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QUANTIZED FIELD FORMULATION OF THE FREE-ELECTRON LASER

IN THE HEISENBERG PICTURE*

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The phase and amplitude operator equations valid for field intensities ranging from a single photon state to an intense laser state are derived by means of quantized field theory. Using the Dirac equation, driving current operators, which are expressed by radiation and electron fields, are separated into spontaneous, stimulated, and spin terms. Then, utilizing the semiclassical nature of the electron state, coherence condition and spectral equations are derived. From the spectral phase equation, a delay-time scaling for oscillator operation is obtained in good agreement with experiments.

1. Field Theoretical Approach

In a free-electron laser (FEL) oscillator experiment, the build-up from noise to the initial intensity increase is best described by quantized field theory. To describe the interacting electron and photon field correctly, electrons and radiation are represented by a quantized Dirac field and a quantized radiation field, respectively. In our formulation, we take the Heisenberg picture to highlight the transition between quantum mechanical and classical states. In this picture, the operators evolve with time and the state vector is fixed.

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The number of electrons and the number of photons are operators in the field theory. The number of photons evolves according to the field equation. Because the typical density of electrons in the beam pulse is low for the known energy spread, the wave packets of individual electrons are well separated spatially. In this context, electrons may be treated classically. However, their positions are not precisely known.

2. Quantized radiation field

For a simple harmonic-oscillator state characterized by wave number vector k and photon polarization α , the creation and annihilation operators satisfy the commutation relation

$$[a_{k\alpha}, a_{k'\alpha'}^\dagger] = \delta_{kk'} \delta_{\alpha\alpha'}.$$

An occupation-number operator $N_{k,\alpha}$ defined as $N_{k\alpha} \equiv a_{k\alpha}^\dagger a_{k\alpha}$ satisfies a commutation relation with the photon-phase operator $\phi_{k\alpha}$:

$$[N_{k\alpha}, \phi_{k'\alpha'}] = i \delta_{kk'} \delta_{\alpha\alpha'}. \quad (1)$$

From Eq. (1), we easily derive the relation

$$[e^{i\phi_{k\alpha}}, N_{k'\alpha'}] = \delta_{kk'} \delta_{\alpha\alpha'}. \quad (2)$$

We now extend the definition of creation and annihilation operators by including the phase operator as defined in ref. [1] p. 34, which also accommodates the simple harmonic-oscillator commutation relation.

$$a_{k\alpha} \equiv e^{i\phi_{k\alpha}} f_{k\alpha} e^{-i\omega_{k\alpha} t}, \text{ and} \quad (3)$$

$$a_{k\alpha}^\dagger \equiv f_{k\alpha} e^{-i\phi_{k\alpha}} e^{i\omega_{k\alpha} t},$$

where the amplitude operator $f_{k\alpha}$ is defined as $f_{k\alpha} \equiv \sqrt{N_{k\alpha}}$.

In a transverse gauge, the normalized vector potential of the free radiation field is represented as a sum of harmonic oscillators with field-amplitude operator and phase operators:

$$\hat{A}_R(\vec{x}, t) = \left[\frac{1}{\sqrt{LS}} \sum_{k\alpha} c \sqrt{2\pi/\omega_{k\alpha}} g_{k\alpha}(x, y) \hat{e}^\alpha e^{i(kz - \omega_{k\alpha}t + \phi_{k\alpha})} + \text{Hermitian Conjugate} \right] \quad (4)$$

The normalized profile function $g_{k\alpha}(x, y)$ describes the transverse dependence of modes. The interacting radiation field is represented the same as the free field, but a slow time dependence is included both in the field amplitude and in the phase operators. The normalization volume is LS where L and S are the undulator length and cross-sectional area, respectively. The commutation relation for the amplitude and phase operators is a boundary condition before the interaction Hamiltonian modifies them.

3. Radiation field energy

The energy density of the radiation field is given by Eq. (4) as a sum of harmonic oscillators associated with (k, α) . In terms of the B field, it is

$$\mathcal{H} = \vec{B} \cdot \vec{B}^+ / (4\pi) \\ = \pi/c^2 / (2L) \sum_{k\alpha} (k^2/\omega_{k\alpha}) |g_{k\alpha}|^2 (e^x f_{k\alpha} f_{k\alpha}^+ e^{-x} + e^x f_{k\alpha}^+ f_{k\alpha} e^{-x}) \quad (5)$$

where x is the phase $i(kz - \omega_{k\alpha}t + \phi_{k\alpha})$.

