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MASTER

OPTIMAL GEOTHERMAL RESOURCE EXTRACTION FOR ELECTRIC POWER APPLICATIONS

Peter D. Blair
Energy Center
University of Pennsylvania
Philadelphia, PA 19104

Thomas A.V. Cassel *
Energy Center
University of Pennsylvania
Philadelphia, PA 19104

ABSTRACT

This paper reports on a phase of ongoing work at the University of Pennsylvania concerning the analysis of capital investments in the development of geothermal electric power facilities. **The phase being addressed deals with determining the optimal rate of extraction of geothermal resources for producing electric power. The paper formulates the mathematical approach for estimating an optimal time-path of production to maximize the net present value of a rate sensitive reservoir.

INTRODUCTION: GEOTHERMAL RESERVOIR RATE SENSITIVITY

In geothermal resources, as in petroleum resources, the amount of usable resource may depend upon the time-path of production. This implies that there exists a maximum efficient rate (MER) of production, above which there will be a significant reduction in the total resource recoverable from the reservoir.

In oil and gas production, increasing the extraction rate may affect the amount of recoverable resource by reducing reservoir pressure. For instance, in natural water driven reservoirs, the rate at which subterranean water displaces oil (or gas), and thereby maintains reservoir pressure, is determined by such factors as the relative size of the aquifer and the permeability and porosity of the reservoir rock. If oil is extracted at a faster rate than water displaces that oil, then the natural reservoir pressure is reduced.

To some extent, petroleum reservoir pressure drawdown can be alleviated by pumping water, steam or other fluids into the aquifer, thus extending the useful life of the reservoir and increasing the total amount of resource ultimately recoverable. Therefore, in a rate sensitive petroleum reservoir, the total recoverable resource depends on the time-path of investment in pressure maintenance capital as well as on the time-path of production. The optimal extraction rate for a reservoir may be estimated by determining the interdependent time-paths of investment and production which maximize the discounted profit stream over a reasonable planning horizon. These considerations are dealt with in some detail by Kuller and Cummings [1], McKie and McDonald [2], Sweeney [3], Clark [4], Gordon [5], and others.

In the case of geothermal resources, the situation is somewhat analogous in that the extraction rate may affect both fluid replacement and thermal recharge of the reservoir and, hence, the pressure and temperature of the resource. The literature illustrates empirical evidence of such rate sensitivity at some geothermal sites (Ref. 6 and 7). For example, the effect of extraction rate on geothermal resource temperature is illustrated in Figure 1 for a reservoir in southern California. Pressure maintenance techniques, such as the water or steam injection practices of the petroleum industry, are not feasible with geothermal resources because such practices would cool the reservoir.

* Presently on leave of absence from Bechtel International, Inc., San Francisco, CA, 94119.

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Figure 1. PREDICTED EFFECT OF EXTRACTION RATE ON HEBER GEOTHERMAL RESERVOIR TEMPERATURE (Source: Bechtel Corp. Ref. 6)

The pressure and temperature characteristics of geothermal well-head products, unlike petroleum well-head products, directly affect the usability of the resource for electric power generation. Figure 2 illustrates the economic consequences of a declining resource temperature. The energy conversion efficiency (the inverse of the "fluid rate" in Figure 2) of a geothermal power plant will decline as the resource temperature declines. Electric utilities are unlikely to tolerate a time-wise decay in plant power rating; thus, the resource producer will be required to supply increasing quantities of geothermal fluid to the plant - most likely at a decreasing price - as the fluid temperature declines (see Figure 3). The power plant must be designed for the lowest resource temperature (i.e., highest fluid flow rate) it will encounter in its operating life; thus plant investment costs will increase as increasing resource production causes a decline in resource temperature.

Later in this paper, a series of equations is developed expressing these operational and economic consequences of rate sensitivity in a geothermal reservoir. The net effect of reservoir rate sensitivity is that the net present value (NPV) of an investment opportunity in the reservoir will be dependent on the time path of fluid extraction which is, in turn, dependent on the time path of investment in the well field. It is desirable, in such circumstances, to identify the maximum efficient rate (MER) of production which will maximize the NPV of investments in the reservoir. The formulation of this problem is the subject of this paper.

