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THE STRUCTURE AND DYNAMICS OF PATTERNS OF BENARD CONVECTION
CELLS*

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The Structure and Dynamics of Patterns of Bénard Convection Cells

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Abstract: Bénard-Marangoni convection, in containers with large aspect ratio, exhibits space-filling cellular structures, highly deformable, but crystallized. They contain dislocations and grain boundaries generated and moved by elementary topological transformations, and are subjected to a weak shear stress due to the earth's rotation. The cellular structure and its fluctuations are analyzed from a crystallographic viewpoint, by using two complementary approaches. One is a global analysis of cellular structures in cylindrical symmetry. Their structural stability and defect pattern are obtained as topological mode-locking of a continuous structural parameter. The other, a local, molecular dynamics of the cells, gives a realistic parametrization of the forces and the transformations by generalizing the Voronoi cell construction in one extra dimension.

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1. Introduction: Topological representation of hydrodynamic patterns

This is an attempt to solve a structural problem of hydrodynamics by using crystallographic methods and concepts, in terms of cells, topological defects like dislocations, their interaction and their motion. Conversely, it can be read as an application of crystallography to extremely soft and deformable, albeit crystalline materials. Such materials are known to exist: colloidal crystals,¹ bubble rafts,² magnetic bubbles,³ etc.

Here, we shall investigate the cellular structure produced by Bénard-Marangoni⁴ convection. A fluid heated from below exhibits convection motion above a certain temperature threshold. Buoyancy makes the hotter fluid rise, and surface tension on the (upper) free surface of the fluid, which is stronger where the fluid is colder pushes it down. When the size of the container is much larger than the thickness of the fluid ($\approx$ cell size), one observes a two-dimensional cellular pattern (a froth). On the free surface, the fluid propagates from the hotter cell centers to the colder vertices of the froth.

Hot points repel each other like soft disks. Some cells divide. Just above the convection threshold, the structure is ordered (one recognizes reticular planes, dislocations, etc.), and one anticipates a perfect hexagonal structure (triangular packing of repelling disks) for the ground state, as was indeed pointed out by Bénard himself.⁵ However, this perfect hexagonal structure is never observed:

1) There are always defects (pentagonal and heptagonal cells), well in excess of the 6 pentagons required by topology.⁶

2) The whole structure rotates, with the period of Foucault's pendulum⁷. This indicates that cellular structure and individual cells are sheared by the earth's rotation, through the Coriolis force acting on the fluid propagating on its free surface. The trajectory of the center of an individual cell is an outgoing spiral,⁷ suggesting that shear energy is dissipated outwards from the middle of the container, where new cells are formed.

Dissipation of shear in crystals is carried naturally by dislocations gliding along planes. Cylindrical symmetry, imposed by Coriolis forces and by the shape of the container, should make dislocations align and glide on concentric circles. An indirect proof of the presence of dislocations is the observation that, while the cell rotates about itself with the period of Foucault's pendulum, it undergoes sudden jogs of $\sim 2\pi/5$ (when the spiral trajectory intersects a glide circle).⁷

Topological defects, dislocations gliding to dissipate shear strain, are crystallographic concepts. All these features are manifest in the convective cell structures, except the glide circles which have yet to be observed. We can therefore translate a problem of pattern formation and stability in hydrodynamics (described by hydrodynamic field equations which are complicated to write down and to solve) into a much simpler many-body problem. The bodies are discrete objects (the cells), which repel each other. We want to describe the ground state and elementary excitations. This is a standard problem in crystallography, but here we

have a) cylindrical symmetry, b) a very soft (deformable, malleable) structure. (b) implies that topological defects can be created and moved easily. (a) imposes the presence of these defects. The ground state must be structurally stable, i.e., invariant under small fluctuations of its parameters.⁸

2. Elementary Topological Transformations

Structural defects are created and moved by elementary topological transformations.⁹ Consider a two-dimensional cellular froth with C cells, separated by E edges or interfaces, meeting at V vertices. E and V are secondary, topological elements partitioning space between the physical hot points C . Edges are free to expand or shrink at the mercy of the motion of the hot points, leaving the structure topologically invariant until one of them disappears, inducing an elementary topological transformation (ETT) (Fig.1): There are only two types of ETT, neighbor switching ($T1$), or cell disappearance ($T2$, and its inverse). Cellular division or mitosis (the topological agent of growth of biological tissues), is the composition of $(T2)^{-1}$ with a few $T1$. It is observed, for example, on Lake Natron¹⁰ (a soda lake in Tanzania) and in the process of formation of Bénard-Marangoni cellular structures.