Then using the commutation relation Eq. (2) and retaining only the nonvanishing terms in the periodic integration, we obtain the total

radiation energy by spatial integration over the length L and cross-sectional area S .

$$H = \int_0^L dz \int_S ds \mathcal{H} = \sum_{k\alpha} \hbar \omega_{k\alpha} (N_{k\alpha} + 1/2) \quad (6)$$

4. Circularly polarized radiation field and classical connection

The circularly polarized field operator is obtained from Eq. (4).

With the polarization vector definitions $\epsilon^\pm = \mp 1/\sqrt{2} (\hat{\epsilon}_x \pm i\hat{\epsilon}_y)$,

$$\hat{A}_h = -c/\sqrt{LS} \sum_k \sqrt{2\pi\hbar/\omega_{k+}} \hat{\epsilon}^+ e^{i(kz - \omega_{k+}t + \phi_{k+})} f_{k+} g_{k+} + \text{H.C.} \quad (7)$$

At large occupation number $N_{k\alpha}$, the commutation relation between the phase and amplitude operators loses its significance. For a plane wave ($g_{k+} = 1$) in the classical limit, the field reduces to

$$\hat{A}_h = c\sqrt{4\pi\hbar/(LS\omega)} [\epsilon_x \cos(kz - \omega t + \phi_k) - \epsilon_y \sin(kz - \omega t + \phi_k)] f_k \quad (8)$$

Comparing Eq. (8) to a classical vector potential A , we obtain a relation for the radiation amplitude between classical and quantum states.

$$A_{k\alpha} = c\sqrt{4\pi\hbar/(LS\omega)} f_{k\alpha} \quad (9)$$

The dimensionless vector potential amplitude $\underline{a}_{k\alpha}$ is expressed as

$$\underline{a}_{k\alpha} = eA_{k\alpha}/(m_0 c^2) = e/(m_0 c) \sqrt{4\pi\hbar/(LS\omega)} f_{k\alpha} \quad (10)$$

5. The field equation

The field equation for the vector potential operator is

$$\square \hat{A} = -(4\pi/c) \hat{j} \quad (11)$$

Substituting the radiation field into the left-hand side of Eq. (11) and neglecting the second derivatives of $f_{k\alpha}$ and $\phi_{k\alpha}$ with respect to t and z , we have

$$(\partial_z^2 + \nabla_{\perp}^2 - \partial_{ct}^2) \hat{A} = K \sum_{k\alpha} 2g_{k\alpha}/\sqrt{\omega} \left\{ i k \epsilon^{\alpha} e^x [\partial_z f_{k\alpha} + \omega/(ck)(\partial_{ct} f_{k\alpha})] \right. \\ \left. - k \epsilon^{\alpha} e^x [(\partial_z \phi_{k\alpha}) f_{k\alpha} + \omega/(ck)(\partial_{ct} \phi_{k\alpha}) f_{k\alpha}] \right\} + \text{H.C.} \quad (12)$$

$$\text{where } x = i(kz - \omega_{k\alpha} t + \phi_{k\alpha}) \quad , \text{ and} \quad (13)$$

$$K = \sqrt{2\pi/c^2/(LS)} \quad (14)$$

The electron current is expressed in terms of Dirac fields.

$$j_{\mu} = iec\bar{\psi}\gamma_{\mu}\psi \quad (15)$$

With the $p_{\mu} = (e/c)A_{\mu}$ prescription in the Dirac equation, the current is decomposed into three terms as in ref. [1], p. 107 (Gordon decomposition).

$$j_{\mu} = i e \hbar / (2m_0) (\partial \bar{\psi} / \partial x_{\mu} \psi - \bar{\psi} \partial \psi / \partial x_{\mu}) - e^2 / (m_0 c^2) A_{\mu} \bar{\psi} \psi \\ - e \hbar / (2m_0) \Sigma_{\nu} \partial (\bar{\psi} \sigma_{\nu\mu} \psi) / \partial x_{\nu} \quad (16)$$

The first term is a current of electron probability density. We will call this convective current j_{μ}^{sp} , a "spontaneous current." The second term is an electron-photon interaction current. This we will call the "stimulated current" j_{μ}^{st} . The electromagnetic field A_{μ} in the stimulated current consists of the radiation field and the undulator field. The third term is a current having a magnetic moment caused by the electron spin.

