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	STATEMENT OF PROBLEM Develop information which will provide the basis for structural analysis and design rules for systems and components which utilize snubbers as supports. Results will be used to assure that dynamic response characteristics of snubber supported systems and components will be bounded within acceptable limits.		
	ABSTRACT: The sensitivity of mechanical and hydraulic snubber parameters to system displacements, stresses and forces are analyzed. Acceleration threshold, clearance and friction are evaluated for mechanical snubbers while hydraulic snubber investigations include lock velocity, bleed rate, unlock loading, clearance and friction. The back-up structure is influential for both types of snubbers and although not a snubber parameter, per se, is treated like a parameter. Forcing functions are utilized, and include both harmonic and time history seismic inputs to the mathematical models. Mathematical models are used to simulate snubber characteristics. Special mathematical techniques are developed for economical use in piping programs. Acceptable parameter ranges are established, based on criteria for the various mechanical and hydraulic snubber characteristics.		
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1. INTRODUCTION

The objectives of this program under NRC contract FIN #B3076-8 are to perform analytical evaluations and parametric studies to:

- (1) Identify structural and performance parameters which significantly affect snubber dynamic response characteristics;
- (2) Determine the sensitivity of the snubber response and the corresponding effects on the snubber supported system to variations in each parameter identified above; and
- (3) Develop simplified analyses techniques and design rules which will bound the response of the system within acceptable limits.

The areas of work covered in (1) and (2) were completed this fiscal year, FY 78, and work under (3) is planned for completion in FY 79.

Simple analytical models with simple loadings were used initially to establish dimensionless response parameters permitting insights into the basic problem. The use of steady-state harmonic response calculations was invaluable as a tool. After the simple models and loadings were examined the more sophisticated models and loadings were studied. Figure 1.1 illustrates both the simple and more sophisticated models.

Snubber parameters, e.g., acceleration threshold parameter and bleed rate, were considered invariant quantities, i.e., unaffected by feedback with the restrained system. Although it is recognized that there is some interaction between the various snubber parameters and system response, these effects were not considered in this study. The work does not consider variables that effect parameter magnitudes such as hydraulic fluid properties, entrapped air, and inertia effects.

Figure 1.1 Structural Models

The first two objectives of the program are related to response sensitivity, i.e., the amount of change in response due to a corresponding change in a parameter. The load or response magnitude is considered when evaluating or determining the acceptable parameter ranges that maintain acceptable response.

The evaluation of system response considers not only displacement response but also stress and snubber reaction load response characteristics. In general, displacement response is important when considering the response of components such as pumps, steam generators, and valves. Stress and reaction load responses, on the other hand, are often related better to piping systems.

The following five tasks are used as a basis for meeting the program objectives.

TASK 1 - IDENTIFY PARAMETERS AND DETERMINE PROBABLE RANGES OF PARAMETER VALUES

This task involved determining which snubber parameters affect dynamic response. Significant parameters associated with mechanical snubbers include the acceleration threshold parameter, clearance, and friction. Parameters of significance with hydraulic snubbers include lock velocity, bleed rate, friction, and clearance. The identification of parameters and probable ranges were made based on Nuclear Regulatory Commission (NRC) and vendor suggestions.

Consideration was also given to stiffness and dynamic effects of the snubber back-up support structure. These effects are influential in attenuating or modifying parameter response sensitivity.

TASK 2 - DEVELOP SNUBBER ANALYTICAL MODELS

Under this task mathematical models were established for the hydraulic and mechanical snubbers. Using the mathematical model for a snubber, a solution technique based on a modified modal analysis procedure was developed. This technique is useful in incorporating snubber parameters since it is not susceptible to convergence problems, and requires minimal computer time. Special purpose computer codes were used for limited studies while a general purpose piping code SUPERPIPE, Reference 1, was used for more complex analyses.

TASK 3 - SELECT APPROPRIATE TIME HISTORY FORCING FUNCTIONS

The selection of forcing functions is important if meaningful results are to be obtained. Harmonic loadings provide insights into physically understanding the problem. Seismic loadings are then used in evaluating or relating response sensitivity to actual conditions. The four most used, strong motion seismic events for the design of facilities have been employed as "benchmarks" for this study. They include: El Centro (California-1940); Taft (California-1952); San Fernando (California-1971); and Washington (Washington-1949). These seismic events represent varying earthquake magnitudes and frequencies.

TASK 4 - EVALUATE RESPONSE SENSITIVITY FOR PARAMETERS ESTABLISHED IN TASK 1

This part of the study consisted of forming response curves for the various structural models and for the various loadings with the aim of graphically presenting the response sensitivity data. Consideration is given to displacement, stress and loads.

TASK 5 - ESTABLISH AN ACCEPTABLE RANGE OF PARAMETERS THAT WILL BOUND SYSTEM RESPONSE WITHIN ACCEPTABLE LIMITS

Establishing the acceptable parameter ranges requires the consideration of all the findings of the previous tasks. This is one area where the actual magnitude was considered in the time history forcing functions for the four seismic events utilized. The acceptable ranges were based on the dynamic response characteristics of displacement, stress and snubber loadings.

The results or conclusions obtained from this study are based on a limited observation contained herein. Experimental and analytical work by other authors was not considered. The effort scheduled for FY 79 will supplement this work and as a result, future refinement of the acceptable ranges for the various parameters is expected.

2. SUMMARY

This study interrogated the externally evident parameters associated with mechanical and hydraulic snubbers. The basic approach taken was to first use simple and fundamental models. As more sophisticated systems were developed analytical comparisons were made with the fundamental investigations. Summaries of each parameter are presented below to highlight results of the snubber sensitivity study for FY '78.

ACCELERATION THRESHOLD PARAMETER (A.T.P.) — Analytical studies of simple systems reveal that the acceleration threshold parameter has the potential of increasing system response rather than reducing the response as might be expected when snubbers are used. Analytical studies indicate that an amplification may exist when compared to the base motion when the forcing frequency is less than the natural frequency of the snubbed system or when low forcing frequencies (< 3 Hz) are applied to the system. For the more complex seismic inputs, amplification also occurs in some cases. Acceptable ranges for this parameter depend on the frequency response characteristics of the system being restrained.

VISCOUS PARAMETERS (LOCK VELOCITY, BLEED RATE, RELEASE CRITERIA) — Response sensitivity of the viscous parameters is not greatly dependent on the forcing frequency or on the magnitude of the applied load. The release criteria (condition when velocity changes from bleed velocity to free velocity) has the least effect on response of the viscous parameters. Lock velocity has the greatest effect on system response. The response of highly loaded systems (based on benchmark seismic loadings) are in general reduced to 5 percent of the unrestrained response for realistic bleed rates when the lock velocity is less than 1 in/sec. A realistic bleed rate (< 1 in/sec) reduced response to acceptable limits since the snubber represents a highly "damped" or critically damped restraint.

FRICTION LOADS — The study of friction (coulomb) indicates that small friction loads have negligible effect on system response. For the case of harmonically excited simple system, the response is affected significantly only after the friction load exceeds 40 percent of the applied load. The same trends were observed for nonharmonic (seismic) loadings.

CLEARANCE — Clearance reduces the effectiveness of other snubber parameters in reducing the response of the system (stress and displacement). Snubber reaction loads, however, may be increased or decreased by clearance depending on the magnitude of the load and the amount of clearance, compared to the free displacement (no support) condition.

BACK-UP STIFFNESS — Since the "effective" stiffness of a snubber is generally greater than its back-up support structure, the snubber response characteristics, e.g., damping properties, acceleration limits, etc., may be "washed out" by flexible supporting structures. The results of the work presented in this study indicate that the combined "effective" stiffness of the back-up structure and snubber must be at least twenty times greater than the piping or component stiffness to be totally effective in reducing response. In terms of hydraulic snubber viscous parameters the support stiffness should be greater than $1.2 (C\Omega)$ and 1000 C/M where C is the snubber rated load (lb) divided by the bleed rate (in/sec), Ω is the predominant forcing frequency in rad/sec, and M is the support structure effective mass.

SUPPORT DYNAMIC INTERACTION — Since the "effective" snubber stiffness is in general quite large, the snubber acts as a "rigid" connection between the supporting structure and supported system. A classical resonant condition can occur if the forcing frequency coincides with the combined system-support natural frequency. Based on this situation it is advisable to have a supporting structure with a higher natural frequency than the supported system. Ideally it is desirable to have the supporting structure with a natural frequency that is twice that of the supported structure. In addition to the specific parameters mentioned above the study indicated other results some of which are discussed below.

The response analysis of a specific single degree of freedom system with a snubber having negligible clearance indicated that the response resulting from each of four different seismic inputs was nearly the same. Therefore, it can be concluded that the response of a component is not greatly dependent on the seismic input, whereas the response of a piping system may or may not be. This depends on the dynamic amplification resulting from the forcing frequency and its coincidence with the natural frequency.

Although snubber response appears to be insensitive to changes in component load and forcing frequency, this may not be the case for the piping systems.

Table 2.1 shows the range of snubber parameters that ensure a properly designed system will operate within acceptable limits. These ranges are based solely on the work presented to date in this report and may be revised pending results of the tasks scheduled for FY '79.

TABLE 2.1

ACCEPTABLE LIMITS FOR SNUBBER PARAMETERS

Acceleration Threshold Parameter	<0.04 g
Lock Velocity	<60 in/min
Bleed Rate ($C = \frac{\text{rated load}}{\text{bleed rate}}$)	>(10) ⁴ lb-sec/in
$(C/M = \frac{C}{\text{mass}})$	>(10) ³ sec ⁻¹
Friction Load	40% rated load
Back-Up Structural Stiffness	
Viscous Parameters ⁽¹⁾	$k > 1.2(C\Omega)$
Effective Snubber Stiffness ⁽²⁾	$K/K^* > 20$
Clearance	
No Other Parameters ⁽³⁾	$(\delta_G/\delta_F) < 0.15$
With Other Parameters ⁽³⁾	$(\delta_G/\delta_F) < 0.10$

NOTES:

- (1) C = Rated Load/Bleed Rate (lb-sec/in)
 Ω = Predominant Forcing Freq. (rad/sec)
- (2) k = Support Stiffness (lb/in)
 $K^* = [\text{Pipe Flexibility}]^{-1}$ (lb/in)
- (3) δ_G = Clearance Gap (in)
 δ_F = Unrestrained Response (in)

3. SNUBBER PARAMETERS

The scheme adopted for identifying snubber parameters was to isolate parameters applicable to mechanical snubbers, hydraulic snubbers and those common to both. Following analyses of the individual parameters, combinations are addressed.

3.1 The Mechanical Snubber

The mechanical snubber is a device that operates on the principle of limiting the acceleration of any pipe movement, relative to the snubber base, to a threshold value of a specified "g" level, Figure 3.1. Should a disturbance accelerate the pipe in either direction, a braking force is applied within the arrestor to restrict the acceleration to specified limits. Thermal expansion which is a gradual movement is not restricted. A particular feature of the snubber is that its performance is independent of the force being applied. The design of the unit is completely symmetrical. Braking action in both tension and compression loading is identical.

3.1.1 Acceleration Threshold - This snubber parameter is unique to the mechanical arrestor. It represents the limit acceleration of pipe movement relative to the base of the snubber. For the "ideal" mechanical arrestor, unrestrained pipe movement is permitted when pipe accelerations are less than a specified "g" level. At acceleration rates equal to or greater than the threshold parameter, the snubber reacts with a load that does not permit pipe movement to accelerate at a rate greater than the threshold value. The acceleration threshold is a "built-in" feature of the snubber. It cannot be adjusted or modified. The acceleration threshold level is normally set from 0.02 g to 0.08 g. Snubbers can be designed for other values that might be desired. Consider the Pacific Scientific Snubber, one that is in extensive use. Here the threshold values are basically a function of the capstan spring rate and the inertia mass, Figure 3.1. Other type mechanical snubbers are assumed to have the same acceleration threshold characteristics.

Figure 3.1 Cutaway of Typical Pacific Scientific Arrestor

At impact loads greater than the rated capacity, the capstan spring may slip as it engages the torque transfer drum. This "safety" device prevents failure of the device and permits accelerations greater than the acceleration threshold limit. Since snubbers should not be used above their rated capacity, this situation is not being considered.

3.1.2 Clearance - The primary source of clearance is established by setting the ball nut mechanism. This is where the linear translational motion caused by a seismic disturbance is converted to rotational motion of the ball screw and drum assembly.

3.1.3 Friction - Breakaway or friction loads are created when parts rub or roll upon each other. Mechanical snubbers have fairly low breakaway loads, usually about 1 percent of the rated capacity. These loads are generated in the ball nut mechanism, bearing sleeves, and in the telescoping cylinder. The friction loads are sensitive to corrosion and certain types of contamination. Situations may arise where friction or breakaway forces exceed this 1 percent value.

3.2 The Hydraulic Snubber

The hydraulic arrestor is a velocity sensitive device used for restraining pipe motion during dynamic loading while permitting free thermal motion. A control valve permits unrestricted motion at low velocities, whereas resistance to motion is encountered at high velocities. The operational characteristics of the hydraulic snubber are related in general to two parameters - the locking velocity, and the bleed velocity. The locking velocity is the velocity at which free flow of the hydraulic fluid through the valve is stopped, and resistance to movement develops. At this velocity, a poppet valve closes and the fluid flow is restricted through a bleed orifice. When the hydraulic fluid is channeled through the "bleed" system, restraining forces are created which are proportional to the bleed velocity, Figure 3.2. The bleed and locking velocities can be set independently of each other and can be changed from the nominal values that have been set at the factory.

Figure 3.2 Schematic of the Snubber Cylinder and Control Valve

3.2.1 Lock Velocity - The lock velocity is that actuator or piston velocity at which free motion of the actuator stops and restricted motion begins. Prior to a seismic event, all motion occurring with a velocity less than this value will be unrestrained. The actuator will also lock when acceleration exceeds a given level.

3.2.2 Bleed Rate - The bleed rate is the restrained motion that occurs after snubber lock up occurs (see Locking Velocity). The piston velocity is proportional to the actuator load, that is, a velocity dependent force is produced as a result of this action.

The physical properties of the fluid have significant effects on the operation of hydraulic snubbers. The stiffness properties of snubbers are particularly sensitive to the effective bulk modulus and, of course, the bulk modulus is quite sensitive to air entrapment and temperature. If air pockets exist in the system, dead bands may also exist.

3.2.3 Clearance - For hydraulic snubbers this is unrestrained motion, at piston velocities greater than the locking velocity. It occurs because of mechanical tolerances at end fittings and entrapped air in the hydraulic fluid. The unrestrained motion acts as a clearance or dead band.

3.2.4 Friction - The friction or breakaway loads result from the actuator rod bushing and seal, and the piston seal. Corrosion effects tend to increase these loads.

3.2.5 Unlock Loading - This is the actuator load due to hydraulic pressure which causes the snubber to become unlocked; that is, free flow of hydraulic fluid takes place in the snubber as the poppet valve opens.

3.3 Parameter Combinations

The snubber parameters that affect response can be divided into two categories. The first category represent those parameters which affect the relative motion between the pipe (component) attachment and the arrestor support. And

the second category represent those parameters which effect the snubber support motion. Although the second category of parameters may not be related directly to the snubber component, it is important that these characteristics be recognized.

Some parameters that may be considered in the first category include: acceleration threshold parameter, dead bands (gaps due to tolerances), linear stiffness characteristics due to the mechanical structure, nonlinear stiffness characteristics due to fluid compression or air entrapped in the hydraulic system, and viscous parameters such as lock velocities and bleed rates. Parameters that may be considered as belonging to the second category include: back-up or structural response characteristics, nonlinearities due to snubber orientation (single snubber usage), and clearances or local support flexibility associated with installation such as loose connections.

Figures 3.3.1 and 3.3.2 show analytical schematics of the mechanical and hydraulic snubbers. Included in these schematics are the snubber support mass and stiffness properties, and the snubber attachment flexibility characteristics. The figures describe "generalized" snubbers with various possible parameters.

