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ON-LINE STRUCTURAL RESPONSE ANALYSIS:
USING THE EXTENDED KALMAN ESTIMATOR/IDENTIFIER

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ABSTRACT

This report discusses the development of on-line state and parameter estimators used to analyze the structural response of buildings. The estimator/identifier is an extended Kalman filter (EKF), which has been applied with great success in other technological areas. It is shown that the EKF can perform quite well on simulated noisy structural response data.

I. INTRODUCTION

The need to analyze the vibrational response of buildings is essential in the design of earthquake resistant structures.[1] Practical techniques to determine actual structural response are still in their developmental stage and no one approach has been universally accepted.[2,3] It is important for the Nuclear Regulatory Commission (NRC) to have knowledge of proposed structural responses in order to evaluate the structural integrity of nuclear reactor vessels in the present and future power or research facilities. The above issues are only a few which motivate the development of techniques to analyze structural response data.

In this report, we assume that the structure can be modeled by its normal modes, i.e., we start with a modal model. Clearly, structural modeling is a complex process and simplifying assumptions must be involved to achieve this model but for the purpose of this study the modal model is adequate. We discuss the application of Kalman estimator/identifiers to the simulated response of an eight mode structural model. A state description of the structure is developed in Section II, which can be used to estimate the structure model response characteristics from noisy data. In Section III an on-line state estimator is developed to estimate or reconstruct the model state response of the given structures. The estimator/identifier is developed in Section IV and applied to both structures. In all cases, data and simulation results are presented.

II. STATE REPRESENTATION OF COMPLEX EXPONENTIAL SIGNALS

In this section we consider the development of state space models for complex exponential signals directly from the series expansion. We develop a coordinate system which can be written by inspection of either the frequency or time representation. This form will prove beneficial in simulation and estimation.

The basic structural identification problem to be solved is - - -given a structural response signal (exponential) from a noisy measurement instrument, estimate the normal modes of the structure. In many other applications ([4],[5]), the signal of interest, say $f(t)$, can be also represented by a complex exponential power series. In electromagnetics as well as structures, this representation has physical interpretation. It is well-established in fields like control, estimation, and more generally signal processing that the state space form is convenient from a theoretical and practical viewpoint.

Consider the time domain exponential signal representation given by

$$f(t) = \sum_{i=1}^{n_1} \left[A_i e^{-(\sigma_i + j\omega_i)t} + A_i^* e^{-(\sigma_i - j\omega_i)t} \right] + \sum_{i=1}^{n_2} b_i e^{-\lambda_i t}, \quad (2.1)$$

or taking the Laplace transform of (2.1), we obtain the equivalent frequency domain signal

$$F(s) = \sum_{i=1}^{n_1} \left[\frac{A_i}{s+\sigma_i+j\omega_i} + \frac{A_i^*}{s+\sigma_i-j\omega_i} \right] + \sum_{i=1}^{n_2} \frac{b_i}{s+\lambda_i} \quad (2.2)$$

Equation (2.2) can be written as

$$F(s) = \sum_{i=1}^{n_1} \left[\frac{2\operatorname{Re}\{A_i\}s + (2\operatorname{Re}\{A_i\}\sigma_i - 2\omega_i\operatorname{Im}\{A_i\})}{s^2 + 2\sigma_i s + (\sigma_i^2 + \omega_i^2)} \right] + \sum_{i=1}^{n_2} \frac{b_i}{s+\lambda_i} \quad (2.3)$$

We would like to take either $f(t)$ or $F(s)$ and write down the state space form directly. Before we obtain the solution to the general problem, let us consider a more restricted and tractable signal.

Recall that the linear time invariant, state space representation for a scalar system [6] is given by

$$\dot{x}_t = Fx_t + gu_t$$

$$y_t = hx_t \quad (2.4)$$

where the state, input and outputs are $x \in \mathbb{R}^n$, $u, y \in \mathbb{R}^1$, and $F \in \mathbb{R}^{n \times n}$, $g, h^T \in \mathbb{R}^n$.

Thus, the problem stated above can be precisely stated as:

Given the $(\{\sigma_i\}, \{\omega_i\}, \{\lambda_i\}, \{b_i\}, \{A_i\})$ of (2.1) or (2.2), find (F, g, h) of (2.4).

For simplicity, assume we have $n_1=2$, $n_2=0$ in (2.2), then

$$F_i^C(s) = \frac{A_i}{s+\sigma_i+j\omega_i} + \frac{A_i^*}{s+\sigma_i-j\omega_i} \quad (2.5)$$

Using the transformations given in Siljak [7], it can be shown that a state model corresponding to (2.5) is given by

$$\dot{x}_i^C = F_i^C x_i^C + g_i^C u +$$

where

$$F_i^C = \begin{bmatrix} -\sigma_i & \omega_i \\ -\omega_i & -\sigma_i \end{bmatrix}^{++}, \quad g_i^C = \begin{bmatrix} g_{i1}^C \\ g_{i2}^C \end{bmatrix}, \quad h_i^C = [h_{i1}^C \ h_{i2}^C] \quad (2.6)$$

In general, we would like to know how to select the components of g_i^C and h_i^C so that we can write (2.6) from (2.5) by inspection. If we simply derive the transfer function of (2.5) in terms of (2.6) and equate coefficients, we can specify various combinations. Writing out (2.6) we have

$$\dot{x}_i^C = \begin{bmatrix} -\sigma_i & \omega_i \\ -\omega_i & -\sigma_i \end{bmatrix} x_i^C + \begin{bmatrix} g_{i1}^C \\ g_{i2}^C \end{bmatrix} u \quad (2.7)$$

$$y_i^C = [h_{i1}^C \ h_{i2}^C] x_i^C$$

Recall that [6]

$$F_i^C(s) = h_i^C (sI - F_i^C)^{-1} g_i^C = h_i^C \frac{\text{adj}(sI - F_i^C)}{|sI - F_i^C|} g_i^C \quad (2.8)$$

$$\text{adj}(sI - F_i^C) = \begin{bmatrix} s + \sigma_i & \omega_i \\ -\omega_i & s + \sigma_i \end{bmatrix} \quad \text{and} \quad |sI - F_i^C| = (s + \sigma_i)^2 + \omega_i^2 \quad (2.9)$$

†Here the time subscript t is suppressed.

††Note also that this form is no longer applicable when repeated roots occur; however, in most structure repeated roots and not practically realizable.

So we construct (2.8) using (2.7) and (2.9), i.e.,

$$F_i^C(s) = h_i^C (sI - F_i^C)^{-1} g_i^C = \frac{[h_{i1} \ h_{i2}] \begin{bmatrix} s + \sigma_i & \omega_i \\ -\omega_i & s + \sigma_i \end{bmatrix} \begin{bmatrix} g_{i1} \\ g_{i2} \end{bmatrix}}{(s + \sigma_i)^2 + \omega_i^2}$$

or

$$F_i^C(s) = \frac{(g_{i1}^C h_{i1}^C + g_{i2}^C h_{i2}^C)(s + \sigma_i) + (g_{i2}^C h_{i1}^C - g_{i1}^C h_{i2}^C)\omega_i}{(s + \sigma_i)^2 + \omega_i^2} \quad (2.10)$$

Now, equating (2.10) with (2.5) we see that

$$g_{i1}^C h_{i1}^C + g_{i2}^C h_{i2}^C = 2\text{Re}[A_i] \quad (2.11)$$

$$g_{i1}^C h_{i1}^C - g_{i2}^C h_{i2}^C = -2\text{Im}[A_i]$$

So, with 2 equations (RHS known) and 4 unknowns, we must specify 2 unknowns to obtain a unique solution. We specify g_i^C so that the subsystem of (2.6) is controllable and likewise h_i^C for observable (see [6] for definitions, etc.).