Vertices are trivalent (coordination $z=3$) except at the critical point of a $T1$ transformation, a "Four-Corner Boundary". Not only structural stability, but also dynamics imposes $z=3$. Repelling hot points triangulate space, which is partitioned, for example, by drawing perpendicular

bisectors. The three bisectors (E) of a triangle meet at a point (V), which is $z=3$.

Euler's identity,⁹

$$C - E + V = 1 \quad (1)$$

for a finite planar cellular froth (the cell at ∞ is not included), and valencies relations, $zV=2E$, $\langle n \rangle C=2E$ (each edge joins 2, $z=3$ -valent vertices, and separates 2 cells, $\langle n \rangle$ -sided on average), imply that

$$\langle n \rangle = 6 - (\text{boundary corrections}). \quad (2)$$

Cells are (nearly all) hexagonal, as observed by Bénard.⁵ Also, $6C=2E=3V$.

ETT change the number of sides of the cells involved in the transformation. A 5- (resp. 7-) sided cell is a positive (negative) disclination. (A pentagon is produced in a hexagonal lattice by cutting out a $2\pi/5$ wedge and reglueing. The plane buckles into a cone, and the pentagon is a source of positive curvature. Similarly, a heptagon is produced by adding a wedge, and is a source of negative curvature. The plane buckles into a saddle).

A dipole pentagon-heptagon is a dislocation (Fig.2) (produced by cutting out a rectangular strip, and reglueing. The plane remains flat (uncurved), but it has a step. The source of the step is the dislocation). A single T1 in a hexagonal froth generates a pair of dislocations, which glide apart upon applying shear stress, by successive T1, i.e., by a purely local process. Similarly, one single cell division (Fig.3) generates a pair of dislocations. Further divisions in the neighboring cells make the two dislocations climb away from each other like a defective zipper, leaving behind a layer of new cells. Again, this is a local mean of adding material (unlike the cutting scenario above). It is the way that our intestine grows.¹¹

ETT are local transformations, which, like collisions in gases, keep the cellular network in statistical equilibrium. The observable manifestation of this local balance is given by Aboav's relation.^{9,12}

3. The DAISY, ideal crystal in cylindrical symmetry.⁸

The ideal, infinite $z=3$, planar crystal is a perfect hexagonal lattice. It does not have cylindrical symmetry. The ideal, $z=3$ crystal with cylindrical symmetry is the DAISY, shown in Fig.4. It is a computer construction (by a single algorithm given in eq.(3) below) of the structure of several *compositae* (composite flowers like daisies, asters, etc.), which is indeed cylindrical and crystallographic ("atoms" are identical, $s=1,2,\dots$ florets, sprouting from a central stem and pushing out their elder brothers, labelled by larger s).^{8,13}

One notices immediately concentric circles of dislocations (dipoles 5/7) which are the glide circles necessary to dissipate shear. They are boundaries between perfect, hexagonal grains. It is well-known that grain boundaries are array of dislocations. But here, the array is quasiperiodic.¹⁴ The perfect, hexagonal grains have reticular "planes" which are the visible spirals or parastichies in composite flowers. All but a few percent of *compositae* have Fibonacci numbers of spirals 1,1,2,3,5,8,13,21,34,...., and the remaining few follow the similar Lucas sequence, 1,3,4,7,11,18,29,..... (The outer grains have higher numbers than the inner ones, running through the sequence in an automatic and self-similar fashion.^{8,13,15}

We have therefore a cylindrical cellular structure which is topologically stable (§4) and universal (Fibonacci, and Lucas sequences). This universality, noted by Leonardo, Kepler, Goethe,...., remains the major outstanding puzzle of phyllotaxis (plant architecture).^{13,15}