3.3.1 Mechanical Snubber Model - For the mechanical snubber the schematic shows two stiffness terms, one a displacement dependent characteristic $k(X)$, the other a velocity dependent stiffness characteristic $k(\dot{X})$, a load dependent parameter C_f , which is a velocity dependent reaction, and X_L which represents a load due to motion constraints. It should be noted that the spring constants are not necessarily linear. They could be either hardening, softening or a combination of both. The respective clearances for each of these parameters are also shown. The displacement dependent stiffness characteristic represents the flexibility of the internals of the snubber, i.e., the telescoping cylinder, the ball screw shaft, and threads. The velocity dependent stiffness $k(\dot{X})$ is a nebulous entity but is probably best visualized as resistance to changes in motion of the rotating inertia mass. The C_f , or velocity dependent load may actually represent the frictional resistance of moving parts such as screw shaft - ball nut friction, or sliding interfaces like the telescoping cylinder. Some of the

Figure 3.3.2 Hydraulic Snubber Model

frictional characteristics may be coulomb type; however, they result in an "effective" velocity dependent loading. The acceleration threshold reaction is an acceleration dependent parameter which results from the interaction between the capstan spring and the inertial mass.

The clearance δ_{CX} and $\delta_{C\dot{X}}$ associated with structural stiffness of the overall component may be related to tolerances between internal parts and gaps in end attachments. The clearance δ_{CC} which is primarily due to friction is usually small since friction exists even without relative motion. The acceleration constraint clearance $\delta_{C\ddot{X}}$ is related to slop or tolerances between the ball screw shaft, and the ball nut. It is difficult to categorize these clearances according to origin, consequently clearances are lumped together as a single item.

3.3.2 Hydraulic Snubber Model - For the hydraulic snubber, the load parameter C represents a viscous load proportionality factor relating load to velocity. This is an important parameter that results from the bleed velocity setting for the snubber, and to a lesser extent the lock velocity and "free flow" pressures exerted on the hydraulic fluid. It is the classical viscous damping coefficient with one exception; it may be a function of velocity and displacement. Frictional forces may also be considered in this category. The flexibility parameters $k(X)$ and $k(\dot{X})$ are similar to those of the mechanical snubber, that is, a combination of "external" structural flexibilities and "internal" flexibilities. The internal flexibilities, however, are dependent on the properties of the hydraulic fluid such as air entrapment and entrainment, bulk modulus, and viscosity. The spring constants are not necessarily linear, and could be either, hardening, softening, or a combination of both.

Clearances from external sources such as installation tolerances are treated the same for both hydraulic and mechanical snubbers. Internal sources differ; the greatest source of clearance is from entrapped or entrained air in the hydraulic snubber. Since clearances are related to the hydraulic fluid they will be considered as part of the nonlinear restorative characteristic of the fluid.

4. STRUCTURAL/ANALYTICAL MODELS

The selection of a mathematical model involves consideration of system response characteristics, load types, restraint characteristics, and the interaction with other dynamic systems, e.g., the snubber and supporting structure. Utilization of simple structural models formed the basis for meeting the goals of the study. The single degree of freedom lumped mass oscillator was used intensively in the study. The simple oscillator model enables one to readily isolate parameters and establish dimensionless combinations including both system response characteristics and snubber parameters. The simple oscillator also minimizes the number of variables that affect system response.

The response of the lumped mass oscillator when related to a more complex system can be viewed as an anchor motion or base excitation input to the system which is restrained by the snubber.

When a simple system is used to develop response sensitivity characteristics, one must realize that input and restraint effects become more pronounced than they would be for a complex system. For example, consider the single degree of freedom lumped mass systems shown in Figure 4.1, here each system has the same characteristic equation,

$$m\ddot{X} + C\dot{X} + kX = f(t).$$

The response magnitude is a function of $f(t)$ and is different for each model shown. The load expressions for each of the six cases are shown in Table 4.1.

The response sensitivity to the viscous parameter, C , can be studied using at least five different structural models. Although it is possible to logically single out one model as more realistically representing a given situation, the reference sensitivity of snubber parameters suggests that any one or all of the models can be used to meet the goals of this program.

Figure 4.1 1-D.O.F. Boundary Condition Variations

TABLE 4.1
 FORCING FUNCTION PERMUTATIONS

Case	$f(t)$
1	$kX_B [\sin \Omega t]$
2	$kX_B [\frac{1}{2} \sin \Omega t + (C\Omega/k) \cos \Omega t]$
3	$kX_B [\frac{1}{2} \sin \Omega t]$
4	$kX_B [\sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$
5	$kX_B [\frac{1}{2} \sin \Omega t + \frac{1}{2} \sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$
6	$kX_B [\frac{1}{2} \sin \Omega t + \frac{1}{2} \sin (\Omega t - \phi) + (C\Omega/k) \cos \Omega t]$

Depending on the component or system configuration, the snubber may affect the system response by restraining motion, by acting as a load or input attenuator, or a combination of the two. Component sensitivity studies are based on various types of loadings and supports. This allows an overall response sensitivity to be established. Studies consider base excitation, force inputs, and displacement acceleration inputs.

In addition to the simple oscillator, more complicated structures have been studied, such as, a multi-degree of freedom lumped mass model, simple beam models and piping loops. These more sophisticated models are used to verify the results of the study of simple systems. Consideration of the more sophisticated system permits insights into multi-mode effects and reduced response sensitivity.

The lumped mass models represent a series of models with different natural frequencies and forcing functions. Figures 1.1(c) and 1.1(d) represent two of several beam models used in this study. Figure 1.1(e) represents the "typical" piping loop with components that are found in reactor piping such as valves, springs, straight pipe runs and elbows. The valve weights, spring rates, and lengths with dynamic characteristics - in particular, the natural frequency and displacement and stress responses - approximate those found in typical reactor installations. Without the snubber installation the first three natural frequencies of this model are 2.311 Hz, 4.191 Hz and 7.251 Hz. With the snubber installed the first three frequencies are 5.854 Hz, 9.881 Hz and 11.292 Hz.

It is recognized that the static and dynamic properties of the back-up structure contribute to or modify the response characteristics of the snubber. Compliance with the dynamic coupling of the systems are investigated as a separate task in this study.

5. FORCING FUNCTIONS

Use of harmonic forcing functions in simple oscillators allows the development of dimensionless expressions thus reducing the number of independent variables. With the response characteristics dependent on the forcing frequency, the use of actual seismic inputs is essential to the development of a comprehensive understanding of snubber parameter sensitivity.

Seismic loadings selected for this study represent the four most widely used seismic inputs for the design of nuclear facilities (Table 5.1).

TABLE 5.1
SEISMIC EVENTS INPUTS

Event	Location	Richter Magnitude	Max G
Imperial Valley Earthquake	El Centro, California	6.7	.348
Kern County Earthquake	Taft, California	7.7	.196
San Fernando Earthquake	San Fernando, California	6.4	.253
Western Washington Earthquake	Olympia, Washington	7.1	.280

Information was obtained from the CAL TECH Earthquake Engineering Research Laboratory, Reference 2, with test instrument corrected and fully documented digitized acceleration, velocity and displacement data. This data was processed through the Rockwell computers and the CRT plots are shown for the various seismic disturbances in Figures 5.1 through 5.9. Acceleration traces are shown in Figure 5.1 through 5.4 and displacement traces are shown in Figure 5.5 through 5.8. The response spectral data for these seismic events are shown in Figure 5.9.

The records used in the investigation were measured only in the United States and predominantly in California. The conclusions are applicable to the west coast of the United States. Slower ground motion attenuation rates in the east and mid-west suggest that ground motions in these regions could have different frequency contents. Nevertheless, the seismic design of major structures in the east and mid-west have of necessity, been based on measured records from the west. This practice will no doubt continue until a sufficiently large ensemble of strong motion records can be obtained from the east and mid-west.

Figure 5.1 El Centro Acceleration Seismic Input

Figure 5.2 San Fernando Acceleration Seismic Input

Figure 5.3 Washington Acceleration Seismic Input

Figure 5.4 Taft Acceleration Seismic Input

Figure 5.5 El Centro Displacement Input

Figure 5.6 Taft Displacement Input

Figure 5.7 San Fernando Displacement Input

Figure 5.8 Washington Displacement Input

6. SUPPORT CHARACTERISTICS

6.1 Back-up Structural Stiffness

Typical dynamic analyses of components and piping systems consider the snubber as a rigid restraint. This is considered particularly valid when the motion during a dynamic disturbance is very small and applies when clearances are nonexistent, or loads are low compared to the rated capacity of the snubbers. The response of the system, however, can be influenced by the flexibility of the supporting structure.

Consider an infinitely long pipe supported at uniform distances.

The fundamental mode of this system can be found using a single span.

The second mode can be found by changing the boundary conditions from pin-pin to fixed-fixed.

Supports on the infinitely long pipe can be related to anchors in a general piping system. The resonant frequency of the system can best be changed by placing a snubber at the center of the span.

Using the two systems in the figure, the effects of snubber stiffness on system frequencies can be evaluated by plotting the dimensionless parameters of restrained natural frequency/unrestrained natural frequency, (f_n/f_1) , and the spring stiffness/pipe stiffness, (k/K^0) as shown in Figure 6.1.1. The flexibility at the point of snubber attachment is α . The pipe stiffness, K^0 , is evaluated on the basis of pipe flexibility at the snubber location. K^0 is, therefore, a function of not only EI (modulus of elasticity and bending moment of inertia) and ℓ for a general system but also a function of hanger and anchor locations and geometry.

For the snubber to be active in changing the system frequencies, its "effective" stiffness needs to be at least twenty times greater than the pipe stiffness (K^0). That is, the snubber is most effective in altering the response of the system when $(k/K^0) \geq 20$. This is the case when response can be directly related to system natural frequencies. This can be verified by noting system response, rather than system natural frequencies, as a function of the stiffness parameter, (k/K^0) . The results for a simple beam model are shown in Figure 6.1.2 where peak transient response for an undamped system is shown as a function of "effective" snubber stiffness. Figure 6.1.3 presents the results for the El Centro seismic loading.

Figure 6.1.1 Snubber Stiffness vs Natural Frequency

Figure 6.1.2 Response Spectra for Simply Supported Beam

Figure 6.1.3 Beam Response to El Centro Seismic Input

Two different harmonic forcing functions and one seismic loading were examined to evaluate the effects of resonance on the forcing frequency. In all cases, the response was found to be insensitive to k for $(k/K^0) > 10$. This same trend is expected to exist regardless of the complexity of the structure, provided reasonable engineering design practices are used. For example, if the snubber were placed very close to either support, the response sensitivity to (k/K^0) would become less pronounced. This would represent poor engineering choice because the effect on the response would be minimal.

The hydraulic snubber efficiencies require additional consideration of the backup structure. This can best be demonstrated by the dashpot-spring arrangement subjected to a harmonic excitation,

This system crudely represents a hydraulic snubber "C" with a back-up support stiffness "k" associated with it. The steady state force in the snubber is,

$$F = CX_B\Omega \left[\left(\frac{a^2}{1+a^2} \right) \cos \Omega t + \left(\frac{a}{1+a^2} \right) \sin \Omega t \right],$$

where,

$$a = \left(\frac{k}{C\Omega} \right)$$

The development of the above relationship is given in Section 9.

If efficiency of the support is defined as the ratio of the maximum load developed to the maximum possible load, the following is obtained,

$$\eta = \sqrt{\frac{a^2}{1 + a^2}} = \sqrt{\frac{\left(\frac{k}{c\Omega}\right)^2}{1 + \left(\frac{k}{c\Omega}\right)^2}}$$

This expression gives a method for determining support stiffness for a given forcing function, and type of hydraulic snubber. Figure 6.1.4 shows a plot of the efficiency (η) versus ($k/c\Omega$). For low efficiencies a minimal effect is experienced for reducing response. On the other hand effectiveness is enhanced in reducing response for high efficiencies. If a minimum 0.75 efficiency is desired from Figure 6.1.4, it can be seen that k must be greater than $1.2 (c\Omega)$.

Knowing k and ($c\Omega$), Figure 6.1.5 can be used to evaluate the effectiveness of the snubber.

6.2 Support Dynamics

In the discussion of back-up structural stiffness, it was presumed that the dynamic characteristics of the supporting structure did not attenuate the input loading or couple dynamically with components and piping systems. Dynamic coupling, however, can occur and the response of the system can be greatly influenced by its dynamic characteristics.

The response characteristics of structures should be considered when designing components and piping systems. This practice is generally used in the design of nuclear facilities. Even when detailed dynamic analyses are not required the spacing of piping supports and snubbing devices are often selected so that the frequency of the piping will be significantly different from that of main structures. Building structures typically have a resonant frequency range of 2 → 4 Hz. For this reason, component and piping systems are designed to have a natural

Figure 6.1.4 Hydraulic Snubber Efficiency

Figure 6.1.5 Support Efficiency

frequency of at least 8 Hz. Although a higher frequency would be desirable, 8 Hz is considered to be a realistic and acceptable value.

The basic structural feature selected to represent dynamic characteristics of structures supporting snubbed components and systems is one where the ground motion input is applied at all anchors and all snubber locations. The model is an expansion of the generalized model as illustrated below.

The analytical solution for the response of the system can be found in Section 9, Analytical Methods.

Consider the structural model shown in the figure below:

$$\frac{C}{M} = 10,000 \text{ sec}^{-1}$$

$$\frac{m}{M} = 1.0$$

$$\frac{1}{2\pi} \sqrt{\frac{K}{m}} = 8 \text{ Hz}$$

Viscous parameters are considered in this structural mode. The parameters K (stiffness) and M (mass) represent the equivalent properties of the system for which the response is studied and k and m those for the supporting structure. The system and supporting structure natural frequency are $\frac{1}{2\pi} \sqrt{\frac{K}{M}}$ and $\frac{1}{2\pi} \sqrt{\frac{k}{m}}$, respectively. Coupling is accomplished using the snubbing device. In this example, the "uncoupled" supporting structure has a natural frequency of 8 Hz and is excited by a 6 Hz harmonic base excitation.

The maximum response can be evaluated for various values of supporting structure natural frequencies (k/m). Thus the effects of support frequency in combination with the viscous parameters can be observed with the effects of viscous properties. The results of this study are shown graphically in Figure 6.2.1.

The response of the masses, representing the two ends of the snubber, are nearly equal. A response peak is noted similar to a classical response-frequency curve. The small difference in relative motion between both ends of the snubber relative to the overall motion of the masses suggests that the effective stiffness of the snubber is much greater than the stiffness of the component system or the supporting structure. Since the effective stiffness is much greater than the apparent dynamic stiffness of the supporting structure the overall response of the component system (K, M) can be approximated from the model shown in the figure below.

Figure 6.2.1 Support Dynamic Characteristics

The response of the system can be approximated by

$$\delta = \frac{\ddot{x}_B}{\frac{K}{M} \left(\frac{1}{1+\beta} \right) + \frac{k}{m} \left(\frac{1}{1+\beta} \right) - \Omega^2},$$

where,

$$\beta = \left(\frac{m}{M} \right).$$

The natural frequency of the coupled system is

$$f_{ns} = \frac{1}{2\pi} \sqrt{\frac{K}{M} \left(\frac{1}{1+\beta} \right) + \frac{k}{m} \left(\frac{\beta}{1+\beta} \right)}.$$

When the forcing frequency approaches f_{ns} the response will increase as a resonant condition is created.

7. DETAIL PARAMETER STUDIES

7.1 Acceleration Threshold Parameter

The acceleration threshold parameter (A.T.P.) is a characteristic that is unique to the mechanical snubber. It limits acceleration of the pipe movement relative to the base of the snubber.

The effects of the A.T.P. on system response were studied using the single degree of freedom lumped mass oscillators and simple beam models shown in Figure 7.1.1 and the more complex piping system shown in Figure 1.1(e). The investigation considered steady-state and transient response characteristics resulting from harmonic loadings, and transient response characteristics resulting from seismic inputs.

Consider the simple system shown below excited by a harmonic loading. The response of the system was evaluated by numerical methods (Section 9). The transient response for the first few cycles of loading is shown in Figure 7.1.2.

The response of the lumped mass system does not respond at either the forcing frequency or the natural frequency, but responds with a complex response which appears to be the summation of two harmonics, one the forcing frequency and the

Figure 7.1.1 A.T.P. Structural Models

Figure 7.1.2 Oscillator Response Due to A.T.P.

other a much lower frequency component. This lower frequency component is referred to as the shadow frequency.