Case I: Fix $g_i^C \ni (F_i^C, g_i^C)$ is controllable, find h_i^C .

$$g_i^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \rho [g_i^C \ F_i^C g_i^C]^\dagger = \rho \begin{bmatrix} 0 & \omega_i \\ 1 & -\sigma_i \end{bmatrix} = 2 \quad (\text{controllable for } \omega_i \neq 0)$$

$$h_i^C = [-2\text{Im}A_i \quad 2\text{Re}A_i]$$

†Here $\rho(\cdot)$ means rank of $(\cdot)$.

Case II: Fix $h_i^C \ni (F_i^C, h_i^T)$ is observable, find g_i^C .

$$h_i^C = [0 \ 1], \quad \rho \begin{bmatrix} h_i^C \\ h_i^C F_i^C \end{bmatrix} = \rho \begin{bmatrix} 0 & 1 \\ -\omega_i & -\sigma_i \end{bmatrix} = 2 \text{ (observable for } \omega_i \neq 0)$$

$$g_i^C = \begin{bmatrix} 2\text{Im}A_i \\ 2\text{Re}A_i \end{bmatrix}$$

Case III: $g_i^C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $h_i^C = [2\text{Re}A_i \ 2\text{Im}A_i]$

Case IV: $h_i^C = [1 \ 0]$, $g_i^C = \begin{bmatrix} 2\text{Re}A_i \\ -2\text{Im}A_i \end{bmatrix}$

So, we choose, say Case II, then starting from (2.1) with $n_1=1, n_2=0$, we have

$$\dot{x}_i = \begin{bmatrix} -\sigma_i & \omega_i \\ -\omega_i & -\sigma_i \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y_i = [-2\text{Im}A_i \ 2\text{Re}A_i]x = [-2\rho_i^* \ 2\rho_i]x \quad (2.12)$$

Now, using this form for the n_1 , complex subsystems of (2.1) and using the observable canonical form for n_2 simple poles (see Padulo and Arbib [6]), we obtain the general form

$$\dot{x} = Fx + gu$$

$$y = hx$$

for

$$F = \left[\begin{array}{ccc|ccc} F_{11}^C & & 0 & & & \\ & \ddots & & & 0 & \\ & & F_{n_1}^C & & & \\ -0 & - & - & \bar{F}_{11} & - & \bar{0} \\ & 0 & & & \ddots & \\ & & & 0 & & F_{n_2} \end{array} \right], \quad g = \begin{bmatrix} g_1^C \\ \vdots \\ g_{n_1}^C \\ \bar{g}_1 \\ \vdots \\ g_{n_2} \end{bmatrix}, \quad h = [h_1^C | \dots | h_{n_1}^C | h_1 \dots h_{n_2}]$$

where

$$F_i^C = \begin{bmatrix} -\sigma_i & \omega_i \\ -\omega_i & -\sigma_i \end{bmatrix}, \quad g_i^C = c_i, \quad h_i^C = [h_{i1} \ h_{i2}] = [-2\text{Im}A_i \ 2\text{Re}A_i],$$

e_i is unit column vector,

$$\text{and } F_i = -\lambda_i, \quad g_i = 1, \quad h_i = b_i \quad (2.13)$$

$$n = 2n_1 + n_2$$

The connection diagram for this system is shown in Figure 1.

Let us now consider an example from a structural model of a building.

FIGURE 1 FLOW DIAGRAM FOR THE i -TH COMPLEX EXPONENTIAL AND REAL SUBSYSTEM

Example 1: In this example, we have six complex pole pairs, simulating the first 6 modes of an 8 mode building model, $n_1=6$, $n=12$. The data is shown in Table I. Each subsystem is characterized by

$$\begin{aligned} F_1^C &= \begin{bmatrix} 0.3196 & 6.3844 \\ -6.3844 & 0.3196 \end{bmatrix}, g_1^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_1^C = [-2(5.0307 \times 10^{-4}) \quad 2(4.8972 \times 10^{-5})] \\ F_2^C &= \begin{bmatrix} 0.948 & 18.4363 \\ -18.4363 & 0.948 \end{bmatrix}, g_2^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_2^C = [-2(1.2309 \times 10^{-4}) \quad 2(3.3512 \times 10^{-5})] \\ F_3^C &= \begin{bmatrix} 1.5441 & 30.8429 \\ -30.8429 & 1.5441 \end{bmatrix}, g_3^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_3^C = [-2(5.76252 \times 10^{-5}) \quad 2(2.5668 \times 10^{-5})] \\ F_4^C &= \begin{bmatrix} 2.0875 & 41.6996 \\ 41.6996 & 2.0875 \end{bmatrix}, g_4^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_4^C = [-2(3.0163 \times 10^{-5}) \quad 2(1.8347 \times 10^{-5})] \\ F_5^C &= \begin{bmatrix} 2.5599 & 51.1357 \\ 51.1357 & 2.5599 \end{bmatrix}, g_5^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_5^C = [-2(1.63434603 \times 10^{-5}) \quad 2(1.1688 \times 10^{-5})] \\ F_6^C &= \begin{bmatrix} 2.9439 & 58.8295 \\ 58.8295 & 2.9439 \end{bmatrix}, g_6^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, h_6^C = [-2(9.7117 \times 10^{-6}) \quad 2(4.4075 \times 10^{-6})] \end{aligned}$$

and our composite system is given by

$$\dot{x} = \begin{bmatrix} F_1^C & & & & & 0 \\ & F_2^C & & & & \\ & & F_3^C & & & \\ & & & F_4^C & & \\ & & & & F_5^C & \\ 0 & & & & & F_6^C \end{bmatrix} x + \begin{bmatrix} e_2 \\ e_2 \\ e_2 \\ e_2 \\ e_2 \\ e_2 \end{bmatrix} g, \quad y = [h_1^C | h_2^C | h_3^C | h_4^C | h_5^C | h_6^C] x$$

this completes the example.

<u>MODE</u>	<u>σ_i</u>	<u>ω_i</u>	<u>$\text{Re}[A_i]$</u>	<u>$\text{Im}[A_i]$</u>
1	-0.31962	6.3843445	4.8971593×10^{-5}	5.0306702×10^{-4}
2	-0.94799	18.436263	3.3511927×10^{-5}	1.2308962×10^{-4}
3	-1.5441	30.8429	2.5668428×10^{-5}	5.7625165×10^{-5}
4	-2.0875	41.699615	1.8346731×10^{-5}	3.0163211×10^{-5}
5	-2.5599	51.135703	1.1688100×10^{-5}	1.6434603×10^{-5}
6	-2.9439	58.829464	4.4074538×10^{-6}	9.7116721×10^{-6}

TABLE I: Example 1 Data

Before we discuss the simulation, let us consider an alternate way of displaying (2.1), since we have no knowledge of which coordinate system is more appropriate for identification purposes. Instead of displaying the real and imaginary parts of the complex poles directly, suppose we desire only the quadratic polynomial of which we have an exact solution for the roots. We can use the familiar companion matrix or controllable form for each complex and simple pair and then construct a composite system analogous to (2.2).