6. Electrons as classical entities

Electrons can be treated classically because electron wave-packets have little overlap for a typical experimental situation. For example, the mean interparticle distance in the Los Alamos FEL experimental beam pulse is about 10^{-5} cm, and the z-position ambiguity for $dE/E=10^{-3}$ is about 10^{-9} cm from the uncertainty principle. Defining a jth electron coordinate as $z_j = \beta_{zj}ct + \delta_j$, and using an electron transverse velocity $\vec{U}_j$ spectrally decomposed as $\vec{U}_j = \sum_l \eta_l e^{-i\omega_l t} \vec{U}_{lj}$ (with frequency ω_l , amplitude η_l , and unit vector $\vec{U}_{lj}$), the transverse spontaneous current becomes

$$\vec{J}_{\perp}^{sp} = e/\gamma \sum_{j=1}^n \delta(z - z_j) \sum_l \eta_l e^{-i\omega_l t} \vec{U}_{lj} \quad (17)$$

We evaluate currents at the retarded time τ defined by a group velocity $c\beta_g$ of the radiation field. The interpretation of the field operator is the same except (z,t) is replaced by (z,τ) ; τ is defined as $\tau = t - z/(\beta_g c)$. (18)

For waveguide modes, the group velocity β_g of the radiation is determined by a dispersion relation with cutoff p and is given as

$$\beta_g = (1/c)(\partial\omega/\partial k) = ck/\omega = k/\sqrt{k^2 + p^2} \quad (19)$$

However, to simplify the argument, we will assume $\beta_g = 1$ in the following discussion. With these new variables, the spontaneous current, Eq. (17), is represented as

$$j^{sp} = e/\gamma \sum_j h_j \delta[z - \beta_{zj} c\tau/(1 - \beta_{zj}) - \delta_j/(1 - \beta_{zj})] \\ \cdot \sum_{j=1}^n \vec{U}_{2j}/(1 - \beta_{zj}) \exp [-i(\omega_0/c)(c\tau + \delta_j)/(1 - \beta_{zj})] . \quad (20)$$

7. Quantization of undulator field

The undulator field is also quantized so that the stimulated current drives the field at a correct strength. The normalization is such that the total energy of the undulator field is a sum of harmonic oscillators. In the near-axis approximation, where the x- and y-dependence of the field can be neglected, the classical helical-undulator field is written as

$$\vec{A}_w = -(B_0/k_w)[\cos(k_w z)\hat{x} + \sin(k_w z)\hat{y}] . \quad (21)$$

When quantized, the helical-undulator field operator is given as

$$\vec{A}_w = K\vec{F}_H \exp(ik_w z)a_w + \text{H.C.} , \quad (22)$$

where $\vec{F}_H$ is defined as $-\sqrt{2/(k_w cS)} \hat{e}^-$, and a_w is an annihilation operator. The field normalization is taken over the cross section S and undulator length L.

For the classical linear-undulator field, with transverse aperture L_x and L_y ,

$$A_x = -(B_0/k_w) \cosh(k_w y) \cos(k_w z) \quad \text{and} \quad A_y = A_z = 0 . \quad (23)$$

The corresponding quantized field operator is

$$\vec{A}_w = K\vec{F}_w \exp(ik_w z)a_w + \text{H.C.} , \quad (24)$$

where $\vec{F}_w = -2\sqrt{LS}/c \cosh(k_w y) x / \sqrt{L_x} [L \sinh(2k_w L_y) - L_y \sin(2k_w L)]$.

8. Field equation

In a retarded frame, the field equation, after neglecting the electron-spin driving current, becomes

$$\begin{aligned} & \kappa \sum_{k\alpha} 2g_{k\alpha} / \sqrt{\omega} [i\hat{\epsilon}^\alpha e^{x_{k\alpha}} (\partial_z f_{k\alpha}) - k\hat{\epsilon}^\alpha e^{x_{k\alpha}} (\partial_z \phi_{k\alpha}) f_{k\alpha}] \\ & = 4\pi K e^2 / (m_0 \gamma c^2) (\psi^\dagger \psi) \left\{ \sum_{k\alpha} g_{k\alpha} / \sqrt{\omega} \hat{\epsilon}^\alpha e^{x_{k\alpha}} f_{k\alpha} \right. \\ & \quad + \vec{F} \exp[-ik_w(\beta_z c\tau + \delta)/(1 - \beta_z)] a_w \\ & \quad \left. - m_0 c^2 / (eK) \sum_{\vec{q}} \eta_{\vec{q}} (\vec{U}_{\vec{q}}/c) \exp[-ik_w(\beta_z c\tau + \delta)/(1 - \beta_z)] \right\} , \end{aligned} \quad (25)$$