EXTRACTION ECONOMICS OF DEPLETABLE RESOURCES: AN OVERVIEW

Optimal resource extraction theory has evolved from two perspectives: (1) resource value to an individual firm and (2) resource value to the overall society. The former, on which we concentrate in this paper, has been developed theoretically by Peterson [9], Lewis [10], Heal [11], Scott [12], Gaffney [13] and others. More recently the theory has been applied to specific resources, primarily oil and gas, e.g., Cummings and Burt [14]. The latter approach, the definitive treatment of which is given in Hotelling [15], has been extended considerably in recent years (Herfindahl [16], Solow [17], and others).

Kuller and Cummings [1] have developed the theory of optimal extraction applied to individual firms to account for time paths of production and investment in petroleum recovery. This approach incorporates fundamental technology parameters in a model for optimal petroleum extraction. A similar (albeit considerably expanded) treatment for geothermal electric power production is presented here.

As discussed earlier, a principal determinant of the optimal extraction rate is reservoir performance. We can characterize the concept of reservoir performance as the following: * first assume a well field is described by the following parameters:

- U_t volume of resource extracted during period t
- v_t capital investment in period t
- X total recoverable resource from reservoir
- K_t capital stocks available at beginning of period t
- F_t capacity constraint on recovery during period t
- C_t production cost during period t

Time paths of production and investment are denoted, respectively, by the following:

$$\bar{U}_t = \{u_1, u_2, \dots, u_t\} \quad (1)$$

$$\bar{V}_t = \{v_1, v_2, \dots, v_t\} \quad (2)$$

* this formulation was first presented applied to oil and gas by Kuller and Cummings [1].

Figure 2. EFFECT OF GEOTHERMAL RESERVOIR TEMPERATURE ON POWER PLANT COST AND PERFORMANCE (Source: EPRI Ref. 8)

Figure 3. EFFECT OF GEOTHERMAL RESERVOIR TEMPERATURE DECLINE ON FLUID FLOW REQUIREMENT FOR CONSTANT POWER PLANT OUTPUT (Source: Bechtel Corp. Ref. 6)

For a planning horizon of $t = 1, 2, \dots, \tau$ periods we state three relationships for reservoir performance:

1. Total resource production over τ cannot exceed X .

$$\sum_{t=1}^{\tau} u_t \leq X \quad (\bar{U}_\tau, \bar{V}_\tau) \quad (3)$$

$$\text{where } \frac{\partial X}{\partial u_t} \leq 0 \text{ and } \frac{\partial X}{\partial v_t} \geq 0$$

The value of X depends upon the time paths of production and investment. In geothermal production, later investments are likely for additional wells to maintain resource utilization, which are analagous in oil and gas to later "artificial lift" investments to maintain resource production.

2. Resource production is limited by installed capital (upper bound).

$$u_t \leq F_t (\bar{U}_t, \bar{V}_t, K_t) \quad (4)$$

$$\frac{\partial F_t}{\partial u_t} \leq 0 \quad \frac{\partial F_t}{\partial v_t} \geq 0 \quad \frac{\partial F_t}{\partial K_t} \geq 0$$

The level of installed capital is a function of available capital stocks and the resource characteristics which are in turn affected by past and current values of u and v .

3. Production costs depend on past production and investment.

$$C_t (\bar{U}_t, \bar{V}_t, K_t) \quad (5)$$

$$\frac{\partial C_t}{\partial u_t} \geq 0 \quad \frac{\partial C_t}{\partial v_t} \geq 0 \quad \frac{\partial C_t}{\partial v_s} \leq 0 \quad \frac{\partial C_t}{\partial K_t} \leq 0$$

($s \neq t$)

We develop this cost function for geothermal electric power production in the next section. Any single firm j exploiting a field over the planning horizon $t = 1, \dots, \tau$ under a given resource price (p_t) will seek to maximize the value of a stream of discounted profits over the planning horizon:

$$\text{Max } Z_j = \sum_{t=1}^{\tau} \left[p_t u_{jt} - C_{jt} (\bar{U}_t, \bar{V}_t, K_{jt}) \right] \delta_t \quad (6)$$

where δ_t is the discount factor $(1+r)^{-t}$; r is the discount rate.

