- A SIMPLE MODEL OF BEAM TRANSPORT IN LOW-PRESSURE ION-FOCUSED REGIMES*

R. J. Briggs

Lawrence Livermore National Laboratory, University of California

Livermore, CA 94550

ABSTRACT

This paper presents a simple model of self-focused beam transport at very low gas pressures, where fractional electric neutralization is incomplete and secondary electrons are expelled from the beam region

DISCLAIMER

This document represents the work of a contractor or agency of the United States Government. Neither the United States Government nor any agency thereof nor any of their recipients, including the contractor or agency, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

*The Lawrence Livermore National Laboratory is operated by the University of California for the Department of Energy under contract number W-7405-ENG-48.

This work is performed by LLNL for the Department of Defense under DARPA (DOD) ARPA Order 3718, Amendment #33 and 34, monitored by NSWC under Contract No. N60921-81-LT-W0009, Amendment No. 1 and N60921-81-LT-W0026.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
074

INTRODUCTION

Stable self-focused transport of a relativistic electron beam at low gas pressure was demonstrated many years ago in the Astron beam experiments.¹ In this propagation regime, secondary electrons are quickly expelled from the beam region by the strong space-charge electric field. A net focusing force on the beam electrons is provided by the ion density that builds up inside the beam, and instabilities such as the (electron-electron) two-stream interaction are avoided as long as the product of gas pressure and pulse length is low enough to keep secondary electrons from accumulating inside the beam.

This transport mode has potential application to stabilization of transverse oscillations inside an induction linac,² inexpensive and/or disposable beam transport systems, and beam conditioning prior to injection into higher pressure gas cells. The purpose of this note is to present a simple model of this transport mode that should be useful as a first approximation to more complete descriptions.

BEAM ENVELOPE DESCRIPTION

We assume secondary electrons leave the beam region instantaneously, without causing additional (avalanche) ionization,³ so the net space charge inside the beam becomes partially compensated by the ion density that builds up according to

$$\frac{\partial n_i}{\partial \tau} = \nu n_B \quad (1)$$

with ν the primary impact ionization rate, $\tau = t - z/c$, and $n_B = I_0 / e c \pi a^2$ the beam density. With a collapsing beam envelope described by $a(\tau)$, the usual beam envelope approach would involve following the time-dependent radial profile of the ions and averaging the focusing force over the beam cross-section. With a square (top-hat) beam profile, however, we note that the ion density inside the beam remains constant in r if the beam radius collapses in a monotonic fashion (see Fig. 1). In this case, the time-dependent fractional electrical neutralization inside the beam, $f_e = n_i / n_B$, is rigorously given by

$$f_e = a^2 \int_0^\tau \frac{\nu d\tau}{a^2} \quad (2)$$

for the case of a constant beam current that turns on at $\tau = 0$. (We will also use Eq. (2) with more general radial profiles; it should be a reasonable model equation in those cases, but note that it does not correspond to a profile average of the time-dependent pinch force.)

Fig. 1. Ion density accumulation with a collapsing beam.

To calculate a specific self-consistent equilibrium beam envelope profile in τ , we assume an emittance that is constant in τ , so⁴

$$\begin{aligned} a^2 &= q^2 \frac{I_a}{I_0 f_e} \\ &= a_0^2 / f_e \end{aligned} \quad (3)$$

with $I_a = 17 \gamma$ kiloamps, a_0 the radius of the beam when it is fully-neutralized, and q the beam emittance (without the factor of π). The f_e denominator should actually be replaced by $f_e - 1/\gamma^2$, but we take $\gamma \gg 1$ and ignore this slight correction to the $\tau = 0$ beam front location. The simultaneous solution of Equations (2) and (3) gives

$$f_e = v\tau/2 \quad (4)$$

and

$$\frac{a}{a_0} = \left(\frac{2}{v\tau} \right)^{1/2} \quad (5)$$

Note that this "trumpet-shaped $a(\tau)$ profile (see Fig. 2) is maintained without change as the beam propagates; inclusion of gas scattering and energy loss can of course lead to radial expansion and nose erosion as with the usual self-focused beam at high pressures.

The physical explanation of this "erosion-less" equilibrium solution is similar to the corresponding model calculation in the local conductivity limit.⁵ After the initial free expansion and "evaporation" of most of the very front electrons has occurred, a very low density population consisting of those electrons initially aimed straight ahead builds up at the front, and

Fig. 2. Beam profile in τ .

these electrons generate an ion density capable of very weakly focusing the subsequent beam sections. In the absence of scattering, these front electrons can continue down the system without any further "evaporation", and the weakly focused regions near the front can maintain a (large) equilibrium radius with no further expansion. (These general conclusions are not altered by including a finite escape time of the secondary electrons, although the details of the front part of the profile are altered.) In the very low pressure regimes of interest here, gas scattering is indeed quite weak, so these equilibrium profiles are of some physical importance. We also note that the energy loss profile is not concentrated at the very front part of the beam in the present propagation mode, as shown in the following section, so the energy loss by itself does not lead to strong nose erosion in the present case.