Recall the controllable form (see Padulo and Arbib [6] for details), i.e. given

$$F(s) = \frac{b_n s^{n-1} + b_{n-1} s^{n-2} + \dots + b_1}{s^n + a_n s^{n-1} + \dots + a_1}$$

then

$$A = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & 0 & 1 \\ -a_1 & -a_2 & & & -a_n \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, \quad c = [b_1 \quad \dots \quad b_n] \quad (2.14)$$

where (A,b,c) is a state space representation like (F,g,h) of (2.13). Of course, we have only 2×2 A_i^C 's for each complex pole pair and 1×1 A_i^C 's for each simple pole (identical to F_i in (2.13). Using (2.14) and identifying coefficients of (2.3), we write by inspection

$$A_i^C = \begin{bmatrix} 0 & 1 \\ -(\sigma_i^2 + \omega_i^2) & -2\sigma_i \end{bmatrix}, \quad b_i^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \text{and} \\ c_i^C = [(2\text{Re}[A_i]\sigma_i - 2\text{Im}[A_i]\omega_i) \quad (2\text{Re}[A_i])] \quad (2.15)$$

The composite system (see Figure 2) is written identically to (2.13) with $(\Lambda_i^C, b_i^C, c_i^C)$ replacing (F_i^C, g_i^C, h_i^C) . Note the trade off between the two coordinate systems. The (F, g, h) model displays real and imaginary parts of the pole, while the (A, b, c) model displays the coefficients of the quadratic characteristic polynomial. Which form is best to use from an identification viewpoint is not known at this time, and some computer studies may be of interest. To extend this work to multidimensional systems (u and y vector, g and h matrices) would require further investigation.

Using the data in Table I, and the composite form developed in (2.13) we constructed a general complex exponential simulator called STRCIM (STRuctures CIMulation). STRCIM solves the continuous state/discrete measurement stochastic equations

$$\begin{aligned} \dot{x}_t &= Fx_t + gu_t + w_t \\ z_k &= h_k x_k + v_k \end{aligned} \quad (2.16)$$

where (F, g, h) are given in (2.13) and $\{w\}$ and $\{v\}$ are Gaussian processes with respective covariances $Q_t \in \mathbb{R}^{n \times n}$ and r_k .

Figure 2 Flow Diagram for the i -th Exponential Subsystem Using THE CONTROLLABLE CANONICAL FORM

Taking the structure of Example 2.1, with parameters of Table I, we constructed a simulation using STRCIM.

Generically, for each sub-block we have

$$\begin{aligned}\dot{x}_{i1} &= -\sigma_i x_{i1} + \omega_i x_{i2} \\ \dot{x}_{i2} &= -\omega_i x_{i1} - \sigma_i x_{i2} + u\end{aligned}$$

and

$$y_i = -2\rho_i^* x_{i1} + 2\rho_i x_{i2}$$

where

$$\begin{aligned}\rho_i^* &= \text{Im}A_i \\ \rho_i &= \text{Re}A_i\end{aligned}$$

The analytical solution for both the state and output of our generic complex exponential representation follows. The above state representation takes the general form

$$\begin{aligned}\dot{x}_i &= F_i^C x_i + g_i^C u \\ y_i &= h_i^C x_i\end{aligned}\tag{2.17}$$

where

$$F_i^C = \begin{bmatrix} -\sigma_i & \omega_i \\ \omega_i & -\sigma_i \end{bmatrix}, \quad g_i^C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad h_i^C = [-2\text{Im}A_i, 2\text{Re}A_i].$$

It is well known [6] that (2.17) has solution

$$x_i(t) = e^{F_i^C(t-t_0)} x_i(t_0) + \int_{t_0}^t e^{F_i^C(\alpha-t_0)} g_i^C u(\alpha) d\alpha\tag{2.18}$$

For this example we drop the c superscript.

and

$$y_i(t) = h_i^C e^{F_i^C(t-t_0)} x_i(t_0) + \int_{t_0}^t h_i^C e^{F_i^C(t-\alpha)} g_i^C u(\alpha) d\alpha \quad (2.19)$$

Now, if we let $u(\alpha) = \delta(t)$, then the impulse response of (2.17) is given by:

$$x_i(t) = e^{F_i^C(t-t_0)} x_i(t_0) + e^{F_i^C(t-t_0)} g_i^C$$

$$y_i(t) = -2\text{Im}A_i x_{i1}(t) + 2\text{Re}A_i x_{i2}(t) \quad (2.20)$$

To find the analytic solution for the state transition matrix, we use the method of Laplace transforms [6]

$$e^{F_i^C t} = \mathcal{L}^{-1} \{ (sI - F_i^C)^{-1} \}$$

$$= \mathcal{L}^{-1} \begin{bmatrix} \frac{s+\sigma_i}{(s+\sigma_i)^2 + \omega_i^2} & \frac{\omega_i}{(s+\sigma_i)^2 + \omega_i^2} \\ \frac{-\omega_i}{(s+\sigma_i)^2 + \omega_i^2} & \frac{s+\sigma_i}{(s+\sigma_i)^2 + \omega_i^2} \end{bmatrix}$$

or

$$e^{F_i^C t} = \begin{bmatrix} e^{-\sigma_i t} \cos \omega_i t & e^{-\sigma_i t} \sin \omega_i t \\ -e^{-\sigma_i t} \sin \omega_i t & e^{-\sigma_i t} \cos \omega_i t \end{bmatrix} \quad (2.21)$$

Substituting (2.21) into (2.20) we see that

$$\begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix} = \begin{bmatrix} e^{-\sigma_i t} (\cos \omega_i t (x_{i1}(t_0)) + \sin \omega_i t (x_{i2}(t_0) + 1)) \\ e^{-\sigma_i t} (-\sin \omega_i t (x_{i1}(t_0)) + \cos \omega_i t (x_{i2}(t_0) + 1)) \end{bmatrix} \quad (2.22)$$

and

$$y_i(t) = -2\text{Im}A_i x_{i1}(t) + 2\text{Re}A_i x_{i2}(t) \quad (2.23)$$

These equations represent the analytic solution to the state equations of (2.16) without process or measurement noise.

Let us now reconsider our example.

Example 2: From the results of Example 1, we have

$$\begin{aligned} i=1 & \begin{cases} \dot{x}_{11} = -\sigma_1 x_{11} + \omega_1 x_{12} & y_1 = -2\rho_1^* x_{11} + 2\rho_1 x_{12} \\ \dot{x}_{12} = -\omega_1 x_{11} - \sigma_1 x_{12} + u \\ \sigma_1 = 0.3196, \omega_1 = 6.3844, \rho_1^* = 5.0307 \times 10^{-4}, \rho_1 = 4.8972 \times 10^{-5} \end{cases} \\ i=2 & \begin{cases} \dot{x}_{21} = -\sigma_2 x_{21} + \omega_2 x_{22} & y_2 = -2\rho_2^* x_{21} + 2\rho_2 x_{22} \\ \dot{x}_{22} = -\omega_2 x_{21} - \sigma_2 x_{22} + u \\ \sigma_2 = 0.948, \omega_2 = 18.4363, \rho_2^* = 1.2309 \times 10^{-4}, \rho_2 = 3.3512 \times 10^{-5} \end{cases} \end{aligned}$$

$$i=3 \quad \left\{ \begin{array}{l} \dot{x}_{31} = \sigma_3 x_{31} + \omega_3 x_{32} \quad y_3 = -2 \rho_3^* x_{31} + 2 \rho_3 x_{32} \\ \dot{x}_{32} = -\omega_3 x_{31} - \sigma_3 x_{32} + u \\ \sigma_3 = 1.5441, \omega_3 = 30.8429, \rho_3^* = 5.76252 \times 10^{-5}, \rho_3 = 2.5668 \times 10^{-5} \end{array} \right.$$

$$i=4 \quad \left\{ \begin{array}{l} \dot{x}_{41} = \sigma_4 x_{41} - \omega_4 x_{42} \quad y_4 = -2 \rho_4^* x_{41} + 2 \rho_4 x_{42} \\ \dot{x}_{42} = -\omega_4 x_{41} - \sigma_4 x_{42} + u \\ \sigma_4 = 2.0875, \omega_4 = 41.6996, \rho_4^* = 3.0163 \times 10^{-5}, \rho_4 = 1.8347 \times 10^{-5} \end{array} \right.$$

$$i=5 \quad \left\{ \begin{array}{l} \dot{x}_{51} = -\sigma_5 x_{51} + \omega_5 x_{52} \quad y_5 = -2 \rho_5^* x_{51} + 2 \rho_5 x_{52} \\ \dot{x}_{52} = -\omega_5 x_{51} - \sigma_5 x_{52} + u \\ \sigma_5 = 2.5599, \omega_5 = 51.1357, \rho_5^* = 1.6434503 \times 10^{-5}, \rho_5 = 1.1699 \times 10^{-5} \end{array} \right.$$

$$i=6 \quad \left\{ \begin{array}{l} \dot{x}_{61} = -\sigma_6 x_{61} + \omega_6 x_{62} \quad y_6 = -2 \rho_6^* x_{61} + 2 \rho_6 x_{62} \\ \dot{x}_{62} = -\omega_6 x_{61} - \sigma_6 x_{62} + u \\ \sigma_6 = 2.9439, \omega_6 = 58.8295, \rho_6^* = 9.7117 \times 10^{-6}, \rho_6 = 4.4075 \times 10^{-6} \end{array} \right.$$