In an n firm unitized field the problem becomes the following:

$$\text{Max } Z = \sum_{j=1}^n Z_j = \sum_{t=1}^{\tau} \sum_{j=1}^n \left[p_j u_{j t} - C_{j t} (\bar{U}_t, \bar{V}_t, K_{j t}) \right] \delta_t \quad (7)$$

so that

$$K_{j k, t+1} = K_{j k t} - D_{j k t} (u_{j t}, v_{j k t}, K_{j k t}) \quad (8)$$

where $D_{j k t}$ is the net depreciation of firm j 's capital component k during period t . We assume these are $k = 1, 2, \dots, q$ capital components to overall investment, i.e., $v_{j t} = \{v_{1 j t}, v_{2 j t}, \dots, v_{q j t}\}$

$$\frac{\partial C_{jt}}{\partial v_{jkt}} \beta_t = B_D + B_E \quad (13)$$

for a given j , k , and t .

FORMULATING THE GEOTHERMAL EXTRACTION PROBLEM

The optimal geothermal extraction problem is now formulated by incorporating the technological characteristics of producing electric power from a rate sensitive reservoir (discussed in the Introduction) into the general model of optimal production and investment developed in the previous section. Particular attention is focused on the effect of temperature decline on the optimal rate of extraction and its effect on production costs.

Golabi and Scherer [23] have developed a fairly detailed model of optimal extraction of geothermal energy for non-electric applications. However, their approach assumes no increase in brine flow is required due to temperature decline in the geothermal reservoir. In electric power applications, increased brine flow must be considered to accommodate the electric utility constraint of constant power plant output despite time-wise decline in brine quality.

Translating the previously discussed framework of depletable resource economics to the geothermal problem, the objective becomes one of finding the time-path of electric power production that maximizes the net present value (NPV) of the resource producer's investment. In order to apply the theory, the following assumptions are made:

1. The downhole temperature (T) is known and is a function of the time-path of geothermal fluid production:

$$\bar{U}_{p,k} = U_{p,1}, U_{p,2}, \dots, U_{p,k}$$

where $U_{p,k}$ is the fluid produced during period k . $U_{p,k}$ may be expressed as:

$$U_{p,k} = U_k CF_k$$

where U_k is the fluid requirement for all power plants on line in period k and CF_k is the average capacity factor (i.e., fraction of time in operation) of the power plants in period k .

Thus,

$$T(\bar{U}_{p,t}) = f_1(U_1 CF_1, U_2 CF_2, \dots, U_t CF_t)$$

which is an implicit function of time.

2. The fluid production from a new (clean) well is a function of downhole temperature:

$$W = f_2(T) \quad (\text{lbs/hr. well})$$

3. The electric power producer is a price taker and will take all electric power that can be produced at busbar price P_t^* (mills/kWh) (assumed to be a function of time to account for escalatory and inflationary effects).
4. The geothermal fluid is assumed to be priced as the difference between the above-mentioned market busbar price and plant-related costs:

$$P'_{jt} = P_t^* - S_{jt}$$

where S_{jt} is the total plant-related cost attributed to plant j (mills/Kwh)

The plant related costs with which we are concerned are those of amortized capital costs and recurring operating costs (exclusive of "fuel"). We

assume that plants are designed to meet end-of-life conditions of fluid temperature, thus:

$$S_{jt} = \begin{cases} f_3(T_{t_j+L}) & , t_j < t < t_j+L \\ 0 & , \text{otherwise} \end{cases}$$

where t_j is the time at which plant j comes on-line and L is the plant life. The fluid price measured per lb of fluid supplied is then the following:

$$P_{jt} = P'_{jt} \eta_j(T_t) \quad (\$/lb)$$

where η_j is the net specific energy $\left(\frac{\text{Watt.Hr}}{\text{Lb of Brine}}\right)$ of plant j , a function of temperature. A representative temperature relationship of net specific energy (corresponding to the 200 MW plant depicted in Figures 2 and 3) is shown in Figure 4.