ELECTROMAGNETIC FIELDS

Maxwell's equations for this circularly-symmetric system take the form

$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\mu_0 \frac{\partial H_\phi}{\partial t} \quad (6)$$

$$-\frac{\partial H_\phi}{\partial z} = \epsilon_0 \frac{\partial E_r}{\partial t} + J_r \quad (7)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) = J_z + \epsilon_0 \frac{\partial E_z}{\partial t} \quad (8)$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_r) + \frac{\partial E_z}{\partial z} = \frac{\rho}{\epsilon_0} \quad (9)$$

With the assumed dependence $E_z(r, \tau)$, with $\tau = t - z/c$, we have

$$\frac{\partial}{\partial z} = -\frac{1}{c} \frac{\partial}{\partial \tau} \quad (10)$$

We will assume

$$J_z \approx J_B \quad (a)$$

$$\epsilon_0 \left| \frac{\partial E_z}{\partial \tau} \right| \ll J_B \quad (b)$$

$$\left| \frac{\partial E_z}{\partial z} \right| \ll \frac{\rho}{\epsilon_0} \quad (c)$$

The last two approximations allow us to drop the terms containing E_z in Eq. (8) and Eq. (9). The first assumption relies on the escape of secondary electrons being purely radial (thereby contributing to J_r , which cannot be neglected if we are to keep charge conservation preserved). With these approximations, the transverse fields have the following form [from Eqs. (8) and (9), noting that $\rho = \rho_B + \rho_i$].

I. Uniform beam, $r \leq a$. Here $\rho = \rho_B(1 - f_e)$ inside the beam, so

$$E_r = \frac{(\mu_0/\epsilon_0)^{1/2} I_B}{2\pi a^2} (1 - f_e) r \quad (11)$$

$$H_\phi = \frac{I_B}{2\pi a^2} r \quad (12)$$

For $r > a$, the radial electric field has a complicated form because of the nonuniform ion charge density outside the instantaneous beam radius a (τ) (see Fig. 1). The magnetic field is given by

$$H_{\phi} = \frac{I_B}{2\pi r} \quad (13)$$

for $r > a$.

11. Bennett profile. Here

$$J_B = \frac{I_B}{\pi a^2 (1 + r^2/a^2)^2} \quad (14)$$

In this case, we will take $\rho_i/\rho_B = f_e$ for all r , with f_e given by Eq. (4) (i.e., assumed independent of r) and the scale radius given by Eq. (5). We then have

$$E_r = \frac{(\mu_0/\epsilon_0)^{1/2} I_B r}{2\pi (a^2 + r^2)} (1 - f_e) \quad (15)$$

and

$$H_{\phi} = \frac{I_B r}{2\pi (a^2 + r^2)} \quad (16)$$

The axial electric field can be calculated for general radial profiles when approximations (a)-(c) are made. From Eq. (6), we have

$$\frac{\partial E_z}{\partial r} = \mu_0 \frac{\partial H_{\phi}}{\partial \tau} - \frac{1}{c} \frac{\partial E_r}{\partial \tau} \quad (17)$$

using Eq. (9). Operating on Eq. (17) by $(1/r \partial/\partial r r)$ on both sides, and noting that

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial E_r}{\partial \tau} &= \frac{1}{\epsilon_0} \left(\frac{\partial \rho_B}{\partial \tau} + \frac{\partial \rho_i}{\partial \tau} \right) \\ &= \frac{1}{\epsilon_0} \left(\frac{\partial \rho_B}{\partial \tau} - v \rho_B \right) \end{aligned} \quad (18)$$

and

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial \tau} &= \frac{\partial j_B}{\partial \tau} \\ &= c \frac{\partial \rho_B}{\partial \tau} , \end{aligned} \quad (19)$$

we have

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial E_z}{\partial \tau} = v \mu_0 j_B \quad (20)$$

We take a radial boundary condition corresponding to a conducting wall ($E_z = 0$) at $r = b$. Note that an outer boundary condition of some type must be specified with this model, since we must physically account for the manner in which we "terminate" the outward radial flow of the secondary electrons.

I. With a square beam profile, we have

$$\begin{aligned} E_z &= - \frac{v \mu_0 I_B}{2\pi} \left[\ln \frac{b}{a} + \frac{1}{2} \left(1 - \frac{r^2}{a^2} \right) \right] , \quad r < a(\tau) \\ &= - \frac{v \mu_0 I_B}{2\pi} \ln \frac{b}{r} , \quad b > r > a(\tau) \end{aligned} \quad (21)$$

Note that the only τ -dependent term in Eq. (21) is the beam radius $a(\tau)$. When $a(\tau) > b$, we have

$$E_z = - \frac{v\mu_0 I_B}{4\pi} \left(\frac{b^2 - r^2}{a^2} \right) \quad (22)$$

[There is no problem with $a > b$; the electrons "outside b " can be considered lost without altering the solution. The significance of $a(\tau)$ in this case is a scale length for the current density inside $r = b$].