The simulator was run using a negative impulse input to the system. This is easily accomplished by setting the proper initial conditions to unity (see [6] for details). In STRCIM the state is defined to be

$$x^T = [x_{11} x_{12} x_{21} x_{22} x_{31} x_{32} x_{41} x_{42} x_{51} x_{52} x_{61} x_{62}]^T$$

with (impulse) initial condition

$$x^T(0) = [0-1 \mid 0-1 \mid 0-1 \mid 0-1 \mid 0-1 \mid 0-1]$$

The response of each pair of poles corresponding to each mode is shown in Figures 3 thru 5. The overall impulse response is shown in Figure 6 for no measurement noise (see 2.16), and levels of $r_1 = 4.9 \times 10^{-11}$, $r_1 = 1.225 \times 10^{-10}$.

We define the following signal-to-noise ratio (SNR) for our signal in conjunction with Lager [5]

$$SNR = \frac{(\text{peak of first positive cycle})}{(\text{noise standard deviation})}$$

In Figure 6, the SNR is shown for various sets of simulated data.

This completes the state description of the "modal" model. In the next section, we design a state estimator to "reconstruct" the modes from noisy measurement data.

Also a negative h_1^C , was used inadvertently, i.e., $h_1^C = 2\text{Im}A_1$.

FIGURE 3: Simulated State Response for Mode 1 and Mode 2.

FIGURE 4: Simulated State Response for Mode 3 and Mode 4.

FIGURE 5: Simulated State Response for Mode 5 and Mode 6.

FIGURE 6: Simulated Noisy Impulse Response of Six-Mode Model

III. STATE ESTIMATOR DESIGN

Before any attempt at parameter estimation is initiated, we choose to construct a state estimator or reconstructor in order to gain some experience and insight into the dynamics of these systems. Recall that our dynamic model takes the general form

$$\begin{aligned}\dot{x}_t &= Fx_t + gu_t \\ y_t &= hx_t\end{aligned}\tag{3.1}$$

where F , g , and h are given in (2.13).

For the purposes of this study, we constructed the STRCIM simulator which generated process (state) and measurement data of (2.16). Simulations using (2.16) enable us to generate data with modeling uncertainties or random driving functions using $\{w_t\}$ as well as measurement uncertainties in instrumentation using $\{v_k\}$. Some typical simulation outputs were shown in Figure 6. Here we saw the noisy measurements for the case of small and large SNR ratios.

The design of state estimator is straightforward using the extended Kalman filter (EKF) algorithm for a linear system (Table II) and the DYNREST code developed by Lawrence Livermore Laboratory [8]. Examining the relations we must supply both system and measurement Jacobians before the algorithm can be implemented. The Jacobians are given by

$$F(x_t) := \frac{\partial f(x_t)}{\partial x_t} = F, \text{ and } H(x_t) := \frac{\partial h(x_t)}{\partial x_t} = h \text{ of (2.13).}$$

The linear Kalman filter was implemented using DYNЕСТ, first, using the simulated data for tuning purposes, then using data generated from a structures simulator, SAP IV [9]. The structure simulated was 8 stories with only 8 modes (16 states); however, a reduced order model of 6 modes (12 states) was used to see how well the estimator could track the states. Typical noisy SAP IV output data is depicted in Figure 7 for the identical SNR generated from STRCIM. Note that for these runs the sign error in h_1^C was corrected and a positive impulse input was used in accordance with the 8 story structure model simulated data.

FIGURE 7: Noisy Measurement Data from SAP IV Structure Simulator.

TABLE II: SUMMARY OF CONTINUOUS-DISCRETE EXTENDED KALMAN FILTER

Prediction:

$$\hat{x}_{k+1|k} = \hat{x}_{k|k} + \int_k^{k+1} [f(\hat{x}_\alpha|k) + g(u_\alpha)] d\alpha$$

$$\tilde{\pi}_{k+1|k} = \int_k^{k+1} [F(\hat{x}_{k|k}) \tilde{\pi}_{\alpha|k} + \tilde{\pi}_{\alpha|k} F^T(\hat{x}_{k|k}) + Q_\alpha] d\alpha$$

Innovation:

$$\epsilon_{k+1} = z_{k+1} - h(\hat{x}_{k+1|k})$$

$$R_{k+1|k}^e = H(\hat{x}_{k+1|k}) \tilde{\pi}_{k+1|k} H^T(\hat{x}_{k+1|k}) + R_{k+1}$$

Correction:

$$K_{k+1} = \tilde{\pi}_{k+1|k} H^T(\hat{x}_{k+1|k}) (R_{k+1|k}^e)^{-1}$$

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \epsilon_{k+1}$$

$$\begin{aligned} \tilde{\pi}_{k+1|k+1} &= [I - K_{k+1} H(\hat{x}_{k+1|k})] \tilde{\pi}_{k+1|k} [I - K_{k+1} H(\hat{x}_{k+1|k})]^T \\ &\quad + K_{k+1} R_{k+1} K_{k+1}^T \end{aligned}$$

where

$$F(\hat{x}_{k|k}) := \left. \frac{\partial f(x)}{\partial x} \right|_{x=\hat{x}_{k|k}}, \quad H(\hat{x}_{k|k}) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=\hat{x}_{k|k}}$$

and

$$\tilde{\pi}_{k+1|k} := \text{Cov}(\tilde{x}_{k+1|k}) \text{ for } \tilde{x}_{k+1|k} := x_{k+1} - \hat{x}_{k+1|k}$$

$$x_0 \sim N(\hat{x}_{0|0}, \tilde{\pi}_{0|0})$$

Runs of the state estimator were accomplished on simulated (STRCIM) data where no model mismatching occurred (See Figures 8 and 9) and then with "truth" data generated from SAP IV (see Figure 10). In these figures we have selected three representative modes in order to display the filter tracking capability for various time constants. In Figure 8 we see that the for no model mismatch the filter tracks quite well. Quantitatively, the tracking or estimation error is depicted in Figures 8 and 9 along with the 2σ bounds predicted by the filter. It is clear from these plots that the filter with a perfect model does excellently. The model mismatch is indicated by the statistical properties of the innovations sequence, i.e., $\{e_k\}$ is a zero mean, white sequence, when the filter is operating properly. In Figure 9, we see that the innovations do satisfy this criteria, i.e., for whiteness only 5% of the $R^{\hat{e}}$ can exceed the 2σ bound. Note that the tracking error curves are generated from $(\hat{x}_{\text{error}} = x_{\text{true}} - \hat{x})$ and require that x_{true} is known. Since we generated the data using STRCIM, x_{true} was available, but this is not the case for the SAP IV data. We can see for the case of using the SAP IV data, that the filter does reasonably well for these modes and that the model mismatch is not too severe as indicated by the innovations statistics. As expected the filter predicted variance ($\hat{\pi}_{ss}$) indicates that the filter performs better when the perfect model is employed. In the next section, we use the extended Kalman filter for state and parameter estimation.