The net specific energy also provides a relationship for computing the total fluid flow required to support a given plant during period t

$$u_{jt} = \frac{W_j}{\eta_j(T_t)} \quad (\text{lbs})$$

where W_j is the rated capacity of power plant j in mw; u_{jt} is the brine flow rate at time t . The corresponding revenue to the resource producer is then the following (for all plants $j = 1, 2, \dots, m_t$):

$$R_t = \sum_{j=1}^{m_t} P_{jt} u_{jt}$$

where m_t is the total number of plants on line at time t .

The total investment required in the well field during time period t must cover the capacity required for every plant in the field:

$$U_t = \sum_{j=1}^{m_t} u_{jt}$$

where U_t is the total brine flow to all plants during period t .

Investments during any period are required to compensate for three factors: (1) reduction in power plant efficiency due to decline in downhole temperature, (2) reduction in brine flow rate per well due to aging (e.g., plugging) and (3) reduction in flow per well due to reservoir drawdown (reduced aquifer pressure related to downhole temperature decline). The sum of all investments plus operating and maintenance expenses form the geothermal field's cost function.

The total well field investment in period t is given by

$$v_t = \alpha_t \left[\frac{U_{t+\lambda}}{w_o T_{t+\lambda}} - \frac{U_{t+\lambda-1}}{w_o T_{t+\lambda-1}} \right]$$

where α_t is a factor reflecting the cost of drilling a new well and constructing attendant service facilities for that well (\$/well); λ is the time lag from well investment to on-line production. The first term in the brackets defines the number of wells (equivalent new) required to support production during period $t+\lambda$; the second term gives the number of wells required in the preceding period. Hence the difference is the total number of new wells for which investment is required during period t (in order to come on line during period $t+\lambda$).

Figure 4. EFFECT OF GEOTHERMAL RESERVOIR TEMPERATURE DECLINE ON POWER PLANT PERFORMANCE (Source: Bechtel Corp. Ref. 6)

The well field production cost function then becomes:

$$C_t = v_t + \beta_t \left(\sum_{i=1}^l a_i v_{t-\lambda-i} \right) + \gamma \left[\left(\sum_{s=t-l}^t v_s \right), t \right]$$

where β_t reflects the cost to recondition old wells and/or to add new wells to replace aged ones; l is the useful life of a new well; a_i is the fraction of investment $v_{t-\lambda-i}$ that requires rehabilitation during period t . Note that the following relationship must hold:

$$a_l \beta_t = \alpha_t$$

γ is the operating and maintenance cost function which varies with both the level of production and time. We characterize γ , the operating and maintenance costs as a function of currently installed production capacity and time. The objective function of equation for the geothermal problem then becomes the following:

$$\text{Max NPV} = \sum_{t=1}^{\tau} \left\{ \left(\sum_{j=1}^{m_t} p_{jt} u_{jt} \right) - C_t \right\} \delta_t$$

where τ is the planning horizon and m_t is the total number of power plants on-line during period t .

The constraints are of the same general form as those of equations 8 through 11. The geothermal extraction problem in terms of the well-field parameters described above is then the following:

Select the time-path of installing power plants, (i.e.,

$$\bar{w}_\tau = \left\{ \sum_{j=1}^{n_1} w_j, \sum_{j=n_1+1}^{n_1+n_2} w_j, \dots, \sum_{j=n_{\tau-1}+1}^{n_{\tau-1}+n_\tau} w_j \right\}$$

where n_t is the number of new plants installed at time t ; w_j is the power rating of plant j ; and τ is the planning horizon), in order to maximize:

$$\begin{aligned} & \sum_{t=1}^{\tau} \left\{ \left(\sum_{j=1}^{m_t} \left[n_{jt} (p_t^* - s_{jt}) \right] \right) \frac{w_j}{n_{jt}} \right. \\ & - \alpha_t \left[\frac{U_t + \lambda}{w_{oTt+\lambda}} - \frac{U_t + \lambda-1}{w_{oTt+\lambda-1}} \right] \\ & \left. - \beta_t \left[\sum_{i=1}^l a_i v_{t-\lambda-i} \right] + \alpha_t \right\} \end{aligned}$$