Two-dimensional phase transitions (e.g., melting) are mediated by the unbinding of pairs of topological defects, dislocations (or vortices) and, possibly, disclinations.¹⁶ A representation (Fig.4) of the ground state structure, emphasizing the position and function of topological defects, is a prerequisite to understand the Bénard-Marangoni structures observed experimentally, which have increasing amount of disorder as the temperature increases above threshold. It is easy to melt the structure, harder to pinpoint the transition(s).¹⁷

4. Structural stability and universality of the DAISY

Fluctuations of a structure can be represented by a dynamical map; universality and stability of the structure are then consequences of universality and stability of dynamical maps.

A crystallographic structure must be labelled and described by a local reference frame. In conventional, translationally invariant systems, the frame is the fundamental domain or unit cell, and each atom is labelled by the position of its unit cell (i.e. by D integers in D dimension), with, if necessary, an index giving its position within the cell. In cylindrical symmetry, we are limited to one integer $s=1,2,\dots$, labelling each cell from the center outwards (from younger to older florets). The cell's center is then given in cylindrical coordinates by

$$\theta(s) = 2\pi \lambda s \tag{3}$$

$r(s)$ = monotonic increasing function of s .

Cells are then drawn by democratic partition of space between centers (Voronoi construction:⁹ interfaces are perpendicular bisectors between neighboring centers, and every point inside a cell is closer to this cell's center than to any other's). This produces the DAISY of Fig.4. The structure is parametrized essentially by one quantity, the divergence angle $2\pi\lambda$, $\lambda \in [0,1]$. The cells lie on a generative spiral $r(\theta/2\pi\lambda)$, not visible in Fig.4 because successive florets are not neighbors. (The pitch of

the generative spiral is smaller than the cell radius). Only visible are the reticular spirals (parastichies), which are ordered as $\{\lambda s\}$. ($\{n\}$ is the fractional part of n , $n=\{n\} \bmod 1$. The ordering follows from the fact that eq.(3), projected on $r=\text{cst}$, is a circle map). A minor adjustment in the function $r(s)$ straightens the spirals while retaining the ordering, glide circles, etc..⁸

Neighbors $s, s+B_m$, have approximatively the same azimuth $\theta(s+B_m)\approx\theta(s)$, and $\lambda\approx A_m/B_m$. If these were equalities (λ rational), cells centers would lie on the same radii, and the Voronoi cellular structure would resemble a spider web.⁸ λ is therefore irrational, and representable uniquely as a continued fraction,

$$\lambda=1/[c_1+1/[c_2+1/(c_3+1/\dots)]]\approx 1/[c_1+1/[\dots+1/c_m]]=A_m/B_m \quad (4)$$

($0 < c_i, A_m, B_m, s \in \mathbb{Z}^+$ are natural numbers). A_m/B_m is called the m 'th convergent to the irrational λ . B_m is the number of cells in a circular shell (one cell thick), or the number of reticular spirals. It labels neighbors. (A_m is the number of turns in the generative spiral necessary to fill the shell).

An hexagonal cell in a grain has six neighbors, and belongs to three families of reticular spirals). Its neighbors are labelled by three B_m 's, denominators of three convergents to λ . Thus, the local reference frame is the triangle of labelled neighbors ($s+B_m, s+B_{m-1}, s+B_{m-2}$) to a given cell s (Fig.5). One goes from s to $s+B_m$ either directly, or jogging through the

other two parastichies and the final label must be independent of the path and of the origin cell, $s+B_m=(s+B_{m-2})+B_{m-1}$, thus,

$$B_m = B_{m-1} + B_{m-2} \quad (5)$$

which is the relation generating Fibonacci, Lucas, etc., sequences. The triangular relation (5) is crystallographically central and overriding.