FIGURE 8: State Estimation using STRCIM data ($r=4.9 \times 10^{-11}$) for Mode 1 and Mode 3.

FIGURE 9: State Estimation (Mode 6) and Innovations using STRCIM data ($r=4.9 \times 10^{-11}$)

FIGURE 10: State Estimation and Innovations Using SAP IV data ($r=4.9 \times 10^{-11}$)

IV. ON-LINE STATE AND PARAMETER ESTIMATION

In this section, we discuss the development of an on-line state and parameter estimator using the extended Kalman filter algorithm of Table II. Basically, the technique consists of modeling the constant parameters as "augmented" states in the state equations and obtain pseudo state (parameter) estimates. This technique is well known [10] and more current methods have recently been developed to improve convergence [11]. The parameter estimation or identification problem can be solved with the EKF algorithm; however, it is computationally burdensome and can diverge. A more acceptable but burdensome technique is the maximum likelihood algorithm [12] which is currently under development at LLL.

The general parameter estimation/identification problem is,

Given a dynamical system in state forms

$$\dot{x} = f(x, \theta) + g(u, \theta) + w$$

and measurement system

$$z_k = h(x_k, \theta_k) + v_k$$

as in (2.16) with θ a q -dimensional vector of unknown parameters, find the best estimates (minimum variance) of $\hat{x}, \hat{\theta}$ of x and θ .

In order to use the EKF algorithm, we must rewrite the linear state and measurement equations of (2.13), i.e.,

$$\dot{x}_{i1} = -\sigma_i x_{i1} + \omega_i x_{i2}$$

$$\dot{x}_{i2} = -\omega_i x_{i1} - \sigma_i x_{i2} + u$$

and

$$y_i = -2\rho_i^* x_{i1} + 2\rho_i x_{i2} \quad i=1,2,\dots,6$$

$$y = \sum_{i=1}^6 y_i$$

by defining the new state vector

$$x^T := [x_{11} x_{12} | x_{21} x_{22} | x_{31} x_{32} | x_{41} x_{42} | x_{51} x_{52} | x_{61} x_{62} | \sigma_1 | \omega_1 | \sigma_2 \omega_2 | \sigma_3 \omega_3 | \sigma_4 \omega_4 | \sigma_5 \omega_5 | \sigma_6 \omega_6].$$

Substituting the corresponding states for the parameters in (4.1), we obtain

$$\dot{x}_1 = -x_{13} x_1 + x_{14} x_2$$

$$\dot{x}_2 = -x_{14} x_1 - x_{13} x_2 + u$$

$$\dot{x}_3 = -x_{15} x_3 + x_{16} x_4$$

$$\dot{x}_4 = -x_{16} x_3 - x_{15} x_4 + u$$

$$\dot{x}_5 = -x_{17} x_5 + x_{18} x_6$$

$$\dot{x}_6 = -x_{18} x_5 - x_{17} x_6 + u$$

$$\dot{x}_7 = -x_{19} x_7 + x_{20} x_8$$

$$\dot{x}_8 = -x_{20} x_7 - x_{19} x_8 + u$$

$$\dot{x}_9 = -x_{21} x_9 + x_{22} x_{10}$$

$$\dot{x}_{10} = -x_{22} x_9 - x_{21} x_{10} + u$$

$$\dot{x}_{11} = -x_{23} x_{11} + x_{24} x_{12}$$

$$\dot{x}_{12} = -x_{24} x_{11} - x_{23} x_{12} + u$$

$$\dot{x}_i = 0 \quad \text{for } i = 13, 14, \dots, 24$$

and

$$y = [-2\rho_1^* 2\rho_1^* | -2\rho_2^* 2\rho_2^* | -2\rho_3^* 2\rho_3^* | -2\rho_4^* 2\rho_4^* | -2\rho_5^* 2\rho_5^* | -2\rho_6^* 2\rho_6^*] \underline{x} \quad (4.2)$$

The equations of (4.2) take the general nonlinear form

$$\begin{aligned} \dot{x}_t &= f(x_t) + g(u_t) + w_t \\ z_k &= h(x_k) + v_k \end{aligned} \quad (4.3)$$

where w_t , v_k are Gaussian with covariance P_t and P_k , respectively.

The equations of (4.2) and (4.3) are nonlinear stochastic differential equations which can be solved recursively with the EKF algorithm (see Table II). In order to implement the filter for this problem, we require the analytic system and measurement Jacobians, $F(x_t)$ and $H(x_t)$. The structure of the system Jacobian is shown in Figure 11 and the measurement Jacobian is

$$H(x_t) = [h \ 0_{12}^T].$$

The parameter estimator was constructed and run as before using the data from STRCIM for tuning and then the SAP IV data. In all STRCIM runs the initial parameter values for the states were selected to be in 10% error, i.e., $\sigma_i + 1.1 \sigma_i$ and $\omega_i + 1.1 \omega_i$. Using the parameters of Example 1 and the measurement data of Figure 6, the estimator was run. We again selected modes 1, 3 and 6 to display for comparison

FIGURE 11: System Jacobian for EKF Estimator/Identifier

purposes. We found some tracking degradation in the initial state estimates due to the high initial uncertainty in the parameters. Once the parameter estimates converge, then the state estimates improve (see Figures 12 and 13).+ The initial uncertainty is also reflected in the innovations sequence. The parameter estimates all converged within 4 seconds of problem time and modes 1, 3, and 6 are depicted in Figure 14 thru 16. The performance of the filter for two distinct settings of initial error covariance and process uncertainty (Q) is shown in Tables III and IV. We see that the filter tracks the frequency ω_i -parameters excellent with a high of about 2% error; however, it does not do as well for the damping σ_i -parameters. As seen in Table IV, an attempt to adjust the initial error $\tilde{\pi}_0$ for Mode 6 results in some improvement (39% to 13% error) for that mode, but tracking deterioration in some of the other modes results (e.g., 5 goes from 1.8% to 52% error). The problem with tracking the higher damping ratios is that the filter poles corresponding to these estimates must be set a much wider bandwidths than the bandwidth of the particular mode. The setting is required in order for the filter to quickly track the parameter before all of the data on that mode dies out of the measured response. This completes the computer "tuning" runs for the on-line estimator/identifier. In the next section we discuss some of the detailed simulation runs.

The estimator could have been re-run over the response data many times using the final estimates of the previous run, as initial estimates of the subsequent run. We expect the parameter estimates to improve; however, this was not done in this study.

FIGURE 12: Degraded State Estimates Using STRCIM data ($r=4.9 \times 10^{-11}$) and EKF Identifier

FIGURE 13: Degraded State Estimates and Innovations Using STRCIM data ($r=4.9 \times 10^{-11}$) and EKF Identifier.