To describe this shadow frequency, consider the steady-state response of a single degree of freedom, lumped mass as a result of an harmonic excitation, without the influence of the acceleration limit - acceleration threshold parameter. The steady-state response, velocity and acceleration can be expressed by

$$\begin{aligned} X &= A_0 \sin(\Omega t - \phi), \\ \dot{X} &= A_0 \Omega \cos(\Omega t - \phi), \\ \ddot{X} &= A_0 \Omega^2 \sin(\Omega t - \phi) \end{aligned}$$

where, ϕ is a phase angle between the applied force and the response.

The system acceleration is an harmonic function which is 180° out of phase with the displacement. If the acceleration were plotted as a function of time it would be a continuous sine wave shown below.

This assumes an Acceleration Threshold Parameter where, $\ddot{X}_L \ll A_0 \Omega^2$.

Applying the acceleration limits to the system acceleration response curve, the system acceleration response becomes that shown by the solid line in the figure below.

If $\ddot{x}_L \ll A_0 \Omega^2$, the acceleration characteristics become, for the first cycle

Where τ is the period of the forcing function. If the acceleration is integrated from $0 \rightarrow \tau$ one finds that there is a net movement or shift of the mass from the neutral position as shown in Figure 7.1.3.

Figure 7.1.3 A.T.P. Residual Amplitude

The mass undergoes an incremental displacement $\Delta X = 1/4 \ddot{X}_L \tau^2$. If this situation were to continue to exist, the mass would continue to move away from the neutral position at a rate of $1/4 \ddot{X}_L \tau^2$ /cycle. As the mass moves away from its neutral position an additional force equal to kX is applied to the mass causing a shift in amount of time that $+\ddot{X}_L$ applies and $-\ddot{X}_L$ applies. For the first cycle,

$$\ddot{X}_L \text{ exists when } 0 \leq t \leq \tau/2$$

and,

$$\ddot{X}_L \text{ exists when } \tau/2 \leq t \leq \tau$$

For the second cycle, in which the applied load (resulting acceleration) is changed because of the added force (kX) the following situation exists,

$$\ddot{X}_L \text{ exists when } 0 \leq t \leq (\tau/2 - \Delta)$$

and,

$$\ddot{X}_L \text{ exists when } (\tau/2 - \Delta) \leq t \leq \tau$$

The Δ value for each cycle of loading varies because the total force ($F \sin \Omega - kX$) varies. The value of Δ increases during each cycle of loading until the maximum displacement (zero velocity) of the shadow response occurs, at which time, Δ reaches a maximum $\Delta_{\max}$. The cycle then continues with Δ decreasing until the time shift reaches its negative maximum $-\Delta_{\max}$ at which time the shadow response is at its minimum position. The shadow response represents the time integral of the acceleration, as shown below.

This shows how the shadow response is created. The higher frequency perturbations seen in Figure 7.1.2 have a period equal to the period of the forcing function.

Considering the same lumped mass oscillator used in the previous example the response characteristics have been determined for various frequency ratios β , where

$$\beta = \frac{\text{Forcing Frequency}}{\text{Natural Frequency}}$$

This is shown in Figure 7.1.4 as a function of the A.T.P. The curve indicates the response sensitivity to the A.T.P. One can see that the response sensitivity is a function of forcing frequency and A.T.P. value.

Utilizing numerical analysis procedures the response spectrum for a single degree of freedom lumped mass is shown in Figure 7.1.5. The response of the system as a function of the forcing frequency is shown for several acceleration threshold parameter values. The dashed line shown on this figure represents the

Figure 7.1.4 Response Characteristics Due to Acceleration Threshold Parameter

Figure 7.1.5 Response Spectrum for Lumped Mass Oscillator

response if no snubber were present, ($\ddot{X}_L = \infty$). The results presented for this one case indicate that it is possible to obtain a greater response, with an ideal snubber possessing an acceleration threshold characteristic, than one would get if no snubber were present.

The results for this particular case indicate that for low frequency harmonic excitations (< 2 Hz), the "restrained" response of the simple oscillator will exceed the unrestrained response for A.T.P. values that were considered. This situation also exists at high frequencies, however, with a diminished response amplitude. Therefore, the low frequency response characteristics are more important.

To develop greater insight into the response sensitivity of simple systems towards the A.T.P. the following evaluation is made. In the subsequent analyses dimensionless ratios are utilized.

The lumped mass systems are subjected to a harmonic base excitation, where $\ddot{X}_L$ represents the A.T.P. If $X_L \ll |X|$, where $|X|$ is the amplitude of the acceleration of the system without the A.T.P., the response of the system can be evaluated in terms of dimensionless parameters.

Beginning with the differential equations of motion for each system,

$$[\text{Sys (a)}] \quad \ddot{\delta} + \omega^2 \delta = X_B \Omega^2 \sin \Omega t, \quad \dots (7.1.1a)$$

and,

$$[\text{Sys (b)}] \quad \ddot{X} + \omega^2 X = X_B \Omega^2 \sin \Omega t, \quad \dots (7.1.b)$$

the following differential equation of motion representing the response for the system can be developed. (See Section 9, Analytical Methods.)

$$\ddot{Q} = \phi \operatorname{sgn} |\sin \Omega t - Q| \quad \dots (7.1.2)$$

For system (a)

$$Q = \left(\frac{\omega}{\Omega}\right)^2 \frac{\delta}{X_B}, \quad \varphi = \left(\omega^2 \frac{\ddot{X}_L}{|\ddot{X}_B|}\right), \quad \dots (7.1.3a)$$

and, for system (b),

$$Q = \left(\frac{X}{X_B}\right), \quad \varphi = \left(\Omega^2 \frac{\ddot{X}_L}{|\ddot{X}_B|}\right) \quad \dots (7.1.3b)$$

Equation (7.1.2) is solved using computerized numerical techniques. The results obtained from the solution of Equation (7.1.2) are shown in Figure 7.1.6. The response curve shows the peak values of response (X/X_B) as a function of $(\ddot{X}_L/|\ddot{X}_B|)$. Although the pseudo response $\ddot{Q}$ (see Equation 7.1.2) appears to be a function of both φ and Ω , the Ω can be eliminated since φ also contains the frequency parameter. Therefore, the remaining parameters are (X/X_B) and $(\ddot{X}_L/|\ddot{X}_B|)$. The relative displacement δ for system (a) and the absolute displacement X for system (b) are interchangeable in Figure 7.1.6.

The insights that can be derived from Figure 7.1.6 is limited, yet important. The observation most pertinent to this study concerns the response parameter (X/X_B) . Results of the study indicate that $(X/X_B) > 1$ for certain values of $(\ddot{X}_L/|\ddot{X}_B|)$. The implications of this are not obvious at first. It appears, however, that there can be response amplification in the presence of A.T.P., (X_L) , consequently it is possible that response for a given system can be increased due to the A.T.P.

Next, consideration was given to non-harmonic input, specifically seismic ground motion. For the case of the lumped mass model, the sensitivity of the dynamic response to A.T.P. has been evaluated and presented in Figure 7.1.7. The El Centro ground acceleration was applied to a 6 Hz and 12 Hz lumped mass oscillator. The results indicate that as the $\ddot{X}_L$ value increases, the response increases. However, the response of the 12 Hz oscillator indicates, shaded area of Figure 7.1.7, that for certain values of the acceleration threshold parameter,

Figure 7.1.6 Acceleration Threshold Parameter Harmonic Input

Figure 7.1.7 A.T.P. Response Due to El Centro Input

($\ddot{X}_L$), the response was greater than if the A.T.P. were not considered. Also shown in Figure 7.1.7 is the response of both systems in the absence of $\ddot{X}_L$. Although the response is greater for specific values of $\ddot{X}_L$ for the 12 Hz system, the total response is an order of magnitude smaller than the response for the 6 Hz system.

The next step in this study was to develop a structural model that would permit the response characteristics of simple beam models to be studied as a function of the effects of the acceleration threshold parameter. An analyses procedure that could solve this problem would be general in nature and be applicable to piping analyses. The analyses procedure that was developed is presented in Section 9.

The basic model incorporated lumped mass finite element techniques, utilizing modified modal analyses procedures. The result for the model are shown below are presented.

At this point in the study, it is appropriate to look at the results of one specific system. The system described above was subjected to an harmonic loading while attention was given to the transient displacement response. Although, only a few cycles of response were studied, trends could be easily detected.

The model shows a simple beam that was used for purposes of studying variations of the snubber parameter, $\ddot{X}_L$, acceleration threshold parameter. Figure 7.1.8 presents the results of this preliminary study. Shown on this figure are the response traces for the initial transient phases of loading for various $\ddot{X}_L$ values.

Figure 7.1.8 Beam Response Due to Acceleration Threshold Parameter

Figure A shows the response when $\ddot{X}_L = 0$, or when the snubber acts as a rigid support. Figure B shows the response when $\ddot{X}_L = 0.1 \text{ in/sec}^2$. The superimposed dashed line in this figure is the response of the beam at the point where the snubber is attached. From this figure, one can see how the low cycle shadow response starts to appear as a superimposed response. Figure C indicates an even more pronounced effect which is seen in the increased response and Figure D shows the total response when the snubber response is at its apparent maximum.

When $\ddot{X}_L = 5 \text{ in/sec}^2$, the steady-state response, that is, the component responding at the forcing frequency, is completely dominated by the low cycle shadow response. The response of the point being forced follows very closely the response of the snubber. As $\ddot{X}_L$ is increased, the snubber becomes "inactive" during certain portions of the applied loading. At this time, the response begins to take on a complex form as shown in Figure E. And finally when the snubber is removed ($\ddot{X}_L = \infty$), the response becomes predictable.

It is apparent from this example that if the standard engineering practice of representing a snubber as a rigid restraint were used ($\ddot{X}_L = 0$) - A, the response could be underpredicted by a factor as high as 10. ($\ddot{X}_L = 5 \text{ in/sec}^2$) - D.

7.2 Viscous Parameters

Viscous properties represent a series of three parameters that are related to hydraulic snubbers and are "lumped" together. These parameters are: 1) lock velocity; 2) bleed rate; and 3) unlock criteria. Their functions are to initiate snubber operation, control snubber reaction, and disengage the snubber during dynamic loadings.

The operation of the snubber can be described briefly as follows: Pipe movement is permitted until the velocity reaches the locking velocity, at which time the snubber engages. This locking velocity is dependent on the viscosity of the hydraulic fluid and the preset compression of the poppet spring. After lockup, additional displacement of the snubber piston is through the bleed

mechanism. The bleed velocity is a function of the load on the piston. The flow rate through the bleed orifice varies approximately linearly with the load on the piston rod. After lockup occurs, the piston velocity at the rated load of the snubber is defined as the "bleed rate" of the snubber. When the piston load drops below a certain value the poppet spring opens and free flow occurs with the snubber disengaged.

The viscous parameters are highly dependent on the physical properties of the hydraulic fluid, i.e., operation of the snubber is sensitive to both the viscosity and bulk modulus of the fluid. Since the scope of the study involves only response sensitivity to the snubber parameters, this study will not include hydraulic fluid physical variations or any other considerations directly involved with operation of the snubbing device that control the magnitudes of the viscous parameters. It is recognized that temperature and fluid volume have an effect on hydraulic snubber performance, however, this study only considers the parameter and not the cause.

The bleed rate is a load dependent property which is defined in terms of the rated load of the snubber. Since bleed rate and rated load are connected with each other a new parameter referred to as the damping parameter "C" is introduced. This parameter has units $\left[\frac{\text{FORCE-TIME}}{\text{LENGTH}}\right]$ which are the same as the classical viscous damping coefficient and represents the ratio of the snubber rated load to bleed rate at the rated load,

$$C = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

Therefore, throughout the following analysis this term will be used in place of bleed rate.

Considering the range of viscous parameter values of hydraulic snubbers, i.e., the rated load capacities and nominal bleed rate values, the snubbers have viscous damping parameters C that are much greater than the critical damping value for that of the system. Consideration of resonant influences is not required in the investigation of viscous parameters. The effect of resonance on system response may have a slight effect for large $\dot{x}_L$, lock velocity values.

This effect is not considered significant in the practical range of lock velocities ($\dot{X}_L < 1.0$ in/sec).

For commercial snubbers the range of interest for the viscous damping parameter is $10^4 < C < 10^6$ lb-sec/in. The practical range of interest for the lock velocity is $0.01 < \dot{X}_L < 1.0$ in/sec.

Consider the single degree of freedom lumped mass oscillator, excited by a harmonic base motion. The initial transient response for this system is shown in Figure 7.2.1. Since the system is critically damped the transient response does not contain free vibration components. The transient response consists of the steady-state component plus an exponential decay component. Due to the initial shift developed during the first cycle of loading the maximum response occurs during the first loading cycle for harmonic excitations. The actual peak-to-peak response maximum occurs later in time. Consequently a reasonable estimate for the maximum displacement would be twice the steady-state amplitude. Any trends that are established are based on steady-state harmonic analysis and reflect response trends for the transient response regime.

With the large number of permutations of the various system parameters, the present study was limited to a few selected cases. Since the dynamic response appears to be more sensitive to lock velocity than to the other viscous parameters, this parameter is discussed first.

Since applied force, forcing frequency, and natural frequency are not viscous parameters the first task was to determine the effects on system response. The response data presented in Figure 7.2.2 is used to evaluate forcing frequency effects on the lock velocity response sensitivity. In this study various harmonic excitations were considered in connection with a given dynamic system. The response at small lock velocities ($\dot{X}_L < 0.03$ in/sec) for the frequencies considered is practically independent of lock velocity. This is reasonable since the snubber is active during almost the entire excursion. In this case the response can be approximated by

Figure 7.2.1 Transient Response for Harmonic Input
 (Critically Damped System)

Figure 7.2.2 Forcing Frequency Effects on Viscous Response

$$\frac{x}{x}_{\text{stat}} \approx \frac{1}{2\xi \left(\frac{\Omega}{\omega}\right)}$$

where

ζ = Critical damping ratio $\gg 1$

Ω = Forcing frequency

ω = Natural frequency

The response for large lock velocities ($\dot{x}_L > 10$ in/sec) is also independent of lock velocity. In this case the response velocity does not exceed the lock velocity and snubber restraint is not experienced. For this region the response is

$$\frac{x}{x}_{\text{stat}} = \frac{1}{\sqrt{\left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]^2 + \left(2\xi \frac{\Omega}{\omega}\right)^2}}$$

Between the two extremes of lock velocity, $0.03 < \dot{x}_L < 10$ in/sec, the response sensitivity to the lock velocity appears to be similar for each of the different harmonic forcing frequencies studied. The free response amplitude is reduced by 95 percent for lock velocities less than 1 in/sec (Figure 7.2.2).

An important question in seismic design is, "What range of locking velocity will limit the response of the system to acceptable limits?" For practical purposes it is reasonable to presume that any snubbing device that reduces motion to five percent of its unrestrained value is acceptable. Based on these limited observations locking velocities less than 1 in/sec will perform this task satisfactorily.

Since the previous results were obtained for a specific load value, (F/m), the next step was to determine what effects the applied load has on system response. The results shown in Figure 7.2.3 are used for making this determination. As in the previous example a harmonic loading was employed. However,

Figure 7.2.3 Load Effects on Viscous Response

in this example the applied frequency is fixed and is equal to the undamped natural frequency of the oscillator.

At low lock velocities, $\dot{X}_L < 0.03$ in/sec, the response is proportional to applied load, (F/m). The response is similar to that of a "classical" highly damped lumped mass oscillator. For very high lock velocities, the response is also proportional to the applied load, with the same response characteristics as a single degree of freedom undamped lumped mass oscillator. For the intermediate values of lock velocity the applied load, (F/m), appears to have a less significant effect on response than at the lock velocity extremes. It appears from these limited observations that the unrestrained free response amplitude will be reduced by 95 percent for lock velocities less than 1 in/sec regardless of the applied load, (F/m).

Since large motion seismic events may produce ground accelerations which approach 1 g a practical value of (F/m) can be deduced from this. A typical loading is

$$\frac{F}{m} = \frac{W}{W/g} (1.g \text{ loading}) = g = 386.4 \frac{\text{in}}{\text{sec}^2}$$

and

$$\left(\frac{F}{m}\right) \approx 350 \rightarrow 400 \left(\frac{\text{in}}{\text{sec}^2}\right)$$

Figure 7.2.4 indicates the response sensitivity for a different viscous damping parameter, $C/m = 10^3$ (sec^{-1}). This value is considerably lower than those encountered in typical systems, however, it indicates similar sensitivity trends to those observed for $C/m = 10^5$ (sec^{-1}).