II. With a Bennett profile, we have

$$E_z = - \frac{v\mu_0 I_B}{4\pi} \ln \left(\frac{b^2 + a^2}{r^2 + a^2} \right) \quad (23)$$

for all cases.

A check of the basic approximations (b) and (c) can now be done using these field solutions. Condition (c) is the most stringent since it corresponds to

$$\epsilon_0 \left| \frac{\partial E_z}{\partial \tau} \right| \ll (1 - f_e) J_B \quad (24)$$

Using the Bennett profile solution, Eq. (23), as a representative case we find

$$\frac{v^2 a^2}{8c^2} \left(\frac{b^2}{b^2 + a^2} \right) \ll f_e (1 - f_e) \quad (25)$$

The approximations are good in general as long as $va/c \ll 1$, as it should be since the escape time of the secondary electrons is of order c/a (or longer), and the ion density buildup time $1/\nu$ should be much longer than this escape time for the model to make any sense. In the limit $f_e \rightarrow 0$ or $f_e \rightarrow 1$, the approximations used in the field solutions are violated. With $f_e \ll 1$, we need

$$\frac{vb}{c} < (8f_e)^{1/2} \quad (26)$$

Even with $f_e \sim 10^{-3}$, v only require $vb/c \lesssim 0.1$.

The field solutions should therefore have a broad range of validity in all cases of interest. For example, with $b \approx 10$ cm and air or nitrogen ($v \approx 10^9$ p with p the pressure in torr), Eq. (26) is satisfied for $\tau > p/36$ nanoseconds.

DISCUSSION

With either the square or Bennett profiles, the axial electric field is roughly constant in τ and r when $b \gg a(\tau)$:

$$E_z \approx - \frac{v_0 I_B}{2\pi} \ln \frac{b}{a} \quad (27)$$

The value of v depends on the gas type; for nitrogen $v = 10^9$ p and we note that

$$\frac{E_z}{p} \approx 20 \left(\frac{I_B}{10 \text{ KA}} \right) \ln \frac{b}{a} \frac{\text{KeV}}{\text{cm Torr}} \quad (28)$$

This value of E_z/p is, by itself, almost sufficient to insure that the secondary electrons are in a "runaway regime"³ at currents of 10 KA or greater.

The energy loss of electrons in a pulse propagating in the low pressure "ion focused regime" is distributed in a roughly uniform fashion over the population. The ultimate range of a pulse propagating in this mode is therefore given by (for N_2 or air):

$$R \approx \frac{E_0}{E_z} = \frac{E_0(\text{MeV})}{2p \ln b/a} \left(\frac{10 \text{ KA}}{I_B} \right) \text{ meters} \quad (29)$$

For example, the FX-25 beam ($I_B = 10$ KA, $E_0 = 1.5$ MeV) would lose its full energy in around 1 meter at 0.3 tcrr (with $b/a = 5$). The data from this experiment is not in disagreement with this calculation since none of the beam front (which did appear to be propagating in an IFR mode) survived for over a meter or so.

For ATA, operation at N_2 pressures of 1-10 μ should allow useful transport of this very low I/I_A beam. The range at 10 KA beam currents would be of order 2-20 kilometers in this case, so minimal energy loss should occur in any transport application with ATA.

The energy loss per meter given by Eq. (27) can be directly related to the kinetic energy given to the secondary electrons as they are accelerated into the wall at $r = b \gg a$ by the space charge electric field, plus the electric field energy associated with the ions left behind the beam pulse. When $f_e \ll 1$, the energy imparted to the secondary electrons dominates the beam's energy loss since negligible field energy (of order f_e^2) is left behind in that case.

REFERENCES

Beal, J.W., R.J. Briggs, and L.D. Pearlstein, "Beam Research Program - Recent Results and Status," UCID-16169, December 1972.

Briggs, R.J., "Suppression of Transverse Beam Breakup Modes in an Induction Accelerator by Gas Focusing," UCID-18633, April 7, 1980.

Briggs, R.J., "Modeling Beam Front Dynamics at Low Pressures," UCID (to be published).

Lee, E.P. and R.K. Cooper, "General Envelope Equation for Cylindrically Symmetric Charged-Particle Beams," Particle Accelerators, Vol.7, pg. 83-95, 1976.

Briggs, R.J., Beam Research Internal Memo MFE/BR/80-56, March 17, 1980.