subject to:

$$K_{jkt}, t_n = K_{jkt} - D_{jkt} (u_{jt}, v_{kt}, k_{jkt})$$

$$u_{jt} \leq F_{jt} (\bar{u}_t, \bar{v}_t, K_{jt})$$

$$\sum_{s=1}^{\tau} \sum_{j=1}^{m_s} u_{js} \leq x(\bar{u}_\tau, \bar{v}_\tau)$$

$$u_{jt} \geq 0 \quad N_{jkt} \geq 0$$

where: $m_t = \sum_{t'=t-L}^t n_{t'}$ = number of plants on line at time t

L = power plant service life.

The necessary and sufficient conditions for solution of the geothermal optimization problem, thus formulated, follow Kuhn-Tucker theory (ref. Hadley [8] or Himmelblau [9]). These conditions may be examined as interpretations of the generalized Lagrangian solution to the problem as presented above.

CONCLUSIONS AND CONTINUING WORK

This paper formulates the production optimization problem for a rate sensitive geothermal reservoir and develops the approach to a solution for this formulation. Current work on this problem centers around identifying the conditions under which the optimization problem is solvable and interpreting those conditions. The geothermal problem is not as straightforward as the analogous oil and gas problem because of the impacts of temperature decline phenomenon on geothermal power plant design, resource price and, consequently, the optimal rate of extraction.

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* this formulation was first presented applied to oil and gas by Kuller and Cummings [1].

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similarly:

$$K_{jt} = \{K_{j1t}, \dots, K_{jqt}\}$$

The maximization of future profits problem is also subject to the capacity and resource constraints.

$$u_{jt} \leq F_{jt}(\bar{U}_t, \bar{V}_t, K_{jt}) \quad (9)$$

$$\sum_{t=1}^T \sum_{j=1}^n u_{jt} \leq X(\bar{U}_T, \bar{V}_T) \quad (10)$$

$$u_{jt} \geq 0, v_{jkt} \geq 0 \quad (11)$$

The necessary and sufficient conditions for a solution to this kind of problem are results of Kuhn-Tucker theory (Hadley [8] or Himmelblau [9]).

Under this formulation of the optimal extraction problem Kuller and Cummings derive several relationships characterizing optimal paths for production and investment. These are presented as interpretations of the results of a generalized Lagrangian solution to the optimization problem. The condition for optimal production is when the present value of marginal net income equals total user costs. User costs can be interpreted as "the present value of future profit foregone by a decision to produce a unit of output today" (Scott [10]). Typical components of total user costs in reservoir problems are the following:

1. Stock User Costs - marginal scarcity value of the resource (G_s)
2. Boundary User Costs - effect of capacity limits on production (G_B)
3. Capital Consumption User Costs - marginal productivity of available capital (G_C)
4. Production User Costs - effects of current production characteristics (G_P)

The nature of these costs is discussed by Kuller and Cummings [1], Scott [20], Smith [21] and Cummings [22]. We can state the general relationship describing optimal production as the following:

$$\left(P_t - \frac{\partial C_{jt}}{\partial u_{jt}} \right) \beta_t = G_s + G_B + G_C + G_P \quad (12)$$

The solution to this equation is an optimal production rate at time t where the present value of the net income to a firm (the left hand side of equation 2.12) equals the total user cost of that production incurred by the firm (the right hand side).

The optimal rate of investment is that at which the present value of marginal costs of investments in capital type k by firm j is equal to the total benefits to the reservoir realized by that investment. These benefits include:

1. Present value of the future stream of direct benefits (B_D)
2. External impacts of investment on costs and recoverable stock (B_E)

The optimal investment in capital stock k in time t by firm j can thus be characterized by the following:

$$\frac{\partial C_{jt}}{\partial v_{jkt}} \beta_t = B_D + B_E \quad (13)$$

for a given j , k , and t .