The next step was to evaluate response sensitivity for various C (bleed rate) values as a function of lock velocity. Two sets of data were generated for this purpose and both studies utilize harmonic excitations. The input for these

Figure 7.2.4 Load Effects on Viscous Response

studies, the results of which are shown in Figures 7.2.5 and 7.2.6 was a harmonic base acceleration with a peak acceleration of 0.348g and a frequency of 6 Hz. This acceleration level was selected because it has the same maximum input level as the El Centro earthquake. The present study was extended to consider this earthquake for additional comparisons. Figure 7.2.5 shows the response results for a 6 Hz natural frequency system while Figure 7.2.6 shows the results for a 12 Hz resonant system.

Inspection of Figures 7.2.5 and 7.2.6 indicates that the response characteristics are nearly identical for lock velocities less than 1 in/sec independently of the damping value. For heavily damped systems the response is independent of the system natural frequency. It can be shown that the steady state relative displacement can be approximated by

$$\delta = \left(\frac{m}{C\Omega}\right) |\ddot{X}_B| = \left(\frac{\Omega m}{C}\right) X_B \quad ,$$

where

δ = Relative displacement (in)

$|\ddot{X}_B|$ = Base acceleration (in/sec²) ($\Omega^2 X_B$)

C = Damping (lb-sec/in)

Ω = Harmonic forcing frequency (rad/sec), and

m = System mass (lb-sec²/in) .

However, as the lock velocity increases the response becomes increasingly sensitive to the natural frequency of the system and when the lock velocity is sufficiently large the response becomes that of an undamped oscillator whose steady state response can be expressed as

Figure 7.2.5 Bleed Rate Effects on Viscous Response

Figure 7.2.6 Bleed Rate Effects on Viscous Response

$$\delta = \frac{x_B \beta^2}{(1 - \beta^2)} = \frac{|\ddot{x}_B| \beta^2}{\Omega^2 (1 - \beta^2)}$$

where

Ω = Forcing frequency (rad/sec)

ω = Natural frequency (rad/sec)

$\beta = \Omega/\omega$

$x_B, |\ddot{x}_B|$ = Base displacement and accelerations, respectively - (in), (in/sec²)

It appears that the response characteristics are almost identical for both systems for lock velocities less than 1 in/sec, however, for higher lock velocities the response becomes sensitive to the resonant characteristics of the system.

Since the response amplification was small for the 12 Hz system, relatively low lock velocities are required to reduce the response to 95 percent of the unrestrained value - $\dot{x}_L < 0.2$ in/sec. It is anticipated that later studies will show that since the actual response is low, a detrimental effect will not be experienced for much larger lock velocities. Further insights can be obtained if response is plotted as a function of the viscous parameter $C \equiv (C/m)$ for various lock velocities. Figure 7.2.7 indicates that for a given lock velocity response may increase as the bleed rate is reduced (C is increased). This situation appears to exist when the lock velocity exceeds 0.3 in/sec.

The results presented for the limiting cases $\dot{x}_L = 30$ in/sec and $\dot{x}_L = 0$ in/sec can be explained, however, a physical interpretation of this phenomenon for intermediate values of $\dot{x}_L$ is not available at this time.

Figures 7.2.8 and 7.2.9 indicate response characteristics for the systems investigated in Figures 7.2.5 and 7.2.6, however, the input is the El Centro earthquake ground motion rather than harmonic input. The system response

Figure 7.2.7 Lock Velocity Effects on Viscous Response

Figure 7.2.8 Bleed Rate Effects on Viscous Response
(Seismic Input)

Figure 7.2.9 Bleed Rate Effects on Viscous Response

characteristics are very similar for El Centro motion and for harmonic inputs (both 0.348 g maximum acceleration) as can be seen from Figures 7.2.5, 7.2.6, 7.2.8 and 7.2.9. For realistic viscous damping parameters ($10^4 < C < 10^6 \frac{\text{lb-sec}}{\text{in}}$) the response appears to be insensitive to the damping (bleed rate) for lock velocities greater than 0.03 in/sec (0.18 in/min). The response can be represented by a straight line on log-log paper as indicated in Figures 7.2.8 and 7.2.9. Although the maximum input g levels are the same for both models, the El Centro response is slightly greater than the harmonic input because the harmonic response represents the 1st cycle maximum response whereas the El Centro maximum response represents the maximum during the entire loading. Plotting the first cycle maximum response, the actual maximum response will occur for critically damped systems for all cases where $\dot{x}_L$ is sufficiently low enough so that the snubber can apply a restraining load. For the cases studied, $\dot{x}_L < 10 \text{ in/sec}$ or 600 in/min. Figures 7.2.10 and 7.2.11 show the transient response during the first 20 seconds of the seismic disturbance.

Taking a specific example where the system frequency is 6 Hz and $C/m = 10,000 \text{ sec}^{-1}$, the response due to four different seismic disturbances is calculated and plotted in Figure 7.2.12. Response sensitivity and magnitude are similar for lock velocities that are not representative of extreme values. The largest differences in response occur at the extremes, i.e., when the response is independent of the lock velocity (fully engaged or fully disengaged). This characteristic property is also exhibited in Figure 7.2.3, where the response due to harmonic excitations is studied, and in Figures 7.2.8 and 7.2.9 where seismic input is studied. The results of this particular investigation suggest that the variations in frequency content and magnitude of the four seismic disturbances studied have very little effect on response sensitivity; the most significant factor is the amount of time that the velocity of the component causes the snubber to be engaged. For example, if the velocity of the component never exceeds the snubber lock velocity, the snubber will have no effect on system response, or if the unrestrained response velocity is large compared to the snubber lock velocity, the response will be insensitive to changes in lock velocity.

Figure 7.2.10 Response to El Centro Ground Motion Acceleration for
 12. Hz Lumped Mass Oscillator

Figure 7.2.11 Response to El Centro Ground Motion Acceleration for 6. Hz Lumped Mass Oscillator

Figure 7.2.12 Seismic Effects on Viscous Response

Most of the studies concerning the viscous parameters have been made using the single degree of freedom oscillator. A logical extension of these studies would be to consider the snubber parameters in relation to a "typical" piping loop. The response analyses of a system enables one to evaluate sensitivity changes resulting for increased structural complexity. The system that will be used is shown in Figure 7.2.13. Although this model does not represent an actual system, it is typical of piping loops in that it has many features found in typical loops. It contains spring hangers, valves, rigid anchors, and a piping routing (although very simple).

Assume a typical hydraulic snubber is utilized in the model, i.e., a 50,000 pound rated load, a 10 in/minute bleed rate and a 10 in/minute lock velocity. The response of the system due to El Centro seismic excitation is studied. Three conditions are examined. Case 1) investigates the response without a snubber; Case 2) investigates the response with a typical (realistic) hydraulic snubber; and Case 3) investigates the response with a rigid snubber. The results of the study are summarized in the Table below. Deflections and stresses are presented for keypoints.

	W/O SNUBBER	WITH SNUBBER	RIGID SUPPORT
Stress @ Fixed Anchor	12846 PSI	4691 PSI	3147 PSI
Stress @ Snubber	4910 PSI	301 PSI	2336 PSI
Disp. @ Snubber	1.595 in.	.306 in.	0 in.

Consider comparing the results of the complex structural model with response characteristics developed from a simple 1-degree of freedom oscillator shown in Figure 7.2.9. Using the entire mass of the piping loop in evaluating c/m its value is $1,100 \text{ sec}^{-1}$. Based on the data presented in Figure 7.2.9, the ratio of the restrained response to free response is,

$$\frac{\delta_R}{\delta_F} = \frac{.35}{2.00} = .175$$

Based on the response, of the piping loop the same ratio is

$$\frac{\delta R}{\delta F} = \frac{.306}{1.595} = .191$$

The agreement appears good for this particular case. Consequently, the results for the simple single degree of freedom oscillators are relevant with regard to complex systems.

7.3 Clearance

Clearance is associated with nearly all snubber designs and with snubber installation hardware. The actual magnitude varies from negligibly small values to accumulative values ranging from 0.125 to 0.250 inch (extreme cases). The clearance effects become particularly important when load reversals take place. For this case, the actual clearance becomes equal to the full "plus to minus" range of the dead band, hence large clearance values may need to be considered.

When determining the effects of clearance on the snubbed system, the clearance parameter is nearly always considered with another snubber parameter. The consideration of the clearance parameter in the absence of other snubber parameters does not represent a realistic situation since snubbers in general are not rigid nor is the back-up support structure completely stiff. Certain insights can be gained, however, by investigating the effects of clearance with a rigid support.

Figure 7.3.1 shows the structural model that was used to determine the sensitivity of load, stress, and displacement to snubber gap. Stresses were evaluated for a series of gaps ranging from 0 (fixed support) to infinity (no support). The dimensionless ratio ($\delta G / \delta F$) is plotted on the abscissa. For values of ($\delta G / \delta F$) > 1, the response is unaffected by the snubber gap because it is greater than free response.

MODE	NATURAL FREQUENCY - Hz	
	WITH SNUBBERS	WITHOUT SNUBBERS
1	5.854	2.311
2	9.781	4.191
3	11.292	7.254
4	15.161	10.005
5	21.161	15.527

Figure 7.2.13 Simple Piping Loop

$\ddot{X}_B(\tau) \equiv$ El Centro Earthquake

Figure 7.3.1 Clearance Effects On Stress

If the restraint were not truly rigid as is the case with a snubber, the stress would be reduced for a given (δ_G/δ_F) value as shown in Figure 7.3.1. The reduction in snubber reactor load (Figure 7.3.2) reduces the stress at point B. Increased support (snubber) flexibility does not, in general, reduce stresses at all points in a system. Figure 7.3.3 shows the effect of clearance on displacement response. The response at the snubber increases, with structural support flexibility. If the flexibility of the snubber is very large (soft spring), the natural frequency of the system may be affected and the response characteristics of the system will change.

Displacement response is reduced when the snubber is added to the system. This may not be the case for stresses, in particular, at the snubber location. The principle on which the snubbers are designed is that if the motion of a point in a system is reduced the overall response is reduced. Since the clearance parameter permits added motion, the overall reduction of system response is diminished with clearance. Situations can exist where response is either increased or decreased by the introduction of clearance.

Consider the lumped mass system shown below:

Figure 7.3.2 Clearance Effects on Snubber Reaction

Figure 7.3.3 Clearance Effects on Displacement Response

In this illustration, it is possible to create a situation where response X_2 increases as δ_c increases, and another situation where X_2 decreases as δ_c increases. This is accomplished by changing the forcing frequency f_Ω . Figure 7.3.4 shows the results.

Clearance affects response by attenuating the snubber parameter. Situations can occur, however, where the response is affected as a result of changes in the resonant frequency of the system. This is shown in the previous example.

The following study considers the effect of clearance and viscous parameters on the response of a simple system excited by El Centro seismic event. The investigation considers two values of the viscous parameter C (or C/M).

Consider the system shown:

The clearance is in series with the viscous parameters. The response is calculated as a function of lock velocity. This model was selected because the response is more sensitive to this parameter than other viscous parameters. Figures 7.3.5 and 7.3.6 show the results of the study.

Based on these observations, the component response is restricted to the clearance band for lock velocities less than 1 in/sec. The response increases as the lock velocity increases until the undamped response is reached. As the clearance increases, a decreased sensitivity of response to the viscous parameters is experienced. It appears from these observations that clearances reduce the response sensitivity to a given parameter, in this case the viscous parameter.

IF ($X_1 \equiv$ FIXED) $f_n = 1.61$ Hz
 IF ($X_1 \equiv$ FREE) $f_n = 1.00$ Hz

Figure 7.3.4 Clearance Effects on System Response

Figure 7.3.5 Effects of Clearance on System Response, $C/M = 10^6 \text{ sec}^{-1}$

Figure 7.3.6 Effects of Clearance on System Response, $C/M = 10^3 \text{ sec}^{-1}$

7.4 Breakaway Characteristics

Breakaway loads are those loads that restrain movement until the reaction load reaches the "breakaway" or release value, at which time, motion can take place. The motion is opposed by a force equal to the breakaway value. Movement ceases when the reaction load becomes less than the breakaway value.

There are two frictional characteristics to consider. First, there is the initial breakaway load or static friction, and second, the dynamic friction which is the resisting load during movement. The static friction is, in general, greater than the dynamic friction. The response of the system is greatly dependent on the dynamic breakaway loads (friction) and less dependent on static friction. The maximum loading for seismic disturbances usually occurs several seconds after the earthquake begins. It is probable that the static restraint force will be exceeded during the early stages of the seismic disturbance and the dynamic frictional characteristic will apply when the greatest loading occurs.

Den Hartog, Reference 3, studied the breakaway frictional characteristics for harmonic steady-state loadings for simple lumped mass oscillators. His work considered one variance in the frictional load which in some cases caused the motion to stop in each cycle. As a result of this work, response curves were generated showing the response sensitivity with frictional loads. Figure 7.4.1 shows the response characteristics for a simple lumped mass system in terms of dimensionless parameters. This response curve indicates that the resistive force F_R has little effect on steady-state response when its value is less than 40 percent of the applied load, P .

A detailed look at the transient phase of the harmonic loading (Figure 7.4.2) indicates that the maximum total response probably occurs during the first cycle of loading and may approach twice the steady-state value. The cyclic range may be greater during the steady-state phase of the response.

Figure 7.4.1 Friction Effects on Steady State Harmonic Response

Figure 7.4.2 Transient Response Considering Friction Loads

The response sensitivity considering breakaway frictional characteristics and seismic base acceleration is shown in Figures 7.4.3 and 7.4.4. The response is shown for two different dynamic systems, one having a natural frequency of 6 Hz and the other of 12 Hz. The results show similar trends or characteristics as those established for the harmonic loadings.

7.5 Stiffness Parameter

The load-deflection characteristic of a snubbing device is referred to as the snubber stiffness. The stiffness parameter may be built into the snubber by virtue of subcomponent flexibilities or it may be associated with mounting hardware or structural support flexibilities.

In the above figure the overall stiffness is expressed as:

$$k_{eq} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}}$$

The individual stiffness or flexibilities could represent, for example, k_1 (support flexibility), k_2 (free air compressibility), k_3 (attaching hardware flexibility), and k_4 (piston flexibility). If k_1 is small, the overall stiffness of the snubber will be small, and the remaining large k 's will have little effect on the overall stiffness of the snubber.

Figure 7.4.3 Response Sensitivity Due to Friction for Seismic Loadings (12 Hz System)

Figure 7.4.4 Response Sensitivity Due to Friction for Seismic Loadings (6 Hz System)

The steady state load deflection relationship is shown above for a hydraulic snubber subjected to a harmonic excitation (a fixed frequency). One way of determining the effective stiffness is to draw a straight line between the positive and negative peak responses as shown.

This represents the effective or average stiffness. The instantaneous stiffness is the instantaneous slope of the response curve, i.e., $k = k(t)$. The effective stiffness range for a 2½-inch bore hydraulic snubber is approximately 100,000 to 300,000 lb/in.

The stiffness characteristic of snubbers is the one most important parameter affecting dynamic response of piping systems. This applies to the "effective" stiffness of the snubbing devices, Figure 7.5.1, or to "normal" linear stiffness characteristics, i.e., simple linear springs. The "effective" stiffness of a snubber is a function of the following parameters: fluid bulk modulus, clearance, lock and bleed velocities, acceleration and velocity threshold parameters, and snubber support structure characteristics.

Figure 7.5.1 Response Envelope For 1-D.O.F. Oscillator

Although hydraulic and mechanical snubbers dissipate energy, this is not the primary mechanism for limiting the overall response of a system. The snubbing device actually works by limiting the response of the system at the point of attachment or in a less direct way by modifying the natural frequency characteristics of the system. Since the frequency or response characteristics of a system are sensitive to support stiffness, the "effective" stiffness of snubbers become important.

Consider the simple single mass, undamped, oscillator system shown in Figure 7.5.1. The response envelope or spectra for this system is altered by the effective snubber stiffness where the frequency response characteristics increase as a result of the additional stiffness introduced by the snubber. This can be seen in Figure 7.5.1 from the curve labeled $(\Omega/\omega) = 2$. In this situation, the snubber stiffness tends to drive the resonant or natural frequency of the structure towards the forcing frequency, increasing system response. The same situation would exist if a base excitation was employed rather than an applied oscillating force.