where $x_{k\alpha} = -i\omega_{k\alpha}\tau + i\phi_{k\alpha}$.

and where the electron field-density operator $(\psi^\dagger \psi)$ operates on β_z and δ for individual electrons.

9. Laser, electron spectra, and coherence condition

Our radiation field is box normalized in the z-direction with the period equal to the undulator length L and over transverse area S. This box normalization quantizes k to

$$k_j = 2\pi j/L \quad , \text{ where } j \text{ is an integer.} \quad (26)$$

The corresponding retarded time τ is then quantized to

$$\tau_j = L/(cj) \quad . \quad (27)$$

We define the integration period at the maximum τ_1 :

$$\tau_1 = L/c \quad (28)$$

Equation (25) describes a spectrally decomposed radiation field. The evolution of the individual field amplitude and phase at each (k, α) is obtained by taking a Fourier component obtained by multiplying Eq. (25) by $\exp(i\omega_j \tau)$ and integrating over the period τ_1 . The result is

$$\begin{aligned} & \sum_{\alpha} (g_{j\alpha} / \sqrt{\omega_j}) [i k_j \hat{\epsilon}^{\alpha} e^{i\phi_{j\alpha}} (\partial_z f_{k\alpha}) - k_j \hat{\epsilon}^{\alpha} e^{i\phi_{j\alpha}} (\partial_z \phi_{j\alpha}) f_{j\alpha}] \\ &= 2\pi \epsilon^2 / (m_0 c^2 \gamma) (\psi^+ \psi) \left[\sum_{\alpha} (g_{j\alpha} / \sqrt{\omega_j}) \hat{\epsilon}^{\alpha} e^{i\phi_{j\alpha}} f_{j\alpha} + \vec{F} \left\{ \sin [(\omega_j - \omega_p) \tau_1 / 2] \right. \right. \\ & \quad \left. \left. / [(\omega_j - \omega_p) \tau_1 / 2] \right\} \exp [i(\omega_j - \omega_p) \tau_1 / 2 - i k_w \delta / (1 - \beta_z)] a_w \right] \\ & \quad - 2\pi e^2 / (c K q) (\psi^+ \psi) \sum_{\ell} n_{\ell} \vec{U}_{\ell} \left\{ \sin [(\omega_j - \omega_s) \tau_1 / 2] / [(\omega_j - \omega_s) \tau_1 / 2] \right\} \\ & \quad \exp [i(\omega_j - \omega_s) \tau_1 / 2 - i(\omega_j / c) \beta_g \delta / (1 - \beta_z)] \quad . \end{aligned} \quad (29)$$

where $\omega_p = k_w \beta_z c / (1 - \beta_z)$, $\vec{F}$ is either $\vec{F}_H$ or $\vec{F}_w$

$$\text{and } \omega_s = \omega_{\ell} / (1 - \beta_z) \quad . \quad (30)$$

The spontaneous term strongly drives the radiation when $\omega_j = \omega_s$.

The driving strength for both spontaneous and stimulated currents at a particular frequency is determined by the spectral bandwidth as a function of the electron transit time τ_1 across the undulator, which is inversely proportional to the undulator length.

