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RECHARGE AND REPRESSURIZATION
IN THE REPFLO MODEL

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1. INTRODUCTION

The question we address in this short report is how recharge and repressurization can be taken into account in the REPFLO code. We do not attempt a definitive discussion of the problem. Rather, our first goal is to determine whether any treatment of these effects is required, and if so, to provide a crude model that requires no major restructuring of the code.

The REPFLO formulation is based on the assumption of saturated flow. This assumption is, of course, invalid during the recharge phase. Once recharge and repressurization have occurred, however, the code will recover whatever validity it originally had. The possibility exists, therefore, that the effects of recharge and repressurization can be treated as transients whose primary effect is to shift the time origin. This point of view will be useful if the time scales for recharge and repressurization are short compared to the time scale for waste migration.

We tentatively assume that there are three distinct time scales in the problem: (1) A pressure relaxation time. This is the time required for the pressure to relax to a new distribution after the boundary conditions are changed. This relaxation time comes into play twice in the problem. The pressure distribution relaxes to a quasi-steady value after the repository is constructed, remains nearly constant during recharge, and relaxes to a new steady state during repressurization. We assume that the construction time is short compared to the relaxation time, and that this in turn is

short compared to the recharge time.

(2) The recharge time of the repository. We assume that this is long compared to the pressure relaxation time, but short compared to the waste migration time.

(3) The waste migration time, i.e., the time required for waste to start reaching the biosphere in significant quantities. We assume this is the longest time scale in the problem.

2. PRESSURE RELAXATION

The finiteness of pressure relaxation times is a consequence of the finite compressibility of water and of the formation matrix in which it is held. If the media were incompressible, the speed of sound would be infinite. Information as to changes in boundary conditions would travel at infinite velocity, and relaxation would be instantaneous, since no mass transfer would be required to effect the relaxation. When the compressibility is finite, however, the speed of sound is finite, and additional time is required for the mass motion that accompanies compression or decompression.

Typical sound speeds in water and solids are $\sim 5000 \text{ m sec}^{-1}$. Accordingly we assume that the information that conditions have changed is transmitted instantaneously, within the present context, and that the relaxation time scale is determined by the speed at which water can flow through the porous medium to accommodate the changed requirements of pressure distribution. Simultaneously with the water flow the formation matrix may expand or contract, so an effective compressibility for the formation must also be included in the calculation.

The picture we take as a working hypothesis is this. Originally a steady-state pressure and flow distribution exist. These distributions are perturbed by the excavation, which occupies about a decade in time to complete. The repository volume is now a region of lowered hydraulic potential, and water starts to flow from the formation into the repository, and at the same time pressure relaxation in the formation flow field begins to occur. If the pressure relaxation time is sufficiently short, a new quasi-steady pressure and flow distribution will be established before recharge has pro-

gressed significantly. These conditions will hold until recharge is complete, or nearly so, at which time repressurization will occur, and conditions in the formation will relax to the distributions appropriate for the presence of the saturated, pressurized repository.

Pressure relaxation is essentially a diffusive process, by which we mean that when perturbed, the pressure obeys an equation of the same form as the diffusion equation during its relaxation to a new steady state. We shall give a very crude discussion intended only to make the diffusive behavior intuitively plausible; more complete discussions can be found in the literature (see e.g. Bear, Ref. 1, p. 202 et seq., especially Eq. 6.3.40)

We assume Darcy's equation

$$\vec{q} = -k \vec{\nabla} \phi \quad (1)$$

where $\vec{q}$ is the Darcy velocity, k the hydraulic conductivity, and ϕ is the hydraulic potential. Next we define the current $\vec{j}$ of water of density ρ

$$\vec{j} = \rho \vec{q} = -k\rho \vec{\nabla} \phi \quad (2)$$

In saturated flow, the mass of water per unit volume, m , is

$$m = n\rho \quad (3)$$

where n is the porosity of the formation, i.e., n is the ratio of the interstitial volume to the total volume of the formation.

The continuity equation takes the form

$$\vec{\nabla} \cdot \vec{j} = - \frac{\partial m}{\partial t} \quad (4)$$

From (2)

$$\vec{\nabla} \cdot \vec{j} = - k\rho \nabla^2 \phi - \{\vec{\nabla} \phi\} \cdot \{\vec{\nabla}(k\rho)\} \quad (5)$$

The second term on the right can be shown to be small compared to the first in conditions likely to be of interest, so we make the approximation

$$\vec{\nabla} \cdot \vec{j} = - k\rho \nabla^2 \phi \quad (6)$$

From (3) we have

$$\frac{\partial m}{\partial t} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t} \quad (7)$$