Ideally, to dampen the oscillations of the single degree of freedom system, one would choose an effective snubber stiffness of $K_e \gg k$. The resonant frequency of the system would be much greater than the forcing frequency and the response would deteriorate to that of a static loading.

The "effective" stiffness properties are, in general, nonlinear for snubbers. This nonlinearity results from asymmetric response characteristics, e.g., in hydraulic snubbers the actuator rod is on one side of the piston. Nonlinear load deflection characteristics can have one of the following type forms.

The load-deflection characteristic of TYPE ③ involves the clearance parameter, and is addressed in the section on Clearances. Nonlinear stiffness characteristics similar to those indicated by TYPES ① and ② are the starting points for determining the sensitivity of response to the parameter γ , which will be referred to as the bilinear stiffness ratio.

The figure above for TYPE ① shows the load deflection characteristics for the support system. Inasmuch as K and γK can be interchanged without changing the conclusions we will investigate the range where $0 \leq \gamma \leq 1$.

Figure 7.5.2 shows how the natural frequency of a single lumped mass varies as a function of the bilinear ratio γ . Considering the typical linear system, where $\gamma = 1$, the results indicate that the rate of change of the natural frequency as a function of γ is greatest when $\gamma = 1$, that is, $(\partial f_n / \partial \gamma)_{\max}$ for a linear spring. Consequently, a change in natural frequency can be expected as bilinear effects are introduced.

For steady-state harmonic loading, Figure 7.5.3 shows how the system steady-state response varies with the bilinear ratio parameter γ . The response is similar to a linear system and the maximum response occurs when the forcing frequency is equal to, or approaches, the natural frequency of the system.

The primary effects are, (1) sensitivity of the natural frequency to the bilinear stiffness ratio γ , and (2) the nearness of the forcing frequency to the "modified" natural frequency.

The response peak-to-peak displacement is twice the maximum response X^* when $\gamma = 1$, and less when $\gamma \neq 1$. This should be considered when evaluating the total response range.

Figure 7.5.2 Resonant Frequency Versus Bilinear Stiffness Ratio

Figure 7.5.3 1-D.O.F. Response Characteristics (Bilinear Spring)

To determine the steady state forced response of the dynamic system shown above, consider the equation

$$M\ddot{X} + K^*X = F \sin \Omega t \quad \dots (7.5.1)$$

K^* is an equivalent linear spring. It replaces the bilinear spring while approximately producing the same response - the so called average energy approach.

For the single degree of freedom oscillatory system shown above, the natural frequency can be determined from

$$\omega_n^2 = \frac{K^*}{m} \quad \dots (7.5.2)$$

However, the natural period of vibration is

$$f_n = \frac{1}{\pi} \sqrt{\frac{K}{m}} \left/ \left(1 + \sqrt{\frac{T}{\gamma}} \right) \right.$$

or,

$$\omega_n = \frac{2\pi}{\pi} \sqrt{\frac{K}{m}} \left/ \left(1 + \sqrt{\frac{T}{\gamma}} \right) \right. \quad \dots (7.5.3)$$

Squaring (7.5.3) gives

$$\omega_n^2 = 4 (K/m) / \left(1 + \sqrt{\frac{1}{Y}} \right)^2 \quad \dots (7.5.4)$$

Equating the frequency equations (7.5.2) and 7.5.4) yields

$$\frac{K^*}{m} = 4 \left(\frac{K}{m} \right) / \left(1 + \sqrt{\frac{1}{Y}} \right)^2 \quad \dots (7.5.5)$$

and solving for K^* ,

$$K^* = \frac{4K}{\left(1 + \sqrt{\frac{1}{Y}} \right)^2} \quad \dots (7.5.6)$$

Substituting equation (7.5.6) into equation (7.5.1) gives

$$MX + \left[\frac{2}{\left(1 + \sqrt{\frac{1}{Y}} \right)} \right]^2 KX = F \sin \Omega t \quad \dots (7.5.7)$$

Solving for the steady-state response

$$X = \frac{F}{K \left[\frac{2}{\left(1 + \sqrt{\frac{1}{Y}} \right)} \right]^2 - M\Omega^2} \quad \dots (7.5.8)$$

Or in terms of the natural frequency,

$$X = \frac{(F/K)}{\left[\frac{2}{\left(1 + \sqrt{\frac{1}{Y}} \right)} \right]^2 - \left(\frac{\Omega}{\omega_{no}} \right)^2} \quad \dots (7.5.9)$$

where

$$\omega_{no}^2 = \left(\frac{K}{m} \right)$$

Equations (7.5.8) and (7.5.9) represent the steady-state response in the +X direction. To find the response (steady-state) in the -X direction, equate the potential energies for $X < 0$ with that when $X > 0$,

$$\frac{1}{2} kX^2 = \frac{1}{2} \gamma K(X^*)^2 \quad \dots (7.5.10)$$

X^* represents the motion in the -X direction. Therefore from equation (7.5.10),

$$\left(\frac{X^*}{X}\right)^2 = \frac{1}{\gamma}$$

and solving for X^* ,

$$X^* = \sqrt{\frac{1}{\gamma}} X \quad \dots (7.5.11)$$

If $0 < \gamma < 1$, the maximum response is

$$X^* = \sqrt{\frac{1}{\gamma}} X \quad \dots (7.5.12)$$

Considering equation (7.5.9) and

$$\frac{X^*}{F/k} = \frac{X^*}{X_{\text{static}}}$$

for the static case

$$\frac{X^*}{X_{\text{static}}} = \frac{\sqrt{\frac{1}{\gamma}}}{\left[\frac{2}{\left(1 + \sqrt{\frac{1}{\gamma}}\right)} \right]^2 - \left(\frac{\Omega}{\omega_{no}} \right)^2} \quad \dots (7.5.13)$$

The steady-state response can graphically be represented by:

Equation (7.5.13) does not consider damping. This can be incorporated as follows

$$\frac{X^*}{X_{st}} = \sqrt{\frac{1}{\gamma}} \frac{1}{\sqrt{\left\{ \left[\frac{2}{1 + \sqrt{\frac{1}{\gamma}}} \right]^2 - \left(\frac{\Omega}{\omega_{no}} \right)^2 \right\}^2 + \left[2\zeta \left(\frac{\Omega}{\omega_{no}} \right) \right]^2}} \dots (7.5.14)$$

Equations (7.5.9), (7.5.13), and (7.5.14) represent envelope values for the response. These results are for an average energy approach. In reality there is beating phenomenon that occurs as shown below. This results from $K \neq \gamma K$.

This beating period is a function of the forcing frequency and the natural frequency.

$$\tau_B = \frac{1}{f_\Omega - f_n}$$

$$\tau_B = \frac{1}{f_n - \frac{\sqrt{K/M}}{\pi \left(1 + \sqrt{\frac{1}{\gamma}} \right)}} \quad \dots (7.5.15)$$

The figure above shows the steady-state response of the system. The total displacement range is

$$X_R = (X + X^*) = \frac{\sqrt{\frac{1}{\gamma}} (F/K) (1 + \sqrt{\gamma})}{\left[\frac{2}{1 + \sqrt{\frac{1}{\gamma}}} \right]^2 - \left(\frac{\Omega}{\omega_{no}} \right)^2} \quad \dots (7.5.16)$$

The results indicate a lumped mass, supported by a bilinear spring system, has response characteristics similar to that of a linear system. The bilinearity stiffness ratio effects the system response by changing the natural frequency. The response characteristics of a system with a specific γ ratio are similar to a linear system with forcing frequencies. The principle difference between a linear and bilinear supported system is the asymmetric response that occurs about the neutral position.

The results obtained from the simple, single degree of freedom analysis indicate the same trends should exist for more complicated lumped mass systems or continuous beam models. Since the bilinear system response can be evaluated in terms of an effective spring rate K^* , the response trends can be expected to follow those of linear supported systems. A more detailed discussion is presented later in this section.

The stiffness or load-deflection characteristics described in TYPE (2)a is an effect that is commonly referred to as a "hardening" spring, that is, as the deflection increases so does the stiffness. The "hardening" spring characteristic is probably the most common naturally occurring effect, and is most often found in hydraulic and soil systems. There is another stiffness property referred to as the softening spring. For this, the stiffness decreases with increasing deflection. This effect is described as TYPE (2)b. This situation is probably less common in nature than the hardening effect. The most common examples can be attributed to nonlinear geometric effects and inelastic behavior. As far as snubber systems are concerned, the hardening characteristic is the most relevant.

The response of a simple single degree of freedom oscillation having "hardening" or "softening" stiffness characteristics has been studied in great detail by Duffing, Reference 4.

Duffing's equation for a system with a nonlinear restoring force is,

$$m\ddot{X} + k(X \pm \kappa X^3) = F \sin \Omega t \quad \dots (7.5.17)$$

Where the reaction load is,

$$R = k(X \pm \kappa X^3) \quad \dots (7.5.18)$$

When the upper sign of κ is used, the system is said to have a "hardening" spring, while the lower sign means the system is restored by a "softening" spring.

HARDENING (TYPE (2)a)

SOFTENING (TYPE (2)b)

The steady state response frequency equation for the single degree of freedom system is

$$1 \pm \frac{3\kappa^2 X^2}{4} - \frac{\Omega^2}{\omega_n^2} = (\pm) \frac{F}{kX} \quad \dots (7.5.19)$$

The parenthetical plus-or-minus ($\pm$) sign refers to X being in phase (+) or π radians out of phase (-) with the exciting force F .

By letting the excitation displacement approach 0 in equation (7.5.19), the relation between the natural frequency and the amplitude of the non-linear system can be found.

$$\omega_n = \sqrt{\frac{k}{m}} \sqrt{1 \pm \frac{3\kappa^2 X^2}{4}} \quad \dots (7.5.20)$$

Referring to equation (7.5.19) the response frequency relationship for a hardening spring is

$$\left(1 - \frac{\Omega^2}{\omega^2}\right) + \frac{3\kappa^2 X^2}{4} - \frac{F}{kX} = 0$$

or,

$$\left(\frac{3\kappa^2}{4}\right)X^3 + \left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]X - \frac{F}{k} = 0 \quad \dots (7.5.21)$$

and for a softening spring is

$$-\left(\frac{3\kappa^2}{4}\right)X^3 + \left[1 - \left(\frac{\Omega}{\omega}\right)^2\right]X - \left(\frac{F}{k}\right) = 0 \quad \dots (7.5.22)$$

Phenomena that occur as a result of the "hardening" or "softening" support characteristics are the jump phenomena. Jump phenomena for "softening" springs are illustrated as follows

With increasing frequency of excitation, the amplitude gradually increases from "a" until "b" is reached. It then jumps to a larger value at "c" and diminishes as the forcing frequency is increased. In decreasing the frequency from some point "d", the response increases to some point beyond "c", to point "e" where the response will drop to point "f", and continue to decrease from that point. Stability analysis shows the middle branch is unstable (shaded area).

For a "hardening" spring, the jump phenomena are illustrated as follows

With increasing excitation frequency, the amplitude increases beyond "f" to some point "e" where the response drops to "c" and continues to decrease. If the excitation frequency decreases from point "d", the response will increase to point "b" where it will jump to point "f" and then continue to decrease. Just as with the "softening" spring, the middle branch is unstable.

When the drop-jump phenomenon occurs, it is usually preceded by an accidental unsteadiness or extraneous disturbance. When the instability occurs there is a phase change between the response and the forcing function which gives rise to a transient superimposed motion at the time of the drop-pump.

The effects of the "hardening" (softening) parameter κ^2 on dynamic response can be evaluated using the frequency response equation

$$\pm \left(\frac{3\kappa^2}{4} \right) X^3 + \left[1 - \left(\frac{\Omega}{\omega} \right)^2 \right] X - \left(\frac{F}{K} \right) = 0$$

Figures 7.5.4 and 7.5.5 show the variation of response amplitude X as a function of κ^2 , parametric with forcing frequency and magnitude, (F/k) and (Ω/ω) , constant. These plots apply to a single degree of freedom system.

Figure 7.5.4 Hardening Parameter Effects on System Response

Figure 7.5.5 Softening Parameter Effects on System Response

8. PARAMETER RANGES

Once the parameters affecting system response are determined, the next step is to establish parameter ranges that will assure system responses are bounded within acceptable limits.

Parameter ranges may be controlled directly by the snubber manufacturer, or by installation practices and procedures. Examples of parameters controlled directly by the customer or manufacturer are the acceleration threshold, lock velocity, and snubber stiffness characteristics. Parameters which can best be related to installation practices would include, end fitting clearances, back-up support stiffnesses, positioning (orientation), and environmental considerations such as radiation and temperature. The clearance parameter is unique in that it can be influenced by both the manufacturer and by installation procedures. Other parameters may be influenced more by probabilistic occurrences, rather than by manufacturer or installers (designers) control. This would include air entrainment or entrapment, and increased friction caused by corrosion, and contamination. It is the intent of this work to consider ranges of parameters that exceed the limits of current design practice.

Probable parameter ranges were established by using vendor literature and information obtained by NRC. These ranges were extended to consider values that would be pertinent to this study. The minimum range of parameters addressed by this study are presented in Table 8.1.

The establishment of acceptable parameter ranges is not an easy task in light of all the variables involved. Consideration must be given to back-up structure effects, load magnitudes, displacement-stress-load responses, parameter combinations, and response characteristics of the supported component or system. The purpose of this section is to discuss which results were used in establishing acceptable parameter ranges. The conclusions are based solely on the data generated in this study.

TABLE 8.1
 SNUBBER PARAMETER RANGES

<u>Parameter</u>	<u>Range to be Investigated</u>
Dead Band	.0 to .125 in
End Fitting Tolerancd	.0 to .125 in
Breakaway Loads	1 to 3%
Frequency Range	3 to 33 Hz
Acceleration Threshold	0 to .2 g
Lock Velocity	0 to 1 in/sec
Bleed Rate (@ Rated Load)	.001 to 1 in/sec
Snubber Support Stiffness	10 ² to 10 ⁶ lb/in

The back-up structural support dynamic and stiffness characteristics influence snubber performance used as a basis for establishing parameter ranges. The following describes the basis for establishing acceptable parameter ranges.

8.1 Viscous Parameters

The viscous parameters that effect response are lock velocity and bleed rate. The results of the study indicate that the forcing frequency does not have a significant effect on response, Figure 7.2.2, nor does the applied load, Figure 7.2.3, for realistic values of bleed rate. Response curves such as those indicated in Figures 7.2.5 and 7.2.6 are representative of system response. Component responses are maintained within acceptable limits provided the lock velocity is less than 1 in/sec. Figure 7.2.6 does not substantiate this value based on percent reduction of response, however, it does support this conclusion based on actual maximum response. Therefore, the established limit appears reasonable. The bleed rate is related to the viscous constant C as follows.

$$C = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

In general, C values are large, e.g., a 2,000 lb snubber with a bleed rate of 6 in/min has a viscous constant value of $2 (10^4)(\text{lb-sec/in})$. Sample studies done to date indicate that if reasonable lock velocities are maintained ($< 1 \text{ in/sec}$) that a bleed rate less than the lock velocity will assure a bounded response. Without putting excessive constraints on the viscous parameter, the following limits will be established,

$$(C/m) > 10^3 \text{ sec}^{-1}$$

or

$$C > 10^4 \frac{\text{lb-sec}}{\text{in}}$$

8.2 Friction Loads

The effects of friction on system response are negligible, i.e., the friction loads must be at least 40 percent of the applied load before dynamic response is significantly affected. Based on the results developed for harmonic, Figure 7.4.1, and seismic, Figure 7.4.3, the frictional characteristics have insignificant effects on dynamic response when less than 40 percent of the rated load of the snubber.

8.3 Acceleration Threshold Parameter

The establishment of an acceptable A.T.P. range is a complex issue. There apparently is considerable interaction between system response characteristics and the A.T.P. as indicated in Figure 7.1.4. The possibility of increased rather than reduced response, Figures 7.1.4, 7.1.6, and 7.1.7, complicates the issue. The results shown graphically in Figure 7.1.6 indicate that the response is nonlinear with load magnitude ($\ddot{X}_B$). Based on the results presented in Figure 7.1.6, an A.T.P. ($\ddot{X}_L$) equal to .001 g would assure a 90-95 percent reduction in free response. This appears restrictive, however, if wise design practices are used in realistic limit of .04 g should bound the response within acceptable limits. This limit may be modified when further studies are completed.