A grain is characterized by the reference frame $\{B_m, B_{m-1}, B_{m-2}\}$. At a grain boundary, two families of reticular spirals go through, and the third is replaced by a higher B_{m+1} (better convergent), through dislocations which do indeed add material. There are two possibilities,^{8,15}

- $$\begin{aligned} (1) \quad \{B_m, B_{m-1}, B_{m-2}\} &\rightarrow \{B'_{m+1}, B_m, B_{m-1}\} && \text{regular transition} \\ (2) \quad \{B_m, B_{m-1}, B_{m-2}\} &\rightarrow \{B''_{m+1}, B_m, B_{m-2}\} && \text{singular transition} \end{aligned} \quad (6)$$

with B'_{m+1} and B''_{m+1} consistent with eq.(5). Only the former gives the observed Fibonacci or Lucas phyllotaxis. λ is then noble ($\{c_i\}=1$, $i \geq i_0$). Noblest is the golden mean $1/\tau = (1-\sqrt{5})/2$ ($i_0=1$) generating the Fibonacci sequence. The singular transition involves intermediate convergents, $c_i \neq 1$, and non-noble λ .^{8,15}

Consider a given grain $\{B_m, B_{m-1}, B_{m-2}\}$, that is, a set of three rational convergents to λ , with A_m/B_m bounded by the other two. It can easily be shown that any $\lambda \in]A_{m-1}/B_{m-1}, A_{m-2}/B_{m-2}[$ generates the same ordering of B_m successive cells or reticular spirals: $s, s+B_{m-2}, s+2B_{m-2}, \dots \pmod{B_m}$, or $s, s+B_{m-1}, \dots \pmod{B_m}$, clockwise or anticlockwise, i.e. the same structure $\{\lambda s\}$. This is a topological mode-locking: The structure is locked on that given by the rational convergent A_m/B_m , for any λ within the interval $]A_{m-1}/B_{m-1}, A_{m-2}/B_{m-2}[$. The possible structures (A_m/B_m) follow an incomplete devil's staircase as a function of λ , exactly as in conventional mode-locking of two coupled oscillators¹⁸ where the abscissa of the staircase represents the ratio of their natural frequencies, and the ordinate, the observed ratio, locked to rationals.

At a grain boundary, it is necessary for the structure to lock on rationals with larger denominators $B_m \rightarrow B_{m+1}$ (6) (Recall that B_m is the number of cells filling a circular shell. This number must increase as one goes outwards, in order to keep the cell size ($\approx$ fluid thickness) constant). In ordinary mode-locking, the coupled oscillators achieve a finer tuning if instead of locking on p_1/q_1 (one oscillator making p_1 oscillations in q_1 periods of the other, before the two return in phase) or on p_2/q_2 , they lock on the new rational frequency,

$$(p_1+p_2)/(q_1+q_2) \equiv p_1/q_1 \oplus p_2/q_2 \quad (7)$$

(Farey sum). Note that the denominators $q_1+q_2 = q_1 \oplus q_2$ satisfy the triangular rule (5). Eq.(7) generates and orders all the rationals $\in [0,1]$ uniquely as in Fig.6, the Farey construction.¹⁹

Figure 6 can be seen as a tiling by triangles, with a triangle representing grain $\{B_m, B_{m-1}, B_{m-2}\}$. A grain boundary forces a bifurcation. The triangle is replaced by either one of the two Farey offsprings, coinciding with the regular or singular transition of eq.(6) (Fig.7).

It is easy to introduce an energy. The energy measures the strain necessary to confine λ within the interval yielding the required structure. Structure-conserving fluctuations of λ can be represented by dynamical maps (sending the interval into itself, with A_m/B_m , and the interval boundaries A_{m-1}/B_{m-1} , and A_{m-2}/B_{m-2} as fixed points), e.g. by homographic functions. Homographic functions with rational fixed points have remarkable properties (they are involutions and the fixed points have marginal stability). The energy necessary to fix the structure is then a simple functional of the fluctuation map, exactly as a Ginzburg-Landau free energy is a functional of some order parameter.

An energy can similarly be associated with each alternative E^{ST} , E^{RT} at a bifurcation, and with the operation funnelling a divergence λ from the ST side of a bifurcation into a regular transition (E^F). One finds readily that $E^{RT} < E^F < E^{ST}$ and this is sufficient to justify the universality (and the structural stability) of *compositae*, which always select the lowest energy alternative (RT) at every bifurcation encountered upon

growth. This preference has not been tested on Bénard-Marangoni structures as yet.

