," Argonne National Laboratory, Technical Memorandum ANL-CT-81-22, May 1981.
2. Chen, S. S., Jendrzejczyk, J. A., and Lin, W. H., "Fluidelastic Instability of Rectangular Tube Arrays Subjected to Liquid Cross Flow," Proc. of the Fourth Int. Conference on Pressure Vessel Technology, London, England, May 19-23, 1980, Paper No. C86/80, pp. 193-200 (1980).
3. Chen, S. S., "Vibrations of a Row of Circular Cylinders in a Liquid," Trans. ASME 97, pp. 1212-1218 (1975).
4. Chen, S. S., "Vibration of Nuclear Fuel Bundles," Nucl. Eng. Des. 35, pp. 399-422 (1975).
5. Chen, S. S., and Jendrzejczyk, J. A., "Experiments on Fluidelastic Vibration of Tube Bundles," J. Mech. Design, Trans. ASME 100, pp. 540-548 (1978).

Distribution for ANL-CT-81-29Internal:

E. S. Beckjord
 C. E. Till
 R. Avery
 R. A. Lewis
 R. S. Zeno
 P. R. Huebotter
 G. S. Rosenberg
 M. W. Wambsganss
 A. R. Brunsvold
 B. L. Boers
 S. S. Chen (55)
 H. Chung
 H. Halle
 J. A. Jendrzeczyk

W. P. Lawrence
 W. H. Lin
 T. M. Mulcahy
 D. N. Bounelis
 M. Weber
 R. A. Valentin
 C. A. Kot
 W. T. Sha
 S. H. Fistedis
 ANL Patent Dept.
 ANL Contract File
 ANL Libraries (2)
 TIS Files (6)

External:

DOE-TIC, for distribution per UC-79k (146)
 Manager, Chicago Operations Office, DOE
 Director, Technology Management Div., DOE-CH
 E. Gallagher, DOE-CH
 President, Argonne Universities Association
 Components Technology Division Review Committee:
 A. Bishop, U. Pittsburgh
 F. W. Buckman, Consumers Power Co.
 R. A. Greenkorn, Purdue U.
 W. M. Jacobi, Westinghouse Electric Corp., Pittsburgh
 M. A. Schultz, Mashpee, Mass.
 E. E. Ungar, Bolt, Beranek and Newman, Inc., Cambridge, Mass.
 J. Weisman, U. Cincinnati