We now assume an equation of state for water adequate for our purposes is (Ref. 1, Eq. 2.3.4)

$$\rho = \rho_0 \{1 + \beta(p - p_0)\} \quad (8)$$

where (Ref. 2, p. 4)

$$\beta \approx 5 \times 10^{-11} \text{ cm}^2 \text{ dyne}^{-1} \leftrightarrow 5 \times 10^{-6} \text{ m}^{-1} \quad (9)$$

and we shall express p in meters. Upon differentiating (8) and dropping the subscript on ρ , we obtain

$$\frac{\partial \rho}{\partial t} = \beta \rho \frac{\partial p}{\partial t} \quad (10)$$

We note from (9) that a change in head of 1000 m produces a change of less than one percent in ρ , so we expect that in the present class of problems changes in density will be quite small and the linear approximation (8) applies.

We define a coefficient of compressibility, α_p , for the formation in analogy with (8). The correct quantity for use in finding $\frac{\partial n}{\partial t}$ depends on the application of interest. For our problem, the quantity that seems most likely to apply is "the effective rock compressibility" (Ref. 1, p. 52, Eq. 2.7.3). In terms of this, we have

$$\frac{\partial n}{\partial t} = \alpha_p \frac{\partial p}{\partial t} \quad (11)$$

According to Bear

$$\alpha_p \sim (10^{-5} \text{ to } 10^{-6}) \frac{1n^2}{1b} \sim 5 \times 10^{-6} \text{ m}^{-1} \quad (12)$$

which is about the same as the value of β given in (9).

For $\frac{dm}{dt}$ we now have

$$\frac{dm}{dt} = n \frac{\partial \rho}{\partial t} + \rho \frac{\partial n}{\partial t} = n \rho (\beta + \alpha_p) \frac{\partial p}{\partial t} \approx 10^{-5} n \rho \frac{\partial p}{\partial t} \quad (13)$$

By definition of the hydraulic potential

$$p = \rho g \phi - \int_0^z \rho g \, dz' = \rho g (\phi - z) \quad (14)$$

An adequate approximation for our purposes is obtained by differentiating (14) with respect to t and neglecting $\frac{\partial \rho}{\partial t}$. Then

$$\frac{\partial p}{\partial t} = \rho g \frac{\partial \phi}{\partial t} \quad (15)$$

Finally, from (4), (6), (13), (14) and (15)

$$\frac{\partial \phi}{\partial t} - \frac{k}{ng\rho (\beta + \alpha_p)} \nabla^2 \phi = 0 \quad (16)$$

this equation is in absolute units. It can be written in the dimensionally simpler form

$$\frac{\partial \phi}{\partial t} - \frac{k}{n(\beta + \alpha_p)} \nabla^2 \phi = 0 \quad (17)$$

where now β and α_p have dimensions of inverse length (see Eqs. 9 and 12).

Eq. (17) is the diffusion-type equation for pressure relaxation to which we referred earlier. The effective diffusion constant, or diffusivity, is

$$D = \frac{k}{ng\rho(\beta + \alpha_p)} \quad (L^2 T^{-1}) \quad (18)$$

This is sometimes written

$$D = \frac{k}{S} \quad (19)$$

where

$$S = ng\rho(\beta + \alpha_p) \quad (L^{-1}) \quad (20)$$

is called the specific volume storativity; it is the increase in the weight of water stored in unit bulk volume of the formation per unit increase of water pressure.

We are now in a position to estimate relaxation times. From (9) and (12)

$$S = 10^{-5} n \quad (m^{-1}) \quad (21)$$

We assume for the porosity the smallest value quoted by Bear for shale and limestone (Ref. 1, p. 46).

$$n \approx 10^{-2} \quad (22)$$

Hence

$$S \approx 10^{-7} \text{ (m}^{-1}\text{)} \quad (23)$$

For the hydraulic conductivity of the formation, we assume

$$k = 10^{-10} \text{ (m sec}^{-1}\text{)} \approx 3 \times 10^{-3} \text{ (m yr}^{-1}\text{)} \quad (24)$$

This is the value assumed in the REPFLO test problem, and is characteristic of "good" limestone (Ref. 1, p. 136).

Then the diffusion coefficient is

$$D = \frac{k}{S} \approx 3 \times 10^4 \text{ (m}^2 \text{ yr}^{-1}\text{)} \quad (25)$$

This is a value that might be typical for relatively impermeable media.

The diffusion equation (16) or (17) is

$$\frac{\partial \phi}{\partial t} = D \nabla^2 \phi \quad (26)$$

Now let τ denote the relaxation time, and λ a length characteristic of the problem. Then with order of magnitude accuracy,

$$\tau \sim \frac{\lambda^2}{D} \quad (27)$$

For the test problem, $\lambda \sim 300$ m. Hence

$$\tau \sim \frac{10^5}{3 \times 10^4} \sim 3 \text{ yr} \quad (28)$$

A very short time indeed.