Either choice at the bifurcation is compatible with the triangular eq.(5), and yields a grain boundary sequence of dislocations (d) and isolated hexagons (h). Dislocations are defects, sources of strain which repel each other, and their number is imposed by the convergents B_m of λ . A smooth, quasiperiodic sequence of d and h results.^{14,20} Shear is absorbed by a dislocation glide on this quasiperiodic structure through a T1, as,

$$\dots d \underline{d} h d h \dots \rightarrow \dots d h \underline{d} d h \dots , \quad (9)$$

(glide of the underlined dislocation by a cell's diameter). The action is then repeated on another dislocation, as in an ideally efficient, $1/\tau$ -full parking lot (d are the cars, h the empty spaces).

This concludes the global description of Bénard-Marangoni structures under shear. We have been able to accommodate structure-conserving fluctuations. The lowest topological excitations are T1 on grain boundaries (9), which dissipate shear efficiently. The next lowest topological excitations are T1 creating new dislocation pairs, at first near the grain boundaries, then within the grains. Next, are cellular divisions which create dislocation pairs (Fig.3) and additional cells. But, at that level of excitation, one should shift to a local description of the dynamics of cellular structure.

5. Local dynamics of cells

We now discuss the local dynamics of the cellular structure, not as a field theory, but as a many-body problem where the cells (hot points) are the bodies, interacting through some reasonable repulsive two-body potential. We require that the representation automatically generates a space-filling, cellular froth, and that ETT in the froth correspond to elementary local displacement of the bodies.

A simple generalization of the Voronoi (democratic) partition of space between seeds (hot points) produces a froth with realistic structure (isotropic cells), capable of accommodating T2 or cellular divisions.²¹⁻²³ Consider circles instead of points as seeds, and define the (Laguerre) distance of a point P to a circle C (hypersphere in $D > 2$ dimension) as the length of the tangent to C through P (power of P with respect to circle C). The locus of points at equal distance between two circles is a straight perpendicular line, the radical axis (hyperplane if $D > 2$), which generalizes perpendicular bisector to unequal circles. Like perpendicular bisectors in a triangle, the three radical axes of three circles are concurrent. They are the interfaces of the Laguerre froth. The larger the circle seed, the larger its corresponding cell in the froth. Laguerre froth reduces to Voronoi's if all the seed circles are equal. Small fluctuations in circle radii already generate realistic cell shapes. Generalization to 3 or more dimension is straightforward, as is the analysis of a section of the froth²¹ (itself a Laguerre froth), important in the analysis of 3D polycrystalline aggregates in metallurgy (stereology).

The dynamical evolution of the froth is much more manageable than its construction.²³

The radical axis between two circles of radii r_1 and r_2 depends on $r_1^2 - r_2^2$ only. This remark gave Telley²³ the idea of representing seeds as identical, vertical paraboloids (umbrellas) in one extra dimension, which intersect the horizontal, physical space as the Laguerre circles. The altitude of the physical plane is fixed but arbitrary. The umbrellas are the bodies of our problem, each specified by the 3 coordinates $\mathbf{q}=(\mathbf{x},z)$ of their apex. Here, $\mathbf{x}$ is the coordinate of the circle's center in the 2-dimensional physical space, and z , the height above physical space, measures the circle radius, $z \sim r^2$, or the cell size.

Now for the ETT. The set of seeds looks like a Yosemite mountain profile (Fig.8). A paraboloid below the horizon (a seed too small) does not generate any cell in the froth. Conversely, a paraboloid which is too high (a seed too large) obscures smaller paraboloids nearby and gobbles up their representative cells. So, a T2 topological transformation occurs whenever the z coordinate of a paraboloid is pushed below the horizon. Conversely, when a new paraboloid rises above the horizon, it divides the cell containing its apex. T1 transformations are produced by moving nearby seeds horizontally rather than vertically. ETT are therefore naturally induced by motions of the bodies, T1 by horizontal ($\mathbf{x}$) motion, T2 or cell division by vertical (z) motion.