FORMULATING THE GEOTHERMAL EXTRACTION PROBLEM.

The optimal geothermal extraction problem is now formulated by incorporating the technological characteristics of producing electric power from a rate sensitive reservoir (discussed in the Introduction) into the general model of optimal production and investment developed in the previous section. Particular attention is focused on the effect of temperature decline on the optimal rate of extraction and its effect on production costs.

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Translating the previously discussed framework of depletable resource economics to the geothermal problem, the objective becomes one of finding the time-path of electric power production that maximizes the net present value (NPV) of the resource producer's investment. In order to apply the theory, the following assumptions are made:

1. The downhole temperature (T) is known and is a function of the time-path of geothermal fluid production:

$$\bar{U}_{p,k} = U_{p,1}, U_{p,2}, \dots, U_{p,k}$$

where $U_{p,k}$ is the fluid produced during period k . $U_{p,k}$ may be expressed as:

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Thus,

$$T(\bar{U}_{p,t}) = f_1(U_1CF_1, U_2CF_2, \dots, U_tCF_t)$$

which is an implicit function of time.

2. The fluid production from a new (clean) well is a function of downhole temperature:

$$W = f_2(T) \quad (\text{lbs/hr. well})$$

3. The electric power producer is a price taker and will take all electric power that can be produced at busbar price P_t^* (mills/kWh) (assumed to be a function of time to account for escalatory and inflationary effects).
4. The geothermal fluid is assumed to be priced as the difference between the above-mentioned market busbar price and plant-related costs:

$$P'_{jt} = P_t^* - S_{jt}$$

where S_{jt} is the total plant-related cost attributed to plant j (mills/Kwh)

The plant related costs with which we are concerned are those of amortized capital costs and recurring operating costs (exclusive of "fuel"). We

assume that plants are designed to meet end-of-life conditions of fluid temperature, thus:

$$s_{jt} = \begin{cases} f_3(T_{t_j+L}) & , t_j < t < t_j+L \\ 0 & , \text{otherwise} \end{cases}$$

where t_j is the time at which plant j comes on-line and L is the plant life. The fluid price measured per lb of fluid supplied is then the following:

$$P_{jt} = P'_{jt} \eta_j(T_t) \quad (\$/lb)$$

where η_j is the net specific energy $\left(\frac{\text{Watt.Hr}}{\text{Lb of Brine}}\right)$ of plant j , a function of temperature. A representative temperature relationship of net specific energy (corresponding to the 200 MW plant depicted in Figures 2 and 3) is shown in Figure 4.

The net specific energy also provides a relationship for computing the total fluid flow required to support a given plant during period t

$$u_{jt} = \frac{W_j}{\eta_j(T_t)} \quad (\text{lbs})$$

where W_j is the rated capacity of power plant j in mw; u_{jt} is the brine flow rate at time t . The corresponding revenue to the resource producer is then the following (for all plants $j = 1, 2, \dots, m_t$):

$$R_t = \sum_{j=1}^{m_t} P_{jt} u_{jt}$$

where m_t is the total number of plants on line at time t .

The total investment required in the well field during time period t must cover the capacity required for every plant in the field:

$$U_t = \sum_{j=1}^{m_t} u_{jt}$$

where U_t is the total brine flow to all plants during period t .

Investments during any period are required to compensate for three factors: (1) reduction in power plant efficiency due to decline in downhole temperature, (2) reduction in brine flow rate per well due to aging (e.g., plugging) and (3) reduction in flow per well due to reservoir drawdown (reduced aquifer pressure related to downhole temperature decline). The sum of all investments plus operating and maintenance expenses form the geothermal field's cost function.

The total well field investment in period t is given by

$$v_t = \alpha_t \left[\frac{U_{t+\lambda}}{w_o T_{t+\lambda}} - \frac{U_{t+\lambda-1}}{w_o T_{t+\lambda-1}} \right]$$

where α_t is a factor reflecting the cost of drilling a new well and constructing attendant service facilities for that well (\$/well); λ is the time lag from well investment to on-line production. The first term in the brackets defines the number of wells (equivalent new) required to support production during period $t+\lambda$; the second term gives the number of wells required in the preceding period. Hence the difference is the total number of new wells for which investment is required during period t (in order to come on line during period $t+\lambda$).