ζ_1 -Estimated Damping Parameter for Mode 1

ζ_1 -Damping Parameter Estimation Error for Mode 1

ω_1 -Estimated Frequency Parameter for Mode 1

ω_1 -Frequency Estimation Error for Mode 1

FIGURE 14: Parameter Estimates for Mode 1 Using
STRCIM Data ($n=4.9 \times 10^{-11}$)

FIGURE 15: Parameter Estimates for Mode 3 Using STRCIM data ($r=4.9 \times 10^{-11}$)

FIGURE 16: Parameter Estimates for Mode 6 Using STRCIM Data ($r=4.9 \times 10^{-11}$)

TABLE III: Parameter Estimates for Six-Mode Model

<u>Mode</u>	<u>α_{True}</u>	<u>$\hat{\alpha}$</u>	<u>$\hat{\alpha}$</u>	<u>%Error</u>	<u>ω_{True}</u>	<u>$\hat{\omega}$</u>	<u>$\hat{\omega}$</u>	<u>%Error</u>
1	0.3196	0.3306	0.01	3.4	6.384	6.383	0.001	0.01
2	0.9480	0.9965	.049	5.1	18.44	18.40	0.04	0.22
3	1.5440	1.716	0.172	11.1	30.84	30.90	0.06	0.19
4	2.0880	2.261	0.173	8.2	41.7	41.7	0	0
5	2.5600	2.515	0.045	1.8	51.14	52.11	0.97	1.9
6	2.9440	1.806	1.14	38.7	58.83	58.65	0.18	0.3

$$\% \text{ Error} = \frac{\hat{\alpha}}{\alpha_{\text{True}}} \times 100, \quad \tilde{\alpha} = \alpha_{\text{True}} - \hat{\alpha}$$

10% Initial Parameter Error

TABLE IV: Parameter Estimates for Six-Mode Model

(Adjust Mode 6)†

<u>Mode</u>	<u>α_{True}</u>	<u>$\hat{\alpha}$</u>	<u>$\tilde{\alpha}$</u>	<u>%Error</u>	<u>ω_{True}</u>	<u>$\hat{\omega}$</u>	<u>$\tilde{\omega}$</u>	<u>%Error</u>
1	0.3196	.3321	0.013	3.9	6.384	6.378	0.006	0.09
2	0.9480	1.001	.053	5.3	18.44	18.79	0.05	0.27
3	1.5440	1.509	0.035	2.3	30.84	30.77	0.07	0.23
4	2.0880	1.783	0.305	14.6	41.7	41.63	0.07	0.17
5	2.5600	3.895	1.34	52.1	51.14	50.94	0.2	0.39
6	2.9440	2.558	0.386	13.1	58.83	59.22	0.39	0.7

$$\% \text{ Error} = \frac{\tilde{\alpha}}{\alpha_{\text{True}}} \times 100, \tilde{\alpha} = \alpha_{\text{True}} - \hat{\alpha}$$

10% Initial Parameter Error

†Adjust only initial error covariance effecting real part of mode 6, i.e., $\tilde{\pi}_{66}$.

VI. SIMULATION RESULTS

In this section we discuss the results of running the estimator/identifier over an ensemble of 30 simulated measurement data sequences using both the STRCIM data as well as the SAP IV data. The ensemble was generated by restarting the simulator random number generator at different seed values; thus, the member noise statistics are identical but the individual sample values differ. In all cases the estimator used the 6-mode model. Initially, the filter was tuned (adjusting $\hat{\pi}$, Q , R) to track a particular member of the ensemble and then run over the ensemble with a 10% initial parameter error. Three cases were run: (1) STRCIM for SNR=100; (2) STRCIM for SNR=40; and (3) SAP IV for SNR=100 all for 350 data points at a 0.01 sampling interval.

Simulation results for the first case (SNR=100, STRCIM) were impressive (see Table V). The largest mean parameter error occurred on the highest damped mode 6 with ~10% error while the lowest damped mode 1 had the smallest error of ~0.06%. These results were expected, since the lowest mode dominates the response $z(t)$ after all of the other modes have died out, therefore, it's estimate improves as more and more data is processed. During the initial phase of the identification, the highly damped modes exhibit the least relative error, since they dominate the response. This damping phenomena suggests that the modes should be continuously excited (e.g., a square wave input $u(t)$) so that the esti-

TABLE V: ENSEMBLE PARAMETER ESTIMATES AND ERRORS FOR STRCJM DATA (SNR=100, N=30)

Mode	True(σ_i)	Mean($\hat{\sigma}_i$)	STD($\hat{\sigma}_i$)	Mean($\hat{\tau}_i$)	Error	STD($\hat{\tau}_i$)	Min $\hat{\tau}_i$	Max $\hat{\tau}_i$
1	.3196	.3198	.00103	-.00062	0.062	.00323	-0-	.0060
2	.9480	.9533	.01173	-.00557	0.557	.012515	.0004	-.0338
3	1.5441	1.5619	.06218	-.01155	1.155	.04027	-.0013	-.0810
4	2.0875	2.1474	.11432	-.02869	2.869	.05476	-.0012	-.1262
5	2.5599	2.5873	.10487	-.04976	4.976	.04097	-.0022	-.1198
6	2.9439	3.2374	.15070	-.09970	9.97	.05120	-.0017	-.1955

Mode	True(ω_i)	Mean($\hat{\omega}_i$)	STD($\hat{\omega}_i$)	Mean($\hat{\tau}_i$)	Error	STD($\hat{\tau}_i$)	Min($\hat{\tau}_i$)	Max($\hat{\tau}_i$)
1	6.3844	6.3834	.00091	.00014	.014	.00014	-0-	.0005
2	18.4363	18.4378	.01223	-.00008	.008	.00066	.0001	.0014
3	30.8429	30.8395	.04889	.00011	.011	.00159	.0001	.0033
4	41.6996	41.7117	.15358	-.00028	.028	.00367	.0001	-.0117
5	51.1357	51.2757	.24030	-.00273	.273	.00470	.0001	-.0112
6	58.8295	58.9928	.79684	-.00277	.277	.01354	.0001	.0336

where $\hat{\tau}_i := \frac{\hat{\tau}_i - \tau_i^{\text{True}}}{\tau_i^{\text{True}}}$, 10% initial parameter errors

mator/identifier can update its parameter estimates with more and more data). The subject of optimal inputs for identification has been treated by Mehra [13] and is beyond the scope of this discussion. In all cases the frequency terms were all less than 1% error with the maximum occurring again at the mode 6 ($\sim 0.3\%$) and the lowest at Mode 2 ($\sim 0.01\%$). Typical σ -parameter estimates in this case for Modes 1 and 6 are depicted in Figure 17, and the parameter data corresponding to this particular run is shown in Table VI where $x_s = \hat{\sigma}$ and $\hat{\omega}$. Small sample statistical tests were run over the ensemble ($N=30$) data and these results are shown in Table VII. From this table we see that the EKF estimator is generally biased.† Note that the confidence interval estimates do not contain the population parameter because of this bias.

As expected, we see a large degradation in the parameter estimates when the SNR decreases. The low frequency (σ_1) damping average error is small at 1.7% with a high error in Mode 3 of $\sim 53\%$ (see Table VIII). The frequency component errors are somewhat higher than case 1, but are still adequate with a high average error of $\sim 2\%$ in Mode 6. A typical run over this data is depicted in Figure 18 and the results are also shown in Table VI. Small sample statistics on the parameter estimates are given in Table IX. The estimator produced biased results for all σ 's, but performed somewhat better for the ω 's. It is interesting to note that for the

†The statistical test for bias is shown in Appendix A.