8.4 Support Stiffness Requirements

Current design practice does not consider support stiffness requirements. This study indicates that with relatively high "effective" stiffness of snubbers, the back-up structural support stiffness properties may effect snubber response characteristics. Back-up structural stiffness requirements have been established in this study so that the snubber can be "effective." For example, it would be unreasonable to attach a snubber having an effective stiffness of $0.3(10^6)$ lb/in to a supporting structure having an effective stiffness of 5,000 lb/in for a massive component. If the back-up support stiffness is very low there could be a dynamic interaction that would affect response. Considering the combined "effective" stiffness of both the back-up structure and the snubber, the "effective" stiffness should be at least twenty times greater than the stiffness ($1/\text{flexibility}$) of the pipe at the snubber location. The factor of twenty can be verified from Figures 6.1.1, 6.1.2, and 6.1.3. In terms of the viscous constant, where C is the ratio of the rated load to bleed rate, the back-up structure stiffness should be $1.2 (C\Omega)$ where Ω is the predominant forcing frequency in rad/sec. Figure 6.1.4 shows the response efficiency of the support. The efficiency represents the actual load transferred into the base compared to the maximum load that can be transferred.

8.5 Clearance

Clearance is always associated with another snubber parameter. Limited observations indicate that clearance attenuates the ability of the snubber parameter to reduce the displacement response amplitude, Figures 7.3.3, 7.3.5, and 7.3.6. Sufficient information has not been generated to date to establish absolute clearance limits for all the snubber parameters. However, based on observations, clearances are estimated to be 15 percent of the unrestrained response for rigid supports and 10 percent of the unrestrained response for snubbers.

9. ANALYTICAL METHODS

The method used for solving the transient response problem must be capable of considering parameters such as nonlinear restorative forces $F_R = F(X)$, viscous forces $F_V = F(\dot{X})$, clearances, and motion constraints. Analytical techniques must have the capabilities of considering these parameters singly or in any combination.

Although the list of snubber parameters is considerably longer than those parameters listed above, the snubber parameters do fall into one of the above categories. For example, the nonlinear stiffness characteristic associated with entrained air in hydraulic fluids is categorized as a nonlinear restorative force while the velocity dependent load associated with this fluid is categorized as a viscous force.

The analytical method that is chosen must be general in nature, so that large component or piping system with complex geometries may be analyzed. It must be flexible enough to allow for modification so additional parameters may be added. It must be efficient so that computer costs are to be kept at a minimum. The method that can best be used to satisfy the previously mentioned requirements is a modified modal analysis procedure utilizing a load correcting algorithm.

Prior to describing the analysis method that will be used to solve the general problem, some of the analysis procedures that were abandoned in favor of the modified modal analysis procedure will be discussed.

Equations of motion for nonlinear systems can usually be solved approximately by using step-by-step integration procedures. Many well known numerical methods involve extrapolation or interpolation formulas for the solution which are applied in a series of small but finite time intervals. Some of the more popular methods are based on assumed acceleration functions between integration time steps. Of particular importance are the constant, average, and linear acceleration methods - the later two involving interaction procedures. The

acceleration analysis method works quite well for simple 1 or 2 degree of freedom systems but encounters stability problems when the complexity of the structural model increases or the natural frequencies become high. Many elaborate analysis procedures have been developed to overcome most of these problems; however, their complexity and long running computer times do not make these methods suitable for the solution of the snubber problem. Another major drawback of the acceleration analysis method is the selection of an integration time interval that will produce a stable or at least conditionally stable solution. In many cases very small integration time steps are required for a stable solution, and even smaller time steps may be required for an accurate solution. Figure 9.1 shows how the accuracy of a simple single degree of freedom solution varies as a function of the integration time interval for the constant acceleration method.

The modal analysis procedure has an advantage over general step-by-step integration procedures in that many of the stability problems can be eliminated quite easily for large systems of equations. Although the modal analysis procedure requires the evaluation of system natural frequencies and modeshapes, standard analysis methods can easily be adopted for this purpose. When analyzing complex systems it is often necessary to calculate only a few modes to get reasonably accurate results; consequently, the solution time is reduced considerably.

Due to the nature of the nonlinear parameters used to describe the snubbers, standard modal analysis procedures cannot be used to solve the problems. However, modifications can be made to this standard procedure to permit the adoption of this procedure.

The modal analysis technique has become a popular analysis tool since the advent of finite element analysis. The snubber analyses procedure developed here will be incorporated in an existing finite element statics and dynamics piping program SUPERPIPE. The development of the procedure begins with the differential equations of motion in terms of the mass and stiffness matrices as shown in Equation (9.1).

Figure 9.1 Response Versus Time Increment Sensitivity

$$\begin{bmatrix} M & 0 \\ 0 & M^* \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{X}_S \end{Bmatrix} + \begin{bmatrix} K_{11} & K_{1S} \\ K_{S1} & K_{SS} \end{bmatrix} \begin{Bmatrix} X \\ X_S \end{Bmatrix} = \begin{Bmatrix} F(t) \\ R^* \end{Bmatrix} \quad \dots (9.1)$$

$X_S, \ddot{X}_S$ = Known displacement and acceleration (excluding snubber locations)

$X, \ddot{X}$ = Unknown displacement and acceleration

$\{F(t)\}$ = Specified forces

$\{R^*\}$ = Unknown reaction forces

Separating (9.1) into parts we obtain two matrix equations,

$$[M]\{\ddot{X}\} + [K_{11}]\{X\} = \{F(t)\} - [K_{1S}]\{X_S\} \quad \dots (9.2a)$$

and

$$-\{R^*\} + [K_{S1}]\{X\} = -[M^*]\{\ddot{X}_S\} - [K_{SS}]\{X_S\} \quad \dots (9.2b)$$

where the known quantities are on the right side of the equations.

By solving (9.2a) for $\{X\}$ all the displacements can be determined.

Examination of (9.2a) indicates that the homogenous portions involve only those degrees of freedom that are unknown. Therefore, if modal analysis procedures are to be used the frequencies and mode shapes are determined for the system with all known movements removed or fixed.

$$[M]\{\ddot{X}\} + [K]\{X\} = \{0\} \quad \dots (9.3)$$

The modeshapes and frequencies are determined for the homogenous differential equation shown in (9.3).

To solve (9.2a) assuming the modal coordinate transformation,

$$\{X\} = [A]\{Z\}, \text{ and } \{\ddot{X}\} = [A]\{\ddot{Z}\} \quad \dots (9.4)$$

Where $[A]$ = Matrix of eigenvectors normalized such that $[A]^T[M][A] = [I]$,

$$[M][A]\{\ddot{Z}\} + [K][A]\{Z\} = (F(t) - [K_{1S}]\{X_S\}) \quad \dots (9.5)$$

By multiplying both sides of Equation (9.5) by $[A]^T$ we obtain

$$[A]^T[M][A]\{\ddot{Z}\} + [A]^T[K][A]\{Z\} = [A]^T(f(t) - [K_1S]\{X_S\}) \dots (9.6)$$

From the principle of orthogonality,

$$\begin{aligned} [A]^T[K][A] &= [\omega_n^2] \\ [A]^T[M][A] &= [I] \end{aligned}$$

Therefore, Equation (9.6) becomes

$$[I]\{\ddot{Z}\} + [\omega_n^2]\{Z\} = \{P(t)\} \dots (9.7)$$

This is a series of uncoupled equations, each representing the response of a single degree of freedom oscillator,

$$\ddot{Z}_i + \omega_{ni}^2 Z_i = P_i(t) \dots (9.8)$$

Knowing Z_i , $\dot{Z}_i$, and $\ddot{Z}_i$ we can proceed to solve for X_i , $\dot{X}_i$, and $\ddot{X}_i$, where

$$\{X\} = [A]\{Z\}, \quad \{\dot{X}\} = [A]\{\dot{Z}\}, \quad \{\ddot{X}\} = [A]\{\ddot{Z}\} \dots (9.9)$$

The modal analysis procedure is limited to those problems where boundary restraints do not vary during the course of the analysis.

For the nonlinear snubber problem, boundary restraints do vary with time. A modification to this procedure is employed where $[A]$ is also a function of time.

A procedure which permits the use of modal analysis techniques has been adopted, the procedure utilizes a technique whereby reaction loads are determined so that specified displacements or accelerations can be made without modifying the modeshape matrix $[A]$ or system frequencies.

The procedure involves the solution for a given time increment, for example, $X(t)$, $\dot{X}(t)$ are known and it is desired to calculate $X(t + \Delta t)$ and $\dot{X}(t + \Delta t)$.

The solution is obtained using the normal modal analysis techniques described. The results at $(t + \Delta t)$ are compared with whatever boundary restraints are specified. If, for example, an acceleration limit is set at $\ddot{X}_L$, all snubber locations are checked to see if

$$|\ddot{X}|_i \geq \ddot{X}_L \quad \dots(9.10)$$

If this situation exists one realizes that X_i at $(t + \Delta t)$ has to be

$$X(t + \Delta t)_i = X(t) + \underbrace{\dot{X}(t)\Delta t + \frac{1}{2} \ddot{X}_L \Delta t^2}_{\Delta X_i} \quad \dots(9.11)$$

Basically X_i is unknown if $|\dot{X}|_i < \ddot{X}_L$, and X_i is known if $|\dot{X}|_i \geq \ddot{X}_L$.

Having determined X_i for all "m" snubber locations where $|\dot{X}|_i \geq \ddot{X}_L$ one obtains the snubber specified displacement vector,

$$\{\Delta X\} \quad \dots(9.12)$$

Returning to Equation (9.3), the homogeneous part of the differential equation is formed which includes active degrees of freedom at the snubbers,

$$[M]\{X\} + [K]\{X\} = \{R\} \quad \dots(9.13)$$

where

$R_i = 0$ - At all non-snubber locations,

$R_i = 0$ - At all snubber locations where $|\dot{X}|_L < \ddot{X}_L$

and $R_i \neq 0$ - At m locations where $|\dot{X}|_i \geq \ddot{X}_L$

Knowing $\{\ddot{X}(t)\}$, $\{\dot{X}(t)\}$, and $\{X(t)\}$, we solve equation (9.13) by modal analysis techniques for changes in X_i at the m snubber locations, by incrementing on a one by one basis the non-zero R loads. Thus we can find the "dynamic" influence coefficients at the m active snubber supports.

$$\frac{\partial X_1}{\partial R_1} = \frac{\Delta X_1}{\Delta R_1} \quad \dots (9.14)$$

$$\frac{\partial X_2}{\partial R_1} = \frac{\Delta X_2}{\Delta R_1}$$

$$\vdots \quad \vdots$$

(etc.)

The total deflections can be written as

$$\Delta X_1 = \frac{\partial X_1}{\partial R_1} \Delta R_1 + \frac{\partial X_1}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_1}{\partial R_n} \Delta R_n$$

$$\Delta X_2 = \frac{\partial X_2}{\partial R_1} \Delta R_1 + \frac{\partial X_2}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_2}{\partial R_n} \Delta R_n$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\Delta X_n = \frac{\partial X_n}{\partial R_1} \Delta R_1 + \frac{\partial X_n}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_n}{\partial R_n} \Delta R_n$$

or in matrix form,

$$\{\Delta X\} = \left[\frac{\partial X}{\partial R} \right] \{\Delta R\} \quad \dots (9.15)$$

Since we know $\{\Delta X\}$ from equation (9.12)

$$\{\Delta R\} = \left[\frac{\partial X}{\partial R} \right]^{-1} \{\Delta X\} \quad \dots (9.16)$$

One can now proceed to determine the response at $(t + \Delta t)$ and satisfy the requirements of equation (9.10), or others that may be imposed on the system, by solving equation (9.2a) with the additional load term $\{\Delta R\}$.

Hence knowing $\{X(t)\}$, $\{\dot{X}(t)\}$, $\{F(t + \Delta t)\}$, and $\{\Delta R\}$,

$$[M]\{\ddot{X}\} + [K]\{X\} = (\{F(t)\} - [K_1 S]\{X_S\}) + \{\Delta R\} \quad \dots(9.17)$$

can be solved for $\{X(t + \Delta t)\}$, $\{\dot{X}(t + \Delta t)\}$, and $\{\ddot{X}(t + \Delta t)\}$. Here,

- $\Delta R_i = 0$ - All non-snubber locations
- $\Delta R_i = 0$ - All snubber locations $|\ddot{X}|_i < \ddot{X}_L$
- $\Delta R_i \neq 0$ - All snubber locations where $|\ddot{X}|_i > \ddot{X}_L$

The solution for the next time increment can be solved by the same method.

Returning to Equation (9.8), the solution to the uncoupled equations can be found exactly if $P(t)$ is known as a function of time. $P(t)$ is not necessarily limited to harmonic functions, but exact solutions can be obtained for other functions such as ramps, steps, impulses, etc. However, seismic or dynamic disturbances cannot be expressed mathematically as a function of time, but rather as a series of pairs of points defining the excitation vs time. An exact solution will not be obtained but instead an approximate solution will be obtained, one that will be a function of the integration time step that has been chosen.

The solution of Equation (9.8) for an arbitrary excitation can be solved by using techniques presented by Duhamel, commonly referred to as a Duhamel's Integral, the Convolution Integral, or Superposition integral. Briefly, the forcing function is broken up into a series of constant load steps as shown below.

Knowing $\{\ddot{X}(t)\}$, $\{\dot{X}(t)\}$, and $\{X(t)\}$, we solve equation (9.13) by modal analysis techniques for changes in X_i at the m snubber locations, by incrementing on a one by one basis the non-zero R loads. Thus we can find the "dynamic" influence coefficients at the m active snubber supports.

$$\frac{\partial X_1}{\partial R_1} = \frac{\Delta X_1}{\Delta R_1} \quad \dots (9.14)$$

$$\frac{\partial X_2}{\partial R_1} = \frac{\Delta X_2}{\Delta R_1}$$

$$\vdots \quad \vdots$$

(etc.)

The total deflections can be written as

$$\Delta X_1 = \frac{\partial X_1}{\partial R_1} \Delta R_1 + \frac{\partial X_1}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_1}{\partial R_n} \Delta R_n$$

$$\Delta X_2 = \frac{\partial X_2}{\partial R_1} \Delta R_1 + \frac{\partial X_2}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_2}{\partial R_n} \Delta R_n$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\Delta X_n = \frac{\partial X_n}{\partial R_1} \Delta R_1 + \frac{\partial X_n}{\partial R_2} \Delta R_2 + \dots + \frac{\partial X_n}{\partial R_n} \Delta R_n$$

or in matrix form,

$$\{\Delta X\} = \left[\frac{\partial X}{\partial R} \right] \{\Delta R\} \quad \dots (9.15)$$

Since we know $\{\Delta X\}$ from equation (9.12)

$$\{\Delta R\} = \left[\frac{\partial X}{\partial R} \right]^{-1} \{\Delta X\} \quad \dots (9.16)$$

One can now proceed to determine the response at $(t + \Delta t)$ and satisfy the requirements of equation (9.10), or others that may be imposed on the system, by solving equation (9.2a) with the additional load term $\{\Delta R\}$.

Hence knowing $\{X(t)\}$, $\{\dot{X}(t)\}$, $\{F(t + \Delta t)\}$, and $\{\Delta R\}$,

$$[M]\{\ddot{X}\} + [K]\{X\} = (\{F(t)\} - [K_1 S]\{X_S\}) + \{\Delta R\} \quad \dots(9.17)$$

can be solved for $\{X(t + \Delta t)\}$, $\{\dot{X}(t + \Delta t)\}$, and $\{\ddot{X}(t + \Delta t)\}$. Here,

- $\Delta R_i = 0$ - All non-snubber locations
- $\Delta R_i = 0$ - All snubber locations $|\ddot{X}|_i < \ddot{X}_L$
- $\Delta R_i \neq 0$ - All snubber locations where $|\ddot{X}|_i > \ddot{X}_L$

The solution for the next time increment can be solved by the same method.

Returning to Equation (9.8), the solution to the uncoupled equations can be found exactly if $P(t)$ is known as a function of time. $P(t)$ is not necessarily limited to harmonic functions, but exact solutions can be obtained for other functions such as ramps, steps, impulses, etc. However, seismic or dynamic disturbances cannot be expressed mathematically as a function of time, but rather as a series of pairs of points defining the excitation vs time. An exact solution will not be obtained but instead an approximate solution will be obtained, one that will be a function of the integration time step that has been chosen.