Figure 4. EFFECT OF GEOTHERMAL RESERVOIR TEMPERATURE DECLINE ON POWER PLANT PERFORMANCE (Source: Bechtel Corp. Ref. 6)

The well field production cost function then becomes:

$$C_t = v_t + \beta_t \left(\sum_{i=1}^{\ell} a_i v_{t-\lambda-i} \right) + \gamma \left[\left(\sum_{s=t-\ell}^t v_s \right), t \right]$$

where β_t reflects the cost to recondition old wells and/or to add new wells to replace aged ones; ℓ is the useful life of a new well; a_i is the fraction of investment $v_{t-\lambda-i}$ that requires rehabilitation during period t . Note that the following relationship must hold:

$$a_i \beta_t = \alpha_t$$

γ is the operating and maintenance cost function which varies with both the level of production and time. We characterize γ , the operating and maintenance costs as a function of currently installed production capacity and time. The objective function of equation for the geothermal problem then becomes the following:

$$\text{Max NPV} = \sum_{t=1}^{\tau} \left\{ \left(\sum_{j=1}^{m_t} p_{jt} u_{jt} \right) - C_t \right\} \delta_t$$

where τ is the planning horizon and m_t is the total number of power plants on-line during period t .

The constraints are of the same general form as those of equations 8 through 11. The geothermal extraction problem in terms of the well-field parameters described above is then the following:

Select the time-path of installing power plants, (i.e.,

$$\bar{w}_\tau = \left\{ \sum_{j=1}^{n_1} w_j, \sum_{j=n_1+1}^{n_1+n_2} w_j, \dots, \sum_{j=n_{\tau-1}+1}^{n_{\tau-1}+n_\tau} w_j \right\}$$

where n_t is the number of new plants installed at time t ; w_j is the power rating of plant j ; and τ is the planning horizon), in order to maximize:

$$\begin{aligned} & \sum_{t=1}^{\tau} \left\{ \left(\sum_{j=1}^{m_t} \left[n_{jt} (p_t^* - s_{jt}) \right] \right) \frac{w_j}{n_{jt}} \right. \\ & - \alpha_t \left[\frac{U_t + \lambda}{w_o T_{t+\lambda}} - \frac{U_t + \lambda - 1}{w_o T_{t+\lambda-1}} \right] \\ & \left. - \beta_t \left[\sum_{i=1}^{\ell} a_i v_{t-\lambda-i} \right] + \alpha_t \right\} \end{aligned}$$

subject to:

$$K_{jk}, t_n = K_{jkt} - D_{jkt} (u_{jt}, v_{kt}, k_{jkt})$$

$$u_{jt} \leq F_{jt} (\bar{u}_t, \bar{v}_t, K_{jt})$$

$$\sum_{s=1}^{\tau} \sum_{j=1}^{m_s} u_{js} \leq x(\bar{u}_{\tau}, \bar{v}_{\tau})$$

$$u_{jt} \geq 0 \quad N_{jkt} \geq 0$$

$$\text{where: } m_t = \sum_{t'=t-L}^t n_{t'}, \quad = \text{number of plants on line at time } t$$

L = power plant service life.

The necessary and sufficient conditions for solution of the geothermal optimization problem, thus formulated, follow Kuhn-Tucker theory (ref. Hadley [8] or Himmelblau [9]). These conditions may be examined as interpretations of the generalized Lagrangian solution to the problem as presented above.

CONCLUSIONS AND CONTINUING WORK

This paper formulates the production optimization problem for a rate sensitive geothermal reservoir and develops the approach to a solution for this formulation. Current work on this problem centers around identifying the conditions under which the optimization problem is solvable and interpreting those conditions. The geothermal problem is not as straightforward as the analogous oil and gas problem because of the impacts of temperature decline phenomenon on geothermal power plant design, resource price and, consequently, the optimal rate of extraction.

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