FIGURE 17: Damping Parameter Estimates for Modes 1 and 6
Using STRCIM Data (SNR = 100)

TABLE VI: Typical Ensemble Member Run for Case Studies

Case 1: STCIM Run (SNR=100)

1	SIGMA(I)	J	XS(J)	ERR	1	OMEGA(I)	J	XS(J)	ERR
1	0.3196	13	0.3196	0.0000	1	6.3844	14	6.3836	0.0001
2	0.9480	15	0.9617	-0.0144	2	18.4363	16	18.4429	-0.0004
3	1.5441	17	1.6672	-0.0797	3	30.8429	18	30.8237	0.0006
4	2.0875	19	2.2619	-0.0835	4	41.6996	20	41.6799	0.0005
5	2.5599	21	2.7737	-0.0835	5	51.1357	22	51.0269	0.0021
6	2.9439	23	3.3244	-0.1293	6	58.8295	24	59.3827	-0.0094

Case 2: STCIM Run (SNR=40)

1	SIGMA(I)	J	XS(J)	ERR	1	OMEGA(I)	J	XS(J)	ERR
1	0.3196	13	0.3225	-0.0089	1	6.3844	14	6.3855	-0.0002
2	0.9480	15	0.9416	0.0067	2	18.4363	16	18.4557	-0.0011
3	1.5441	17	1.5246	0.0126	3	30.8429	18	30.9036	-0.0020
4	2.0875	19	2.1543	-0.0320	4	41.6996	20	41.5307	0.0040
5	2.5599	21	2.7761	-0.0845	5	51.1357	22	51.2076	-0.0014
6	2.9439	23	3.4874	-0.1846	6	58.8295	24	60.5023	-0.0284

Case 3: SAP IV (SNR=100)

1	SIGMA(I)	J	XS(J)	ERR	1	OMEGA(I)	J	XS(J)	ERR
1	0.3200	13	0.3273	-0.0228	1	6.3925	14	6.3778	0.0023
2	0.9492	15	0.9577	-0.0195	2	18.9600	16	18.8688	0.0048
3	1.5460	17	1.5086	0.0242	3	30.8820	18	30.8894	-0.0002
4	2.0902	19	1.9172	0.0828	4	41.7520	20	41.9492	-0.0047
5	2.5632	21	2.4221	0.0551	5	51.2000	22	50.7240	0.0093
6	2.9489	23	2.9992	-0.0170	6	58.9050	24	58.1667	0.0125

TABLE VII: SAMPLE STATISTICS FOR PARAMETER ESTIMATES (SNR=100, STRCIM, N=30)

<u>Parameter</u>	<u>True Value</u>	<u>95% Confidence Interval</u>	<u>True Parm In?</u>	<u>T-Statistic</u>	<u>Biased? (> 2.045)</u>
1	0.3196	0.3198 ± 0.00038	Yes	1.064	No
2	0.9480	0.9533 ± 0.00437	No	2.475	Yes
3	1.5441	1.5619 ± 0.02315	Yes	1.568	No
4	2.0875	2.1474 ± 0.04258	No	2.870	Yes
5	2.5599	2.6873 ± 0.03906	No	6.654	Yes
6	2.9439	3.2374 ± 0.05613	No	10.667	Yes
1	6.3844	6.3834 ± 0.00034	No	6.079	Yes
2	18.4363	18.4378 ± 0.00456	Yes	0.654	No
3	30.8429	30.8395 ± 0.01821	Yes	0.384	No
4	41.6996	41.7117 ± 0.05720	Yes	0.430	No
5	51.1357	51.2757 ± 0.08950	No	3.192	Yes
6	58.8295	58.9928 ± 0.29678	Yes	1.123	No

TABLE VIII: ENSEMBLE PARAMETER ESTIMATES AND ERRORS FOR STRCIM DATA (SNR=40, N=30)

Mode	$\text{True}(\sigma_i)$	$\text{Mean}(\hat{\sigma}_i)$	$\text{STD}(\hat{\sigma}_i)$	$\text{Mean}(\tilde{\sigma}_i)$	$\% \text{Mean}(\tilde{\sigma}_i)$	$\text{STD}(\tilde{\sigma}_i)$	$\text{Max}(\tilde{\sigma}_i)$	$\text{Min}(\tilde{\sigma}_i)$
1	.3196	.3251	.00554	-.01724	1.724	.01731	.0607	.0012
2	.9480	1.0860	.14622	-.1435	14.35	.1556	.6120	.0052
3	1.5441	2.3686	1.07828	-.5275	52.75	.6983	3.0006	.0003
4	2.0875	2.9587	.63466	-.4173	41.73	.3040	1.3637	.0797
5	2.5599	3.3961	.61279	-.3266	32.66	.2394	1.0981	.0202
6	2.9439	3.7587	.42949	-.2768	27.68	.1459	.5991	.0761

Mode	$\text{True}(\omega_i)$	$\text{Mean}(\hat{\omega}_i)$	$\text{STD}(\hat{\omega}_i)$	$\text{Mean}(\tilde{\omega}_i)$	$\% \text{Mean}(\tilde{\omega}_i)$	$\text{STD}(\tilde{\omega}_i)$	$\text{Max}(\tilde{\omega}_i)$	$\text{Min}(\tilde{\omega}_i)$
1	6.3844	6.3776	.00455	.00107	0.107	.00071	.0031	.0
2	18.4363	18.4746	.05469	.00207	0.207	.00297	.0102	.0002
3	30.8429	30.7475	.55344	.00310	0.310	.01794	.0659	.0
4	41.6996	41.2649	1.06143	.01043	1.043	.02544	.1038	.0007
5	51.1357	50.2906	2.64549	.01652	1.652	.05173	.1823	.0005
6	58.8295	57.6746	2.19647	.01963	1.963	.07133	.1641	.0051

FIGURE 18: Damping Parameter Estimates for Modes 1 and 6 Using STCIM Data (SNR = 40)

TABLE IX: SAMPLE STATISTICS FOR PARAMETER ESTIMATES (SNR=40, STCIM, N=30)

<u>Parameter</u>	<u>True Value</u>	<u>95% Confidence Interval</u>	<u>True Parm In?</u>	<u>T-Statistic</u>	<u>Biased?(>2.045)</u>
1	0.3196	0.3251 $\pm$ 0.00206	No	5.438	Yes
2	0.9480	1.0860 $\pm$ 0.05446	No	5.169	Yes
3	1.5441	2.3586 $\pm$ 0.40161	No	4.137	Yes
4	2.0875	2.9587 $\pm$ 0.23637	No	7.519	Yes
5	2.5599	3.3961 $\pm$ 0.22823	No	7.474	Yes
6	2.9439	3.7587 $\pm$ 0.15996	No	10.391	Yes
1	6.3844	6.3776 $\pm$ 0.00170	No	8.186	Yes
2	18.4363	18.4746 $\pm$ 0.02037	Yes	1.118	No
3	30.8429	30.7475 $\pm$ 0.20613	Yes	0.944	No
4	41.6996	41.2649 $\pm$ 0.39533	No	2.243	Yes
5	51.1357	50.2906 $\pm$ 0.98532	Yes	1.748	No
6	58.8295	57.6746 $\pm$ 1.5630	Yes	1.507	No

particular tuning run (Table VI) the errors etc., were quite small; however, they were not representative of the errors over the ensemble. This suggests that one should be cautious in making conclusions for parameter estimators over a single member run.

In the final case we ran the reduced-order estimator (6 mode) model against "truth" data generated by the 8 Mode SAP IV simulator. The ensemble contained ten sample waveforms generated identically as before with a sampling rate of 0.02 sec. The estimator was adjusted to compensate for the model mismatch by varying Q until the innovations were white. A typical member run for this case is shown in Figure 19 with the run data in Table VI. Parameter tracking data is given in Table X. We see that the estimator did reasonably well for the damping parameters with the high error of ~4% on Mode 2 and a low of 0.046 surprisingly on Mode 6. The frequency estimates were all good with a high of ~2% for Mode 6. The small sample statistical analysis is shown in Table XI. The estimator again appears biased; however, it appears more biased for the frequency parameter in contrast to the STRCIM runs which appear more biased for the damping parameters. We expect the bias in this case, however, because of the model mismatch. It is possible to estimate the bias on-line as shown Friedland [14] by augmenting the states.