The solution of Equation (9.8) for an arbitrary excitation can be solved by using techniques presented by Duhamel, commonly referred to as a Duhamel's Integral, the Convolution Integral, or Superposition integral. Briefly, the forcing function is broken up into a series of constant load steps as shown below.

At time (t) a step excitation $F(t)$ acting from (t) to $(t + \Delta t)$ exists. The response of the single degree of freedom oscillator described by Equation (9.8) can be expressed as

$$X(t) = A_0 e^{-\zeta \omega_n t} \cos(\omega_d t - \gamma) - \frac{F^*}{\omega_n \omega_d} e^{-\zeta \omega_n t} \cos(\omega_d t - \psi) + \frac{F^*}{\omega_n^2} \quad (9.18)$$

where

$$A_0 = \sqrt{\dot{x}_0^2 + 2 \zeta \omega_n x_0 \dot{x}_0 + \omega_n^2 x_0^2} / \omega_d$$

$$\tan \gamma = \frac{x_0 + \zeta \omega_n x_0}{\omega_d x_0}$$

$$\tan \psi = \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

$$\omega_d = \frac{\omega_n}{\sqrt{1 - \zeta^2}}$$

$x_0, \dot{x}_0$ = Initial conditions, i.e., the displacement and velocity respectively at t

ζ = Critical damping ratio

F^* = Constant force during time $t \rightarrow (t + \Delta t)$

If the force F^* does not vary during Δt the solution can be calculated exactly. Therefore to solve the snubber problem, Δt is chosen so that $F(t)$ does not vary significantly during Δt . As a matter of fact, if $F(t)$ does not vary with time, Δt can be increased without significantly affecting the results. Consequently, the computation may be reduced drastically. Knowing the condition at t , equation (9.18) can be used for determining the response at $(t + \Delta t)$. The displacement, velocity, and acceleration can be expressed as follows:

$$x(t + \Delta t) = A_0 e^{-\zeta \omega_n \Delta t} \cos(\omega_d \Delta t - \gamma) - \frac{F^*}{\omega_n \omega_d} e^{-\zeta \omega_n \Delta t} \cos(\omega_d \Delta t - \psi) + \frac{F^*}{\omega_n^2} \dots (9.19a)$$

$$\dot{x}(t + \Delta t) = e^{-\zeta \omega_n \Delta t} \left\{ \frac{-\zeta}{\sqrt{1 - \zeta^2}} \omega_d A_0 \cos(\omega_d \Delta t - \gamma) - A_0 \omega_d \sin(\omega_d \Delta t - \gamma) \dots (9.19b) \right.$$

$$\left. + \frac{F^*}{\omega_n} \left(\frac{\zeta}{\sqrt{1 - \zeta^2}} \right) \cos(\omega_d \Delta t - \psi) + \frac{F^*}{\omega_n} \sin(\omega_d \Delta t - \psi) \right\}$$

$$\ddot{x}(t + \Delta t) = -A_0 \left(\frac{\zeta}{\sqrt{1 - \zeta^2}} \right) \omega_d e^{-\zeta \omega_n \Delta t} \left(-\omega_d \sin(\omega_d \Delta t - \gamma) - \zeta \omega_n \cos(\omega_d \Delta t - \gamma) \right) \dots (9.19c)$$

$$-A_0 \omega_d e^{-\zeta \omega_n \Delta t} \left(\omega_d \cos(\omega_d \Delta t - \gamma) - \zeta \omega_n \sin(\omega_d \Delta t - \gamma) \right)$$

$$+ \frac{F^*}{\omega_n} \left(\frac{\zeta}{\sqrt{1 - \zeta^2}} \right) e^{-\zeta \omega_n \Delta t} \left(-\omega_d \sin(\omega_d \Delta t - \psi) - \zeta \omega_n \cos(\omega_d \Delta t - \psi) \right)$$

$$+ \frac{F^*}{\omega_n} e^{-\zeta \omega_n \Delta t} \left(\omega_d \cos(\omega_d \Delta t - \psi) - \zeta \omega_n \sin(\omega_d \Delta t - \psi) \right)$$

The analysis approach has been discussed in general terms in the preceding section. The following section will be concerned with the computational procedures. Analysis procedures for handling nonlinear restorative forces, viscous forces, clearances, and motion constraints will be discussed. The various procedures will be combined to form the total snubber analytical model (computer code).

This particular form of loading can be separated into two categories:

- (1) nodal movements that are constrained throughout the loading cycle; and
- (2) nodal movements that are constrained during portions of the loading cycle.

The first class or category of constraints are those that can be handled by standard modal analysis techniques. These movements are in general the specified loading such as pipe anchor movements or support motions. The solution for this type of input is presented in the previous section and since this technique is not unique, a detailed description of the procedure is omitted. However, the second category is unique to the snubber analysis and will be presented in detail.

The detail of the analysis procedure is in the form of a flow chart. The snubber parameter that the analysis procedure applies to is the acceleration threshold parameter. Basically, the nodal accelerations are limited to a specified limit $\ddot{X}_L$. The flow chart in Figure 9.2 indicates the general approach for solving the problem when considering acceleration limits.

Viscous loads are those loads or reactions that are a function of velocity. The force may be proportional to the velocity as it is in classical viscous damping or it may be any generalized function $F_V = F_V(\dot{X})$. The calculation and implementation of viscous snubber reactions is quite simple inasmuch as the boundary conditions for the piping or component system do not vary with time. Viscous loadings are created by hydraulic snubbers, and it should be noted that there is no connection between these viscous effects and the modal damping referred to in Equation (9.8). This damping (percent critical damping) refers to the piping or component system damping and not the discrete damping of the snubbers. The flow chart in Figure 9.3 indicates the general approach for solving the problem of viscous damping forces.

Figure 9.2 Response Due to Acceleration Threshold Parameter

FIGURE 9.2 (Cont.)

- STEP ①: All anchor motions or specified displacements, loads and system parameters are known at $t = 0$.
- STEP ②: The selection of a time increment is made based on either input data frequency characteristics, system natural frequency characteristics, or both. In general, the time increment is constant throughout the analysis, but this is not a restriction. The time increment (Δt) may be varied as the solution progresses to minimize computer time.
- STEP ③: Knowing the conditions at t and the loads at $(t + \Delta t)$ the response at $(t + \Delta t)$ can be calculated. [See Equations (9.10), (9.8), and 9.4).]
- STEP ④: After calculating the response at $(t + \Delta t)$ each snubber location motion is checked to see if the acceleration is greater than the acceleration threshold parameter (limit acceleration). If all accelerations are less than or equal to $\ddot{X}_L$, the solution calculated at $(t + \Delta t)$ is correct. If not, go to STEP ⑥.
- STEP ⑤: Increment the time and proceed to calculate the response for the next integration time interval.
- STEP ⑥: Since certain accelerations exceed the limit values, the predicted displacements from STEP ③ will not be correct. The response at these locations can be calculated since $\ddot{X}_L$ is known. Knowing what was calculated from STEP ③ and what should be, from STEP ⑥, the displacement increment is known:

$$\Delta X = X(t + \Delta t) - X(t)$$

Therefore at each snubber location where the acceleration exceeds $\ddot{X}_L$, the displacement increment ΔX is known for the time step going from $t \rightarrow (t + \Delta t)$.

FIGURE 9.2 (Cont.)

- STEP ⑦: In order to calculate $(\partial X_i / \partial F_j)_i$ terms, a unit change impulse load is applied at snubber location j (when $\ddot{X} > \ddot{X}_L$) and the change of displacement ΔX is noted at each snubber location where $\ddot{X}(t + \Delta t) > \ddot{X}_L$. This solution is represented by Equation (9.13). Therefore in "m" locations have snubber responses where $\ddot{X}(t + \Delta t) > \ddot{X}_L$ then a (mxm) matrix of $(\partial X_i / \partial F_j)$ terms will be formed.
- STEP ⑧: Having obtained the matrix of "flexibility" terms $(\partial X_i / \partial F_j)$, the snubber reaction loads ΔR can be calculated since ΔX_i are known from STEP ⑥.
- STEP ⑨: The snubber reaction loads calculated in STEP ⑧ are added to the applied load vector $F(t + \Delta t)_i$ and the response calculation of STEP ③ is repeated. The desired response determined in STEP ⑥ will not be obtained.

Figure 9.3 Response Due to Viscous Parameter
 (Without Consideration of Unlocking)

FIGURE 9.3 (Cont.)

- STEP ①: All anchor motions or specified displacements, loads, and system parameters are known at $t = 0$.
- STEP ②: The selection of time increment is made. (See STEP ② - Motion Constraints.)
- STEP ③: The calculation of the viscous force at $(t + \Delta t)$ is based on the velocity at t . This approach adds an insignificant error provided Δt is not large - which removing the need for an iterative solution scheme.
- STEP ④: The viscous force is added to the applied force to get the total force acting on the system at $(t + \Delta t)$.
- STEP ⑤: The response at $(t + \Delta t)$ is calculated based on the applied force obtained in STEP ④ and the response at t , i.e., $X(t)$, $\dot{X}(t)$, and $\ddot{X}(t)$.
- STEP ⑥: The integration time interval is incremented and the next integration step is initiated.

A problem that needs to be addressed for viscous hydraulic snubber reaction loads is implementing the criteria when the snubber "unlocks". First, there is the locking velocity parameter that dictates when the snubber becomes active. Next, there is the bleed rate which effects the response when the snubber is active. Finally there is the criteria which indicates the snubber becomes inactive. Several criteria can be established for causing the snubber to become inactive, some of these include:

- a) Reaction load $< R_{CRIT}$
- b) $X_i < \dot{X}_{CRIT}$
- c) $X < X_{CRIT}$

The problem is one of establishing which parts of the response cycle have an "active" snubber. This complicates the flow diagram analysis procedure shown in Figure 9.3. The procedure used for this analysis is shown in Figure 9.4.

Nonlinear restorative loads are those that are other than linear functions of displacement response $F_R = F_R(X)$. The linear restorative forces are in general treated as linear spring rates, while the higher order terms are treated as nonlinear restorative forces. The consideration of these forces in the dynamic analysis is identical to the procedure used for solving the viscous problem. The procedure differs only in STEP (3) (Figure 9.3) where the nonlinear translational load is calculated rather than the velocity dependent load. The snubber characteristic that is more exemplified by this nonlinearity, is caused by hydraulic fluid with entrained air.

Clearances are usually associated with other response parameters that act in conjunction with them. For example a clearance may exist with viscous damping, or nonlinear restoring forces; or, a clearance may exist between the system and a rigid anchor. Clearances may be symmetric or asymmetric about an equilibrium point. Basically a clearance represents a dead band in the response-reaction characteristics of a snubber. The analysis procedure used when clearances are considered is basically similiar to that when no clearance

exists. However, there exists a check in which the response is compared to clearance values at a specific time, and if the response is in the dead-band region, corrective steps or load modifications are not made. For example consider the viscous damping parameter procedure. If clearance is included the flow chart is modified from that shown in Figure 9.3 to that shown in Figure 9.5.

Although the previously discussed analysis technique works well when damping effects are small, numerical problems are encountered when the discrete damping parameter $C = \text{RATED LOAD}/\text{BLEED RATE}$ is large. For a single degree of freedom lumped mass model, a large value appears to be $C > 20 M\omega_n$. The numerical instabilities can be minimized by using very small integrating time steps; however, for most practical problems the running time would be prohibitively long. Therefore, to solve problems when viscous damping parameters are large, an alternate analysis procedure has been adopted. This procedure, which requires a load correction step for each integration step, is stable for large discrete viscous damping parameters, $C \gg 20 M\omega_n$.

Figure 9.4 Response Due to Viscous Parameters (Includes Unlocking)

Figure 9.5 Response Due to Viscous and Clearance Parameters

Looking at a single degree of freedom lumped mass, we can proceed to develop an analysis procedure that will not be sensitive to the magnitude of C . The analysis technique is based on constant velocity between integration time steps.

The response at $(t + \Delta t)$ can be expressed by

$$X_i(t + \Delta t) = X_i(t) - \dot{X}_{B_i}(R)\Delta t \quad \dots(9.20)$$

The same response can be expressed in terms of the reaction loads,

$$X_i(t + \Delta t) = X_i(F, t, R) = X_i(R) \quad \dots(9.21)$$

since F, t are constant during Δt .

Equating these expressions,

$$X_i(t) - \dot{X}_{B_i}(R)\Delta t = X_i(R) \quad \dots(9.22)$$

For the general system consisting of many modes, there is one equation for each snubber support. Basically we have " m " nonlinear equations in which R must be solved.

In order to solve these equations, we take and rearrange the equation as

$$X_i(t) - \dot{X}_{B_i}(R)\Delta t - X_i(R) = \Psi_i \quad \dots(9.23)$$

where $\Psi_i = 0$ when R is correct.

The figure on the previous page shows a graphical representation of the function $\Psi(R)$. A first approximation is

$$\frac{\Psi(R_0)}{R_0 - R} = \frac{\partial \Psi}{\partial R} (R_0) \quad \dots (9.24)$$

where

$$\left(\frac{\partial \Psi}{\partial R} \right)_i = \frac{\partial}{\partial R} \left[X(t) - \dot{X}_B(R) \Delta t - X_i(R) \right] = \frac{\partial \dot{X}_B}{\partial R} \Delta t - \frac{\partial X}{\partial R} (R_0)$$

and

$$\left(\frac{\partial \Psi}{\partial R} \right)_i = - \frac{1}{C} \Delta t - \frac{\partial X}{\partial R} (R_0) \quad \dots (9.25)$$

Since we are concerned with the general solution, Equation (9.6) becomes

$$\left(\frac{\partial \Psi}{\partial R} \right)_{ij} = - \frac{1}{C} \Delta t - \sum \frac{\partial X_i}{\partial R_j} (R_{0j}) \quad \dots (9.26a)$$

therefore,

$$\left[\frac{\partial \Psi}{\partial R} \right] = \left[\frac{\Delta t}{C} \right] + \left[\frac{\partial X}{\partial R} \right] \quad \dots (9.26b)$$

Writing Equation (9.24) in matrix form,

$$\frac{\Psi(R_0)}{R_0 - R} = \frac{\partial \Psi}{\partial R} (R_0)$$

$$\Psi(R_0) = \frac{\partial \Psi}{\partial R} (R_0) (R_0 - R)$$

and,

$$\{\Psi(R_0)\} = \left[\frac{\partial \Psi}{\partial R} \right] (\{R_0\} - \{R\}) \quad \dots (9.27)$$

Solving for $\{R\}$,

$$\{R\} = \{R_0\} + \left[\frac{\partial \Psi}{\partial R} \right]^{-1} \{\Psi(R_0)\} \quad \dots (9.28)$$

Substituting (9.26b) into (9.28):

$$\{R\} = \{R_0\} + \left[\frac{\Delta t}{C} + \left[\frac{\partial X}{\partial R} \right] \right]^{-1} \{\Psi(R_0)\} \quad \dots (9.29)$$

Equation (9.29) is a recursion formula that can be used to solve for the snubber reaction loads $\{R\}$.

Further examination of Equation (9.29) reveals that since $\left[\frac{\partial X}{\partial R} \right]$ is a linear matrix, the equation is linear. Therefore $\{R\}$ can be solved after one iterative load correction.

One can see, that as "C" becomes large, the $\left[\frac{\Delta t}{C} \right]$ matrix becomes small, and since $\left[\frac{\partial X}{\partial R} \right]$ is stable, the solution will remain stable.