After a threshold time τ_{delay} , coherent driving starts to contribute significantly at one of the spontaneous frequencies.

$$\omega_j = \omega_p = k_w \beta_z c / (1 - \beta_z) \quad (31)$$

For a simple helical orbit in a helical undulator, an electron oscillates once around the transverse plane as it travels one undulator period λ_w . The transverse electron velocity follows the transverse electric-field vector as they exchange maximum energy. The transverse electron frequency is

$$\omega_\perp = k_w c \beta_z \quad (32)$$

By using the equation of motion, one can show that the electron transverse velocity is related to the undulator field strength B_0 by the equation:

$$\beta_\perp = e B_0 \lambda_w / (2 \pi m_0 c^2 \gamma) \quad (33)$$

Using Eqs. (32) and (33) and substituting to Eq. (30), the coherence condition emerges, namely,

$$\lambda_s = \lambda_w / \left\{ \beta_z (1 + \beta_z) \gamma^2 \right\} \left\{ 1 + [e B_0 \lambda_w / (2 \pi m_0 c^2)]^2 \right\} \quad (34)$$

10. Phase and Amplitude operator equations

Separating the real and imaginary parts of the spectral component of the field, in Eq. (29), we obtain the following phase and amplitude equations.

$$\begin{aligned} \partial \phi_{k\alpha} / \partial z = & -2\pi e^2 / (m_0 \gamma c^2 k) (\psi^\dagger \psi) \left[1 \right. \\ & + \sqrt{\omega_{k\alpha}} / (g_{k\alpha} f_{k\alpha}) \hat{F} \cdot \hat{\epsilon}^\alpha \cdot \cos(\phi_{k\alpha} + X) [\sin(\delta_{kp}) / \delta_{kp}] a_w \\ & \left. - m_0 c^2 \sqrt{\omega_{k\alpha}} / (g_{k\alpha} f_{k\alpha} K) \cdot \eta_\perp (\vec{U}_\perp / c) \cdot \hat{\epsilon}^\alpha \cos(\phi_{k\alpha} + Y) [\sin(\delta_{ks}) / \delta_{ks}] a_w \right] \quad (35) \end{aligned}$$

$$\partial f_{k\alpha}/\partial z = -2\pi e^2 \sqrt{\omega}/(m_0 \gamma c^2 g_{k\alpha} k) (\psi^\dagger \psi) \left[\vec{F} \cdot \hat{e}^\alpha \sin(\phi_{k\alpha} + X) \cdot [\sin(\delta_{kp})/\delta_{kp}] a_w \right. \\ \left. - m_0 c^2/(eK) \sum_{\mathbf{l}} \eta_{\mathbf{l}} (\vec{U}_{\mathbf{l}}/c) \cdot \hat{e}^\alpha \sin(\phi_{k\alpha} + Y) \sin(\delta_{ks})/\delta_{ks} \right], \quad (36)$$

$$\text{where } \delta_{kp} = (\omega_k - \omega_p) \tau_1/2,$$

$$\delta_{ks} = (\omega_k - \omega_s) \tau_1/2,$$

$$X = -\delta_{kp} + k_w \delta/(1 - \beta_z), \text{ and}$$

$$Y = -\delta_{ks} + (\omega/c) \beta_g \delta/(1 - \beta_z).$$

11. Interpretation of the field equation

With the state vector for the system near the vacuum state and a small photon occupation number at the coherent frequency, the commutation relation between phase and amplitude requires the total uncertainty of the photon phase. Therefore, the averaged field amplitude does not grow along z during the initial stage of start up. However, in the phase equation, the field-amplitude operator appears in the denominator of both the spontaneous and stimulated driving currents. This implies that, at small field amplitude, the photon phase motion is large and is uncertain.

Let us consider the ideal helical undulator. The field equations for the amplitude and phase operators (written in terms of the quantized, classically dimensionless, undulator-field operator $\tilde{a}_w$) are

$$\partial \phi_{k-}/\partial z = -2\pi e^2/(m_0 \gamma c^2 k) (\psi^\dagger \psi) \left[1 \right. \\ \left. - [m_0 c^2/(eK g_{k-})] \sqrt{\omega_k/2} \cdot \tilde{a}_w/f_{k-} \cos(\phi_{k-} + X) \sin(\delta_{kp})/\delta_{kp} \right. \\ \left. - [m_0 c^2/(eK g_{k-})] \sqrt{\omega}/f_{k-} \cdot \sum_{\mathbf{l}} \eta_{\mathbf{l}} (\vec{U}_{\mathbf{l}}/c) \cdot \hat{e}^- \cos(\phi_{k-} + Y) \sin(\delta_{ks})/\delta_{ks} \right], \text{ and} \quad (37)$$

$$\partial f_{k-}/\partial z = 2\pi e \sqrt{\omega}/(\gamma k g_{k-} K) (\psi^\dagger \psi) \left[\tilde{a}_w/\sqrt{2} \sin(\phi_{k-} + X) \cdot \sin(\delta_{kp})/\delta_{kp} \right. \\ \left. + \sum_{\mathbf{l}} \eta_{\mathbf{l}} (\vec{U}_{\mathbf{l}}/c) \cdot \hat{e}^- \sin(\phi_{k-} + Y) \sin(\delta_{ks})/\delta_{ks} \right]. \quad (38)$$