Longer relaxation times might occur for other problems. A value for λ of ~ 1000 m instead of the 300 m assumed would increase τ by a factor of ten. A value much less than 10^{-10} m sec $^{-1}$ for k seems rather unlikely. The value of S is determined by compressibilities, which are not subject to much variation, and by the porosity n . The porosity might be an order of magnitude larger than our assumed value of 10^{-2} , which would increase τ by another factor of ten. Finally, we conclude

$$\tau < 300 \text{ yr} \quad (29)$$

should cover almost all cases, but the result $\tau \sim 3$ yr for the test problem (Eq. 28) should be a fairly good approximation.

In view of the fact that we are interested in waste isolation for periods of at least 10^4 years, we do not believe that relaxation effects have any significant role to play in site evaluation, and will have very little more to say on the subject in the remainder of this report.

3. RECHARGE

Next we wish to estimate the time to recharge the unsaturated repository, and verify the assumption that the recharge time is much longer than the repressurization time. As before we take the REPFLO test problem as an example.

First neglect the fact that repressurization is occurring at all, and suppose that the potential field has its unperturbed configuration. The Darcy velocity in the formation is constant at its ambient value, which in the test problem is

$$q_a = 3 \times 10^{-4} (\text{m yr}^{-1}) \quad (30)$$

The volume of water entering unit length of a pipe of radius a , per unit time, is

$$Q_a = 2aq_a = 6 \times 10^{-4} a \text{ (m}^2 \text{ yr}^{-1}) \quad (31)$$

The interstitial volume per unit length of the pipe is

$$V = \pi a^2 n \quad (\text{m}^2) \quad (32)$$

The time to fill the interstitial volume, in this approximation, is

$$\Delta t_1 = \frac{V}{Q_a} \sim 5 \times 10^3 \frac{n}{a} \text{ (yr)} \quad (33)$$

If we take the porosity of the repository backfill to be $n \sim 10^{-1}$, and use $a = 3\text{m}$, then

$$\Delta t_1 \sim 1500 \text{ yr} \quad (34)$$

The result (34) is interesting from two points of view. First $\Delta t_1 \gg \tau$, which suggests that not much recharge occurs during relaxation, and second Δt_1 is short enough that it does not appear likely that recharge is going to have much effect on waste isolation times, at least for configurations and regimes similar to the test problem.

We based the estimate (34) on the assumption that no relaxation had occurred. Since $\Delta t_1 \gg \tau$, however, a more meaningful estimate would be obtained by assuming that relaxation had proceeded to completion. A very simple means exists for doing this in the case of the test problem. Since the unsaturated repository is effectively at zero potential, its drawdown is 30.5 m. The potentials of the upper and lower aquifers are as before. All that is necessary is to set the repository drawdown at 30.5 m, and use all the standard REPFLO formulas to get the flow from the formation into the repository. The only difference is that now this flow goes to charge the repository, rather than wending its way to the upper aquifer through the network.

We have shown (Ref. 2, Eqs. 4.13-20) that the net flow, per unit length to a repository room is

$$Q = \frac{2\pi kD}{f(a,b,c)} = \frac{2kDc}{b} \quad (35)$$

According to the above paragraph, we must take for the drawdown D the background potential 30.5 m. Hence,

$$D = D_o = 30.5 \text{ m} \quad (36)$$

The spacing between aquifers is $2b$, and their potential difference is $2D_o$, hence

$$\frac{kD}{b} = q_a, \quad (37)$$

where q_a has the same value as that given in (30). Comparing (35) and (31), we see that the effect of relaxation is to increase the recharge flow by the factor c/a , which is just the ratio of the spacing between rooms to their radii. In the test problem, this ratio is about ten, so we are led to revise the result (34) to

$$\Delta t_2 \sim 150 \text{ yr.} \quad (38)$$

Finally, we expect repressurization to take about as long as depressurization, so it does not appear that the entire unsaturated phase of the flow is of significant duration, at least in the regime characteristic of the REPFLO test problem.

4. SUMMARY AND CONCLUSIONS

We have concluded from our examination of the REPFLO test problem that recharge and repressurization occupy such short intervals that they are not likely to be important considerations in repository evaluation. Although we used the test problem as an example, we believe our conclusions have a considerable range of validity.

In our analysis we made use of several equations from our own analysis in Ref. 3. Only trivial modification is required to permit calculation of recharge times directly from data that are required as input to REPFLO. These times could be added to all output times, so they would constitute only a shift in the origin of the time scale.

Should this simple program be implemented, however, we recommend a more thorough study of its validity, and incorporation of flags in the output to warn of violations or questionable validity.

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