There is a duality between cell seed paraboloids (umbrellas) and vertices of Laguerre froth, i.e. between hot and cold points in the convective structure:²³ Vertices can also be represented by the same

paraboloid as the cell seeds, but inverted (opening upwards). The bottom $\mathbf{q}^*=(\mathbf{x}^*,z^*)$ of the inverted paraboloid lies on the intersection of its (3) adjacent paraboloids. The converse is also true, but more stringent: the umbrella apex $\mathbf{q}$ lies on the intersection of its 6 adjacent, inverted paraboloids. This non-random coincidence indicates clearly the restrictions necessary to produce a froth.²³ Any set of C seeds (bodies) produces a Laguerre froth, specified by $3C$ coordinates $(\mathbf{x},z)$ in 2 dimensions. But the froth has $V=2C$ vertices, which would have required $4C$ coordinates if chosen independently. Note that independent orientation only of every interface ($E=3C$) also gives the required number of degrees of freedom. Physically, edge orientation (crack) may be the relevant degree of freedom in brittle fragmentation or geological jointing processes.

Duality yields the driving forces on the hot points (seeds), if the energy E is proportional to the total interfacial length, as is clearly the case in soap froths (surface tension) and in metallurgical aggregates (grain boundary energy), less obviously so in fluids. Displacement of a seed paraboloid $\mathbf{q}$ displaces, geometrically, the nearby vertex paraboloids $\mathbf{q}^*$, hence changes the interfacial length and the energy of the froth. This produces a force $\mathbf{F}=-\nabla_{\mathbf{q}}E$ on $\mathbf{q}$. Neighboring seeds are fixed, so that only the n -sided central cell $\mathbf{q}$ is deformed, with its vertices sliding on the fixed incident interfaces (n degrees of freedom) to minimize the energy cost of moving $\mathbf{q}$.

Laguerre's froth, with paraboloid seeds²³ constitutes therefore an excellent model for understanding and simulating the dynamics of a cellular structure. It identifies the independent degrees of freedom

(cells, hot points in one extra dimension), generates elementary topological transformation by simple, local motion, and computes the forces by a realistic, local geometric algorithm. In a cylindrical container, it should relax to the DAISY ground state of §4, even in the limit of negligible shear, in a cylindrical container, but this remains to be tested.

6. Conclusions

A crystallographic representation of cellular convective structures in cylindrical symmetry or under shear has been presented. The "atoms" are the hot points, cells of the froth. Structural integrity is only disturbed by elementary topological transformations. ETT generate structural defects (pairs of dislocations) and make them move (glide by T1, climb by cellular division). Global (ground state structure, dislocation glide on grain boundaries), and local (Laguerre-Telley modeling of the cellular structure) analyses of the structure, its stability and its dynamics have been presented.

Two tests remain to be carried out: (i) Obtain experimentally DAISY as the structure taken up by Bénard-Marangoni convection at a higher rate of shear. (ii) Combine global and local analyses, by calculating the ground state of the Laguerre froth in cylindrical symmetry, or by melting the DAISY through thermal fluctuations under Laguerre-Telley dynamics. (This was done in ref. 8 by imposing random fluctuations of the cell's seeds position $\mathbf{x}$ only, neglecting forces.)

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Figure captions

Fig.1 Elementary topological transformations: a) Neighbor switch (T1).
b) Cell disappearance (T2).

Fig.2 Topological dislocation (pentagon-heptagon pair).

Fig.3 Creation and climb of a dislocation pair by successive cell divisions.

Fig.4 The DAISY. Cells are all hexagonal except o=pentagon, +=heptagon, dipole o+=dislocation. Grain boundaries are quasicrystalline. (From Ref. 8)

Fig.5 Reference frame, showing labels of cells and its neighbors, and demonstrating the triangular relation (5).

Fig.6 Farey construction of rationals. A triangle defines a grain, structurally locked-in.

Fig.7 Bifurcation at a grain boundary. Here, the regular transition is on the left (8/13), the singular transition on the right (7/11).

Fig.8 The "Yosemite" horizon of paraboloid seeds. The horizontal axis represents the physical plane. Seed **q5** is below the horizon and does not generate a cell in the froth. From Telley.²³

a)

b)

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

END

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