FIGURE 19: Damping Parameter Estimates for Modes 1 and 6
Using SAP IV Data (SNR = 100)

TABLE X: ENSEMBLE PARAMETER ESTIMATES AND ERRORS FOR SAP IV DATA (SNR=100, N=10)

<u>Mode</u>	<u>True(σ_i)</u>	<u>Mean($\hat{\sigma}_i$)</u>	<u>STD($\hat{\sigma}_i$)</u>	<u>Mean($\tilde{\sigma}_i$)</u>	<u>%Mean($\tilde{\sigma}_i$)</u>	<u>STD($\tilde{\sigma}_i$)</u>	<u>Max($\tilde{\sigma}_i$)</u>	<u>Min($\tilde{\sigma}_i$)</u>
1	0.32003	0.32685	0.00054	-0.02131	2.131	0.00172	0.0243	0.0192
2	0.94919	0.99054	0.02162	-0.04357	4.357	0.02276	0.0812	0.0172
3	1.54603	1.51981	0.02030	0.01695	1.695	0.01314	0.0479	0.0044
4	2.09021	2.05969	0.16086	0.01461	1.461	0.07696	0.1195	0.0218
5	2.56321	2.56568	0.11913	-0.00097	0.097	0.04648	0.0716	0.0176
6	2.94894	2.95028	0.06476	-0.00046	0.046	0.02194	0.0374	0.0018

<u>Mode</u>	<u>True(ω_i)</u>	<u>Mean($\hat{\omega}_i$)</u>	<u>STD($\hat{\omega}_i$)</u>	<u>Mean($\tilde{\omega}_i$)</u>	<u>%Mean($\tilde{\omega}_i$)</u>	<u>STD($\tilde{\omega}_i$)</u>	<u>Max($\tilde{\omega}_i$)</u>	<u>Min($\tilde{\omega}_i$)</u>
1	6.3925	6.37788	0.00098	0.00229	0.229	0.00016	0.0026	0.0021
2	18.96	18.87580	0.01777	0.00442	0.442	0.00094	0.0054	0.0027
3	30.882	30.86026	0.07952	0.00070	0.070	0.00256	0.0059	0.0002
4	41.752	41.92164	0.22588	-0.00406	0.406	0.00539	0.0140	0.0010
5	51.2	51.06726	0.39348	0.00259	0.259	0.00767	0.0115	0.0001
6	58.905	57.73525	0.52917	0.01986	1.986	0.00897	0.0341	0.0109

TABLE XI: SAMPLE STATISTICS FOR PARAMETER ESTIMATES (SNR=100, SAP IV, N=10)

<u>Parameter</u>	<u>True Value</u>	<u>95% Confidence Interval</u>	<u>True Parm In?</u>	<u>T-Statistic</u>	<u>Biased?(>2.045)</u>
1	0.32003	0.32685 $\pm$ 0.00349	No	39.930	Yes
2	0.94919	0.99054 $\pm$ 0.01398	No	6.048	Yes
3	1.54603	1.51981 $\pm$ 0.01313	No	4.085	Yes
4	2.09021	2.05969 $\pm$ 0.10403	Yes	0.600	No
5	2.56321	2.56568 $\pm$ 0.07704	Yes	0.066	No
6	2.94894	2.95028 $\pm$ 0.04188	Yes	0.065	No
1	6.3925	6.37788 $\pm$ 0.00063	No	47.176	Yes
2	18.96	18.87580 $\pm$ 0.01492	No	14.984	Yes
3	30.882	30.86026 $\pm$ 0.05142	Yes	0.865	No
4	41.752	41.92164 $\pm$ 0.14607	No	2.375	Yes
5	51.2	51.06726 $\pm$ 0.25446	Yes	1.067	No
6	58.905	57.73525 $\pm$ 0.34221	No	6.99	Yes

For these particular signals it is worth noting that the SNR varies as a function of time. If we define an energy-type SNR as

$$\text{SNR}_1 = \frac{\int_0^T z^2(\alpha) d\alpha}{r^2} \quad (6.1)$$

where recall that $z = Hx+v$ and $\text{Cov}(v)=r$.

Plots of SNR, are shown in Figure 20 for two values of r . We see that the SNR, is maximum during the initial time (~5 sec) and decreases thereafter. This phenomenon is expected since all the modes are present initially adding to the signal level and as they terminate the signal level must decrease. Again, this suggests that a different input signal be applied to increase the SNR periodically enabling most identification algorithms to track the parameters. In an on-line application, however, it may not be feasible to vibrate the structure in this fashion. This completes the discussion of the performance of the EKF estimator/identifier on structural response data. In the next section, we summarize these results.

FIGURE 20: Energy Signal-to-Noise Ratio for STRCIM and SAP IV Simulated Responses

VII. SUMMARY

This report has discussed the development of a particular state space description, which can be obtained by inspection from exponential series time or frequency representations. We have shown that Kalman estimators can be used to reconstruct the state with good precision for the case of perfect and non-perfect estimation models. The performance of the filter implies that it would be feasible to incorporate algorithms into ground motion measurement instruments to improve accuracy and precision as well as possible on-line "structural failure" detectors.† The extended Kalman estimator/identifier was applied to the model identification problem and it was shown that reasonable results could be expected for both damping and frequency modal components. The low magnitude damping modes could be estimated with high confidence, while the high magnitude damping modes could not, unless the EKF was adjusted significantly for the particular mode. This adjustment typically caused tracking degradation in other parameters. Results of ensemble runs indicate that the EKF identifier tends to yield precise but biased estimates supporting previous analysis [11]. However, it did appear that reasonable results could be obtained. It was mentioned that better results could be expected using the well-known maximum likelihood algorithm [12]; however, this algorithm is computationally more burdensome and not an on-line technique.

†Structural failure detectors are devices similar to those implemented by Candy, et al [15] in detecting the theft of special nuclear material. These concepts will be discussed in a future report.

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APPENDIX A: SMALL SAMPLE STATISTICAL ANALYSIS

In this appendix, we discuss the statistical hypothesis test used to test the EKF estimator/identifier for biased estimates. The hypotheses under test are

$$\begin{aligned} \mathcal{H}_0 : E(\hat{\theta}) &= \theta_{\text{True}} \quad (\text{unbiased}) \\ \mathcal{H}_1 : E(\hat{\theta}) &\neq \theta_{\text{True}} \quad (\text{biased}) \end{aligned} \quad (\text{A-1})$$

where $\hat{\theta}$ is the parameter estimate of θ and each $\hat{\theta}_i$ is i.i.d and $N(\theta, \sigma_\theta^2)$.

The sample mean is used as the test statistic, i.e.,

$$\overline{(\hat{\theta})} = \frac{1}{N} \sum_{i=1}^N \hat{\theta}_i \quad (\text{A-2})$$

where $\overline{(\hat{\theta})} \sim N(\theta, \frac{\sigma_\theta^2}{N})$ (see Anderson [16] for details). The probability of a Type I error (reject $\mathcal{H}_0$ when $\mathcal{H}_0$ is true) is given by α the significance level of the test, i.e.,

$$P(|\overline{(\hat{\theta})}| > \tau) = \alpha$$

where τ is the threshold. Using tables it is convenient to shift the parameters

$$P\left(\left|\frac{\overline{(\hat{\theta})} - \theta_{\text{True}}}{\sigma_\theta / \sqrt{N}}\right| > \left|\frac{\tau - \theta_{\text{True}}}{\sigma_\theta / \sqrt{N}}\right|\right) = \alpha \quad (\text{A-3})$$

If we use the sample variance $\hat{\sigma}_\theta^2$ to estimate σ_θ^2 , then we use the T-distribution [16] with test statistic

$$t = \frac{(\hat{\theta}) - \theta_{\text{True}}}{\hat{\sigma}_{\theta}/\sqrt{N}} \quad (\text{A-4})$$

Finally, the test for biased estimates is given by

$$t \begin{matrix} \mathcal{H}_1 \\ > \\ \mathcal{H}_0 \end{matrix} \quad \tau \quad (\text{A-5})$$

selecting $\alpha = 0.05$ (miss probability) corresponding to a probability of detection of 0.95 we obtain $\tau = 2.045$ from the T-distribution tables. Results of this test are shown in Tables VII, IX and XI.