At the initiation of oscillation, both the spontaneous and stimulated parts in the phase equation are driven at nearly equal strength, and their coefficients are very large (10^{13}) when there are only a few photons. However, when an observation is made, the quantum phase ambiguity cancels out these terms. On the other hand, the amplitude operator may increase its magnitude, although the expectation value is also cancelled because of the quantum phase ambiguity. After a large number of passes through the undulator-cavity system, the probability of observing a large amplitude increases (that is, $1/f_k \rightarrow 0$). Accordingly, the spontaneous and stimulated driving terms diminish in strength, and the rate of change of the phase operator [Eq. (37)] is reduced. The rate of change of the amplitude operator starts to lose its chaotic behavior. A choice of start-up criteria is not definitive, but we set the spontaneous and the stimulated terms to have the same strength as the constant first term in Eq. (37). This is done by formally integrating Eq. (38), neglecting the phase-dependent fluctuation factor, and substituting into Eq. (37). This procedure produces a scaling:

$$Z = c\tau_{\text{delay}} \propto (m_0 c^2 \gamma k) / [2\pi e^2 \langle \psi^\dagger \psi \rangle] = (m_0 c^2 \gamma) / [e^2 \langle \psi^\dagger \psi \rangle \lambda_L] \quad (39)$$

An overall multiplication factor M (of approximately 1) on the right-hand side of Eq. (39) is undetermined:

$$\tau_{\text{delay}} = M (m_0 c^2 \gamma) / [e^2 \langle \psi^\dagger \psi \rangle \lambda_L] \quad ; \quad (40)$$

choosing $M = 1/(2\pi)$, the experimental results are reasonably well explained.

12. Examples of delay-time scaling compared to oscillator experiments $M = 1/(2\pi)$

<u>Conditions</u>	<u>Eq. (40)</u>	<u>Measurement</u>
(a) 43 MeV, $(\psi^\dagger\psi) = 2 \times 10^{11}/\text{cm}^3$, $\lambda_L = 3.2 \mu\text{m}$	24.3 μs	31 μs
(b) 20 MeV, 40 A peak, $\lambda_L = 10.4 \mu\text{m}$, $r = 1.0 \text{ mm}$	2.9 μs	<10 μs
(c) 3 MeV, 1.25 A, $\lambda_L = 400 \mu\text{m}$, $r = 3.0 \text{ mm}$	3.5 μs	4 μs

(a): Stanford experiment, (b): Los Alamos experiment,

(c): UCSB experiment, r : beam radius

13. Conclusion

Using the quantized field theory with the Heisenberg picture, the convective current is identified as a spontaneous electron current. The stimulated current is obtained by quantizing the undulator field. Both the spontaneous and the stimulated currents show a classical spectral dependence. The phase uncertainty of light quanta at small occupation number is connected with the observation of fast phase motion of the stimulated emission at low intensity. The delay time is associated with a state where this phase change is near a minimum that is due to the possible increase of the large photon occupation number after multiple passes through the cavity.

14. Glossary of Terms.

k : photon wave number of wave length λ_L ($= 2\pi/\lambda_L$)

ω : photon frequency ($= ck$)

$A_{k\alpha}^\dagger, A_{k\alpha}$: creation and annihilation operators for photon

$N_{k\alpha}$: photon number operator at wave number k and polarization α .

$f_{k\alpha}, \phi_{k\alpha}$: amplitude ($\equiv \sqrt{N_{k\alpha}}$) and phase operators

$(\psi^\dagger\psi)$: electron density operator.

k_w : undulator wave number

L, S : undulator length and cross-sectional area.

$a_w^\dagger, a_w$: undulator creation and annihilation operators.

$\tilde{a}_w$: dimensionless classical undulator field operator

$$= \frac{e}{m_0 c^2} k |F| A_w$$

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Reference

- [1] J.J.Sakurai, Advanced Quantum Mechanics (Addison-Wesley, Reading, Massachusetts, 1967).