9.1 Analysis Procedure for Considering Support Dynamic Characteristics

Assume for purposes of analysis that the snubber reaction load is represented simply as R , where R is a function of variables such as accelerations, velocities, or displacement with a nonlinear relationship to load. The two differential equations of motion are:

$$M\ddot{X}_1 + (X_1 - X_B)K = -R \quad \dots(9.1.1a)$$

$$m\ddot{X}_2 + (X_2 - X_B)K = R \quad \dots(9.1.1b)$$

Expressing the response in terms of the relative displacements δ_1 and δ_2 , the differential equations become

$$M(\ddot{\delta}_1 + \ddot{X}_B) + K\delta_1 = -R \quad \dots(9.1.2a)$$

$$m(\ddot{\delta}_2 + \ddot{X}_B) + K\delta_2 = R \quad \dots(9.1.2b)$$

where

$$X_1 = \delta_1 + X_B \quad X_2 = \delta_2 + X_B \quad \dots(9.1.3a)$$

$$\dot{X}_1 = \dot{\delta}_1 + \dot{X}_B \quad \dot{X}_2 = \dot{\delta}_2 + \dot{X}_B \quad \dots(9.1.3b)$$

Assume the reaction load R is a function of acceleration as it pertains to the acceleration threshold parameter. That is

$$R = R(\ddot{\delta}_1, \ddot{\delta}_2) \quad \dots(9.1.4)$$

The acceleration of each mass assuming $R = 0$ is

$$\ddot{\delta}_1 = -\ddot{X}_B - \left(\frac{K}{M}\right)\delta_1 \quad \dots(9.1.5a)$$

$$\ddot{\delta}_2 = -\ddot{X}_B - \left(\frac{k}{M}\right)\delta_2 \quad \dots(9.1.5b)$$

Imposed on the system is the following condition,

$$|\ddot{\delta}_1 - \ddot{\delta}_2| \leq \ddot{X}_L \quad \dots(9.1.6)$$

If $|\delta_1 - \delta_2| \geq X_L$ then $R \neq 0$, and

$$\ddot{\delta}_1 - \ddot{\delta}_2 = -R\left(\frac{1}{M} + \frac{1}{m}\right) - \left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \quad \dots(9.1.7)$$

Also,

$$\delta_1 = \delta_2 + X_L \operatorname{sgn} \left[-\left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \right] \quad \dots(9.1.8)$$

letting

$$\emptyset = -\left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \quad \dots(9.1.9)$$

Then,

$$\ddot{\delta}_1 - \ddot{\delta}_2 = \ddot{X}_L \operatorname{sgn}(\emptyset) = -R\left(\frac{1}{M} + \frac{1}{m}\right) - \left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \quad \dots(9.1.10)$$

Solving for R,

$$R = \left(\frac{mM}{m+M}\right) \left[\left(\frac{k}{m}\right)\delta_2 - \left(\frac{K}{M}\right)\delta_1 - \ddot{X}_L \operatorname{sgn}(\emptyset) \right] \quad \dots(9.1.11)$$

Solving for δ_1 from equation (9.1.2a) and equation (9.1.11),

$$\delta_1 = \left(\frac{1}{M+m}\right) \left[(M+m)\ddot{X}_B - K\delta_1 - K\delta_2 + m\ddot{X}_L \operatorname{sgn}(\emptyset) \right] \quad \dots(9.1.12)$$

And from Equation (9.1.8),

$$\ddot{\delta}_2 = \ddot{\delta}_1 - \ddot{X}_L \operatorname{sgn}(\emptyset) \quad \dots(9.1.13)$$

If we let

$$\beta = \left(\frac{m}{M}\right) \quad \dots(9.1.14)$$

The following expressions can be used to calculate the system response and the snubber reaction load.

$$\ddot{\delta}_1 = \ddot{x}_B - \left(\frac{K}{M}\right)\left(\frac{1}{1+\beta}\right)\delta_1 - \left(\frac{k}{m}\right)\left(\frac{\beta}{1+\beta}\right)\delta_2 + \left(\frac{\beta}{1+\beta}\right)\dot{x}_L \operatorname{sgn}(\phi) \dots (9.1.15)$$

and

$$\left(\frac{R}{M}\right) = \left(\frac{\beta}{1+\beta}\right)\left[\left(\frac{k}{m}\right)\delta_2 - \left(\frac{K}{M}\right)\delta_1 - x_L \operatorname{sgn}(\phi)\right] \dots (9.1.16)$$

And finally,

$$\ddot{\delta}_2 = \ddot{\delta}_1 - \ddot{x}_L \operatorname{sgn}(\phi) \dots (9.1.13)$$

where

$$\phi = -\left(\frac{K}{M}\right)\delta_1 + \left(\frac{k}{m}\right)\delta_2 \dots (9.1.9)$$

If the snubber reaction load is velocity dependent, as for a hydraulic snubber, the following analysis is applicable.

The differential equations of motion are the same as previously developed.

$$\ddot{\delta}_1 + \left(\frac{K}{M}\right)\delta_1 = -\ddot{x}_B - \frac{R}{M} \dots (9.1.2a)$$

$$\delta_2 + \left(\frac{k}{m}\right)\delta_2 = -x_B + \frac{R}{\beta M} \dots (9.1.2b)$$

where

$$\beta = \frac{m}{M} \dots (9.1.14)$$

Since the load is velocity dependent,

$$R = C(\dot{\delta}_1 - \dot{\delta}_2) \dots (9.1.17)$$

or,

$$\frac{R}{M} = \frac{C}{M}(\dot{\delta}_1 - \dot{\delta}_2) \quad \dots(9.1.18)$$

where

$$C = \text{Viscous Damping Coefficient} = \frac{\text{RATED LOAD}}{\text{BLEED RATE}}$$

During the time increment Δt , the response changes from $\dot{\delta}_1 \rightarrow \dot{\delta}_{1\text{act}}$ and $\dot{\delta}_2 \rightarrow \dot{\delta}_{2\text{act}}$. Also during the time increment the snubber reaction also changes. The velocity at t and $(t + \Delta t)$ can be determined from the following expressions:

$$\dot{\delta}_{1\text{act}} = \dot{\delta}_1 - \left(\frac{\partial \dot{\delta}_1}{\partial R}\right) \frac{R}{M} \quad \dots(9.1.19a)$$

$$\dot{\delta}_{2\text{act}} = \dot{\delta}_2 + \left(\frac{\partial \dot{\delta}_2}{\partial R}\right) \frac{R}{M} \quad \dots(9.1.19b)$$

where

$$\dot{\delta}_{1\text{act}}, \dot{\delta}_{2\text{act}} = \text{Correct velocities at } (t + \Delta t)$$

$$\dot{\delta}_1, \dot{\delta}_2 = \text{Velocity at } (t + \Delta t) \text{ when } R = 0$$

$$\left(\frac{\partial \dot{\delta}_1}{\partial R}\right), \left(\frac{\partial \dot{\delta}_2}{\partial R}\right) = \text{Partial derrivatives}$$

Substituting (9.1.19a) and (9.1.19b) into (9.1.18) the following expression is obtained:

$$\left(\frac{R}{M}\right) = \left(\frac{C}{M}\right) \left[(\dot{\delta}_1 - \dot{\delta}_2) - \left(\frac{R}{M}\right) \left(\frac{\partial \dot{\delta}_1}{\partial R} + \frac{\partial \dot{\delta}_2}{\partial R} \right) \right] \quad \dots(9.1.20)$$

or, rearranging and solving for $\left(\frac{R}{M}\right)$,

$$\left(\frac{R}{M}\right) = \left(\frac{C}{M}\right) \frac{(\dot{\delta}_1 - \dot{\delta}_2)}{1 + \left(\frac{C}{M}\right) \left(\frac{\partial \dot{\delta}_1}{\partial R} + \frac{\partial \dot{\delta}_2}{\partial R} \right)} \quad \dots(9.1.21)$$

The above expression for the reaction load can then be substituted into Equations (9.1.2a) and (9.1.2b) and the solution can be obtained by conventional analysis procedures.

9.2 Analysis of Acceleration Threshold Parameter (System A)

Given the single degree of freedom system shown above, in which $\ddot{X}_L \ll |\ddot{X}|$, where $|\ddot{X}|$ is the amplitude of the acceleration for the unsnubbed system, the differential equation of motion is

$$m\ddot{X} + kX = kX_B \sin \Omega t \quad \dots(9.2.1)$$

Expressing the response in terms of relative components, where

$$\delta = X - X_B \quad \dots(9.2.2a)$$

$$\dot{\delta} = \dot{X} - \dot{X}_B \quad \dots(9.2.2b)$$

$$\ddot{\delta} = \ddot{X} - \ddot{X}_B \quad \dots(9.2.2c)$$

then Equation (9.2.1) becomes

$$m\ddot{\delta} + k\delta = -m\ddot{X}_B$$

or

$$\ddot{\delta} + \omega^2 \delta = -X_B \Omega^2 \sin \Omega t \quad \dots(9.2.3)$$

Solving Equation (9.2.3) for the relative acceleration,

$$\ddot{\delta} = \Omega^2 X_B \sin \Omega t - \omega^2 \delta \quad \dots (9.2.4)$$

However, since the acceleration is limited to $\ddot{X}_L$, and since $\ddot{X}_L \ll |\ddot{X}|$, the acceleration of the mass can be expressed as

$$\ddot{\delta} = \ddot{X}_L \operatorname{sgn} |\Omega^2 X_B \sin \Omega t - \omega^2 \delta| \quad \dots (9.2.5)$$

The following equation is equivalent to Equation (9.2.5)

$$\ddot{\delta} = \ddot{X}_L \operatorname{sgn} \left| \sin \Omega t - \frac{\omega^2 \delta}{\Omega^2 X_B} \right| \quad \dots (9.2.6)$$

Letting

$$Q = \frac{\omega^2 \delta}{\Omega^2 X_B} \quad \dots (9.2.7)$$

then

$$\ddot{Q} = \frac{\omega^2 \delta}{\Omega^2 X_B} \quad \dots (9.2.8)$$

Substituting (9.2.7) and (9.2.8) into (9.2.6) the following expression is obtained:

$$\ddot{Q} = \left(\frac{\omega^2 X_L}{\Omega^2 X_B} \right) \operatorname{sgn} \left| \sin \Omega t - \frac{\omega^2 \delta}{\Omega^2 X_B} \right| \quad \dots (9.2.9)$$

or

$$\ddot{Q} = \phi \operatorname{sgn} |\sin \Omega t - Q| \quad \dots (9.2.10)$$

where

$$Q = \left(\frac{\omega}{\Omega} \right)^2 \left(\frac{\delta}{X_B} \right) \quad \dots (7.2.11a)$$

$$C = \omega^2 \left(\frac{\ddot{X}_L}{|\ddot{X}_B|} \right) \quad \dots (9.2.11b)$$

The same analysis procedure can be followed for the single degree of freedom oscillator shown below.

The results are as follows:

$$\ddot{Q} = \phi \operatorname{sgn}|\sin \Omega t - Q| \quad \dots(9.2.10)$$

where

$$Q = (X/X_B) \quad \dots(9.2.12a)$$

$$\phi = (\Omega^2 \frac{\ddot{x}_L}{|\dot{x}_B|}) \quad \dots(9.2.12b)$$

The solution to Equation (9.2.10) can be obtained by numerical techniques on a computer. It should be pointed out that due to the nature of this equation very small integration time steps are required.

9.3 Viscous Snubber Efficiency

Given the system shown in the above figure. The load transferred into the base is the same as the load in the dashpot which is the same as the load in the spring.

$$F_B(t) = kX_O(t) = C(\dot{X}_B - \dot{X}_O) \quad \dots (9.3.1)$$

The snubber efficiency η is defined as the ratio of the snubber load F_d to the maximum snubber load, where the snubber load is

$$F_d = C(\dot{X}_B - \dot{X}_O) \quad \dots (9.3.2)$$

therefore,

$$\eta = \frac{|F_d|}{|CX_{B\Omega}|} \quad \dots (9.3.3)$$

The force in the dashpot is

$$F_d = C(X_B - X_O) \quad \dots (9.3.4)$$

and the force in the spring is

$$F_S = kX_O \quad \dots (9.3.5)$$

At any instant the forces are equal, therefore

$$C(\dot{X}_B - \dot{X}_O) = kX_O \quad \dots (9.3.6)$$

or

$$\dot{X}_O + \left(\frac{k}{C}\right)X_O = \dot{X}_B$$

or

$$\dot{X}_O + aX_O = X_B\Omega \cos \Omega t \quad \dots (9.3.7)$$

where

$$a = \left(\frac{k}{C}\right) \quad \dots (9.3.8)$$

The solution of (9.3.7) can be found by Laplace Transform methods or other techniques. Solving for $x_0(t)$,

$$x_0(t) = \frac{x_B}{1 + \left(\frac{k}{C\Omega}\right)^2} \left[\left(\frac{k}{C\Omega}\right) \cos \Omega t + \sin \Omega t - \left(\frac{k}{C\Omega}\right) e^{-\left(\frac{k}{C}\right)t} \right] \dots(9.3.9)$$

or disregarding transient terms,

$$x_0(t) = \frac{x_B}{1 + \left(\frac{k}{C\Omega}\right)^2} \left[\left(\frac{k}{C\Omega}\right) \cos \Omega t + \sin \Omega t \right] \dots(9.3.10)$$

Therefore,

$$\dot{x}_0(t) = \frac{x_B \Omega}{1 + \left(\frac{k}{C\Omega}\right)^2} \left[\cos \Omega t - \left(\frac{k}{C\Omega}\right) \sin \Omega t \right] \dots(9.3.11)$$

Substituting (9.3.11) into (9.3.4) the following expression is obtained,

$$F_d = C x_B \Omega \left[\left(\frac{a^2}{1 + a^2}\right) \cos \Omega t + \left(\frac{a}{1 + a^2}\right) \sin \Omega t \right] \dots(9.3.12)$$

From (9.3.3) and (9.3.12),

$$\eta = \left| \frac{F_d}{C x_B \Omega} \right| = \sqrt{\left(\frac{a^2}{1 + a^2}\right)^2 + \left(\frac{a}{1 + a^2}\right)^2} \dots(9.3.13)$$

$$\eta = \sqrt{\frac{a^2}{1 + a^2}} \quad \text{WHERE} \quad a = \left(\frac{k}{C\Omega}\right)$$

Therefore, the support efficiency is

$$\eta = \sqrt{\frac{\left(\frac{k}{C\Omega}\right)^2}{1 + \left(\frac{k}{C\Omega}\right)^2}} \dots(9.3.14)$$

10. CONCLUSIONS AND RECOMMENDATIONS

Basic information on snubber response sensitivity was obtained, for mechanical and hydraulic snubbers, using simple analytical models. Sophisticated analytical models, utilizing hydraulic snubber parameters, were also investigated. This data can now be used for the development of simplified analyses and design rules. The results obtained from the analyses of the simple models appeared to be consistent with the results of the more complex models. In future work, it is recommended that analyses of these more complex models be extended to include the following parameters:

1. Mechanical Snubber Acceleration Threshold Parameters (ATP);
2. Mechanical Snubber ATP Combined with a Clearance;
3. Hydraulic Snubber Parameters Combined with a Clearance.

This should verify that the parameter sensitivity developed for the simple models is valid for the more sophisticated models and verify, or refine, the acceptable parameter ranges in Table 2.1. In FY 79 detailed time history studies, using the snubber mathematical models developed herein, will be compared with various simplified analytical procedures, to determine their applicability. Comparisons will be made based on stress response, snubber loadings, and displacement. The simplified analyses procedures that will be developed should assure that system response will be bound within acceptable limits.

NOMENCLATURE

- M, m = Mass (lb-sec²/in)
 K, k = Spring rate (lb/in)
 K^*, k_e = Effective spring rate (lb/in)
 K^o = Pipe stiffness (lb/in)
 Ω = Forcing frequency (rad/sec)
 f_Ω = Forcing frequency (Hz)
 ω_n = Natural frequency (rad/sec)
 f_n = Natural frequency (Hz)
 ζ = Critical damping ratio
 γ = Bilinearity stiffness ratio
 $X, \dot{X}, \ddot{X}$ = Translational displacement, velocity, and acceleration respectively (in., in/sec, in/sec²)
 $\tau, t, \Delta t$ = Time and time increment respectively (sec)
 ℓ = length (in)
 τ_B = Beating period (sec)
 κ^2 = Hardening (softening) coefficient
 $\ddot{X}_L$ = Acceleration threshold parameter (in/sec²)
 F, P = Force (lb)
 C = Viscous damping coefficient (lb-sec/in)
 $\dot{X}_B$ = Bleed velocity (in/sec)
 X_B = Base excitation (in)
 $\dot{X}_L$ = Lock velocity (in/sec)
 α = Flexibility (in/lb)
 R = Reaction load (lb)

δ_C = Clearance (in)

F_d, C_F, F_v = Damping forces (lb)

∞ = Infinity

$\delta, \dot{\delta}, \ddot{\delta}$ = Relative displacement, velocity and acceleration respectively
(in., in/sec, in/sec²)

F_r = Friction load

η = Support efficiency

$a = (k/C\Omega)$

β = Forcing frequency/natural frequency

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